Fundamentals of Transport Processes II
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Lecture - 13
Mechanical Energy Conservation

Welcome to lecture number 13 of our course on Fundamentals of Transport Processes. In the last 12 lectures we have derived the equation of motion for fluid flow, and now it is time it to start solving them, so before we go to solution let us explain a few everything that we have so far.

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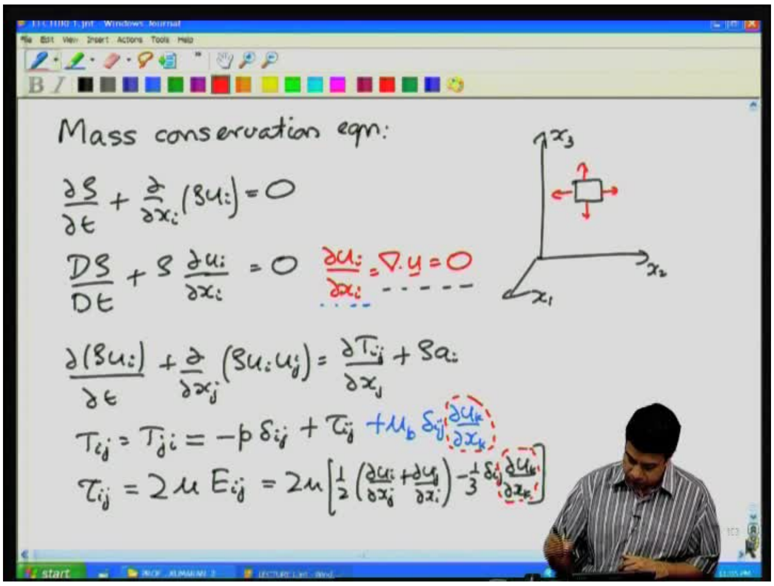

And so the mass conservation equation, the mass conservation equation can be written in the Eulerian reference frame, in a reference frame in a fixed reference frame at each point in the fluid as partial rho by partial t plus the divergence of rho u, is equal to 0. This just reflex the fact that in a moving material reference frame in which all points are moving with the same speed velocity as the fluid velocity at that point, there should be no net change of mass within a differential volume.

Alternatively, in terms of the substantial derivative, this can be written as d rho by d T plus rho partial by partial x i of the velocity is equal to 0 d rho by d T plus rho times divergence of the velocity fillers equal to 0. And if the fluid is in compressible the equation just reduces to partial u i by partial x i this is equal to the divergence of the velocity is equal to 0, important to keep in mind when I write it in this fashion this is valued only in a Cartesian coordinate system.

The divergence operator has different forms in different coordinate systems, because in this spherical and cylindrical coordinate systems for example, the coordinate the the unit vectors depend up on position and so you got to take the derivative, so the unit vectors would position as well. So, this is the incompressible mass conservation equation divergence of velocity is equal to 0; that means, that the radial part the isotropic part of the rate of deformation tenser is 0, locally in every volume there is no volumetric expansion or compression.

Next the momentum conservation equation can be written in a couple of different forms partial of rho u i by partial t plus T by T x j of rho u i u j is equal to minus is equal to partial T i j by partial x j plus rho a i. Where T i j is the stress tenser second tenser T i j is the force per unit area acting in the i direction at a surface whose is in the j direction at each point in the fluid.

So, there are nine components of the stress tenser which are defined at each and every point within the fluid from the angle of momentum conservation equation we know that T i j is a symmetric tenser. So, T i j is equal to T j i and as with our derivation in the case of the rate of deformation tenser, where we separate it out into a symmetric anti-symmetric and isotropic part, so there was an isotropic part which was basically something times an identity tenser.

Then there was a symmetric traceless part, a symmetric tenser which had the property that the sum of the diagonal elements was equal to 0, plus an anti-symmetric part, so in this case the tenser is symmetric. So, I can only separate out into two parts, one is an isotropic part proportional to the identity tenser, and this second is the symmetric traceless part.

So, this for a static fluid the isotropic part is just basically the negative of the pressure, because the isotropic part is basically the force per unit area acting along along the outward unit normal for a differential volume. The isotropic part is the force per unit area acting along the outward unit normal for each and every volume, the pressure as we classically define at acts along the inward unit normal therefore, the isotropic part in the absence of flow is just minus p time delta i j.

Then there is a second part which is non 0 only when there is flow tau i j, if this tenser symmetrical trace less in other words if the flow does not add to the isotropic part. Then this symmetric trace less tenser was postulated to be a linear function of the velocity gradient, now if it is a linear function of the velocity gradient you know that the velocity gradient the rate of deformation tenser can be separated out into symmetric traceless isotropic and anti-symmetric.

This tau i j is symmetric traceless it is only a function of the velocity gradient and it is linear it has to be a liner function of the velocity gradient not a quadratic function, it cannot be just proportional to the velocity itself, because if I move the entire equipment to the constant speed. There should be no internal stresses that are generated, it cannot be proportional to the anti symmetric part of the rate of deformation tenser, because at a given location the anti symmetric part represents the solid body rotation around that point.

And when a solid body revolves the distances between points, material points in that body do not change, so there is no deformation, so it cannot be function of the symmetric anti-symmetric part. The only thing it can be a function is the symmetric traceless part, and this if it is a linear relationship then the only relation that it can have is a constant times the symmetric traceless tenser be consistent and use the notation E i j symmetric traceless tenser.

And that coef constant coefficient sitting in front is two times the viscosity, it is two times the viscosity, so that is the symmetric traceless part of the rate of deformation tenser. It is possible that due to flow if you fluid was compressible in general you could also have an isotropic part due to the velocity the rate of deformation tenser. In an isotropic part due to the rate of deformation tenser has to be of course, be proportional to the divergence of the velocity itself.

So, in general you could also have another contribution, which as a coefficient times partial u i by I am sorry by partial x k, you could have another contribution if the fluid was compressible, because as you can see this is the divergence of the velocity.

However, since we are restricting our discussion to incompressible fluids this contribution is 0, and all you are left with is two mu times E i j where E i j.

As you recall was a symmetric traceless part of the rate of deformation tenser, which was half that is the matrix plus it is complex conjugate minus the trace. And since the divergence of the velocity is 0, this term is also equal to 0, so those where are Nevier stokes mass and momentum conservation equations.

