Fundamentals of Transport Processes

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Lecture No. # 08

Unidirectional Transport

Cartesian Co-ordinates-1

Welcome to this the 8th lecture, on the fundamentals of transport processes. And this is when we get down to business, we actually start solving problems on the temperature, velocity, and concentration fields in simple situations.

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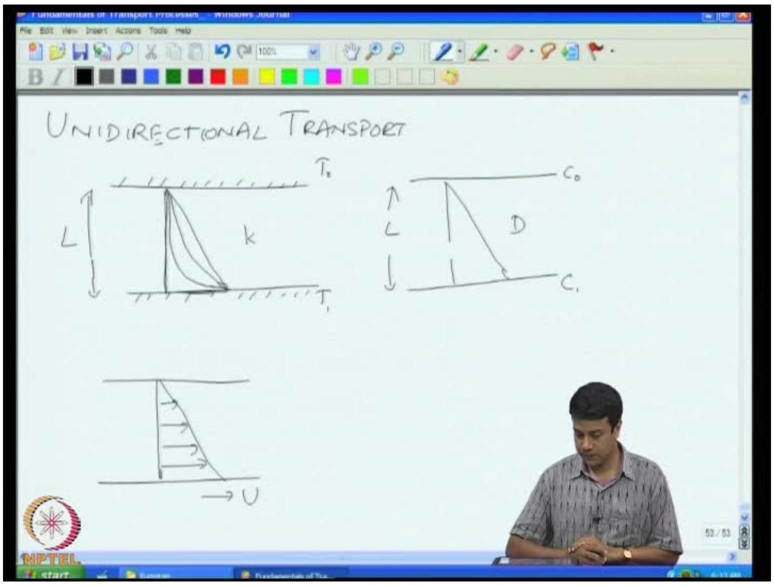

The simplest situation that one can consider is the example of the transfer between two flat plates. I have two flat plates, separated by a distance L. In the heat transfer problem, I will have a fluid of thermal conductivity k between the two plates and I will keep the temperatures of the two plates at two different temperatures, that is, T naught and T 1. Then I want to know, what is the temperature in-between the two plates?

In the case, of steady state, it is quiet easy; it is just a linear variation of temperatures between the two plates. Finally, at steady state, the temperature would be linear, it would start at T naught over here and go to T 1 over here, the linear variation temperature, but that is not the only situation. I could have a situation where, for example, the entire system was at temperature T naught at the starting point and I instantaneously supplied heat to the lower plate. So that the temperature at the lower plate became T 1 and then, one can ask, what is the variation of temperature?

Obviously, at very early times, the temperature cannot be at constant throughout, but then I have increased this temperature to T 1. Therefore, at early times, I have the variation that looks like this. Finally, in that long time limit, I will go to the linear temperature. So, we want to consider both steady and unsteady situations. The equivalent mass transfer problem is to have a medium between the two plates - the concentration on the lower one is C 1 and the concentration of the upper one is C naught, and I have a medium with some diffusion coefficient D separated by length L. So, I have two porous plates. I fix the concentration on those two plates and I want to find out what is the concentration in-between. The momentum transfer problem is, look at this one. I have two plates, this is at 0 velocity and I said this is moving with the velocity (( )). If I wait for sufficiently long, I will get a linear velocity profile between these two plates. Obviously, I have to calculate all of these: temperature, concentration, and velocity profiles that is - the velocity fields, the variation with respect to position not only at the final state but also of these transient states.

I explained in the introductory lectures, that there are two things involved here - one is the balance laws, these balance laws are for mass momentum energy, and those, which are returned for a small differential volume. Basically, what they will tell you is how the concentration varies with location in a small differential volume. Once I have that, I now have to find out in the problem itself I have not given the concentration variation for every point; I have only given the value at two boundaries. I know what the variation is in each differential volume from which the form of the variation of each differential volume. I have to construct the total concentration, temperature, and velocity fields across the entire system and that involves the process of integration as I told you. So, we will go through those two separate steps first in this lecture.

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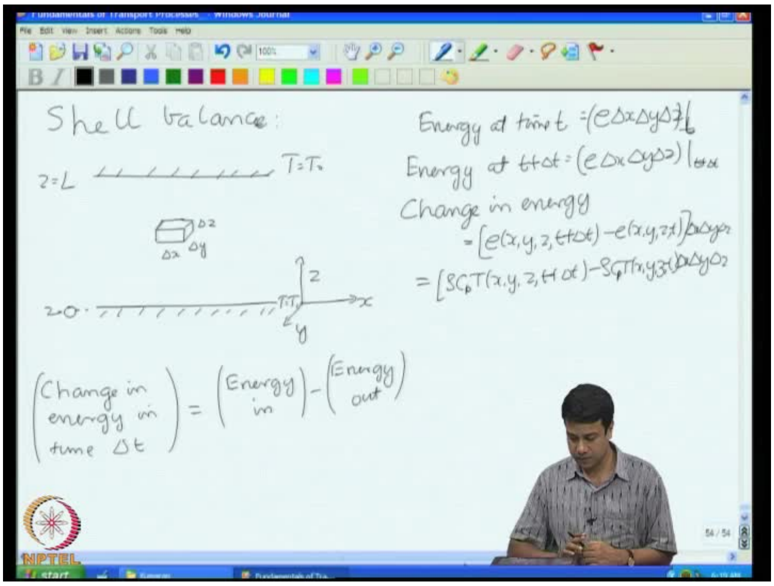

