Fundamentals of Transport Processes II
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Lecture - 24
Inertia of Low Reynolds Number

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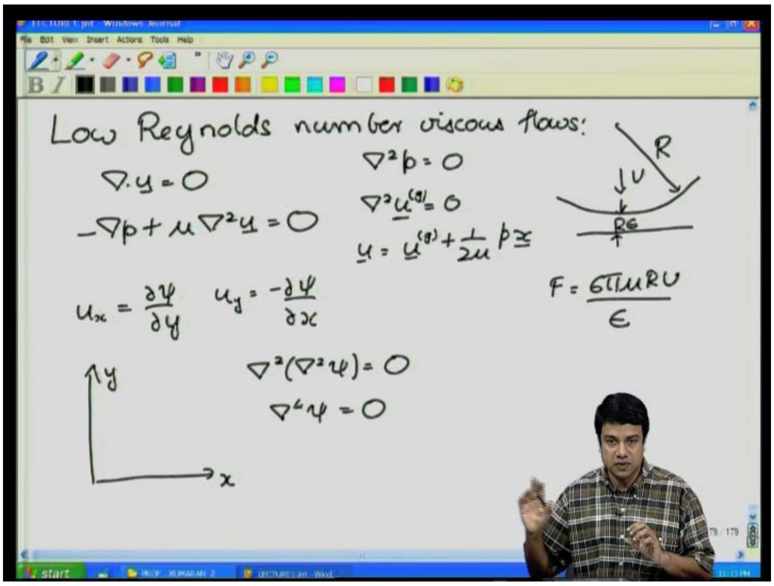

So, welcome to lecture number 24 of our course on fundamentals of transport processes, we were just about finishing up on our discussion of viscous flows in the low Reynolds number, limit is a low Reynolds number viscous flows. And as I told to the governing equations or the stokes equations, the divergence of velocity is equal to 0, mass conservation incompressible fluid, and the momentum conservation equation. So, this were the governing equations, and if we had solved it in various situations both for external flows outside an object, I showed you how we can reduce this stokes equations to two Laplace equations for the general part of the velocity profile and for the pressures and so on. And the total velocity is equal to the general part plus 1 by 2 mu p times the position vector in the coordinate system, and we looked at various interpretations of this.

The solution, because particularly easy when you are solving around a spherical object, because there are no other vectors in the system. Therefore, the solution has to be linear in the velocity of the object and in one of the spherical harmonic solutions, we looked at how to derive spherical harmonic solutions? Vector, tensor solutions just by taking the gradients of the fundamental solution, the fundamental solution is just 1 over R in two dimensions; you can try to work it out yourself you will find that in two dimensions, the fundamental solution is log of R in three dimension, the fundamental solution is 1 over R, and by taking repeated gradients you can get all vector, tensor, and any higher order tensor solution for the equations. And just from linearity and reversibility, we know that the solution has to be linear in one of those scalar, vector, tensor solutions and linear in the velocity of the object itself.

And on this basis one can construct solutions with satisfied boundary conditions, and use these to determine the force on an object, that is moving in a viscous flow a force on a sphere was of course, given by stocks law. In that case there was a net force on the particle, and the velocity decade as 1 over R far away, so we have a net source the decay is proportional to 1 over R.

We also construct two other situations, where either we have a rotational flow far away or an extensional flow far away, in those two cases there was no net force, however there was a net torque or a net force moment. The velocity field due to these decade as 1 over R square, and we saw how to construct the solutions due to these force moments, where the symmetric and the anti symmetric force moments. Resulting in respectively, a net symmetric force moment is due to an extensional flow far away, a symmetric traceless rate of deformation tensor. The anti symmetric force moment is due to the rotational flow far away, and in both those cases the solution decay is 1 over R square, these decays are general in the sense at their applicable to any form of the any shape of object.

There will be higher order terms of course, which we have neglected but, the slowest decaying terms, if there is a net force in the object has to goes as 1 over R, there is no net force, it has to goes as 1 over R square, if there is net force moment and so on. We also saw, how to reduce the stokes equation in two dimensions to a bi-harmonic equation? so in that case, if you have a two dimensional flow in the x y Cartesian coordinate system. u x you can write it as in terms of the stream function, and the resultant mass conservation equation is trivially satisfied, when the velocities are expressed in terms of the stream function. The momentum conservation equation when I express the velocity in this terms of the stream function, gives me the bi-harmonic expansion del square del square psi is equal to 0, or alternatively it is called the bi-harmonic equation.

The forth order derivative acting on side psi is equal to 0, and we also saw how to solve this equation for the particular case, for the flow around a corner. And finally, we looked at the flow and between surfaces that are nearly in contact with each other, the specific problem that we solved was the flow was the velocity of a sphere settling in a fluid, for the case where the radius of the sphere was much larger than the distance between two surfaces. And in this case we used scaling principles, because this is a nearly unidirectional flow, the distance between the surfaces is small compared to the lateral extent. And therefore, the flow is nearly unidirectional, at any point if you do the scaling analysis, you will find that the velocity profile is close to a parabolic profile. And using that one can find out, what is the pressure difference between the between the centre of this gap? and the ambient pressure and that difference basically drives the flow.

From that you find out, what is the force acting on this object? and it turns out to be 6 pi mu R U by epsilon, so the force increases proportional to 1 over epsilon, as the distance between the two surfaces becomes smaller and smaller. So, this in summary is what we done so for in stokes flow, there is one further little bit that we need to do, and that is when we solved the stokes flow equations. We assumed that the convective effects could be neglected everywhere in the fluid, that is diffusion is dominant, diffusion is dominant means that the the momentum defuses instantaneously throughout the entire domain.

However, the domain is infinite in extent, if you take for example, the flow around the spherical object can the momentum really defuse everywhere instantaneously, when the domain itself is infinite in extent. Is there going to be some distance beyond which inertial effects will become important, even though the Reynolds number is very small, so let us look at that question, can I really consider the diffusion is instantaneous throughout the domain, even when the domain is very large. In other words, if I have a sphere in a very large volume of fluid, and I push the sphere some distance, I will apply a force on this sphere, that is going to be a distempers to the velocity field.

Stokes equations are quasi study that means that the disturbance to the velocity field depends only upon the velocity, instantaneous velocity of the sphere. However, the information cannot really travel instantaneously, if I change the velocity of the sphere, I cannot expect that even very very far away, there is should be a velocity disturbance which have instantaneously coinciding with the velocity disturbance on the sphere.

