Fundamentals of Transport Processes II
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Lecture - 1
Review of Fundamentals of Transport Processors I

Welcome to this new course on fundamentals of transport processes two. This is a follow on course to the earlier course on fundamentals of transport processes one. So, that course should be considered as a prerequisite for this course. So, those of you who haven't gone through that course there are certain things there that we will be using in this course. So, I would urge all of you after this lecture to go and look back at what all we did in that course to bring yourself up to date with all the material that was done. So, in this lecture, I will just try to reiterate some of the important concepts that we developed in the first course, on fundamentals of transport processes one.

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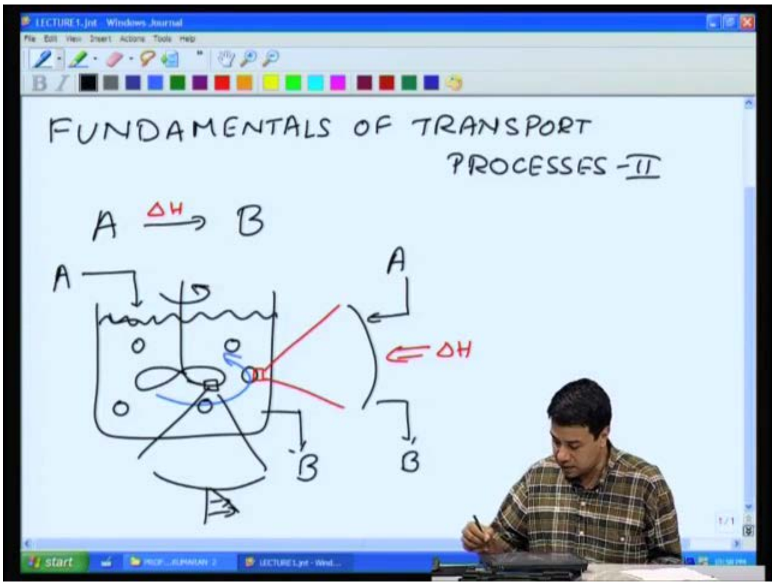

So, this is the name of our present course, in the first part we looked at the transport mostly of mass and energy in chemical systems. The motivation for that was that whenever, we talk about some process which converts some raw materials into some useful finished products. We require raw materials to be converted into products, so if we have some raw material A which is going to some product B, we can do the experiment in a test tube of course, and convert A to B this might require some additional heat energy to be put in order to facilitate the process. Some heat energy to be taken out if it is an exothermic reaction.

However, when we run when we try to produce this on the large scale, in economically feasible ways, in economical ways in order to produce mass a large amounts of chemicals. We run into the difficulty that you need to make sure that that the reactants get to the place where they have to be converted, the products leave and the amount of energy that is required is supplied at that particular location. And all of this requires that in addition we also need to provide some way of mixing the products factors.

So, we took in the first in the in the fundamentals of transport processes one, 3 of the typical cases in which we discussed each of these process. One was a stirred tank reactor with some fluid in it, in which the product came in I am sorry the reactant came in the product left and the stirring was necessary in order to mix the entire system. Now, this is was an homogeneous reaction, so you had some catalyst particles there which were catalyzing the reaction.

Now, for the reaction to takes place the reaction actually takes place on the catalyst surface, for this reaction to take place it is necessary to ensure that at the location where the reaction is taking place, the reactants come in the products leave. And there is also sufficient supply of heat energy incase it is an endothermic reaction, or sufficient removal of heat energy incase it is an exothermic reaction. So, the transfer of mass of the reactants to the surface at the location, locally where the reaction is taking place the transfer of products out from the surface the transfer of energy in or out of the surface. These are essential for making sure that the reaction takes place as designed.

Similarly, in this case you also have some kind of stirring taking place there is some kind of a circulation of fluid, and that circulation of fluid takes place because of this impeller. At the impeller surface itself it is necessary to make sure, that there is sufficient momentum transfer to the fluid. So, that the fluid is stirred and that is the process of momentum transfer, the shear stress that is exerted by the impeller on the fluid causes the fluid to rotate. And this transfer of momentum ensures that sufficient mixing takes place that these reactions can be carried out.

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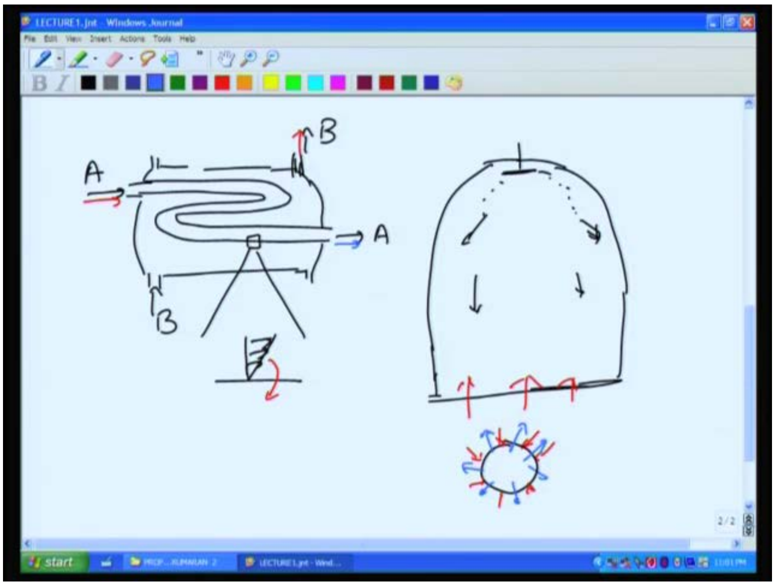

In a similar manner, we took the example of a heat exchanger a shell and tube heat exchanger. In this case you have a shell side and the tube side, and let us say hot fluid comes in at the tube side and cold fluid goes out and cold fluid comes in, and hot fluid goes out. In this case the heat is coming in with the fluid that is coming in which is at a higher temperature, the fluid is coming in is at higher temperature the fluid is going out is colder. The heat is coming in with the fluid that heat is transferred across the surface of the heat exchanger to the outer shell side fluid, and this heat is carried out by the shell side fluid.

So, heat is coming in on one side is going out on the other side, and however this heat has to be transferred across the surface. And if you look at the surface itself we find that there is one fluid that is passing through over here, there is another fluid on the other side. And this heat has to be transferred from this fluid to this one and that transfer has to take place perpendicular to the direction in which flow is taking place.

So, this is a problem of heat transfer and of course, we also considered the case of a spray dryer, if we go back and recall in that previous lecture, we have a spray dryer which was spraying little droplets of some food product. And these droplets were cooling and drying as they came down, if you recall this example was done for the particular case of a spray dryer, where we have some food products which are sensitive. So, that the difference in temperature between the hot air that is coming in and the droplets cannot be more than a certain amount. And for this reason this dryer has to be very large the drop has to travel long distance before it dries.