First, how do you write balance for a small differential volume? It is called a shell balance and the volume has to be chosen in such a way that it correspond to the symmetry of the configuration that we are considering. In this particular case, I have two infinite plates - these are. So, the first thing is to choose a coordinate system that I can use to analyze the problem. In this particular case, I just have two flat plates - so I can choose a coordinate system, as a Cartesian coordinate system x, y and the distance between the plates is a z coordinate.

At this location z is equal to 0 and this is at z is equal to capital L - these are the two flat plates, the boundaries of the domain I am considering. The plates themselves are considered to be infinite in the x y plane. Basically, these two plates are of infinite extent in the x and y directions, there are 2 boundaries - z is equal to 0, and z is equal to L. So, I prescribe the temperature T is equal to T naught here and T equal to T 1 at the lower boundary. So, that is what is given to me. How I am going to solve this problem? First I consider a small differential volume between these two plates, this has extent delta x in the x coordinate, delta y in the y coordinate and delta z in the z direction.

Since, the system is of infinite extent in the x y plane, there will be no variations of the temperature in the x and y directions. So, the temperature is at constant in both the x and y directions and is only a function of the z coordinate. If it is a steady state, it is not a function of time.

For the general unsteady problem, the temperature will also be a function of time as well as the z coordinate. So, let us try to derive an equation for the temperature, which tells us how the temperature will vary with respect to the z coordinate as well as with respect to time.

The fundamental principle here is the energy conservation principle. So, the energy conservation principle states that the change in energy, in a small time delta T within this volume; the change in energy within a small time delta T is going to be equal to energy in minus the energy out. So, this is the fundamental principle, plus any sources or any sinks minus sink; the sources could be due to an exothermic reaction; the sink could due be an endothermic reaction. For the present, we will not concern as well as far as sources and sinks. We will assume there will be no heat generation or absorption within the fluid and therefore we only have the transport of energy across the surfaces of this differential volume. What is the change in energy in a time delta T? The change in energy in the time delta T is equal to the energy per unit volume is specific energy, times the volume. So, this is equal to the specific energy times the volume which is: delta x, delta y, delta z at the time t, change in energy. This is energy at time t. Energy at t plus delta t will be equal to e times delta x, delta y and delta z at t plus delta t. Therefore, change in energy will be equal to e at x, y, z, t plus delta t minus e at t times delta x, delta y, delta z. So, that is the change in energy within the time interval delta t. This specific energy, I can also write it as the density times, the specific heat times temperature. This can also be written as row C p times T at x, y, z, t plus delta t minus row C p t at x, y, z t times delta x y z - that is the change in energy within in the differential volume.

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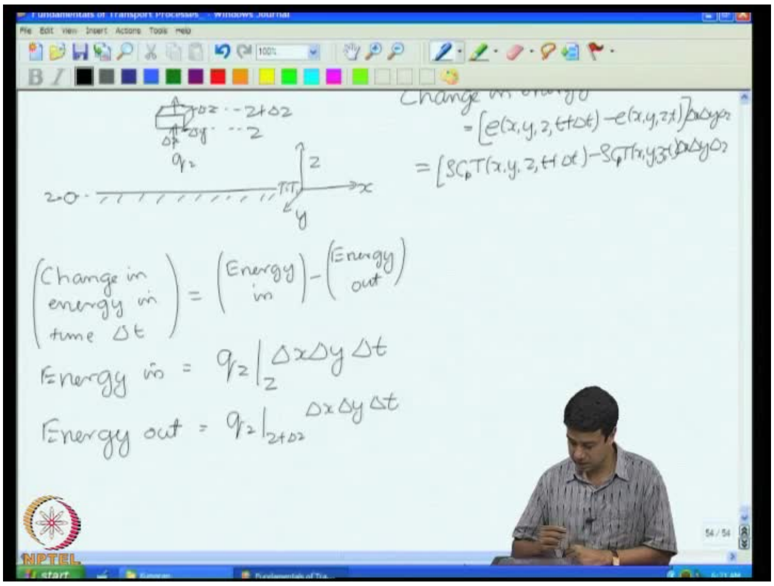

What is the energy in? If we consider the plus z coordinate, the heat flux upward as positive, then the energy in is coming in at this bottom plane the energy is coming in at this bottom plane z. So, energy in is going to be equal to flux in the z direction at the location z. The flux at the location z, flux is the energy per unit area, per unit time. So (( )) total energy in has to be flux times the area times the time interval - so the total energy in has to be in flux times delta x, delta y that is the area of the bottom surface times the time. So that is the flux.

