Fundamentals of Transport Processes II
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Lecture - 14
Unidirectional Flow

So in the last lecture, we were getting down to the business of solving the equations that we had derived, and I had shown you that you to get non trivial solutions for the pressure field, even in the absence of flow; that was hydrostatics. The pressure in general is non-zero simply, because it could have body forces acting. The next simplest case, is a unidirectional flow, where the flows only along one direction, and in general the gradients are only along the other direction.

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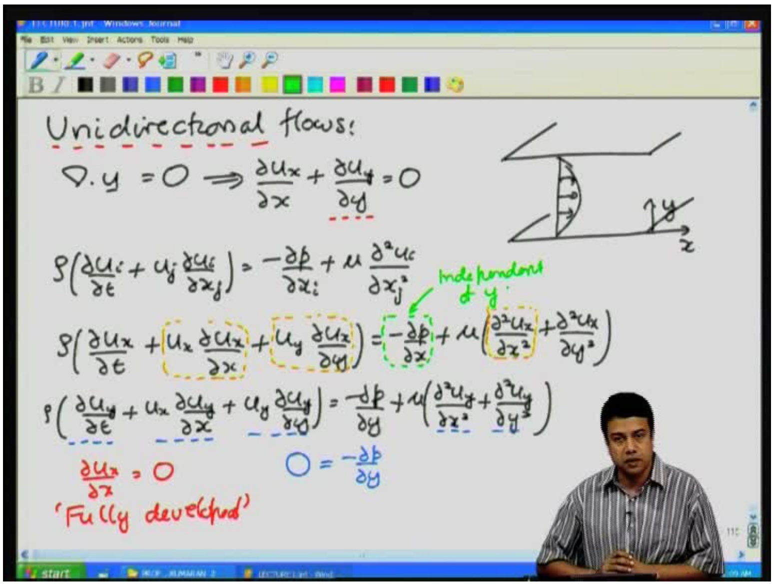

Let us look at how these arise. The simplest case is the flow in a channel. So let us take this as the x direction, this as the y direction. Consider this as a two dimensional channel, which is effectively infinite in the third dimensions, so you should think of this as a slot. This is effectively infinite in the third dimension, and across the two ends of the slot, there is some pressure difference that is applied, due to which there is fluid flow in this channel. And now the velocity is only the x direction, and because you are imposing 0 velocity boundary conditions at the two walls, there is a gradient in the y direction. So let us first try to write down the complete momentum and mass conservation, navies stokes mass and momentum conservation equations, for this particular case. So in this case, the velocity field, means sorry the mass conservation equations del dot u is equal to 0, which if I write it out in these two dimensions, I will neglect variations in the third direction for simplicity, you could write it easily with all three dimensions as well.

So this just becomes equal to partial u x by partial x plus partial u i by partial y is equal to 0. The second is, the momentum conservation equations, these are now vector equations, so rho times partial u i by partial t plus u j. Let us neglect the body forces for the present, we will assume there is no gravity here. So these are now two equations for the x and y directions. The index i takes values both x and y, so you will get rho times partial u x by partial t plus u j partial u i by partial x j, there is a summation over j; that means that you have to add up values for j is equal to x and j is equal to y so this becomes u x partial u x by partial x plus u y partial u x by partial y i is x in this case, so I just get minus partial p by partial x plus mu into partial square u x by partial x square Laplacian of u x, so this becomes partial square u x by partial y square.

So that is a complete momentum conservation equation in the x direction. Similar equation can written for the y direction; so rho times partial u i by partial t plus u x partial u y; that is the complete momentum conservation equation in the y direction. Now for this two dimensional coordinate system, if the flow is unidirectional, if the flow is unidirectional that means that u y is equal to 0, flows only in the x direction. So if u y is equal to 0 and flows only in the x direction; that means that, in this equation as you can see. In the y momentum conservation equation, each term has u i in it, each term has u i in it; that means that, all of those terms are 0. Therefore, the momentum conservation equation the y direction, just reduces to minus partial p by partial y is equal to 0.

No dependence of pressure on the cross stream direction; partial p by partial y is equal to 0. It is also an implication for the mass conservation equation. As you can see here, if u i is equal to 0 this term is equal to 0; that means that partial u x by partial x is also equal to 0; that means that, for this unidirectional flow, just from the mass conservation condition alone, you cannot have a variation of the x velocity, along the x direction. The x velocity is independent of the downstream direction;

that is partial u x by partial x is equal to 0. This is what is called a fully developed flow, this is what is called a fully developed flow The fact that the flow is unidirectional, implies that it is fully developed; that is because there is no velocity u i, it is necessary that partial u x by partial x has to be equal to 0. Now that has implications for the x momentum conservation equation.

Because the flow is fully developed, this term is equal to 0; u x partial u x by partial x is identically equal to 0, because u x is independent of x. Similarly, this term is 0, because u i is equal to 0, this term is 0 because u i is equal to 0 and finally, this term is equal to 0, because partial u x by partial x is equal to 0. So for a fully developed fluid, another important implication, since partial p by partial y is equal 0 partial p by partial x is independent of y. So this pressure gradient term, since the pressure itself is independent of y, pressure itself is independent of the cross stream direction; that means if the pressure gradient is also, independent of y, independent of y the pressure itself is also independent of y.

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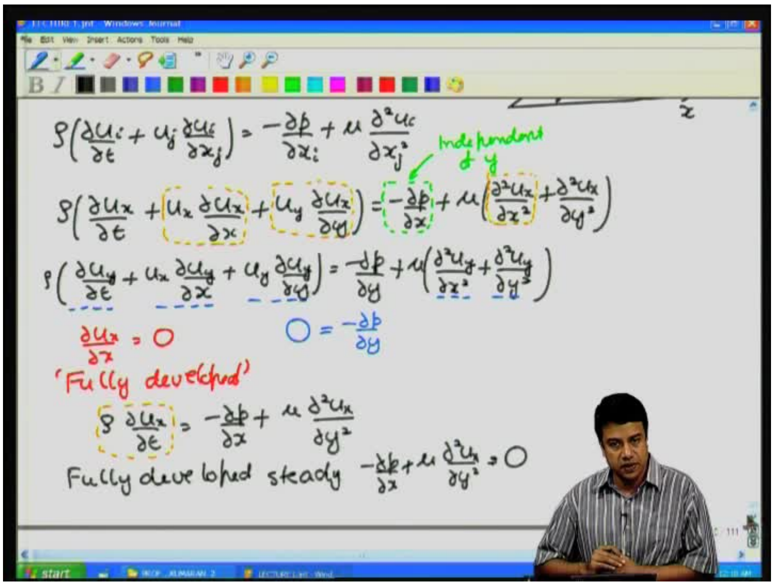

Therefore my x momentum conservation equation reduces to rho partial u x by partial t is equal to minus partial p by partial x plus mu partial square u x by partial y square, for this simple one dimensional flow. So this is the equation for a fully developed flow. In addition if the flow is steady; steady means time independent if the flow is fully developed, and it is steady; that means that, this term is also equal to 0. So for a fully developed steady flow, minus partial p by partial x plus mu partial square u x by partial square is equal to 0. And if you recall these were the exact equations we had for unidirectional flows, for a full developed steady flow, with partial p by partial x being independent of y. So this is the equation, is quite easy to solve it.

