Fundamentals of Transport Processes II
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Lecture - 21
Flow in a corner

Welcome to the course of fundamentals of transport processes 2. We are now at lecture number 21, just about half way through the course. And just to recap we had started off with the integral theorems, and vector calculus. And differential operators using these integral theorem and differential operators, we had derived it the mass momentum and energy, mass and momentum conservation equations for a netroneum fluid. To write the stress linear function of the rate of the de formation tensor, and then we had these for the navier stroke equations for one scalar equations, for divergence of velocity equals to 0 for mass conservation, for an incompressible fluid, where the density is constant, and the three momentum conservation equations for the three components of the velocity, and these have to be solved for the velocity field a vector with three components as well as the pressure. So, that is where we were. We had then scale the momentum conservation equation and found that the ratio of the inertial and the viscous terms is the Reynolds number.

The equations themselves are non-linear equations, so it is difficult to obtain the general solution. In fact this is something common to all of the transport processes, the equation can be posed mathematically with boundary conditions can be posed mathematically. However, that does not exist a general mathematical solution procedure which will work for all problems. The solution procedure inherently involves a physical understanding of the physical situation and simplification, based upon those in order to obtain solution which are specific to well defined limits.

Of course, one could solve this equation numerically on a computer using various procedures however this does not give us the physical understanding and the objective. In this course was to develop a physical understanding of the fundamental forces that influences the fluid flow in well defined parameter receivers. So, we had first looked at the limit of low Reynolds number, where the inertial terms in the conservation observations are neglected, and the conservation equation has just the pressure and the viscous terms.

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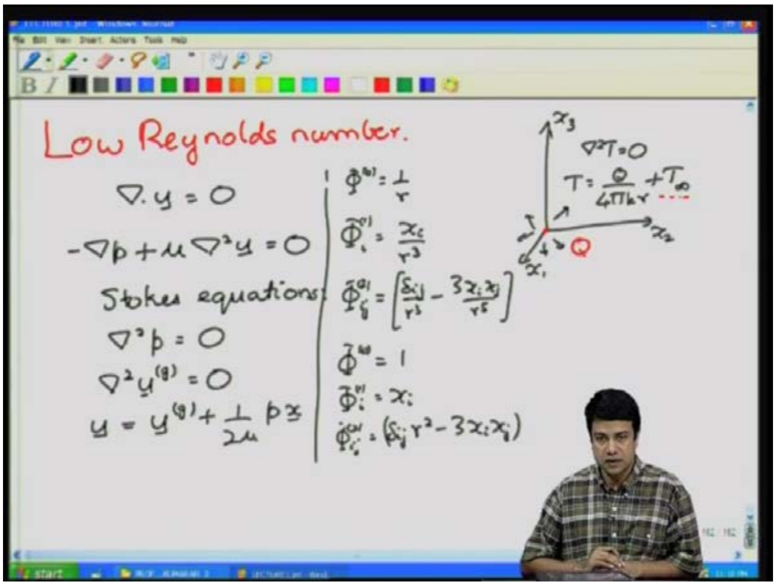

So, conservation equation, were the diversion of velocity, del dot mu is equal to 0 minus crap p plus mu del square mu is equal to 0. Let me take small numbers inertial forces are small compare to viscous forces, alternatively convective transport of momentum is small compared to diffusive moment transport of momentum. Therefore, the flow is dominated by diffusion, in the case of heat and mass transfer, when the flow was diffusive dominated, we found that the concentration or the temperature equations satisfies the temperature fields, satisfy the Laplace equations. In this case, for these stokes, equations the solution can still be reduced to 2 Laplace equations. The first is del square p is equals to 0, this is obtained just by taking the divergence of the momentum conservations equation and using the fact that the velocity is equals to 0.

And the second equation, was for the general part of the velocity field, and the total velocity field consists of the general solution u general plus 1 over 2 mu p times x factor, where x is vector and p is the scalar. So, we had obtain how to do this so we solution for these procedure equation is to solving 2 Laplace equations one for a scalar and one for the vector. And we had seen how to obtain scalar vector tensor and higher order solutions of these Laplace equations. It started off with the fundamental solution with the point source, we had done that earlier del square p is equal to 0 a point source at the origin.

The temperature is equals to 3 by 4 by k times the radius square, r is the distance from the origin. So, if I have a coordinate system with the point source at the origin, this is an axis symmetric co ordinate system. You know that is the point source the amount of heat coming out at per unit time is Q total amount of heat coming out per unit time is q. We have to solve del square t is equal to 0 in the absence of convection around the point source. The solution for that t is equal to Q by 4 pi k r plus temperature far from the point source. Therefore, fundamental solution of Laplace equation of this case is equals to 1 over r constant divided by r but, of course, we are specifying the solutions only 2 within the multiplicative constant.

I can take repeated gradients of this, because the 5 naught is the solution of Laplace equation 5 naught equal to del square of 5 naught is equals to 0 everywhere. I take the gradient of this equation, I find that the del square the Laplace of n gradient of 5 naught is also the solution. Therefore, gradient of 5 naught is also the solution for Laplace equation that is the vector solution. Because when I take the vector of the gradient of this scalar I get a vector. 5 1 i is equal to x i by r cubed once again to within a multiplicative constant.