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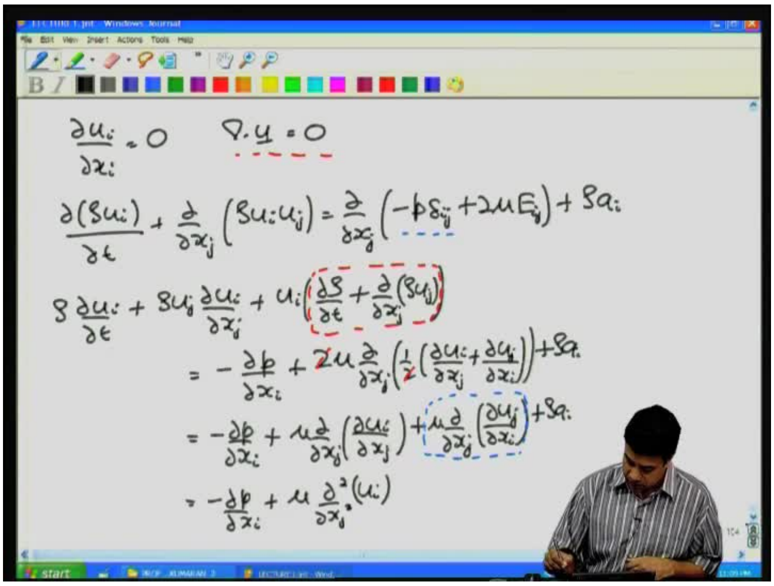

To write it out in detail once again the mass conservation equation is just that partial u i by partial x i is equal to 0 or divergence of the velocity is equal to 0, the momentum conservation equation I can write in two ways partial of rho u i by partial t plus partial by partial x j of rho u i u j is equal to partial by partial x i of minus p and I should take j here delta i j plus 2 mu E i j plus rho a i.

And this thing can be simplified a little bit, as you know if I subtract out u i times the mass conservation equation from this, I can expand out the left hand side to get rho partial u i by partial t plus rho u j partial u i by partial x j plus u i into partial rho by partial t plus partial by partial x j of rho u j. Just use the chain rule for differentiation for the left hand side, this entire term is identically equal to 0 because that is the mass conservation equation.

On the left right hand side i have minus partial by partial x j of p times delta i j partial by partial x j of p times delta i j non 0 only when i is equal to j. So, this just equal to minus partial p by partial x i plus coefficient of viscosity, if that is a constant you can take it out 2 mu partial by partial x j of half partial u i by partial x j plus partial u j by partial x i plus rho a i these 2 half and 2 will cancel out.

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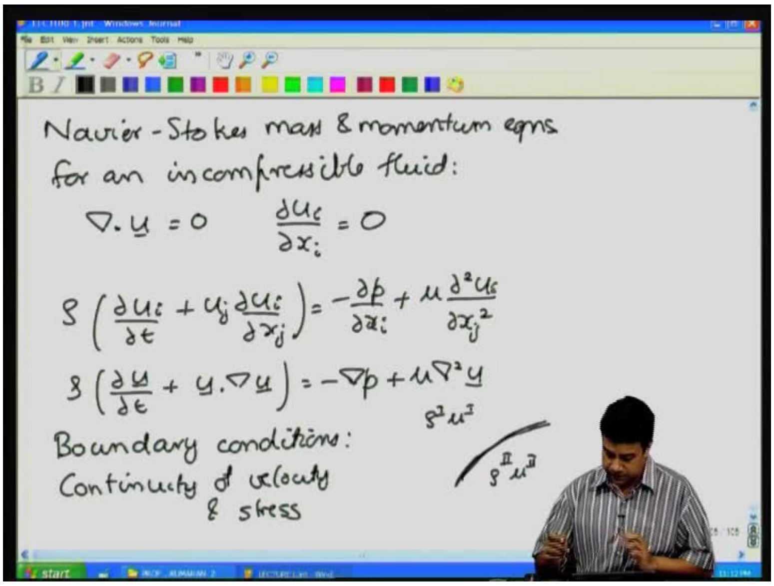

And I Can write this as minus partial p by partial x i plus mu partial by partial x j of partial u i by partial x j plus partial u j by partial x i plus rho a i, in this second term here x i and x j are independent coordinates. So, you can interchange the order of differentiation partial by partial x j of partial u j by partial x i is also partial by partial x i of partial u j by partial x j and partial u j by partial x j is just the divergence of the velocity which as we know is 0, the divergence of the velocity is equal to 0.

Therefore, my final equation becomes minus partial p by partial x i plus mu d square by d x j square of u i d square by d x j square is just the Laplacian del square of this vector. So, my final Navier stokes mass and momentum equations for an incompressible fluid the divergence of the velocity is equal to 0 partial u i by partial x i is equal to 0 repeated index dot product scalar.

And rho into partial u i by partial t plus u j partial u i by partial x j is equal to minus partial p by partial x i plus mu partial square u i by partial x j square, rho into partial u vector by partial t plus u dot grad note that the index j is repeated, so u dot grad u vector is equal to minus gradient of p plus mu del square u vector. Vector equation three components of the velocity. So, there are three equations here plus 1 mass conservation scalar equation 3 un 4 unknowns, the 3 components of the velocity one pressure, and so you can solve for all of them.

And finally, I discussed boundary conditions at the end of the last class, it is a second order differential equation in the spatial coordinates first order in time, since it is second order in the spatial coordinates you need two boundary conditions for each coordinate. The boundary conditions basically at an interface at an interface between let us say two let us say I have fluid 1 with rho 1 mu 1 viscosity mu 1 and rho 2 mu 2, at the interface itself you require that there should be continuity continuity of velocity and stress.

That means, continuity of velocity means at the interface, the velocity in fluid 1 has be equal to the velocity in fluid 2 that is all 3 components of the velocity in fluid 1 have to be equal to all 3 components the appropriate components of the velocity, in fluid 2. Then there also has to be a force balance that is the normal forces on both sides have to be equal and the tangential forces on both sides have to be equal, so that is what is meant by continuality of velocity and stress.

