Fundamentals of Transport Processes II
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Lecture - 17
Flow around a Sphere

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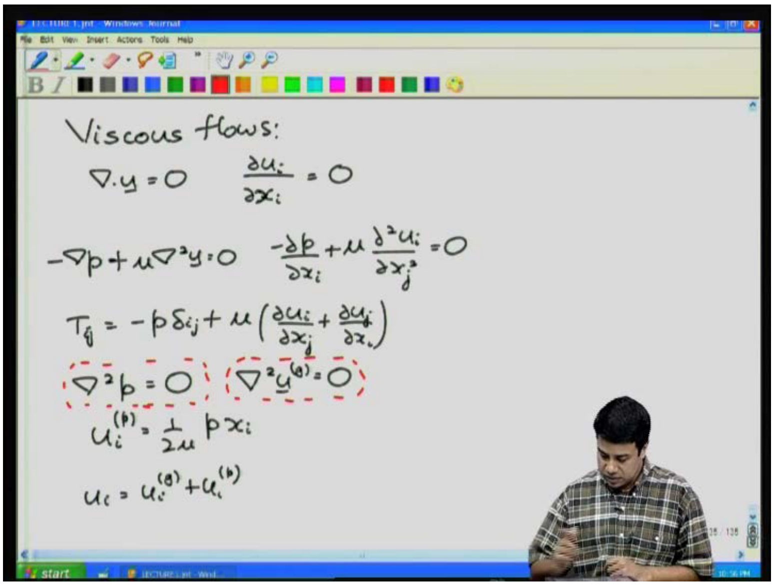

This is lecture number seventeen of our course on fundamentals of transport processes welcome. We were going through viscous flows in the limit of low Reynolds numbers, where inertial effects are negligible, and so the inertial terms in the momentum conservation equation, can be neglected. And I had shown you that for these flows, the conservation equations are; mass conservation equation, the divergence of velocity is equal to 0, and the momentum conservation equation without the inertial terms. So, the divergence of the total stress is equal to 0 or minus partial p by partial x i, and the stress is given by T i j minus p delta i j plus mu times, two mu times the symmetric part of the rate of the tenser, so mu times partial u i by partial x j plus, and we had discussed a scheme for solving these equations. The first thing you to you take the divergence of the momentum conservation equation, and you will get del square p is equal to 0. That is because when I take the divergence of the momentum conservation equation.

The divergence of del square u, is the same as the Laplacian of the divergence of u, which is 0. So, to solve the equation, first I solve for the pressure field del square p is equal to 0, and then if you know the pressure, you can insert that into the momentum conservation equation to get an in homogenous equation for the velocity. So you have an in homogenous equation, for the velocity minus grad p plus mu del square u is equal to 0. The way to solve it is by separating out the velocity into a general solution plus a particular solution. The general solution is the solution of the homogenous equation, without the in homogenous part. So for the general solution, the equation is del square u general is equal to 0; that is the general solution satisfies the homogenous equation, without the pressure term.

The particular solution is any one solution, which satisfies the entire equation, you can choose any one; the one that is most convenient to you. The general solution contains the constants of integration, particular solution does not have to contain any, and we found one particular solution that satisfies this equation, and that is that the particular solution, is equal to one by two mu p times x i, so that was the particular solution. And this total solution u i is equal to u i general plus u i particular, so that is the final solution that you get. So for these viscous flows, the problem reduces to one of solving two Laplace equations; one for the scalar pressure, and the other for the vector velocity. You can solve these two Laplace equations, and then put them in, and then satisfy boundary conditions you have the entire solution, so that is the way that we will try to solve these equations.

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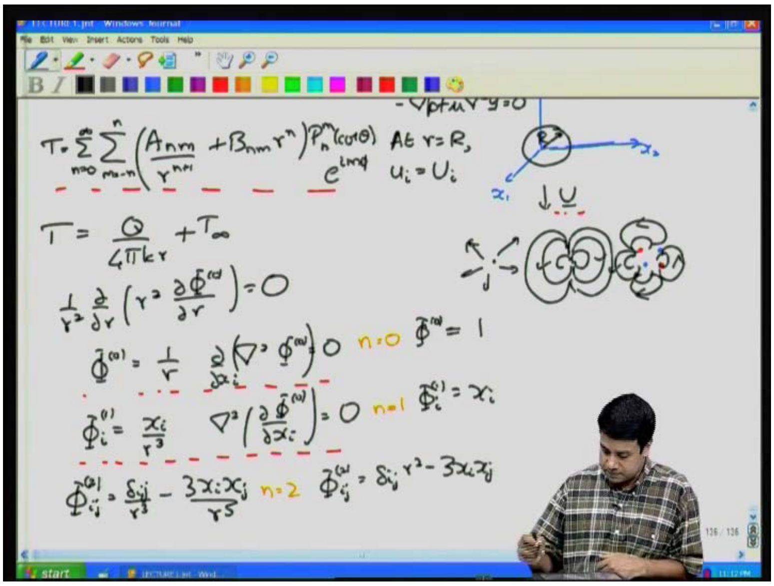