And if you look locally at the surface of the droplet itself at the surface of the droplet itself, the heat energy has to first go in and it has to evaporate the water within this droplet. And ones it evaporate this water this water has to now come out of the droplet, and this is a combined process of heat transfer of heat in the latent heat required for the evaporation of the water in this tray dryer and mass transfer out. The mass that comes out is because the water is evaporated in order to give you a dry powder after this spray drying is over.

So, these were prototypical examples that we took, each of these cases there is a flux of either heat or mass going in or going out of some surface. There is also transport due to the fluid that is coming in itself so for example, the previous reactor. Example, there is transfer along with the fluid of the reactants in at this point there is transfer of reactors in along with the fluid there is transfer of products out along with the fluid.

However, if you look at the catalyst surface itself there is transport that is happening perpendicular to the surface because the surface is impenetrable the fluid itself cannot travel perpendicular to the surface. However, there is a transfer of mass and heat perpendicular to the surface. So, that motivated us to distinguish between two dominant mechanisms by which transport takes place.

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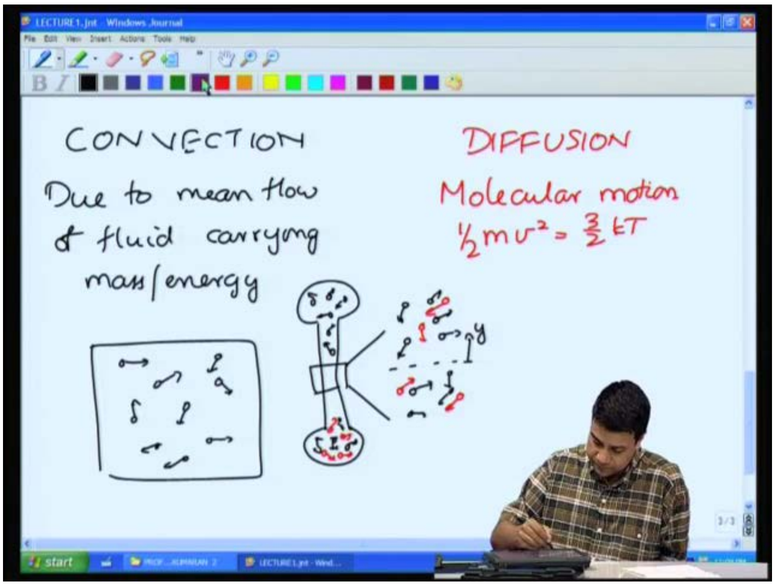

CONVECTION

Due to mean flow of fluid carrying mass/energy

DIFFUSION

Molecular motion

$\frac{1}{2}mv^2 = \frac{3}{2}kT$

One is convection and the other is diffusion, convection is the process by which transfer takes place because of the flow of the fluid that is carrying, either the mass or the heat. In this in these examples that we took the convection is the heat that is coming in with the heat exchanger at the inlet over here, heat that is coming with the heat exchanger. Heat that is leaving the heat exchanger with the fluid could be somewhat convective transfer processes in this case in this spray dryer case.

For example, the heat is coming in with the gas that is coming in to dry the droplets, and water is of course, going out because it gets collected by the gas. The water vapor gets entrained by the gas and it leaves along with the gas, so that is transport along with the fluid itself as is convection.

Similarly, here flow in of reactants at the inlet is convection, products out is convection. And similarly, if you have some heating or cooling that is taking place in the entire reactor itself that is getting carried along with the fluid, either at the inlet or the outlet that is convection. The transport that takes place perpendicular to the surface not due to the mean fluid flow is diffusion. So, convection is due to mean flow of fluid carrying mass, or energy. Diffusion is not due to mean flow it is due to molecular motion, so it is a molecular process.

In a fluid we have molecules, which are in rapid fluctuating motion the average energy of the molecules is of course, given by the equi-partition of energy. So, we know very well that for the molecular velocities, the equi-partition dual holds for the translational velocities half m v square is equal to the 3 by 2 k T. And we had calculated some of the gas velocities in the previous course the velocity of the gas molecules is comparable to the velocity of the speed of sound in that particular medium.

So, in air for example, sound travels at around 300 meters per second. Similarly, the molecules of the gas themselves have fluctuating velocities of the order of 300 meters per second, in hydrogen it is a little larger because the mass of hydration is much smaller its about 2000 meter second. In denser gases it will be less, but that is a roughly the magnitude of the fluctuating velocities of the molecules.

So, these molecules are all in fluctuating motion, now if the concentration of some reactant product were uniformly spread throughout the molecules of that would also be in fluctuating motion. And on average the molecules has equal quality of moving upwards and downwards and therefore, on average you will have no net motion of molecules.

However, if there is a gradient in the concentration, we took the specific example in the last class of the transport between two bulbs. We have a carrier gas the molecules are in fluctuating motion, and we have a trace constituent which is, which has a higher concentration on one side than on the other side. We assume that the concentration of this trace molecules is sufficiently small that the motion of this molecules will not shift the center of mass.

So, in that case what is going to happen we should look at some particular region here I have a surface here and I have molecules that are in fluctuating motion. And if I have a concentration gradient of this trace solute, if there is a variation in concentration of this trace solute. I can plot the concentration as a function of this distance from the surface and plot the concentration as a function of distance from the surface, I will just take this off.

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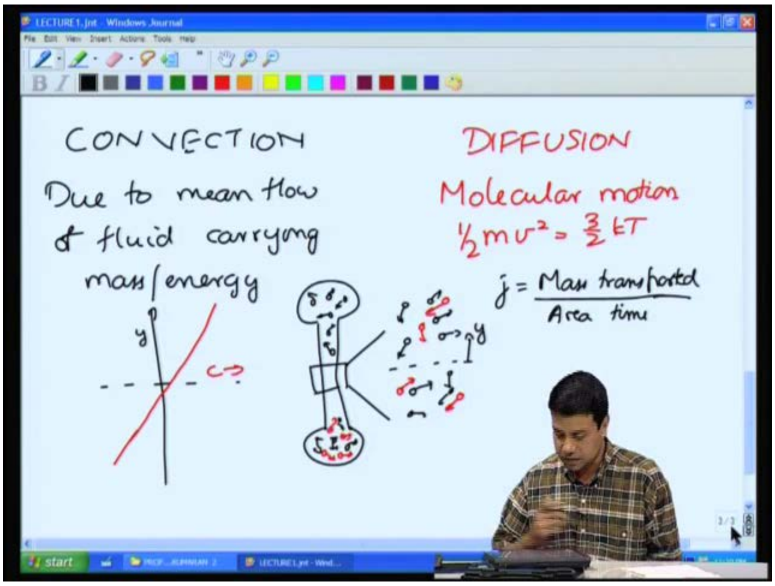

CONVECTION

Due to mean flow of fluid carrying mass/energy

DIFFUSION

Molecular motion

$\frac{1}{2}mv^2 = \frac{3}{2}kT$

$j = \frac{\text{Mass transported}}{\text{Area time}}$

So, I have the surface here, if I plot concentration as a function of distance from the surface. I will have some variation in concentration across the surface there will be some variation of concentration. So, the concentration on top here is higher than the concentration below. Now, the molecules themselves are in fluctuation motion here they travel equal velocities both upwards and downwards. So, there is no preference for travelling in any particular direction.

