Fundamentals of Transport Processes II
Prof. Kumaran
Department of Chemical Engineering
Indian Institute of Technology, Bangalore

Lecture - 16
Viscous Flows Part – II

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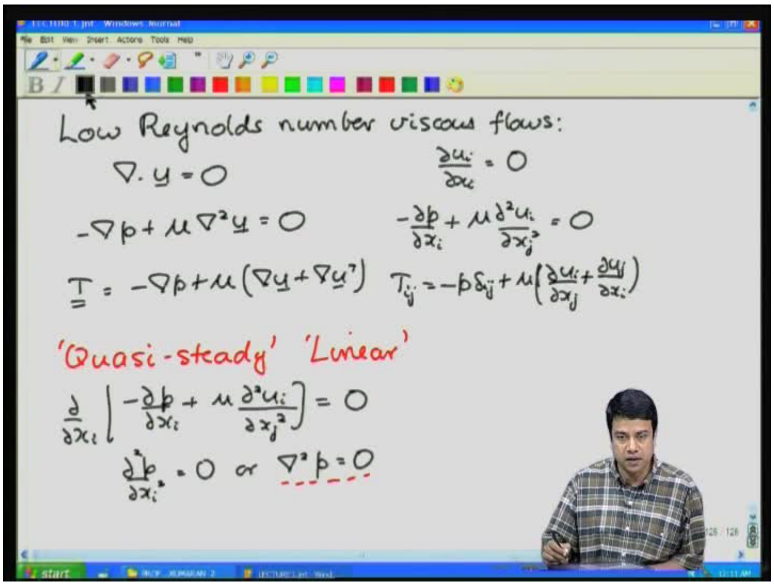

So, welcome to this lecture number sixteen of our course on fundamentals of transport processes two. The last lecture, we were looking at the low Reynolds number hydrodynamics. The equations and their solutions in the limit of very low Reynolds number, where we neglect the inertial terms, in the conservation of equation. These are also called as viscous flows or creeping flows, and the fundamental equations for these flows are basically, the mask conservation equation, and the momentum conservation equation, where we neglect completely, the inertial terms in the conservation equation. And the stress answer, is given by minus the grade into the pressure plus mu into grade u plus grade u transpose. It is more convenient to write it in an additional notion, this equal to 0 and t i j, so these are the conservation equations.

These equations are quasi steady, no dependence on time, in the equations themselves. Therefore, you not solving a partial derivative in time, where the difference in the value or some quantity, in this case the velocity between two different time instances comes in. That means at the given instant time, the velocity field is completely specified by the velocities, or stress at the boundaries, at that instant in time. So for that reason it is called as quasi steady.

As I said this is a consequence of the fact that you are assuming the dominates. Therefore, the diffusion takes place of the entire domain instantaneously. The other important point, is that this equations are linear, the mass conservation equation is linear in the velocity, the momentum conservation equation, is linear in the velocity of the pressure, stress is linear in the velocity of pressure, and this has important consequences that we discussed in the last lecture. The flow in the limit of very low Reynolds number is reversible; that means that if I reverse the direction of, either the velocity or the stress, on all boundaries.

Then the velocity at each point in the fluid exactly reverses if I double the velocity at or the stress on the all the boundaries, the velocity and pressure at each point within the fluid increases by factor of 2. And because of this combination of the linearity and reversibility, we had discussed many interesting consequences in the in the previous lecture. Because of linearity and reversibility, one can also use linear super position; that is if I have some particular configuration, where I have some objects, or some surfaces ,moving with some velocities and some directions, I can separate that out into sub problems. In this sub problems all the boundaries have to remain exactly the same, the velocities can be separated out of different parts, and assign to different sub problems. I solve each of those sub problems, add them up, and I get the solution for the original problem.

So that is one the consequences of linearity and reversibility, in the limit of very low Reynolds number. And I had shown you in the last lecture that many of the consequences that we are commonly used to, for flows that we experience in everyday life, do not apply at the micro skill for low Reynolds number flows, simply because of this difference, the equations are linear. And because the equations are linear, one is guarantee that the solutions exist, subject to appropriate boundary conditions, and that the solutions are unique; that is you cannot have two possible solutions, for the exact same, set of boundary conditions that are specified. So that consequences the implications of this linearity, and reversibility, or that for a given set of equations, and for a given set of boundary conditions, there exist exactly one solution, and that solution is unique. They guarantee the existence; you cannot have a situation, where no solution exists. And last class, we had processed to try to solve this equations, diffusion dominated resume, you would expect the equations to be, in the form of Laplace equations. So at post these equations in the form of Laplace equations, and they tempted to solve them.

So first things first, how did we do that. So the conservation equations are settled, for the mass and momentum conservations equation. First thing I did was take the diversions of the momentum conservation equations. So divergent means repeated index, and when I take the divergent are Laplacian of the velocity, because the you can interchange the out of differentiation. The divergent are Laplacian of the velocity, is equal to the Laplacian of the divergent of the velocity. Divergent of velocity equal to 0, and therefore, I just get the equation for the pressure, as the Laplacian of the pressure, is equal to zero or del square p equal to 0. How do I find out the velocity of the field? Since I have been able to separate out the pressure, and find the solution for their pressure explicitly, I can go back to the momentum conservation equation, where I consider the pressure as an homogeneous term. Therefore, the original conservation equation that I had, I can consider the pressure as an homogeneous term.

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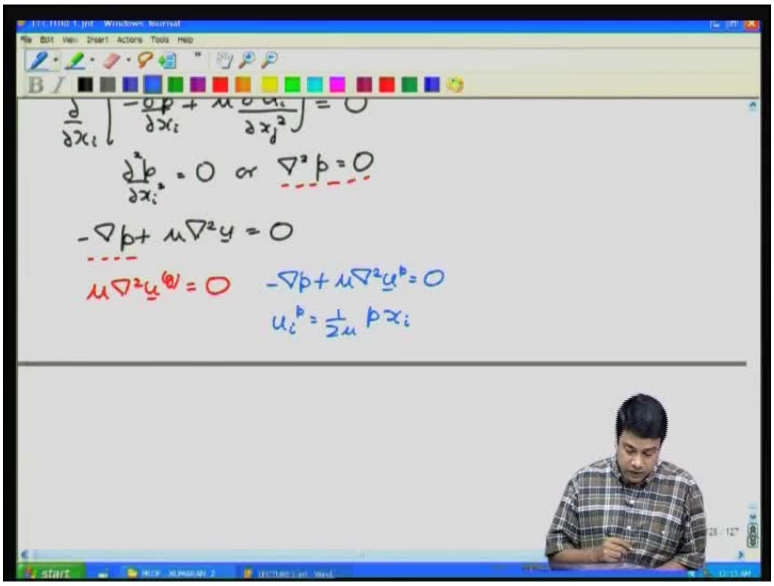

