Fundamentals of Transport Processes II
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Lecture - 2
Introduction

Welcome to the Fundamentals of Transport Processes – II; this is the second lecture in this course. In the first lecture, I have briefly reviewed the prerequisite for this course that is the fundamentals of transport processes 1, briefly what we did there and how. I hope you have had a chance to go back and look at those things because we will be using similar material here as well. In this lecture I would like to focus on what we will be doing in this course, most of this course will be comprised of fluid mechanics that is how do you calculate the velocity field for fluid flow, given the forces that are acting on that fluid.

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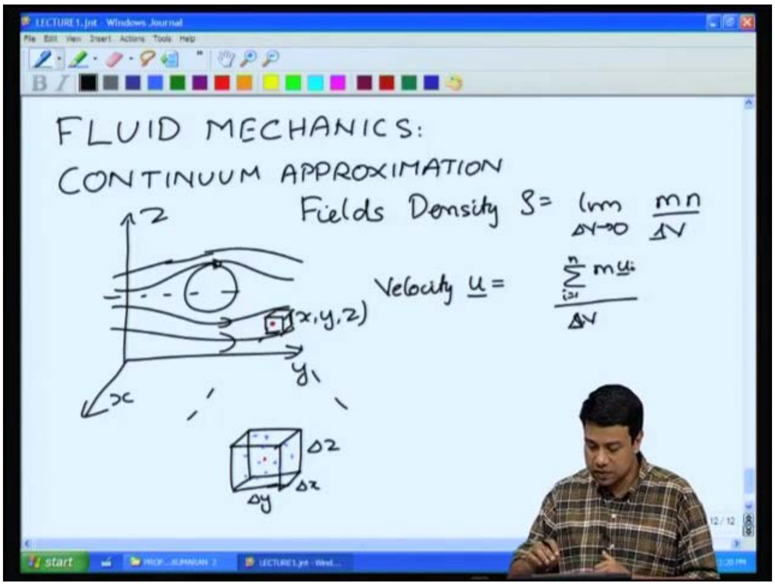

Now, in the last course we had dealt primarily with transport of mass and energy. Transport of mass I must emphasize is confined to the case of very dilute solutions, where the solute concentration is small, so that the transport of solute does not affect the center of mass motion of the fluid as a whole. In this course we will deal primarily with momentum transport that is fluid mechanics trying to predict the velocity field of the flow. The big difference between this and the previous course as you would have anticipated is that velocity is a vector, it has 3 components this is in contrast to concentration, and temperature fields for mass and heat transport respectively, which are both scalars. So, we could write out equation for just 1 quantity.

Whereas in this case for the velocity field it has 3 components, more importantly those components depend upon the coordinate system that you are using to analyze the problem. If you analyze it in different coordinate systems the components of the velocity will in general be different. So, that is something that has to be kept in mind since there are 3 components 1 might think simplistically that I could just write down 3 equations for each of those components, and solve them. However these equations depend upon what coordinate system you are using to analyze the problem.

In the previous fundamentals of transport processes course, we had talked about the transport of heat and mass these are given by the fluxes, the heat flux and the mass flux. These fluxes are vectors they are in the direction of variation of concentration and direction of variation of temperature. So, in the case of scalar quantities, such as heat and mass the fluxes are in the form of vectors. You can anticipate that in the case of vector quantities, such as the fluid velocity the flux is going to be more complicate and so we will develop techniques in this course to analyze, the velocity as a vector itself. And the flux of the velocity as just 1 object independent of the coordinate system, that is being used to analyze the problem. And for that we need to develop specialized skills and i will come back to that, but first what is the framework that we are going to be using?

The framework is basically what is called the continuum approximation, which I had discussed in the last fundamentals of transport processes course as well. That is we assume that the concentration, temperature and velocity are continuously varying functions of position; that is they have 1 particular value at each and every location in space, those values can also change in time.

So, the idea is to identify these fields, the concentration, temperature and velocity fields as continuously varying functions in space and time. And see what constraints are imposed by conservation laws and constitutive relations and how these fields can vary. These constraints give us conservation equations which are in the form of differential equations, telling you as you go from point to point in space and as you progress in time, how these fields vary. And if you can solve those differential equations you can get by some kind of an integration procedure, the entire velocity field, concentration field or temperature field.

Now, once you have a continuously varying function the advantage is that you can use all of the machinery that has been developed in calculus for differentiation integration, but as in the case of the fundamentals of transport processes 1 it's a little more that and we will discuss what methods we developed for solving these filed equations. Of course, the concept of field itself is an approximation, so if I have a fluid say flowing past some object I am defining velocity and temperature fields as continuously varying functions around this object. And what do I mean by the value of the temperature or the velocity at 1 particular point, the value at 1 particular position. So, if I have a coordinate system so if I have some particular position x, y, z.

What is the meaning of saying the temperature at this point or the density at this point has some specific value. What that means is that I go to this particular location sit on this location and construct a small volume around this location, construct a small volume around this location. So, this volume can be any shape in this particular case I will just take a cubical shape. So, this is the location and at this particular location I construct a small volume of distance let us say delta x, delta y, delta z and I count what is the total number of molecules of the fluid within this differential volume.

And so if I take the total number of molecules within this differential volume, my fluid density is defined as the summation over this entire volume of the mass of the molecule of each molecule over all the molecules within this volume divided by the volume itself. So, this is equal to the mass of the molecule types and number of molecules divided by the volume that is the density.

