Fundamentals of Transport Processes II
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Lecture - 27
Two-dimensional Potential Flow - Part I

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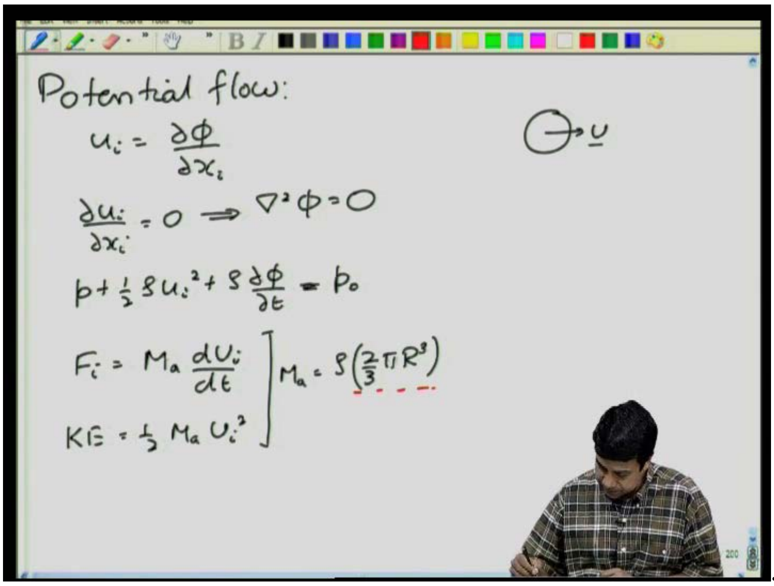

So we are at lecture number 27 of our course on fundamentals of transport processes, where we were discussing potential flows higher Reynolds number flows invisid, irrotational. The vorticity is equal to 0; that means that the anti-symmetric part of the rate of deformation tensor is equal to 0, since del cross u is equal to 0 u can be expressed as the gradient of the potential. So u is written as the gradient of a potential, this potential is the velocity potential and the mass conservation equation is equivalent to writing del square phi is equal to 0.

So phi satisfies the Laplace equation and the momentum conservation equation can be reduced to the Bernoulli equation as we had shown in the previous lectures p plus half rho u i square plus rho times partial phi by partial t is equal to p naught. In the body force is provided that body force is can be expressed as the gradient of the potential, that potential can be incorporated within the pressure itself as we had shown in the previous lectures.

So using this we had solved the equations for the flow around a sphere, moving with the constant velocity u. We found out that the force F i is equal to the added mass times d U i by d t. The kinetic energy is equal to half added mass times U i square in both cases the added mass is equal to the density of the fluid times 2 by 3 pi R cubed, 2 by 3 pi R cubed is one half of the mass of fluid displaced. I am sorry, 2 by 3 pi R cube times rho is one half of the mass of fluid that is displaced by the sphere and we got the rather paradoxical result that for a sphere that is moving at constant velocity, the net force exerted by the sphere on the fluid is identically equal to 0.

I had justified that on the basis that since there are no viscous treasures, there is no dissipation of energy therefore, you should not required to do any work in order to move that sphere at constant velocity. In this lecture, we will see that this is a more general in relation, for any object that is moving at constant velocity the net force exerted on that object is equal to 0.

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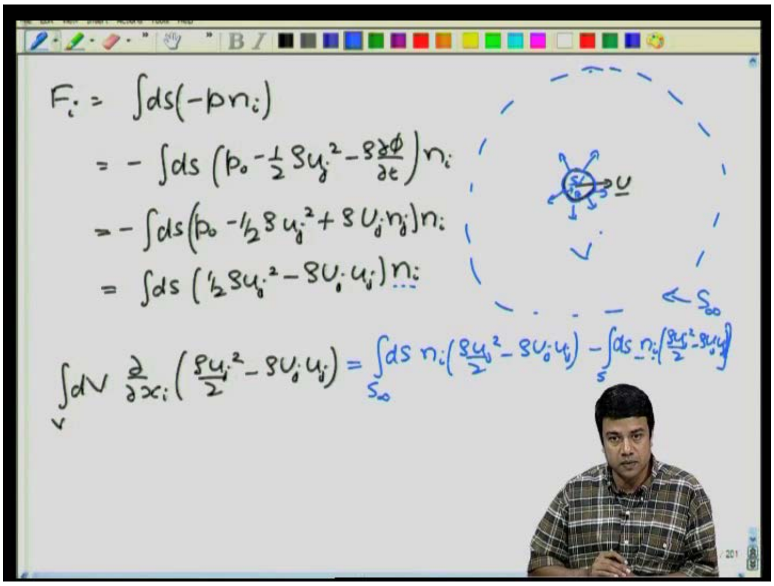

So let us take an object of a general shape let us take an object of a general shape moving with a velocity u, once again I work in a coordinate system, which is fixed at the centre of the object of course, I could equivalently have the object at constant velocity, I am sorry the objects stationary and the fluid flowing in at a constant velocity far away from the object, the result will end up being the same. We will solve it for this particular configuration in this lecture and later when we do two dimensional potential flows we will solve it for the other configuration.

I fix my origin at the centre of this object and I find out the velocity potential of course, the velocity potential for this case is going to be a complicated expression for a sphere I got a nice simple velocity potential, for this particular cases going to be a complicated expression. But in either case the force exerted the force exerted F i is going to be integral over the surface of p times n i times a unit normal is equal to minus integral ds of p naught minus half rho u i square. Because the object is moving with a constant velocity the origin of the coordinate system is translating I have this additional contribution to the total time derivative of the potential. So this is equal to minus rho d phi by d t.

Because this object is going at a constant velocity d phi by d t is not equal to 0 even though the velocity is a constant, even though d u d t is equal to 0, d phi d t is not equal to 0 because the object is translating. Therefore, if I am sitting at a fixed location, a fixed observation point as an object is moving the potential at this point changes because the distance from the centre of the object changes. Even though I am sitting at a fixed location, because the origin of the coordinate system is moving, this is going to be a change in the potential at that point.

We had calculated that in the previous lecture and that turns out to be minus integral ds of p naught I am sorry if I have a unit normal here minus half rho u i square plus rho capital U summation, see this is a different index because I cannot have the same index appearing three times. So this is u j square minus rho times capital U j times small u j times n i of course, p naught by an integrated over this entire volume p naught times the unit normal integrated over the entire surface has to be equal to 0 because a constant pressure cannot exert a net force.