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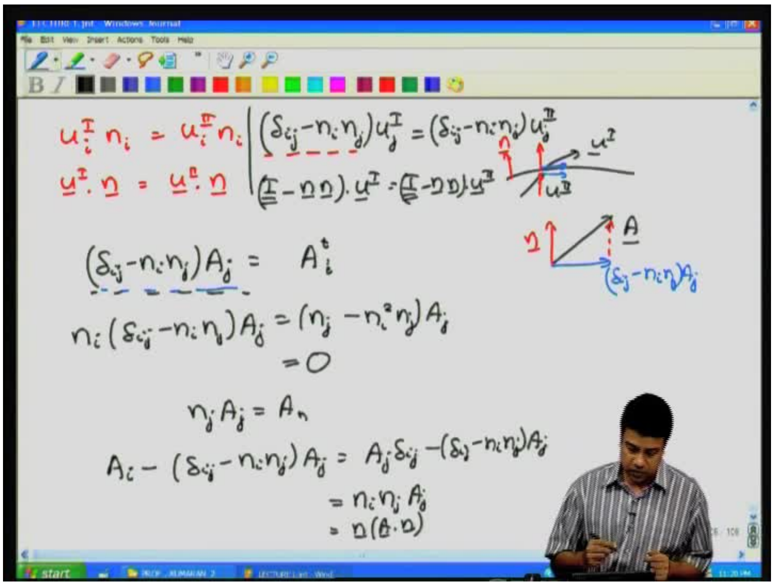

These are general they hold for any fluid, the continuity of velocity basically means that if I have an interface like the velocity in fluid 1 and the velocity in fluid 2 had that interface the velocity in fluid 2. At a given point on the interface the velocity in fluid 1 and the velocity in fluid 2 have to be exactly the same, which means that the normal components of the velocities have to be the same and the tangential components of the velocities in the two fluids have to be exactly the same. Now, you can define the unit normal to the surfaces n, n is in general the unit normal to the surface going from one fluid to the other.

So, if the velocities are equal; that means, that the normal components are equal; that means, that u 1 i n i is equal to u 2 i n i or alternatively in vector notation u 1 dot n is equal to u 2 dot n, that is the normal component the component of the velocity at the interface along the direction of the normal in fluid 1 has to be equal that in fluid 2. Otherwise, you will have a velocity discontinuity at that point fluid 1 is moving faster or slower than fluid 2 fluid 2 at that point and the velocity is double valued, velocity is a single valued function of position.

Therefore, you cannot have two values of the velocity at given location, even if that location is an interface that is one thing, the second is that tangential components also have to be equal, how do we write down the tangential components. Tangential component is perpendicular to normal, it is in the plain perpendicular to the normal can be in any direction in that plain perpendicular to the normal the way you get the tangential component of a vector is to take delta i j minus n i n j times u j 1.

So, delta i j minus n i n j, where n is the components of the unit vector perpendicular to the surface the unit normal, so it has it has three components along with three directions, so I goes from 1 to 3 the resultant is a vector. So, delta i j minus n i n j times any vector is a resultant is a vector is called the tangential component of that of the original vector it is called this, because if i take the dot product of this with the unit normal. If I take the dot product of this with the unit normal, I will end up getting 0 let us see how that happens, so this is a vector.

So, this is a vector j is repeated, so it is it is a dot product the free index is i the free index is i therefore, if I want dot this with the unit normal I have to multiply it by n i. So, multiply it by n i what i get is n i into delta i j minus n i n j times A j n i times delta i j is n j, because it is nonzero only when i is equal to j, n i times n i n j is n i square n j this whole times n j. What is n i square? n i square as you know, when you have index repeated two times; that means, there is a summation, so n i square is n 1 square plus n 2 square plus n 3 square.

The sum of the three components is the squares the the sum of the squares of the three components of that unit vector the vector is a unit vector; that means, that the sum of the squares of the three components has to be equal to 1. So, n i square is equal to 1, therefore, I have n j minus n i square n j, so this whole just reduces to 0 unit normal times this vector is equal to 0; that means, that this vector is perpendicular to the unit normal.

This perpendicular vector is perpendicular to the unit normal and so this is the tangential component the normal component is just given by is equal to n i A i that is the I am sorry I should put as, now what do you get when you subtract out this from the original unit vector. and I subtract out this one from the original unit vector I will get A i minus delta i j minus n i n j a j, so I have my original unit vector, I have original vector A, this is a unit normal, I told you that this 1 delta i j minus n i n j A j is tangential unit vector.

If I take A minus this tangential vector, I should get a vector that is along the unit normal, because you subtract out these two you should end up with a vector which is along that unit normal. And that vector you can write A i as A j delta i j A i can always be written as A j times delta i j because delta i j is 1 only, when i is equal to j minus delta i j minus n i n j into A j. you can subtract out these two quiet easily to just get simply n i n j A j, which is equal to the magnitude of a along, n this is just equal to n vector n i free index; that means, that depends a vector direction times a dot n.

So, this is the vector along the unit normal whose magnitude is equal to a dot n the normal component, so sum of these two basically will end up giving you the total vector the component along the normal, plus this tangential component which is delta i j minus n i n j times a j. So, this thing is called the transverse projection operator, it gives you the component of the vector which is perpendicular to the direction n that is the tangential component of the velocity.

That has to be equal on both sides that has to be equal on both sides, so this is equal to delta i j minus n i n j times the vector in the second fluid or this is also, often written as the identity tenser minus n n dotted with u is equal to minus n, n dotted with u the second fluid. So, that is how you have to write the boundary conditions of the tangential component of the velocity.