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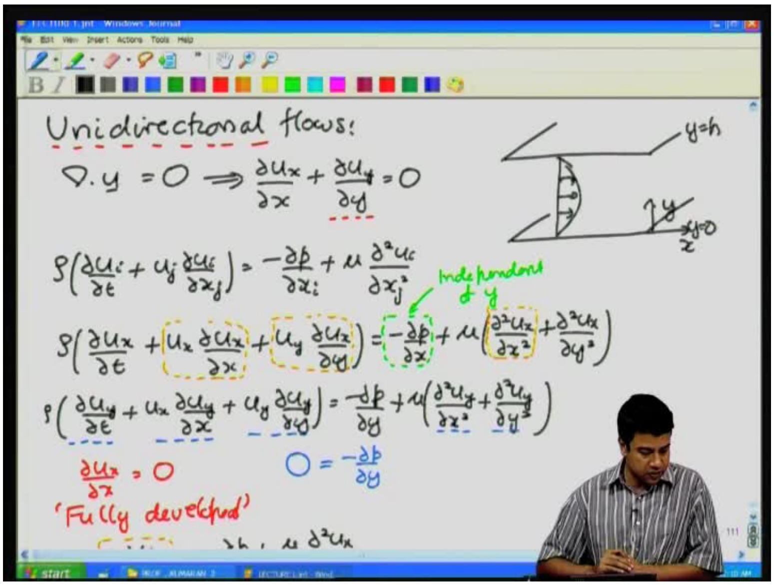

For definiteness, let us take two location, two boundaries at y is equal to 0 and y is equal to h, the two boundaries of the slot at y is equal to 0 and y is equal to h, we required the non slip condition at those surfaces, the velocity has to be equal to 0 at y is equal 0, and y is equal to h.

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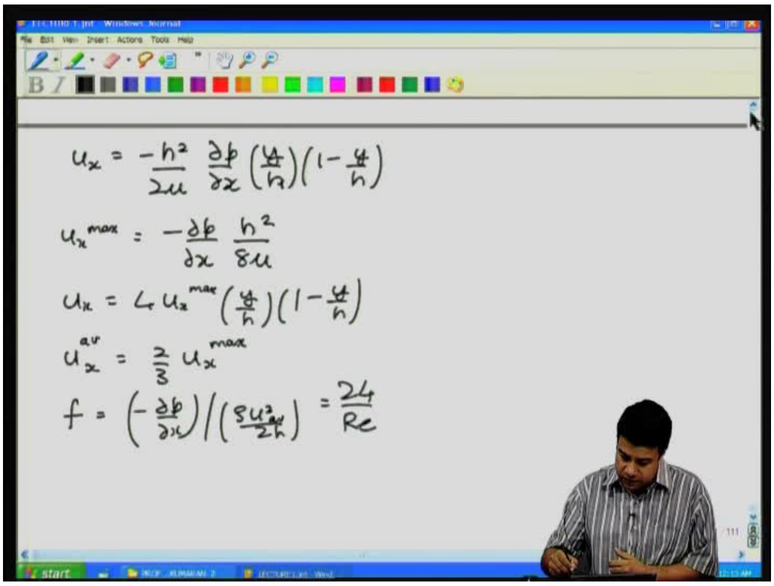

And on this basis you can quite easily solve the equations, to get the exact velocity profile, resulting solution that you will get for the velocity profile is u x is equal to minus h square by 2 mu partial p by partial x into y by h into 1 minus y by h quadratic solution for the velocity profile. There is a maximum value of the velocity, at y is equal to h by 2, the velocity is a maximum, at the center of the cannel. The velocity is maximum, exactly at the center of the channel h by 2, and that maximum component velocity is given by u x max is equal to minus partial p by partial x into h square by 8 mu, and therefore, u x is equal to 4 u x max into y by h into 1 minus y by h, and you can as easily calculate the average velocity by integrating across the cross section and dividing by total h, and you will find that u average, in the x direction, is equal to 2 by 3 of the maximum velocity, is equal to 2 by 3 of the maximum velocity and on this that basis you can calculate the frication factor. The frication factor f for this flow is defined as; minus delta p by delta x by rho u square square by 2 h, which is end up giving you 24 by the Reynold’s number.

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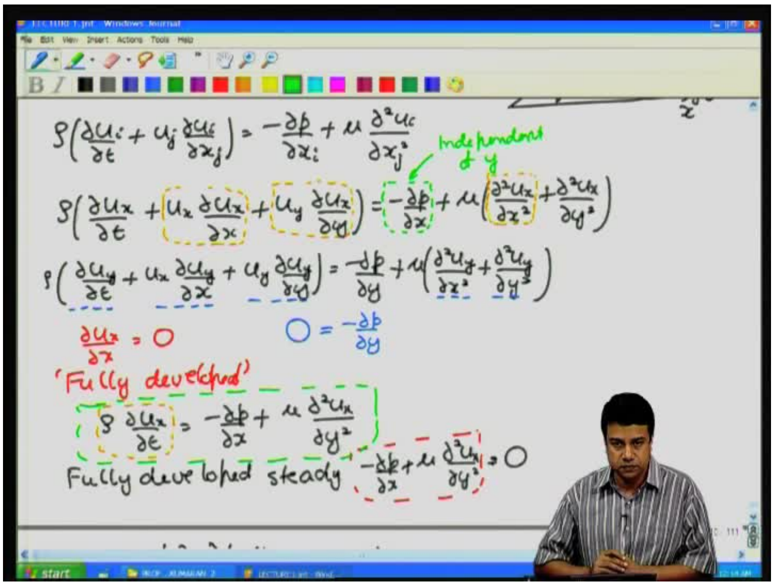

So this once you know that for a unidirectional flow, the equation has to be of this form, then it is quite easy to solve. You just get a quadratic equation, because from the two equations, we know that the pressure is independent of y as well, and then you can solve to get two equations. If the flow is fully developed, but not steady, the equation that you get, is this one. If the flow is fully developed, but not study derivative as well, that if you recall we had done, in detail in fundamentals of transport processes one, where I showed you how you get boundary layer solutions in the limit where, the inertial terms are dominating. In that limit you will get boundary layer solutions, at the wall, and the flow near the center will be a plug flow, if you recall we had done that in, detail in the fundamentals of transport processes.