So I get the final force as integral over the d surface ds of half rho u j square minus rho U j u j times n i. So I have to evaluate this. Note that this n i is the outward unit normal this n is the outward unit normal to the object. Now consider the following integral consider the following integral, integral over the fluid volume integral over the fluid volume of partial by partial x i of rho u j square by 2 minus rho capital U j small u j.

So this is an integral over the entire fluid volume. So to place this in context I am having an object which is moving at the velocity U and I have got an expression for the force.

This is an irregular object I have got the expression for the force. If I consider this integral over the entire fluid volume which goes all the way from the surface of the object to a surface far away at infinity, fluid volume this is the fluid volume V which goes all the way from the surface of the object to the surface far away at infinity. This is the divergence of something integrated over the volume that is equal to integral over the bounding surfaces of n dot this thing.

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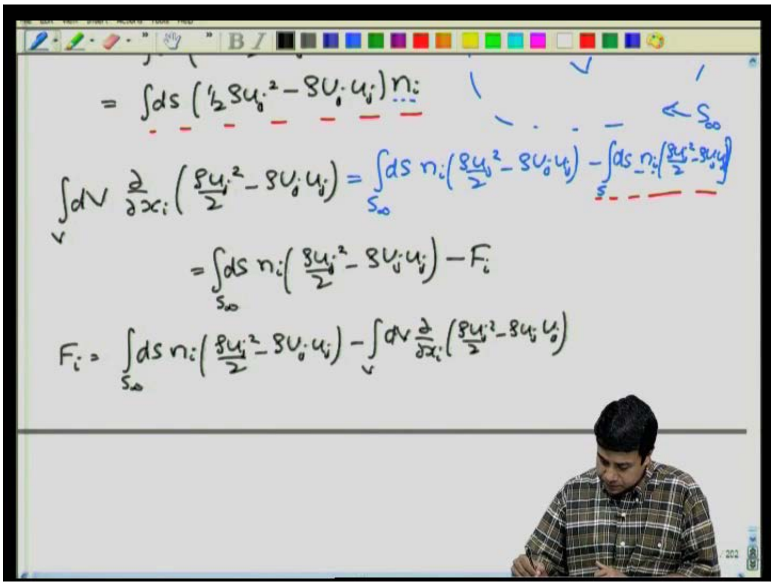

So this is equal to integral over the surface this one surface at s infinity as we saw, this another surface on the surface of this sphere. So there is an integral over the surface at infinity ds of n i into rho u j square by 2 minus rho capital U j small u j minus the integral over the surface s ds n i one has to be careful here, because for the surface that is on the surface of this sphere the fluid surface that is on the surface of the sphere the unit normal's pointing opposite to the unit normal to this sphere, the unit normal is pointing opposite to the unit normal of the sphere.

So therefore, whereas, I should strictly speaking add up the contributions on the two surfaces. If I take this n i to be the unit normal for the sphere itself if I take this n i to be the unit normal for the sphere itself this becomes minus integral ds n i of the same thing, where I have defined my n as the outward unit normal to the surface of the sphere on this integral ds and you can easily verify that this second term is just the force on the sphere, the second term here, the second term that I have here is just the force on the sphere that is here of course, the first term is an integral at infinity, the second term is the integral over the surface of the sphere.

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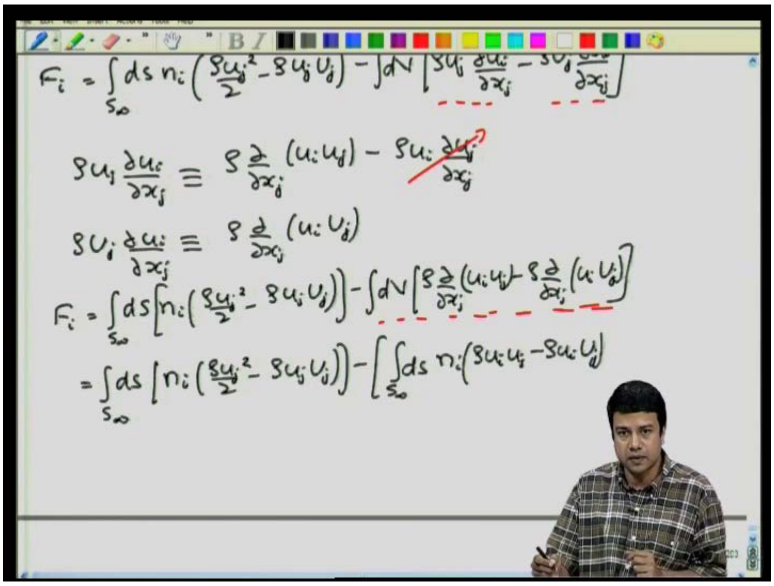

So this is going to be equal to integral ds of the surface at infinity n i into rho u square by 2 minus rho minus F i. So this gives me the force F i in terms of this volume integral over the fluid volume and the surface integral over the surface at infinity therefore, this gives me F i is equal to integral over the surface at infinity minus integral over the volume minus the integral over the volume fluid volume. We can simplify this integral over the fluid volume by taking, by differentiating it, differentiate using chain rule, partial by partial x i of half rho u j square is going to be equal to rho u j partial u j by partial x i, partial by partial x i of rho u j times capital U j capital U is the velocity of the object it is a constant therefore, this just becomes equal to minus rho u j partial capital U is a constant, I have here partial u j by partial x i.

I have here partial u j by partial x i since the flow is a irrotational the rate of deformation tensor is symmetric therefore, the rate of deformation tensor partial u j by partial x i is equal to its transpose because we have an irrotational flow, the anti-symmetric part of the rate of deformation tensor is 0 therefore, the rate of deformation tensor is symmetric. That means, that the rate of deformation tensor partial u j by partial x i is equal to its transpose partial u i by partial x j.

So this becomes equal to integral over the surface at infinity ds n i minus integral over the volume of rho u j partial u i by partial x j minus rho times capital U j partial u i by partial x j. I just replaced the rate of deformation tensor by its transpose because we know that the rate of deformation tensor is symmetric. Now I have to two terms here, these two terms let us just simplify them a little bit. We know that rho u j partial u i by partial x j can be written as rho times partial by partial x j of u i u j minus rho u i partial u j by partial x j.

