Fundamentals of Transport Processes II
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Lecture - 26
Potential Flow around a Sphere

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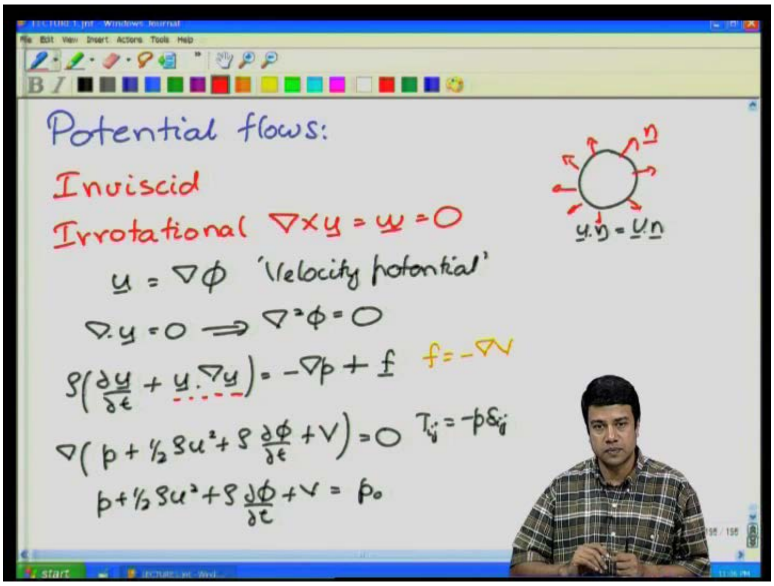

So welcome to this lecture number 26 of our course on fundamentals of transport processes. We were discussing high Reynolds number flows, potential flows, if you recall the previous section was on low Reynolds number flows where, inertial effects are neglected. This section is on high Reynolds flows where viscous effects are neglected.

So these are potential flows, they have two characteristics; one is they are inviscid that is the viscosity is equal to 0. So we completely neglect the viscous terms in the conservation equation and there also irrotational del cross u which is the vertisity is equal to 0 everywhere in the flow. So therefore, since the vertisity is equal to 0, the anti-symmetric part of the rate of deformation tensor is equal to 0 everywhere in the flow. We derived the conservation equations for this case, since the flow is a rotational the velocity can be written as the gradient of a potential because the curl of a gradient of a scalar is always equal to 0.

Once we do that we get the mass conservation equation, the divergence of velocity is equal to 0 reduces to del square phi is equal to 0, phi is called the velocity potential phi is called the velocity potential. The momentum conservation equation in the absence of inertia reduces to rho times partial u by partial t plus u dot grad u is equal to minus grad p neglect the viscous terms and therefore, you just get the body force. And we had we had simplified this non-linear term for the case where the vertisity is 0 or the anti-symmetric part of the rate of deformation tensor is equal to 0. And once we do that we get an equation of the form the gradient of p plus half rho u square plus rho partial phi by partial t plus the potential, here we are assuming that the force is given by the gradient of a potential. So it is a conservative force f is equal to minus grad v. So the force is a conservative force.

So this whole thing is equal to 0 or p plus half rho u square plus rho partial phi by partial t plus v is equal to some constant p naught. So the gradient of some function is equal to 0 that function itself is equal to a constant everywhere within the flow field of course, there pressure itself is known only to within an unknown constant. So we had to specify the pressure at some boundary and then you know the pressure everywhere within the flow. As I said we have neglected the viscous terms of the conservation equation and therefore, it is not possible to satisfy the zero normal velocity and zero tangential velocity boundary conditions at the surface.

The original Navies Stokes equations that we had was second order differential equations in the velocity. When we neglected the viscous terms we basically get a first order equation in the velocity or a second order equation in the potential, in the original equations we could satisfy both tangential and normal velocity or stress conditions. In the modified equations we can only satisfy the normal velocity and normal stress conditions.

The stress itself in this case is given by the stress tensor is equal to just the pressure part, since we have neglected the viscosity there is no viscous stress in this particular case, the stress is determined purely by the pressure and it is isotropic it is an isotropic tensor and therefore, at solid surfaces we can only satisfy the boundary condition that the normal velocity is equal to the velocity of surface along the unit normal itself where capital and is the unit normal to the surface.

This is the equivalent of the zero normal velocity boundary condition where n is the normal and we cannot satisfy the tangential boundary condition because, we reduced the equation from a second order to a first order physically, the reason is because when we neglect viscosity we neglect the diffusion of momentum, in the absence of momentum diffusion there cannot be any transfer of momentum perpendicular to the flow.

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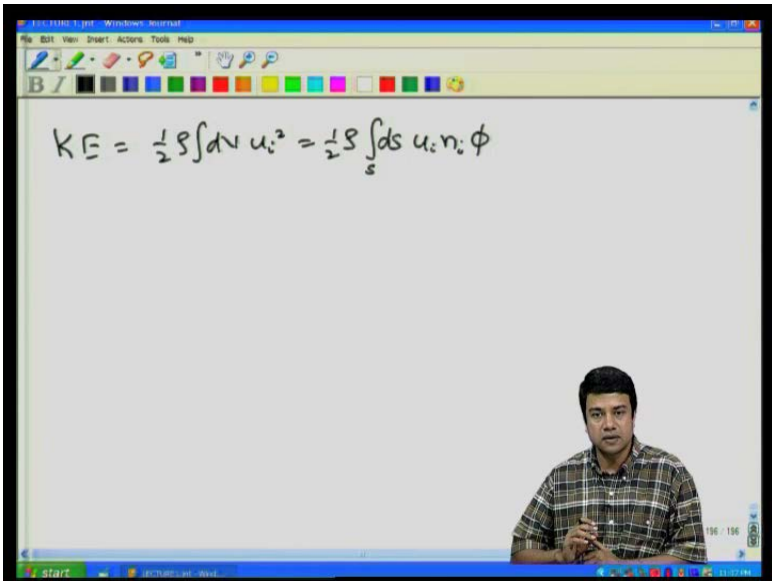

There is of course, convective transport of momentum along the flow direction, however there is no momentum transport perpendicular to the flow and the no-slip condition of the zero tangential velocity boundary condition at the surface requires a transfer of momentum perpendicular to the flow. So that the flows gets stopped at the surface, since we do not have that mechanism, we do not have a zero tangential velocity boundary condition for these potential flows.

