Fundamentals of Transport Processes II
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Lecture - 25
Potential Flow

So, this is the lecture number 25 of our course on Fundamentals of Transport Processes. We were discussing Potential Flows high Reynolds number flows in-viscid in the rotational. So, if you recall we first discussed some integral theorems of vector calculus. So, that we could derive the conservation equations with out reference, to an underlying coordinate system. And then we derived the Navier Stokes equations, incompressible Navier Stokes equations for the mass conservation and momentum conservation.

So, one scalar equation for the mass conservation, plus three vector equations for momentum conservation. There four variables, the three components of the velocity and the pressure, and we looked at various boundary conditions that have to be specified for these equations. We then scaled the momentum conservation equation, got the dimensionless Reynolds number ratio of inertia and viscosity. And then we looked at, simplified cases in the limits of low and high Reynolds numbers.

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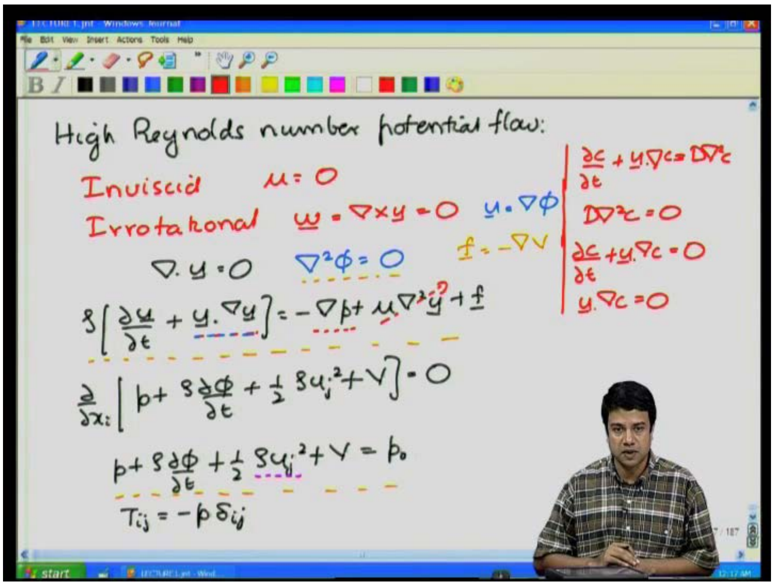

We have completed our discussion of the viscous, stokes flow solutions and then we had started discussion of the high Reynolds number potential flow solutions. So this is high

Reynolds number, potential flow. Two requirements for potential flow, one is in-viscid and the other is irrotational. Irrotational implies that the vertices which is del cross u is equal to 0 everywhere, with in the fluid. And in-viscid means we setting the viscosity equal to 0.

So, the original navies stokes equations for del dot u is equal to 0 and rho into partial u by partial t plus u dot grad u equal to minus grad p plus mu del square. Potential flow this term is being set equal to 0 and we could use the irrotationality condition to simplify this term. Since the flow is a rotational del cross u is equal to 0, I can always express the velocity as the gradient of a potential, I can always express the velocity as the gradient of a potential.

Then the mass conservation equation, now just becomes laplacian of the potential equal to 0 because, I get del dot grad phi is equal to 0, the divergence of the gradient of phi is equal to 0 or del square of the potential is equal to 0. And if you recall we could simplify this term in the momentum conservation equation from the condition that the flow is irrotational. And once I do that I get an equation of the form the gradient of I should have plus the force here, p plus rho partial phi by partial t plus half rho u j square plus any potential is equal to a constant is equal to 0.

The gradient of this entire function is equal to 0; that means that this function is equal to a constant. The gradient of this entire function is equal to 0 that means, that this function is equal to a constant. So, p plus rho d phi by d t plus is equal to this constant, note that originally my equation was a vector equation. This equation is actually a vector equation, where I have used here the fact that f vector is equal to minus the gradient of a potential. In in this simplification I have used the fact that f is conservative.

And therefore, it can be expressed as minus of the gradient of a potential. So, coming back to my earlier point momentum conservation equation originally was a vector equation. I reduced it to an equation which was basically the gradient of a scalar function is equal to 0, gradient is a vector; however, since the gradient is equal to 0; that means, that this function is equal to a constant. So, from this vector equation I have reduced it to just one equation here. The original momentum conservation equation had three equations for each for three components of the velocity whereas, my reduced equation is just a scalar equation.

I have two scalar equations laplacian of potential is equal to 0 and this equation for the pressure. Solved for two scalars of the potential and the pressure, the velocity of course, is given by the gradient of the potential. So, this is the simplification in the potential flow case, the stress tensor for the potential flow is just given by minus p times the identity tensor the stress tensor is just equal to minus p times the identity tensor there is no viscous part.

And as I have mentioned in the last lecture because, there is no viscous part, one cannot satisfy the tangential velocity or tangential stress boundary conditions at the surface. The tangential velocity boundary condition that is in order for the flow to come to a stop at the surface at a stationary surface, we require momentum diffusion from the surface. In our potential flow approximation we set the viscous term equal to 0 there is no momentum diffusion. Therefore, the velocity field does not decrease to 0 at the surface.

Therefore we have to solve these two equations subject to the normal velocity boundary condition and the normal stress boundary condition. As I had explained in the last lecture the potential is still linear in the velocity, because I have expressed u is equal to grad phi the pressure; however, is no longer linear in the velocity because proportional to half u square. We have a term that goes as half u square. So, because of that the pressure is no longer linear in the velocity, it is a non-linear function the velocity.

Just to go back and make a reference to our previous course we had looked at, transporter mostly heat and mass. In that case we have divided the regimes into two types. So the heat or mass conservation equation in that case will be of the form partial c by partial t plus u dot grad c is equal to d Del square c so, that would be the form of the mass conservation equation.