Note that in my definition I have delta v at the bottom. So, this is the mass per unit volume, in the limit as the volume goes to 0 in the volume as the in the limit as delta x, delta y, delta z become smaller and smaller this volume shrinks down to a point at the same time. Since, the volume is decreasing the number of molecules is also decreasing in such a way, as the in the limit delta x, delta y, delta z goes to 0 this limiting value has a unique limiting value that is in the limit as delta x delta y delta is going to 0, volume itself is shrinking, but however the number of molecules over there is also decreasing. In such a way that the ratio of the 2 goes to a constant value as delta x, delta y, delta z are going to 0. So, that is the meaning of density at 1 particular location, what about the velocity field?

The velocity field this as I said is a vector this velocity field is a vector. So, for this vector I will use the notation u under bar to denote that it is a vector. So, here what I do is I go to this differential volume each molecule within this differential volume has 1 particular velocity or 1 particular momentum vector. So, I add up over all the molecules of the mass of each molecules times the velocity of each molecule, I add up the mass of each molecule times the velocity of each molecule.

And then I divide it by delta v, this is what is called as the momentum density this is what is called the momentum density that is in this case mean the momentum per unit volume, which is equal to the mass density times the mean velocity. So, if I have the mass density from here and I do this calculation and find out the momentum density, the velocity is just this whole thing divided by the mass density in the limit once again as the volume goes to 0 limit.

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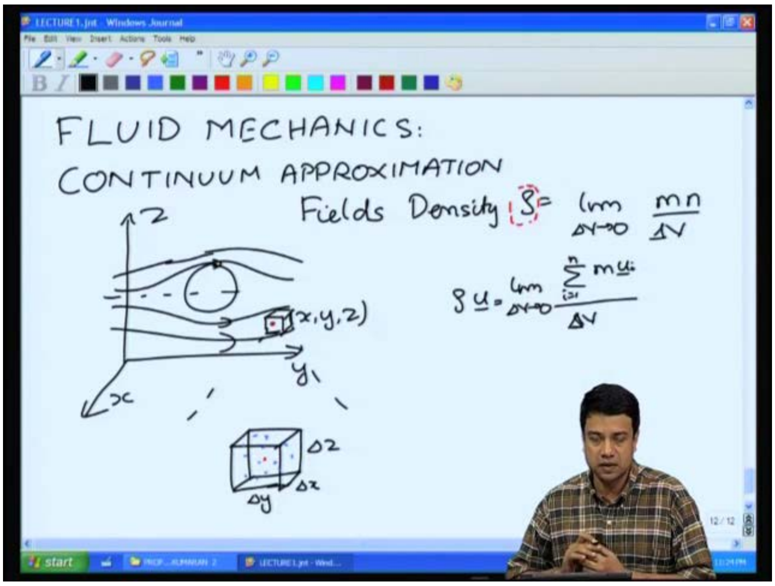

So, that is what is meant by saying the mean velocity at a particular point, so that is the local point wise value of the velocity at that particular location. Note that this is a vector velocity so if I for example, if I take the velocity of molecules in the x direction or in the y direction. Some molecules will have positive velocity some molecules will have negative velocity you have to add all of those up, if the fluid is at rest it does not mean the molecules are at rest. What it means is that the sum of the velocities of all molecules is equal to 0 not that their individual molecular velocities are equal to 0. So, this is a vector addition it gives you a vector momentum density and that divided by the density itself is the vector mean velocity of the fluid, the fluid mean velocity.

Now, of course, you cannot just take this volume going to 0 because as you make the volume smaller and smaller, there is going to come a stage at which, there is either 1 particle in that volume or no particles in that volume. So, there is a limit to which you can take this delta v going to 0, this delta v that you are assuming has to be sufficiently small that it is small compared to what are called the macroscopic scales in the flow. In this particular instant that macroscopic scale will be the sphere diameter. For example, the flow through a pipe that macroscopic scale will the be the pipe diameter. You have to make sure that the length scales of the differential volume are small compared to the macroscopic scales.

However they still have to be large compared to the microscopic scales they have to be large compared to the molecular diameter for liquids, large compared to the mean free path the time between collisions for gas molecules. So, as you make this differential volume smaller and smaller in such a way that the volume the characteristic length is small compared to the macroscopic scales. But still large compared to the molecular sizes you are going to come to a stage at which you can define the density and the mean velocity as quantities, which are independent of the size of the volume with the stipulation that it is large compared to the microscopic scales. But still small compared I am sorry large compared to molecular scales, but still small compared to microscopic scales.

So, the typically in the case of gases or liquids in fundamentals of transport processes 1 we had actually calculated mean the free path of a gas. And that mean free path turns out to be of the order of 0.1 to 1 micrometers. So, that is pretty small any typical length scale that we encounter in process applications starting from millimeters to centimeters meters is going to be sufficiently large, that 1 can use the continuum approximation.

Continuum approximation cannot be used when the size has become sufficiently small that it is comparable to molecular scales, either the mean free path or the distance between or the molecular diameter. In that case we have to actually do a complete molecular based simulation of the entire system. So, that is the only limitation of the continuum approximation. So, here I have defined for you the density the mean velocity as fields and I have sort of tried to explain what those fields mean. The other distinguishing feature of this velocity field is that the velocity is a vector.