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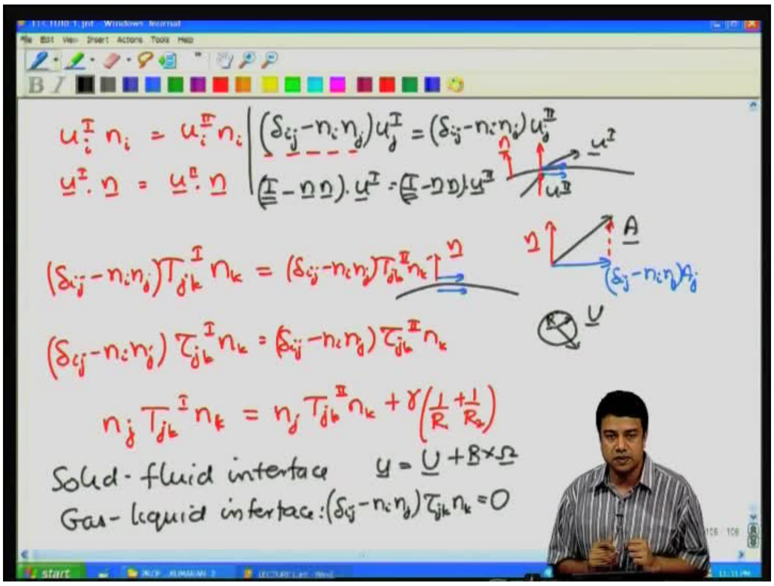

The other boundary condition that you need is the continuity of the stress at the surface that is the force per unit area both normal and tangential to the surface have to be equal on both sides. The tangential stresses have to be equal, because you cannot have a net force that is exerted by the interface itself, but however, you can have a net force normal to the surface exerted by the interface, that is the force of surface tension.

So, the surface tension force acts normal to the surface, now for the tangential component of the stress the force acting at that surface n with unit normal n, so at this surface that I have with unit normal n, the force per unit area that is acting is the stress tenser dotted with n the force per unit area that is acting is a stress tenser dotted with n. So, the force per unit area in fluid 1 is equal to T i j 1 times n j and we said that the tangential component of the force per unit area in fluid 1 has to be exactly the same as the tangential component of the force per unit area in fluid flow 2, T i j 1 n j.

The tangential component of this I get by just multiplying it by the transverse projection operator, so i will have, so I should have some other symbol here, when I use it transverse projection operator. So, I will i will write this as T j k and k into the transverse projection operator which is delta i j minus n i n j that in fluid 1 there is a stress that is along the tangential direction on both sides of the interface.

The force along the tangential direction on both sides of the interface have to be equal, this is equal to delta i j minus n i n j into T j k 2 n k, so that that is the tangential stress balance condition on both sides of the interface the tangential forces have to be equal. Further, tangential component of the stress there is no contribution due to the pressure, because pressure always acts perpendicular of the surface.

So, there is no contribution to the pressure due to the tangential component of the stress therefore, the tangential component along I can write this as del delta i j minus n i n j into the shear stress tau j k 1 n k is equal to delta i j minus n i n j into tau j k 2 n k. So, the 2 components of shear stress tangential to the surface have to be equal on both sides, finally, for the normal component the force acting at the interface itself is equal to T j k 1 n k.

The normal component of this is this dotted with the unit normal, the normal component force acting at the in is just this dotted by the unit normal, that is this dotted with n j. This on the other side it is n j T j k 2 n k; however, the two need not be equal because the interface itself can exert a surface tension force per unit area due to the curvature of the surface.

That surface tension force also has to be in corroborated here, this surface tension force is in general written as the coefficient of surface tension times 1 by r 1 plus 1 by r 2, where r 1 and r 2 are the principle radii of curvature along the surface the maximum and the minimum. These two maximum and minimum are always along orthogonal directions on the surface, so if I we have a surface that looks something like this a curved surface.

I will have two directions along which the radii of curvature or either maximum or minimum, and these are two perpendicular directions and along those two directions, if you calculate what is 1 by r 1 plus 1 by r 2 that times a surface tension coefficient gets me the difference in the trace along the between the two fluids. This difference in stress can also general be written in terms of the divergence of the unit normal, the unit normal is given by n and the divergence along the surface gives you.

So, if I define if I have unit normal n is equal to unit normal n that is perpendicular to the surface, I can take the surface divergence del S of n which is equal to the diversion of the unit normal along two orthogonal directions along the surface itself. And that divergence of n can also be used instead of 1 by r 1 plus 1 by r 2, that is a little bit outside the scope of this course, so I will not discuss that in detail but, just basically this surface tension force can also be expressed in terms of the divergence of the unit normal at the surface.

However, it is strictly speaking proportional to 1 by r 1 plus 1 by r 2 where r 1 and r 2 are the two principle curvatures, so this is for the interface between two fluids two fluids in general. If you have an interface between a fluid and a solid, the solid is assumed to be rigid; that means, that no matter how much the force is exerted on that solid it does not deform, it is a solid object.

It is not a like a fluid which deforms, continuously upon application of stress the solid will deform to some exert and then stop, so in a study state the solid will have some deformation, but it will not continuous deform under flow. In those cases where the boundary condition that is used is that the velocity of the fluid at the surface is equal to the velocity of the surface solid itself, so for a solid fluid. All you end force is that the velocity of the solid is equal to the velocity of the fluid at the surface of the solid is equal to the velocity of the solid itself; that means, that the velocity of the fluid at the solid interface.

So, if I have some some solid object here which is moving with some velocity U and we call that as capital U, capital U is the velocity of the solid; that means, each point on the surface of that solid is moving with exactly the same velocity, because it is a rigid object. On the other hand if it is rotating in general you will have a translational velocity plus a rotational velocity of the solid around it is axis for rotation, in any case for the solid fluid interface the boundary condition that you impose is that the velocity of the fluid at the surface is equal to the velocity of the solid itself.

If it is just purely translating, if it is rotating as well if it is rotating as well then the velocity on, the surface is equal to omega cross r, where r is the distance from the center of rotation to that point on the surface. So, in that case you define this as r cross omega, where omega is the angle of velocity and r is the distance from the center of ration to that point on the surface, basically the velocity of the fluid at the surface is equal to the velocity with which the solid surface is moving at that point.

Once you do that there is now no necessity to impose the stress boundary conditions, because the solid will generate whatever stress internally is required to balance the fluid stress at the interface. That is the basic idea of the no slip condition at a fluid solid interface it is always valid, unless for some reason the length scale of the flow becomes comparable to the mean fluid path. Now, for a gas, liquid interface for a gas liquid interface the viscosity of a gas is typically much smaller than the viscosity of the liquid, typically the gas will have a viscosity that is between one hundredth to one thousandth the viscosity of a typical liquid.

So, in that case what one can do is to save that the shear stress which is generated by the viscosity of the gas is negligible compared to the viscosity of the liquid, so what you do is to set the tangential stress at the in the liquid equal to 0, at the liquid gas interface. So, for the shear stress you'd impose the condition that delta i j minus n i n j times tau j k n k is equal to 0 for the liquid, because the gas has a negligible viscosity, so it is exerting a shear stress.