For these potential flow solution equations we had we had proved various theorems for example, you can show that the solution is unique, you cannot have two different solutions of the potential flow equation that satisfy the same boundary conditions. You can show that for the velocity in the fluid to be non-zero, you have to have a non-zero normal velocity at the bounding surfaces because the kinetic energy as you recall the kinetic energy for the flow can be written just as a surface integral half rho integral over the volume of u square.

Using the fact that the velocity is the gradient of a potential I can write this as half rho integral over the bounding surface as of this volume of u i times the unit normal times the potential u dot n times phi therefore, the if the normal velocity at all bounding surfaces is equal to 0, the kinetic energy is 0 and therefore, the velocity is equal to 0 at each point in the flow. We also showed that for a potential flow solution the kinetic energy of the flow is smaller than the kinetic energy of any other flow which does not necessarily satisfy the potential flow conditions.

So therefore, the potential flow has the minimum kinetic energy of all possible flows that satisfies the conservation equations call the minimum energy theorem and finally, we were solving the potential flow solutions for the flow around a solid object, as usual we take the sphere as the simplest solid object because we can use tensor symmetries in order to determine the velocity profile of the sphere. So let us go back to that solution for the flow around a sphere. So I have a sphere that is translating with some velocity u vector.

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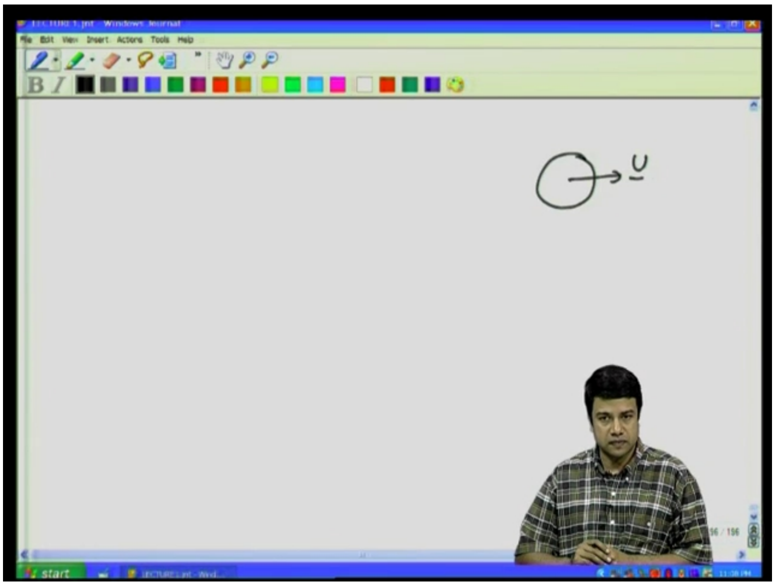

In potential flow and I need to find the solution for the potential flow equations. The boundary conditions as I mentioned are u i n i is equal to capital U i n i or u dot n. For the fluid is equal to the sphere velocity u dotted with the unit normal at that particular location on the surface and of course, we require that u goes to 0 as the distance goes to infinity as the distance from the surface of the sphere becomes larger and larger, since this sphere is moving in a quiescent fluid. We require that the velocity goes to 0 as r goes to infinity and this is the local unit normal at each point in the surface.

Now without loss of generality of course, you can place your the origin of the coordinate system at the center of the sphere. So that the surface of the sphere is basically given by r is equal to capital R. So I use a spherical coordinate system in which the center of the coordinate system is at the center of the sphere.

And then I have my radius vector r and angle theta the configuration is axis symmetric as you go around this axis which is parallel to u that I use the velocity direction itself as the axis from my coordinate system that is the simplest thing to do. Once you have chosen that as the axis there should be no variation as you go around that axis. So therefore, there is no dependence upon the phi coordinate.

So it becomes an axis symmetric problem. We solved the potential flow equations in the last lecture del square phi is equal to 0 with this boundary condition, solutions are of two types the growing and the decaying harmonics of course, we cannot have the growing harmonics because we require the velocity to go to 0 far away therefore, the solution has to be linear in the decaying harmonics. In addition the velocity field is linear in the velocity of the sphere and this is the gradient of potential is equal to the velocity, the potential is also linear in the velocity of the sphere. So therefore, my potential function has to be something that is linear in the velocity of the sphere as well as one of the harmonics.

It has to be linear in the velocity of the sphere as well as in one of the harmonics. The only way to get a potential that is linear in the velocity of the sphere as well as one of the harmonics is to multiply it by dot the velocity vector with the first vector sorry call harmonic, where A is some constant which will be determined from the boundary conditions. So since this vector spherical harmonic dotted with capital U gives you a scalar. This is the only possible solution, you cannot get solutions of any other form, this satisfies the Laplace equation because the vectors spherical harmonics is a solution of the Laplace equation.

So this is the final solution for the velocity field. Now of course, we determine this constant A by finding out what is the normal velocity boundary condition on the surface of the sphere the unit normal is the outward radial direction, it is along the radial direction therefore, the unit normal around the surface of the sphere can be written as the displacement vector, at the displacement vector to that point on the surface divided by the radius, since the unit normal is along the displacement vector and it has unit modules it is the unit normal is equal to the displacement vector divided by its magnitude which is the radius and the surface of the sphere.

Of course at the surface of the sphere this is just going to be equal to x i by capital R, capital R is the radius of the sphere. So we can impose this boundary condition u i n i is equal to capital U i n i as you recall u i is equal to partial phi by partial x i is equal to A U j into delta ij by r cubed minus 3 x i x j by r power 5 therefore, the boundary condition requires that A U j this is into the unit normal, unit normal in this case is equal to x i by r is equal to capital U i x i by r at r is equal to capital R. So that is the boundary condition from that boundary condition you find out the value of A.