For definiteness, let us look at a specific problem, and that is a sphere settling in a fluid, a sphere of some radius r settling in a fluid, with some velocity capital u, and we want to find out what is the velocity field around the sphere, and what is the net force exerted on the sphere, due to the fluid around it. Since you have one sphere in a settling fluid and there is no other boundaries, it is most convenient, to use a coordinate system, whose centre, is at the centre of the sphere. If you do that, the system is spherically symmetric, the configuration is spherically symmetric. The sphere velocity, of course, it is not spherically symmetric, it is in one direction alone. So, let us choose a coordinate system, which is centered at the centre of the sphere. And as I told you, we solve this in vector notation, without reference to the underlined coordinate system. So, I have to solve the stokes equations, for this case del dot u is equal to 0 minus grad p plus mu del square u is equal to 0, with the no slip condition on the surface of the sphere.

No slip condition, the velocity of the fluid, on the surface of the sphere, is equal to the velocity of the sphere itself. So, at a surface; that is r is equal to capital R, the velocity of the fluid has to be equal to the velocity of the sphere itself; that is that the fluid velocity is equal to the sphere velocity, the sphere velocity is capital u as specified over here, in one particular direction. We do not have to specify that direction beforehand, we will just assume that it is moving in one particular direction. So, we have to obtain solutions for the velocity field, and the pressure field, Laplace equations for the pressure, and for the general solution for the velocity field. So, we have to obtain solutions for del square p is equal to 0 del square u i general is equal to 0. Put them both together, and I get the final solution, and then select the constants in that solution, in order to satisfy the boundary condition. The way to obtain general solutions for the Laplace equation in terms of general scalars vectors, and tensors, we had discussed in the last lecture. If you recall, in our previous course on fundamentals of transport processes one.

We had actually obtained the general solution of the Laplace equation in two ways; one was using separation of variables, and the r theta phi coordinates, and that case we got a solution in terms of the Legendre polynomials in the theta, in the azimuthal angle and cos and sin in the meridional direction, and in the radial direction you had groin and decaying harmonics. The decaying harmonics decreased with r as 1 over r power n plus one, the groin harmonics increased as r power plus n. So, we had solutions that was the form A n m by r power n plus 1 plus B n m r power n p n m of cos theta times e power i m phi. So, that was one way that we had obtained for a particular case of a temperature field that satisfied the Laplace equation, where A n m and B n m are a constants, and m goes from minus n to plus n, and n is equal to 0 to infinity. So, those were the solutions that we obtained by separation of variables techniques. We also obtained solutions, by superposing sources and syncs.

So the first solution for n is equal to 0 1 by r was the solution due to a point source of heat. Solution due to the point source of heat was equal to q by 4 pi k r plus t infinity, so that was for point source. Then if you superpose a source and a sync, close to each other, you get a dipole, and the temperature field due to that decreases as 1 over r square. You get three such linearly independent solutions; one in which the source and sync are separated along the z axis, second along the x, and the third along the y axis, so that gives you the dipole, and then you also get the quarter pole movements, in which you have two sources, and two syncs, in such way that the net source, is equal to 0, the net dipole is equal to 0, and you are ended up with a quarter pole movement. So, those were two ways that we did in the course on fundamentals of transport process one. In this lecture, in the previous lecture, in our present course, we had looked at another way of getting solutions, which unified both of these. So the way of getting the solution, was to first take the source solution for the source, as the fundamental solution. We know that in a spherical coordinate system, if the solution is spherically symmetric; that is there is no dependence on theta and phi coordinates.

The equation, the Laplace equation reduces to, and that has a solution, which to within an undetermined constant is just 1 over r. So, this satisfies phi naught satisfies the equation del square phi naught is equal to 0. To get a vector solution, I take the gradient of this entire equation; I take the gradient of this entire equation. This is still equal to 0, if you take, if del square phi naught is equal to 0, its gradient is also equal to 0. Therefore, you get del square partial phi naught by partial x i is equal to 0, just by interchanging the order of differentiation. So, obviously the gradient of phi naught is also a solution of the Laplace equation, and that is a vector solution, it is the gradient of phi naught, so it is a vector solution. And we had that, we had obtained that constant once again to within, an undetermined constant as x i by r cubed. If you recall when we did this, we used the identity that, partial r by partial x i is equal to x i by r, because r is equal to square root of x 1 square plus x 2 square plus x 3 square.