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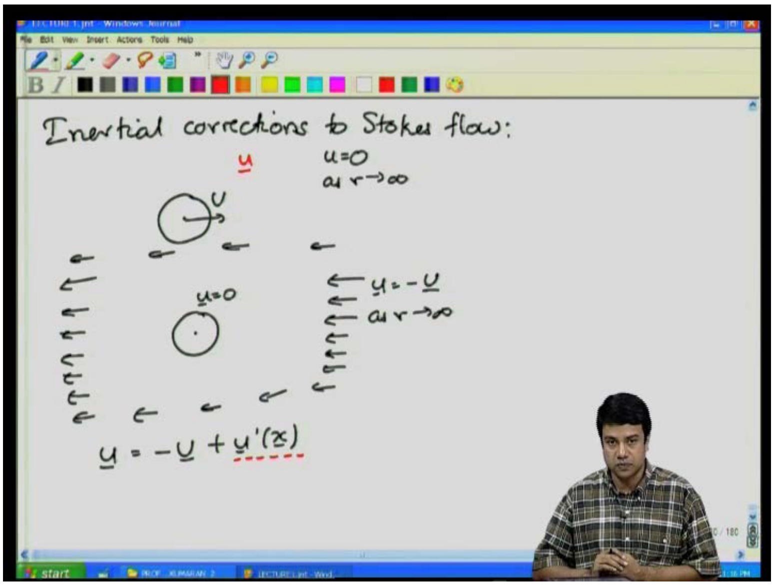

So, let us look at that question, what are the inertial corrections to stokes flow? so, far what we been doing is to consider a sphere moving with a velocity u in some direction, in a fluid where the velocity is equal to 0, as r goes to infinity. Where I fix my origin at the center of the sphere, so that is the problem that we make solving so far, what I prefer to do for discussing inertial corrections, because when you when there are inertial corrections the quasi study approximation no longer is valid. Therefore, I cannot just sit on the surface of the sphere, and assume that nothing changes outside, we will come back to that when we discuss potential flow in the next lecture.

But for the present rather than having the sphere moving with a velocity u, and the fluid stationary far away, I had prefer to discuss a system where the sphere, the velocity of the sphere is equal to 0 on the surface. And the fluid is coming in with a velocity minus u far away, r goes to infinity, u is equal to minus u, as r goes to infinity, minus u far away from the sphere at all points. So, there is a uniform velocity far away from the sphere, the sphere of course is stationary therefore, one would expect there is a velocity disturbance in the vicinity of the sphere. The sphere is stationery, and in order to keep it stationery, you have to exert a force on the sphere, because there is a net force exerted on the fluid, there will be a velocity disturbance due to the sphere.

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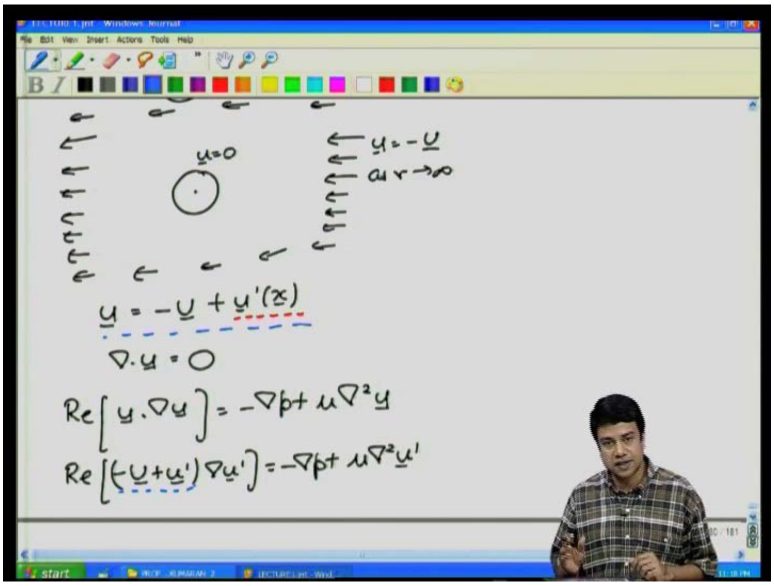

So therefore, the velocity field I can write it as, u vector is equal to minus u velocity far away plus a disturbance due to the presence of the sphere, u prime; u prime itself is function of of x, u vector is a function of x, capital U is a constant, so the velocity with which the fluid is moving far away from the sphere. So, now this configuration exactly equivalent to the sphere moving in the fluid stationery, but it is easier to analyze this particular problem. Because, this is just Galilean transform, it involves just translating the entire equipment with a velocity minus u, the forces that are exerted should be identical, and the velocity u prime that I have here.

The velocity u prime that I have here, there the velocity disturbance should be identical to the velocity u, that I have in this configuration, because in this in the top configuration the fluid velocity goes to 0 far away. Therefore, the velocity disturbance is just u, in the bottom configuration the fluid is minus u far away and therefore, the velocity disturbances u prime, and the two have to be equal. So, let us go back to the stokes equations now, what happens when the Reynolds number is small? but, not 0, my equations become del dot u is equal to 0, a Reynolds number times. In this bottom configuration, it is a steady configurations, so the, so the time derivative of the velocity disturbance has to be equal to 0. Note that capital u itself is independent of positioning time, small u prime use a function of position but, since the configuration is steady, it is independent of time.

So, I will get R e into u dot grad u is equal to minus grad p plus mu del square u, and I substitute for u from this expression in terms of minus u plus u prime, note that the gradients of capital U are 0. Therefore, any term that contains the gradient of capital U has to be 0 into grad u prime, so that is the equation when expressed in terms of the velocity disturbance. Note that this conductive term has both the steady velocity far away as well as the velocity disturbance. Now, let us estimate the various terms in this equation, as I said the velocity u prime is identical to the velocity, that you have in the fluid for the case, where the sphere is moving and the fluid is stationery.