The second term is of course, velocity times the divergence of the velocity partial u j by partial x j is the divergence of the velocity, since the flow is incompressible the divergence of the velocity has to be equal to 0 and therefore, I have rho u j times partial u i by partial x j is rho times partial by partial x j of u i times u j, as for the second term rho u j partial u i by partial x j, since capital U is a constant anyway I can take it into the derivative because capital U is a constant anyway. So I can take it into the derivative. So this becomes equal to the rho partial by partial x j of u i times capital U j.

So substitute these n the simplifications n and I will get the force F i is equal to integral as a infinity ds n i into rho u j square by 2 minus rho u j capital U j minus integral dV of rho partial by partial x j of u i u j minus rho partial by partial x j of u i times capital U j. So that is a simplification for this case, you look at this second term here, look at the second term here, both the terms here are the divergence of something integrated over a volume, both terms are divergence of something integrated over a volume. So that can of course, will reduce to a surface integral, the integral of the unit normal times the surface, partial by partial x j of u i u j integrated over the volume is equal to n j times u i u j integrate over the surface.

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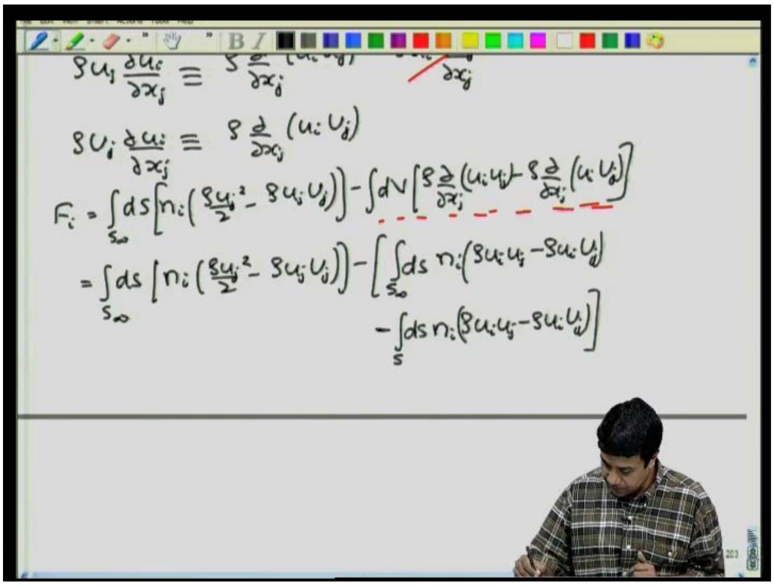

Of course, now there are two surfaces, one is the surface on the object itself; the other is the surface at infinity. So therefore, for this fluid volume V there are two bounding surfaces, one surface is on the object, the other surface is far away at infinity therefore, this volume integral has to be summed over those two surfaces. So therefore, if I convert this into a surface integral what I will get is integral over the surface at infinity ds n i into rho u square by 2 minus rho minus I have two surface integrals here, integral the surface at infinity ds n i into rho u i u j minus rho u i times capital U j. So this is over the surface at infinity and I have the next over the surface integral over the surface of the object, note that in this case I am taking the unit normal outward. And therefore, I have an negative sign, I had taken the unit normal inward to the object or outward to the fluid, this would have be in a positive sign.

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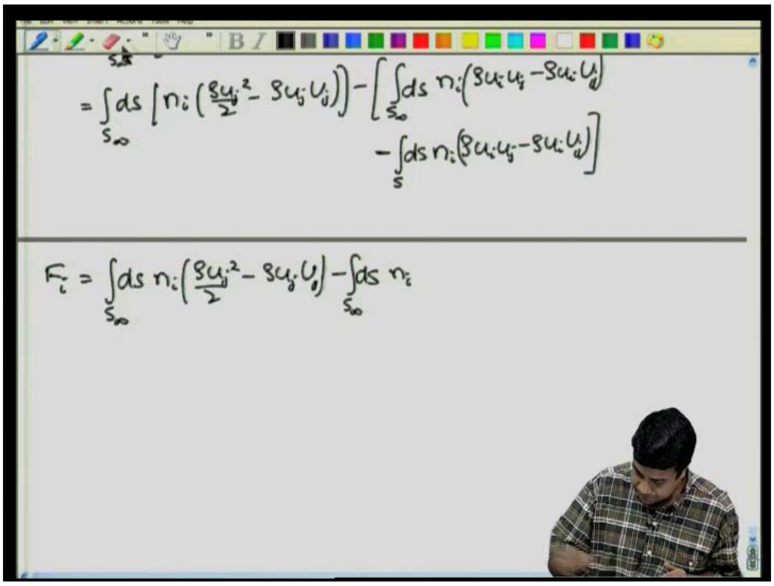

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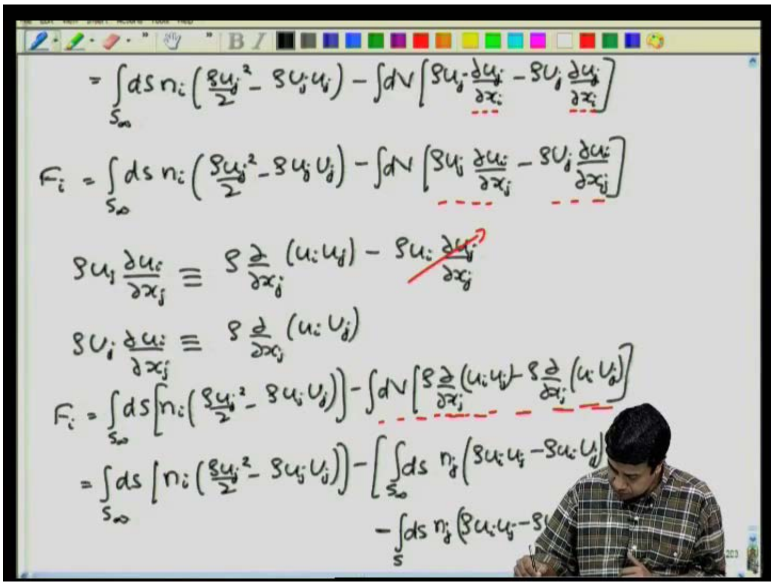