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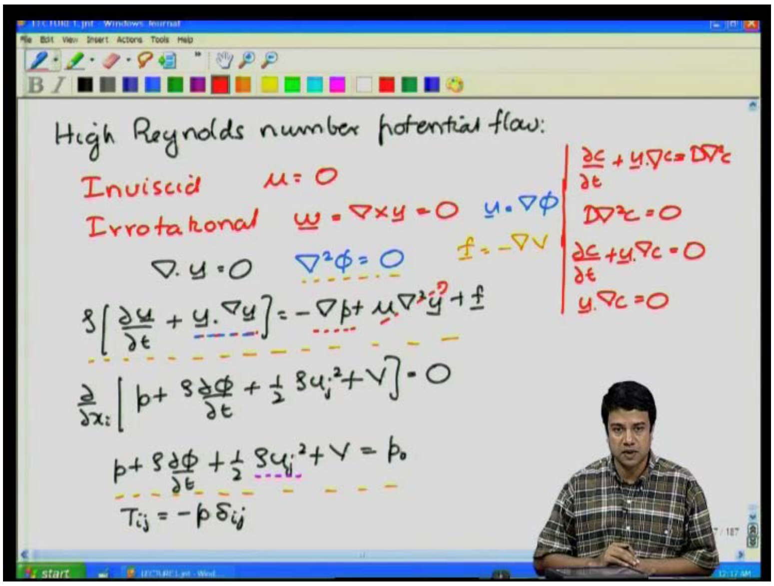

C is a scalar, is equal to d Del square c and there we had divided in to two different limits. One is the diffusion dominated regime where we, just solve d Del square c is equal to 0. One could also have a convection dominated regime in the high pickled number limit, in the low pickled limit of course you just solved Del square c is equal to 0. The high pickled number limit is convection dominated regime in which you solve this equation partial c by partial t plus u dot grad c is equal to 0.

And if the flow as steady the equation would just reduce to u dot grad c is equal to 0. The solution of this equation is basically that there is no variation in the concentration along the stream lines. So, if I am going along the stream lines the concentration is remaining a constant along that stream line so, in that case the solution in that case was trivial that the u dot grad c is equal to 0 basically says, that if I go along the velocity stream line. There is no variation in concentration along that stream line.

So, in that case there was really no problem to solve once, I new what the velocity field was I know exactly what the concentration is everywhere, just on the condition that there is no variation in velocity along the stream line. In this case of course, there is a more it is a more complicated system the reason is because, I have this additional fluid pressure. Here in the case of mass and heat transfer there is no equivalent of the fluid pressure, which tends to maintain a constant density through out the flow.

In the case of fluid mechanics, there is a fluid pressure, which maintains a constant density throughout the flow. Therefore, I get a non-trivial result for the relationship between the pressure and the velocity. So, we solving the same convection dominated regime except that we have this additional fluid pressure here, and that makes all the difference. That is why you have this potential flow regime in fluid mechanics we do not have equivalent problems in heat and mass transfer because of the fluid pressure.

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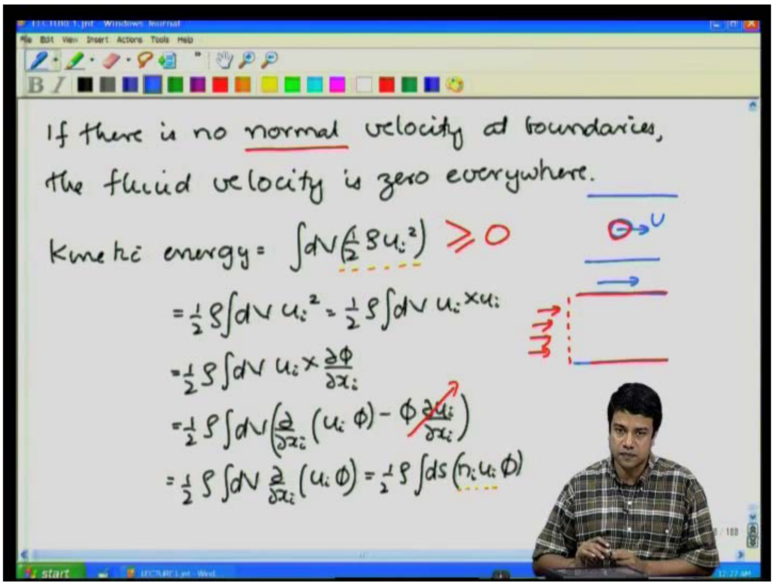

For this potential flow solution we can show exactly various things. The simplest thing to show is that, there is a non zero fluid velocity with in the fluid, only if there is a non zero normal velocity at the boundaries of the fluid. Alternatively one can show that if, there is no normal at boundaries the fluid velocity is zero everywhere, It should emphasize that there is no normal velocity at the boundaries. So, if at all bounding surfaces the normal velocity is identically equal to 0 then the fluid velocity everywhere, with in the fluid has to be equal to 0. This is in contrast to for example; a case of viscous flows where for example, in the flow through the channel example. We could have a velocity that is non zero I am sorry this is we we could have, the velocity that is non zero even though the normal velocity of the surface is equal to 0.

In this particular case the bottom plate is stationary the top plate is moving with a velocity u but, that velocity u is tangential to the surface, it is not normal. So, there is no normal velocity of any of the surfaces but, despite that there is a fluid velocity in the fluid. The reason is because there can be diffusion of momentum perpendicular to the flow direction and that causes the fluid to flow.

In the case of a potential flow we are neglecting this momentum diffusion and because of that, unless there is a normal velocity there cannot be any fluid flow. This is an exactly result and I will briefly show you how that consider. The total kinetic energy is the integral over the entire volume of the kinetic energy per unit volume. The kinetic energy is mass times the velocity square the local kinetic energy per unit volume is mass per volume, density, times the velocity square.

Is equal to half rho u i square. This is always a positive quantity, it is positive semi definite, it is always greater than or equal to 0 when the density is positive in u square is a positive number. Since the density is a constant I can write this as half rho integral d v times u i square. This equal to half rho integral d v u i into u i, u i (( )) u i square for one of the u i's i express the velocity in terms of the gradient of the potential. So, I will write this as half rho integral d v u i times partial phi by, partial x i.

Integrate this by parts integrate this by parts I will get half rho integral d v partial by, partial x i of u i phi minus phi partial u i by partial x i. Integration by parts, we know that the divergence with the velocity the second term on the right hand side has to be equal to 0 from the incompressibility condition. The second term in the right hand side is the divergence of the velocity so, this has to be equal to 0 just by incompressibility. So, that is equal to half rho integral over the volume of partial by partial x i of u i phi.