However, the concentration of molecules is higher in the upper side, so on average you will have the motion of molecules from the upper to the lower side. So, this is my distance y, so this and this is the concentration c you will have on average the motion of molecules from the upper to the lower side. It will go from the region of higher concentration to a region of lower concentration, and this process is called molecular diffusion and this is responsible for the transport of both mass momentum and energy in these fluid systems.

Now, we had estimated what is the flux of molecules due to this diffusion process. The flux was defined as j is equal to mass transported across the surface mass transported across the surface per unit area, per unit time. That is the amount of mass transported across the surface per unit area per unit time. So, that is related to the way in which the concentration varies across the surface. The relationship between these two is what is called the Fick's law of diffusion, so in the simplest form the way that it is written is that.

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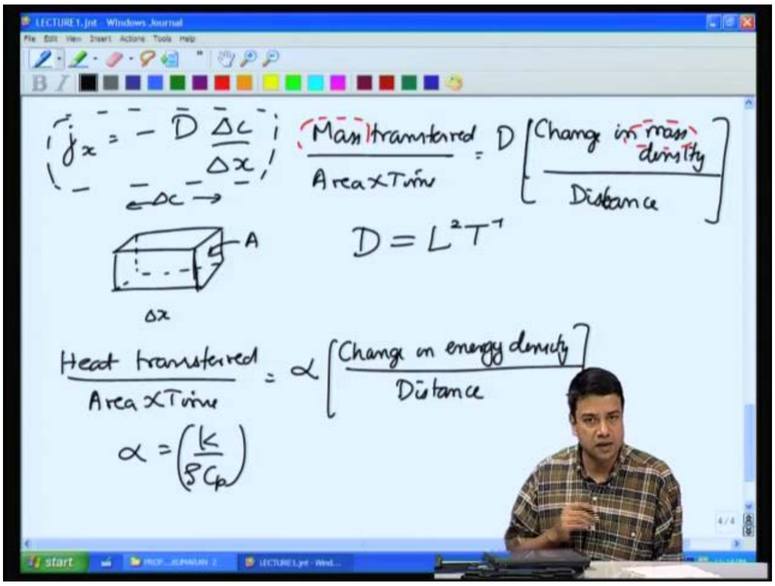

$$j_x = -D\frac{\Delta c}{\Delta x}; \quad \frac{\text{Mass transferred}}{\text{Area} \times \text{Time}} = D\left[\frac{\text{Change in mass density}}{\text{Distance}}\right]$$

$$D = L^2T^{-1}$$

$$\frac{\text{Heat transferred}}{\text{Area} \times \text{Time}} = \alpha\left[\frac{\text{Change in energy density}}{\text{Distance}}\right]$$

$$\alpha = \left(\frac{k}{\rho C_p}\right)$$

The flux j in the x direction is equal to minus D delta c by delta x. So, let us let us look at this in a little more detail. Let's say that I have some particular element of fluid here ok a slab of fluid in which I have different concentrations on the two sides of the slab, I have a difference in concentration delta c across the slab. The distance across which there is this difference is delta x and this cross sectional area of the slab is A. Now, obviously the amount of mass that is being transported that is going to be proportional to the area itself, if i have more area I will have more mass transported.

So, it makes sense to define the mass transported per unit area per unit time as the flux. So, that is why I defined the flux as the mass transported per unit area per unit time. Now, the mass transported per unit area per unit time has to decrease as the distance between the two ends become bigger, so it has to inversely proportional to the distance delta x. So, this is the equation for the Fick's law for diffusion and all flux, so this basically says relates the mass transported per unit area into time is equal to a diffusion coefficient, diffusion coefficient times change in mass density, concentration is a change in mass density.

Density divided by the distance across which that change takes place, divided by the distance across which the change takes place. Mass density has the dimension of mass per unit volume, mass density has dimension of mass per unit volume so on the left hand side I have mass per unit area per unit time, right hand side mass per unit volume divided by distance. That means that the diffusion coefficient has dimensions of length square T inverse, this is true for all diffusion coefficients.

In the case of heat transfer instead of mass instead of mass I substitute heat. So, my equation for the thermal diffusion becomes heat transferred per unit area into time is equal to the thermal diffusion coefficient alpha times change in energy, change in energy density divided by distance. That is energy density is the energy per unit volume, the thermal energy per unit volume in that fluid divide by distance. And once again the thermal diffusivity has to have the dimensions of length square per unit time, and if you recall we had made a we had a obtained a relation between the thermal diffusivity and the heat conductivity, as alpha is equal to k by rho C p.

Similar is the case with momentum diffusion, momentum diffusion will be the focus if this second part where we will deal with the transport of momentum, the equations for the transport of momentum in the fluids. So, I will come back to that a little later. Now, this diffusion coefficient it has dimensions of length square per unit time, from microscopic arguments we have derived the value of this diffusion coefficient in gases if you recall.

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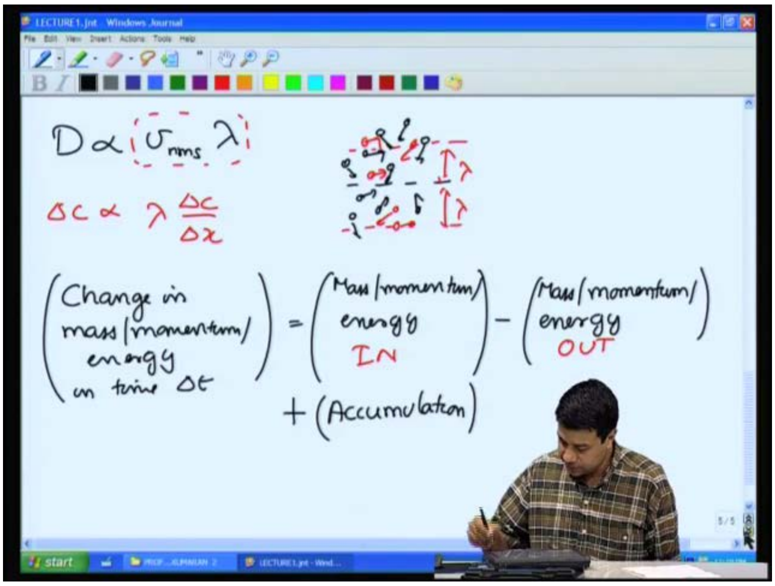