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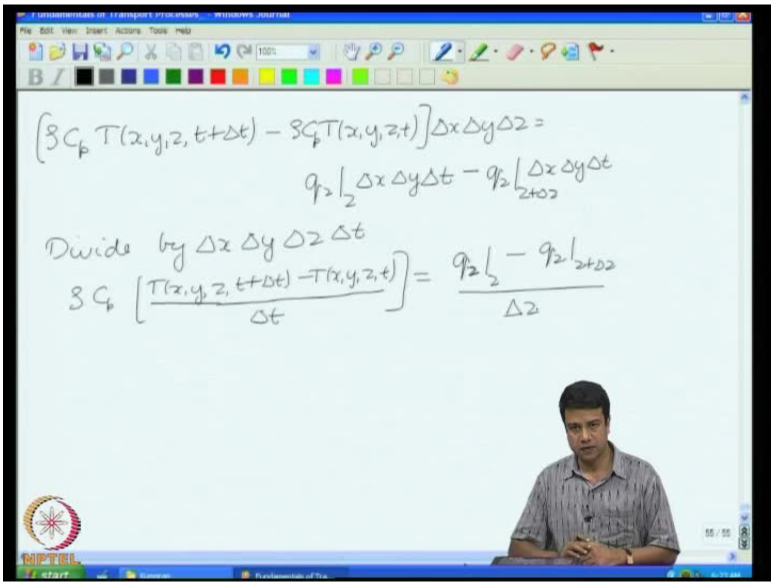

Flux q z has been defined positive, if it is acting in the upward direction. So, the flux q z is positive, is in the plus z direction. What is the energy out? The energy that is going out at the upper surface: the upper surface is at the location z plus delta z. Therefore, energy out is equal to q z at z plus delta z times delta x delta y delta t. So, this gives me the energy in and the energy out. So, change in energy within a time delta t has got to be equal to be the energy in minus the energy out. Change in an energy in a time delta t row C p T at x, y, z, t plus delta t. I am assuming for the moment that the density and the specific heat or not functions of position. The temperature is the only variable that is variable in position. This multiplied by the volume. This is equal to the energy in which is equal to q z at z times the area - cross sectional area perpendicular to the z axis the cross sectional area times minus the energy going out of the top surface q z at z plus delta z times delta x delta y delta t. Now, divide by delta x delta y delta z delta t dividing by the volume and by the time interval.

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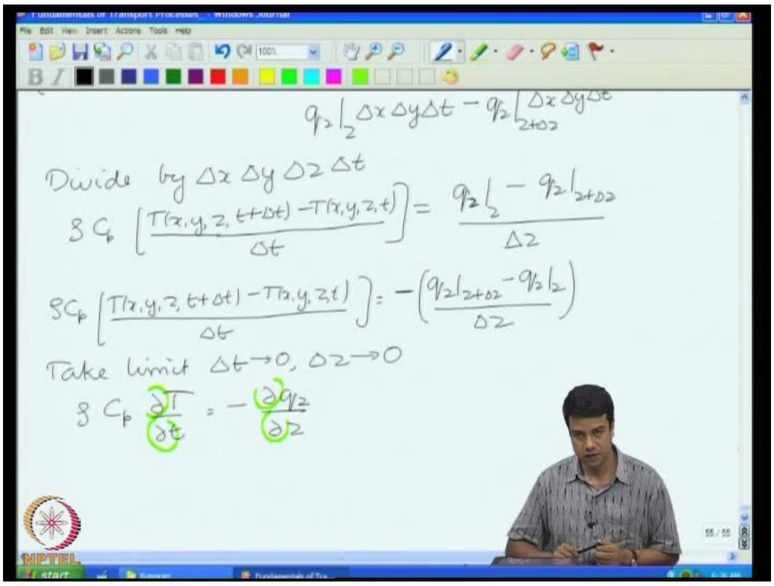

Once you do division, what you will get is that row C p into T at x y z t plus delta t minus T at x, y, z, t, the whole thing divided by delta t is equal to q z at the location z minus q z at z plus delta z by delta z. I can now rewrite this equation a little bit, on the right hand side, I can write this as minus of q z at z plus delta z minus q z at z divided by delta z.

I take the limit delta t going to 0 and delta z going to 0, I take the limit of both delta t going to 0 and delta z going to 0. Therefore, my final equation becomes row C p partial T by partial t is equal to minus…

Note: that I have here these partial signs, what we take the partial derivative to respective time as indicated over here, you are keeping the positions. The constants x, y and z are the constants. You are taking the limit as delta x, delta y and delta z go to 0, but I will take the partial derivative keeping x, y, and z as constants. I will take the partial derivatives with respect to z; I will keep the other independent variables as constants in other words I am keeping x, y and t as constants.