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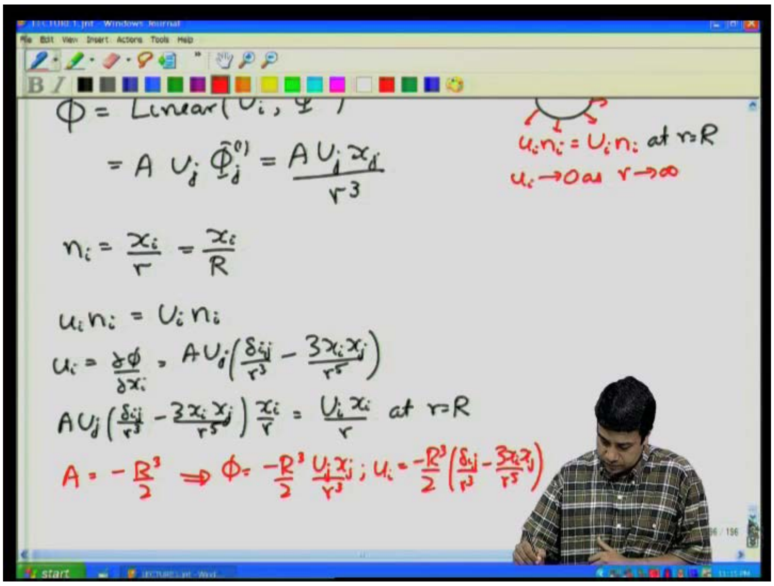

We solved this in the previous lecture and we got that A is equal to minus R cubed by 2. You can easily verify that for A is equal to minus R cubed by 2 this boundary condition is identically satisfied therefore, this gives the solution for the potential minus R cubed by 2 by r cubed and the velocity that gives the solution for the potential and for the velocity. Next we had determined the total kinetic energy of the flow in the last lecture.

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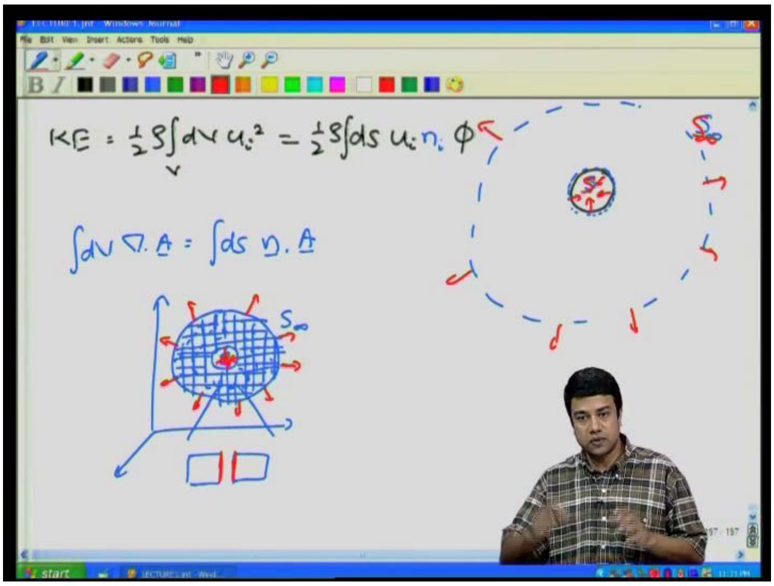

So as I said the total kinetic energy half rho integral over the volume of d v times u i square, this is the integral over the entire fluid volume, that is this the integral over all of the fluid that is located outside this sphere all of the fluid that is located outside this sphere. I can also write this as half rho integral ds of u i n i phi and let me just put this n i in blue here for a reason, this n i have put it in blue for a reason because that is the outward unit normal to the fluid volume. The fluid is outside this sphere extending up to infinity, the fluid is outside the sphere extending up to infinity obtaining the surface integral over this fluid volume, one has to take this carefully.

I am going through this in detail because we will see such volume integrals again and again in our analysis of potential flow. The fluid volume is the volume that is in between the surface of the sphere in between the surface of the sphere and the surface at infinity very far away from the sphere. I will call this surface of the sphere as s and this as s infinity. So the fluid is in between the sphere and far away the surface far away.

So I have to do this integral over these two surfaces and these two surfaces there is a fluid in between these two surfaces, recall that when we when we did the divergence theorem and we did the divergence theorem just to make this concept clear, we said that integral over the volume of d v of del dot A is equal to integral over the surface ds of n dot A that was the divergence theorem. How did we prove it? If you recall we took some volume with an outward unit normal n we took this volume with an outward unit normal n, then we divide this volume into small little bits.

And we looked at two adjacent volumes we looked at two adjacent volumes and we found out contributions to integral of divergence of A over these two adjacent volumes reduce to surface integral over the surfaces of these two volumes. Now for these two volumes they have this common surface they have this common surface, the flux through the surface is the same because what leaves one volume comes into the other volume right. So therefore, the value of A at these two surfaces is exactly the same.

Whereas the unit normal's the outward unit normal's are in opposite directions. The outward unit normal's are in opposite directions whereas, the value of the vector A at that particular location was the same therefore, the integral over these internal surfaces which are between two adjacent volumes exactly cancel out and all I am left with is the integral over the outside surface all I am left with is the integral over this outside surface, that give me the divergence theorem. This was done for an object which is what is called singly connected that is you have one object with just one surface surrounding it, this could be extended to multiple surfaces as well as follows I could have for example, one outer surface and one inner surface and there is a volume between these two there this a volume between these two there is an outer surface and an inner surface and I could also what is the value of integral d v divergence of A over this surface I am sorry over this volume which has two surfaces, one surface s inside, the other surface s infinity, outward unit normal to the volume for the surface outside it is directed outward to the volume, for the surface inside this is directed into the sphere that is at the center.