So in this equations, since I have evaluated the pressure, this can be considered as an in homogenous term, and so I get an in homogenous differential equation; that is linear in the velocity that velocity can be separated out into two parts; a general solution for the homogenous equation, without this in homogenous term. A general solution for the homogenous equation, without this in homogenous term plus second equation in which, the pressure comes in, in which the pressure comes in. But this has only a particular solution, any one solution that satisfies the equation, does not have to be a general solution, does not have to contain integral constant, can be any one solution that satisfies this equation. And we saw in the last lecture that the particular solution, that satisfies this, this is equal to 1 by two mu p times x i.

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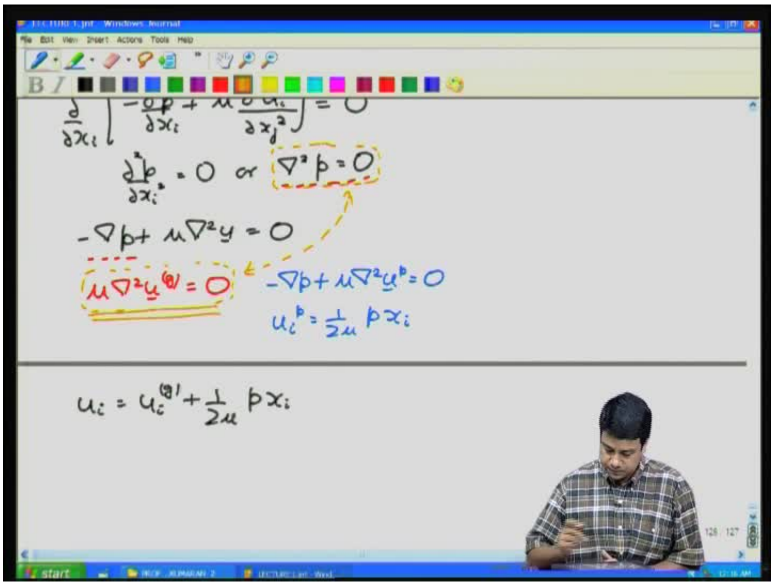

So the final solution for the velocity field, is just u is equal to the general solution plus, let me write it in notation as u i general plus 1 by two mu p times x i. So, in this solution strategy, we have now reduced the equation to two equations, for the pressure, and the general form of the velocity field. This is the equation for the pressure just the del square p is equal to 0, where p is the scalar. This is the equation for the general solution for the velocity field, which says that del square of u general is equal to 0 u general is now a vector, and the final solution for the velocity field is obtained as u is equal to u general plus 1 by two mu times the pressure times x i. And you can see that to solve each of this equations, you have to solve a Laplace equation, pressure is the scalar, so you have solved del square p is equal to 0 for a scalar, u general is a vector, so you solve del square u general is equal to 0, where u general is a vector. Of course, this equation contains three components, it is a vector equation, it contains three components, and this equation is telling you, that the del square of each component of the vector, is equal to 0; that is the solution.

So if you wanted to solve in that way, you have to solve vector, scalar equations, scalar Laplacian equation for each of these; the pressure plus the three components of the velocity, complicated. Now I have to solve four Laplace equations, whereas when we did converter transport process one; you just have to solve one equation, for either the temperature or the concentration field. Not only you have to solve these equations, you have to make sure that the solutions are now consistent with the boundary conditions, as well as the mass conservation condition, because this equation del square u general is equal to 0, does not in general satisfy the mass conservation equation. If you have solutions of this, for which the mass conservation condition is not satisfied.

So for that reason, this is a complicated equation to solve the way did it, in format of transport processes one. So, look at the ways are solving the vector equation, explicitly; that is without splitting it up into two individual components in this lecture, but first we go back, and look at how we solved for the concentration of the temperature fields in fundamentals of transport processes one.

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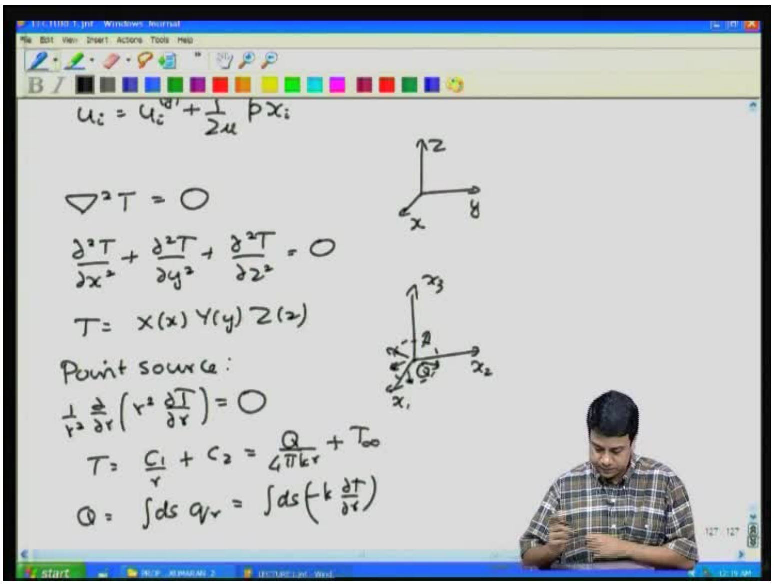

If we recall there, in the case of diffusion dominated transport, you always had to solve del square d was equal to 0, you solve for example, in Cartesian coordinates, then x y z Cartesian coordinates systems this was the equation. And the way you solved it was by separation of variables; t is equal to some function of x. Use the separation of variable to convert this, partial differential equation into three ordinary differential equations. The homogenous boundary conditions reduce to an Eigen value problem, which you had a set of discrete Eigen values and Eigen functions, and orthogonality relations for used for that case.

Now of course, you have three components of the velocity three position coordinates, and trying to solve each of those, will become very complicated especially if you do not have nice boundary conditions for the velocities, on the warning surfaces. The procedure that we will use, was one that we had used earlier, for point source of e. If you recall, you defined the point source, as something that emits, so strength q, which is emitting heat in all directions.