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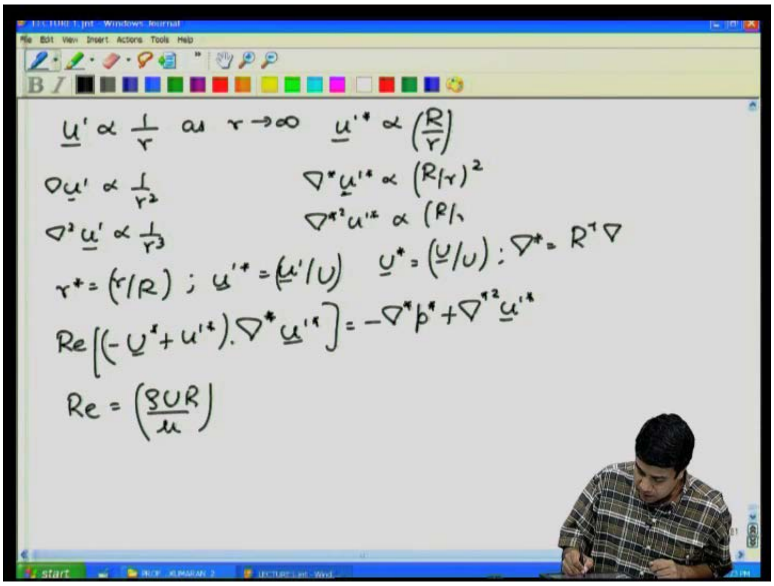

So therefore, since there is a net force exerted by the sphere on the fluid, one would expect that the velocity u prime is proportional to 1 over r, as r goes to infinity, net force of the sphere on the fluid implies, that the velocity has been proportional to 1 over r, because there is a net source of momentum. If you take I should get rid of the mu here, let me just let me write in terms of dimensional equations, u prime is proportional to 1 over r, as r goes to infinity. What about the gradients of u prime? Every time you take a gradient take the derivate of 1 over r, it goes as the next higher power of r, 1 over the next higher power of r. That means that grad u prime should be proportional to 1 over r square, and del square u prime should be proportional to 1 over r cubed.

So, now I have the governing equation, now I will define my scaled coordinates ok. As r star is equal to r by capital R, and u prime star is equal to u prime by capital U, where u is the velocity of the with which the fluid is coming in far away from the sphere. And you can easily verify, once you do this scaling, that the net result that you will get is R e into minus u star plus u prime star dot is equal to minus, where the Reynolds number R e is equal to rho u r by mu. So, that is the Reynolds number and of course, I have defined the scaled u star vectors equal to u vector divided by the scalar magnitude of its velocity u. And the divergence del star is equal to r inverse times the gradient, because capital R is the only length scale in the problem the which is the radius of the sphere, so that is how I have defined the gradient the scaled gradient.

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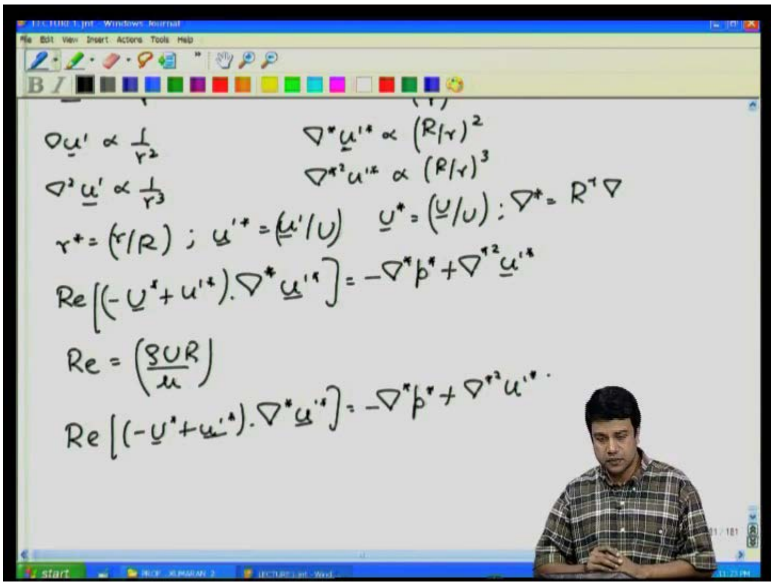

So, in terms of scaled variables u prime vector has to be proportional to 1 over r, what that means is that u prime star has to be proportional to R by r, the u prime star is scaled by capital U. Therefore, it has been proportional to capital R by r is dimensionless, if I take one gradient, if I take one gradient is proportional to R by r the whole square and two gradients proportional to R by r the whole cubed. Now, let us just put in these dependences, let us just put in these dependences in the equation, so I get the first term R e into minus U star plus u prime star dot, so that is the equation, and I will put in the dependences here. This u prime star is of course, independent of the radius r, because it is just a constant velocity.

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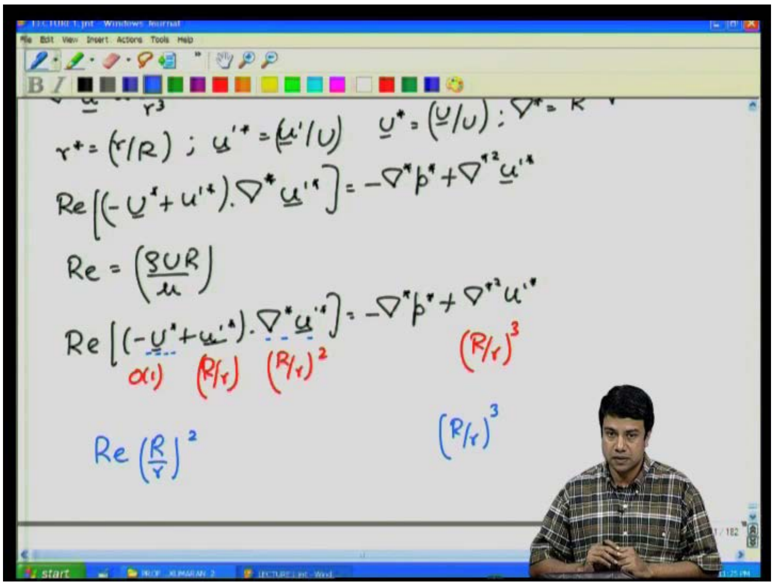

So, this is order 1, it is independent of the radius r, u prime star is proportional to capital R over small r, u prime star is proportional to capital R over small r. So, this is over small r and I have the gradient here, which is R by r the whole square, then there is a gradient of the pressure plus the second derivative of the disturbance to the velocity field. And this goes as R by r the whole cubed, the assumption in the stokes flow equations is that the Reynolds number is small, that the Reynolds number is small.

However, you can see that the largest term on the left hand side, the largest term on the left hand side is proportional to the Reynolds number times u prime star multiplied by the gradient of u star. So, the Reynolds number times R by r the whole square, the largest term on the right hand side at least as far as the velocity gradients are concerned, goes as R by r the whole cubed. So, clearly even when the Reynolds number is small, the term on the left hand side becomes comparable to the term on the right hand side.