$D \propto v_{rms} \lambda$

$\Delta c \propto \lambda \frac{\Delta c}{\Delta x}$

$$\begin{pmatrix}\text{Change in} \\ \text{mass/momentum/} \\ \text{energy} \\ \text{in time } \Delta t\end{pmatrix} = \begin{pmatrix}\text{Mass/momentum/} \\ \text{energy} \\ \text{IN}\end{pmatrix} - \begin{pmatrix}\text{Mass/momentum/} \\ \text{energy} \\ \text{OUT}\end{pmatrix} + (\text{Accumulation})$$

We had seen that the diffusion coefficient is proportional to the root mean square of the gas molecular velocities times the mean free path, which is the distance between molecules in a between collisions in a gas. And physically that comes about because if I have a diffusion process and if there is a concentration gradient of some trace molecule. The molecules that cross this surface come an average distance of mean free path from about the surface, they come on average the distance one mean free path about the surface, they come downwards.

And because they come downwards from a distance of the order of mean free path, the concentration the number of molecules coming down is proportional to the concentration at this particular position. While the molecules going upwards comes from a distance of the order of the mean free path below the surface. So, they come from a distance a significant location distance below the surface, the concentration at these two regions these two places are different and they differ by a value that is proportional to the gradient delta c by delta x delta x is this distance times lambda.

So, this distance is of the order of lambda this is of the order of lambda, the different of concentration goes as lambda times delta c by delta x. The net exchange is proportional to the number density times the concentration itself times the fluctuating velocity. Therefore, you find that the diffusion coefficient is given by the VRMS times the mean free path, this is true for gas molecules.

In gas molecules energy transfer also takes place due to the motion of the gas molecules themselves, and the thermal diffusivity and mass diffusivity are roughly of the same magnitude. The other thing to note is that in both of these cases mass transport takes place down at temperature gradient. So, the concentration on one side is higher than on the other side there will be transfer taking place form the higher concentration to the lower concentration. And for that reason we have this negative sign here in all of this equations for both heat and mass transfer.

So, there are two mechanism that we had studied convection and diffusion, and the next step is to actually use these two mechanisms in order to find out what is the transport rate of mass momentum and energy. Now, that was done by a process that was called shell balances. Basically you write down a balance equation because mass has to be conserved energy has to be conserved momentum has to be conserved.

Therefore, the conservation equation was of the form, change in mass momentum energy in time delta t is equal to IN minus, what leaves plus any accumulation that may be there. So, this is the basic principle the conservation of mass momentum and energy. And this was done by virtue by using what was called shell balances.

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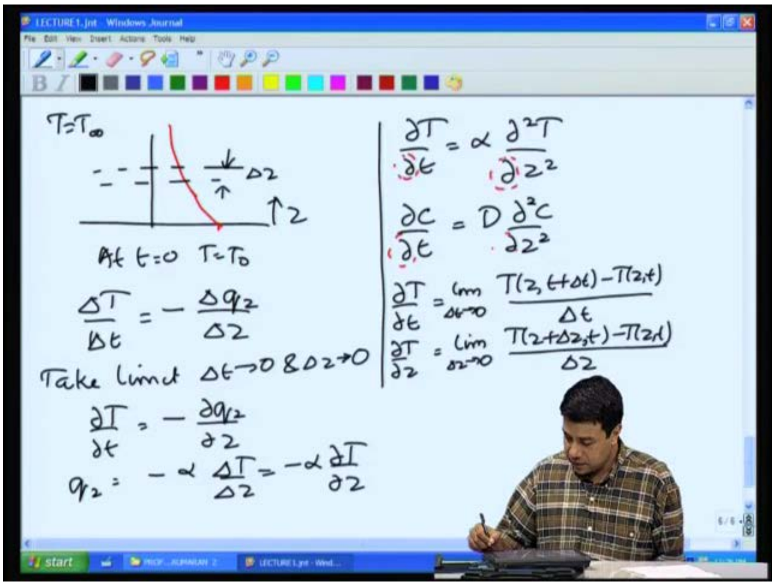

$T = T_\infty$

At $t = 0$ $T = T_0$

$$\frac{\Delta T}{\Delta t} = -\frac{\Delta q_z}{\Delta z}$$

Take limit $\Delta t \to 0$ & $\Delta z \to 0$

$$\frac{\partial T}{\partial t} = -\frac{\partial q_z}{\partial z}$$

$$q_z = -\alpha \frac{\Delta T}{\Delta z} = -\alpha \frac{\partial T}{\partial z}$$

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial z^2}$$

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial z^2}$$

$$\frac{\partial T}{\partial t} = \lim_{\Delta t \to 0} \frac{T(z, t+\Delta t) - T(z,t)}{\Delta t}$$

$$\frac{\partial T}{\partial z} = \lim_{\Delta z \to 0} \frac{T(z+\Delta z, t) - T(z,t)}{\Delta z}$$

So, the simplest case let us say that you had a flat plate with some concentration and it was transporting mass momentum energy into an infinite fluid. So, for example, initially the temperature everywhere T is equal to T infinity and at time T is equal to 0, T is equal to T naught at the surface. So, instantaneously heat the surface at time T is equal to 0, so at some later time energy gets conducted into the medium, then you get some particular energy profile as you go further into the fluid.

How do you find out what this is? So, the way to do that is to construct a small first use a coordinate system, z perpendicular to the surface we look at the thin slab of material delta z called a shell. And you write down the equation for energy coming in to the shell as you going out of the shell the accumulation, in this case there is none because energy is only coming in and going out. Change in energy in time delta t is equal to the energy in minus energy out and by doing that calculation you end up with the diffusion equation for the temperature field.

So, over an instant un of time of delta t of thickness delta z, we find the change delta T by delta t over this time interval delta t is equal to minus the thermal flux the variation of the thermal flux over this time. Delta j the energy flux, so we can call it as the heat flux where q z is the energy travelling in the z direction because the only energy travelling in z direction at the lower surface increases the energy of this volume, that travelling in the z direction the upper surface decreases the energy of the value. And for that reason we find the negative sign.

So, this is the simplest case you do a shell balance and you get this equation, for a shell of thickness delta z, the change in energy over time delta T. And then what you do is that you take the limit as delta t is goes t to 0 and delta z is goes to 0 to get what is called as the partial differential equation, partial T by partial t is equal to minus partial q z by partial z.

The next step is to express the flux in terms of the temperature itself and that is done by the relation between the flux and the variation in temperature with respect to the z direction. So for example, the flux is written as if you recall its minus alpha times delta T by delta z, the flux at any given location for a distance delta z with a variation delta T in temperature across the surface. Take the limit as delta T and delta z goes to 0 this just becomes equal to minus alpha times partial T by partial z. This is the Fourier's law of heat conduction the equivalent of the Fick's law of diffusion that we had just discussed.