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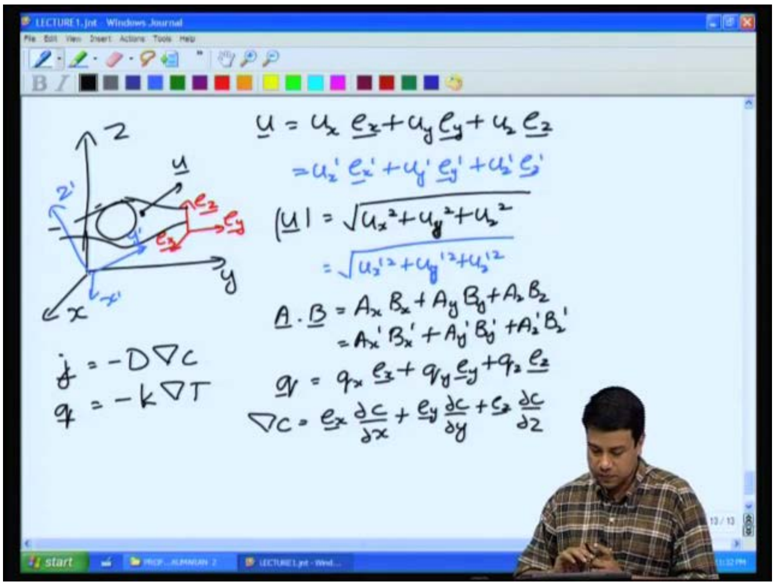

As I said at 1 particular location, the velocity field is a vector it has the magnitude and a direction. And it can be resolved into 3 components, so I can write for example, if velocity u is equal to u x e x plus u y e y plus u z e z it can be written in terms of 3 components, along the 3 different directions where e x e y and e z are the 3 unit vectors, the 3 unit vectors in the 3 directions. So, you can consider this as a combination of 3 scalars u x in the x direction u y in the y direction and u z in the z direction, and that is 1 way to do it to try to write conservation equations for each of this.

However there are properties of vectors which are independent of the coordinate system. So, for example, if I were to use a different coordinate system for this if I were to use a coordinate system consisting of x prime, y prime, z prime with 3 unit vectors in those 3 directions. Then this will also be equal to u x prime times e x prime plus u y prime time e y prime plus u z prime time e z prime, and you get 3 other components. Of course, there should be equations for those as well, but the vector itself has an identity which is separate from the coordinate system that you are using to analyze it.

For example, if I have a flow around a sphere in some particular direction, so in this particular case let us say that this was a velocity field for the flow around a sphere in some particular direction. This vector is a physical quantity that vector has an identity independent of the coordinate system because it is actually gives you the direction of the motion of the fluid at that particular location. So, vectors have identity which are independent of coordinate systems.

So, rather that writing down equations for each particular scalar in this manner, I should be able to write the equations for the entire vector field itself and that will be 1 of the objectives of this course, to derive equations for the vector field itself treating it as an object independent of coordinate systems. For example, there are certain things of vectors which are independent of coordinate systems, for example, the magnitude of a vector is independent of the coordinate system used this also has to be equal to it is independent of the magnitude of the coordinate system used.

Similarly, the dot product between 2 vectors this is also independent of the coordinate system used because the vector is actually a physical thing. So, this I can write it expand it as A x B x plus A y B y plus A z B z this is also equal to this us also equal to A x prime B x prime plus A y prime B y prime plus A z prime B z prime its independent of the coordinate system used. So, in this course rather than work with the components of the vector, we will rather treat the vector as an object itself and try to derive conservation equations for the vector itself rather than for its components. So, that is an important difference between what we will be doing in this course, and what we did previously.

So, there are various vectors velocity is a vector it gives you the direction in which the fluid is moving at that particular location, acceleration is a vector, rate of change of velocity that is a vector it give the particular direction in which there is a change in velocity with respect to time, force is vector, momentum is a vector. There are other vectors which we defined in the previous fundamentals of transport process 1. For example, the heat flux is a vector it gives you the direction in which heat is flowing. So, I can write the flux as q x e x plus q y e y plus q z e z this is a vector gives you the direction in which the heat is flowing.

Similarly, mass flux is a vector we also defined 2 other things as vectors in the previous fundamentals of transport process 1 course, 1 of the important 1s was the gradient. For example, the gradient of concentration grad c this is a vector it is a vector it is a derivative so it's a vector derivative you see the rate at which the concentration is varying as you travel in some direction at some location.

So, this is a flux of the this the fix law of mass diffusion tells you that the flux vector is equal to minus d times grad c flux heat vector equals to minus alpha times k times the gradient of the temperature field. So, these are also vectors. Now, for a scalar quantity its flux is a vector, for a scalar quantity its flux is a vector the concentration is defined at 1 particular location, the flux gives you the direction in which mass is travelling or mass is being transported at that particular location.

So, the flux gives you the direction in which the concentration is being transported, so for a scalar field the vector is a flux for a vector field, what is the flux going to be? So let us discuss that a little bit for a scalar field for a concentration field, the flux is being transported in 1 particular direction at a given location. Let me make 1 notational modification at this point.

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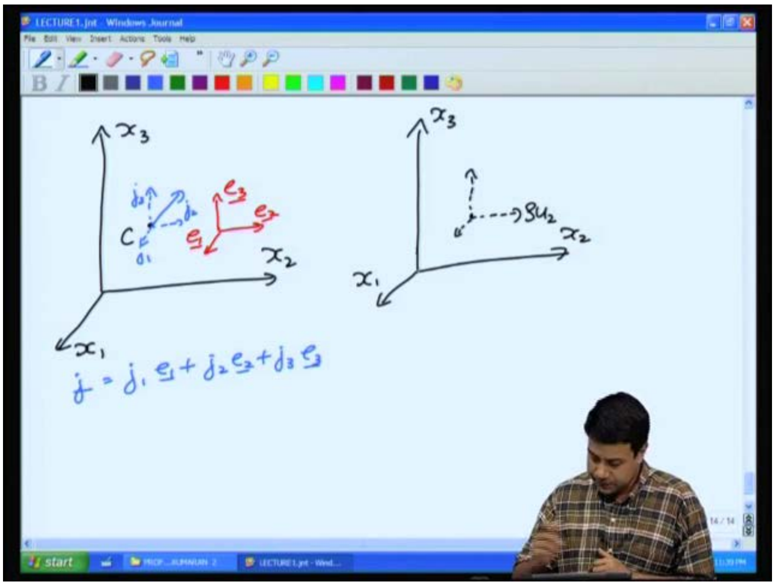

For our future discussion it is going to be more convenient to label the axis as x 1, x 2, x 3 instead of x y and z so from now on we will use these axis labels they will be more convenient for us, when we deal with vector quantities. And at the same location in the same coordinate system, the unit vectors will be defined as e 1, e 2 and e 3. So, these are the unit vectors in the cartesian coordinate system that I will use. And if I can derive the conservation equation in this coordinate system for a vector, without reference to its individual components the conservation equation is a property of that vector itself.