And then to get the next solution, you can once again take the gradient of this vector, take the gradient of this vector. So, partial by partial x j of del square phi 1 i is equal to 0. So, partial by partial x j of phi 1 i also satisfies the Laplace equation i here. Therefore, you get the second solution phi 2 i j is equal to delta i j by r cubed minus 3 r power phi. This is the solution of the equation del square of partial by partial x j of phi 1 i is equal to 0. And then you can take one more gradient, so this solution phi 2 is a second order tenser, take one more gradient you get a third order tenser and so on, so you can get a infinite series of solutions. As I showed you, this one, this first term here, corresponds to a source, the second term, is the dipole, the vector solution, it has three components, three solutions where the dipoles are arranged along the x y and z axis or the x 1 x 2 and x 3 axis. The next, the second order tenser solution is a quarter pole, it is symmetric, and traceless as I showed you, because the trace, of the second order tenser is just del square of phi naught, which has to be 0.

Symmetric and traceless, it has five independent components, and therefore, it exactly corresponds to the five components of this solution, for n is equal to 2 and m is equal to minus 2 minus 1 0 plus 1 and plus 2. So, these were the solutions that were obtained by taking successive gradients of that scalar source solutions. These are the decaying harmonics, the source decreases as 1 over r, the dipole solution decreases 1 as over r square x i by r cubed. So, x i is x 1 which is r sin theta cos phi x 2 is r sin theta sin phi and x 3 is r cos theta divided by r cubed gives me 1 over r square, physically it is clear. Every time you take a gradient the dimension has to go as length inverse times the previous one, because the gradient has dimensions of 1 over length. So, if my fundamental solution is 1 over r, if I take one gradient I should get 1 over r square, take two gradients I should get 1 over r cubed and so on. So those were the decaying harmonics, and we are also take from the decaying harmonic solutions, I had also shown you how to get a series of growing harmonics. So, this was for n is equal to 1, the growing harmonic was just phi naught is equal to just a constant, within a multiplicative constant is just one.

Similarly, phi 1 I am sorry this is n is equal to 0; n is equal to 1 this x i and for n is equal to 2 you get the next solution, so let me just write it here, is equal to delta i j r square minus 3 x i x j. So, the growing harmonics phi naught is just a constant, just constant temperature of course, satisfies Laplace equation trivially, phi 1 is a vector which increases proportional to r, because x i at x i is r cos theta for the z coordinate r sin theta cos phi and r sin theta sin phi. Similarly, you can easily see that phi 2 increases proportional to r square. This was what we had discussed in the previous lecture, hope you had an opportunity to revise it, rather heavy going, but as you will see it makes our subsequent calculations, far more simpler, but there were two reasons why I went through this; one is, the calculations that we do a little later, will become far more simpler. The second really is to assign a physical interpretation to each of these harmonics. So, as we will see, in we saw in the fundamentals of transport processes one that, the first solution n is equal to 0, corresponds to a point source, where there is a net heat coming out, of that point source.

The first solution corresponds to a dipole, which is a source and a sync of equal strength. In the limit as the distance between them goes to 0, but the dipole movement, the force, the source times the distance, remains finite, you get a dipole. And so we had drawn these figures in the previous lecture for a source. For a dipole basically you get something that goes like this. The same as the dipoles in magnetic fields as you recall. Then you have a quarter pole, which is two sources and two syncs, so I have to have two sources and two syncs, in such a way that the dipole movement is equal to 0, so I will put red as the sources, I will put two sources in red, and two syncs in blue. The distance between these two have to be equal, just change this, and then you will end up with a field that looks something like this, it has a four four symmetry, and so on and so you have higher order terms.

So, you have this with four four symmetry, and then you have higher order terms. So, these were the physical interpretations that we gave to all of these terms, in this expansion. Similar physical interpretation holds for these spherical harmonic expansions here; the source and sync terms, and we will see that, when we when we go through the momentum conservation equation, this physical interpretation will come in as a point force, a point force dipole and so on. So, those are the physical interpretations of these terms, and we will see that a point force once again, is like a point source, it gives a disturbance at goes as 1 over r, and dipole goes as 1 over r square and so on. So, that is the reason that we went through this in some detail; the first is that we are dealing with these harmonics, as vectors in themselves, without reference to r theta or phi, so that is one important point. And the other thing is that, because we are dealing with vectors, we can easily get vector solutions, to the equations, tenser solutions to the equations and so on.

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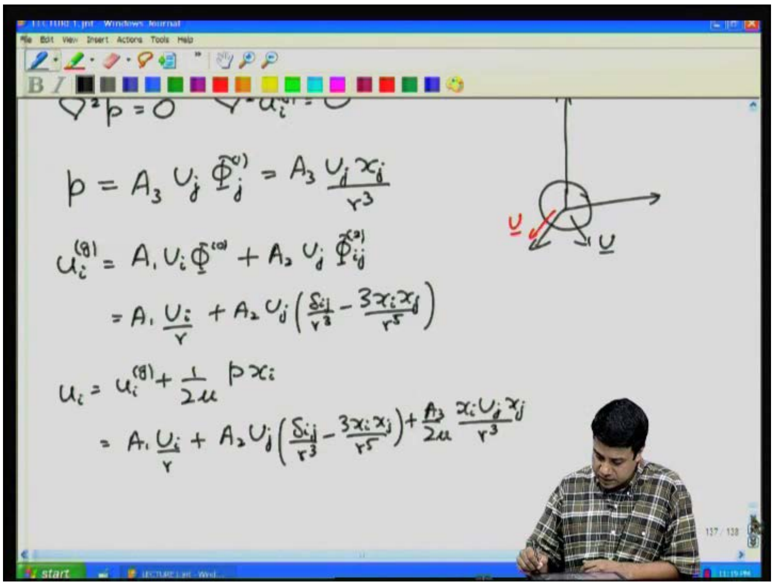