So, with this expression for the flux we can put that into the equation and you get the thermal unsteady diffusion equation, that is partial T by partial t is equal to alpha times d square T by d z square. So, that is the thermal unsteady diffusion equation in one dimension. Similarly, for the concentration field partial c by partial t is equal to D partial square c by partial z square, just to refresh your memories these symbols here these symbols here these partials they represent what are called partial derivatives. We will come back to that in this course, but I just wanted to point it out at this stage just to get things clear at the beginning.

This temperature field concentration field depend upon two independent variables both z coordinate as well as time, in this particular case. And because of this when you take derivatives one has to be clear about which coordinate is being varied and which coordinate is being kept a constant. So, when I say a partial derivative partial T by partial t is equal to T at z t plus delta t minus T at z t divided by delta t in the limit as delta t equals to 0. An important point to be kept in mind when I say partial T with respect to time I said that one particular location in space, and measure the difference of temperature between two instants of time separated by an interval delta T. Difference divided by delta T is the partial derivative.

Similarly, if I wanted to take the partial derivative with respect to z this would be limit as delta z goes to 0, T of z plus delta z minus T sorry coma T minus t of z t divided by delta z. So, I am fixed at one particular instant of time, I take two locations about this point z separated by distance delta z, find the difference in temperature between those two points then divide by an interval. So that is what is meant by a partial derivative. So, in this particular case in one dimensional diffusion case, we got a diffusion equation of this kind you can extend it to other situations. For example, we saw that if you are to do the same in cylindrical coordinates.

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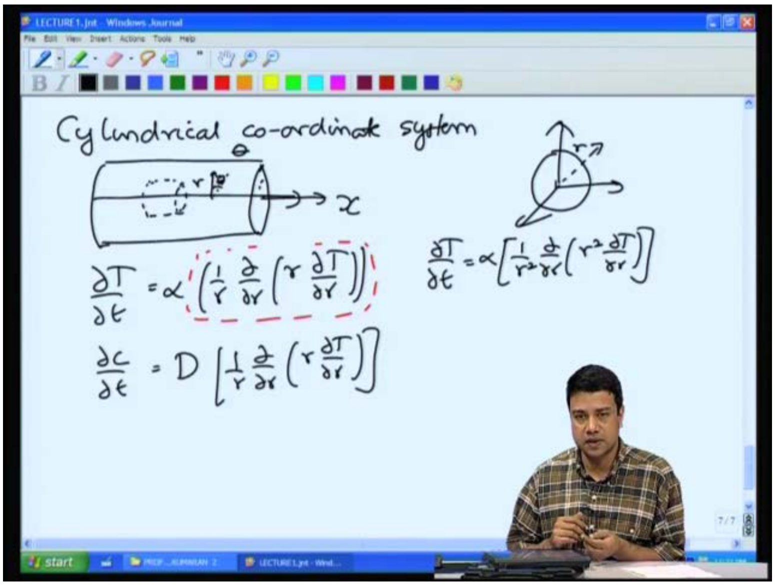

Cylindrical co-ordinate system

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) \right)$$

$$\frac{\partial c}{\partial t} = D \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) \right]$$

$$\frac{\partial T}{\partial t} = \alpha \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) \right]$$

For example, the flow through the transport into pipe for example, so if you wanted to look at heat transfer across the surface of the pipe, it makes sense to take a coordinate system in which the surface of the pipe is a surface of constant coordinate. In the previous case, you are looking at the flat plate and therefore, the plate the surface of the flat plate itself was the surface of constant coordinate. Therefore, we used a coordinate system we used the distance from the plate was the coordinate, in this case it is preferable to use a coordinate system in which the surface of the pipe itself is a surface of constant coordinate.

And the coordinate system that is useful for this is what is called the cylindrical coordinate system, in this there are three coordinate systems one is along the axis, this can be called either the x or the z coordinate I will refer to this as the x coordinate. Then there are two other coordinates one is the distance from the axis. So, the distance from the axis to any location on the within the pipe is called the radial coordinate, and the angle made by this distance vector with respect to some arbitrary axis is what is called the theta coordinate, or the meridional coordinate that is you can take any particular plane as a reference frame plane. The angle made by the distance vector from that plane is called the theta coordinate.

In this particular case if the configuration is axi-symmetric that is there is no variation as you go around at a fixed radius, if you go around at a fixed radius there is no variation in temperature. The equation for the temperature field concentration equation temperature of concentration field becomes partial T by partial t is equal to D into 1 by r d by d r of r partial T by partial r. So, this derivative that we have is different from the derivative we had previously because in the previous case, we just had second with respect to z square.

Similarly, the concentration field I am sorry I should put alpha for the temperature field and for concentration is D into and these additional terms come about because of the dependence of surface area on the distance from the axis. So, as I got outward from the axis as r keeps increasing, the surface area of the differential volume also keeps increasing. So, if I have a shell at this distance the cylindrical surface area of this shell is proportional to is proportional to the radius itself, the cylindrical surface area is equal to 2 pie r that surface area keeps increasing as you go further outward from axis. As r increases the surface area increases.

So, even though the flux maybe a constant the amount of mass transported increases as the surface area increases, and for that reason we had to have additional terms here its not just a simple second derivative in the cylindrical coordinate system. We will see this again when we look at vectors and tensors, we will see how to derive this in terms of the variations of unit vectors with respect to distance one where momentum transport.

Similarly, in spherical coordinate system so for example, if I were trying to calculate the diffusion from a spherical particle its more convenient to use a coordinate system in which the surface of the particles are surface of constant coordinate. In this case I should take one of the coordinates as the distance from the center of the particles itself r, I should take one of the coordinates as the distance from the center of the particle.

Therefore, I will be carrying out my, I will be carrying out my shell balances for spherical shells. Once again for spherical shells the area increases as the radius increases here it increases as r square because the area of the spherical shell is 4 pie r square. For that reason I get once again a different form of the equation in this case I will get partial T by partial t is equal to alpha into 1 by r square d by d r into r square partial t by partial r. So, this was for a spherical shell.

One thing to frame this equations, so this basically gives me a partial differential equation for the variation of temperature and concentration, both in time as well as with respect to distance. So, that is one part of it second part of it, second part is to solve these equations, and in the previous course fundamentals of transport process one, we had seen various ways in which these equations can be solved just to summarize.