In this particular case, because the heat is emitted equally in all directions, the configuration is spherically symmetric, where the spherically symmetric and because of that, you have the equation , in spherical coordinate system, in three dimension becomes 1 by r squared by d r r square partial t by partial r is equal to 0; that is the solution; that is the equation spherical coordinate system. You can solve this quite easily, and get t is equal to some constant by r plus c 2, how do I get that constant out? I get it from the condition that the total heat q, is equal to integral over any surface, if we take any surface over here, integral over that surface of the total flukes in the r direction, this has to be equal to integral over the surface, of minus k times partial t by partial, this gave us the temperature field for the point source, is equal to q by 4 phi k r k plus some constant. This constant can be any value, basically this is determined from the temperature, far away from the object, because as we go far away, r goes to infinity, therefore, temperature has to go to some constant value. So this constant c 2 is basically the temperature far away from the object. So basically the temperature field decreases us 1 over r, as you go far away from the object r.

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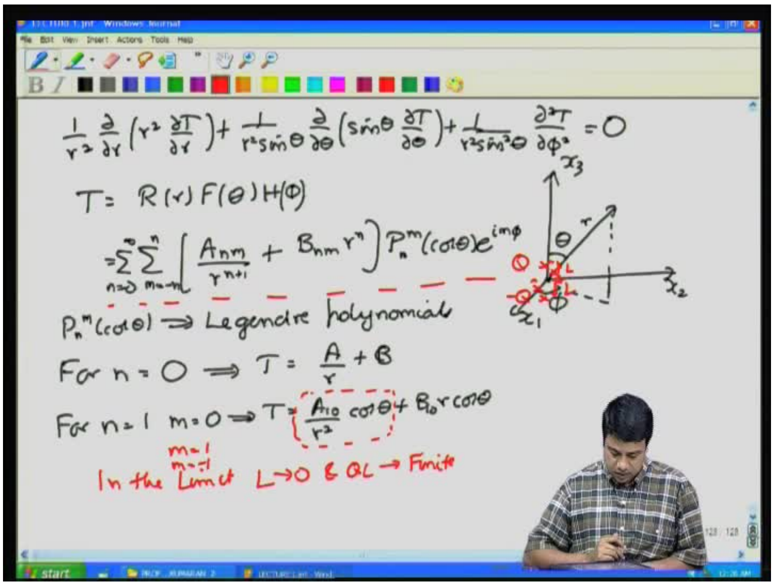

We have done more than this, we are actually solved the Laplace equation in a spherical coordinate system, in the previous lecture, in fundamentals of transport processes one. In that case the equation become 1 by r square d by d r r square d t by d r plus 1 by r square sin theta. So this was the Laplace equation in a spherical coordinate system, r theta phi coordinate system. And we had solve this equation, once again by separation of variables, by separating about t is equal to r of r f of theta times h of phi, and we had got the solution as 1 by r power n plus 1 p n m cos theta e power i m phi. We had got two set of solutions, where p n m of cos theta, where the legendre polynomial. So, you got series of solution in which, the temperature decayed decreases as we went far away. So this was 1 by r power n plus 1 p n m of cos theta e power i m phi plus the second was r power n. So let me just write the general solution n m by r power n plus 1 plus b n m r power n p n m of cos theta e power i m phi.

Where in general we find that m goes from minus n to plus n is equal to 0 to infinity. These discrete Eigen values m and n if you recall, we had got from the condition, that the solution has to be finite at theta is equal to 0, and theta is equal two pi. So these were the solutions that we had got by separation of variables. I had given you physical interpretations of these solutions, so far n is equal to 0, t is equal to some constant by r plus some constant, because for n is equal to 0 m is equal to 0 p n m is equal to one and e power i m phi is also equal to one. For n is equal to one m can go from 0 minus one and plus one, so i m is going from minus n to plus n, so m various between minus one 0 and plus one.

So for n is equal to one for m is equal to 0, we got the solution as t as some constant; a one 0 by r square cos theta plus b 1 0 r cos theta. And if you recall I had given you the physical interpretation of this, the physical interpretation of this first term. The physical interpretation of this first term if you recall, corresponded to the combination of, a source of heat, and the sink of heat plus q minus q located some small distance l above and below, a source and the sink giving you a dipole, located distance plus l and minus l, above and below the z axis, strength plus q and minus q, in such a way that there was no net source, because there is a source plus q, and a sink minus q. The net heat coming out from the domain is 0, in the limit as l goes to 0 and q l being finite.

You get a source dipole, source dipole is something like looks like this, separated along the plus and minus z axis, a small distance from the origin, in such a way that the distance goes to 0, but the source strength, of the sink strength times the distance from its finite you get this second term. There is no net source, therefore, the contribution proportional to 1 over r is equal to 0, but, you get net source and the sink, and that gives you this term, this dipole moment, the source and sink of heat separate by small distance. So n is equal to one, and m is equal to plus 1, we saw that corresponded to two source and sink, separated along the x axis, and form is equal to minus one correspond to along the y axis. And each of the solutions, satisfies the orthogonality condition, so I can use the orthogonality condition to get back the solution for the spherical harmonic expansion. So that is how we had done it in a spherical coordinate system.

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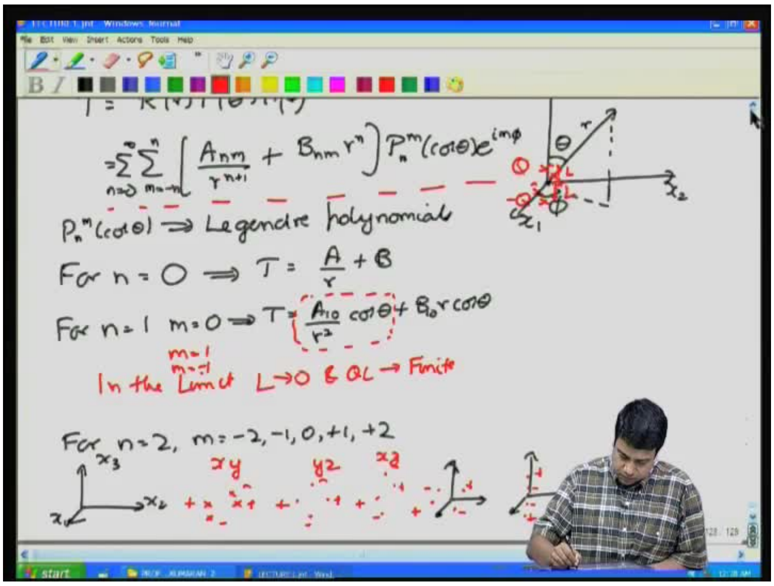