In that case the flow is fully developed, no variation in x, but it is not study, it depends up on time; that is because the pressure, difference applied across the two ends, is time dependent. One can have such a situation, where the pressure is time independent, but the velocity is not, because pressure travels at the speed of sound. So pressure equalizes instantly across the entire, width of the pipe at the speed of sound, whereas the velocity in the pipe is usually much slower. So in many cases, it is a good approximation to consider that the pressure is instantaneously changing its value. The pressure gradient everywhere, is instantaneously changing its value, and the response time of the velocity is much smaller. So it solve for the a time independent velocity profile, assuming that the pressure gradient everywhere, is a constant, independent of position, but dependent upon time.

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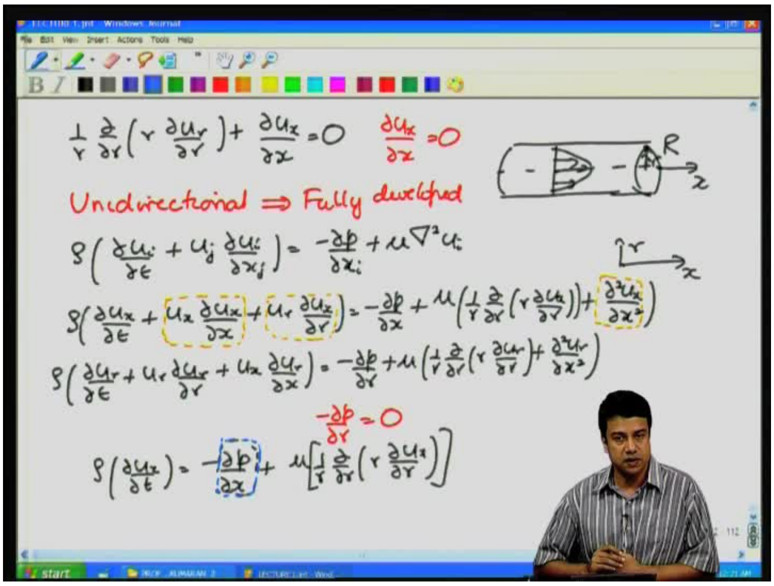

Now a similar thing can be done for a pipe flow, for a pipe flow you can do a similar thing. Let us say you have a pipe with radius r, and you have a pressure difference across, which drives the velocity across the pipe. You can do a similar thing for a pipe flow. I would not go through the detail, through the details of the calculation. In this particular case, we will assume a coordinate system, where x is the stream wise coordinate, and r is the radial coordinate. So you have an r x coordinate system; r is the distance from the axis cylindrical coordinate system, x is along the axis phi is the angle around, the meridional angle. If you assume that the flow is axis symmetric, there is no variation with phi, there is a variation only with r and x. If the flow is unidirectional, there is no variation of the velocity there is no variation of. I am sorry if the flow is unidirectional, there is a velocity, only in the x direction, no velocity in the r direction.

Mass conservation equation in cylindrical coordinates 1 by r d by d r of r partial u r by partial r plus partial u x by partial x is equal to 0, the divergence in a cylindrical coordinate system. Once again if the flow is unidirectional, partial u x by partial x is equal to 0. So once again u x cannot be dependent upon x, and therefore, one has simply fully developed flow in unidirectional implies, fully developed. Next the momentum conservation equation in the x direction, the momentum conservation equation in the x direction Once again rho partial u i by partial t plus u j partial u i by partial x j is equal to minus partial p by partial x i plus mu del square u i. Del square is the Laplacian operator, which will be different now in a cylindrical coordinate system.

Momentum conservation in the x direction; that means i is equal to x so you will get rho into partial u x by partial t plus u x partial u x by partial x plus, times the Laplacian operator acting on the x velocity. Similarly, for the r direction, exact same momentum conservation equation for the r direction, similarly for the r direction. In the r equation, flow is unidirectional; therefore, I just get minus partial p by partial r is equal to 0, it is simple for a unidirectional flow. In the x momentum conservation equation this term is 0, because partial u x by partial x is equal to 0; fully developed flow. This term is 0, because there is no variation, there is no velocity in the r direction, so in the r direction. There is no velocity and therefore, that term is equal to 0. This is equal to 0, because the flow is fully developed. The flow is fully developed so this is equal to 0.

And also since pressure is independent of r d p by d x is also a constant, it is independent of r, it is independent of r d p by d x is also independent of r therefore, the equation that I get is will be of the form rho times partial u x by partial t is equal to minus partial p by partial x plus mu 1 by r d by d r of r partial u x by partial r. Now, p is independent of r therefore, d p d x is also independent of r. A fundamental doubt that is often raised is, does this thing d p by d x, does it have to be a constant, or can it dependent upon x itself, is d p by d x does it have to be a constant, or can it vary with x. In other words, can the pressure is the pressure have to be a liner function of x, or can it be some other complicated function of x.

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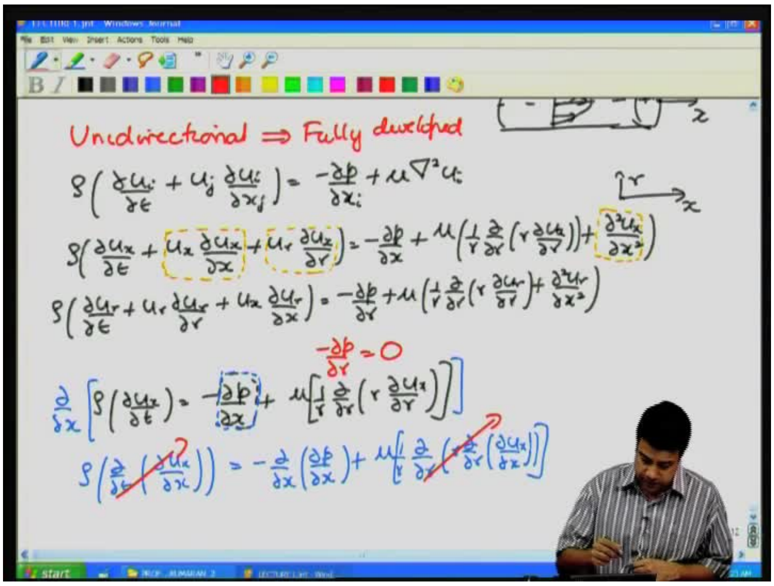

The answer is quite simple, the way you resolve that, is just by taking the derivate of this entire equation, with respective to x. So I take this entire equation, and take d by d x of that. And you know that you can interchange the order of differentiation, the differentiation with t can be interchanged with the differentiation with x. Similarly, the differentiation with r, can be interchanged with the differentiation with x, because they are both independent variables. So therefore, the equation that I get will be rho into partial by partial t of partial u x by partial x minus d by d x of d p by d x plus mu 1 by r d by d r of r d by d r of partial u x by partial x. Just took that took the derivative with respective to x, and it interchange the order of integration into, interchanged the order of differentiation with respective to t and r and with respective to x and r.