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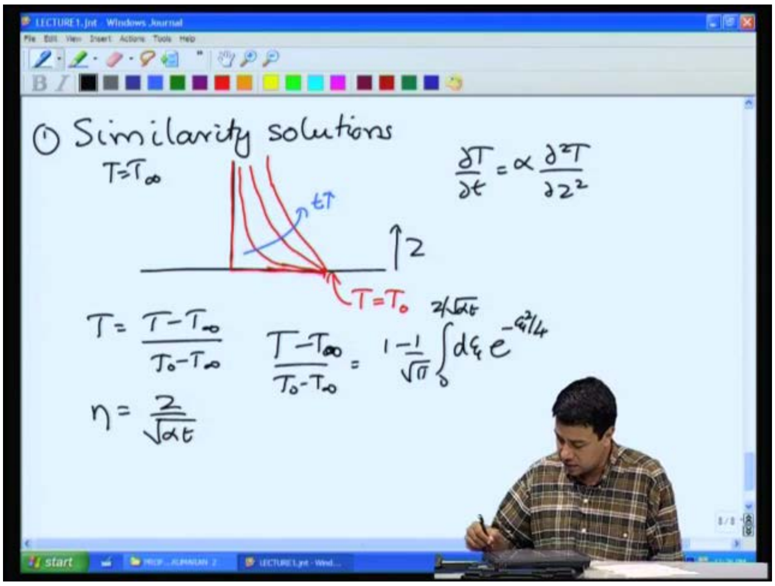

① Similarity solutions

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial z^2}$$

$$T = \frac{T - T_\infty}{T_0 - T_\infty}$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = 1 - \frac{1}{\sqrt{\pi}} \int_0^{z/\sqrt{\alpha t}} d\xi \, e^{-\xi^2/4}$$

$$\eta = \frac{z}{\sqrt{\alpha t}}$$

One was what is called similarity solutions, this was particularly appropriate for the first problem that we solved, and that was the transport from flat surface into an infinite fluid. So, T was equal to T infinity far away and we used as the z coordinate and at time T is equal to 0, the temperature of the surface itself was increased to T is equal to T knot. And because of that if you plot the temperature profile at the surface the temperature of T knot, but at the instant at which the temperature was increased to t knot that increase in energy has not been conducted into the fluid yet. So, the temperature of the fluid was still equal to t infinity which is smaller than T knot.

So, we have a temperature profile that looks something like this and as time progresses conduction takes place, this temperature is still equal to T knot far away it is still equal to T infinity. So, we have a series of curves that looks like this as time progresses. So, this is increase in time, and in order to obtain an analytical solution for this we use the fact that in this particular problem, I can define a scale temperature. And if you recall we had to solve the heat conduction problem, partial T by partial T is equal to alpha times. But since this conduction takes place in to an infinite medium there is nothing that we can scale the z coordinate by there is nothing that we can scale time by because there is no time scale in the problem at time T is equal to 0 you increase the temperature and just leave it there is no final time.

The temperature at the surface is T knot the temperature far away is T infinity, but there is no length scale in between there is no fixed distance at which we have set the temperature equal to some fixed value. Just from dimensional analysis there is only one dimensional scope that we can get in this case, just from dimensional analysis and that is and that is eta is equal to z by root alpha t. Originally the equation was in terms of two independent variables time and z just form dimensional analysis we reduced it just to one in the variable. Therefore, the equation now becomes an ordinary differential equation in this variable eta and on that basis we were able to solve it.

So, just to reiterate all of the equations that we get in this case they are all partial differential equations and there are no general ways of solving partial differential equations. The only way to obtain solutions is to somehow convert these to ordinary differential equations, and then solve them. And similarity solutions procedure offers one way of doing that convert this partial differential equation into an ordinary differential equation, just based upon dimensional analysis. And then get the solution for the temperature.

If you recall we had got the solution for the temperature field as T minus T knot by T knot minus T infinity is equal to 1-1 over root pie integral over 0 to z pie root alpha T d si just e power minus si square by 4. So, that was one way reduce it from two to one variable using the fact just dimensional analysis and the fact that there is no other time scale in this problem. The second was what was called separation of variables, ok?

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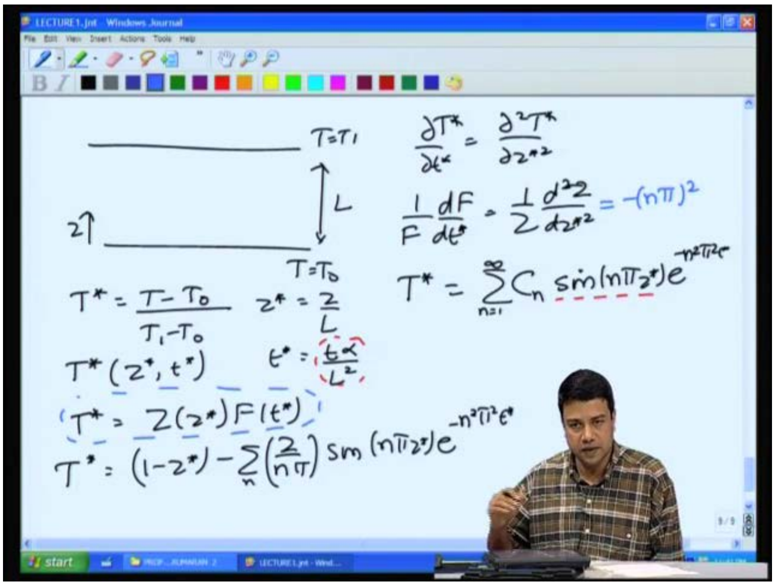

$$\frac{\partial T^*}{\partial t^*} = \frac{\partial^2 T^*}{\partial z^{*2}}$$

$$\frac{1}{F}\frac{dF}{dt^*} = \frac{1}{Z}\frac{d^2 Z}{dz^{*2}} = -(n\pi)^2$$

$$T^* = \sum_{n=1}^{\infty} C_n \sin(n\pi z^*) e^{-n^2\pi^2 t^*}$$

$$T^* = \frac{T - T_0}{T_1 - T_0} \qquad z^* = \frac{z}{L}$$

$$T^*(z^*, t^*) \qquad t^* = \frac{t\alpha}{L^2}$$

$$T^* = Z(z^*)F(t^*)$$

$$T^* = (1 - z^*) - \sum_n \left(\frac{2}{n\pi}\right) \sin(n\pi z^*) e^{-n^2\pi^2 t^*}$$

If I have a channel with one particular two boundary conditions T equal to T knot T is equal to T 1 at one particular distance let us say L in this case there is a length scale in the problem. So, my z coordinate can be scaled by the distance L. So, I can define T star equal to T minus T infinity by T knot minus T infinity z star is equal to z by L. So, in this case we cannot use the similarity solutions, what we do is to use separation of variables procedure.

This T star is a function of z as well as time, so this scale time comes out of the diffusion equation itself the scale time becomes T times alpha by L square. So, this is function of two variables we write it as some function z of z star times some function of t start, the original equation that I had was of the form d t by d t is equal to partial square T by partial z star square. And this is in scale coordinate so the I have already incorporated the thermal diffusion coefficient into this term here.