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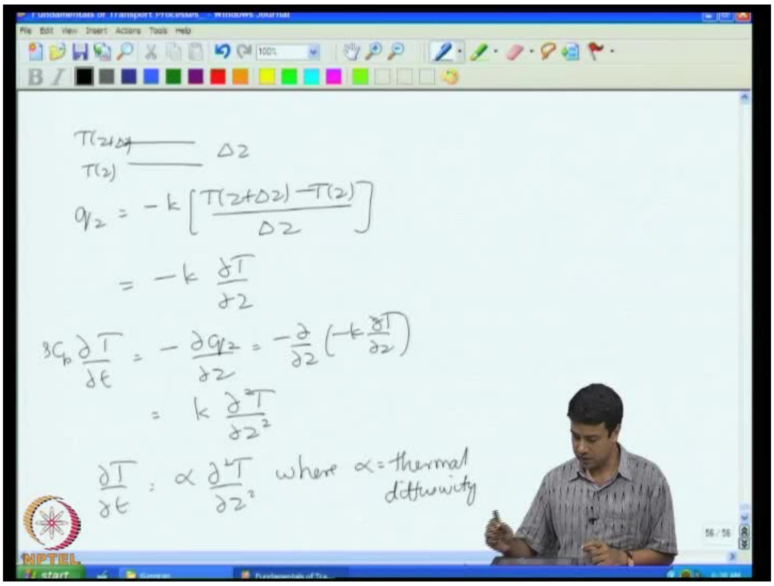

This equation, basically tells me how the temperature changes due to the flux through the system, in order to get an equation for the temperature alone. I need to have some way of calculating the flux in terms of the temperature and that is where the Fourier's law of heat conduction comes in. If I have two surfaces separated by a distance delta z, this is T at z plus delta z and this is T at z. The flux in the z direction is equal to minus k into T at z plus delta z minus T at z by delta z. This is for 1 direction. The hope, the same thing holds for variations and time as well, because the heat energy always goes from a higher temperature to a lower temperature. The flux at time z plus delta z the temperature time at position z plus delta z, if it is higher than z then the flux will be downwards - that is the reason for the negative sign.

If I take the limit as delta z goes to 0, this basically becomes equal to minus k d T by d z, once again the partial derivative, I am taking the variation of temperature with respect to the z coordinate - keeping x, y, and t at constant. Once I have that, if I insert this into the previous equation row C p partial T by partial t is equal to minus partial q z by partial z, which is minus partial by partial z of minus k d T by d z is equal to minus k d square T by d z square.

Assuming here that the thermal conductivity is not a function of position, so you have a material for a, with a constant thermal conductivity. This is the diffusion equation for, I am sorry there should be no negative sign. This is the diffusion equation for heat, I could divide by row C p and I will get d T by d t is equal to alpha d square T by d z square, where alpha is equal to thermal diffusivity. This is the diffusion equation unsteady that you would use for solving one-dimensional problems for the variations of the temperature.

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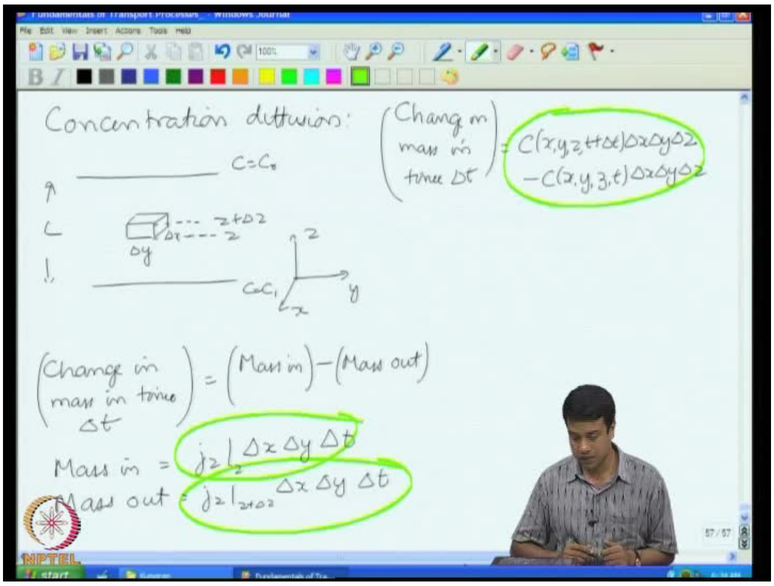