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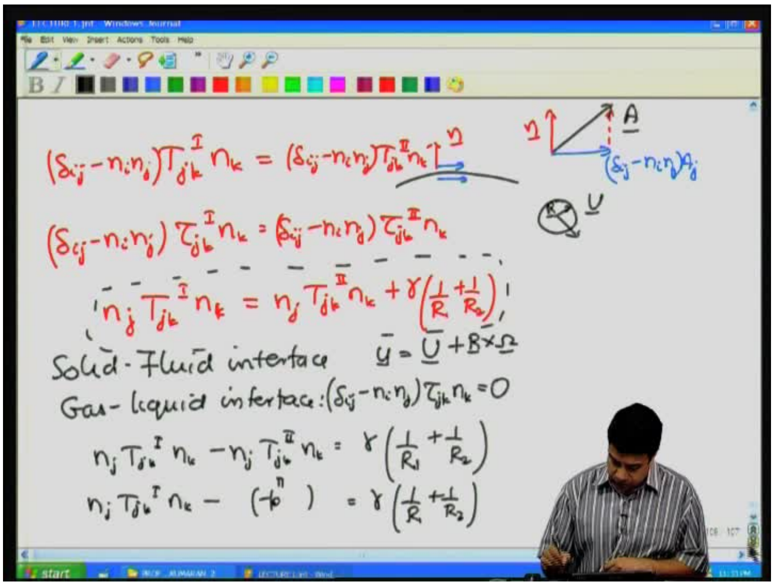

Therefore, the shear stress in the liquid has to come down to that same value as you approach the interface, and the normal stress boundary condition is exactly the same as this one the normal stress boundary condition that would apply, at the gas liquid interface is exactly the same as this 1. So, this is n j T j k 1 n k minus n j T j k 2 n k is equal to gamma into 1 by r 1 plus 1 by r 2, since the viscosity of the gas is negligible the main contribution to the normal stress in the gas is going to be only due to the pressure.

So, that the pressure term just becomes n j into minus p delta j k times n k, where p is the pressure in the gas and this as you can see n j delta j k times n k just n j square, which is 1. So, this just reduces to n j square which is 1, because it is isotropic therefore, I just get minus the pressure in the gas n k minus of this is equal to gamma into 1 by r 1 plus 1 by r 2. Contrast to the liquid solid interface in this case we do not worry about the velocity boundary conditions, the velocity at the gas liquid interface is allowed to slip the velocity need not approach 0, as approach the interface.

What is required is that the shear stress has to be 0, because the viscosity of the gas is negligible, the normal stress difference you neglect the viscous terms. In the normal stress for the gas and you have only the pressure term in the liquid of course, you will have both the viscous and the pressure terms. So, that completes the specification of the governing equations and the boundary conditions for the flow, as I said the next step is to try to solve these equations before proceeding to the solution I would just like to spend some time in deriving the energy balance equation for fluid flow.

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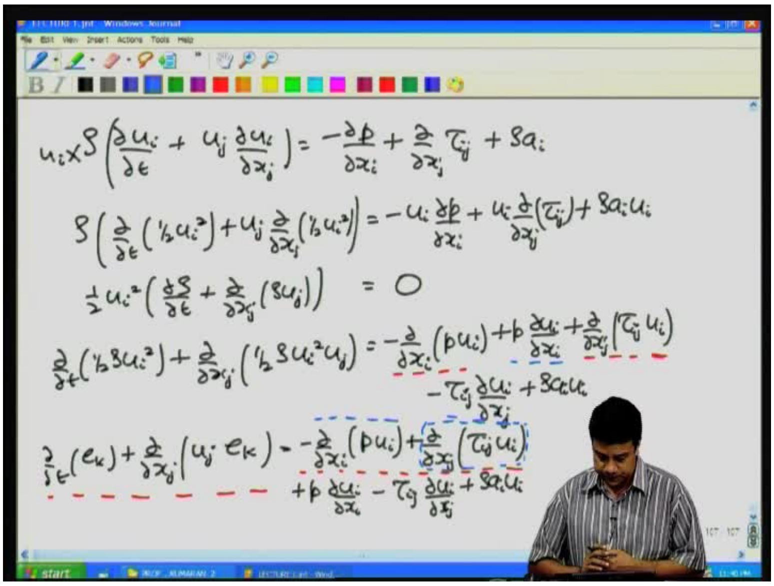

The energy balance equation does not contain any additional information, it is a it is just obtained by dotting the momentum conservation equation with the velocity itself. However, it does contain an important piece of information, which is about the viscose dissipation, the conversion the fluid friction which converts the mechanical energy into heat energy inevitably in any process. So, let us go back and look at the momentum conservation equation, momentum conservation this equation is rho times partial u i by partial t plus u j is equal to minus partial p by partial x i plus partial by partial x j of tau i j the shear stress plus rho times a i.

I get the energy balance equation by multiplying this by u i dotting the entire momentum conservation equation with the velocity itself, to get the mechanical energy balance equation. So, this energy balance equation I just multiply this whole thing by the velocity itself u i and you can see for example, u i times partial u i by partial t is just equal to rho times partial by partial t of half u i square, u i times partial u i by partial t is equal to partial by partial t of half u i square.

Similarly, rho u j times partial u i by partial t when you multiply it by u i once again you just get u j partial by partial x j of half u i square minus u i partial p by partial x i plus u i partial by partial x j of tau i j plus rho a i u i, that is the mechanical energy conservation equation. I can sort of convert this in a into a into a more transparent form by just adding half u i square times the mass conservation equation. So, if I take half u i square times partial rho by partial t plus partial by partial x j of rho u j equal to 0, mass conservation equation itself is 0.

So, if I multiply it by half u square I will still get 0, out of these two and what you get is partial by partial t of half rho u i square plus partial by partial x j half rho u i square u j, the terms on the right i use integration by parts to simplify them. So, I get minus partial by partial x i of p times u i plus p partial u i by partial x i, one integration by parts second integration by parts for the shear stress.