So, it is the divergence of u times potential integrate over the entire volume, can be reduced to a surface integral of the unit normal times the vector. So, integral of a divergence of a vector over the volume is equal to the integral of the unit volume times the vector over all the surfaces that are bounding this volume. Integral over the surfaces of n i u i phi so, this is a surface integral of u dot n the velocity dotted with the normal to the surface times the potential.

So, clearly if u dot n is equal to 0, this integral is equal to 0 this integral is equal to zero if u dot n is equal to 0. That is there is no normal velocity to the surface at any point, then this integral is equal to 0 and u dot n is equal to 0. If u dot n is equal to 0 the kinetic energy is equal to 0 because, the kinetic energy is half rho integral of u dot n times potential. And if the kinetic energy is 0 we can see from here the kinetic energy is equal to 0 the velocity has to be 0 at each point with in the fluid there is, some where the velocity is non-zero. Whether it is positive or negative the kinetic energy will always be positive is equal to is proportional to the square of the velocity.

Therefore, this says that unless if u dot n is equal to 0 in all bounding surfaces the total kinetic energy is 0 and therefore, the velocity is 0 everywhere with in the fluid. That is one has to have a non-zero normal velocity at the surfaces; that are bounding the flow in order to have for the fluid to be moving if the normal velocity is 0 at each and every point within the flow. That means, that the velocity with in the fluid is also zero everywhere the system is stationary.

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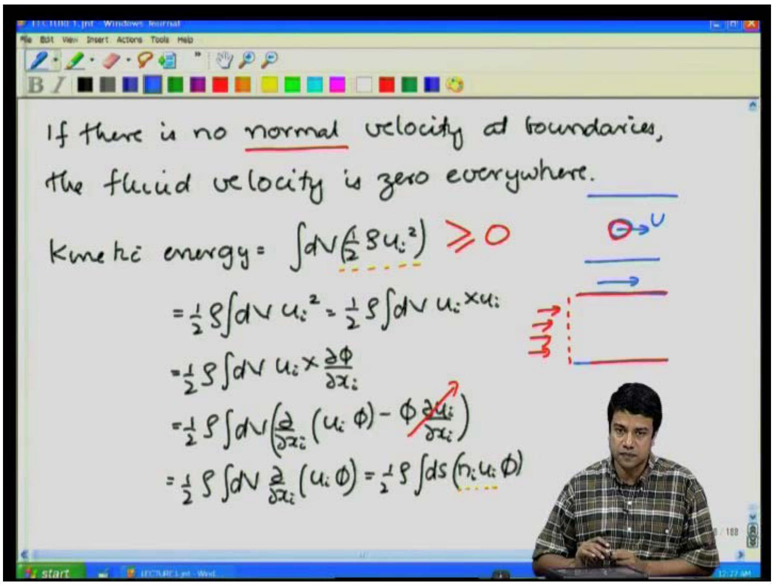

This normal velocity at bounding surfaces can be of various kinds you could have in the case of external flows. For example, you could have a system that the velocity in the walls is equal to 0 but, there is some object that is moving with the constant velocity u. That means that the normal velocity on this surface normal velocity on this surface is non-zero, one could also have flows where there is an in let of fluid. For example, the normal velocity at these surfaces the top and bottom surfaces is equal to 0.

However one could have velocity coming in at an open surface it has velocity coming in at an open surface. So, either you have to have an object moving within the fluid such that the velocity perpendicular to the surface is non-zero. What do you have to have some external source of fluid coming in at at an open surface, only in that case will you have a non-zero velocity acting on the within the fluid; there is no normal velocity everywhere surfaces may be moving tangential.

You could in general have a tangential velocity at a surface like this. That will not cause flow with in the fluid in potential flow, because you have neglected momentum diffusion. One could derive other relationships when we looked at viscous flow I said the equations are linear and because the equations are linear the solutions are unique the solutions exist and they are unique.

That was because the stokes flow was flows are linear the velocity, the pressure is linear in the velocity, the stresses are both linear in the velocity. In this case the pressure is a non-linear function of the velocity but, still it is possible to show that the solution is unique for potential flow. So, we will briefly look at the uniqueness of the solution.

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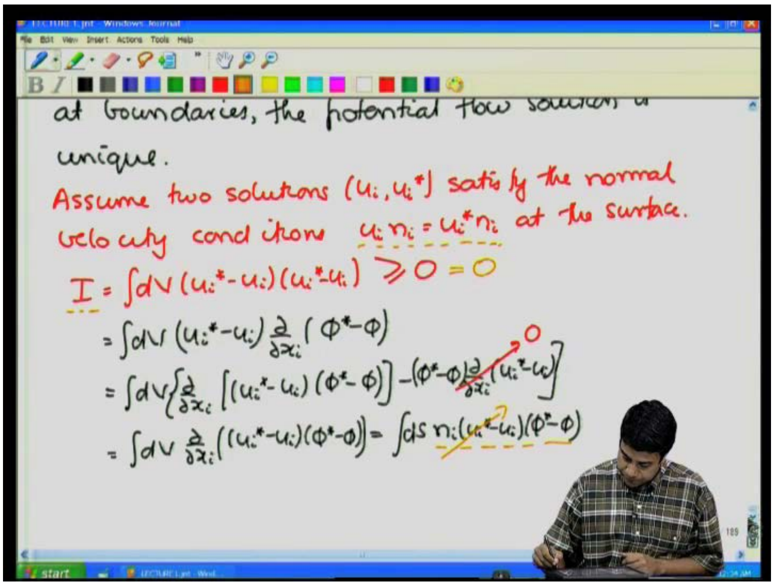

Uniqueness implies that for a given set of normal velocities at the boundaries, there is only one potential flow solution. For we calls this for specified at boundaries let me just say potential flow solution is unique the potential flow solution is unique for a given specified normal velocity boundary conditions at all boundaries. So, in order to prove this we will first postulate the opposite we will first assume there are two solutions.

Satisfy assume there are two solutions of we call it as the normal velocity conditions at the surface. So ,we will first postulate that there are two possible solutions both of which satisfy this normal velocity condition and then we will show, that both of these solutions turn out to be identical to each other. So, if you look at the if you look at the integral, integral over the volume of u i star minus u i into u i star minus u it his is the integral of a square of some number integrated over the volume.