How about the concentration diffusion? I will not go through this concentration diffusion equation in very much detail, because the method for deriving it is very similar to the method for thermal diffusion equation. I have two plates, separated by a distance L, C is equal to C naught on the top plate; C is equal to C 1 and because of the difference in concentration that is a flux or mass going upwards. What we like to do is to determine the entire concentration field, first things first. We choose a coordinate system, once again - x y z coordinates. The planes are infinite in the x y plane - so there is no variation in the x y plane, there is a variation only in the z direction; there could be a variation in time as well. Once again, we take a small differential equation. In differential volume the shell balance in between, the locations z and z plus delta z of height delta z and thickness; delta x and delta y in the two directions. The basic principle in this case is mass conservation principle, that is, change in mass change and time delta t equal to mass in minus mass out that could be sources and sinks of mass as well. For example, I could have a reaction-taking place, which is generating some species; some product species are consuming the reactance in which case the concentration will have sources and sinks in it. For the moment let us not consider that - let us just consider case were the mass itself is conserved and there are no reactions taking place. Change in mass in time t. The mass within this volume is going to be equal to be the concentration times the volume. So, the mass at time t plus delta t is going be equal to the concentration at x, y, z, t plus delta t times the volume. The volume is the delta x delta y delta z, that is, the mass at time t plus delta t.

Change in mass is equal to the mass at time t plus delta t minus the mass at times t. So, this minus the concentration x y z t times delta x delta y delta z. If I consider two time intervals separated by a small delta t, then the change in mass within the time interval has to be equal to the concentration times the volume at time t plus delta t minus the concentration times the volume at time t. What about the mass in; mass out? The mass in is because of the flux that is travelling vertical along the z coordinate, we assume that flux in the positive z direction is positive. So, mass that comes in, has to travel into volume at the location z. So, mass in is going to be equal to the flux at location z, flux is mass transported per unit area per unit time. Therefore, I have to multiply, that by the cross sectional area of that bottom surface delta x, delta y and the time interval delta that is the mass in at the bottom surface at location x, y and z. Mass out at that out surface that is going to be equal to the heat flux, at the top surface times, the area times, time. So, this going to be equal to j z at z plus delta z delta x delta y delta t.

Now, I have the three different terms together mass in, mass out, and now I put all three together to get the conservation equation.

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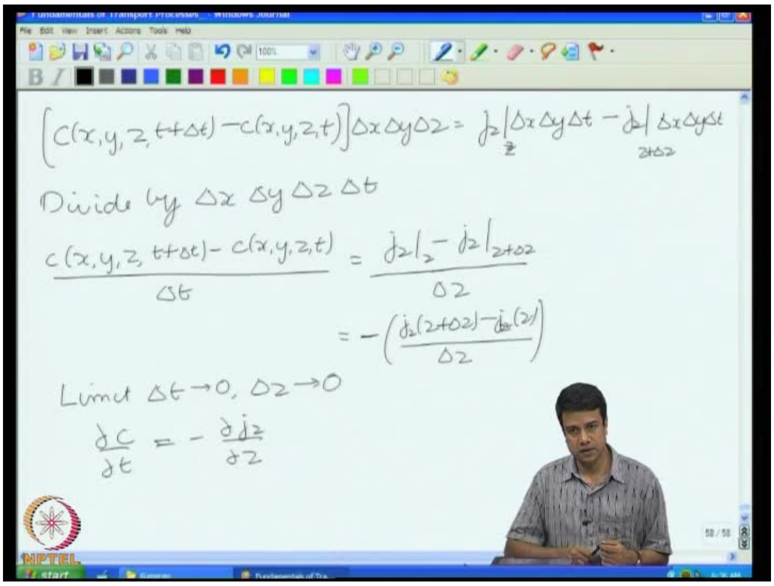

So, the conservation equation will be of the form, C at x ,y, z, t plus delta t minus C at x, y, z, t into the volume delta x delta y delta z is equal to mass in within the time interval delta t mass in is j z times at the location z times delta x delta y delta t minus j z at z plus delt z. The complete form of the mass conservation equation for that differential volume. Divide throughout by volume and time; divide by delta x delta y delta z delta t. I will get C at x, y, z, t plus delta t minus C at x, y, z, t whole divided by delta t, is equal to j z at the location z minus j z at z plus delta z divided by delta z. Once again, I can rewrite the right hand slightly as minus of j z at z plus delta z minus j z at z by delta z.

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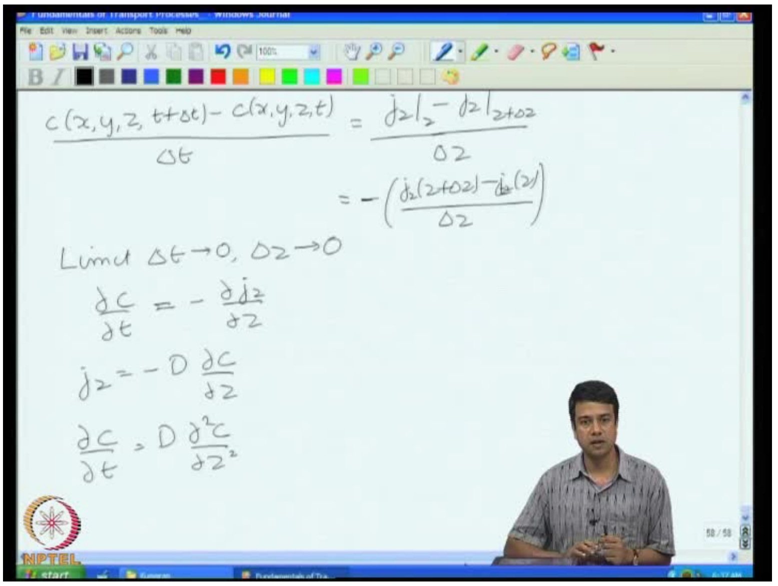