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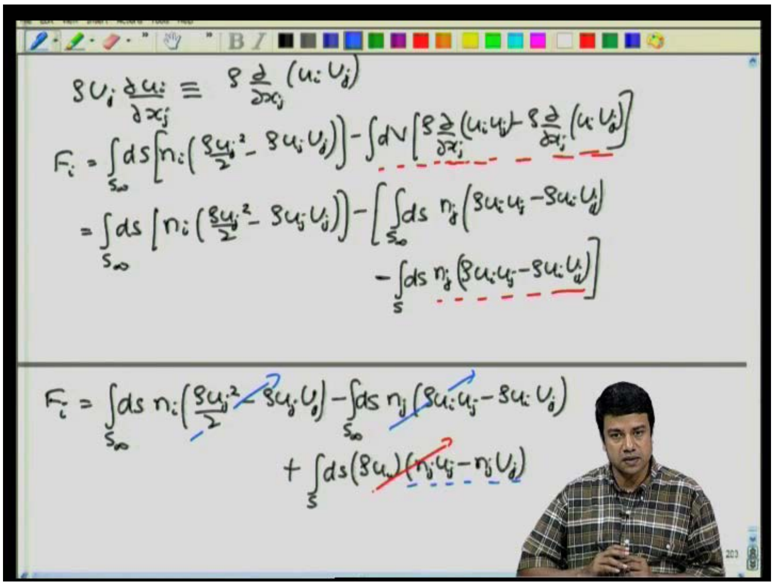

So these are the two terms that come out from this volume integral after I have used the symmetry of the rate of deformation tensor. So I am getting an integral over two surface integrals over surface is at infinity and one surface integral over the object itself let simplify it a little bit here so this gives me the force F i is equal to integral over the surface at infinity ds n i into rho u j square by 2 minus rho u j U j minus the integral over the surface at infinity ds n i in I am sorry this is n j yeah please make that correction here, this index should be j n j because I have a derivative with respect to x j here. Therefore, the unit normal has to have the same index as the derivative. So I should have n j. So this should be n j rho u i u j minus rho u i U j plus the integral over the surface ds of rho u i into n j u j minus n j times capital U j, this putting these two terms together this becomes a rho u i times n j small u j minus n j times capital U j.

This is my final expression for the force and I have taken you through all of this just to prove this one point, that this entire thing has to be equal to 0, on the surface of the object the no normal velocity boundary conditions unit normal dotted with fluid velocity has to be equal to unit normal dotted with object velocity. What I have here is the difference unit normal times object fluid velocity times minus unit normal times fluid velocity n dot small u minus n dot capital U, from the no normal velocity boundary condition on the surface of the object this entire thing has to be equal to 0 because there is a no normal velocity boundary condition on this object.

If two integrals over the surface at infinity the two integrals over the surface at infinity as r goes to infinity the velocity field decreases as 1 over r cubed for a sphere, it decreases exactly as 1 over r cubed for an irregular shaped object you could have other terms as well those are the terms of course, have decay faster as 1 over r power 4, 1 over r power 5 etcetera the 1 over r cubed decay for the velocity field corresponds to the 1 over r square decay for the potential.

If the potential decays as 1 over r square the velocity which is the gradient of the potential decays as 1 over r cubed. The velocity will decay with a lower power only if you have a contribution to the potential proportion to 1 over r, 1 over r is a source term. So if you have a source term you will have a contribution to the velocity which decays 1 over r square. However if there is no net source, if you satisfy the incompressibility condition there is no net source within the fluid, the highest power of decay can be 1 over r square for the potential and therefore, the highest power of decay for the velocity can be 1 over r cubed.

If the velocity decays as 1 over r cubed you can see the first two integrals the surface integrals over the surface at infinity, velocity decreases as 1 over r cubed surface area of a surface, where r goes to infinity increases as 1 over r square, the product of the two 1 over r cubed times 1 over r square is proportional to 1 over r and that product goes to 0 in the limit as r goes to infinity and for that reason the integrals over both of the surfaces. A surface integral over both of these surfaces both of them go to 0 in the limit as r goes to infinity therefore, this has shown that for any shaped object the net force exerted by the fluid on the object provided it is moving at constant velocity has to be equal to 0 when we derived for the specific case of a sphere in the last lecture.

We used symmetry arguments to derive this but, this calculation shows the that is true for any object and I took you through this for a three dimensional object because I will be doing a similar calculation for a two dimensional object, when we look at two dimensional potential flows in the next section. So this calculation basically shows us that the net force exerted on the fluid by any object is 0 provided it is moving at constant velocity. As I mention that is equivalent to what is called the de-Alembert's paradox in the last lecture.

The kinetic energy of course, has to be a constant and once again this is a reflection of the fact that there is no net dissipation of energy in the flow, because you completely neglected viscosity, dissipation of energy of course, only to two viscous dissipation due to the viscous part of the stress tensor, in this case you neglected it. We include only the pressure which is the reversible part of the stress tensor for that reason there is no dissipation and therefore, the net energy of the flows preserved therefore, to move an object at constant velocity you do not need to exert a force.

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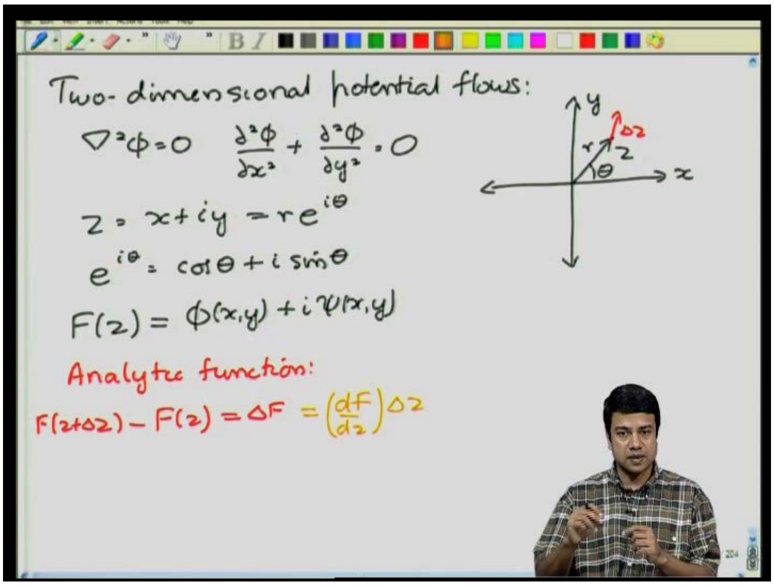

Because when you accelerate the object you do need to exert a force, that force is equal to the added mass times the acceleration. For the particular case of a sphere we found the added mass is equal to one half of the mass of the fluid displaced by the sphere. So let us go to next topic which we will consider two dimensional potential flows, that is we are going to try to solve the Laplace equation in two dimensions del square phi is equal to 0 which corresponds to d square phi by d x square plus and we will solve this using the principle of complex variables, the solutions that we get are in general applicable to any case where the Laplace equation where the the the variable that the field satisfies the Laplace equation. It applies equally well to the diffusion equation for temperature and concentration as well but, in this particular case will apply to the case of potential flow equations. So we have two x and y coordinates, in this two dimensional plane one can define a complex number as z is equal to x plus i y.