This always has to be greater than or equal to 0. There are two potential flow solutions both of which are satisfying the same boundary conditions it the integral over the volume of u i star minus u i the whole square. This integral has to be greater than or equal to 0, if this integral is equal to 0; that means, that u i star is equal to u i everywhere. Because, the square of a number is equal to 0; that means, that number the integral of a square of some function is equal to 0 means that that function is equal to 0 everywhere.

Both of these are potential flow solutions so, therefore I can write this as integral d v times for the second velocity, I will write it as the gradient of a potential. The second velocity the second u i star minus u i, I will write it as the gradient of a potential write it as the gradient of a potential. Once again integrate by parts once again integrate by parts the second integral I have the divergence of u i star minus u i.

Both of these velocities satisfy the potential flow condition mass conservation requires that the divergence of both of these is equal to 0. So, this one goes to 0 and therefore, I get this is equal to integral d v of partial by partial x i of u i star minus u i. The divergence of a vector integrated over a volume is equal to the unit normal times that vector integrate over the surface. The unit normal times that vector integrate over the surface so, this is what is equal to the original function that I had i is equal to this one.

We had postulated two differential velocities profile, velocity profiles u i star and u i. Both of which satisfy the same normal velocity condition at the surface, both of which satisfy the same normal velocity boundary condition at the surface.

This is shown you that this integral u i star minus u i the whole square integrate over the volume is going to be equal to the unit normal times u i star minus u i that is equal to 0. Simply because both velocity satisfy the same normal velocity boundary condition therefore, n dot u star is equal to n dot u therefore, this is equal to 0 therefore, I get the condition that phi is equal to 0. Provided both velocity profiles satisfy the potential flow condition; however, if this integral is equal to 0 it implies that u star is equal to u everywhere with in the flow; that means, that the potential flow solution is unique.

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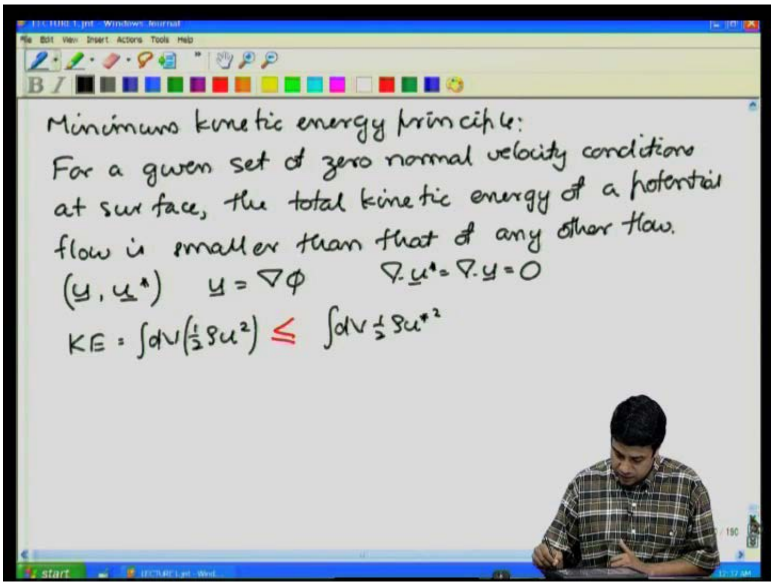

So, for a given set of boundary conditions, normal velocity boundary conditions one can show that the potential flow solution is always unique. One can also show what is called the minimum kinetic energy principle but, the minimum kinetic energy principle takes as this follows, for a given set of conditions at surfaces the kinetic energy the total kinetic energy of a potential flow is smaller than that of any other flow. So, let us say that we have two solutions for the velocity u and u star we have, two solutions for the velocity out of which the velocity u can be written as a gradient of a potential u star not necessary.

So, could be any solution, it could be a solution which has some rotational weight for example, u star is not necessarily a solution of the potential flow equations; however, it has to satisfy the incompressibility condition. So therefore, I require that del dot u star is equal to del dot u is equal to 0. Both velocity profiles satisfy the incompressibility condition but, the velocity u in addition satisfies the potential flow equations.

For this one can show that the kinetic energy which is integral d v of half rho u square for the potential flow is always smaller than the kinetic energy for the general flow. That is the minimum kinetic energy principle that is the kinetic energy for a potential flow is always smaller than the kinetic energy for a general velocity profile. So, in order to prove this what we do is, we take the difference between the two kinetic energy for the general velocity profile minus minus that for the potential flow is equal to half rho integral d v of u i star square minus u i square.

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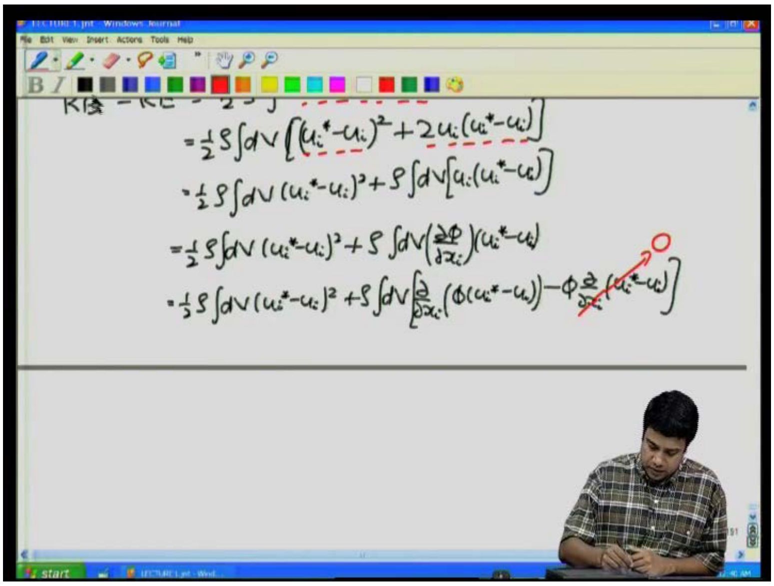

So, have to prove that this is always a positive number. We can simplify this as half rho integral d v of two terms u i star minus u i the whole square plus two u i into u i star minus u i. We just expand out this term u i star minus 2 u i u i star plus u i square plus add the second term two u i u i star minus two u i square. I will get back this original equation so far, so good. So, this first term here is always positive this first term here is always positive what about the second term what about the second term.