I can take another gradient to get 5 2 I j in a second order tensor is equal to delta i j by r cube minus 3 x i x j by r bar 5. And so on one can take repeated gradient to get higher and higher order to tensors, obviously these are decaying solutions they decrease the value of the solution, decreases as r increases. The distance from the origin increases 5 naught decreases at 1 over r 5 1 s 1 r square 5 2 at 1 over r cube and so on. Corresponding to these are also what are called growing solutions these solution increase as r increases.

So, 5 growing solutions leading order is just a constant, though by the constant is just equals to 1. It is just corresponds to the infinity solutions in my solution for the temperature fields constant. Of course, if you take del square of this you too get 0, 5 1 s obtained by taking 5 1 for decaying solution and multiplying it by r power 2 plus 1 r cubes. So, going solution 5 1 corresponds to i is equals to x i and one can get the next higher solution, 5 2 I j is equal to delta i j r square minus 3 x i x j and so on you will get the higher and higher solutions.

So, any solution for the pressure of the general part of the velocity fields that satisfies the Laplace equation, can be written as a linear combination of these either decaying or the growing harmonics. If you are solving for an external flow outside of an object, then we have to use the decaying solutions, because we require that the velocity has to go to 0 far away. On the other hand for internal flows, inside an, objects, we have to use the growing solutions. Because the growing solutions are finite at r equals to 0 which is within the domain. Whereas, the decaying solutions are not, they diverges at r equals to 0, so based up on this we had solved certain solutions.

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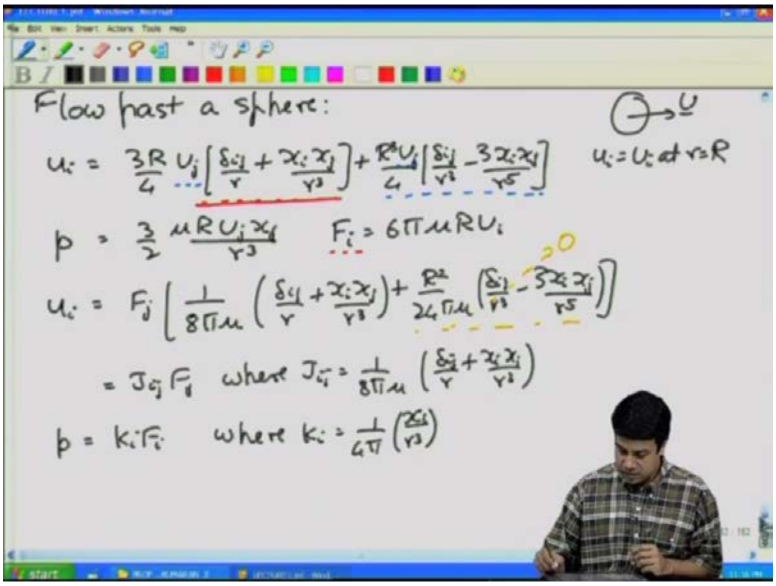

For flow past a sphere for example, flow past a sphere as flow moving on to the constant velocity. And of course, the solution for the velocity and the pressure fields, and the general part of the velocity and, the pressure have to be linear, in one of the harmonics we have to be linear in the velocity of sphere. Because the, stokes flow as i p s i d the linear equation for the fluid velocity and the pressure fields. The stress is linear in the pressure for the fluid velocities. Therefore, the solutions at every points have to be linear, in the fluid velocity and the pressure fields.

Just based upon these simple considerations and the requirements of the fluid velocities has got to be equal to the particle velocity at the surface, at the surface of the sphere. We are actually calculated it the velocity field, 3 r by 4 u j into delta i j by r plus x i x j by r cubed plus r cubed u j by 4 delta i j by r cubed minus 3 x i x j by r power 5. And the pressure was 3 by 2 mu R u j x j by r cubed, let calculate this and collect detail and from that we have calculated the net force exerted by the fluid on the sphere. Which was on instead of minus u the net force excreted by fluid on the fluid is given by 6 pi mu r u i.

Note that there are 2 terms here one which decays proportional to 1 over r as r goes to infinity, and the other which decays proportional to 1 over r cubed. So, obviously if you sufficiently far from this sphere, so position vector is far from the radius of this sphere. The only contribution that would see is the 1 over r, because that is the 1 proportional to 1 over r cubed is much smaller. Now this contribution due to 1 over proportion to 1 over r is like source term, because the temperature of the heat, source the temperature goes as 1 over r. Similarly, the source of momentum that is point force, the velocity of 1 over r, and the velocity can be expressed instead of using velocity of the sphere itself that is velocity of the sphere itself.

I can express it using the force excreted by this sphere, because those two are dependent on each other. That, because I do that I get u i is equal to pi f j by 8 mu, let me write this steps in types force 1 by 8 pi emu delta i j by R plus x i x j by r cubed plus r square by 24 pi mu into delta i j by r cubed plus minus x j by r power 5. In this case may be the velocity terms of the force exerted by the particle on the fluid. If we take the point, particle limit that is capital r is small compared too small r that is the distance from the center of this sphere. This second term goes to 0, because the point particle limit you are taking the small r is very much larger than capital r.