The outward unit normal to the volume that is it goes from the surface, it does not it is in opposite direction to the direction in which this volume is located therefore, it is inverted this point I could do my exact same calculation, divide the surface into large number of volumes similar to what I had for the divergence theorem I could divide into large number of volumes, then calculated integral of d v times del dot A over each of these volumes.

What you will find is that for two volumes that are adjacent to each other, once again those that have a common surface this integral will cancel out those that have a common surface this integral will cancel out, it will not cancel out when there is no common surface when the surface is actually bounding the volume. In this case there are two surfaces bounding the volume, one outside one inside. So therefore, one has to take the integral over these two surfaces what I have called s infinity here and what I call s inside over here, for s infinity the unit normal is facing radically outward, where as for the surface s itself it is directed into the sphere.

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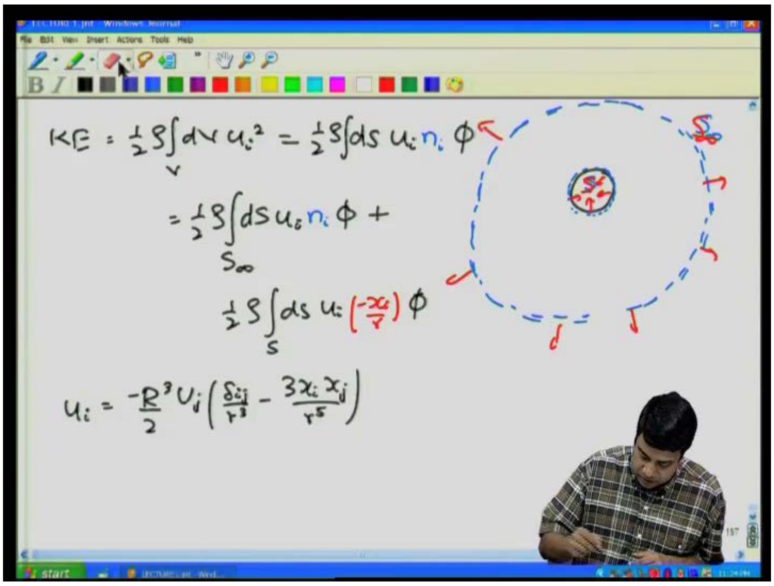

What is outward to the fluid volume is inward to the sphere. So therefore, the unit normal is inward to the sphere for this sphere itself. So for these two integral over these two surfaces, for the integral over these two surfaces this divergence theorem states that, so this has to be is equal to half rho integral over the surface at infinity ds u i n i phi plus half rho integral over the surface S ds u i n i phi, where this n i for the surface s is directed inwards therefore, the the the the value of the unit normal at the surface is directed inwards which means that this unit normal is equal to minus x i by r into phi, because it is directed opposite to the displacement vector in the coordinate system whose origin is at the center of the sphere.

So therefore, we can write this as a sum of two integrals, the first integral over the surface at infinity. If you recall the velocity field that we have just derived u i is equal to minus R cubed by 2 u j delta ij by r cubed minus 3 x i x j by r power 5 as you can see this velocity decreases proportional to 1 over r cubed as r goes to infinity.

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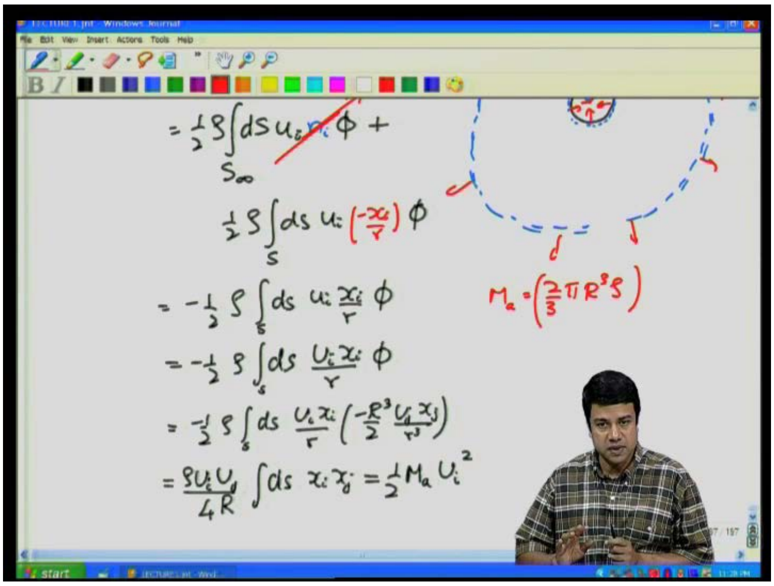

This velocity decays as 1 over r cubed as r goes to infinity, my potential was a dipole and therefore, this is the next higher term the quadruple term. So this velocity decreases as 1 over r cubed. Over this surface at infinity the surface at infinity that I have here as r goes to infinity the surface area increases as r square because the surface of a sphere the surface area is proportional to r square whereas, the velocity itself decreases as 1 over r cubed.

So in the limit as r goes to infinity this gives me a contribution that goes to 0, because the surface is increasing as r square velocity is decreasing as 1 over r cubed the product of the two goes as 1 over r and therefore, it goes to zero in the limit as r goes to infinity. So for that reason this contribution over the surface far away actually goes to 0 as r goes to infinity and I am left with this second contribution minus half rho integral over surface ds u i phi into minus x i by r and I know that on the surface u i n i.

So this u i n i is equal to capital U i times n i I should this sign must take here into phi and therefore, I can write this as minus half rho integral over the surface s of capital U i x i by r times phi because I know that small u i times x i is the same as capital U i times x i and if I substitute my value for phi I will get this as phi goes as minus r cubed by 2.

So I will get minus half rho integral S ds u i x i by r into minus R cubed by 2 u j x j by r cubed and this is taken at the value where capital R is equal to small r because the surface of the sphere is at the location where capital R is equal to small r. So this just becomes equal to rho u i u j by 4 R integral ds x i x j integral over the surface of x i times x j we know how to do is equal to a times delta ij and if you multiply both sides by delta ij finally, you will find that a is equal to 4 phi by 3. So you get 4 by 3 r power 4 times delta i j.