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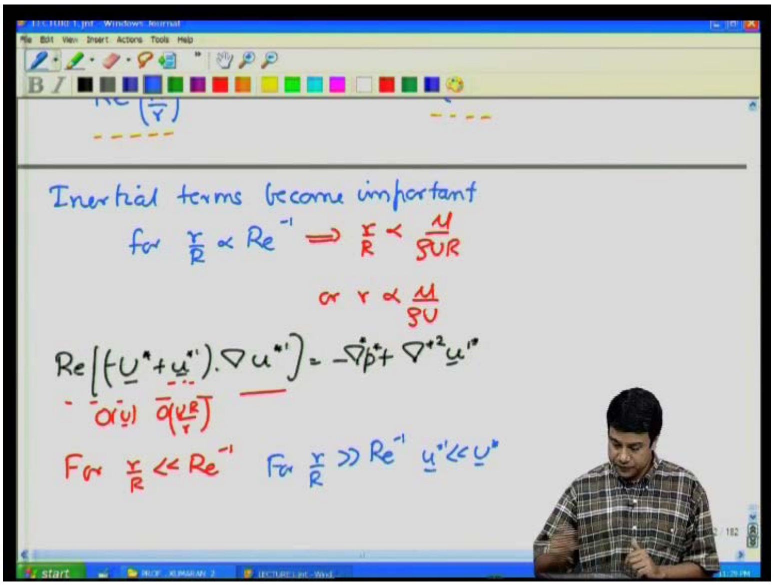

Inertial terms become important for r by R is proportional to R e inverse, so when r by R small r by capital R is proportional to R e inverse, you can see that this term on the left hand side is of the same magnitude as the term on the right hand side, R e inverse inverse of the Reynolds number. So, there is going to come some radius, some radial distance from the object at which the inertial and the viscous terms are going to become comparable to each other. That radial distance is proportional to inverse of the Reynolds number, so that distance becomes larger and larger, as the Reynolds number becomes smaller and smaller.

So, this simple calculations states that even when the Reynolds number is very small, there has to be some large radius at which inertial terms have to become comparable to viscous terms, r by R is proportional to R e inverse. If I write that in terms of the Reynolds number, I get r by R is proportional to mu by rho U R or r is proportional to mu by rho U ok. So this distance at which the inertial terms become of the same magnitude as the viscous terms, does not depend up on the radius of the object itself, it depends only upon the velocity u, as well as the density of the viscosity of the fluid. And once, you go to that sufficiently large distance, it is no longer sufficient to solve only the viscous terms the conservation equation, because the inertial terms the conservation equation have become comparable to the viscous terms.

So, clearly if I have to solve for this particular problem, if I have to solve the complete equations, the equation becomes R e into minus u plus u prime dot grad u prime is equal to minus grad p plus mu. Now, I have to solve this complete equation, if I am sufficiently far away however, one can make a simplification because, this minus u this is always going to be order u, this thing is always going to be order u. Whereas, this term here goes as 1 over r, this terms here goes as 1 over r, we are sufficiently near the object the Reynolds number, if your if the distance for r by R smaller than R e inverse, I can neglect this entire term, I can neglect the entire inertial term of the left for r by R e is much smaller than R e inverse.

On the other hand for r by R larger than R e inverse, I cannot neglect this inertial term, because a inertial term is comparable or larger than the viscous term in that case. However, within the inertial term I have two velocities minus capital U plus u prime, and an r becomes large u prime goes as 1 over r whereas, capital U itself remains independent of distance, because it is a constant it is a velocity of the fluid far away. Therefore, in that case I can neglect u star prime is small compared to u star, so I can neglect u star prime in comparison to u star.

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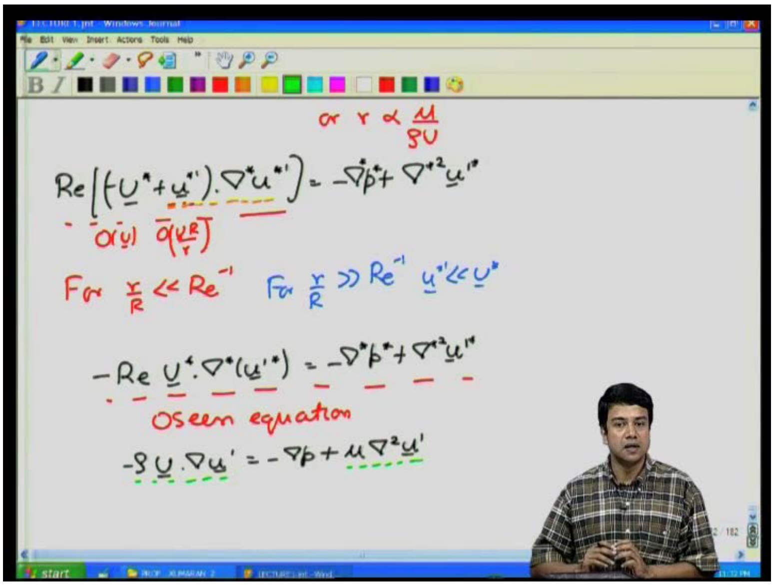

And again a simplified equation for the velocity disturbance, we get a simplified equation for the velocity disturbance, note that I have neglected one term, and that is the disturbance to the velocity field u prime ok in the inertial term. The rational is that when r is small entire inertial term is small anyway, so I can neglected, and r is large u star prime becomes small compared to capital U therefore, it can be neglected. So, this is the simplified equation, it is called the Oseen equation. This includes the effect of inertia far away, while neglecting the velocity disturbance far away.

Because, that is small compared to the study velocity of the which the entire fluid is moving, this equation can be solved the solution procedure is rather complicated. So, the entire oseen equation in dimensional form is given by, rho u dot grad u prime plus mu del square u prime for the disturbance velocity profile. This is still a linear equation, this is still a linear equation that is because I have neglected the non-linear term, which goes as u prime dot grad u prime in this equation here. Hence, I neglected that non-linear term I get still a linear equation, however it is now no longer reversible, the reason is because I have u dot grad u prime.