And if you now write down substitute this I can reduce it to an equation in which, I have 1 over F, d F by d t is equal to 1 over Z d square z by d z square. Now, the left hand side depends only upon time right hand side depends only upon distance and because of that these two have to be constant because if I had to keep z a constant and change time the left hand side will change and the right hand side will not. Unless both of them are individually constants and these constants now have to satisfy the boundary conditions of the two surface. And we saw how to satisfy the boundary conditions, in case you are not familiar with that kindly go and revise it once again.

There are the idea is basically that we right down this T star as a sum for a set of Eigen functions, C n sine n pie z star e power minus n square pie square T star. Where these functions are basically solutions which satisfy the boundary conditions only for integer values of m, these functions satisfy the boundary conditions only for integer values of m. And so these integer values come out with the solution of the equation like all the Eigen functions and I am sorry they are called the Eigen values. And these functions themselves are called the Eigen functions these functions satisfy what are called orthogonality relations.

And these orthogonality relations enable us to determine what these constants are. In this particular case the solution turns out to be T star equals to 1 minus z star the homogeneous part I should change the definition here because I have two different temperatures here. So, the definition here is actually T minus T knot by T 1 minus T knot and this T star becomes 1 minus z star minus summation over n of 2 by n pie sine n pie z star e power minus n square pie square T star.

So, that was the second solution procedure in case you are not familiar with this as I said we will be using this in a in the course once again. So, kindly go back and familiarize yourself with this procedure. Basically once I have done this transformation, I get a differential equation for each of these and each of these has to be a constant. So, each of these has to be constant, the constant has to be a negative because I require that f has to decay to 0 if F is just the transient part this has to decay to 0 as time goes to infinity.

And also it has to satisfy the boundary conditions of these two surfaces on the basis of that I get this minus n pie whole square. Also we looked at oscillatory flows oscillatory systems, where you have a oscillatory variations a time periodic variation, that was done for the particular case of a pipe flow. In that particular case you can, since the equation are linear and since the forcing is oscillatory the response also has to be oscillatory.

So, you can assume an oscillatory function for the time dependence. Since you have already have the oscillatory function for the time dependence, you get an ordinary differential equation for the special dependence. And that was also something we did in the last class in the last course, that is not something that we will be using in the present course. So, I won’t talk too much about that. After that we went and we determined equations which are general which hold for any particular differential volume.

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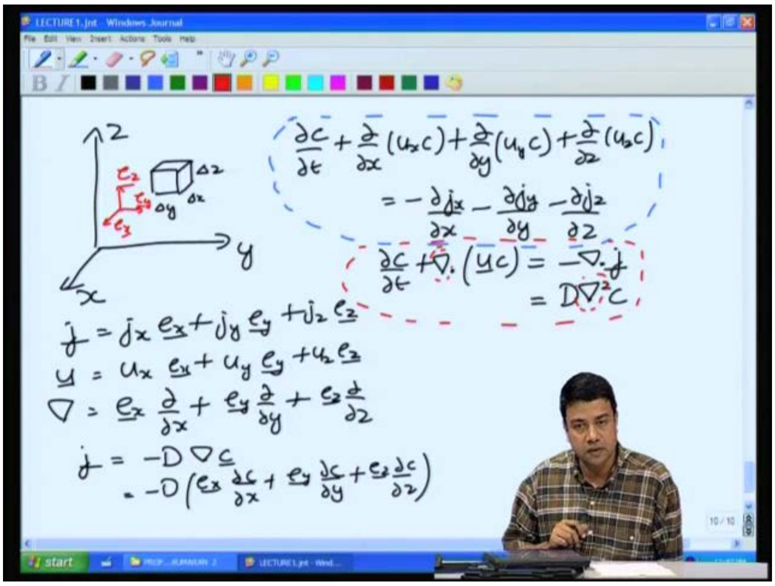

$$\frac{\partial c}{\partial t} + \frac{\partial}{\partial x}(u_x c) + \frac{\partial}{\partial y}(u_y c) + \frac{\partial}{\partial z}(u_z c) = -\frac{\partial j_x}{\partial x} - \frac{\partial j_y}{\partial y} - \frac{\partial j_z}{\partial z}$$

$$\frac{\partial c}{\partial t} + \nabla \cdot (\underline{u} c) = -\nabla \cdot \underline{j} = D \nabla^2 c$$

$$\underline{j} = j_x \underline{e_x} + j_y \underline{e_y} + j_z \underline{e_z}$$

$$\underline{u} = u_x \underline{e_x} + u_y \underline{e_y} + u_z \underline{e_z}$$

$$\nabla = \underline{e_x} \frac{\partial}{\partial x} + \underline{e_y} \frac{\partial}{\partial y} + \underline{e_z} \frac{\partial}{\partial z}$$

$$\underline{j} = -D \nabla \underline{c} = -D\left(\underline{e_x} \frac{\partial c}{\partial x} + \underline{e_y} \frac{\partial c}{\partial y} + \underline{e_z} \frac{\partial c}{\partial z}\right)$$

So, the first was for the cartesian coordinate system in this particular case, you have planes uh surfaces of constant coordinate. We took a small differential volume here of width delta y, delta x, delta z calculated what was coming in and what was going out both due to convection due to fluid motion. As well as due to diffusion the diffusion fluxes and we obtained balance equations in all three coordinates simultaneously. These equations were of the form partial c by partial t plus the derivative of the velocity in x direction times concentration, the derivative in the x direction plus d by dy of u y c, this was equal to the grains of the fluxes minus partial j x minus partial j z partial z.

We had started defining vectors in that previous course they had defined the vector flux j is equal to j x e x plus j y e y plus j z e z velocity vector. We had also defines gradient vector e x times d by d x plus d y. Here I will use e x e y and e z instead of i j and k to the present unit vectors. So, e x is unit vector in the x direction e y in the y direction and e z is the unit vector in z direction, this is the more convenient notation for us for this course.

So, we will be using this with these definitions you can write this equation as partial c by partial t plus del dot the dot product of this operator and this vector u times c, c is a concentration is a scalar u is a vector velocity multiply u and c you get a vector. Take the dot product of this divergence with this one that’s is basically this the other term on the left hand side d by d x of u x c plus d by d y of u y c plus d by d z of u z c.

The right hand side I have minus dot product of this once again this gradient operator with the flux vector j the dot product of the gradient operator with the flux vector j, dot product of gradient of vector we will see is what is called the divergence. Now, the flux itself can be expressed as the gradient of the concentration field, I can write the flux itself vector as minus D times the gradient of the concentration field and this is equal to minus D times e x dc by dx plus ey dc by dy plus ez dc ny dz. And this on the right hand side I just substitute minus d times gradient of c and I get d del square c.