Similarly, for n is equal to 2, m is equal to minus 2 minus one 0 plus 1 pus 2. There are five solutions, for n is equal to 2 minus 2 minus one 0 plus 1 and plus 2. That showed you that each of these corresponds to one particular arrangement of sources and sinks. If you have, this as the x 1 x 2 x 3 axis, you have arrangements in which you have two sources and two sinks, along the x x 1 and x 2 axis. Then you have two sources and two sinks along the x and z plus plus, then we have one along the, and then this corresponds to the x y y z x z; then you have one that is basically called x square minus y square, in which you have a source and sink that is arranged along the x y plan, but, not along the axis. And then finally, you have two sources and sinks, arrange along the z axis.

These was the five solution set we got; combinations of sources and sinks. These were physical interpretations of the result that we got by solving the Laplace equation, in spherical coordinates, by separation of variables procedure. In case you not similar with this, kindly go back to fundamental of transport processes one, and revise what we have done in that particular solution of the Laplace equation. Here we will use the exact same solutions, expect will give them a different physical interpretation, we will use exact same solutions, except will give them a different physical interpretation.

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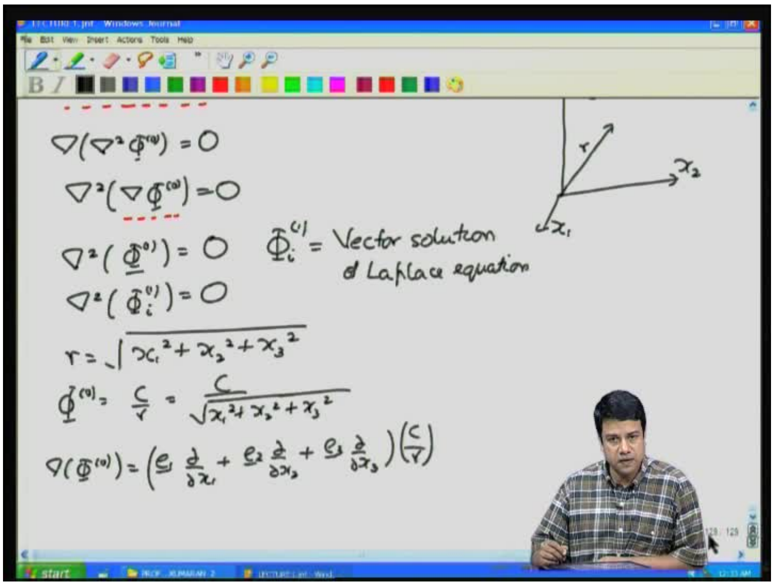

So basically we want solve the laplace equation in spherical coordinates, for del square of something, is equal to 0. I know one solution for this, this thing can be a scalar, can be a vector, can be a tenser, it can be anything. I know one particular solution for this. If this is a scalar, then I know that I have a solution phi is equal to some constant divided by r, plus another constant, we will just define this, correct to one known constant in the field, so this solution I have .Let us call this a fundamental solution. I will put it with the 0 over here, this is the fundamental solution. So if this is the equation, the Laplacian of phi naught is equal to 0, then if I take the gradient of the solution that is also equal to 0. In others words if I take gradient of del square phi naught, this is also equal to 0, because if I take gradient of any function, is that function is equal to 0, its gradient is also equal to 0.

I can rewrite this, interchange the order of differentiation; that is always possible. So I will rewrite this del square of grade phi naught is equal to 0, inter changing the order of differentiation; that means that this term in the brackets here, is also the solution of Laplace equation, del square of phi one vector, this is also a solution of Laplace equation.

In indices notation I will write this as; del square of phi 1 i is equal to 0; that means at this is also a vector solution of Laplace equation, how do I get this vector solution, I have to take the gradient of the fundamental solution. So let us see how we do that, easiest way to take the gradient, is to take it in Cartesian coordinates, easiest way to the gradient is in Cartesian coordinates. This coordinate system this is r and we know that in Cartesian coordinate system r is equal to square root of x 1 square plus x 2 square plus and phi naught is equal to c by r is equal to c by root over x 1 square plus x 2 square plus x 3 square; and how do I take the gradient? Gradient of phi naught is equal to e 1 d by d x 1 of one over r is a constant, times the gradient of the fundamental solution. Now this is pretty easy to do, because I have seen constant by r, which is one by x 1 square plus x 2 square plus x 3 square. For future use, we will just define the definition of the gradient itself.

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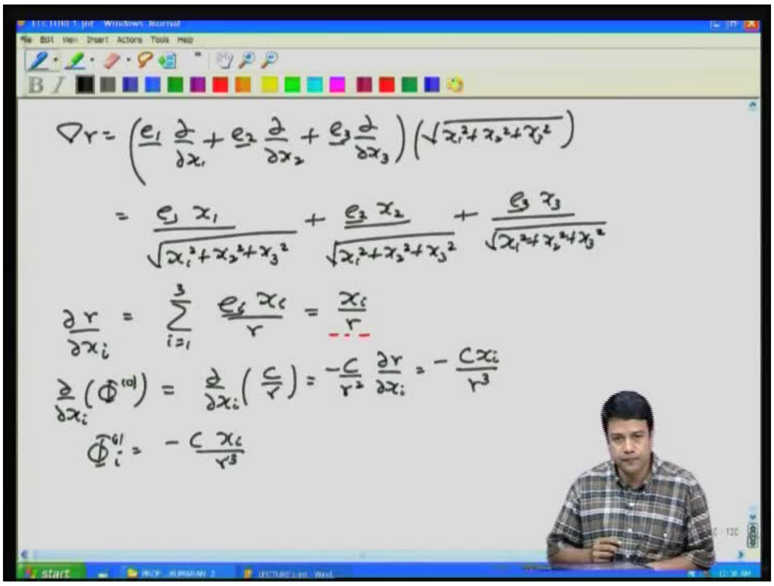

The definition of the gradient of r, is equal to e 1 d by d x 1 plus e 2 of square root of x 1 square plus, this becomes e 1 x 1 by root of plus. So I can write this as sine square root of x 1 square plus x 2 square plus x 3 square just r, you can write this as summation of i e x i by r. In out In our indicial notation, we of course, don write down the summation and the unit vector, so this just is equal to x i by bar r, so this is equal to partial r by partial x i the gradient of r, is equal to x i by r.