So, one is that all solutions had to be linear in the spherical harmonics, so I was trying to solve equations del square p is equal to 0, and del square for u i general is equal to 0, so those were the two equations I was trying to solve; that means that both p and u i general, have to be some linear combinations, of these spherical harmonics. The spherical harmonics are complete and orthogonal; therefore, the there is no projection of one spherical harmonic on to the other. Therefore, I have to a linear combination, which includes each one, a constant times each spherical harmonic individually, you cannot have products of spherical harmonics in the expansion. So, that was one thing that I used, the solution can always be expanded, in the series of spherical harmonic solutions. The other thing that I can use, as I emphasized two lectures ago, is that if I have, a sphere moving with a velocity u, then the solution for both the velocity and the pressure, have to be linear in u. So, if I multiply u by factor of two ,the velocity and pressure at each point will also multiplied by factor of two. If I reverse the direction of u, the velocity and pressure at each point, within the fluid will also reverse, this sphere is spherically symmetric.

So, if for example, the velocity direction was changed to this one, if the sphere moving in a velocity, which was moving in this direction, the velocity of the fluid and pressure at each point should once again be linear, in this vector u. So, I have to get a solution, which is linear in the spherical harmonics, as well as linear in this vector u, which is the direction in which the, sphere is moving, the velocity with which the sphere is moving .So, my pressure, has to contain terms that are linear in u, as well as linear in one of the spherical harmonics. Pressure is a scalar, u is a vector. The only way that I can get a pressure which is linear in u, and linear in one of the spherical harmonics is to write this pressure, as some constant times u dotted with a vector spherical harmonic. If I multiply this by a scalar, I will get a vector, and pressure is a scalar, so it cannot be a scalar. If I dot this with a second third higher order tenser, I will get something that is not a scalar. So, the only possible combination, which was linear in u, as well as linear in one of the spherical harmonics, is this one; the vector u dotted with the first solution, the dipole solution.

So, this is equal to a 3 u j x j y r cubed, because x j by r cubed is the first solution. Of course, the first solution can be taken either as plus x j by r cubed or minus x j by r cubed, will not make a difference, because that is only going to change the sign of a 3 constant. The final solution which satisfies the boundary conditions, will end up remaining the same, regardless of whether I take this vector solution as plus x j by r cubed or minus x j by r cubed .Similar thing I need to do for the velocity, the general part of the velocity u i general, u i general is also linear in capital U, the vector capital U, and its linear in one of the spherical harmonics. So, how do I write that, it has to be linear in u, one way to get it is by taking u itself, and multiplying it by the scalar spherical harmonic. The scalar spherical harmonic is, phi naught, so this gives me a vector, I multiply u scalar by, I am sorry I multiply the vector u by a scalar to get a vector. I can also get this by dotting u with a second order tenser. If I take u and dotted with the second order tenser I will get a velocity that is a vector.

No further solutions are possible, I cannot dot it to the third order tenser, because that me a second order tenser; whereas, my velocity is a vector, similarly and so on, so this is a complete solution; that is the dipole I am sorry that is a quarter pole spherical harmonic phi 2 i j, so these are the complete solutions. And now I have to determine, in these solutions, the constants a 1 a 2 and a 3. I know that u i is equal to u i general plus 1 by 2 mu p times x i, which is equal to a 1 u i by r plus a 2 plus a 3 by 2 mu. So, that is my complete solution for the velocity, in which I now have to determine the coefficients a 1 a 2 and a 3. Now there are two things here, so first of all, these coefficients can of course, be determined from the boundary conditions, but before I go on to the boundary conditions, I still have to make sure that the solution satisfies the incompressibility condition; that is that, it has to satisfies partial u i by partial x i is equal to 0; the incompressibility condition, we have not yet satisfied that with the solution.