Now for a fully developed flow partial u x by partial x is equal to 0; that means that these two terms, are both equal to 0 partial u x by partial x is equal to 0, because unidirectional implies fully developed. Therefore, d by d x of d p by d x is equal to 0. We already know that p is independent of r, and this shows that the second derivative p with respective to x, has to be equal to 0; that means that p has to be a liner function of x or d p by d x has to be a constant. So that is why the pressure gradient will always be a constant, for a unidirectional fully developed flow, so that is the reason why it is always assumed, that this pressure gradient is a constant.

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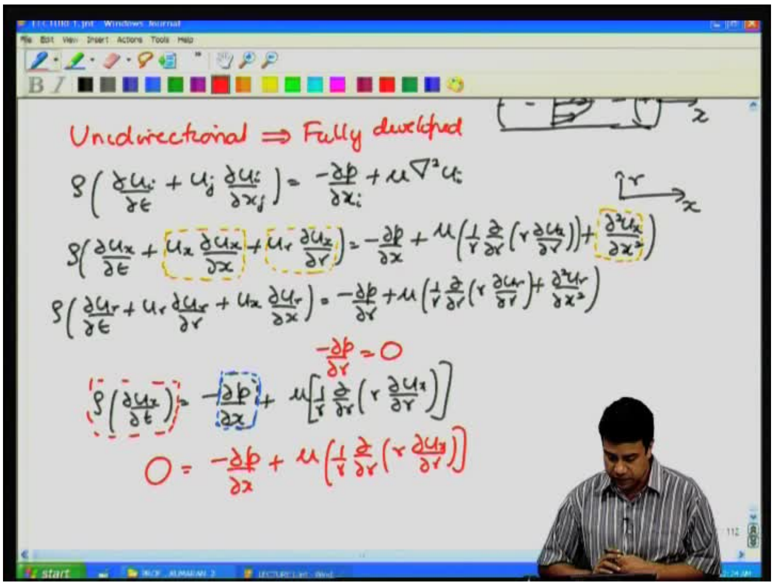

So this is for a fully developed unidirectional flow, if it is also steady, if it is also study, the time derivative is equal to 0, and I get my simplified equation as, minus partial p by partial x plus mu into one by r d by d r of r partial u x by partial r that is equal to 0, and you can easily solve for the conservation equation, to get the velocity profile.

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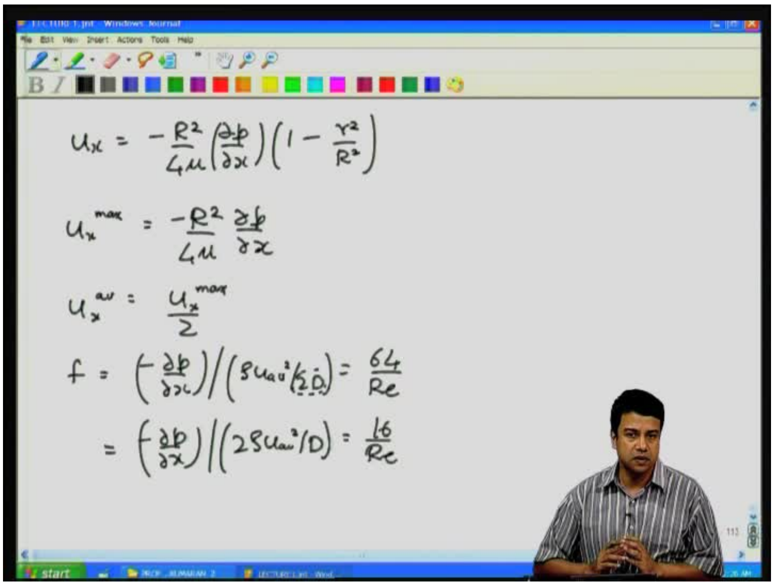

You get the well know solutions u x is equal to minus r square by 4 mu partial p by partial x into 1 minus r square by r square. These are obtained using the boundary conditions that the velocity is equal to 0 at r is equal to capital R. And for the flow in a pipe, the maximum velocity u x max is equal to minus r square by 4 mu d p by d x, u average is equal to the maximum by 2. And the friction factor, which is defined as minus partial p by partial x by rho u average square by 2 d, where d is the diameter of the pipe, is equal to 64 by Reynold's number. This is the this is defined based up on the diameter rho u square average square by 2 is the kinetic energy into divide by the diameter. The other way is to express in terms of the hydraulic radius, the hydraulic radius is d by 4.

So the other frication factor is minus partial p by partial x, by 2 rho u average square by d, is equal to 16 by r e. So it is defined with the radius as the length as the characteristic length in one case, the darcy friction factor, whereas a fanning frication factor is, usually defined with the hydraulic radius, which is the diameter divided by 4. So this gives the frication factor for the flow in a pipe. In the case of an an unsteady flow of course, you just include the time derivate in the equation as well, and one can get boundary less solution, similar to what we had done in the fundamentals of transport processes one. Finally, let us just look at one particular case, where we actually have flow being driven by the body force. Let us just look at one case where the flow is being driven by a body force.

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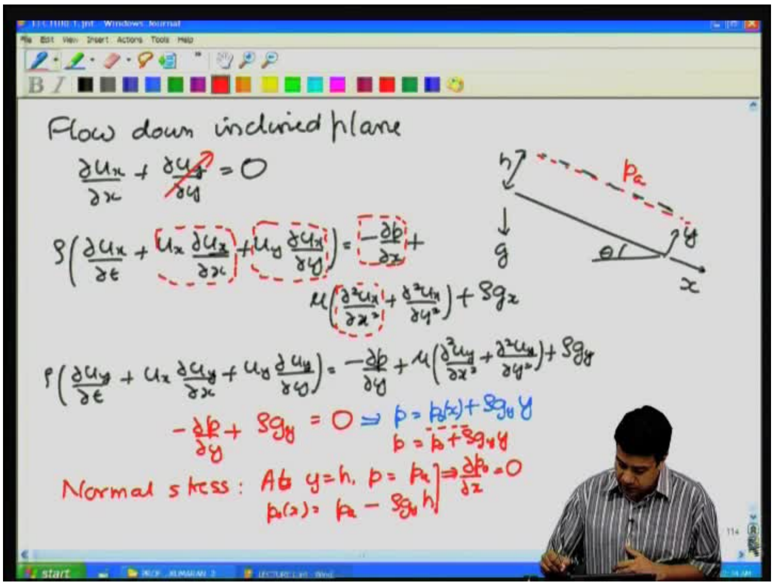

That is, flow down inclined plane. I have a fluid of height h, coming down in inclined plain, in which the gravity vector is acting in the this direction. And of course, since the flow is coming at an angle, this angle is theta, and I put my coordinate system x and y to be, along with and perpendicular to the plain itself. I put my coordinates to be along and perpendicular to the plain itself, those are the flow directions and the cross stream directions. We assume that the velocity the length is sufficiently long that it has reached a fully developed flow. In which case my mass conservation equation partial u x by partial x plus is equal to 0. If it is unidirectional flow, than u i is equal to 0, and therefore, I just get u x is independent of x. Now in my momentum conservation equations for the x and y directions.