So, plus partial by partial x j of rho I am sorry tau i j u i minus tau i j partial u i by partial x j plus rho a i u i just use two integration by parts for these two terms, half rho u i square is the mechanical energy per unit volume, half m u square is the kinetic energy and that divided by volume rho is a mass per unit volume. So, half rho u i square is the flow rate kinetic energy per unit volume, therefore on the left hand side I have partial by partial t of the kinetic energy per unit volume plus partial by partial x j of u j times the kinetic energy.

Note that e kinetic is scalar it is the kinetic energy per unit volume is equal to minus partial by partial x i of p u i plus partial by partial x j of tau i j u i, so those are the first two terms, those are these two terms. And then I have these two terms which came out of the integration by parts, so those will be plus p partial u i by partial x i minus tau i j partial u i by partial x j plus rho a i u i.

Let us look at the physical interpretation of all of these terms on the left hand side I have partial by partial t of the kinetic energy plus partial by partial x j of u times the kinetic energy, by limits rule that is the kinetic energy change in a moving differential volume of fluid, by limits rule this is the kinetic energy change in a moving volume of fluid. These two terms here, they are both divergence of something integrated over and if you take calculate these two terms for a differential volume, these are the divergence of something integrated over that volume, which is the same as n dot that thing integrated over the surface area.

So, these contribute of surface flux the first term here is a surface flux of energy due to the pressure work done at the surface, the first term is the surface contribution of the energy divergence of p u i is basically the the pressure work done at the surface. The the intercool of that times the unit normal will basically give you this, the second term here is a surface work done by the shear stresses, the divergence of tau dot m integrated over the volume is equal to I am sorry the the divergence of tau dot u integrated over the volume is equal to n dot tau dot u integrated over the surface.

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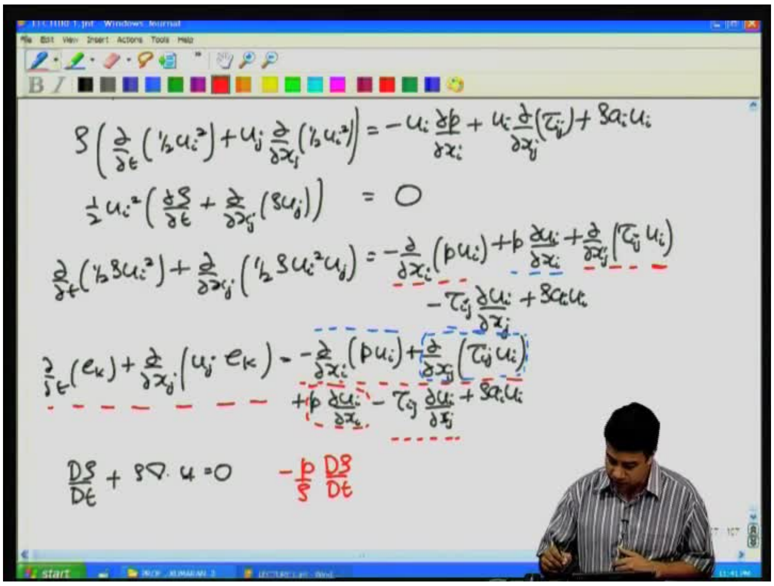

The other two terms at the bottom of course, the last term is just the work done by the body force the acceleration due to the body force, so that is rho a i times u i but, the other two terms here are actually volumetric terms. The first term is actually p times partial u i by partial x i p times divergence of u for the general compressible fluid, the mass conservation equation that we had was that D rho by D T plus rho del dot u is equal to 0; that means, that the del dot u is equal to minus 1 by rho D rho by D t.

Therefore, using that this term can also be written as minus p by rho D rho by D T the work done due to volumetric expansion or compression due to the change in density of the fluid, so that is this term here. This is reversible in the sense that if the density increases the work will be there will be work done, when the density decreases when the volume increase the fluid does work on the outside whereas when the volume decreases or the density increases work is done on the fluid.

So, this is a revisable work done, this final term called an irreversible work and this is the cause for heating within a fluid to viscous friction, you can show that this term always has to be positive, you can show in general that this term always has be positive.

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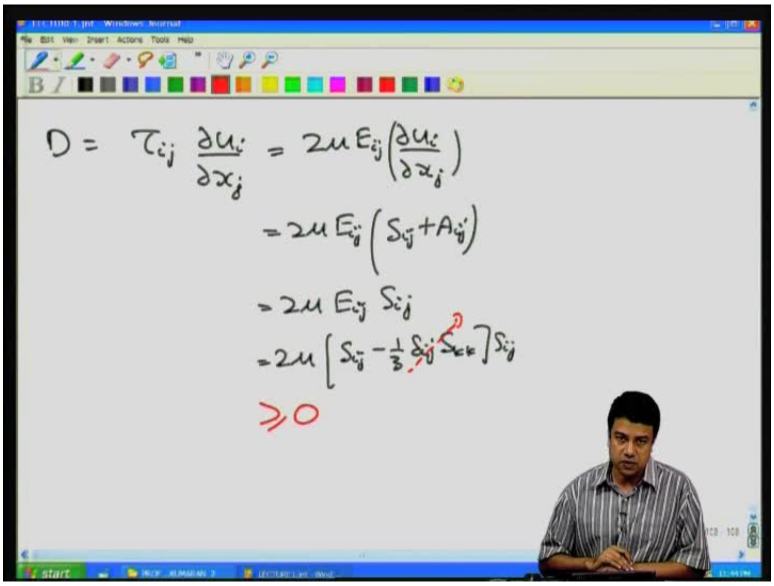

That is the viscous dissipation of energy is equal to tau i j partial u i by partial x j, this is a dissipation rate dissipation of energy per unit volume, and in this equation here this appears with a negative sign this appears with a negative sign here. Therefore, it is always removing energy from the fluid it is always decreasing the kinetic energy of the fluid, so if I expand out this term you will see that this is equal to two mu E i j times partial u i by partial x j E i j is a symmetric trace less tenser E i j is a symmetric trace less tenser.

Therefore, when I multiply E i j times partial u i by partial x j, this partial u i by partial x j is given by the sum of the symmetric plus the anti-symmetric part, the rate of deformation tenser can be separate into symmetric plus anti symmetric part. The product of E i j which is symmetric and E i j which is anti-symmetric is 0 therefore, this can also be written as 2 mu E i j times S i j.