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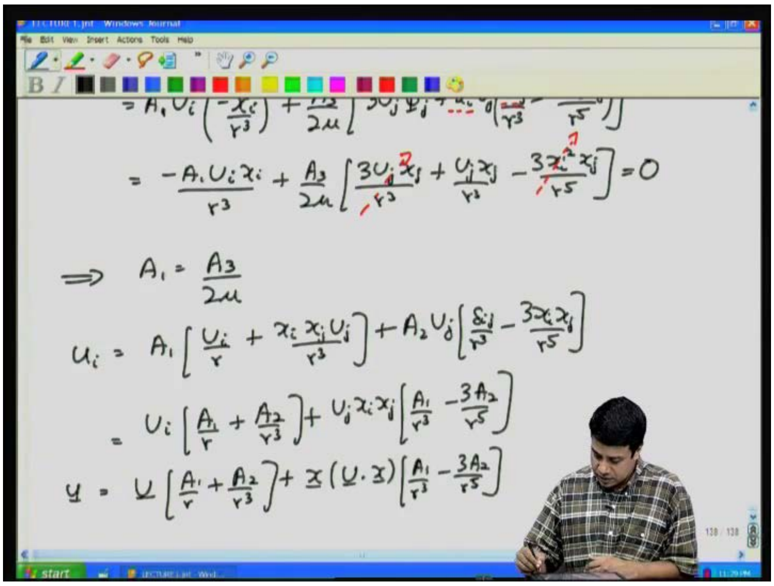

So partial u i by partial x i is equal to 0, so I just take the divergence of the solution that I already have. There is one simplification that you can make here, which I will briefly go through. You know that u i is equal to a 1 u i times phi naught plus a 2 u j times phi 2 i j plus a 3 by 2 mu x i times u j times phi 1 j. If I take the divergence of this, what I will get is that; partial u i by partial x i is equal to a 1 u i partial phi naught by partial x i plus a 2 u j partial by partial x i of phi 2 i j plus a 3 by 2 mu. Now I have two terms here, which I have to use the chain rule for differentiation, so let us just go through that. So, the first term is, I take partial by partial x i of this one, of this x i there, partial x i by partial x i is just delta i i, so we will get u j phi 1 j plus x i u j partial phi 1 j by partial x i. This term is identically equal to 0, this term is identically equal to 0 the reason for that is as follows. We know that partial phi 2 i j by partial x i phi 2 is obtained by taking two derivatives, two gradients, on the fundamental solution.

So, this is equal to partial by partial x i of partial by partial x i of partial phi naught by partial x j, is obtained by taking two derivatives, and I can interchange the order of differentiation here to get partial by partial x j of del square phi naught, this is equal to 0, because phi naught is a solution of the Laplace equation. So, that term in the gradient of the, in the divergence of the velocity field, is identically equal to 0, and because that I am basically left with the divergence of the other two terms. So when I enforce the incompressibility condition, I can neglect this middle term in the expression for the divergence. So, this becomes a 1 u i into partial phi naught by partial x i is just phi 1 i, which is minus x i by r cubed phi naught is 1 over r when I take the derivative of that I get minus x i by r cubed plus a 3 by 2 mu into 3 u j phi 1 j plus x i u j into partial phi 1 j by partial x i is basically just. So, let us just work out partial phi 1 j by partial x i is just phi 2 i j phi 1 j is x i by r cubed so I will get delta i j by r cubed minus 3 x i x j by r power of 5 a delta i j by r cubed minus 3 x i j by r power 5.

So, this is minus a 1 u i x i by r cubed plus a 3 by 2 mu into 3 u j x j by r cubed plus x i times delta i j is just x j, so in this I have the second term here I have x i times delta i j which is just x j so I get plus u j x j by r cubed minus 3 x i square x j by r power 5 x i square x 1 square plus x 2 square plus x 3 square is just equal to r square. So, I have x i square by r power 5 is just 1 over r cubed, and because of that you can easily verify that these two terms will cancel out, and the other two terms are just minus a 1 u i x i by r cubed plus a 3 by 2 mu u j x j by r cubed, both of them are u dot x by r cubed. Therefore, this basically tells me that if this is equal to 0, it implies that a 1 is equal to a 3 by 2 mu. So, the equation for the velocity field now becomes a 1 into u i by r plus x i x j u j by r cubed plus a 2 into a 2 u j into. So, in this expression for the velocity field, I just substituted that a 3 by 2 mu is equal to a 1 in order to satisfy the incompressibility condition.

So, I get a 1 u i by r plus a 1 u j x i x j by r cubed plus this second term. So, that is the expression for the velocity field, I have two constants of integration still remaining a 1 and a 2. Let us just write this in a slightly different form, there are two components here; one is proportional to u i or delta i j u j, the other is proportional to x i x j u j that is x into x dot u, so I have a 1 by r plus a 2 by r cubed plus u j x i x j into a 1 by r cubed minus 3 a 2 by r power 5. Now this is the velocity field, it has two components, so let me just write this out in little more detail, so that becomes clear to you. If I write this in vector form, this becomes u vector into a 1 by r plus a 2 by r cubed plus x i into u j x j is equal to x vector into u dot x x vector into u dot x. So, there are two components, one which is along the u vector itself, the other which is along the x vector, because the i is on x, enforce boundary conditions. So, this was my expression for the velocity; one component along u vector, the other component along the x vector.