In that case you take the x axis as the real axis the y axis as the imaginary axis and any point z has given by x plus i y alternatively, if I take in a polar coordinate system r theta coordinate system this is also given by r e power i theta, where e power i theta as you know is cos theta plus i sin theta this gives us of course, if i substitute this e power i theta is equal to cos theta plus i sin theta I get x is equal to r cos theta y is equal to r sin theta as the usual conversion from polar coordinate system to a cartesian coordinate system.

Now we can have a function of this x and y coordinates in the complex plane. Some function which is a function of this coordinate z, this is in general a complex function this is in general a complex function it will in general have real and imaginary parts, both of these real and imaginary parts will be functions of both x and y. So this function will in general have a real part plus an imaginary part. A functional will have a real part plus an imaginary part, this function is what is called an analytic function. Analyticity in the complex plane is the equivalent of differentiability in the real plane this function is called an analytic function provided that I can define a derivative d F by d z.

So the question is that if i move a small distance delta z from the location z in some direction. If I move a small distance delta z from the distance from the location z in some direction at my original location I had my function F of z my original location I had my function F of z I move to a new location z plus delta z, this function has some other value there F of z plus delta z. The difference between the two is the change in F when you move from z to z plus delta z.

So it is a change in F if you move from z to z plus delta z and a function is called analytic. If this delta F can be expressed as some function d F by d z times delta z. What that means is that the change in delta F the change in f the delta F, which is the change in F when I moved from location z to location z plus delta z depends only upon delta z that is delta x plus i delta y it does not depends separately on x and y it depends on x and y only through this delta z which is x plus i delta y.

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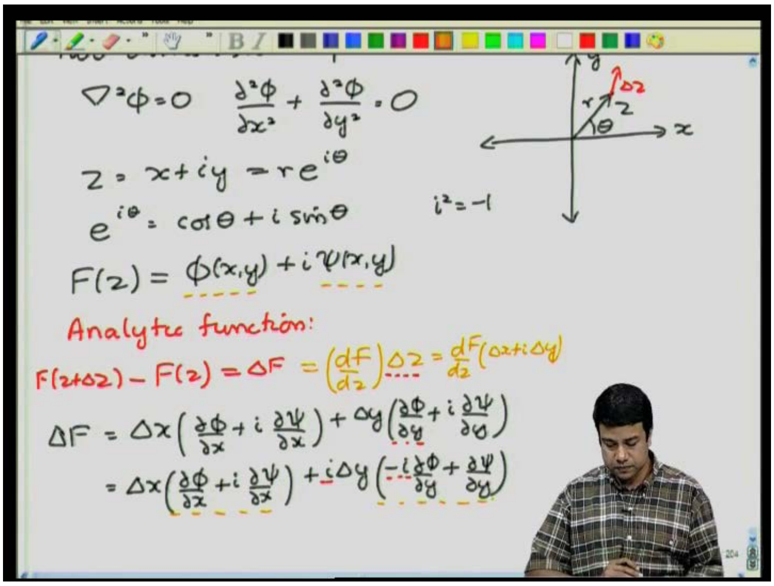

So the requirement that the function has to be analytic imposes conditions on the real and imaginary part of this complex function. So let us look at that. So what is delta F? F delta F the change in F if I if I write F as phi plus i times psi right the change in F is going to be equal to delta x times partial phi by partial x plus i partial psi by partial x plus delta y times partial phi by partial y plus i partial psi by partial y just using the chain rule for differentiation. Delta F is the change in F when it goes a small distance delta z, delta z is equal to delta x plus i delta y; that means, that delta F is going to be equal to the variation of phi with respect to x times delta x i am sorry the variation of F with respect to x times delta x plus the variation of F with respect to y times delta y and partial F by partial x is equal to partial phi by partial x plus i times partial psi by partial x ultimately, I want something that goes proportional to delta z. We want something that proportional to the delta z. So I rewrite this as delta x times plus i delta y times.

Note that the complex square root of minus 1 the imaginary number i has the property that i square is equal to minus 1 which means that in the second expression I have minus I, minus i times i is equal to minus 1 into minus 1 which gives the plus sign over here. So that separation I have made, I have written delta y times partial phi by partial y as i delta y times minus i partial phi by partial y. So if this difference delta F can be written proportional to delta z which is equivalent to d F by d z times delta x plus i delta y.

If it can be written in this fashion it implies that the coefficient of delta x and the coefficient i delta y have to be equal in this expression for delta F, the coefficient of delta x and the coefficient of i delta y have both got to be equal what; that means, is that this has to be equal to this one. Note that F was complex but, phi and psi are real values even though F was complex both phi and psi are real functions therefore, if these two are equal you require that separately the real parts are equal and there imaginary parts are equal.