So, if I want to interchange, if I want to reverse the direction of the velocity everywhere, on the right hand side the velocity the the term on the right hand side is linear in the velocity. And so it reverses whereas, the left hand side term is not, so in this is case it is no longer reversible flows, it contains it is it is now irreversible, that is if you interchange the direction of the velocity within the fluid. I am sorry, if you interchange the velocity at the boundaries, the velocity everywhere within the fluid no longer reverses.

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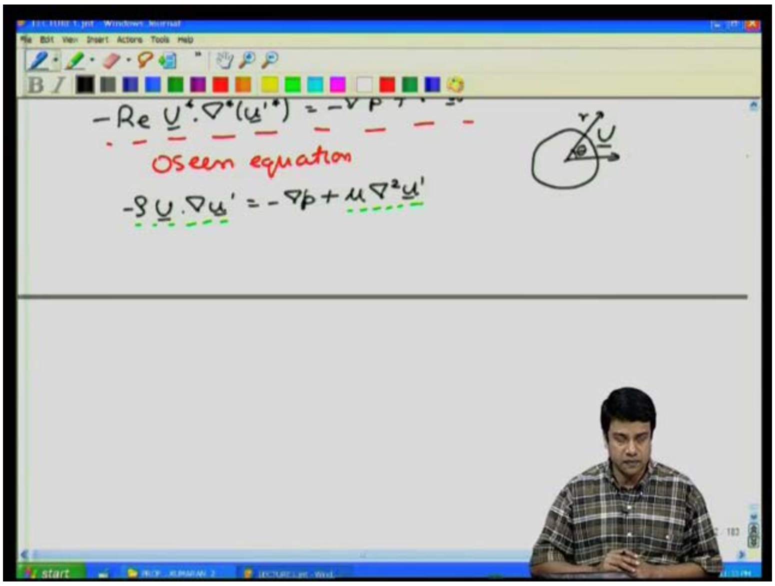

However, it can be solved. The solution is a little complicated mathematically, so I will just give you briefly, how it is done in this case the configuration is axis symmetric, because I have symmetry around the direction of motion of the sphere. The sphere is moving with a velocity u around that axes parallel to u, the configuration is axis symmetric, so one can reduce it so there is no dependence of the angle, the meridional angle going around this velocity direction through the center of the sphere. Therefore, one can solve this in an axis in an axis symmetric coordinate system, in which the coordinates are just the distance r and the angle theta. There is no dependence on the angle around the the, there is no dependence of the angle meridional angle phi around the axis parallel to the velocity vector.

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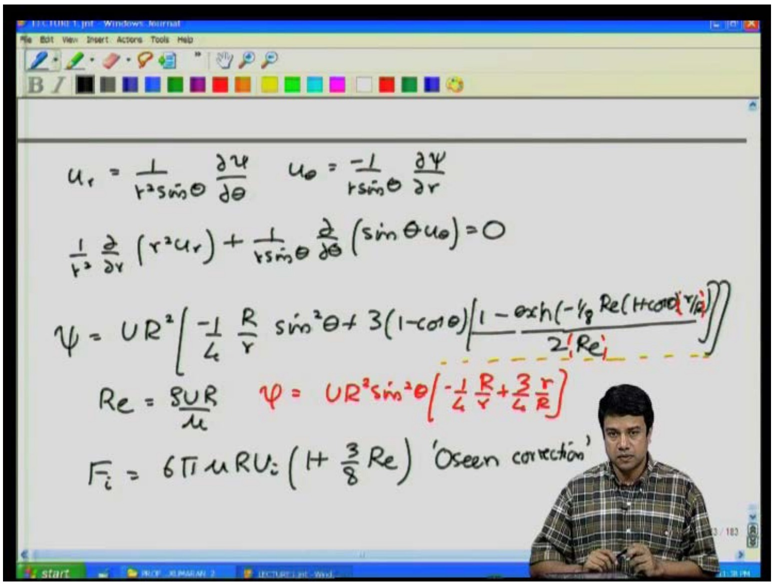

So, in that case it is possible to find a stream function formulation, if I write u r is equal to 1 by r square sin theta partial psi by partial theta; and u theta is equal to minus 1 by r sin theta partial psi by partial r. This will identically satisfy the axi symmetric mass conservation equation that is 1 by r square d by d r of r square u r, so this equation is identically satisfied with this formulation of u r and u theta in terms of the string function. These are inserted in two the conservation equations as usual and you will end up with a bi-harmonic equation, which has to be solved analytically. The solution is not easy to derive but, it is quite easy to verify in terms of the solution is of the form, psi is equal to u R square into minus 1 by 4 R by r sin square theta plus 3 1 minus cos theta 1 minus e power minus 1 by 8 R e into 1 plus cos theta r by R divided by 2 R e.

So, that is the final solution that you get, where R e is equal to rho U R by mu, you can very easily, easily verify that very near the surface of the sphere itself, it will give back the solution that we had earlier. In that case, you are taking the limit where r by R is very much smaller than R e inverse, note that in the exponential I have an r by R here, and R e here. If r by R is much smaller than r e inverse, then the term in the exponential will turn out to be independent of R e, because when I expand this out Taylor’s expansion, I will get a term that is independent of R e.

And the stream function for that case, will just turn out will be equal to U R square sin square theta into this terms are to be the identical solution, that we get for the flow around a sphere. However, the this additional term here results in a correction to the drag law to the drag force in the sphere, there is a connection to the drag force in the sphere due to this additional term. And this correction has been calculated exactly is equal to 6 pi mu R U i 1 plus 3 by 8 times R e, and this is called the Oseen correction, the Oseen correction to the drag force on its sphere due to inertial terms, due to the inertial contribution far away. So, this is the leading order correction in the limit of small Reynolds number, so the summary of the effect of inertia at low Reynolds number is that inertia can be neglected.

If you are sufficiently near the surface of the sphere, if small r by capital R is less than R e inverse in the limit as R e goes to 0, once you go beyond that inertial terms do become comparable to the viscous terms. And you can no longer neglect the inertial terms, the conservation equation however, even though you cannot neglect the inertial terms, you can still reduce it to a linear equation. The oseen equation for the velocity profile, this is still linear in the velocity u prime but, it is no longer reversible, because it contains capital u dot grad of u prime. So, it is no longer reversible but, it is still linear that can be solved, and because of these inertial effects you get a correction to the drag force on the sphere, which goes as 1 plus 3 by 8 times R e. So, that is the summary of the effect of inertia at low Reynolds number, so this completes our discussion of low Reynolds number flows.