Therefore, the gradient of phi naught, I will write it in indicial notation, the gradient of the fundament solution is equal to some constant by r, is equal to minus c by r square partial r by partial x i is in the chain rule of the differentiation, differentiate with set to r first, then differentiate r with respect t to x i, and since the derivative partial r by partial x i is equal to x i by r minus c x i by r cube. So this is the vector solution of the Laplace equation; phi 1 i is equal to minus c x i by r cube.

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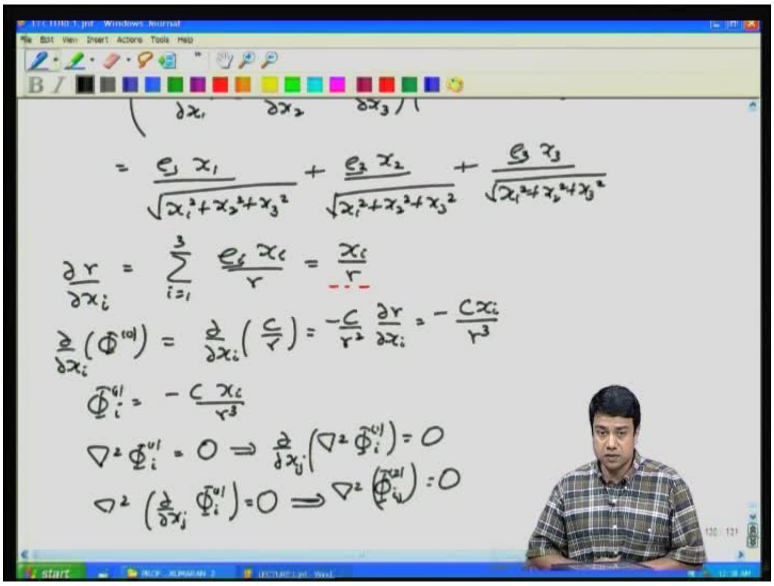

So, this vector solution, contains in it three components, this vector solution contains in it three components x 1 by r cube x 2 by r cube x 3 by r cube, but rather than considering it, rather than considering three different component separately, I just assemble them into a vector. So this is the vector solution of the equation, if i 1 i satisfies the Laplace equation, then its gradient also satisfies the Laplace equation, so that is if del square phi one i is equal to 0.

I take the gradient of the whole thing, I am taking the gradient, no repeated index, so that means that partial by partial x j of phi 1 i is equal to 0, sorry into changing the order of differentiation once again del square of partial by partial x j phi 1 i is equal to 0, which means that del square of phi 2 i j is equal to 0, where phi 2 i j is now a tenser solution of the Laplace equation. So second order tenser, get it by taking the gradient of the first order tenser.

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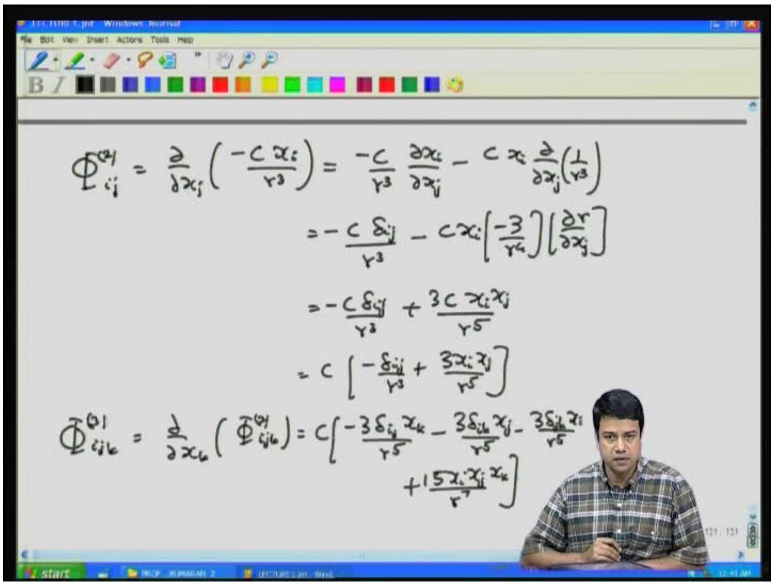

I will define it to only within multiplicative constant; phi 2 i j is equal to partial by partial x j of minus c x i by r cube. Do the differentiation of this using the chain rule, because I have x i in the numerator and r cube in the denominator. So this will give me minus c times partial by partial c by r cube partial x i by partial x j minus c x i partial by partial x j of one over r k partial x i by partial x j is just delta i j so I get minus c delta i j by r cube plus one over, the derivative one over r cube respective x j. What I need to is use chain rule for differentiation, I will get minus c x i into minus 3 by r power 4 into partial r by partial x j minus c delta i j i r cube and partial r by partial x x j is equal to x j by r. So I will get plus 3 c x i x j by r power phi this is equal to c into minus delta i j by r cube plus 3 x i x j by r.

So there is a second tenser solution, second order tenser for the Laplace equation. It automatically contains nine components, and in Cartesian coordinate system, each of these would individually satisfy a Laplace equation. I could do this once more and get a next higher order tenser. I would not go into the details of how the calculation is done, but the next tenser is equal to delta by delta x j of phi 2 sorry take x k. If you do the same procedure expand out by chain rule, and you will get c into minus 3 delta i j x k by r power 5 minus 3 delta i k x j by r power 5 plus 15 x i x j x k by r power 7; that is the next higher solution. There is a third order tenser 27 components 3 power 3, and so you can get the next higher next higher and so on, what do this physically mean.