And finally, the result we got in the last lecture is equal to half times the added mass times u i square, where the added mass is equal to 2 by 3 pi R cubed rho, is equal to 2 by 3 pi R cubed times that density of the fluid. So 2 by 3 pi R cubed is half the volume of the sphere. So 2 by 3 pi R cubed times rho is half the volume of fluid displaced by the sphere. So therefore, the additional kinetic energy due to the fluid flow because when the sphere is moving.

There is a kinetic energy associated with the sphere itself but, there is also an additional kinetic energy associated with the flow of the fluid and this additional kinetic energy is equal to half the mass of fluid displayed by the sphere or it is equal to the density of the fluid times half the volume of the sphere. So this is a general result because even if you had a more complicated object you would have higher order terms in this expansion.

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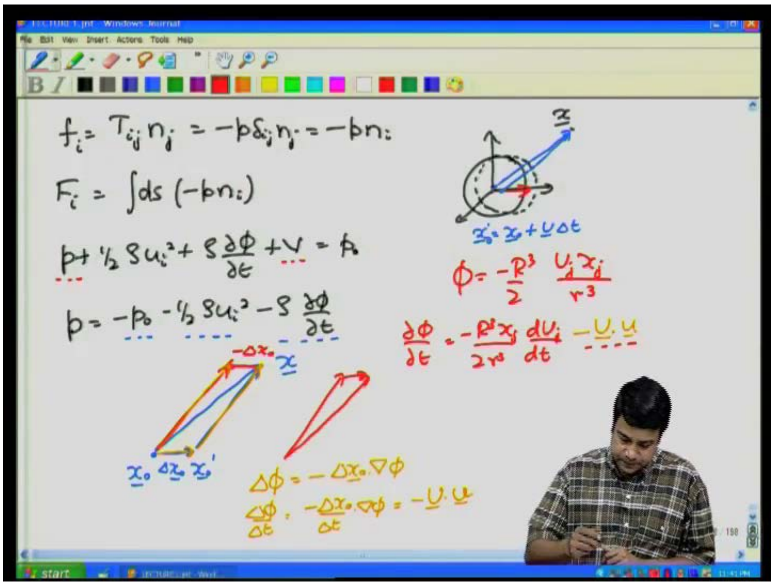

For the potential, however the leading term would still go as 1 over R cubed and if you do the calculation you will find that the kinetic energy is equal to added mass times half u square, where the added mass is some other fraction not half could be some other fraction of the volume of mass of that fluid displaced by the object. So that gives us the total kinetic energy of the flow.

What about the force exerted by the fluid on the sphere or the sphere on the fluid the equivalent of the drag force, that we had calculated previously for viscous flows. In other words to move this sphere within the fluid what is the force that needs to be exerted on the sphere? So let us look at that calculation the force calculation. So I have a sphere which is moving with a velocity U and I want to calculate the net force exerted by the fluid on the sphere and I have of course, a solution for the velocity field due to the for the potential as well as the velocity field.

So that potential solution in the velocity solution both satisfy the zero the normal velocity boundary conditions at the surface and I will just use that to calculate the total force. So how do I calculate the total force? I have to use the equation for the stress f is f i acting at a surface is equal to T ij n j the stress is purely due to pressure. So it is equal to minus p delta ij n j is equal to minus p n i. So this is the force acting per unit area on the surface the total force is calculated as integral over the surface minus p times the unit normal, the pressure of course, is given by the Bernoulli equation.

In this particular case we will neglect the body force since because we will assume there is no gravitational acceleration for the present as I showed you in my Bernoulli equation p plus half rho u i square plus rho partial phi by partial t plus V is equal to p naught. So provided the force is conservative I can combine these two terms to define a new pressure which I will call pi.

So these two terms can in general be combined, the potential plus the fluid pressure can be combined to get pressure which incorporates the body force terms, for this particular calculation we will assume for the present that the potential is identically equal to 0. So with that p is equal to minus p naught minus half rho u i square minus rho partial phi by partial t, a constant integrated so if, so there are three parts here, one is the constant this is the kinetic energy and this is the acceleration this is the acceleration term.

If I take this constant pressure and integrated over the surface I will get 0 because if I take p naught times n i and p not is a constant there is equal force is acting in all directions on the surface and therefore, the net force due to the pressure will end up being equal to 0 because there is a constant acting on all sides of the surface therefore, the net force exerted will be 0. We only non-zero contributions you will get are due to the velocity and the time derivative of the potential due to minus half u rho u square and rho times d phi by d t.

So the velocity of course, we have we know what the velocity is and we can integrate this out over the surface minus half rho u i square integrate over the surface which says that kinetic energy density integrate over that surface. What about the time derivative of the potential? Here is where one has to be careful in going through the calculation, the reason is as follows my sphere is moving with a constant velocity u. So I had got my solution for the potential as phi is equal to minus R cubed by 2 u j x j by r cubed of course, the velocity u could be depend upon time and therefore, you could get a time dependence in the equation for the pressure because d phi by d t there could be a contribution due to the velocity itself.

So I could have a contribution of the form d phi by d t is equal to minus R cubed x j by 2 r cubed times d u by d t. However if we had a steady velocity is the time derivative of the potential equal to 0 we had a steady velocity is the time derivative of the potential equal to 0 or if d u d t is equal to 0 does it mean that partial phi over with respect to t is equal to 0 turns out it is not.

And the reason is as follows, the reason it is not is because I have solved the problem in a coordinate system with origin fixed at the center of the sphere, my origin of the coordinate system was fixed at the center of the sphere and the sphere itself is moving with a constant velocity, this sphere itself is moving with a constant velocity. So therefore, if I look after sometime delta t if I look after sometime delta t this sphere would be at a new position after sometime delta t this sphere would be at a new position and the origin of the coordinate system would also be at a new position, it would at x is equal to x naught plus u delta t.