Take the limit delta t going to 0 and delta z going to 0. This becomes the partial C by partial t equal to minus t j z by z. So, this is the mass conservation equation in terms of the mass flux. Now, to close the equation, we have to write now the mass flux in terms of the concentration. The mass flux in terms of concentration is quiet easy, we have done it before, j z is equal to minus d times partial C by partial Z, just as in the case of heat flux q was equal to minus k times d t v z. In this case, j z is minus c times partial C by partial Z and with this, I have the mass conservation equation partial square C by partial Z square. So, this is the unsteady diffusion equation for the concentration field.

You will notice that, both the equations for the temperature and concentration field at the form d by dt of concentration temperature is equal to a diffusivity times d square by d Z square. Dimensionally it makes perfect sense because d has dimensions of time square by unit time. This is the standard diffusion equation in one dimension. We will see little later, how it changes, when we go to higher dimensions. So, we have to solve either the concentration equation or the temperature equations, the diffusion equation for either one of them in a finite domain bounded by 2 walls, separated by a distance L.

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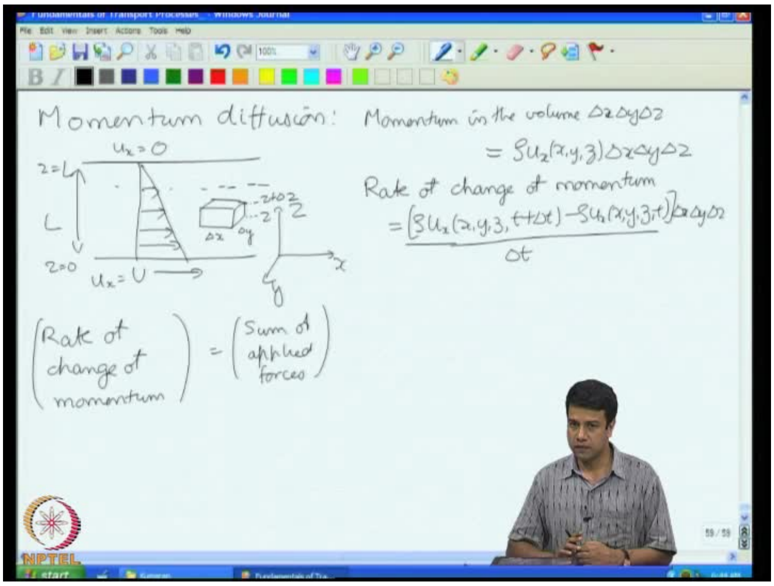

So, we have done mass and momentum. What about it?

We have done mass and energy diffusion - what about momentum diffusion? The fundamental balance law of momentum diffusion is slightly different. So, I will try to go through at in some detail. The configuration typically consists of two plates, I move with the bottom, the plate with the velocity U the top plate is stationary. So, the velocity is equal to U here and U x is equal to 0 on the top plate. At steady state, I will have a linear velocity profile. Let us, use a Cartesian coordinate system as before where x and y coordinates are in the plane of the plates and z is perpendicular to the plates. The distance between the two, in this case, the z is equal to 0, and that z equals to L. So, we use Cartesian coordinates as before x, y and z coordinates. The balance condition for momentum, which we use is slightly different. The balance condition is that the rate of change of momentum is equal to sum of applied forces. It is the same as Newton's, what comes of Newton's laws of motion; rate of change of momentum is equal to sum of applied forces. Important point to note here is, momentum itself is a vector, it has a direction tool. In this particular case, the fluid velocity is in the x direction, which means that the momentum vector is along the x direction.

However, along the x direction, nothing changes (( )) travel along the x direction, keeping z a constant - there is no change in velocity. At every location, along the x as a change at the x coordinate, at every location, the velocity is the same, even though the velocity vector is in the x direction, it does not change in the x direction. Momentum transport takes place along the direction, where there is a change in velocity. In this particular case, the velocity changes only in the z direction, it does not change along the x direction. Therefore, momentum transfer takes place only along the z direction. Rate of change of momentum is equal to sum of applied forces, that is, the fundamental balance law. In this case, as I have said, momentum along the x direction, change in momentum is along the z direction. Therefore, the transfer of momentum the flux of momentum is also along the z direction. There is no change in any quantities along the x direction. We will take a usual differential volume. This is z plus delta z. So, differential volume is bound by z and z plus delta z. Then along the x and y directions, the intervals are delta x and delta y, so this is the configuration that we are considering.