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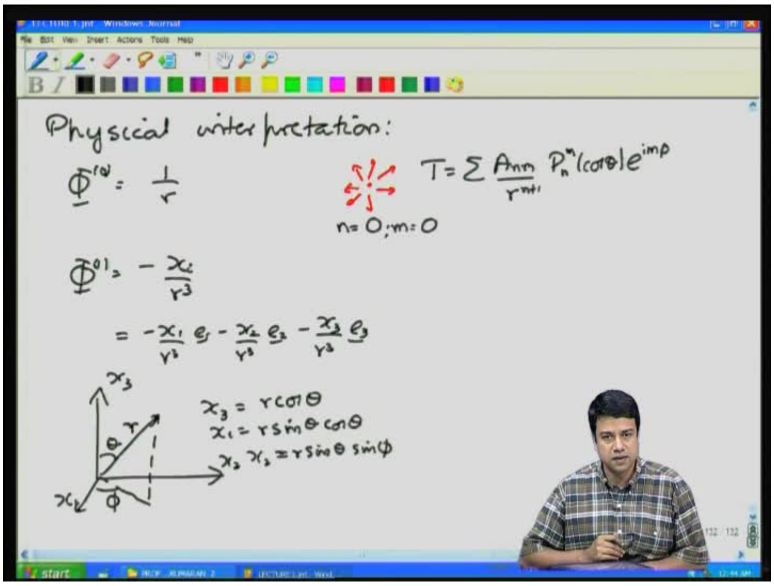

So let us go to the physical interpretation of the spherical harmonic; phi naught, I will put in the constant for now, because this can be multiplied by any constant, they still satisfy the Laplace equation. I showed you that this was equal to the point source; one by r it was spherically symmetric point source. If you recall same as the solution for the source the equation that I had for temperature field is equal to sigma A n m by r power n plus one p n m of cos theta e power i m phi, this was the solution for n is equal to 0 and m is equal to 0.

The next one, was minus x i by r cube that has three solution; that is x i by r cubed x 2 by r cubed and x 3 by r cubed, if I expand out this solutions, in a Cartesian coordinates system that are 3 minus x 1 by cubed e 1 minus x 2 by r cubed e 2 in this spherical coordinates system. You recall the spherical coordinates system that we had, this was the distance r this is theta this is the angle phi, and you that x 3, along the three axis, x 3 is equal to r cos theta x 1 is equal to r sin theta cos phi and x 2 is equal to r sin theta sin phi, this is the expansion of the coordinates x 1 and x 2 x 3 in this spherical coordinate system; x 3 cos theta, because the angle between the x 3 axis and the radius vector equal to theta. The projection is equal to phi x 1 is equal r sin theta cos phi and x 2 is equal to r theta sin phi.

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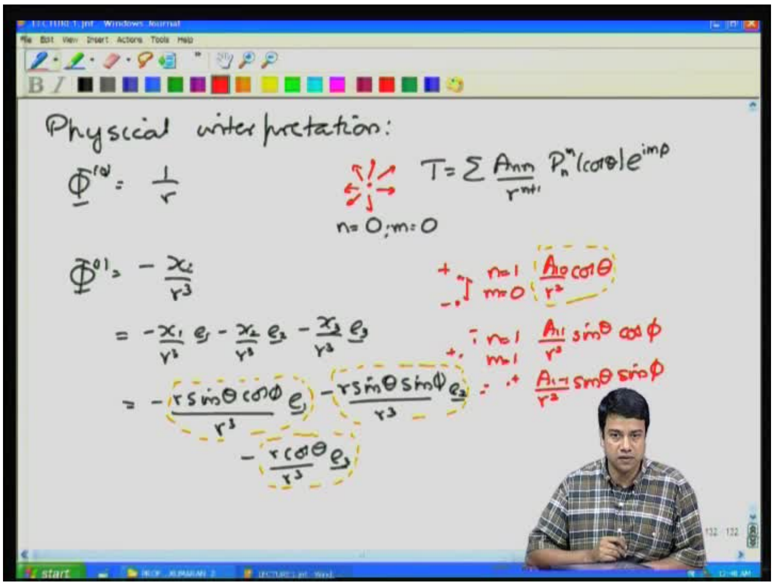

So using this, we get these three solutions for this vector minus r sin theta cos phi by r cubed e 1 minus r sin theta sin phi by r cubed e 2 minus r cos theta r cubed e 3. If you recall this are the exact same solution that we got for sources and the sinks, separated by small distance in this spherical coordinate system. So this x 3 direction, the solution was n is equal to 1 m is equal to 0, some constant a 1 0 a 1 0 by r square cos theta, for separation along the x 3 direction or the z direction. You can see that this one, is identical to the vector solution component along the three direction for the gradient of the fundamental solution, for n is equal to 1 m is equal to 0, get one solution. If you recall, when n is equal to one m i equal to m1inus 1 0 and plus 1; therefore, there are three solutions. The solution for n m is equal to 0 corresponds to exact same solution, in the gradient of the potential, of the fundamental solution, along the x 3 direction.

Then I have two solution for along the x 1 direction n is equal to 1 m is equal to 1 I had a solution which was a one one by r square sin theta cos phi. This one is the identical to the solution here. Note that the negative sign does not really have any physical significance, because if i one is solution negative is also a solution. So this corresponds n equal to one m is equal one, and this one, it is easy to see corresponds to n is equal and m is equal to minus one. This is two sources and sinks separated along the x 2 direction, this plus minus sin theta sin phi. So this, the components of this vector solution of the exact same solution you get for the dipoles with the dipole moment along three coordinate axis. In that case will solve that in terms of (( )) normal, we got three solutions, for n is equal 1, there was solution for m is equal to minus one 0 and plus one. Those three solutions are exactly the components of this vector solution that I get, when I take the gradient of the fundament solution. I get the vector of three component, those components exactly correspond to the solution that I get here.

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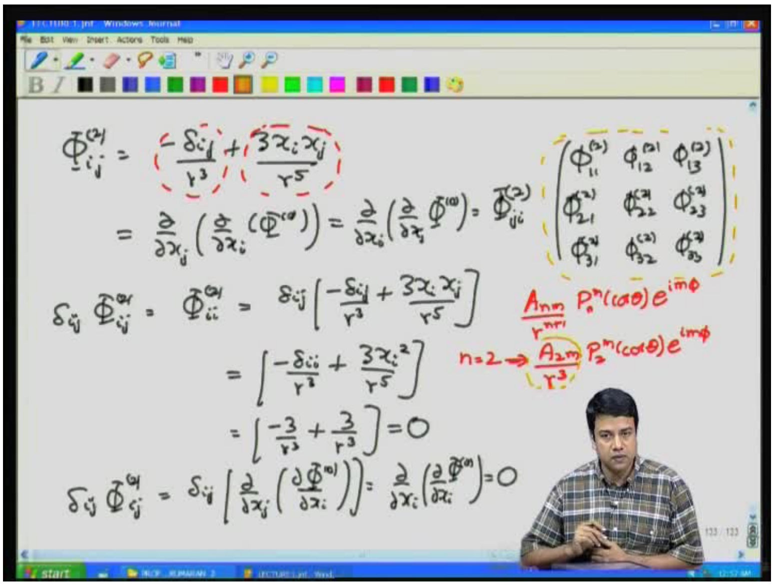