So the origin of the coordinate system is also moving when this sphere moves and I have calculated the potential in this moving coordinate system. What is partial phi by partial t? Partial phi by partial t is the potential the variation potential at a fixed observation if I look at one particular location x.

I call this is x naught right? I look at one particular location x and find out what is the difference in potential between the time t and the time t plus delta t divided by delta t, that is my partial phi by partial t at that location. If the sphere velocity is a constant is that non-zero? Of course,it is not because even though I am sitting at a particular location the sphere has moved therefore, the vector distance between the center of this sphere and the observation location has moved. Even though I am sitting at one particular location because my solution is in a coordinate system whose origin at the center of this sphere, the origin of the coordinate system has moved the observation point has not moved but, because the origin of the coordinate system has moved there is going to be a variation in the potential at their that fixed location.

So that second term has to be taken into account when we calculate the time derivative of the potential, when we are doing the calculation in a reference frame where there is a moving particle. How much is that difference in potential? That can be shown by a simple construction.

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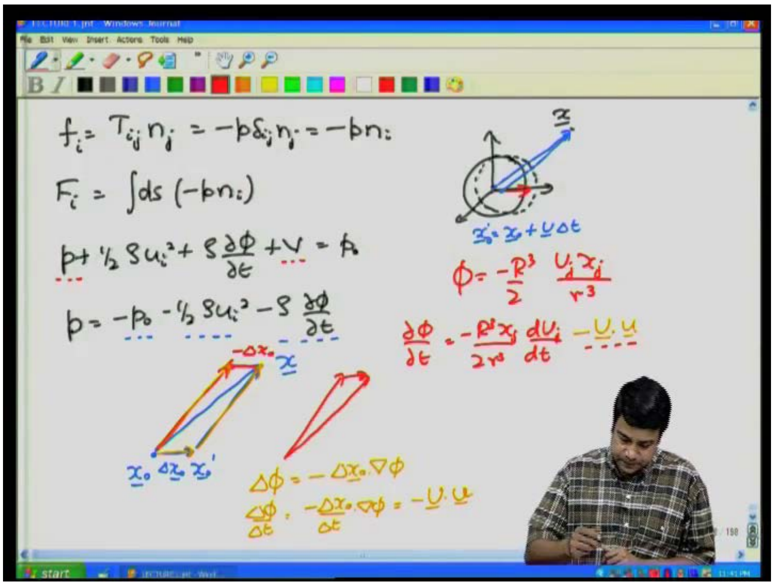

Let us say that my original location was x naught and this sphere moved to a new location x naught prime, where this displacement was delta x naught. My observation point is at the location x, so vector at initial time was is equal to this one, the radius vector after the time delta t when this origin of the coordinate system has moved a distance delta x naught is this one. So the change in radius vector is basically equal to the final minus the initial, the change in radius vector is equal to the final minus the initial, that is this is a change in the radius vector the final one minus the I should I should be careful here that is actually this one.

So this is the final and by initial had a larger angle of inclination. So my initial one had and something like this and this was the change in the radius vector, that same change in radius vector can be obtained. So I am getting a change in radius vector, if I move the point x naught by delta x naught keeping x stationary, that same change in radius vector can be obtained if I keep x naught stationary and move x by distance minus delta x naught.

That same change can be obtained if I keep x stationary and move the observation point by a distance minus delta x naught. You can see that this modified radius vector is identical to this one that I had. So what I get by moving the origin by a distance plus delta x naught is identical to what I get by moving the observation point by a distance minus delta x naught. So therefore, the change in potential is equal to the change in displacement times the gradient of the potential.

So what is the change in potential between this observation point and the observation point after I get after moving that distance, that change in potential is going to be equal to delta phi is equal to minus delta x naught the distance moved times grad phi, that is delta phi change in potential between these two locations is equal to the displacement which is minus delta x naught times grad phi that is taking place in a time delta t.

Therefore, the rate of change of potential which is delta phi by delta t is equal to minus delta x naught by delta t times grad phi, delta x naught by delta t is just the velocity of the sphere because it moves a distance this velocity this sphere moves a distance delta x naught. So delta x naught by delta t is just the velocity of the sphere. You note that this grad x so therefore, this becomes minus the velocity of the sphere and grad phi grad phi is just the fluid velocity because I said that the velocity can be written as the gradient of a potential.

So this becomes minus the sphere velocity dotted with the fluid velocity, this will be the additional contribution whenever you are working in a reference frame in which the origin of the reference frame is moving in time. So I get this additional contribution to partial phi by partial t minus U dot U or minus capital U i dotted with small u i. If you recall even when we did viscous flows we worked in a coordinate system which was fixed at the origin of the sphere in that case of course, the flow was quasi steady, you had neglected the time derivatives and therefore, we could always calculate the stresses because we are neglected the time derivative terms anywhere.

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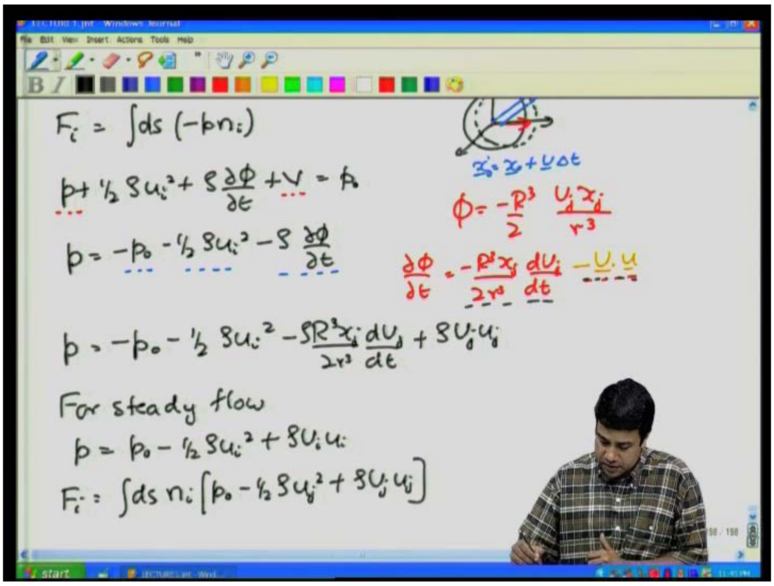

In this particular case there is a time derivative of the potential that enters into the calculation and when you have a time derivative in a reference frame that is moving, you have this additional term which comes in to the time derivative of the potential that has to be included. So therefore, if I calculate the pressure in this reference frame, calculate the pressure using my potential which was calculated in a moving reference frame I get minus half rho u i square minus R cubed x j by 2 I should have a density there plus rho U j times u j.