What is the rate of change of momentum? The momentum in the volume delta x delta y delta z. What is the momentum within that volume? This is equal to the density times u x at x, y, z times the volume, row times u x row is a mass by unit volume. So, row times u x gives me a momentum per unit volume. I multiply that by the volume to get the total momentum, so that is the basic principle. So, rate of change of momentum, change in momentum per unit time.

So, we look at two instances, t and t plus delta t. The rate of change of momentum will be equal to row times u x at x, y, z, t plus delta t minus row u x at x y z t divided by delta t times delta x delta y delta z. The rate of change of momentum in this volume, change in momentum per unit time, the row at u x, the row u x at x y z t plus delta t minus row u x at x y z t times delta x delta y delta z., The whole thing divided by delta t. So, that is the rate of change of momentum in this differential volume. What is about the sum of forces? We have to be a little bit careful here, in how we define things as far as the sum of forces is concerned.

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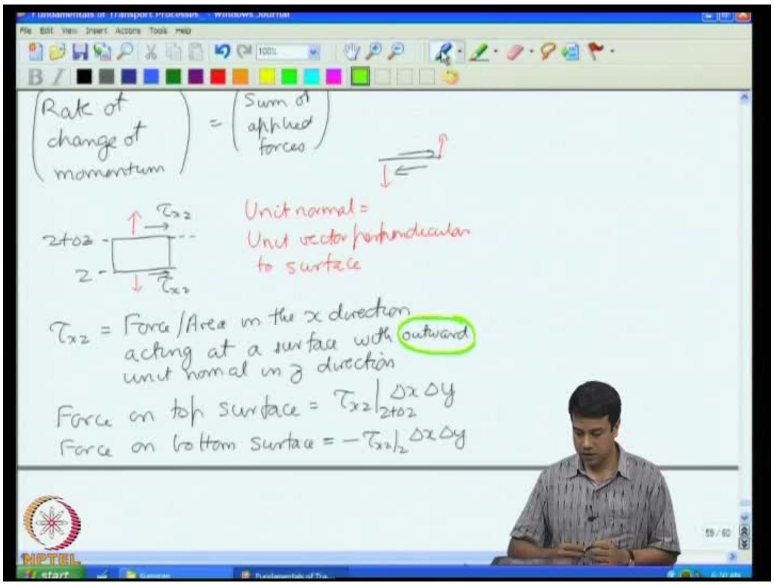

I have some differential volume and there is shear stress acting at the bottom surface, as well as in the top surface, the shear stresses acting on both surfaces. The shear stress tau x z is defined as the force per area in the x direction acting at the surface with unit normal in z direction. The unit normal is the unit vector perpendicular to surfaces. So, unit normal is the unit vector perpendicular to the surface. So, tau x z is the force per unit area in the x direction acting at a surface with unit normal in the z direction. Rate of change of momentum is equal to the sum of applied forces. In this particular case, momentum is in the x direction because it is the x component of the momentum row u x, so at the momentum this will be in x direction.

I want the force on the top surface for my momentum balance equation, for the top surface, the unit normal is in the plus z direction. Therefore, the force on the top surface has got to be equal to the shear stress times the area of the top surface. I define tau x as the force in the x direction. We are interested in momentum change in x direction, so it is appropriate to consider the force in the x direction acting at a surface whose unit normal is in the plus z direction. So, this force on the top surface has to be equal to the tau x z into the area of the top surface delta x delta y.

Except that, this force is now acting on the top surface, on the top surface the location is z plus delta z. The bottom surface was x z. So, this is tau x z at z plus delta z at the location x plus delta z and z plus delta z. Now, if you look at the bottom surface, the unit normal is in the minus z direction. Therefore, tau x z was defined as force per unit area in x direction acting at a surface whose unit normal is in the plus z direction. If the unit normal is in the minus z direction, the force will be opposite, because of Newton's third law which states that if I have at a surface forces on both sides had to be equal and opposite to each other, the balance of action and reaction.

For the top surface, the outward unit normal is in this direction, whereas for the bottom surface it is in the opposite direction; when I go from top to bottom, the direction of the unit normal changes and the direction of force changes, by Newton's third law. Therefore, the force on the bottom surface whose unit normal is in the minus z direction is equal to minus tau x z at z times delta x delta y. To repeat, the principle once again, I have defined tau x z, as the force per unit area, in the x direction acting at a surface, whose unit normal is in the z direction tau x z is the appropriate force to use, because I am writing a balance equation for the momentum and the momentum is in the x direction. If I look at the top surface, the outward unit normal - we should write outward and this is important, this outward is important, outward unit normal to the volume, the unit normal that acts outward from the body. If I look at the top surface, the outward unit normal is in the plus z direction. Therefore, the force on the top surface is equal to the shear stress (( )) tau x z and z plus delta z times delta x delta y.