The same thing can be shown next higher order term; the expansion. phi 2 i j equal to minus delta i j by r cubed plus 3 x i j by r power phi. This is the second order tenser, second order tenser, and therefore it has total of nine components. However, it is also symmetric, if interchange i n j I should get exactly the same result, because if you recall I took the gradient of the fundamental solution two times. So phi 2 i j equal to partial by partial x j of partial by partial x i of phi naught, to gradient two times; that is equal to i can easily interchange the high order differentiation, because x i x j are independent variables. This also equal to partial by partial x i of partial by partial x j of phi naught, is equal to phi 2 j i. In case it is symmetric tenser and therefore, it has six in the current components, also because it is a symmetric tenser, and it is obtained by taking the two gradients of the fundamental solution, it is also traceless.

Traceless some of the diagonal term is equal to 0. How do I get that trace of the tenser. I can get the trace tenser by multiplying it by delta i j. So delta i j phi 2 i j is equal to delta I am sorry delta i j by this solution is non zero only when i is equal to j. So I could replace by j by i in phi 2, then I will get scalar phi 2 i i, only one repeat index, our left hand side you have two repeated indices, so it is scalar. So this equal to delta i j into minus delta i j by r cube plus 3 x i x j by r power delta i j into delta i j is delta i i this is equal to minus delta i i by r cubed plus 3 x i square by r power 5; what is delta i i equal, this equal to summation i, is equal to 1 to 3 of delta i i; there is one summation no unit vector, because it is repeated index. So delta 1 1 plus delta 2 2 plus delta 3 3 is equal to three. So this equal to minus 3 by r cubed plus x i square is x 1 square plus x 2 square plus x 3 square; that is also equal to r square, because x 1 square plus x 2 square plus x 3 square equal to r square.

This becomes equal to plus 3 by r cube, second its cancelled out the r square, where r power 5 is equal to one over r cube, so this equal to 0. So that trace of this second order tenser is identically equal to 0, not a surprise, because if I take the trace of the second order tenser, you will recall that delta i j times phi 2 i j is equal to. If the write this has two gradients, acting on the fundamental solution, in this I can replace i by j, i can replace j by i. Because delta j is naught 0 only when j is equal to i, i will get partial by partial x i of partial by partial x i of phi naught, which is the laplacian of phi naught which has to be equal to 0, in laplace phi naught has to be equal to 0. So therefore, but I have shown you, is that the second order tenser solution phi 2 to i j, it is a second order tenser its symmetric, because interchange of in this makes no difference, it is also traceless. Symmetric matrix has six independent components, out of which if it is traceless, the sum of the diagonal has to be equal to 0, if the some of the diagonal elements have to be equal to 0.

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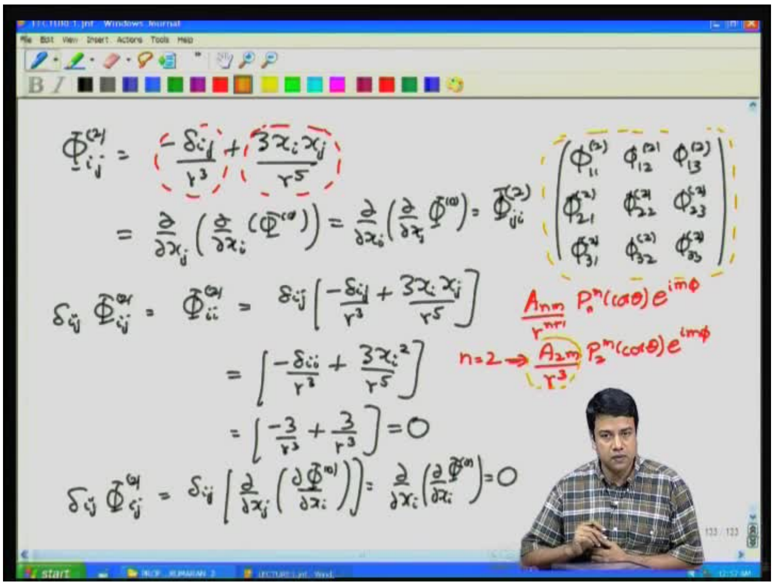

Therefore, this 3 by 3 matrix phi 2 1 1, the second order tenser, symmetric six independent components, it is also traceless, where some of the three diagonals has to be equal to 0. So thus total of five independent components in this second order symmetric traceless matrix, and if you recall the solution A n m by r power n plus 1 p n m of cos theta e power i m phi for spherical harmonic expansion for n is equal to 2; this has a 2 m phi r cubed p e power i m phi, it goes as one over r cubed. You can see that each of these actually goes as one over r cubed, each of these goes one over r cube, the first term is delta i j by r cubed. Second term is three x i x j by r power 5, both x i and x j are r cos theta r sin theta cos phi r sin theta sin phi. Therefore, when I divide by the that two, I will get some function of angle divided by r cubed. Exactly, the same dependence on r that have over here; exactly the same dependence on r that I have over here.

In this at n is equal to 2 m is equal minus 1 I m sorry m goes from minus n to plus n, so this is equal to minus 2 minus 1 0 plus 1 plus 2; five components. Exactly the same as the number of independence component, in this second order traceless tenser, so there five components here. In fact, you can how that each one if these components, in the second order tenser, is a linear combination of the five components in this spherical harmonic solution.

Therefore, there is one to one corresponds between the solutions, scalar, vector, tenser that I get here, with this spherical harmonic solutions that I get here. For n is equal to 0 spherical is symmetric source, n is equal to 1, 3 components corresponds to the vector solution gradient, of the fundamental solution; n is equal to 2, tenser solution has five independent components, because the tenser traceless, corresponds to five component, that I gave, five solution that I get, for n is equal to 2 and m is equal to minus 1 minus 2 minus 1 0 plus 1 plus 2.