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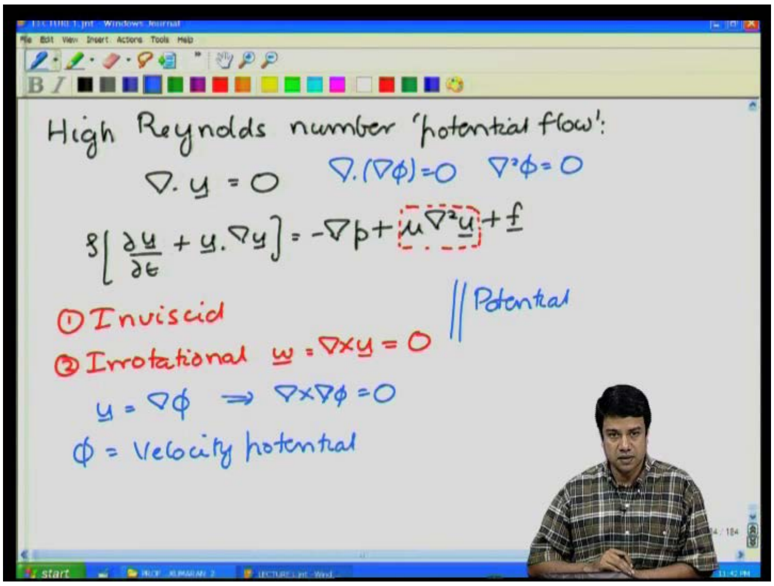

Next, we will go on to flows in the limit of high Reynolds numbers, high Reynolds number what we will call as potential flows, high Reynolds number potential flow. The equations once again the Negus stokes equations del dot u is equal to 0, and rho into partial u by partial t plus u dot grad u is equal to minus grad p plus mu del square u plus anybody force that is exerted. So, that is the mass and momentum conservation equations, in the case of potential flows the assumption is that, the viscous if exhausts small compared to inertial effects. Because, we are in the limit of high Reynolds numbers therefore, the viscous effects can be neglected in comparison to inertial effects.

That is I can neglect this term in the conservation equation in comparison to the inertial terms, that is one approximation Inviscid, that is the flow is inviscid, that is the viscous term is negligible. And therefore, I can neglect it in the moment of conservation equation, the second assumption for potential flow is that the flow is, what is called ir-rotational? the flow is ir-rotational, that is the there is no rotation at any point within the flow. The rotation is related to the verticity is equal to del cross u, that is equal to 0 at each point within the flow, so that is the second assumption for potential flows, that the verticity is equal to 0 everywhere in the flow. Therefore, the curl of the velocity is equal to 0 everywhere in the flow, under these two conditions the flow is called a potential flow ok.

Clearly, the viscosity can be neglected when the Reynolds number is high, in addition you require that there is no rotation locally anywhere within the flow, so that is the inviscid rotational flows are called potential flows. The reason is because as we know, when the curl of the velocity is equal to 0, curl of any vector is equal to 0, the vector can be expressed as the gradient of a scalar. Because, the curl of the vector is equal to 0, vector can be expressed as grad phi such that, because we know that the curl of the gradient of any function is equal to 0.

This phi what is called the velocity potential? it is called the and the velocity is equal to the gradient of the potential therefore, I reduce the velocity from two components can be expressed in terms of just one scalar component, or three components can be expressed in terms of just one scalar component. So, once you express the velocity as the gradient of the potential then you can see that, this reduces to del dot is equal to 0 or the Laplacean of this potential has to be equal to 0. So, that is the equation of motion for the velocity potential, so that is the equation for the potential, the momentum equation of course, gives me the pressure in terms of the velocity fields. That once again can be simplified in this case, using the fact that the flow is irrotational can be simplified, using the fact that the flow is ir-rotational.

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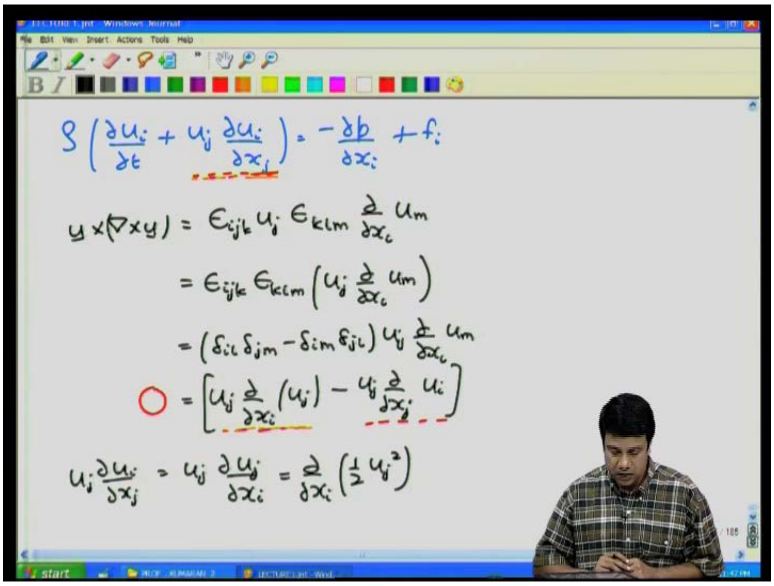

So, let us see how to simplify that, we will go back to an indicial notations, because that is more convenient in this case, and I neglect the viscous term in the conservation equation. The conservation equation is partially u i by partial t plus u j plus the body force, I have neglected the viscous term, we will tried to find a simplification for this term using the fact that the flow is rotational. Therefore, it is a potential flow, simplify this as follows, you should take u cross del cross u in indicial notations. I can write it as epsilon i j k u j times del cross u epsilon k l m partial by partial x l of u m is equal to epsilon i j kepsilon k l m u j partial by partial x l u m, and we know what is epsilon i j k times epsilon k l m it is a product of two anti-symmetric tensors, and it is a being a real forth order tensor.

And this can be written as delta i l delta j m minus delta i n delta j l, and I can write this delta i l times partial by partial x l is u i. Therefore, this will become u j, partial by partial x i of u j minus delta i m, so I get u j, delta j l gives me partial by partial x j of u i. So, that is u cross del cross u, and we know that the flow is a rotational therefore, del cross u has to be equal to 0 everywhere, since del cross u is equal to 0 everywhere, u cross del cross u also has to be equal to 0 everywhere.