So, its independent of coordinate system it's going to be the same weather I use spherical or cylindrical coordinate systems. So, long as my vector quantities and vector derivatives are correctly framed in that other coordinate system. So, we will talk about conservation equations for the quantity itself not for the coordinate system, when it comes to actually solving a problem. Of course, we have to refer to a coordinate system and you have to have a way of doing that doing the transforming these vectors and their derivatives form one coordinate system to another and that we will do as we go through the course.

So, we have a concentration field at one particular location its a continuously varying function at that particular location. So, if c is the concentration I can also define a flux vector at that particular location a flux vector that is the direction of transport of concentration. As I said the flux vector is defined as j vector is equal to j 1 e 1 plus j 2 e 2 plus j 3 e 3 using a coordinate system where x 1, x 2, x 3 are now the axis. These are the 3 components j 1 j 2 and j 3 give you the fluxes in the 3 directions. So, I have j 1 in this direction j 2 and j 3 the 3 components of this flux.

The resultant of these three gives me the direction in which mass is being transported. So, if I have a cross section perpendicular to this particular direction, we will get the maximum transport of mass across that in this case be the direction in which the mass is being transported. Now, let us do the same thing for not for a concentration field, but for a velocity field. So, in this particular case what is getting transported is momentum, momentum is a vector. So, at each particular point I have one momentum vector so this momentum density vector will be rho times u with its 3 components rho u 1, rho u 2, rho u 3 it has 3 components. Each of these components can be transported so the momentum in the x direction can be transported due to molecular diffusion, momentum in the y direction can be transported due to molecular diffusion, momentum in the z direction can also be transported due to molecular diffusion.

For the momentum in the x direction, the x 1 direction as I have defined it here so let us just go back and simplify and just consider the momentum in the x 1 direction alone. This is the momentum in the x 1 direction this momentum in the x 1 direction can be transported in any one of three directions. So, I can have x 1 momentum being transported in the x 1 direction x 1 momentum being transported in the x 2 direction, and x 1 momentum being transported in the x 3 direction. So, there are 3 directions in which this particular component of the momentum rho u 1 can be transported due to molecular diffusion.

Similarly, for the second component rho u 2 this can also be transported in one of three directions. Similarly, the component in the third direction can also be transported in one of three directions. So, each component of the momentum can be transported in 3 directions and there are 3 such components. So, the total comes out to nine components of this particular flux for this vector momentum. This flux can also be defined as a single object, so in this particular case of the mass flux we had a single object a vector which defined the flux completely. It had 1 direction that direction was the direction of transport of mass because mass was a scalar there was only 1 direction for the transport of mass.

In the case of momentum there are 3 components of the momentum and each component can be transported in any one of three directions. Therefore, you have an object which has nine components, in the case of mass transport there was only one direction the direction of transport because concentration was a scalar. In the case of momentum you have two fundamental directions, one is the direction of the momentum itself because momentum is a vector. The second is the direction of transport that is the direction of transport of momentum, basically results in a force acting on some differential volume. So, let us make this a little more precise.

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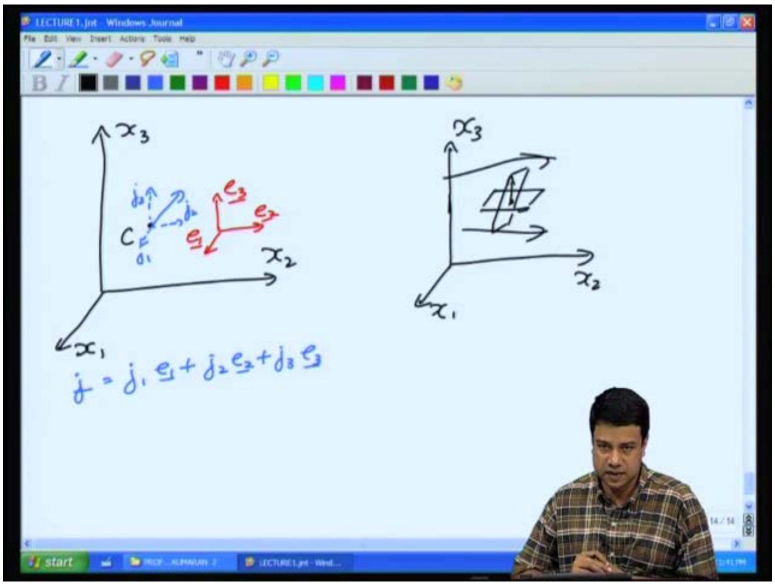

If I have a particular location here and I have a surface, imagine a surface at this particular location within the fluid because of the fluid flow past this surface, there is going to be a force exerted on the surface this imaginary surface within the fluid. This force is due to the transport of momentum, now the force exerted is a vector itself, force exerted is a vector it has three components. However the force itself is going to vary as I change the orientation of the surface. the force itself will vary as I change the orientation of the surface

So if I change if I have the orientation in one particular direction I will get one particular force if I change the orientation to some other direction, I will get some other force. And as I change the orientation in various directions I will get different forces the surface itself the orientation of the surface can be defined, in the terms of the perpendicular to the surface. What is called the unit normal that is unit normal to the surface gives me a direction that unit normal is completely specifies, the orientation of the surface.