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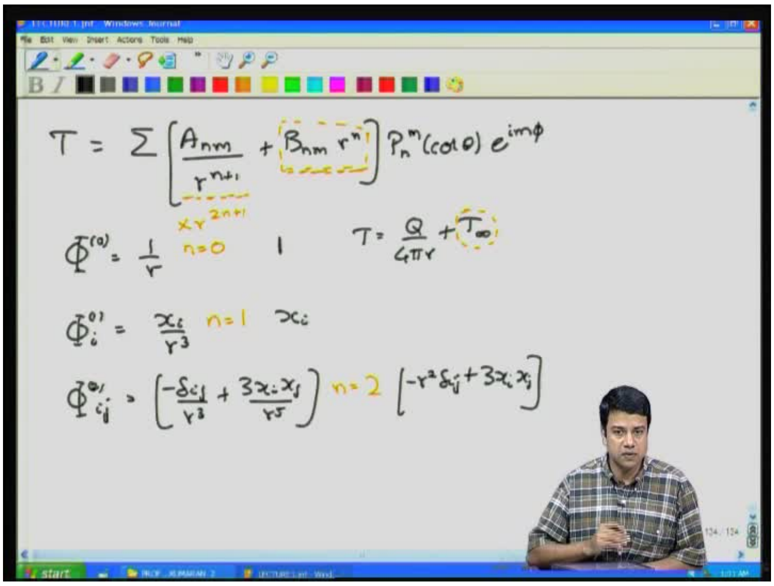

So, this scalar solution, the fundamental solution decreases 1 over r as r becomes large. The vector solution decrease one over r square, x i by r cubed. The tenser solution decreases one over r cubed, corresponds to n is equal to 2 and so on. So, these are the scalar vector tenser solution that you get, if you recall when we did the solution of the temperature field, we not only had that, but we also had second set of solutions. We have a second set of solutions. Therefore, the solutions that we got from the point source are not the only once. I got the solutions from the point source as; phi naught is equal to 1 over r one over r power n plus one with n is equal to 0. For that same value of n, there is another one, which causes r power plus n.

There is another solution, which causes r power plus n, and for n is equal to 0, this is just a constant; r power plus n for n is equal to 0, I get the solution, this solution for this 1, by multiplying the first solution by r power 2 n plus 1. I multiply this by r power 2 n plus one, to get the other solution. Recall these are the decaying harmonics, they decrease as r goes to infinity. These are what are called the grain harmonics; that is the increase as r goes to infinity. So corresponding to the source solution, 1 over r, there is also a grain harmonic solution, which is just a constant, which is not too surprising if you recall when you solve for the temperature field, which is got q by 4 pi r plus t infinity, and this is the constant solution. For the vector solution, one that I got was x i by r cube n is equal to 1, this causes one over r square. The other solution should go as r power plus 1 r power 1 over r power plus 1 so n is equal to 1 means you will get one over r square x i by r cubed. The other one should go as 1 over r power plus 1, you get that by multiplying the first solution by r power 2 n plus 1. In this case, since r power 2 n plus 1 is equal to r cube, the other solution is just x i.

The next higher, is equal to minus delta i j r cubed plus three x i x j by r power phi, this is for n is equal to 2 causes 1 over r cubed, the growing solution i multiply by r power 2 n plus 1, 2 n plus is r power phi. So to multiply by r power phi to get the grain solution, that growing solution will be equal to minus r square delta i j plus three x i x j. So we got the decaying solution by taking successive gradients of the solution for the point source, point source went as 1 over r. We took successive gradients of that solution for the point source, to get all the decaying harmonics. There is another set of solutions for the Laplace equations, is the solution increase, as r goes to infinity. Those are the grain harmonics, and this case you get this by multiplying the decaying harmonics r power to n plus one. In that way, we can get solutions, which are scalar, vectors, and tensors for the Laplace equations.

These are exactly the same, as what we got using spherical harmonic expansion, as well as by looking at point sources dipoles, quarter poles and so on. Since these are the form of vectors, there will be more convenient for us, to express velocity fields, in terms of this. So the next lecture we will use this spherical harmonic expansions, for solving the simplest problem that we can; that is for this settling of its sphere, under gravity in a viscous fluid. You want to solve the nerviest stock equations, in order to find out the velocity field around this sphere, and ultimately to find out what is the drag force on the sphere. If you recall the drag force on this sphere is given by the stocks law, the force is equal to 6 by mu power times u, where r is the radius of the sphere and u is the velocity.

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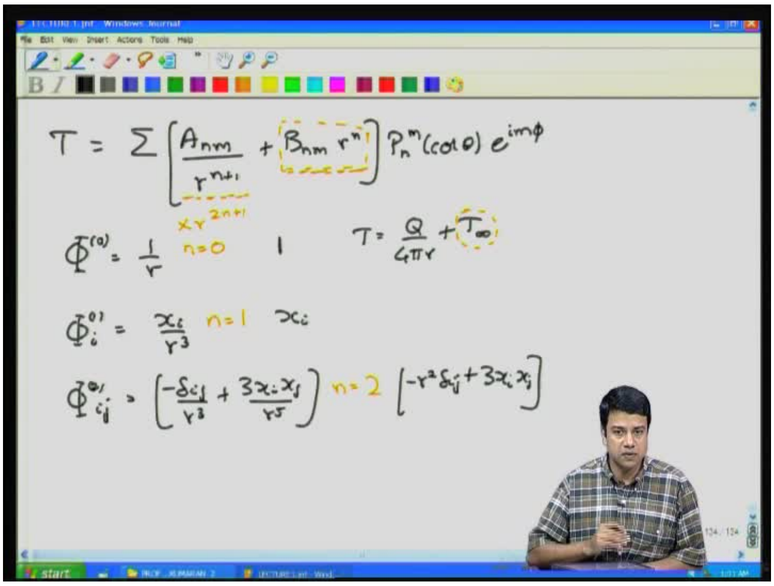

So next lecture, we will look at how we get that solution, and as well the velocity and the pressure field at each point in the fluid, to solve for the velocity and the pressure field subject to, the boundary condition the surface of the sphere using this spherical harmonic expansion. In order to find out, what is the velocity field due to a sphere settling at constant velocity in a fluid, and also to find out, what is the drag force? So should not familiar with this spherical harmonic expansions, please go through this once more. I will revise little, when I start in the next lecture, then I will proceed calculating the velocity field, using spherical harmonic expansion, as well as using the linearity of the stocks flows at low Reynolds number, will see you then.

Thank you.