And you know that E i j is a symmetric traceless part S i j is the symmetric part E i j is the symmetric traceless part, so this is just equal to 2 mu the symmetric traceless part is just S i j minus one third delta i j times the trace, times the trace times S i j. For an incompressible fluid S k k is equal to 0 which is the divergence of the velocity, this is identically equal to 0 therefore, this is identically equal to 0 and you can see that d is equal to two mu times the product of two tensors.

The product of these two symmetric tensors will always be positive, because both tensors are symmetric therefore, whether the half diagonal terms are positive or negative, the product of those two will always end up being positive. Basically, it is equal to S 1 1 square plus S 2 2 square plus S 3 3 square plus 2 times S 1 2 S 2 1plus S 1 3 S 3 1plus S 2 3 S 3 2, so always be positive, so this is always greater than or equal to 0.

In fact, the result is more general than that you can show that even for an for a compressible fluid if you retain this isotropic part show that d is always positive, I would not go through that here it involves a little bit of algebra, but it is quiet simple that even if you have a compressible fluid d is always positive.

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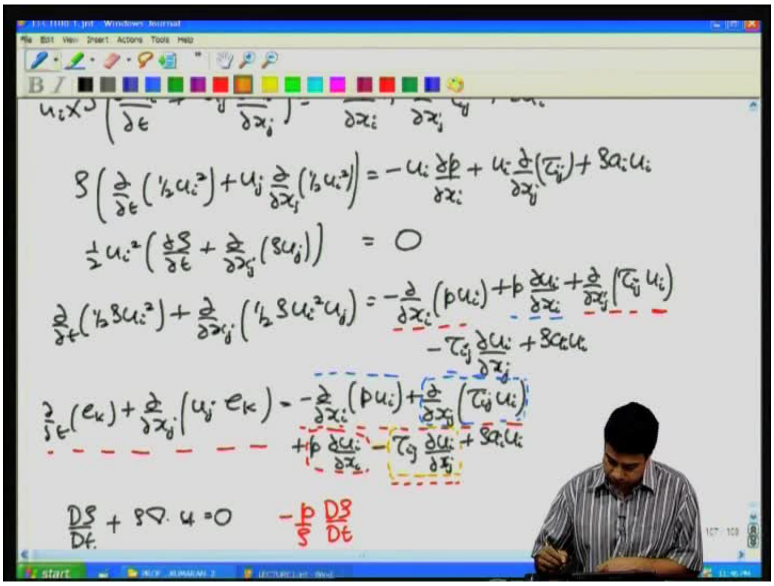

So, therefore, this represents the irreversible dissipation of energy, the conversion of mechanical energy into heat and that is appearing as you can see here, in this equation that conversion is appearing as a sink term, in the mass conservation energy conservation equation. It always tends to reduce the energy of the fluid and therefore, it always dissipates energy converts mechanical energy into heat, and that is what is what causes losses in fluid flow.

The pressure gradient for example, which you apply between the ends of the pipe in order to force fluid through, it that is because the energy is being constantly dissipated within the fluid into heat energy, due to this dissipation term happens only for a viscous fluid. If the viscosity was 0, then I just have the Euler the the the pressure term which was reversible but, however, all fluids have a nonzero viscosity therefore, they always convert energy from mechanical energy to heat energy and that causes dissipation in fluid flows.

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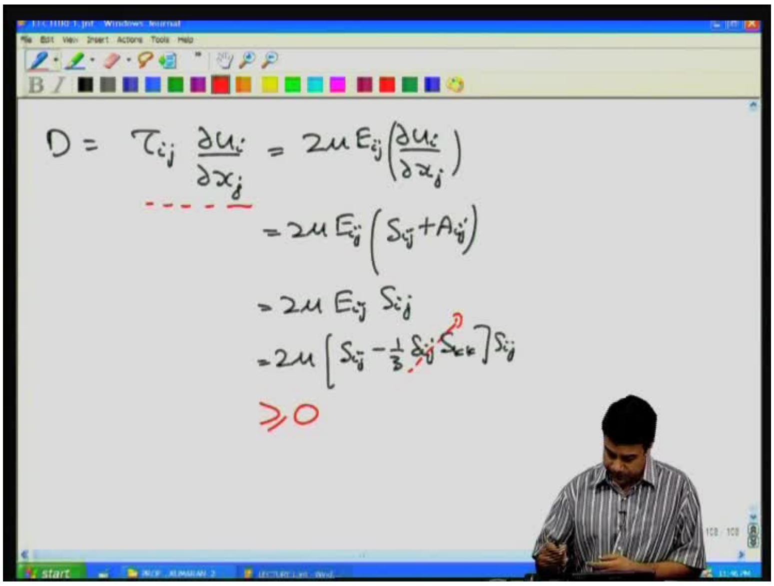

So, that is the energy balance equation as I said it is only for the mechanical energy balance equation, so whenever if you want to estimate how much heat is generated due to the fluid flow the desperation rate in the fluid is basically going to be equal to tau i j times partial u i by partial x j. So, this completes our derivation of the of the conservation equations the constitutive relations the boundary conditions, and I talked briefly about energy dissipation.

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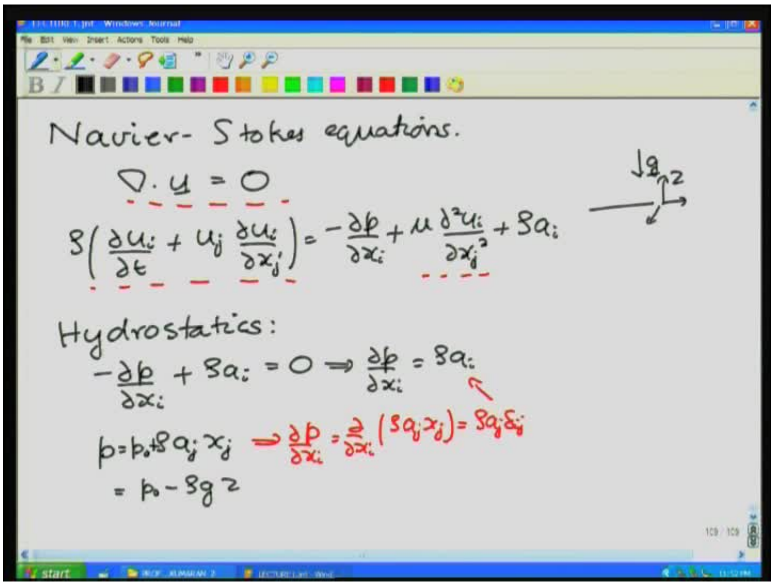

So, that completes our derivation next step is to go head and solve those equations, let us write down our Navier stokes equations again divergence of velocity is equal to 0, rho times partial u i by partial t plus plus mu times partial times u i j square plus rho times acceleration. Note that acceleration is the equivalent of the source or sink terms that you have in the mass movement of conservation equations, this is the equivalent of the diffusion terms the viscous term, these are inertial terms the connected terms.