So, this was the general equation in a cartesian coordinate system this was the equation in a cartesian coordinate system for a cubic differential volume. This form of the equation is far more general this form of the equation is far more general it holds for any coordinate system it holds for any coordinate system. Only thing is that these operators the divergence operator this del square operator called the Laplacian they are different in different coordinate system that is the only difference. We had also explicitly derived constrained to the equations in both cylindrical and spherical coordinate systems.

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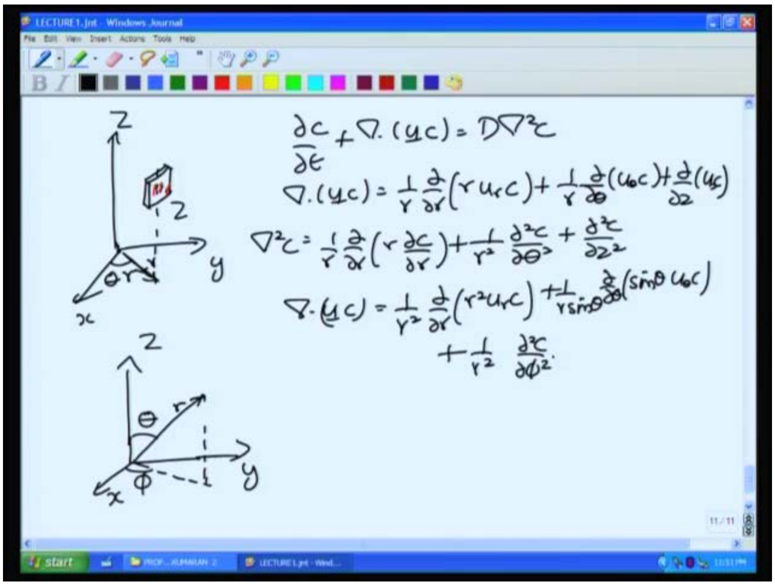

$$\frac{\partial c}{\partial t} + \nabla \cdot (\underline{u}c) = D\nabla^2 c$$

$$\nabla \cdot (\underline{u}c) = \frac{1}{r}\frac{\partial}{\partial r}(r u_r c) + \frac{1}{r}\frac{\partial}{\partial \theta}(u_\theta c) + \frac{\partial}{\partial z}(u_z c)$$

$$\nabla^2 c = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 c}{\partial \theta^2} + \frac{\partial^2 c}{\partial z^2}$$

$$\nabla \cdot (\underline{u}c) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u_r c) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, u_\theta c) + \frac{1}{r^2}\frac{\partial^2 c}{\partial \phi^2}.$$

Just to refresh your memory in the cylindrical coordinate system there is an axis, and the distance from this axis is what is called the radius. So, I have any point here the height above the x-y plane is the z coordinate, the distance from the axis is what is called r and the distance the angle that r makes with any particular axis is called theta. And in this case you have to take the cylindrical shell, the cylindrical shell that has surfaces of constant coordinate.

So, we have surfaces six surfaces just like the cubicle differential volume, it will have six surfaces two of which are perpendicular to the coordinate, that is the one on this side, and the one on the other side the inner one two of which are perpendicular to the theta coordinate that is on the right and the left. And the two of which are perpendicular to the z coordinate top and bottom write differential equation. Write the conservation condition change in mass the differential volume is equal to mass in minus mass out, and you get a differential equation for the cylindrical coordinate system.

And that has the form partial c by partial t plus del dot u c is equal to D del square c where in this case del dot is equal to 1 by r d by d r of r partial r u r c plus 1 by r partial by partial theta. So, this is the definition the del square operator was 1 by r d by d r of r we derive these expressions explicitly from shell balances in (( )). Similarly, we had also done it for the spherical coordinate system, this particular case there are three coordinates one is the distance from the origin this is called r. The second is the azimuthal angle the angle made by the radius vector with any particular axis the generally chosen is the z axis that is theta.

The third is the meridional angle and that is the distance made by projection of this radius vector on to the x y plane, with axis traditional chosen as the x axis that is phi. And in this case once again these operators are redefined, so del dot u c is equal to 1 by r square d by d r of r square u r c plus 1 by r sine theta sine theta partial square c by partial phi square. And finally, we have the del square operator in this coordinate system as del square c is equal to...

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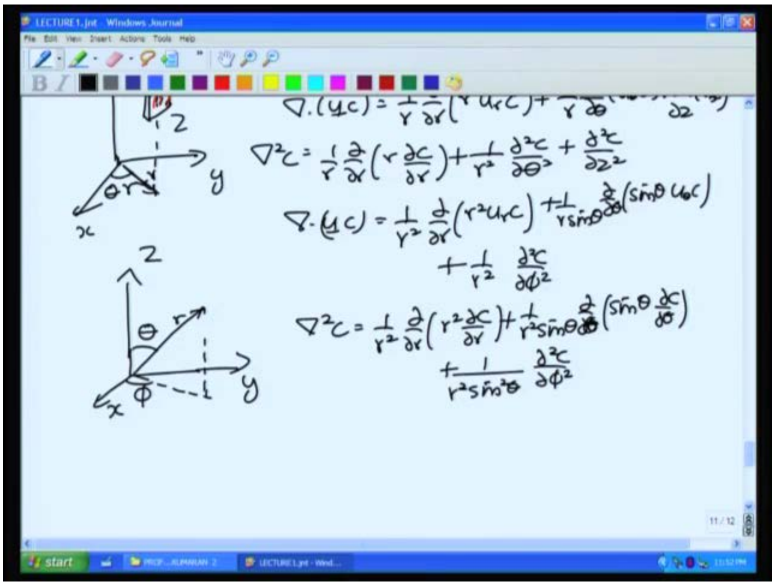

$$\nabla\cdot(\underline{u}c) = \frac{1}{r}\frac{\partial}{\partial r}(r u_r c) + \frac{1}{r}\frac{\partial}{\partial\theta} \qquad \partial z$$

$$\nabla^2 c = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 c}{\partial\theta^2} + \frac{\partial^2 c}{\partial z^2}$$

$$\nabla\cdot(\underline{u}c) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 u_r c\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, u_\theta c) + \frac{1}{r^2}\frac{\partial^2 c}{\partial\phi^2}$$

$$\nabla^2 c = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial c}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial c}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 c}{\partial\phi^2}$$

So, those are the Laplacian and the divergence operators in the different coordinate systems. So, that is where we were and the next lecture I will take some time to motivate, what we are going to do in this course in the context of what I have just discussed for the previous course. This is the background kindly go through it again in case you are not familiar with it, it is important that those who are for understanding the next part of the lecture that you are familiar with all of this. And I will motivate in the next lecture, what we are going to do next. So, we will see you there.