So, I will write this as half rho integral d v u i star minus u i the whole square plus rho integral d v u i into u i star minus u i, and in the second expression we know that the u i can be written as the gradient of a potential. The velocity u can be written as the gradient of a potential, Rho integral d v of u i is the solution of the potential flow equations. So, I can write u i is the gradient of a potential minus u i and by know it by now it will be familiar what we are about to do we going to integrate this by parts.

So, this will be equal to half rho integral d v plus rho when integrate this out by parts in the second integral on the right. The second term contains the divergence of u i star minus u i; however, we postulated with the both u i star and u i or both satisfy the mass conservation equation therefore, the divergence of the velocity turns out to be equal to 0. I am just left with half rho integral d v of u i star minus u i the whole square plus rho.

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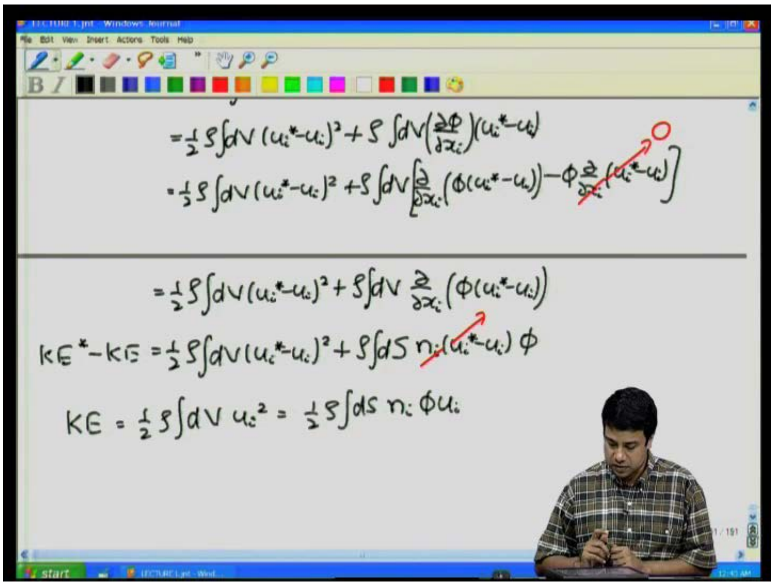

And integral of the over the volume of the divergence of some vectors equal to integral over the surface of a unit normal times that vector. Therefore, I get half rho plus n dot u n dot u star minus u times the potential; however, it said that both the velocities the potential flow solution u as well as, the other solution which does not satisfy the potential flow condition u star both of them satisfy the same normal velocity boundary conditions at all surfaces.

Both the velocity is u star and u satisfy the same normal velocity boundary condition at both surfaces. Therefore, this difference has to be equal to 0 n dot u star minus u at the surfaces should be a surface integral. The integral at the surfaces has to be equal to 0; that means, that the difference k e star minus k e has to be a positive number the difference in kinetic energy between the flow which did not, necessarily satisfy the potential flow conditions and the flow that did satisfy the potential flow conditions.

The difference has to be positive; that means that the potential flow solution has the lowest kinetic energy of any solution. That satisfies the incompressibility condition as well as the normal velocity condition at all boundaries. So, this basically tells us that the kinetic energy of a potential flow has to be minimum. Note that the kinetic energy for the flow can be expressed purely in terms of the surface integrals them selves.

As I showed you previously kinetic energy of the flow is equal to half rho integral over the volume of u i square is equal to half rho integral over the surface of n i phi times u i, n dot u times the potential. That means, that in order to find out the kinetic energy of a potential flow one does not need to know what the velocities everywhere, with in the fluid. It is sufficient to know what is the potential only at the bounding surfaces the potential plus the velocity.

The velocity of course, is the gradient of the potential. So, long as you know the potential and it is gradient at the bounding surfaces you know exactly what the kinetic energy is for the entire flow. So, that is one advantage of dealing with potential flows. So, next let us solve the simplest problem, the simplest potential flow problem that is the flow around a sphere.

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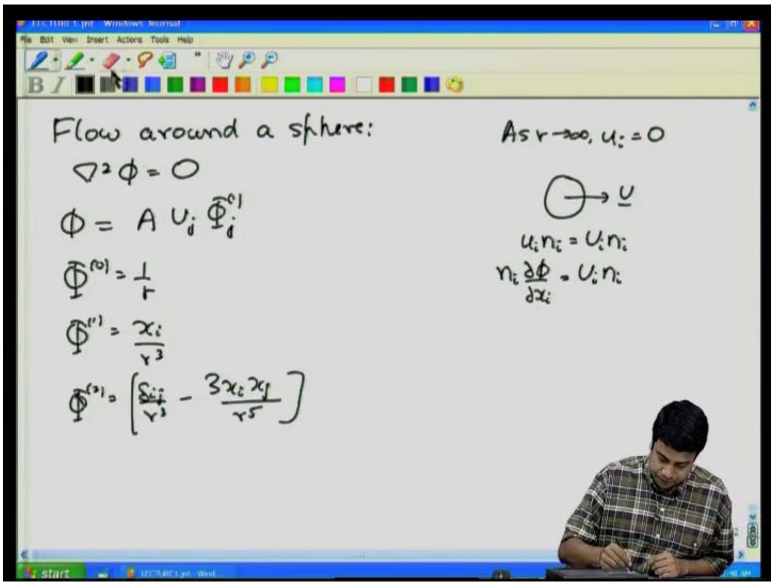

Potential flow solution for the flow around a sphere, I have a sphere that is moving with some velocity u vector. And I want to know what is the velocity field around this sphere. So, I have to solve for the velocity potential del square phi is equal to 0 the laplace of the velocity potential is equal to 0. With the normal velocity boundary conditions that is, u i n i is equal to capital U i note that we cannot satisfy the tangential velocity boundary conditions in this particular case because, you have neglected the viscous stresses.