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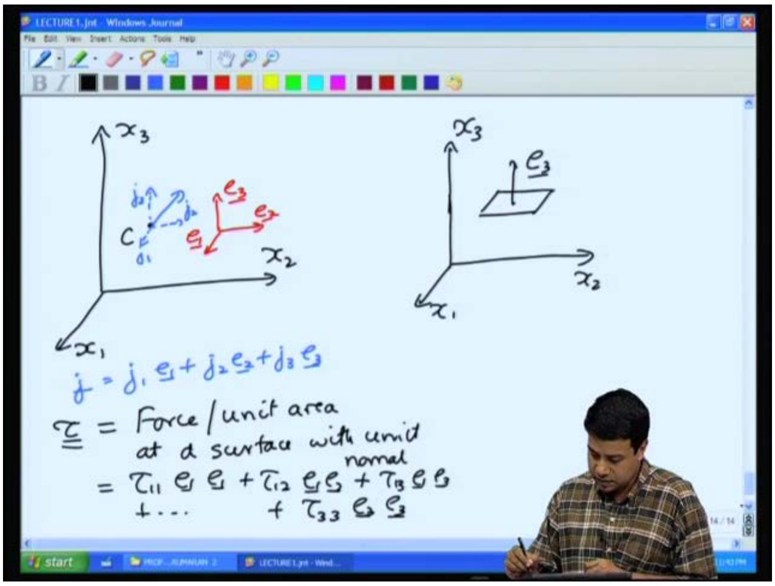

Now, if I have the unit normal for example, in the x 3 direction, so this unit normal is actually e 3 this is e 3. Now, at this surface itself there are forces acting in all 3 directions as I said because of the fluid flow there could be forces acting perpendicular to the surface due to the pressure. For example, there could be forces acting along the surface due to sheer for example, so forces could act in all 3 directions at this particular surface.

Now, the stress is defined such that tau is called a second order tensor, vector has only one fundamental direction. That is the direction of motion of the fluid in case of velocity direction of transport. In the case of concentration this has two directions it's written as the force per unit area at a surface with a unit normal. So, in this particular case j vector had three components j 1 was the flux in the x 1 direction j 2 was the flux in the x 2 direction j 3 was the flux on the x 3 direction. This stress will now have nine components as I said there are 3 components of momentum each component has 3 directions of transport. So, in general I can write this as tau 1 1 e 1 e 1 pus tau 1 2 e 1 e 2 plus etcetera. We have nine components plus tau 3 3 e 3 e 3 so it has a total of nine components, what does each of the components mean?

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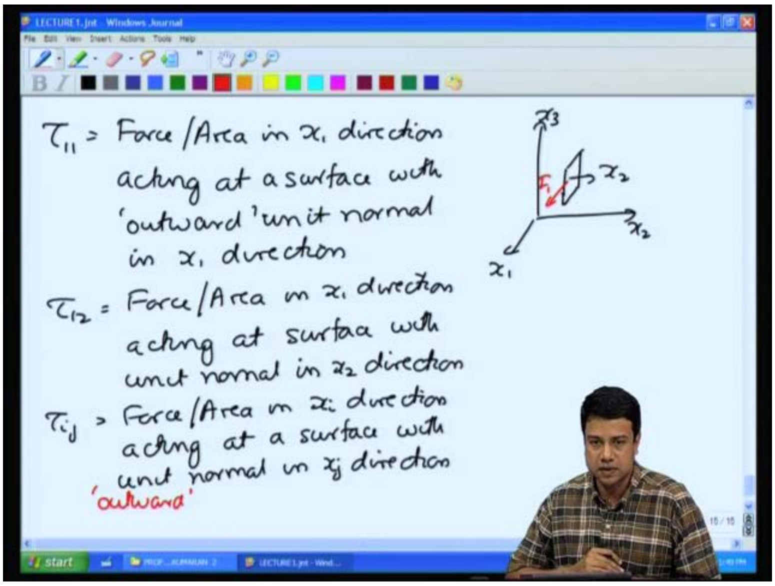

Tau 1 1 is equal to force per area in x 1 direction acting at a surface with in x 1 direction. So, let us spell it out in great detail, so if I want to calculate the component tau 1 at one particular location. Note that this is now a function of both of position and time so at one particular location I go there, I construct a surface with unit normal in the x 1 direction. So, this surface would have unit normal in the x 1 direction and at this surface I measure the force acting on the surface in the x 1 direction divide by the area and I get tau 1 1.

Similarly, tau 1 2 is equal to force per area in x 1 direction acting at surface with unit normal in x 2 direction. So, in this particular case if I want to measure what is tau 1 2 I go to this location once again construct a surface with unit normal in the x 1 direction. And then I measure what is the force acting here in the x 2 direction divide that by the area. So, this basically tau 1 1 gives me the flux of momentum in the x 1 direction, through a surface whose unit normal is in the x 1 direction, tau 1 to gives me I am sorry I should correct this.

So, I construct a surface whose unit normal is in the x 2 direction, so I construct a surface whose unit normal is in the x 2 direction. So, the unit normal is in x 1 x 2 direction and I measure the force in the x 1 direction at this surface, I measure the force in the x 1 direction at this surface divide that by the area and I get tau 1 2. A general formula tau i j is equal to force per area in x i direction acting at a surface with unit normal in x j direction and here I should add outward, outward unit normal in the x j direction.