Then I have a body force here, body force is proportional to the component of the gravity, acceleration due to gravity, which acts along the x direction. So g x is equal to g times sin theta the component of the gravitational acceleration, which is acting along the flow direction. Then I have the cross team velocity plus rho times g y. There is a mistake here and that is that both of these. Both of these have to be y, and this one also has to be y. As usual all terms in the y momentum conservation equation, will end up being which are proportional to the velocity, will end up being 0, because u y is equal to 0 and therefore, the y momentum conservation equation just reduces to minus partial p by partial y plus rho g y is equal to 0.

So, the y momentum conservation minus partial p by partial y plus rho times g y is equal to 0, where g y is equal to minus g y is pointing downwards so g y is equal to minus g cos theta, but we will just assume it is a constant here. This can be solved easily to give me a solution p is equal to p naught plus rho g y times y. Just integrate this once p is equal to p naught plus rho g y times y. This p naught in general, is the value of the pressure at y is equal to 0. It this this p naught is the value of the pressure, at y is equal to 0. When you do the integration, I am integrating a partial differential equation, with respective to y. So the constant of integration that appears, can in general be a function of x. A constant of integration can in general be a function of x, because I am integrating with respective to y. So in this expression if I take the derivative with respective to y, keeping x a constant I just get rho times g y; that satisfies the differential equation. How do we know what the value of p naught is?

We know that at that, at the top interface, this liquid is in contact with a gas, and therefore, the pressure at that top interface has to be a constant. The pressure, the top interface has to be equal to the pressure in the gas, because it is a free moving interface of fluid, flowing down in inclined plain, which is in contact with the atmosphere. So the pressure at the top surface, has to be equal to atmospheric pressure itself; that means that at y is equal to h, at y is equal to h, p is equal to p atmosphere. So this pressure, this constant can be determined from the boundary condition, normal stress condition, at y is equal to h p is equal to p a; that means that p naught of x is equal to p atmosphere minus rho g y h, because at y is equal to h p has to be equal to p atmosphere; that means that the constant of integration, which was dependent upon x, from the boundary condition since p atmosphere, is independent of x coordinate that means that p naught also has to be independent of the x coordinate ,because the pressure has to be atmospheric pressure at that top surface.

And important implication of this, is that partial p naught by partial x is equal to 0; that means that p naught is independent of x, and also the pressure itself, because p is equal to p naught plus rho g y y. So if the pressure p naught is independent of x; that means that p is also independent of x. The the implications of the pressure is also independent of x. Therefore, this term here is equal to 0; that comes from the free interface condition, and from using the y momentum conservation equation. So this has given us an important fact, which we can use in the x momentum conservation equation. Of course, for a unidirectional flow, there are terms in this x momentum conservation equation, which will all go to 0. For example, because u x is independent of x, this term is going to go to 0, because u y is independent of y sorry u y is equal to 0, this term is equal to 0, and this second derivative, is also going to be equal to 0.

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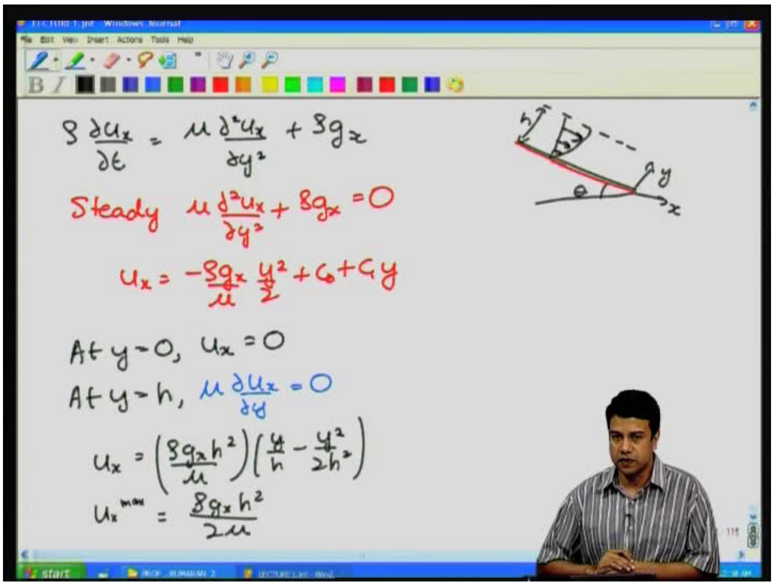

So finally, I will end up with solving the equation rho partial u x by partial t is equal to mu times partial square u x by partial y square plus rho times g x. So this is for a fully developed unidirectional flow, down an inclined plain. If it is also steady, that means that my equation just becomes mu partial square u x by partial y square plus rho g x is equal to 0, for a steady fully developed flow. I can solve this equation u x is equal to minus rho g x by mu into c naught plus c 1 y, where the constant c naught and c 1, have to be determined from the boundary conditions. I am sorry y square by 2 plus c naught plus c 1 y, where c naught and c 1 have to be determined from the boundary conditions. The boundary conditions are, at this bottom surface, I have the no slip condition; that means the velocity v is equal to 0, at that surface. So at y is equal to 0, u x is equal to 0, and at the stop surface, at y is equal to h, the liquid is in contact with a gas, at y is equal to h the liquid in contact with a gas. Therefore, the appropriate boundary condition to apply is that the shear stress, is equal to 0.

The appropriate boundary condition is that the shear stress at that surface partial u x by partial y is equal to 0, which means that partial u x with respective to y is equal to 0. So using these two boundary conditions, one can determine the two constant in the equation. To get the velocity felid u x is equal to rho g x by mu into h square by mu into y by h minus y square by 2 h square. This is a parabolic profile, which has the maximum velocity at y is equal to h, at y is equal h this becomes equal to u x max is equal to rho g x h square by 2 mu. The average velocity will be half of this, the velocity profile that I will get, for the flow down in inclined plain, something that looks like this. Parabolic velocity profile, for which the slope becomes 0, as the top interface is proportional. Now this profile for flow down in inclined plain, is also a very useful one to discuss. One of the other issues that we have not discussed so far, and that is what happens at a liquid, liquid interface.