And therefore, it is not possible to satisfy the tangential velocity boundary conditions we can satisfy only the normal velocity boundary conditions. So, to solve del square phi is equal to 0 with this boundary condition. So, this boundary condition basically says that the unit normal times the gradient of a potential, is equal to some fixed value. Clearly the potential is a linear function of capital u it is a linear function of capital u because when I take its gradient dotted with the unit normal that has to be equal to capital U dot n.

And it self is a unit normal on the surface. So, if the potential is linear function of capital u and it satisfies laplace equations the velocity on the surface of the sphere is equal to u dot n of course, since the sphere is in an infinite medium the velocity far away has to go to the has to be 0. So, velocity far away has to be equal to 0 the velocity at the surface of this sphere it self satisfies the normal velocity boundary conditions, that means that the potential is a linear function of the velocity of the sphere itself, whichever access the velocity of the sphere is the potential will be a in generally a linear function of this access. We know the solutions for the laplace equations growing harmonics and decaying harmonics. In this particular case, we can not have growing harmonics because the velocity has to go to zero far away, the only solutions are the decaying solutions for the velocity profile.

Now for the decaying solutions Ii require that phi has to be some linear combination of these fundamental solution the first harmonic and so, on. So, you get all these multiple solutions the first one is a scalar, the next one is a vector, third one is a second order tensor and so on. Has to be some linear combination of these harmonics has to be linear in one of these harmonics some of linear function of these harmonics and it has to be linear in the sphere velocity.

Because I said because of the no normal velocity boundary conditions the normal velocity boundary conditions require that the potential has to be linear in the velocity capital U. The potential is a scalar, so there is only one way that you can get a solution which is linear in the vector u and in one of the spherical harmonics. Because, only one way that you can get a solution; that is linear in capital U and in one of the spherical harmonics.

And that is just some constant into u j times the vector. This is the only combination we should be linear in the velocity u as well as, linear in one of the spherical harmonics and it will be a scalar solution. So, this the only possible solution for the potential function of course, this satisfies Laplace equation there is an undetermined constant here a which has to be determined from the boundary conditions u dot n is equal to capital u dot n.

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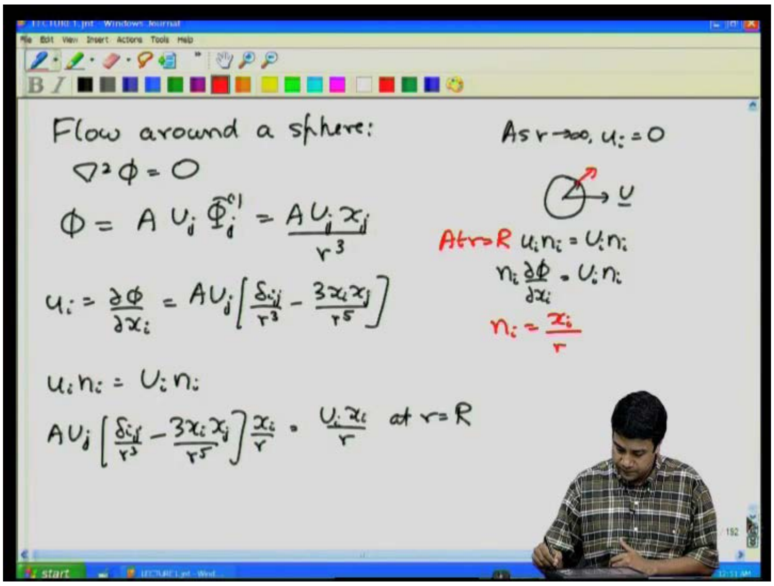

So this solution is going to be equal to A U j x j by r cubed. The velocity is equal to partial phi by partial x i so, we know how to take the gradients of this fundamental solution and the higher harmonics. So, that is the solution of the velocity where a is the undetermined constant, the boundary conditions state that u i n i is equal to capital U i times n i.

On the surface of the sphere since i fixed my origin at the centre of the sphere I fixed my origin at the centre of the sphere on the surface of this sphere the unit normal on the surface of the sphere the unit normal is in the same direction as the position vector. I fix my origin at the centre of the sphere therefore, on the surface of the sphere the unit normal is in the same direction as the position vector.

That means that the unit normal is equal to the position vector divided by it is magnitude. Is equal to the position vector divided by it is magnitude. So, that is n i therefore, I have A U j into delta i j by r cubed minus three x i x j by r power five times x i by r is equal to u i x i by r. This is on the surface of the sphere at the location at r is equal to the radius of this sphere. So, let us put at r is equal to r here where capital R is the radius of this sphere.

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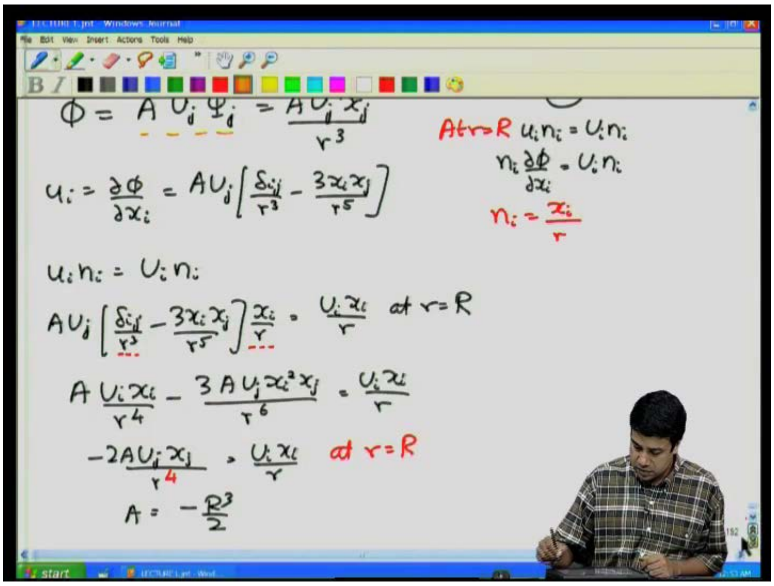

I can multiply these A U j times delta A j gives U i. So, I get A U i x i by r cubed minus three A U j x i square x j by R power five is equal to U i x i by r. x i square which is x one square plus x two square plus x three is equal to R square. And U j x j is U dot x U i x i is also U dot x. So, these are all the same things so, I will get minus two U j x j by r cubed is equal to U i x i by r there is an a here.