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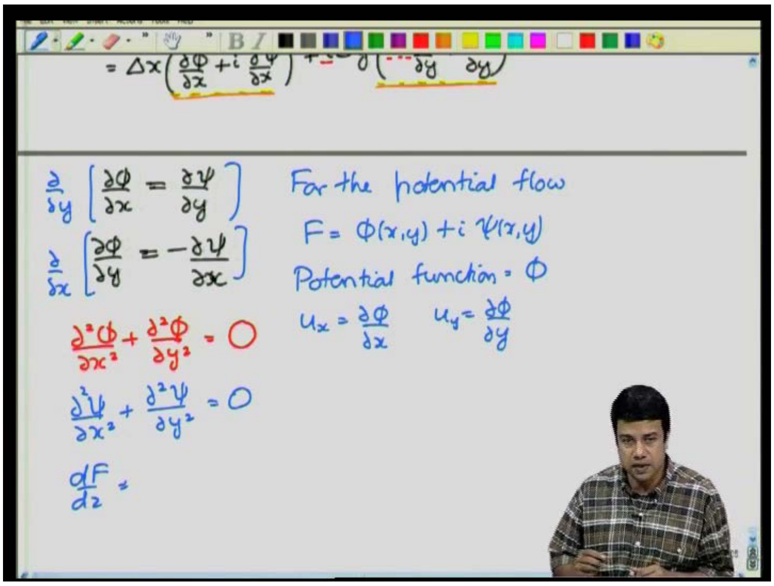

If these two numbers are equal to each other you require separately that the real parts are both equal to each other and the imaginary parts are equal to each other. So therefore, the function is analytic only if partial phi by partial x is equal to partial psi by partial y and partial phi by partial y is equal to minus partial psi by partial x. So this gives you the conditions under which the real and imaginary parts are both equal and complex variables is often referred to as the Cauchy Riemann conditions for analytic functions.

If a function is analytic its real and imaginary parts have to satisfy these conditions for the function to be analytic in the complex plane the Cauchy Riemann conditions, this also has implications for the functions phi and psi itself, the easy way to see it for example, you can do two things here, the first thing you do is the take the first function, you take partial by partial x by partial x of this whole thing and you take partial by partial y the second function and add both of them up.

So on the left hand side you have partial square phi by partial x square plus partial square phi by partial y square. On the right hand side I have the second derivative partial psi by partial x partial y minus partial square by phi by psi by partial x partial y this is identically equal to 0. So this basically tells me that if a function is analytic its real part identically satisfies the Laplace equation in this two dimensional plane, I could do the other thing. So I could do the other way that is I multiplied this way partial by partial y this is by partial by partial x and then subtract the two, we subtract the lower one from the upper one, the second derivatives of phi will cancel out and I will be with left a Laplace equation for psi. So to summarize if a function is analytic both its real and imaginary parts satisfies the Laplace equation. So we are looking for potential flow solutions which satisfy the Laplace equation therefore, this tells us that any complex function any, analytic complex function in the complex plane does satisfy the Laplace equation.

So when you look a potential flow solutions we look for two things, the first is that it has to satisfy the Laplace equation, the second is that it has satisfy the no normal velocity boundary conditions. An analytic function does satisfy the Laplace equation. So it is a valid potential flow solution provided it satisfy the normal velocity boundary condition therefore, any analytic solution function in the complex plane which satisfies the no normal velocity boundary condition is a valid solution for the potential flow equations in that domain subject to those boundary conditions. As I have already shown you solutions for potential flow are unique therefore, you can have only one solution which satisfies the boundary conditions and there is an analytic function in the complex plane therefore, it satisfies the potential flow equations. So for this analytic function since I know that the real part satisfies the Laplace equation without loss of generality, I can assign phi the real part as the velocity potential.

So for the potential flow F is equal to phi of x, y plus i psi of x, y. We know both of them automatically satisfy the Laplace equation. I will choose phi as my potential function. So potential function is phi is equal to phi this satisfies del square phi is equal to 0 what; that means, is that the velocity components u x will be equal to partial phi by partial x and u i is equal to partial phi by partial y. So if I choose my potential this is just my convention you could choose either one as a potential, if you choose the negative of that also as a potential but, as a convention we choose the real part of this complex function as the velocity potential and then you get the two velocity fields based upon the two components of the velocity based upon the derivatives of the potential at that particular location.

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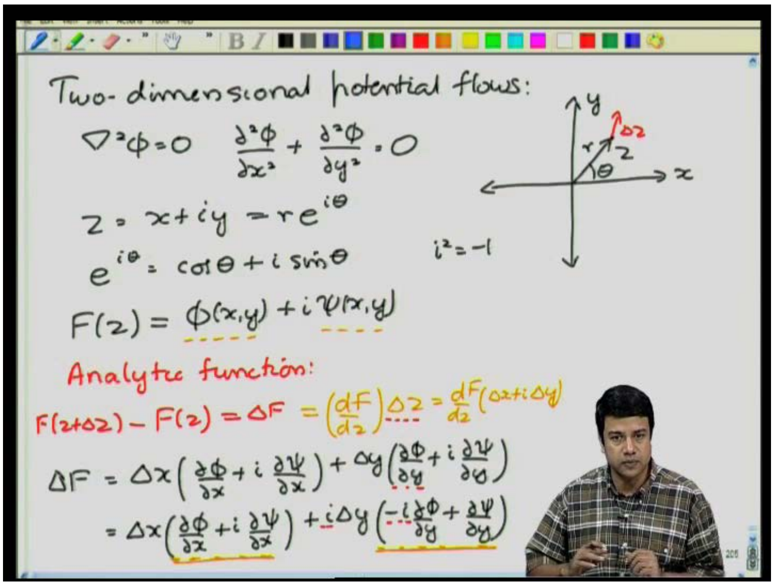

So we know that d F by d z we just calculated d F by d z here is equal to delta x times partial phi by partial x plus i partial psi by partial x plus i delta y times minus i partial phi by partial y plus partial psi by partial y and because our solution is analytic the coefficients of delta x and the coefficients of i delta y are identically equal to each other.

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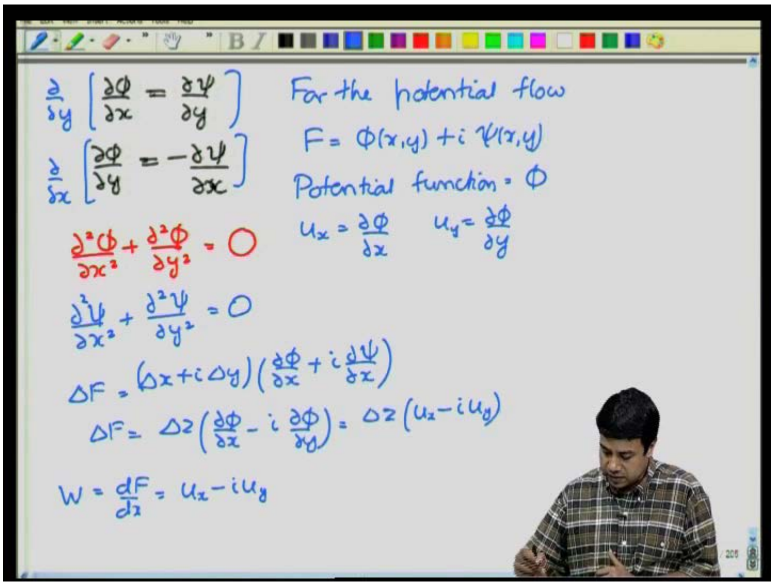

So therefore, I can choose either of these is equal to delta x plus i delta y into partial phi by partial x plus i partial psi by partial x. However i times partial psi by partial x we have the Cauchy Riemann condition here that partial psi by partial x is equal to minus partial phi by partial y. So this is equal to delta x plus i delta y is just delta z into partial phi by partial x minus i partial phi by partial y which is equal to I am sorry I should write the left hand side as delta F here is equal to delta z times u x minus i u y.