In that case this term will actually goes to 0 and you get this is equal to J i j f j, where J i j is equal to 1 by 8 pi mu delta j by R plus x i x j by r cubed. So, this is for a point source delta function in the limit as the radius of the particle goes to 0, but the net force exerted by the particle is still finite even as the radius of the particle goes to 0. And this limits of course, the pressure certain relation p equals to K i F i where using terms of the pressure 1 by 4 pi x j by r cubed. I should put i here so that will be the expression for the pressure, all of this is discussed this is just the review so that is for the particle that is translating.

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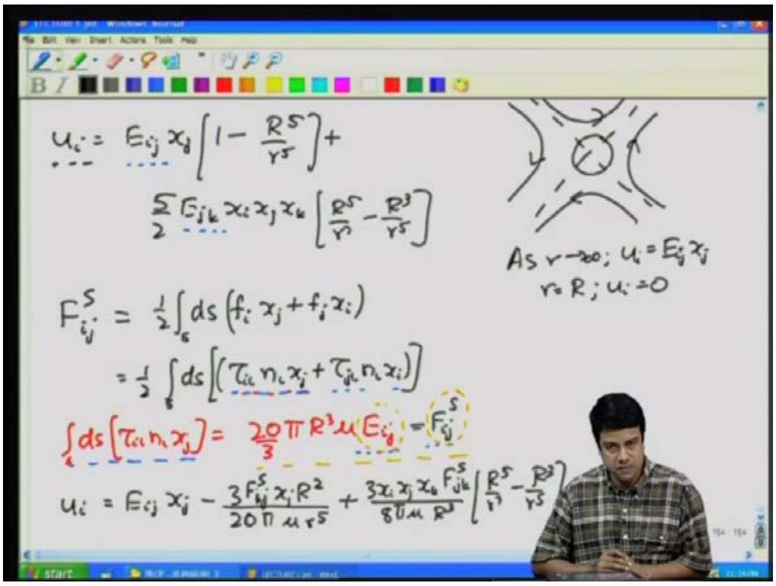

Just very quickly for a particle in a shear flow, this is the particle this is in a fluid which I am imposed a linear for an extensional flow far away from the particle imposed an extensional flow for far away from the particle. So, very far from the particle basically I have an extensional flow particle is sitting at the center that is always possible. Because even if the particle is moving and rotating, I can decompose the flow locally into a steady, translation with the velocity at the center, of the particle, plus a deformation about that center. The deformation about that center can have 2 components, the symmetric traceless part of the rate of deformation tensor which gives me the extensional flow, anti symmetric part which gives me the rotational flow and due to linearity. I can add up the contribution due to each one of these, solve them separately and then add up to the velocity contribution to each one of these.

So, for the extensional part I have already subtracted out the velocity with which the shear is moving, so that I am in a reference frame moving with the same particle as the velocity. And they look around me and see, what is the rate of the deformative tensor relative to the particle center? As r goes to infinity far away from this shear, the velocity is just a extensional flow and at R equals to 0. Of course, if you subtract it out the rotation that means the shear is stationed, so at r equals capital R u i is equal to 0.

Once again we have to solve this problem u i is equal to E i j x j into 1 minus R power 5 by r 5 plus 5 by 2 E j k x i x j x k into R power 5 by r cube minus r cubed by R 5.That was the velocity field net force in this case is identically equal to 0, just by symmetry the force is not equal to any particular direction, because there is no relative velocity of the sphere, with respect to the fluid. That we already solved separately earlier, however there s the net force moment. So, the net force moment is given by I will call it as the symmetric part of the force moment F s i j is equal to half integral over the surface of the particle d s of f i times x j plus f j times x i. This is the symmetric force moment which is equal to half integral over the surface, the force acting on the area is the stress unit normal taw i l n l x j plus taw j l n l x i.

If you recall in the last lecture on our way to calculating the effective viscosity suspension, we are actually calculated one of these. So, this integral we had integral d s taw i l n l x j calculate it over the surface of the particle. We had actually calculate this we get 20 by 3 pi r cubed mu times E i j. We had calculated this on the way while calculating the effective viscosity suspension. So this is the symmetric form of the force format, so I have calculated only one part here I have calculated only one part. However, this results itself is symmetric, because this result is proportional to E i j, which was the symmetric traceless part of the rate of deformation tensor. Therefore, the transpose of this which is the second part is equal to the same thing, because the results itself is the symmetric. So, the transpose is equal to the same thing and therefore, the symmetric part of the force moment, is equal to 20 by 3 pi r cubed mu E i j.

So, the force is equal to 0 but, the force times the displacement in second order tensor integrate over the surface, symmetric part of that is not equal to 0 is not proportional to I j. We had used this to calculate the deformation tensor; however one can do the same thing here. If you recall the when we did the translation of shear in a fluid it first found that velocity terms the velocities of the sphere. The force in terms of the velocities of the sphere and then substituted the, the force instead of the velocity to find the fluid velocity. In terms of the force exerted by this shear, so we can do the same thing here. We can find out this velocity rather than using the symmetric rate of deformation tensor here I can use this force moment to express the velocity of form. If I do that I will get the velocity of the form, it is simple algebra I use this relation, to substitute e in terms of the symmetric part of the force moment.