Because, it is a rotational and therefore, the curl of the velocity vector is equal to 0 everywhere, that means that u dot u cross del cross u also has to be 0 everywhere, that means the u j partial u i by partial x j has be to be equal to u j partial u j by partial x i. And therefore, for this term here what I have here is u dot grad u, u j times partial u i by partial x j, I can substitute instead the first term over here. So, this basically tells me that partial u i by partial x j is equal tou j partial u j by partial x i, you would expected this.

And this basically is telling you that, partial u i by partial x j is equal to partial u j by partial x i, that is the transpose of the rate of deformation tensor is equal to x l, that is the rate of deformation tensor is symmetric that is expected in this case. Because, there is no rotation therefore, the anti-symmetric part of the rate of deformation tensor is equal to 0, I can also write u j times partial u j by partial x i as partial by partial x I of half u j square. Because, if I differentiate this using chain rule, I will just get u j times partial u j by partial x i is in the gradient of half u square, note that u j square is the velocity square u 1 square plus u 2 square plus u 3 square. So that, we will substitute into the equation for potential flow.

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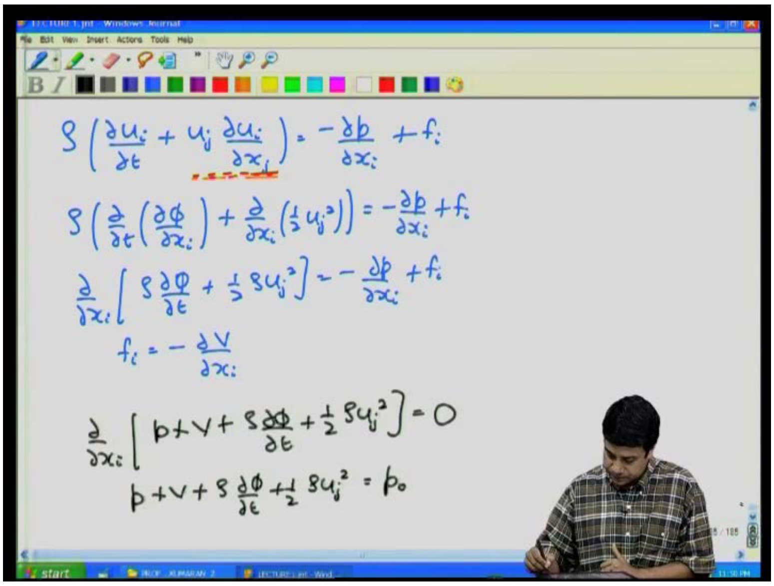

So, let us substitute that in here, for the first time derivative of the velocity, I will substitute partial by partial t of velocity, as I said can be expressed as the gradient of a potential. So, I will substitute the gradient of the potential here, the second term I can write it as the gradient of half u j square is equal to minus partial p by partial x i plus f i. In the first term on the left, I can interchange the order of differentiation, because the time and positioner independent coordinates therefore, this gives me in addition, the density is also constant, the density is a constant everywhere.

So, it can be taken into the gradient, so with this I will get partial by partial x I of rho times partial phi by partial t plus half rho u j square is equal to minus partial p by partial x i plus f i. We have gradients on everything expect the body force, if the body force can also be written as the gradient of a potential, that is it is a conservative body force the body force can be written as, f i is equal to minus the gradient of a potential. For example, in the case of gravitational force the potential would just be rho g times the height. Therefore, the force will be rho times g acting in the direction of the gravity.

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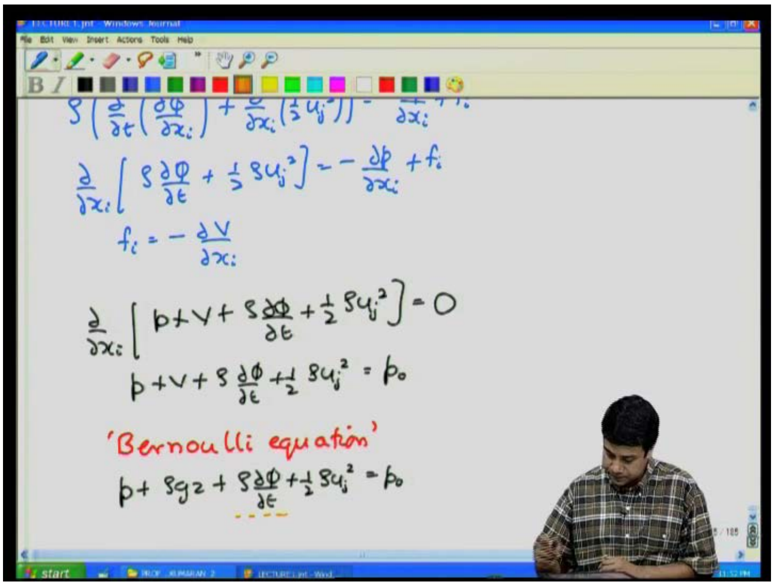

So, if I can write this in terms of the body force, body force times the potential my entire equation becomes, partial by partial x i of partial by partial x i of p plus v plus rho partial phi by partial t plus half rho u j square is equal to 0. Now, this is equal to 0 everywhere within the flow, the gradient is equal to 0 everywhere within the flow, so if the gradient of a function is equal to 0 everywhere within the flow. That means that the function is invariant everywhere within the flow, there is no gradient anywhere, so that is not vary as you move from one location to the other.

That means that this function has to be a constant is equal to a constant for the flow, this is what is known as the Bernoulli equation for gravitational flows. For example, flow under gravity this v is equal to rho g times the height, so in that case the equation would be p plus rho g z plus rho half rho u j square is equal to p not, that would be the Bernoulli equation for and the potential is the gravitational potential, the rho density times the acceleration due to gravity times the height ok.

So, this contains the potential energy rho g z, the kinetic energy half m v square is the kinetic energy, half rho v square is the kinetic energy per unit volume pressure is an energy per unit volume. And you have this additional term here, partial phi by partial t the rate of change of potential, that is the acceleration term if the flow was steady, then this acceleration term would be equal to 0. However, in general this acceleration term is not equal to 0, it has to be included in the potential equation only in the specific case, where the flow is steady this acceleration term is equal to 0.