So once I have the complex function I have the complex function F which is analytic, whose real part is the velocity potential then the derivative of that complex function which I will refer to as the complex velocity it is equal to d F by d x will be equal to u x minus i u y. So this is how the complex velocity which is u x minus i u y is related to the derivative of the complex potential.

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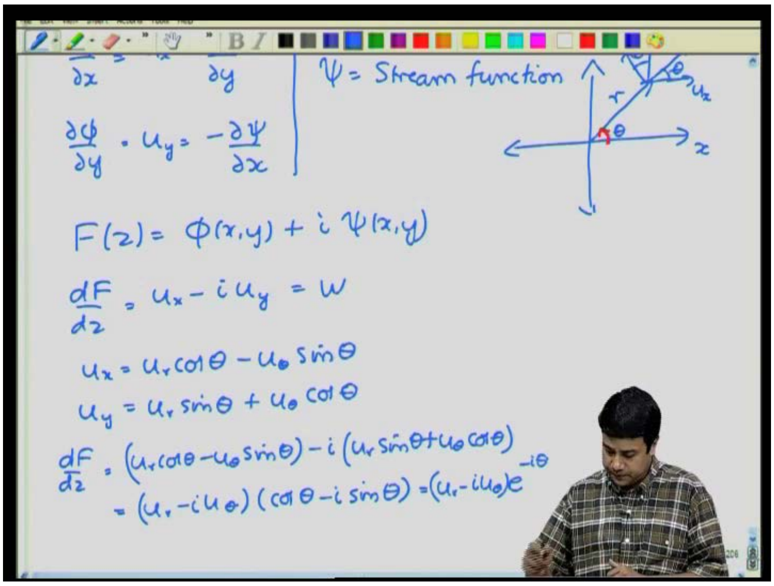

I will refer to F as the complex potential, the real part of F is the actual potential and the imaginary part is this function psi and the derivative of that is the complex velocity, that complex velocity is equal to u x minus i u y, where u x and u y are the two components of the velocity. What is this function psi? We know that from the Cauchy Riemann conditions partial phi by partial x is equal to u x is equal to partial psi by partial y and partial phi by partial y is equal to u y is equal to minus partial psi by partial x.

So what is psi? It is clear from this psi is the stream function, stream function satisfies the condition u x is equal to partial psi by partial y, u y is equal to minus partial psi by partial x therefore, if i define my complex potential F of z is equal to phi of x y plus i and if I identify phi as my velocity potential then psi is the stream function and partial phi by partial x and partial phi by partial y are u x and u y respectively, and from the Cauchy Riemann conditions there have to be partial psi by partial y and minus partial psi by partial x and if I take the derivative d F by d z.

I will get u x minus i u y this is the complex velocity obtain by taking the derivative of the complex potential, note that the derivative of the complex potential with respect to z coordinate it is like taking a gradient except that both of these are now complex objects, where they do not have vector directions, there is complex objects in this complex plane. One can get another interpretation of d F by d z in a polar coordinate system.

So let us just briefly do that for proceeding x and y. So if I am at some location r with an angle theta, this is u x, this is u y, one can find out the components in the r theta coordinate system. The r theta coordinate systems u r is along the r direction, u r is along the direction of increasing r and therefore, it makes an angle theta with respect to the x direction, u theta is along the direction of increasing theta. Note that theta increases in this direction theta increases in this direction, in the anticlockwise direction therefore, u theta has to be in this direction. So this is the direction of u theta.

So therefore, the angle between u theta and u y is also equal to theta. So now this complex velocity u x and u y iI can write in terms of u r and u theta, it is quite easy to see that u x is equal to u r cos theta, u r cos theta because the angle between u x and u r is theta u theta is directed in the direction of minus u x, u theta is in the anticlockwise direction therefore, it is in the direction of minus u x. So this becomes minus u theta sin theta and u y angle between u y and u r is pi by 2 minus theta. So u y is equal to u r sin theta plus u theta cos theta u y is equal to u r sin theta plus u theta cos theta substitute that in here d F by d z is equal to u r cos theta minus u theta sin theta minus i into u r sin theta plus u theta cos theta, you can simplify this you can simplify this quiet easily to get u r minus i u theta into cos theta minus i sin theta and u r cos theta minus i am sorry cos theta minus i sin theta is equal to u r minus i u theta e power minus i theta.

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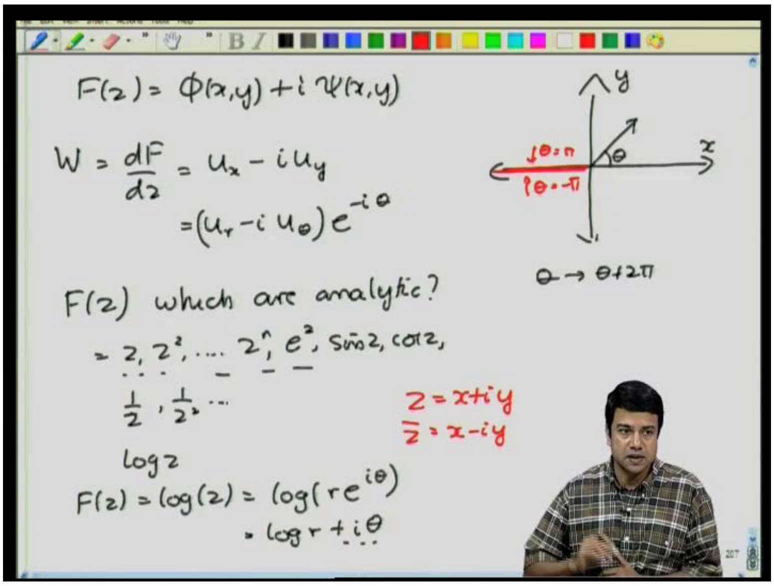