We have the pressure here which is not present in normal heat and mass transfer, and we also have the condition that the divergence of the velocity has to be equal to 0, there is no radial component of the velocity felid. And now we will try to solve these equations in different limits limits, if you recall when we did heat and mass transform we solved it in the limits of where diffusion dominates and where connection dominates.

Similar in this case also, we will solve it in the cases where diffusion dominates and connection dominates, but before that we will just think back and derive some simple results that we got for unit directional flows. In the course on fundamentals of transport processes one just based up on shell balances these equations are more general but, we have to be able to show that these also reduce to those same equations that we obtain using shell balances.

So, if you look at some simple unidirectional transport problems first before we go on to looking at more complicated ways of solving these equations, after all we derived these equations in order to express everything in terms of vector without reference to the underline coordinate system. Whereas, when we did shell balances we always had a coordinate system wrote down a shell that was appropriate for that and then solved it.

So, when we go on to our next solution of low and higher solutions of these equations, we will look at it independent of coordinate systems, first look back at the solutions that we had derived using shell balances and check that we obtain the same solutions here as well. The simplest case that one can consider is when the velocity is completely 0, in that case the equivalent mass and heat transfer cases the temperature of the concentration are uniform everywhere there is no problem to solve.

In this case, however, because we have a pressure there is a problem to solve, hydrostatics velocity is 0 everywhere mass conservation equation is trivially satisfied the momentum conservation equation. I have minus partial p by partial x i plus rho times a i is equal to 0 or the gradient is equal to rho times a i the gradient of the pressure is equal to rho times the acceleration, this I can integrate quiet easily if density and acceleration are independent of possession.

If rho and a i are constants then the equation is quiet easy to integrate the solution is just that p is equal to rho a j times x j, because the density and the acceleration are both constants. So, this solution I can verify that I get the correct result the partial p by partial x i is equal to partial by partial x i of rho a j x j, so rho and a are independent of of coordinate. Therefore, I have partial by partial x i of a j partial x j by partial x i is 1, if j is equal to i where, as partial x 1 by partial x 1 is 0 j is not equal to, so this just becomes equal to rho a j th times delta i j.

And you can verify that this is exactly equal to the right hand side that is required, so it is equal to rho a j x j, whatever when I take gradients, I can always put in a constant, so some constant plus this because when I take the gradient of the constant it becomes 0 any way. So, that is a solution for hydrostatics p is equal to p naught plus rho a j x j, the simplest case if I had for example, a fluid in a gravitational field and the acceleration due to gravity was acting conventionally in the minus z direction, if the acceleration due to gravity was acting conventionally in the minus z direction.

I will just get a equal to p naught minus rho g z the familiar expression in hydrostatics pressure at a point is equal to p constant minus rho g z, where that constant depends up on where you fix your reference location, the location at which z is equal to 0.

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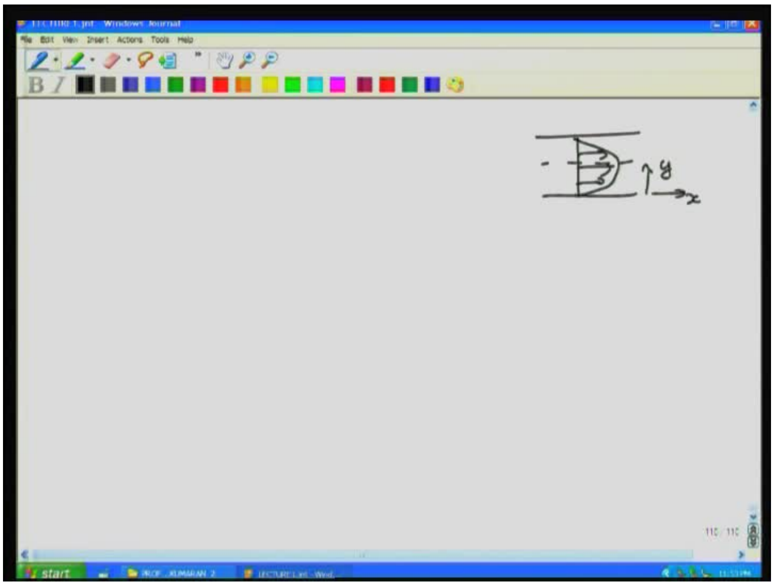

So, this is the fundamentals of hydrostatics all hydrostatics problems can be reduced to exactly this, the next simplest case plus one can consider which we considered earlier as well in fundamentals of transport processes one, is a unidirectional flow. In a unidirectional flow the velocity is only along one direction, the velocity variation in general is only along the second direction.

So, if you have a simple flow in a fluid in a pipe or in a channel in this particular case, if I have a channel flow in which I use my coordinates x and y the velocity, will be along the x direction the velocity gradient will be along the y direction. And of course, in general this is a two dimensional problem, we I have to expand out my navies stokes equations for this two dimensional problem, and many terms will will simply drop out simply because the velocity gradient is only along one direction, the velocity is only along one direction.

So, we will look at that in the next lecture see how the Navies stokes equations can be easily applied to solve one dimensional problems of course, this is the simplification we take the original equations simplify them under these assumptions and then solve the simplified equations. However, the simplified equations that we get should be identical to what we got using shell balances and the solutions should be identical as well.

So, we will continue that in the next lecture, before we gone on to more complicated solutions for example, solutions where there are velocities in all three directions, that we will consider in two limits, one is in the viscous the diffusion dominated limit and the other is in the conventional dominated limit. So, we will start off with simple unidirectional flows in the next lecture before looking at more complicated flows, so will see you then and we will continue in the next lecture.