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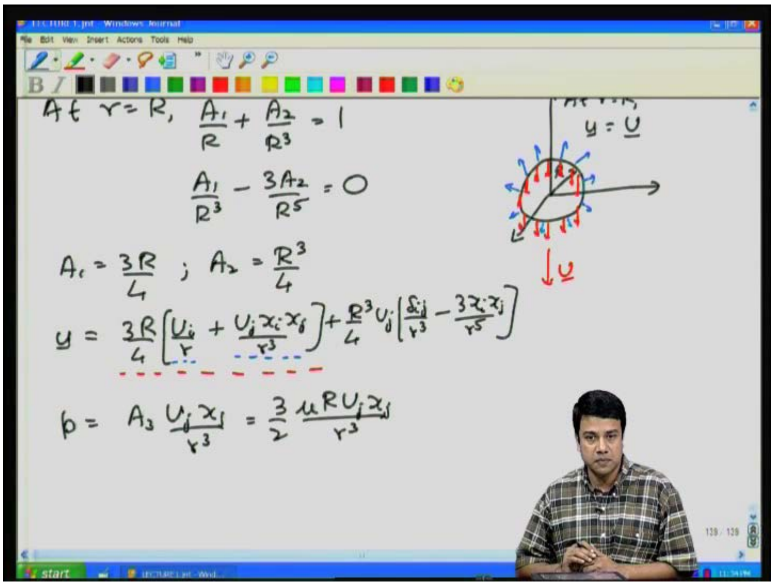

So, the boundary conditions on the surface of the sphere, at r is equal to capital R, u is equal to capital U; that means that u has a component only along capital U vector, so that if my capital U vector is in this direction, if my capital U vector is in this direction; that means that at the surface of the sphere, the velocity has to be equal, and only in the u direction. It is only along u and it has to be equal everywhere on the surface of the sphere, which is the x direction the x direction is along the displacement vector. If I plot the x direction, along the surface of the sphere, it will be along the displacement vector, from the origin at various locations, so that will be the x direction the boundary condition requires that, the velocity is only along u, because the sphere is moving only along that direction; that means if the component of the velocity along the x direction is identically equal to 0, there is no velocity component along the x direction.

So what that means is that, this coefficient is equal to one, because at r is equal to capital R. This coefficient is equal to 1 by r cubed is equal to 1, because that is the velocity along the x direction, and the other component has to be equal to 0, the velocity along the x direction has to be equal to 0, because the velocity is purely along the u direction, only on the surface of the sphere, that is what the boundary conditions tell me. Therefore, I should get a 1 by r cubed minus 3 a 2 by r power 5 is equal to 0. So, you can solve this, and you will get a final solution as a 1 is equal to 3 r by 4 and a 2 is equal to r cubed by 4. You can easily verify that this solution a 1 is equal to 3 r by 4 and a 2 is equal to r cubed by 4, satisfies these two conditions; that is if at the surface of the sphere the velocity is only along the u direction, component of velocity along the x direction is identically equal to 0. Therefore, the velocity fluid velocity vector, is equal to 3 r by 4 into u i by r plus u j x i x j by r cubed plus r cubed by 4 u j into delta i j by r cubed minus 3 x i x j by r power 5. So, that is my final solution for the velocity vector, and this velocity is at each point in the fluid. So, this gives me the complete velocity distribution, at all points within the fluid.

You can see that this velocity vector depends only upon u vector and x vector, not on their individual components. So, in that sense, this is far more general than the velocity that you would have got, by trying to separate out the equation into three components in writing down an equation for each component and so on, so in that sense it is a very general solution. The other point it contains two terms; one of which goes as, 1 over r .Out of these the first term is actually a source term, the first term is actually a source term, which was there in the original solution of the general velocity, the second term actually comes out due to the pressure. Similarly, in the second term is actually a quarter pole term, it goes as 1 over r cubed. The pressure, which was equal to a 3 u j x j by r cubed. If you go back and you look at what a 3 was it was equal to 2 mu times a 1, so this will be equal to 3 by 2 mu r u j x j by r cubed, so that is the equation for the pressure. Now this vector solution of course, I mean it is a solution of the equations, but it does not quite give you a physical insight into how the solution would look.

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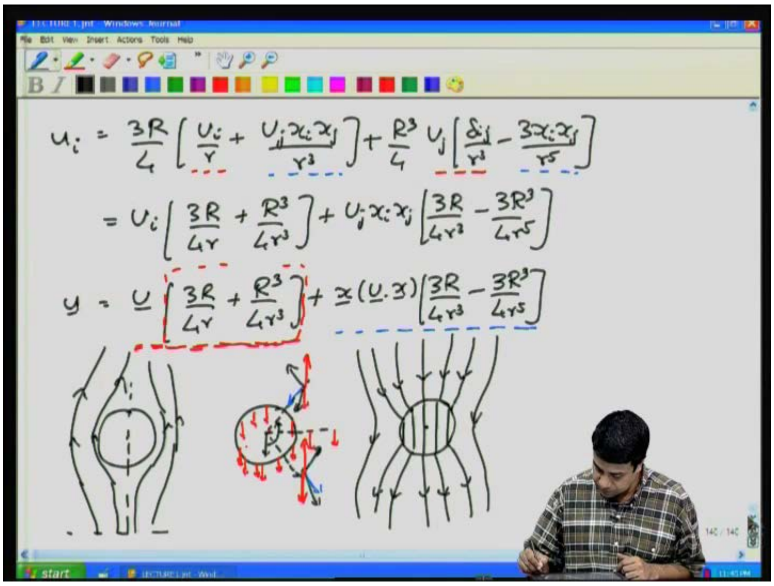