So, these are what are called second order tensors they have two fundamental directions associated with them in the case of the stress, one is the direction of the force at the surface itself. However the force does depend on the orientation of the surface. So, the other direction is the direction of the orientation of the unit normal to that particular surface. So, one is the direction of transport the other is the direction of momentum itself.

So, momentum flux there are two directions one is the momentum direction the direction in which the force is exerted. The second is the direction of transport that is the direction of the unit normal to the surface across which transport is occurring. So, these are second order tensors and once again these will also come into our analysis in this case, as well we will treat as objects in themselves rather than worrying about their components. And this sort of makes it clear why I emphasize that we will be treating vectors themselves as objects because I told you that the velocity vector for which we are writing the conservation equation contains 3 components. And you would think in principle that I could write equations for each of those 3.

However, the flux in that equation now contains nine components so I have to have constitutive relations for nine of those components separately. If I wanted to write equations in component form whereas, the way that we do it here we will treat the velocity itself as the vector object the flux itself as a tensor object. And derive relations between the flux the velocity, and the flux write down conservation equation constitutive relations for these.

So, that is the major advancement that we will do in this course and for that we have to develop the machinery the machinery of vector calculus, I will come back to more precise definitions of stress later on, but the reason I am bringing up at this particular juncture is to just to emphasize, why we are going to be going through all of the advanced calculus that we will be doing a little later. This is one fundamental second order tensor, that we will be using throughout the course. Another fundamental second order tensor is the gradient the velocity field.

When I talked about concentration fields we had defined the concentration gradient as a vector. So, gradient of concentration, gradient of temperature they are both vectors they have 3 components the component in each direction is the partial derivative of the concentration with respect to variation in that direction. So, partial c by partial x if I go a small distance delta x in the x direction keeping delta y and delta z a constant, the variation in delta c divided by delta x. While y and z are kept constant as the partial derivative. So, this gives you a vector for a scalar field the gradient, I could also take the gradient of a vector field and that would give me a second order tensor. The vector that we will use most often trough out this course is the velocity vector because we are writing conservation equations for the velocity field.

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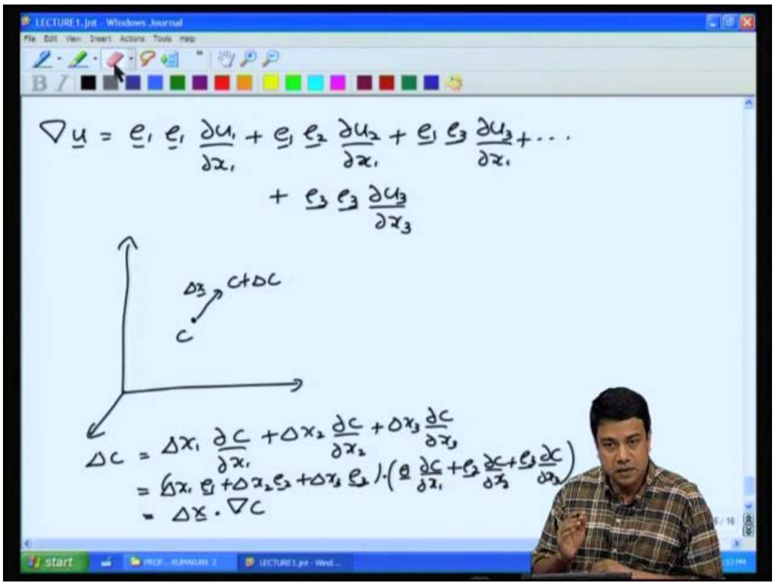

So, I can write the gradient of velocity as a second order tensor, if I write it as gradient of the velocity vector then this becomes e 1, e 2 partial u 2 by partial x 1 plus e 1, e 3 partial u 3 by partial x 1 plus etcetera plus e 3, e 3 partial u 3 by partial x 3. So, this once again has nine components three components of velocity they can vary in any one of three directions. This is a second order tensor the gradient of velocity is a second order tensor it is not the divergence, which is a scalar it is not the curl which is a vector it is the gradient of the velocity, which is a second order tensor.

Physically what does this represent, what did the gradient of the concentration tell you, what did the gradient of the concentration tell you set at one particular point if I go a small distance delta x in some particular direction the variation in concentration is equal to. So, if I have c and c plus delta c here if I go small distance delta x in one particular direction in this concentration field. There is going to be a variation in the concentration that variation in the concentration is equal to delta x 1 partial c by partial x 1 plus delta x 2 partial c by partial x 2 plus delta x 3 partial c by partial x 3 which is equal to delta x 1 e 1 plus delta x 2 e 2 plus delta x 3 e 3 dotted with e 1 partial c by partial x 1 plus e 2 or I can write this as delta x dotted with gradient of c.

So, the variation in concentration when u move a small distance from x to x plus delta x in some particular direction is the dot product of the distance travelled times the gradient of the concentration field. The same thing for the velocity field, rather than dealing with a scalar I am dealing with a vector, but the principle is exactly the same. I have a velocity field velocity vector which is defined at each particular location in this field.