And I will get to the velocity field as E i j x j minus 3 s i j x j r square by 20 pi mu r power 5 plus 3 x i x j x j s, to replace this by symmetric force moment. This is also the symmetric force moment mu all I done is replace s E i j. This symmetric force moment and now if you look at this equation, that term the slowest decaying is actually the one which goes proportional x i x k x j times 1 over R to the 5. Because this decreases 1 over r square this infinity we have 2 terms 1 over decreases to the fourth power as r goes to the infinity.

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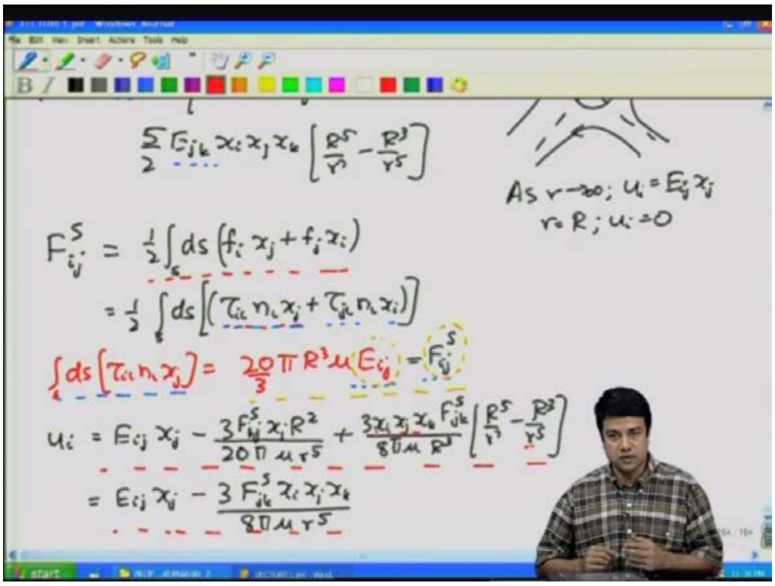

So, in that sense if I retain that alone I will get E i j x j minus 3 F symmetric j k x i x j x k b u 8 pi mu r to fifth v e if I just retain the term which replace the decay r goes to infinity. This term will gets decay when r over to 1 and, when I done it this way you can see once again, the velocity field depends upon only the force moment, it does not depend on the radius of the sphere. In particular if I take this expression I will take this expression and take the limit capital r going to 0 point particle. What I will end up is the term like this, because terms that are proportional to capital r square we will just going to 0, I will end up this one. So, in the limit is capital r goes to 0 point particle limit, but with the symmetric force still finite I will end up this itself. So, this is the solution for the field symmetric point dipole a symmetric force moment in which the force moment is basically, given by this on the surface.

So, that is response to the linear shear flow, an extensional flow the symmetric traceless part of deformation tensor. So, basically this solution for extensional flow far away gives me distance field due to a symmetric force movement plus an additional term which decreases. When you go to the point particle limit other thing, the anti symmetric force movement is basically a response to a rotational flow.

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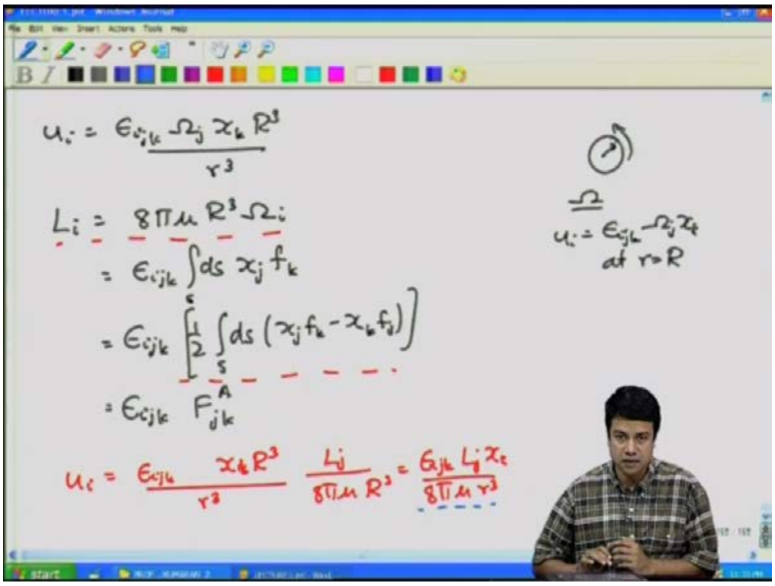

So, for example, we found that the third particle that is rotating with angular velocity omega. So, that u i is equal to f epsilon i j k omega j x k at r is equal to capital R, we had solved this once again. And we obtained velocity fields as u i is equal to f epsilon i j k omega j x k R cubed by r cubed. And the angular taw is equal to 8 pi mu R cubed omega that was the taw what is obtained. Once again I can replace the taw that you can that is equal to f epsilon i j k integral of the surface d s of x j times f j times acting at the surface. Where x j was the position vector of the surface f k was taw n l k n l.