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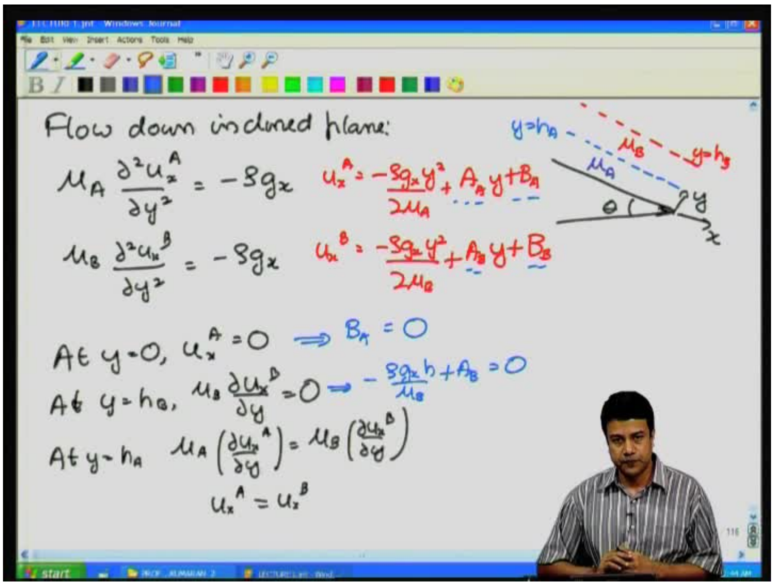

So I have flow down in inclined plain, at an angle theta, but rather than having just one liquid flowing down the inclined plain, I have two layers of liquids, flowing down the inclined plain; this is x, this is y. I have two liquids a and b that are flowing down the inclined plain. So I have liquid a with height h A, and I have a second liquid b of height h, so I have one liquid for which, y is equal to h A at the interface, and the other liquid, for which y is equal to h B. We have two liquids with two different viscosities, mu A here, mu A here and mu B here. So I will assume for simplicity, that the densities of these two are the same, only the viscosities are different. It is quite easy to extend this, to the case where the, both the densities and the viscosities are change.

So for the two fluids, I have my equations are mu A partial square u x A by partial y square is equal to minus rho g x. It should be rho A g x, but I will assume that the densities of both are same. And therefore, for this liquid a I have this relationship, where as for the liquid b my relationship becomes mu B times partial square u x B by partial y square is equal to minus rho g x. The solutions are u x A is equal to minus rho g x y square by 2 mu A plus one constant A A times y plus B A and u x b is equal to minus rho g x y square by 2 mu B plus constant y plus another constant. Now there are four constants; that means that I need four conditions to completely specify, all four of these constants. Boundary conditions, there are two boundaries one at y is equal to 0, which is the boundary between the bottom fluid and the solid surface.

The other at y is equal to h B, boundary between the top liquid and the gas, and there is an interface between these two. So at the bottom boundary I require that, at y is equal to 0 u x A is equal to 0. At the top surface at y is equal to h B, the shear stress has to be equal to 0; that is mu B times partial u x B by partial y has to be equal to 0. And then I have the interface conditions, at the interface between the two liquids the, velocity has to be the same, and the stress has to be the same. There is no normal velocity, so there is no need to impose the velocity condition in the y direction. The only velocity condition that has to be imposed, is in the x direction. There is no normal. The normal stress is only due to pressure, so basically the normal stress condition, will basically tell you the pressure, in both liquids is same as the interface.

The shear stress condition basically is that the velocity A at y is equal to h A. The velocity A times partial u x A by partial y is equal to the velocity I am sorry the viscosity in B times partial u x B by partial y; that is the continuity of stress condition, and this is the continuity of condition u x A is equal to u x B. So these put together, basically give you four equations, with which you can find out what the values of the four constants are, basically it gives you four equations, which can be used to find out what the values of the four constants are. So in this particular case, I am going to enforce the 0, the quality of velocity, as well as the quality of stress, in both fluids, and from that I will find out what all four constants are. So at y is equal to 0 u x A is equal to 0, that will imply that, the constant B A is equal to 0. At y is equal to h B the derivate is equal to 0; that means that minus rho g x h by mu B plus A B is equal to 0.

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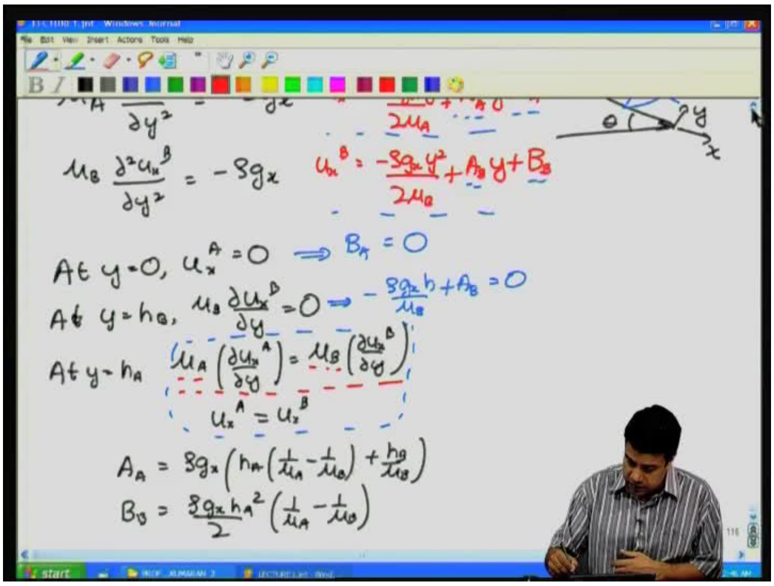

So, these give you two of the constants B A is equal to 0 and A B is equal to rho g x h by mu B, because the shear stress has a top interface is equal to 0. And from these two equation, I will get the two more constants. It is just an excise in algebra, so I would not go through the details, from these two equations I will get the other two constants, and these constants turn out to be A A is equal to rho g x into h A into 1 by mu A minus 1 by mu B plus h B by mu b and B B is equal to rho g x h A square by 2 into 1 by mu A minus 1 by mu B, so that completely fixes all four constants in the equations.

Now these both are quadratic equation. These are both parabolic velocity profiles, both of these are parabolic velocity profiles, and therefore the kind of profile that you will get here, is something that will look like this. There will be a change in the slope at that point, because even though the shear stresses are equal, even though the shear stresses as you can see. The sear stresses on both sides are equal, the viscosities are not equal; that means that there will be, the derivatives of the velocity at the interface will not be equal. You will get a different slope in one liquid, as compare to the other liquid. There will be a discontinuity in, the slope, also velocity at that point the derivative of the velocity.