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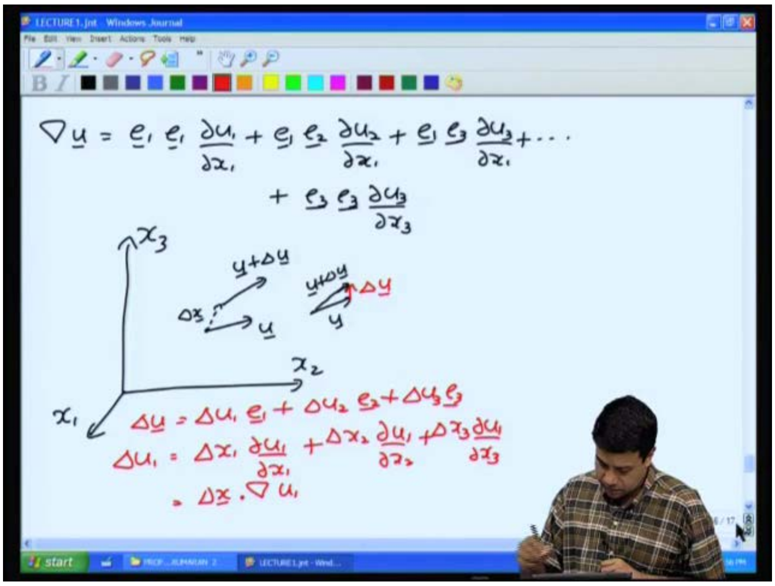

At each position there is a single valued velocity single value implying that the velocity at each particular location is uniquely specified. I have velocity at one particular location u vector, I go small distance delta x in one particular direction. And when I travel this distance at this new location the velocity is equal to u plus delta u the velocity has changed by an amount delta u. Delta u now is a vector delta u is the change in velocity that is the new velocity minus the old velocity.

So, operationally how you would calculate it is using the rules of vector addition. So, this is u and this vector I superpose them at the same initial location, the vector difference now is this one the resultant between the two, this vector difference is delta u it has three components delta u 1, delta u 2 and delta u 3. It has 3 components delta u is equal to delta u 1 e 1 plus delta 2 e 2 plus delta u 3 e 3. Each of these can be defined in terms of the gradient of the velocity field.

So, for example the delta u 1 I can expand it out and write it as delta u 1 is equal to delta x 1 partial u 1 by partial x 1 plus delta x 2 partial u 1 by partial x 2 plus delta x 3 partial u 1 by partial x 3. Or in analogy with the concentration field this is just equal to delta x dotted with grad of u 1 the component u 1.

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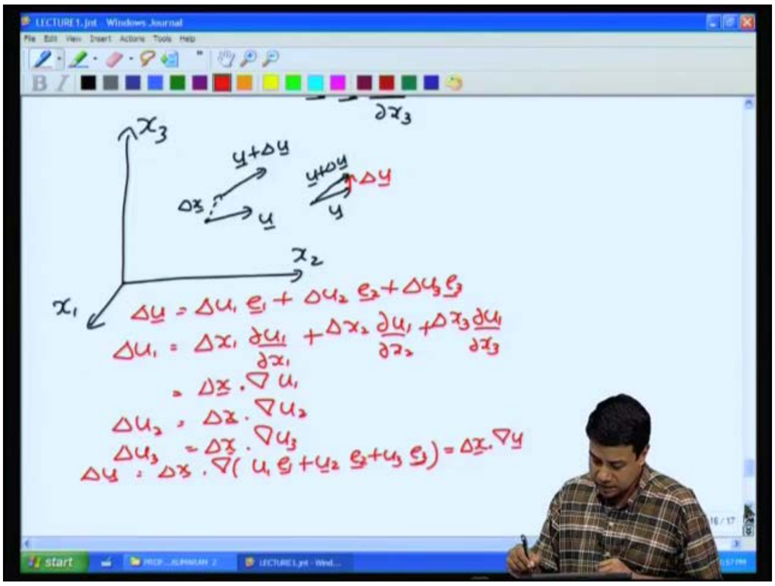

Similarly, delta u 2 is equal to delta x vector dotted with grad of u 2, delta u 3 delta x vector dotted with grad of u 3 add up all of these delta u is equal to delta u 1 e 1 plus delta u 2 e 2 plus delta u 3 e 3. So, that means that delta u is equal to delta x dotted with grad of u 1 e 1 plus u 2 e 2 plus u 3 e 3 which is equal to delta x dotted with grad of u were this gradient of u now contains nine components.

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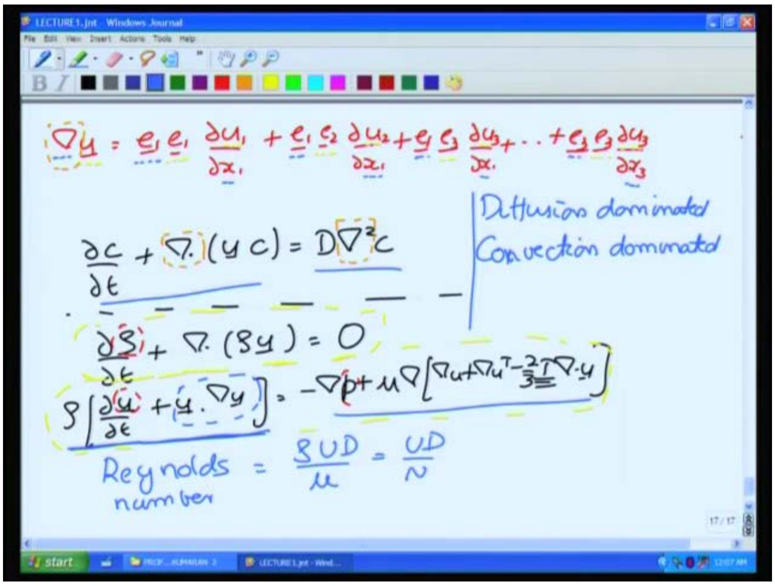

This gradient of u grad u as I would defined it before is equal to e 1 e 1 partial u 1 by partial x 1 plus e 1 e 2 partial u 1 by partial, partial u 2 by partial x 1 plus e 1 e 3 partial u 3 by partial x 1 plus etcetera plus by partial x 3. So, it now has nine components it has 2 indices 2 directions, the first is the direction of the gradient in all of these cases the first one represents the direction of the gradient, the direction of with which you are taking the derivative the direction of with which you are taking the derivative.