One second this is not power four and r power six I should note that I am multiplying throughout by this x i by r x i by r here. So, when I get one over r cubed times x i by r i get one over r power six so this. so this Gives me the value of a therefore, if this is two at r is equal to capital R; that means, that a is equal to minus R cubed by two. So, that gives the be the constant zero normal velocity boundary conditions gives me the constant a in the expression for the potential here.

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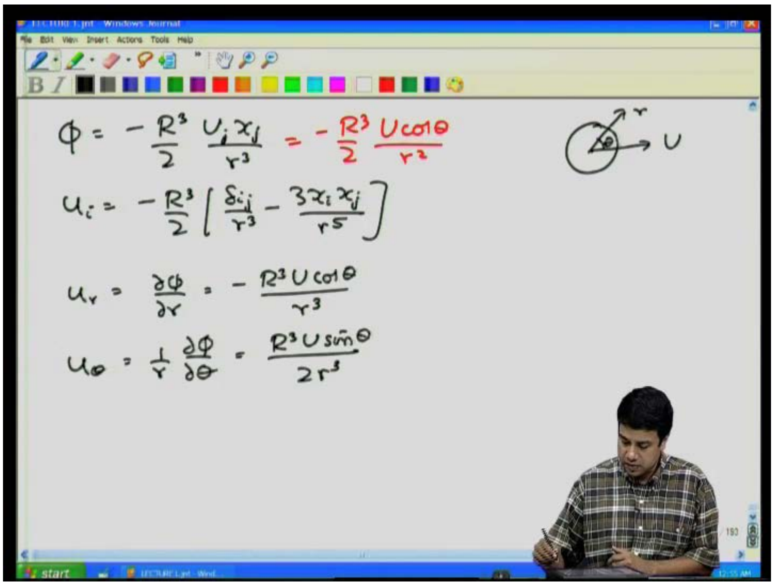

So, my final expression for the potential becomes phi is equal to minus R cubed by 2 U j x j by r cubed. So, that is the potential and the velocity u i is equal to minus R cubed by 2 into delta i j by r cubed minus three x i x j by r power 5. So, those are the potential and the velocity fields, this can be written quite easily in a coordinate system in the r theta coordinate system. If I take U as the axis this is r and this angle is theta U j times h j just u r cos theta.

Because U dot x is equal to U times, r times the cos of the angle between the two. So, this just becomes equal to minus R cubed by 2 U cos theta by r square, and from this I can get the derivatives the velocities in the r and theta directions u r is equal to partial phi by partial r which will be equal to minus R cubed U cos theta by r cubed. And u theta is equal to 1 by r partial phi by partial theta is equal to by 2 R cube. So, these are the two components of the velocity profile u r and u theta for this particular case, next we can evaluate the kinetic energy of the fluid, of the fluid that is moving around the sphere.

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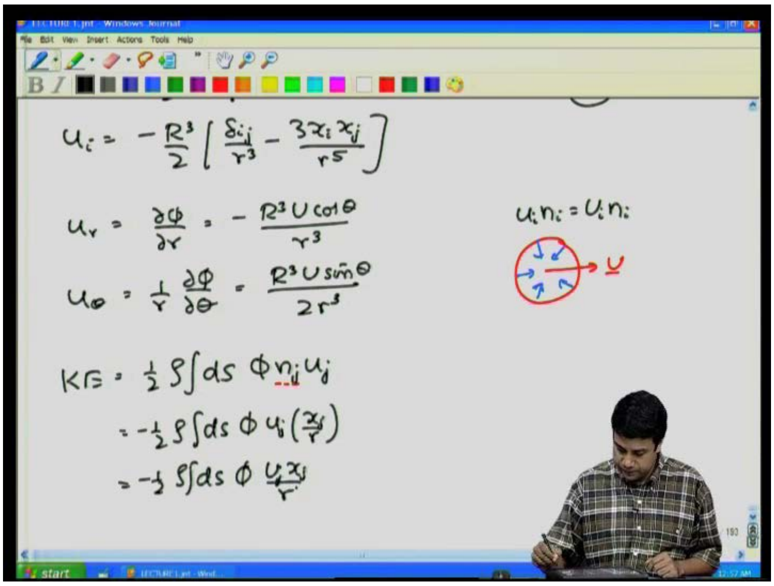

One that the first question that we usually addresses the drag force that requires the calculation of the pressure around the surface of the sphere. So, we will do that in the next lecture but, before we do that I will just briefly here calculate the potential energy. The potential energy is equal to as I said in this particular case this will be integral d v half rho integral over the surface of.

If you recall I just derived for you that the potential energy can be reduced to just an integral over the surface half rho integral over the surface d s of phi n j u j. So, in this, this n is basically to the out ward unit normal the fluid the outward unit normal to the fluid if this n is the outward unit normal to the fluid in our derivation. Because, we integrate over the fluid volume, integral over the divergence of a vector over that fluid volume is equal to the integral of the unit normal times u the outward unit normal over the surface.

In this particular case I have a sphere that is moving with some velocity u the outward unit normal to the fluid actually acts into this sphere. You have to denote normal to the fluid is acting into this sphere therefore; this is equal to minus x j by r on the surface. Minus half rho integral d s phi u j into x j by r; however, we know that small u dot n on the surface is equal to capital U dot n in the surface because I have u i n i is equal to capital U i n i on the surface. So, I can also write this as minus half rho integral d s phi u j x j by r.