The stress acting on this surface so since, this f epsilon i j k is anti symmetric tensor. I can also write this as f epsilon i j k is anti symmetric in this is j and k. Therefore, this anti symmetric tensor multiplied by this tensor x j times f k, second order times tensor x j times f k has a symmetric and anti symmetric part. That this symmetric part multiplied by f epsilon i j k are identically equal to 0. Because the symmetric tensor and the, anti symmetric tensor, equals to 0. Therefore, this can also written as this times the anti symmetric part of this tensor x j f k minus f j x k to get the anti symmetric part of the tensor.

You get the tensor subtract it out the transpose divide by 2 to get the anti symmetric part. So, this is equal to f epsilon i j k times t anti symmetric force movement. Where the anti symmetric force movement is basically, this so I can write this velocity fields in terms anti symmetric force of moment as well. So, the velocity fields expressed in terms of taw, so the velocity fields expressed in terms of the taw u i is equal to f epsilon i j k omega j x k r cubed by r cubed into instead of angle of velocity. I substitute the taw that is L i by 8 pi mu R cubed.

And you can see once again that the R cubed cancels out to give me, this should be j f silon i j k l j x k by 8 pi mu times R cubed. Once again the velocity this term has to be sphere when expressed in terms of the taw in the shear is depen, is not dependent upon on the radius of the sphere R. It depends only upon the taw and the position vector, so even in the point particle limit the velocity disturbance due to this is independent of the radius. And therefore, you get the result for the limit is r goes to 0, you get the taw due to the velocity disturbance due to a point dipole anti symmetric force movement.

And that has, this form it directly proportional to the taw, and the taw can be expressed f silon times the anti-symmetric part of the rate of movement part of the force movement. Therefore, the velocity disturbance due to anti symmetric part the velocity disturbance it decays as 1 over r square. So, we have the disturbance due to force decreases as 1 over r you see the tensor times the force itself. The point force decreases 1 over r and the next higher is the dipole movement, there is no net force exerted by the particle but, the force stated with the terms integrated over the surface is non 0.

So, there is the force type pole that is of two types one is to be a symmetric, and the other is to be anti symmetric. The disturbance due to the symmetric point force is di pole goes as 1 over r square it is identical to a disturbance created by the shear an extensional rate of deformation tensor far away. Anti symmetric part also decreases 1 over r square that disturbance is identical to what you get due to a rotating shear. And of course, you can go to the higher order but, these are the basic types of disturbance, in the limit of low Reynolds number we discussed for.

So, everything we done so far we done or external force flow around particle, we took a sphere even if the particle is more complicated. The slowest decaying part will be of this nature but, ultimately when we found out. The result we found that only the slowest decaying term it is not dependent on the radius of the particle. It will not depends upon the shape of the particle, it will depend only upon the force exerted by the particle. So, long as far as force exerted by the particle is finite as the limit of the characteristic length is 0. You get the same for the disturbance to the velocity field both due to the net force as well as due to the force movement.

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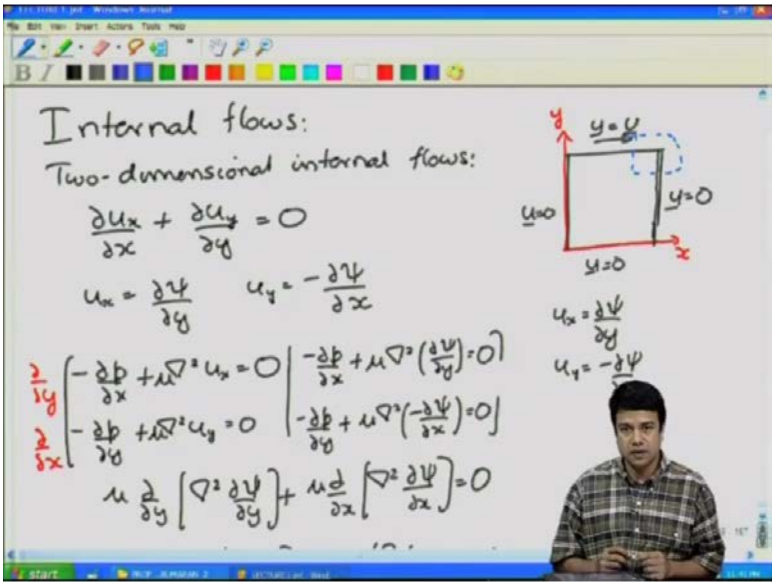

Next let us look at how we would solve for internal flows, the simplest example is for example, is spherical ground with external boundary conditions in the surface of the taw. In this case instead of taking the decaying harmonics, one would take the growing harmonics and solve the equations exactly the same way. So, I would not go to that I will leave that as exercise for you. But, let us look at other types of internal flows in particular, if the flows happens to be a two dimensional internal flow there are certain simplification, that can that one can make in solving the problem. So, let like to that you get two dimensional, the simplest case one can consider is that there is the Cartesian coordinate. A classic example for this is that called the flow and Librium and cavity, where you have 3 walls with velocity equals to 0 and you have one wall moving with the constant velocity. Some constant velocity in the transactional direction you cannot have motion normal to the wall of course, because the flow is compressible. Because the volume of the flow is you cannot have the wall in the transactional direction.