So this first term here is due to the variation of the velocity with respect to time, the second term was because the coordinate system the origin of the coordinate system is moving. So for a steady flow for a steady flow the pressure is just equal to p is equal to p naught minus half rho u i square plus rho U i times u i and of course, the net force is obtained as the F i is equal to integral ds of the unit normal times p naught minus half rho u j square plus rho.

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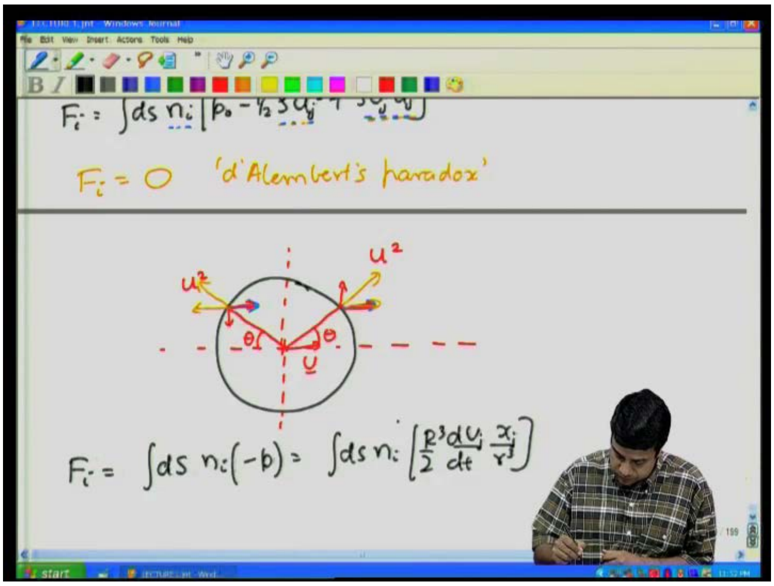

So this is the net force that is exerted on the sphere. Now with this expression of the net force of course, one can calculate of the net force by actually doing the integral by actually taking the solutions for the velocity of the sphere and doing this integral in order to find out what is a net force exerted on the sphere? However it is easy to show just using symmetries that this net force has to be equal to 0 the solution is as follows.

If I have a sphere if I have a sphere which is moving in the direction capital U the velocity the velocity of the flow around this sphere if I look at the velocity of the flow around the sphere, the velocity of this sphere it has to satisfy the normal velocity boundary condition. So; that means, this is the normal velocity of the sphere has to be equal to the normal velocity of the fluid at the surface.

And I will get a normal velocity which looks something like this, because the sphere is moving in this direction. The normal component of the velocity will be along the plus along the along the axis in this direction and along the axis in this direction. You do have a tangential velocity of course, but, you would expect based up on symmetry that tangential velocity is symmetric about this axis. So if I had a tangential velocity that looks something like this over here just based up on symmetry you would expect that the same thing is there on the other side as well.

So would expect a tangential velocity to come down along this axis and go up along that axis, the point is that the magnitudes of these two velocities have to be the same; that means, that if I take two positions which are at an angle theta here and the same angle theta here can I take two positions which are angle theta with respect to the plus velocity axis same angle theta with respect to the minus velocity axis u square on these two sides is exactly the same. So u square here is the same as u square here.

In addition u dot n is also the same because the normal velocity boundary condition, I required that small u dot n is equal to capital U dot n n i am sorry capital U dot n along the surface along, the front surface is the component this component along the front surface. So when we are on the rear surface the component is in the same direction. So small u dot capital U small u dot capital U is exactly the same on these two points on the surface.

So the velocity square is the same on these two points the velocity square is the same on these two points just drawn symmetry u dot n is also the same on these two points just drawn symmetry. If you look at the net force there are two components, one is along capital U here there is perpendicular to capital U. So let us look at the net force along the direction of capital U. So it is equal to half u square plus u dot u times the unique normal, this unit normal is the outward unit normal. So that outward unit normal at the point on the top stream side it is along this direction the downstream side it is along this direction.

The component of the unit normal along the u direction on the upstream side it is here, that is on the down streamside it is pointing backwards, the velocity square and u dot n are both the same; that means, that the pressure on those two sides is the same, unit normal is pointing in or the component of unit normal along u is pointing in opposite directions; that means, that at these two points unit normal times pressure is exactly equivalent magnitude in opposite in direction.

Therefore, the net force due to these two points is equal to 0, the same holds for any point on the circle at an angle theta on the sphere, there is an equivalent point which makes an angle with the downstream side that same angle theta, u square as well as u dot capital U are exactly the same on those two points. The component of the unit normal along the flow direction is opposite on those two therefore, the net force just one symmetry has to be equal to 0. So this basically tells us that for a sphere that is translating at a constant velocity in a fluid the net force is identically equal to 0.

The net force along the direction of u is equal to 0 and because the of the axis symmetry you can see that the net force perpendicular to u also has to be equal to 0 because the flows perfectly axis symmetric around this axis. So for a sphere moving at a constant velocity we find the result that the net force is equal to 0, it goes by the name of the d Alembert's paradox. The net force on a sphere which is moving at constant velocity has to be identically equal to 0, this is true not just for a sphere.

This is true for any object. I will show you that in the next lecture that for any object that you take in three dimensions the net force exerted by the object on the fluid at constant velocity in potential flow is equal to 0, the reason is because in the potential flow equations we have neglected the viscous terms. The viscous terms are responsible for energy dissipation and it is because you need to compensate for the energy dissipation that we need to do work in order to in order to move the object.