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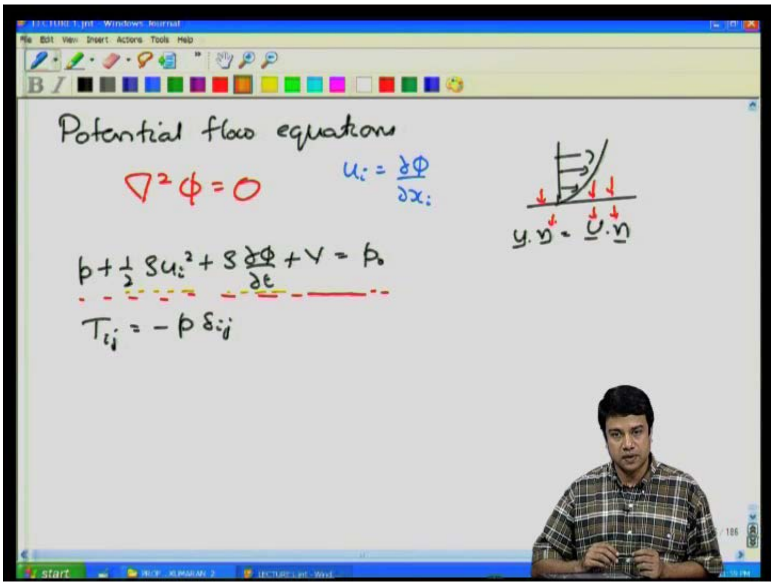

So, these are the potential flow equations, the gradient of the potential is equal to 0, and the Bernoulli equation p plus half plus any potential that is there is equal to p naught. Note that, we have completely neglected the viscous terms in the conservation equation that means, that the stress tensor is just equal to minus the pressure times the identity tensor. So the stress tensor for potential flows equal to minus p delta i j therefore, that is the stress tensor. In addition, we have neglected the viscous term in the conservation equation. The viscous term contained the highest derivative, the second derivative of the velocity with respect to position, and we neglected that viscous term you can no longer apply all the boundary conditions.

That were there in the original problem, as I said originally for the Negus torques equations, it is a second order spatial differential equation in the velocities therefore, you have two velocity boundary conditions at each surface. No slip condition or zero stress condition or some constant stress condition, when we neglected the viscous terms we neglected this highest derivative. And therefore, we can no longer apply the all the boundary conditions, that where there in the original problem, we neglected the viscous stresses. Because, we neglected the viscous shear stress, there is no momentum diffusion from the surface, when there is no momentum diffusion from the surface, one cannot apply a 0 tangential velocity boundary condition at the surface, or a 0 shear stress condition or a constant shear stress condition.

There is no mechanism for transporting momentum along the flow direction, the mechanism for transporting momentum perpendicular to the flow direction, and I am sorry no, no mechanism where transporting momentum perpendicular to the flow direction. Because, we neglected the viscous stresses, which ultimately result in the velocity field coming to 0 at a surface, the momentum diffusion which results in the no slip condition of the surface.

Therefore, we cannot satisfy the tangential velocity boundary conditions any more, the only boundary condition that can we can satisfy is the normal velocity boundary condition, that is that u dot n is equal to capital U dot n. That is the velocity of the fluid along the normal direction to the surface is equal to the velocity of the surface itself, in other words both the surface and the fluid have to move with equal velocity at the surfaces. As for as the tangential velocity boundary conditions concerned, if you neglected viscous stresses, there will be a velocity slip at the surface in the case of potential flows, the tangential velocity in a potential flow will not decrease to 0 at the surface.

The reason is because we do not have the mechanism for reducing the velocity, the diffusion of momentum due to shear stresses. Similarly, we cannot impose the tangential stress boundary condition, because the potential flow has 0 tangential stress we neglect with the viscous terms completely. So, we can only impose the normal stress boundary condition, the normal stress in the case of a potential flow is just equal to minus of the pressure, if it is defined with respect to the outward unit normal. So, therefore this potential flow equations have to be solved with normal velocity and normal stress conditions.

So, first we solve this, I am sorry is corrected, there should be del square of phi is equal to 0, the divergence of the velocity is equal to 0 or the Laplacean of the potential is equal to 0. So, we solve del square phi is equal to 0 with the normal velocity boundary conditions at the surface, in order to obtain a solution for the potential and therefore, for the velocity that solution for the velocity can be inserted in through the Bernoulli equation, to find out what is the pressure acting at the surface. So, this is the way that you would solve in case, there where velocity conditions specified at the bounding surfaces, incase pressure boundary conditions are specified gets a little more complicated, that is satisfy itself both of the equations the Laplacean of the Potential is equal to 0.

And the pressure boundary condition with some initial velocity, and then correct that until you get the pressure, correct the relation between the potential there is a potential itself is linear in the velocity. Because, the potential is given by phi is equal to I am sorry, the velocity is given by d y is equal to partial phi by partial x i, and I am solving the Laplace equation for the potential. Therefore, the velocity is a linear function of the potential, if I increase the velocity by a factor of 2 the potential at all points, will also increase by a factor 2 subject to when unknown constant. Note that, this is the velocity is related to the derivative of the potential, I could always add a constant value to the potential without changing the velocity field.

In other words, I have to specify the potential equal to 0 at some point, then I have a potential at every point within the fluid, in terms of the potential at that particular location subject to an unknown base value. Because, the velocity is only related to the gradient of the potential velocity is linearly potential therefore, if the velocity but, we increase by a factor of two. There where potential gradient goes up by a factor of 2 everywhere within the fluid that means, that this term in the Bernoulli equation will also be linear in the potential.

However, I have a quadratic term here proportional to u square, so if I reverse the velocity everywhere this term does not change sign therefore, the pressure is no longer linear in the velocity, recall the first torque flow. The pressure and the fluid velocity where all linear in the velocity of bounding surfaces in this particular case, the fluid velocity will be linear in the velocity of bounding surfaces. In other words, if I change the velocity by a factor of 2, the fluid velocity at each point will increase by that same factor, the pressure will not because the pressure is non-linear. In fact, you can show that, the solution for these equations are unique, and they satisfy some minimum energy criteria.

So these are the basic equations of potential flow; in the next lecture, we look at some of the properties of these equations, then go on to solve some specific problems. Just to reiterate, neglected the inertial viscous terms in the conservation equation. So, the equations are inviscid, in addition we have consider a rotational therefore, the rate of deformation tensor is symmetric at all points within the flow. In that case, the velocity can be expressed as the gradient of the potential; potential satisfies Laplace equation, the stress is equal to minus p times delta i j, because we have neglected viscous terms. And the pressure is related to the velocity and the potential through the Bernoulli equation. We look in the nest lecture, at how to solve these equations, how to and some of the general properties, so we will see with that.