This is the two dimensional flow where one has the coordinate system x and y, and the mass conservation equation. For this case of the using x and y we will work with x and y mass conservation equation partial u x by partial x plus partial u y by partial y is equals to 0. And we had discussed in the kinematics that absence of a vector will always be expressed in 0 and the curl or something results in no major simplifications in three dimensions. Which, we still have the vector which still we have to take in two dimensions the major simplifications, because the vector is always perpendicular to the plain. The curl of the vector is in the plain the vector itself is perpendicular to the plain. The vector perpendicular to the plain is e z times the steam function psi times. In particular we can express in terms of stream functions u x is equals to partial psi by partial y u y equals to minus partial psi by partial x. You can see the written in this form the equation identically satisfy the incompressibility condition.

So, now this u x and u y in the stream functions can be substituted into the momentum conservation equations. So, the momentum conservation equation in minus y direction are minus partial p by partial x plus del square u x is equal to 0. Where del square is now in two dimensional partial square by partial x square minus partial x by square by partial y square. Then you have minus partial p by partial y plus del square u y is equals to 0 express in terms of stream functions. So, I get minus partial p by partial x plus put a velocity here, we discussed here that terms emu del square emu x and emu del square emu y minus partial p by partial y plus. I can eliminate the pressure from these two equations by simple experiment by taking partial by partial y of the first equation partial by partial x of the second equation subtracting out.

So, I will take partial by partial y in the first equation subtract out partial by partial x of the second equation. Therefore, the second derivative to p of the x and y cancels out. And I will get equation formed mu j square by partial j square 0 mu partial by partial y of del square partial psi by partial y plus del square partial psi by partial x is equal to 0. So, that is the final equation I will get for the stream function, of course, I can see the order differentiation and if you do that, you will get the you can see quiet easily is that mu del square del square psi is equals to 0.

Because the viscosity is not equals to 0 this is just the Casmir, what is called the bi harmonic equation for the stream function. Bi harmonic is the fourth order equation, two Laplace equation acting on the same function is equals to 0. Often this is also written as del power 4 psi is equals to 0. And the boundary condition which you typically imposed are u x is equals to partial psi by partial y and u i equals to partial psi by partial x. So, the equations of both the conditions derivative of both psi with respect to x and y are imposed on all boundaries. Of these have to be solved in order to get the solution. Of course, in Cartesian coordinates were to solve a Laplace equation here, when we did heat and mass transfer, we take partial x square by partial y square of the temperature equals to 0. Identify the motions and the conditions you get the Eigen value correct in the, those two, two boundaries and the other two boundaries get there is a homogenous terms and the constant.

In that are determined from orthogonality relations for the Eigen functions we obtain the Eigen value problem. Rather than solve for the full, Librium cavity, the Librium is little bit complicated, we try to solve a simple one. I will try to solve a simpler case here that is rather than solve for the entire Librium cavity we will focus on one particular, one particular corner of the Librium cavity. Where I have one wall stationary and the other wall moving transactionally and try to find out, what is the velocity field there.

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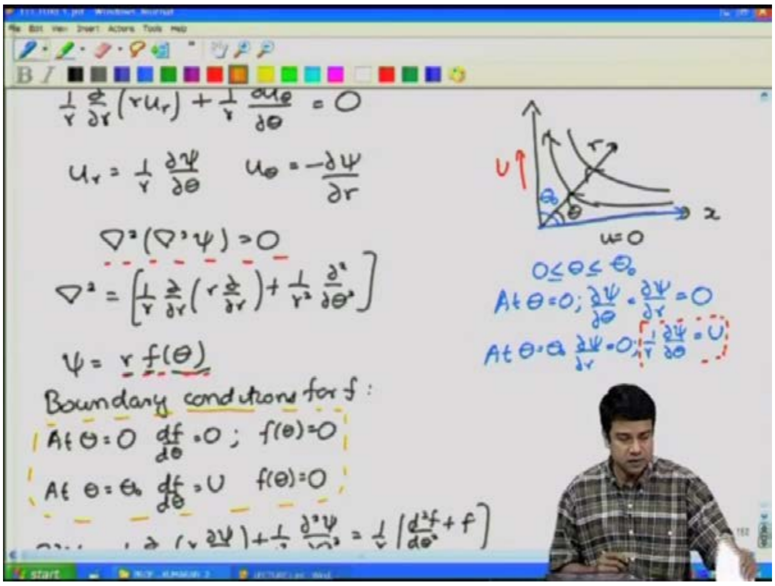

So, the problem I will consider is I will consider the x axis domain this is the x and this is y axis. And this wall is falling with the constant velocity u, this wall is falling with the constant velocity u and the velocity is equals t 0. And this wall, because this moves with the constant velocity u I will end up with stream line which looks something like that, I would like to solve for those. So, now rather than solving the Cartesian coordinate system, it is simpler to solve in a polar coordinate system. So, this is an rth theta coordinate system where the position vector to any distance is the distance from the origin is corner, between these two boundaries. The distance from the origin is r and this angle between the position vector and the x axis is theta. In this polar coordinate system the mass conservation equation is 1 by r partial by partial r of r u r plus 1 by r partial u theta by partial theta is equal to 0 that is the mass conservation equation.