So, let us take a little bit of time, in order to look at the interpretation, of this physical solution, is equal to 3 r by 4 into plus r cubed by 4. We separated out as usual into two components; one which is along u, and the other which is along the x direction. So, this will be equal to u i into 3 r by 4 r plus r cubed by 4 r cubed plus u j x i x j into 3 r by 4 r cubed minus 3 r cubed by 4 r power 5. Just separating out the terms that are proportional to u i, which are these two, and the terms that are proportional to x i x j u j, which are these two. So, this if I write it in vector notation, I will get u vector is equal to u vector into 3 r by 4 r plus r cubed by 4 r cubed plus u j x vector into u dot x into 3 r by 4 r cubed minus. So, this now has two components; one the velocity which is along u vector, and the second is the velocity along x vector.

So let us look at these two terms, the velocity along u vector, and the second is the velocity along the x vector. So, let us take a sphere that is settling in a fluid, and let us assume that, the velocity direction u is in this direction, the velocity direction of u is in this direction. So, at the surface of the sphere itself, velocity is purely in the u direction, at the surface of the sphere itself, the velocity is along u direction alone. At r is equal to capital R as you can see the blue part goes to 0, at r is equal to capital r the blue part is equal to 0 the red part is equal to just one. However if you are away from the sphere, from the surface of the sphere, you will have a non 0, due to the second blue part. The red part is of course, positive everywhere, so for example, at a location, something like this, at a location something like this, the red part is of course, this way, it is going downward.

The blue part is going along x vector, x into u dot x. If u dot x is positive, if u and x are the angle between the two is less than 90 degrees, then you have a positive contribution, along the x vector due to the blue term, and because of that you will have a contribution due to the blue term that looks something like this. And therefore, a resultant velocity vector somewhere here, will be in some resultant direction. When u dot x is identically equal to 0; that is along the 90 degrees, you will have a velocity that is purely in the u direction, so here the velocity will be only in the u direction, and now when you go to a position, on the downstream side of this sphere. If you go to a position on the downstream side of the sphere, a position somewhere here; u vector is of course, along the u direction itself, u vector is always in this direction, u dot x is negative, because the angle between u vector and x vector now, is greater than 90 degrees.

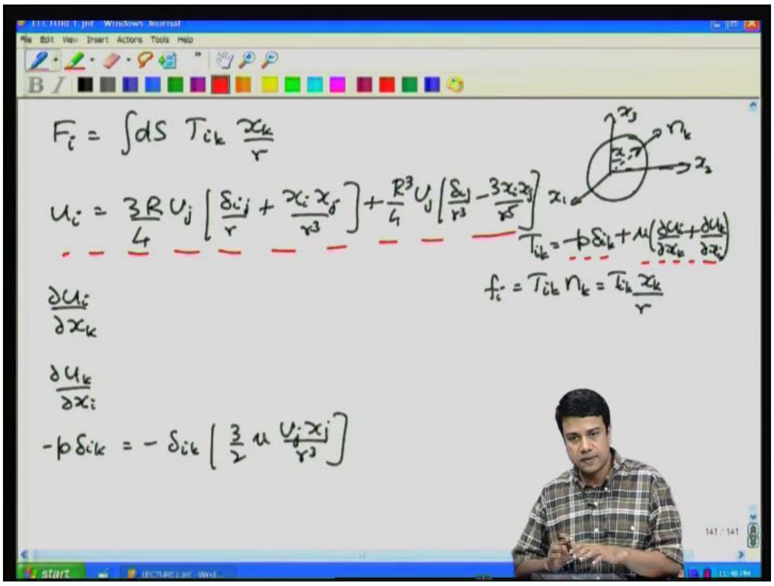
The angle between u vector and x vector here, this is the direction of u vector, x vector is in that direction, so the angle is greater than 90 degree, so u dot x is negative. So, you have a velocity that is in this direction, because u dot x negative, the net velocity is along the inward direction, and the resultant of these two gives me a velocity that looks something like this. So, if I plot it in terms of the velocity of the sphere, if I plot it in terms of the velocity of the sphere. If I look at the fluid streamline, what I will get is something like this. The solid sphere itself is moving at a constant velocity, so the streamlines will all look something like this. On the upstream side of the sphere, you have the velocity going outward, and downstream you have the velocity coming inward. So, that is the kind of velocity profile that you get, due to the settling sphere, as given by our solution of the stokes equations, so this the velocity profile that we get.