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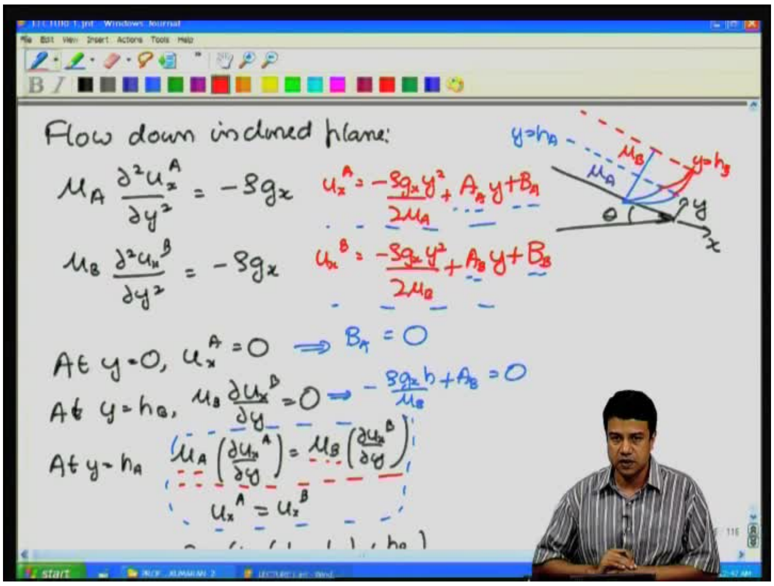

And that discontinuity will of course, depend up on which liquid is more viscous. The liquid which is more viscous will have the lower derivative, because the stresses are equal, the viscosity is higher, the derivate is lower. If the liquid a is for example, more viscous the derivative here will be lower, than you will have a higher derivative in this second fluid, whereas the other way around, you will get something that looks like this, with a lower derivative on top. In either case, the slope at right of the top has to be equal to 0, because the shear stress is equal to 0, at the bottom the velocity is 0. You have two conditions in between, and you have four constants, so you can solve all of these four, to get the velocity profile for an interface problem. So this completes our discussion on unidirectional flows, I have just shows you how can quite easily, from the equations recover all of the, all of the results that we had got in the fundamentals of transport processes one, based upon shell balances.

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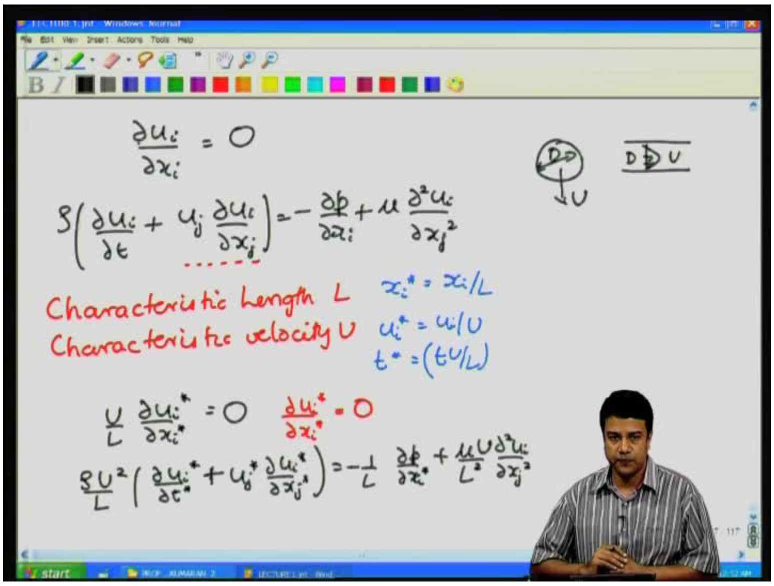

Now, let us look at the equations for a general velocity profile. The equations are; partial u i by partial x i is equal to 0 and rho times partial u i by partial t plus u j partial u i by partial x j. So these are the mass and momentum conservation equations, the navies stokes equations, and let us take a general problem, we would not specify it too closely, to to clearly at the stage. In which we have a characteristic velocity scale U, and a characteristic length scale L. So I have characteristic velocity U, therefore, I can define a scaled coordinate as as x i by L, and a scaled velocity as u i by U. The scale time is quite easy to get, because from a velocity scale U and the length scale L. The only way you can get a time scale, is to define t i star is equal t i times U by L. I am sorry t star this is equal to t times u by l, so it is the characteristic time scale. Let us express this equation in terms of those characteristic length, and velocity scales. The mass conservation equation just becomes U by L partial u i by partial x i is equal to 0. And of course, I can cancel out that U by L, to just get the mass conservation equation as partial u i by partial x i equal to 0, so that is the mass conservation equation.

Momentum conservation equation, on the left hand side I have, lengths velocity square dived by length, I have velocity square divide by length. Therefore, if I take out, that, express this in terms of the scale variable, one can easily verify that, you will get rho u square by L partial u i by partial t plus. The pressure we have not scaled yet, we will come back to that, is equal to minus 1 by L partial p by partial x i plus the vicious term mu times velocity square divide by I am sorry mu times velocity divide by length square. So this becomes mu u by L square partial square u i by partial x j square. Now, these velocity and length scales can be or characteristic to the particular problem. For example, if I had, a sphere of diameter d dropping with a velocity U in our liquid or in a fluid. A sphere of diameter D with a velocity U characteristic length is D characteristic velocity is U. On the other hand if I had pipe flow, a pipe of diameter D, with maximum velocity u then the characteristic diameter is D character length L is D and velocity is U.

Similarly, we could have more complicated cases, but in all those cases, you will have some, something, either the motion of the of an object within the fluid, or the motion of the fluid itself, which will give you a characteristic velocity scale, and there will be some character length scale. Either set by an immersed object, or set by the boundaries in flows through confined geometries, so those are the characteristic length and velocity scales in this case.

Now this momentum conservation equation, I can non-dimentionalize in two ways. You can non-dimentionalize either by dividing by the coefficient of the inertial terms, which are proportional to the density U square by L, or by dividing by the coefficient of the viscous terms mu U by L square, which you chose will depend up on what you except to be the dominant force in this problem. If you except the force, dominant force to be viscous, you dived throughout by mu U by L square. Whereas if you except the dominant force to be inertial, you will dived by rho u square by L. So let us take the first the viscous scaling, we except the viscous force to be dominant.