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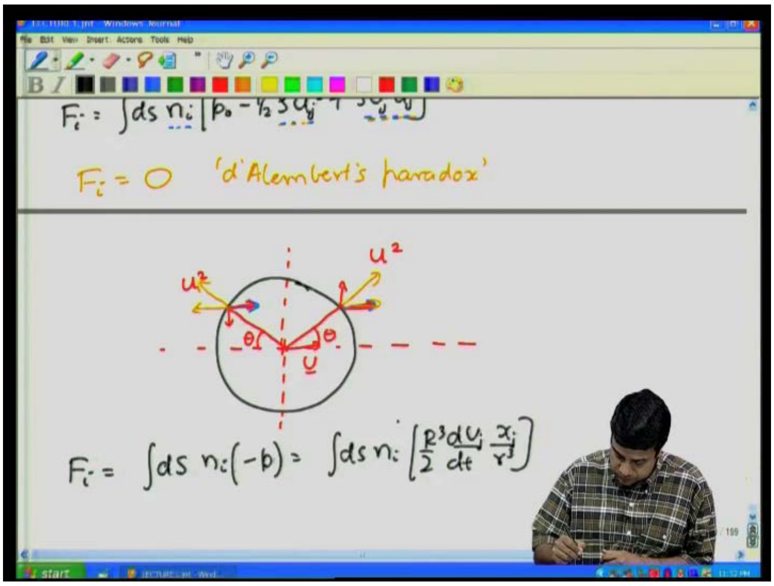

Because the work done due to the motion of the object and due to the force exerted on the object has to balance the dissipation of energy within the fluid. In potential flow we have neglected energy dissipation therefore, there should be no net work done for moving an object at constant velocity because we are moving at a constant velocity the kinetic energy remain of the fluid remains the same.

We have neglected viscous dissipation and if the kinetic energy remains the same; that means, that you do not need do work to move it at constant velocity of course, if the particle is accelerating you do need to work in order to accelerate the particle in the fluid and the work required for acceleration basically comes about by incorporating the time derivative term. So the time derivative term F i for doing for doing work due to acceleration is an integral over the surface of if you recall this is equal to the unit normal times the pressure which is minus rho d phi by d t for an accelerating object where d u d t is not is equal to 0.

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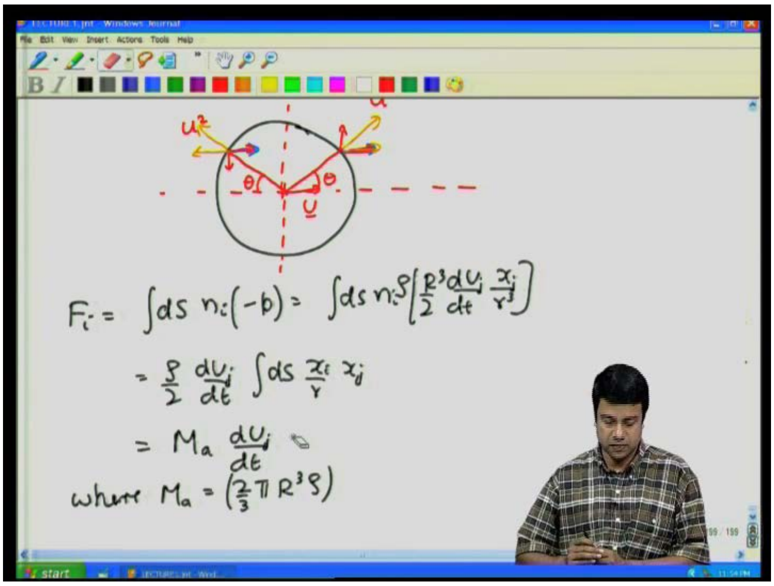

So if you recall for a steady flow we had neglected this term in the equation for the pressure and we showed that the net force due to all the other terms is identically equal to 0. I should remove the p naught here, there should be p is equal to p. We showed that the network due to all of these other terms is identically equal to 0 therefore, the net contribution that comes can come in only due to this particular term.

So this equal to minus p is equal to integral ds n i into R cubed by 2 d u j by d t x j by r cube times rho and once again you can do this integral quite easily. This is calculated on the surface of the sphere and therefore, this R cubed and this is calculated at r is equal to capital R. So this will become integral ds rho by 2 d u j by d t integral ds the unit normal on the surface is equal to x i by r times x j and once again we have integral over the surface of x i x j 4 by 3 pi r power 4 delta ij and I had put that n it is quite easy to see that this is just equal to the added mass times d U j by d t.

Where the added mass is exactly what we had got from our calculation of the kinetic energy 2 by 3 pi R cubed rho by just half the mass of the fluid that is displaced by this sphere. So therefore, the force for an accelerating sphere is equal to the added mass times the acceleration. The kinetic energy is the half added mass times velocity square. So it is all consistent with each other. So this added mass for a sphere is 2 by 3 pi R cubed rho one half of the mass of the fluid displaced by this sphere of course, for objects of other shapes will have a different added mass but, one can be guaranteed that the added mass due to obtained from the force exerted on the sphere as well as from the kinetic energy will end up being the same.

So for a sphere for a sphere moving in three dimensions the net force exerted by the sphere on the fluid is equal to 0. If it is accelerating then there is a net force which is equal to the added mass times the acceleration. What about for objects of other shapes? Is the added is the force required to be exerted by that object also 0 under conditions of steady flow. So that is the question that you will first take up in the next lecture.

We will try to show that this is a general result for an object of any shape moving in any direction under potential flow conditions the net force exerted by the object on the fluid. If the velocity is steady that net force has to be is equal to 0, both along the velocity direction as well as perpendicular to the velocity direction. Once you completed that we will go on to analyzing two dimensional potential flows, where we look only at variations in two directions, there are certain simplifications you can use there, you can use complex variables to get simple solutions for these equations. So that we will look at in the next lecture. So we will continue our discussion of potential flow in the next lecture. We will see you then.