That is the angle between the two plates, I have shown you that there is to be 90 degrees but, it is not to be at any angle. I will solve it to be an general angle theta naught between the two plates. So, the theta naught is the angle between the two plates theta is the angle made by the position vector from the x axis. That means that theta can go from 0 to theta naught, so theta can go from 0 to theta naught. This is stream function for polar coordinate system as well this is quite easy to obtain u r is equal to 1 by r into partial psi by partial theta u theta is equals to u theta is equal to minus partial psi by partial r. So once again, in terms of this in the equation the bi harmonic equation for the stream functions, becomes del square del square psi is equals to 0. Where this operates to del square in polar coordinate system is 1 by r partial by partial r r into partial partial r plus 1 by r square partial square by partial theta square. That itself is self Laplace equation operator in the polar coordinate system we had to take two times we had to operate two times to get the bi harmonic operator.

So, I have to solve this equation in order to obtain the stream function psi, what are the boundary conditions boundary conditions are on. Two axis one is let me take this y of here, because y is y and x are orthogonal axis this angle theta naught could be any value. So, the boundary conditions are on the along one of the boundaries along one of the boundaries. The velocity has to be identically equals to 0 that means that both u r and u theta has to be equals to 0 along with the boundary. That means that at theta is equals to 0 u r is equals to 0 that means that partial psi by partial theta is equal to 0.

And you theta is equal to 0, that means that partial psi by partial r is not also equals to 0 So, that is the boundary condition at theta equal to 0, at the other boundary theta equals theta naught. The velocity along the radial direction is transactional to the surface, the surface along with the radial directions. Therefore, the velocities along with the radial directions that means that u r is not 0 u theta is 0. Just now velocity perpendicular to this plain to which is along the radial direction, so u theta is equals to 0 alongside partial psi by partial r equals to 0 and u r is equal to capital u. That means that 1 by r by partial psi by partial theta is equal to capital u, so those are the boundary conditions u r is equal to capital u. That means 1 by r psi theta is equal to capital u u theta equals to 0 that means the partial psi by partial r is equal to 0.

How do we solve this, you can see from this boundary condition here, you can see from this boundary condition here, 1 by r partial psi by partial theta is equal to u that is constant. That is I would expect psi to be proportional to r on the boundary, therefore, if I wanted to search for a similarity solution. I would look for solution of the firm psi is equal to r times the function of theta. Because I expect on the boundary I would expect psi to be proportional to r, because the partial psi by partial theta is equal to r times u therefore, I can search for a solution in which I is equal to r times theta everywhere.

Now in terms of f what are the boundary conditions, the boundary conditions for f at theta is equal to 0 partial psi by partial theta is equal to 0. In the since partial psi by partial theta is equal to 0, I have d f by d theta is equal to 0 and partial theta, by partial r when you take the derivative of psi with respect to r, I just get f of theta. Therefore, I have f of theta is equal to 0 and at theta equals theta naught partial psi by partial r is still equal to 0. Because u theta is equal to 0 therefore, partial psi by partial r equals to 0 f of naught is 0 and 1 by r partial psi by partial theta and 1 by r partial psi by partial theta is just d f by d theta, this is equal to capital u.

So, those are the boundary conditions for f, I substitute this form, this form of the equation into this bi harmonic equation. Then obtain a solution, for obtain a equation for f of theta. So, let us do that a little quickly, so del square psi is equal to 1 by r d by d r of r partial psi by partial r plus 1 by r square partial square psi r by partial theta square. And if you work it out, in terms of f this works out to be 1 over r into d square f by d theta square plus f just substitute psi in terms of this in order to expression. I had to take another laplacian del square of del square psi is equal to, if you substitute once again, if you this and take a second laplacian you get 1 over r cubed into d power f by d theta power four plus two d square f by d theta square plus f is equal to 0.

So, therefore, my solution for ff theta is determined from the equation this d 4 f by d theta of 4 2 d square f by d theta square plus f is equal to 0. Linear differential equation can be solved quite easily to get f is equal to a cos theta plus b sin theta plus c theta cos theta plus d theta sin theta. Where a b c and d are the 4 constant of integration which has to be evaluated using, the 4 boundary condition that, we have it here these four constants of integration have to be evaluated using the four boundary condition that we are into it.