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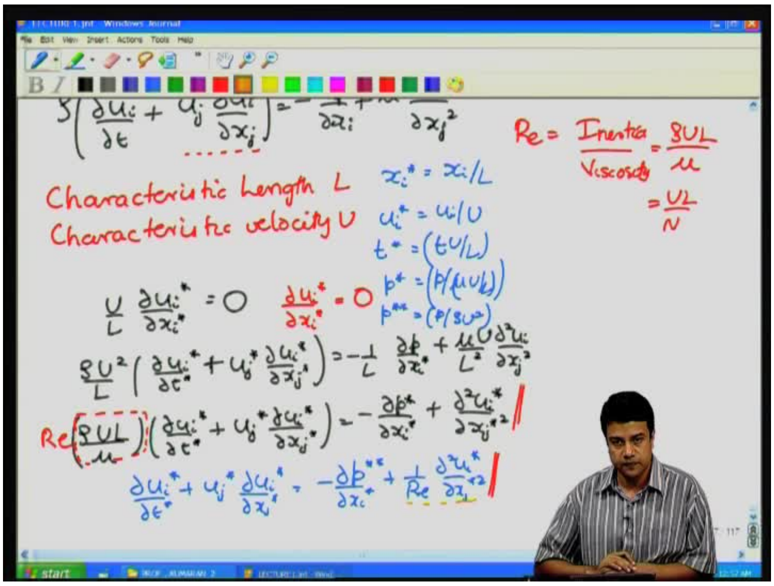

If you take the viscous scaling divide through by mu by L square what I get is rho u L by mu minus partial p star by partial x i star plus mu, I am sorry where my skilled pressure p star is equal to p by mu u by L. So this gives me a dimensionless number here. So this as you all know it is the familiar Reynolds number, r e times, the inertial term is equal to minus the pressure gradient plus the viscous terms in the conservation equation. On the other hand, if I choose to scale it by the inertial terms, if I dived throughout by rho u square by L. This you would do when you except the inertial forces is to be the dominant forces.

If you except the inertial forces to dominate, the viscous forces to be negligible, when you would scale by the inertial terms in the conservation equation. In that case the equation that I would get, is partial u i by partial t plus u j partial u i by partial x j minus partial p. I will call that as an inertial scaled pressure two stars plus 1 by r e partial square mu i by partial x j square, where p star star is an inertial pressure p by rho u square. So the first equation you would use when you except the viscous forces to be dominant, I have a Reynolds number times the inertial forces equal to minus pressure gradient plus viscous forces.

Next you would use when you except inertial forces to be dominant, so that would except a balance between the inertial forces in the equation, and the pressure gradient in the first case, if the viscous forces are dominant, viscous force is the ratio

I am sorry the Reynolds number, is the ratio of inertia by viscosity, is equal to rho u l by is the ratio of the inertial and the viscous forces. If I except the viscous forces to be dominant, the Reynolds number will be small, you would except that there would be a balance between, the pressure gradient and the viscous forces, and the inertial terms can be neglected, the diffusion dominated regime. This Reynolds number can also be written as u l by the kinematic viscosity, the kinematic viscosity is the momentum diffusion. On the other hand if the Reynolds number is large you would except that the inertial forces are dominant. I have a 1 over r e here, multiplying the viscous term, so an r e is large, 1 over r e is small, I would except I could neglect the viscous terms, and only solve for the inertial terms in the conservation equation, balance the inertial terms with a pressure gradient.

So that is the inertia dominated regime, high Reynolds number regime. However as we saw in our previous encounter with, convection dominated transport, even when the Reynolds number is large or (( )) number is large, you would think that you can neglect diffusion, in comparison to convection; however, when you neglect diffusion the equation, transforms from a second order differential equation to a first order differential equation, and you cannot satisfy all boundary conditions. Physically you cannot satisfy boundary conditions, because at the boundaries, the transport that takes place, is only. If you neglect diffusion, there is transport only due to convection, and convicted transport can take place only, along the flow direction. Two transport mass momentum or energy, from or to a bounding surface, you need transport; that is perpendicular to the bounding surfaces; that transport can happen only due to diffusion.

In the case of a fluid mechanics, it is a little more complicated than that, because in general I could have a balance between inertial and pressure forces as well. So I do get non trivial solutions, when I balance inertial and pressure forces. And in those cases, I cannot satisfy tangential velocity boundary conditions. However if I want to stratify tangential velocity boundary conditions, I have once again postulate a thin boundary layer at the surface, where inertial and viscous forces are balanced. The thickness of the boundary layer, has determined, on the basis of the condition that, for that small thickness, there is a balance, when the. Because the thickness is small the gradients are large, and when the gradients are large, there is once again a balance between inertial and viscous forces in this. So that is all going to be our Strategy, for looking at fluid mechanics problems.

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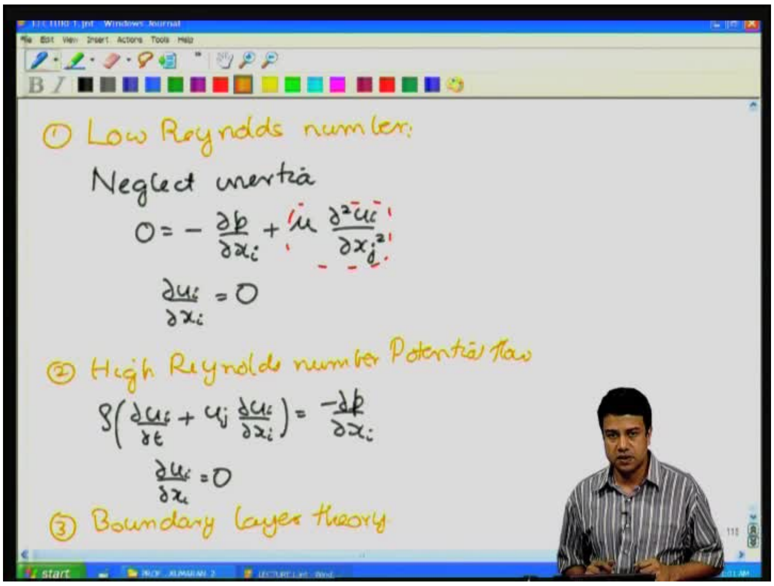

We look at special cases, the first is low Reynolds number. In this case you neglect inertia, and my conservation equation in the absence of the inertial terms, that we set the density is equal to 0, and I just get 0 is equal to minus grad p plus mu times partial square u i by partial x j square, and the divergence of the velocity, the divergence of the velocity is also equal to 0, so that is the first case lower Reynolds number. In the case of momentum mass and energy transfer, I just had this term, in the case of mass and energy transfer I just had this term. For momentum transfer I also have a pressure, and for solving that pressure I have the incompressibility condition. The second case is, high Reynolds number potential flow; in this case you just the rho balance between the inertial terms and the pressure gradient, and the incompressibility condition.

In this case on can no longer stratify the tangential velocity boundary conditions, for the velocity of the stress, neglected the highest derivative. And then the third step will be, to include that very near boundaries, because I realized that transport to boundary has to take place only due to diffusion; and that is boundary layer theory. So this is the broad strategy on the basis of which we will analyze fluid flows, from the navies stokes equations. Next lecture we start with lower Reynold's number flows, so will see you in the next lecture, continue we will start low Reynold's number fluid mechanics in the next lecture. Kindly revise the conservation equations that we have done here, so that we can start next lecture, on viscous flows, in the absence of inertia. We will see you next time.