Sort of clear because as the sphere is moving downwards, this is in a fixed reference ring. As a sphere is travelling out downwards, the velocity upstream has to go outward, because it has to push flow it out of the way, so that the sphere can settle, velocity downstream has to come inwards, because the space that is vacated by the sphere as it moves down, has to be occupied by flowing coming inwards, and if you look near the sphere you will get something which goes like this. So, this is the kind of velocity profile, that you will get in a fixed reference frame. If you are on the other hand, in a moving reference frame; that is moving with the sphere itself. So, you get the solution for the moving reference frame, by subtracting out the velocity with which the sphere is moving. And if you work it out in the moving reference frame, you subtracted out the sphere velocity capital U, so that the velocity of the sphere itself is a constant, I am sorry the velocity of the sphere itself is 0, so that the sphere itself was stationary. So, in that case what has to happen is that, the flow has to go around the sphere.

The sphere itself a stationary, and the flow it is moving upwards, therefore the flow has to go around the sphere like this. In case of the velocity profiles look very different in a fixed and the moving reference frame. If you are moving with the sphere, fluid is going positive upwards, and therefore you have fluid that is diverging as it goes past due. Whereas, if you are in a fixed reference frame, the space below the sphere has to get, the fluid that has to be pushed out, so it goes outwards the space above the sphere the fluid has to come inwards. All of that is captured by this solution, of the equation. So, the velocity that I get, is just by subtracting out u from this, if I subtract out u from this. So, from this vector here, from this vector here, the velocity of the sphere downwards looks something like this.

So, if I set at this point, I have a sphere velocity here, which is less than u, because as you can see when r is less than capital R, this coefficient is less than one. When r is less than capital R this coefficient here, I am sorry when small r is greater than capital R, when you are outside the sphere, this coefficient is less than one. Therefore, I have a large of velocity being added over here, and the net resultant that I get due to these two, these three velocities, will end up being something in this direction. So, it will be outward at the bottom of the sphere, and similarly, if I subtract out this capital R over here, I will get a velocity that is inward behind, that will give me this profile, in a fixed reference way. So, we have got the velocity profile, we have got the velocity, we have got the pressure, how do we calculate the force on the sphere.

(Refer Slide Time: 50:51)



As you know the net force is the integral of the force at each point on the surface. At each point on the surface, there is a stress that is exerted, the stress is equal to minus p delta i j plus mu partial x i. For the present purposes, let me rewrite this a little bit, because my expression for the velocity, has the index j in it. So, rather than using in indices i and j I will use the in indices i and k for the stress. I will use the in indices i and k. So, this is the stress acting at each point on the surface. Then you have this unit normal to the surface k and k, this is the force acting at each point on the surface, is equal to the stress dotted with the unit normal. Therefore, the force acting, is equal to t i k times n k. for this particular case, the unit normal is also equal to the displacement vector divided by its magnitude. So, as soon as I fix the origin, at the centre of the sphere, so as soon as I fix the origin at the centre of the sphere the position vector anywhere on the surface, is just x vector, position vector anywhere on the surface is just x vector, unit normal is x vector divided by its magnitude.

The magnitude of the unit normal is x, is the radius R itself. So, this is just equal to T i k into x k by r, because x vector itself, is the position vector of a point on the surface, that divided by its magnitude gives me the unit vector. That is the force exerted at each point, on the surface. The total force F i is equal to integral of the force exerted on each point on the surface of T i k x k by r. Now I have to evaluate T i k, for that I need the gradient of the velocity, I need the gradient of the velocity I also need the pressure. So, let us write down the gradient of the velocity and the pressure; u i is equal to 3 r by 4 u j into delta i j by r cubed plus x i x j by r power 5 plus r cubed by 4. Now I have to evaluate partial u i by partial x k and partial u k at partial x i and minus p delta i k minus p delta i k is of course, equal to minus delta i k into 3 by 2 mu u j x j by r cubed. So, that is not a problem, but I have to evaluate the gradients of the velocity fields. So, we will have to take the gradients of this velocity field, we will continue this in the next lecture, gradients of the velocity field, multiplied by the unit normal, integrated over the entire surface, in order to get the total force, acting on the surface of the sphere.

So, if the prediction of this total force, as we probably know, this is given by stokes law. We will be able to derive the stokes law solution by this method, without reference to the individual components of the force, but rather just taking a vector, velocity, the stress integrating that, to get the total force. So, kindly go through the derivation in this lecture, and how we took the gradients and so on of the various vector and tenser quantities, and we will continue the evaluation of the total force in the next lecture, from the velocity profile, that we have obtained in detail in this case, important to note that the velocity profile that we obtained, has been done without reference to the underlined coordinated system. So, this is more general, can be used for any, it is obtained in terms of vectors and tensors, and that was facilitated, because we managed to get vector and tenser solutions to the Laplace equations. So, kindly go through this, we will continue the evaluation of the force in the next lecture, we will see you them.