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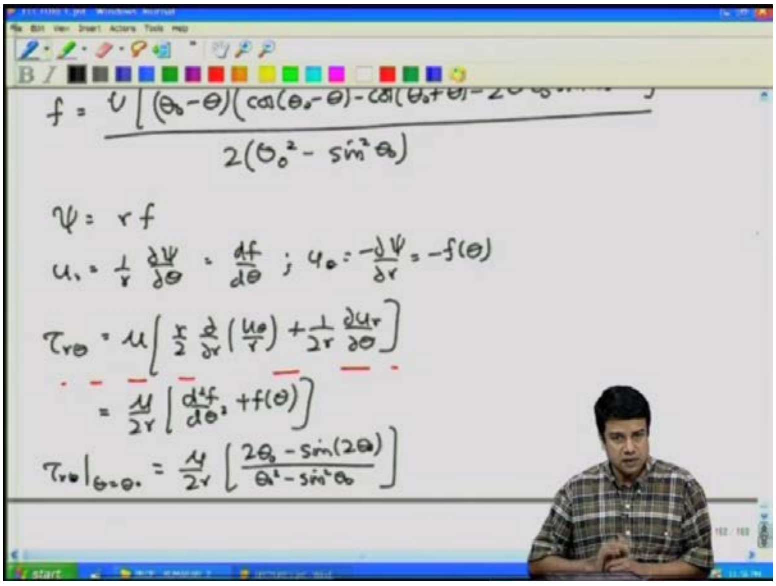

Just little bit of algebra to do it, I would not go through the details. I want the final solution that is f is equal to u into theta naught minus theta cos minus cos of theta naught plus theta minus 2 theta naught sin divided by 2 into that naught square minus sin square theta naught. So, analytical solution is possible for f and the analytical solution for psi is just equals to r times f is equal to r times u times. This is complicated function of theta this gives me the velocity fields. I can take derivatives of this u r so u r is equal to 1 by r partial psi by partial theta is equal to d f by d theta and u theta is equal to minus partial psi by partial r is equal to f of theta so minus here.

So, this gives me the final expression for the velocity fields, the next step is to calculate actually the force acting on the surface. For that I need to calculate the shear stress transactional force that is exerted on this surface. So, for example, I want to show what is the transactional force exerted in the surface. I need to calculate the stress the stress in this case is taw r theta, there is the force acting per unit area in the r direction r direction is this direction.

The r direction r is this direction acting at the surface whose unit is normal in the theta direction, force unit area in the r direction is acting at the surface whose unit is normal in the theta direction. So, this can be evaluated sphere stress taw r theta turns out to be equal to emu r by d by d r of u theta by r plus 1 by r partial u r by partial theta note that this shear stress is not straight forward in a Cartesian coordinate system taw x y is equal to emu into partial x y into partial u y by partial x. The symmetric traceless system far to the definition tensor in this case you have to take into account that the unit vector, in the polar coordinate system also depend upon position. For this one have to go to the standard tables to express the sphere stress in a covering a coordinate system, which incorporates the variation of the unit vector to respect to this position.

And the final expression of that turns out to be this one, and once again this can be expressed in stream function ultimately. In terms of function f it turns out to be mu by r into d square f d square f by d theta square plus f of theta. This expression for this anywhere within the flow in particular, in particular the stress on the wall that is taw r theta at theta equals to theta naught analytical expression for them. It transfer to be equal to mu by r into theta naught minus sin by theta not square minus sin square theta naught. So, this is the expression that you get from the stress, and important point would like to bring your notice is the stress, in the stress flow, actually decreases as proportional to 1 over r. So, in other words as r go to 0 the stress go to infinitely r going to 0, means that I am coming towards the corner. Here r going to 0 means that I am coming towards the corner here I am coming towards to 0 and r coming 0.

Because my origin is that the corner between the two plate one plate is moving other plate is stationary. That means that the velocity is discontinuous at this corner the between two plates and as r goes to 0. Because of this velocity discontinuity the stress actually goes to infinity proportional to 1 over r this is the standard features of all corner flows. And important one of this kind of Librium cavity flows as you go towards the corner the stress increases, one hour over r as r goes to 0. The force per unit area stress of force unit area so to find to total force some plates I have to increase integrate the stress over the radial coordinate to get the total force on plate.

I have to integrate stress over the radial corner, integrates the stress the stress is proportional to 1 over r. That means force is proportional to log of r, so the force is exerted actually infinite in diverse is log of r in the limits of when there is the discontinuity in the velocity in the corner between these two plates. And this is an important issue in all problems which involves all such wall of flows in there is the discontinuity in the velocity and the corner. Of course, there is no resolution to this the stoke flow equation to do this predict, in fact there is discontinuity that is the disadvantages of in stoke flow solutions these this paradox can resolve only.

If we do away, with continue approximation, this assumes that to r goes to the 0, the flow still continues regime. However, as you go closer and closer at some point distance from corner, becomes comparable to mean field path. And that point, continue approximation would break down and because of that you will get finite result. However, if you take the continue approximation all the way r equals to 0 you do that get the results that is singularity. In the net force singularity in the stress as well for all in the, this kind of internal flows. So, this gives you the one part of the solution for this particular case, where I have focused the tensor on non particular corner within the flow, where there is discontinuity in the velocity of the corner, one could solve the entire problem similar manner in using a Cartesian coordinate system.

We would not go through that it is little complicated mathematically; just bring to your notice that, one can solve these kinds of problems in two dimension by using a simplified stream functions. We write the velocity fields in terms of stream functions, this is always possible in two dimensions. Once you have done that you get a bi harmonic equation and that bi harmonic equation can be solved subject to the boundary conditions, in order to obtain the solutions.