Therefore, to summarize if we have a complex function f of z is equal to phi of x, y plus i psi of x, y we identify phi as the potential function. If this is analytic function both phi and psi automatically satisfy Laplace equations. So they are all both solutions of the potential flow equations without loss of generality and by convention we identify phi of this analytic function as the potential for our potential flow. In that case psi automatically becomes the stream function. So the real part is the potential, the imaginary part is the stream function. We define the complex velocity as d F by d z this can be written as in terms of the two velocity components as u x minus i u y alternatively, as u r minus i u theta e power minus i theta. So these are that the velocity components in terms of the complex velocity w which is the derivative of the potential function with respect to psi. So this was the first part of our analysis. How do you get the solutions, which automatically satisfy the potential equation? We showed that any complex function, which is analytic automatically satisfies the potential equations.

However the potential flow equations have to satisfy not just the Laplace equation itself, potential flows which satisfy not just Laplace equation itself but, also the no normal velocity boundary conditions. So we have to find solutions for the potential flow equations which satisfy the zero normal velocity boundary conditions under specific for specific configurations. So we will continue that in the next lecture but, before that let me just talk about complex analytic functions.

What are the forms of F of z which are analytic? In general any function which acts on z itself and does not act differently on x and y is an analytic function, this includes all possible functions such as z, z square etcetera any polynomial z bar and any some of polynomials, if you have a linear sum of polynomial functions it will also be analytic, since for example, exponential of z e power z, sin z, cos z all of these functions can be expressed in Taylor series which involve powers of z, since any power of z is analytic all of these functions also are analytic.

So there are various functions of various kinds any common function which acts on z itself does not act separately on x and y is an analytic function, there are few things that one has to be careful about for example, z power functions of the form 1 by z, 1 by z square etcetera these are also analytic except that they go to infinity at the origin but, they are still analytic. If you take the real and imaginary parts of these function find that they do satisfy the Cauchy Riemann condition 1 over z, 1 over z square and so on only thing is they go to infinity at the origin, then there are other function such as log of z this is also an analytic function but one has to be careful in how it is defined in the complex plane, one has to be careful because if I define F of z is equal to log of z this is equal to log of r e power i theta, z is equal to r e power i theta. So this equal to log r plus i theta. Now the point is that in the complex plane the point at a particular location is defined by r and theta. So this is the angle from 0, this is x and y, this is the angle from the x axis. So I have a point, whose coordinate is r and theta the angle from the x axis is theta, however theta is not uniquely specified for this case, because if I go all the way around then come back to that location theta goes to theta plus 2 pi.

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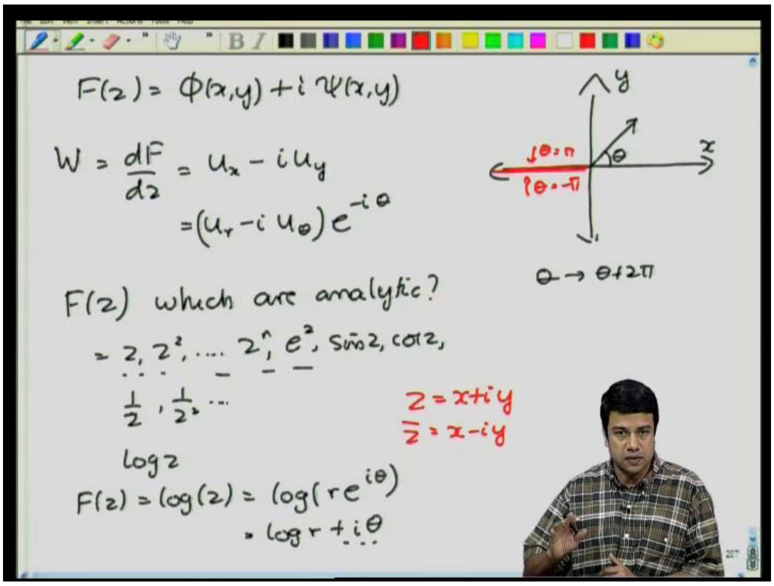

So that is same point is at theta is equal to theta let us say it is some 50 degrees say is theta, then 50 degrees is equal to theta at that point, theta is also equal to 2 pi plus 50 because I can go all the way around and come back. So at one particular location, I have multiple values for the angle theta because I can go on number of times around the origin, come back to that location and I have the same physical point has multiple values of theta. Of course, one cannot have a multi valued function I require that the entire that the all points on this plane have unique values of coordinates, they do have unique values of x and y, but when I expressed in terms of theta it has multiple values that does not make a difference for functions of this kind.

Functions of this kind, even though theta is different, the physical x and y will still be the same. But for this particular function the imaginary part is equal to i times theta and that has multiple values therefore, one has to take the care to ensure that you define a coordinate such as such that you cannot cross one particular axis. So in other words I cannot go around this axis is called a branch cut. So in that case this axis on this side is theta is equal to pi, on this side of this axis theta is equal to minus pi and I cannot cross this axis and therefore, theta is uniquely specified throughout the domain.

So this branch cut is required both for the log function as well as for fractional powers, z power half for example, if I take z power half I get e power i theta by 2. So if I go around theta to theta plus 2 pi that e power i theta by 2 has a different value. So for all those cases one has to take a branch cut in the complex plane, subject to that all of these are analytic functions, the only non-analytic function is one which acts separately on x and y for example, if I take the complex conjugate, if I take z is equal to x plus i y the complex conjugate is equal to x minus i y.

So this is not an analytic function because we are operating separately on x and y, where as in all such functions which operate on z itself, they all turn out to be analytic functions. So we have this range of analytic functions which we can which represent potential flows and we can ask for this particular form of the function what kind of potential flow do I get? So in next lecture we look at that question, I choose a particular form for the function and I try to see, what is the kind of potential flow that results for this particular form of the function and by identifying boundaries on which the normal velocity is equal to zero.

I can find out what kinds of flow result from specific values of these complex functions. So that we will continue in the next lecture, what kind of potential flows results from specific forms of complex functions? We will continue that and look at potential flows in two dimensions using this complex plane. So that will be our next step to look at what kind of potential flows result from specific forms of the complex function? We will continue that in the next lecture. We will see you then.