The second direction is the direction of the velocity vector itself, this velocity vector. So, these are fundamental tensors that we will be dealing with throughout this course. So, I thought, I would take this opportunity here to introduce them in order to give you their physical understanding. Now, once we have these tensors we now have to write down conservation equations as you can see there is already one thing that has already appeared here is this gradient this is a vector derivative it contains three components. But it can be considered an object in itself the vector derivative the gradient is one such there is another one which is the divergence and you already know there is a curl.

So, that is a way of taking derivatives in three dimensional space vector derivatives without reference to the underlined coordinate system, in terms of these vector derivatives we can write down our conservation equations. We already did that in part one where we wrote down for example, for the concentration field we wrote down an equation as partial c by partial t plus divergence of u c is equal to D del square c.

These are all vector derivatives even though we actually calculated them in terms of their components. In this course we will be calculating them as vector derivatives themselves without any reference to the underlined components. So, that is one part of it the other part of it so once we calculate when once we use these vector calculus, we will be able to get equations for the velocity field that are similar to these, they are called the Navier stokes equations.

And I will just write them down here just to introduce them at this point, for an incompressible fluid or let us just take it for a gentle fluid. These have the form by partial t plus divergence of rho u is equal to 0 and rho into partial u by partial t plus u dot grad u minus grad p plus u divergence of you do not really have to have to try to understand these equations too much. Except to realize that there is an equation for the density field this first one is the equation for the density field. The second one is the equation for the momentum field and these contain variables the density rho, the velocity vector u vector and there is a also a pressure in this case p. So, they contain 3 variables.

And they sort of look similar see the left hand side of the momentum equation here, the left hand side of this momentum equation looks similar to this one. The right hand side looks similar to this one important difference there is a pressure in the momentum equations, which is not present in the concentration equations. In the fundamentals of transport process one we actually derived the value of this pressure we actually derived an equation for the pressure using shell balances for the flow through of pipe, here we will do more detailed derivation of the vector equations themselves.

And once we derive these equations using the vector calculus that you develop in the next few lectures, we then have to solve these equations. As usual there is no simple way to solve these equations firstly thee equations are non-linear, in this particular case you have a term that goes as u dot del u. So, this is a non-linear equation so there is no general solution unlike the concentration equation, where you do have a general solution.

And the strategy that we will use is similar to what we use in the last the fundamentals of transport process one, we derive equations in different limiting situations the first limiting situation is diffusion dominated. As I said the left hand side of the equation is basically the convection term and so, if your system is diffusion dominated then the convection terms can be neglected, and you can just solve the diffusion equation itself.

we did that for the concentration field and temperature field, we found out different ways of solving the diffusion equation using separation of variables for example.

Similarly, in the diffusion dominated regime we will solve the equation for the velocity field without taking into count the convection terms. The second is where convection dominated, where we neglect the diffusion terms, but we cannot neglect the diffusion terms. As we saw in the case of the concentration equation, when you neglect the diffusion terms you neglect the highest order derivatives. And therefore, you cannot satisfy all boundary conditions, even when convection is dominating. Convection can only transport concentration along the flow, whereas for transport of mass momentum to a surface at the surface itself there is velocity perpendicular to the flow. So, you cannot have convection acting at the surface itself.

Transport at the surface has to take place due to molecular diffusion and even though convection is dominating, still as you go very close to the surface the molecular diffusion mechanism has to kick in order for transport to take place. Therefore, diffusion will still be important over a small length, whose thickness is determined in such a way that over this length scale, there is a balance between convection and diffusion. Boundary layer theory that we had used in part one, in this particular case it's a little more complicated because even if you neglect diffusion you can still get non trivial solutions because there is a pressure here.

However with just a pressure field you cannot enforce tangential velocity boundary conditions at the surface, for that you still need to postulate a boundary layer. The ratio of the convection and diffusion in this case is equivalent of the Peclet number except that instead of mass or energy diffusion, you have momentum diffusion kinematic viscosity u by rho or u. When the Reynolds number, the Reynolds number is also as you probably know the ratio of inertia and viscosity, when the Reynolds number is small, Reynolds number is small when the velocity is small the characteristic length is small, or the kinematic viscosity is large for highly viscous fluids of a small length scale or velocity scales.

You can neglect the convective terms the diffusion the Navier stokes equations and solve for the diffusion terms alone, the viscous terms alone. When the Reynolds number is large you can neglect the diffusion terms in most of the flow, except very near boundaries because if you neglect diffusion very near boundaries, you cannot enforce the tangential velocity boundary conditions just as you cannot enforce the concentration boundary conditions, or the temperature boundary conditions at high Peclet number. In that case it is necessary to postulate a boundary layer where viscosity is still important even though the Reynolds number is high because the Reynolds number is based on the boundary layer thickness, is such that there is a balance between the convection and diffusion.

So, all of those techniques that were developed for convection dominated and diffusion dominated transport in part one, still apply except that we now have to use it for vector fields. So, that is a broad summary of what we will be trying to do in this particular course. Methods are different we will deal with vectors and tensors as objects within themselves, derive constitutive equations for those objects in themselves, but the physical incite is much the same. There is situations where convection dominates there are situations where diffusion dominates and the interplay between convection and diffusion ultimately determines the rate at which the transport is going to take place.

So, next class we will start developing the machinery of vector calculus that we will need in order for us to derive conservation equations and then solve them later on. So, with that I will end this lecture and we will start our analysis of vector calculus, in the next lecture. So, we will see you then.