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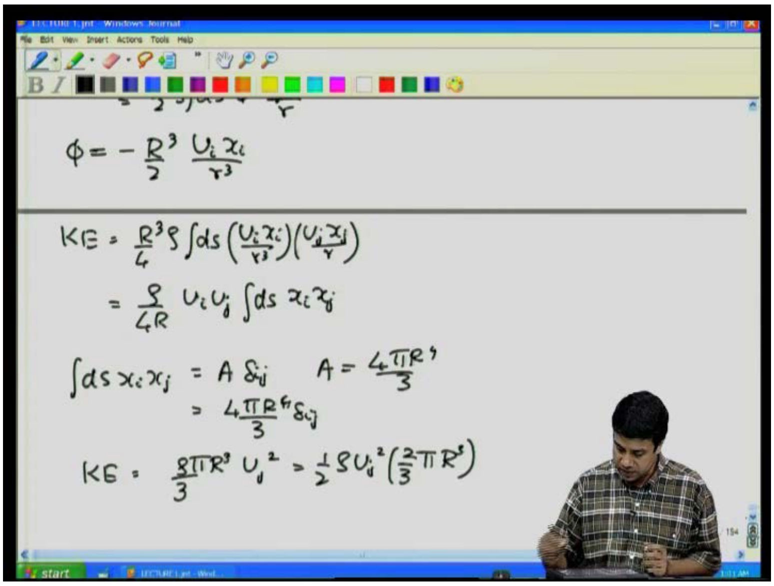

And for this phi I substitute phi is equal to minus R cube by two U j x j by r. We should use another index here by r cubed therefore, the kinetic energy becomes R cube by four times rho integral d s of u i x i by r cubed into u j x j by r. This is integrated over the surface at r is equal to capital R this integral is over the surface at r is equal to capital R. And I can set r is equal to capital R here to get rho by four capital R integral over the surface and once I calculate this integral.

We her we have already calculated this integral, integral over the surface of x i x j is equal to a delta i j multiplied both sides by delta i j and I will get a is equal to four R power four by three four r power four by three is what you get for k. So, this equal to four R power four by three delta i j, with this kinetic energy becomes rho into rho by three R cubed. This becomes four pi pi r cubed u square.

Is equal to half rho u j square into 2 by 3 pi R cubed half rho u square into two by three pi r cubed, rho into 2 by three pi r cubed, rho is the fluid density 2 by 3 pi r cubed is half the volume of the sphere. Because, 4 by 3 pi r cube is the volume of the sphere. So, rho into 2 by 3 pi r cubed is one half of the volume displaced by the sphere, 2 by 3 pi r cubes is one half the volume of the sphere.

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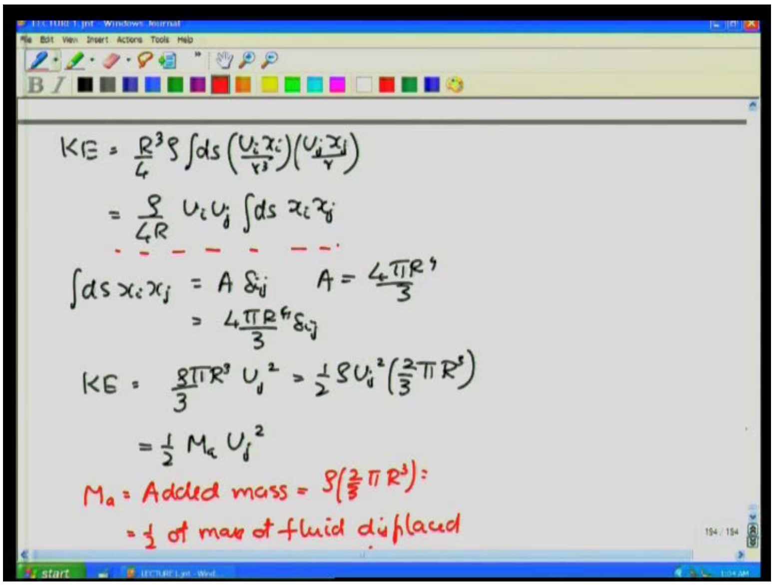

That multiplied by the fluid density rho gives me one half of the mass of the fluid displaced by this sphere. So, therefore, this kinetic energy I can write it as one half M a times u square, m a is what is called the added mass. Note that this kinetic energy is the kinetic energy of the fluid that is flowing around the sphere not the kinetic energy of the sphere itself kinetic energy of the sphere itself will be equal to half times the mass of the sphere times velocity square.

In addition because the fluid is because the particle is moving fluid around it there is could be an additional kinetic energy for the fluid flow around that sphere. That kinetic energy is equal to half m a times u j square, where this M a is equal to the added mass this added mass is rho times 2 by 3 pi r cubed, rho into 2 by 3 pi r cubed is one half of the mass of fluid displaced by the sphere. One half density of fluid times the volume of the sphere.

Therefore, this added mass is equal to one half of mass of fluid displaced by the sphere. So, that is the added mass it is one half of mass of fluid displaced by the sphere. So, in addition to the kinetic energy of the sphere itself there is an additional kinetic energy because there is fluid that is moving around the sphere. Kinetic energy of that fluid moving around the sphere has to be proportional to the density of the fluid of course, the total kinetic energy is equal to half the velocity of the sphere square times one of the mass the fluid displaced by the sphere of the added mass.

So, for this particular sphere moving in a fluid we got the exact result that the added mass is equal to one half of the mass of fluid displaced by the sphere. If we had some other more complicated shape of an object moving through a fluid then the added mass would not be one half it could be some other fraction. But since in potential flow one can always reduce the kinetic energy it was surface integral.

One can always find that the added mass is proportional to the density of the fluid times some fraction or some, some number times the mass of fluid displaced by this sphere. So, this relation just comes out of the fact that the I have a exact result for the potential. And the kinetic energy is just related to a surface integral, in the case of other more complicated shapes see in this case I got u j x j by r cubed as the solution. Potential decreases 1 over r square because, proportional to u j times x j by r cubed therefore, the velocity possess 1 over r cubed. Solution of the potential is a, dipole solution because; there is one over r square, if had a more complicated shape that have other higher order terms.

But, the leading order terms still be the 1 over r square term in contrast to viscous flow where we had a solution proportional to 1 over r. And because of this I get a solution for the kinetic energy as an integral over the surface, and from that I have evaluated the added mass. Next lecture we will continue this we will look at the pressure the force exerted the pressure on the surface and the net force exerted by the sphere on the fluid both under steady motion, as well as when it is accelerated.

As we expect for an accelerating sphere the force will be equal to the added mass times the acceleration of the sphere. Just as kinetic energy is equal to half added mass times the velocity square, we will continue this in the next lecture, we will see with them.