The opposite surface is the downward surface. Force on the bottom surface, the outward unit normal to the bottom surface is in the minus z direction. When I go from above the surface to below the surface, the direction of unit normal changes and from Newton's third law. The forces had to be opposite to each other, that means that the force acting on this bottom surface is minus tau x at x z times delta x delta y. So, we put them all together.

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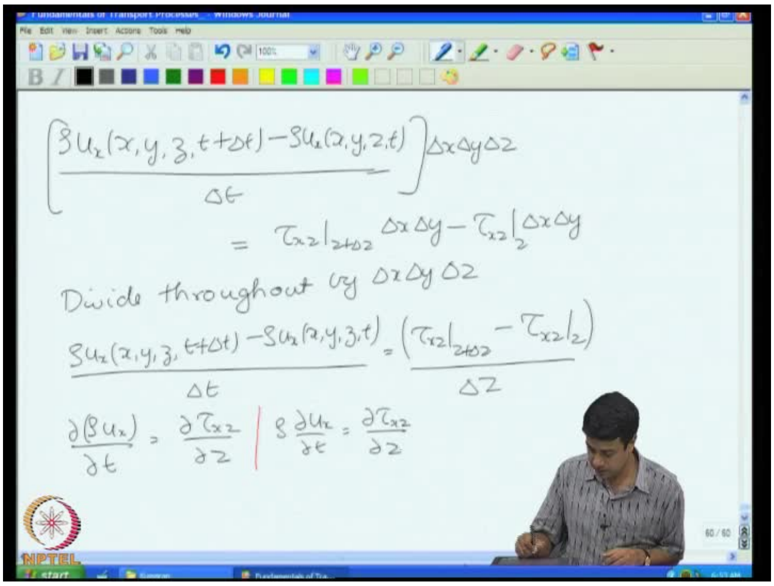

The rate of change of momentum - row u x at x, y, z, t plus delta t minus row u x at x, y, z, t divided by delta t into the volume delta x delta y delta z. This thing has got to be equal to tau x z at z plus delta z times at delta x delta y. Force on the top surface; on the bottom surface, the unit normal is in the minus z direction. The bottom surface the unit normal is in the minus z direction, so this is the balance equation.

We divide throughout by delta x delta y delta z. You will get row u x at x y z t plus delta t minus row u x at x y z t whole thing divided by delta t is equal to tau x z at z plus delta z minus tau x z at z divided by delta z. Now, once again, I take the limit of delta t and delta z going to 0, and my equation becomes - d of row u x of by d t is equal to partial tau x z by partial z. If the density is at constant, it is not a function of time or position, this equation can be rewritten as row partial u x by partial t is equal to partial tau x z by partial z.

Now, I need to write the shear stress in terms of the velocity in, for that I use Newton's laws of viscosity.

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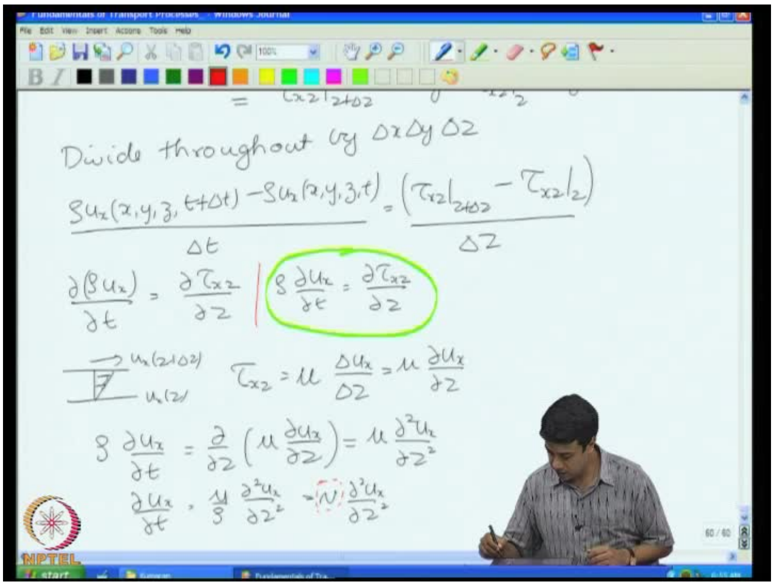

Basically, Newton's law of viscosity states that if I have two plates moving with velocity u x at z plus delta z and u x z. So, there is a linear variation in velocity between these two plates and the shear stress tau x z is equal to the viscosity times delta u x by delta z, the change in velocity divided by the separation of the 2 locations. Take the limit as delta z goes to 0, you will get this as mu times d u x by d z.

So, put that into this differential equation and you will get row times d u x by d t is equal to d by d z of mu d u x by d z. If the viscosity is not a function of position, if we have a fluid with the constant viscosity, this can also be written as mu times d square u x by d z square. This is the diffusion equation for momentum.

In this case, except one should note that - momentum is a vector, it has a direction associated with it. I can divide both sides by the density to get d u x by d t is equal to mu by row times partial square u x by partial Z square (( )). When I take viscosity, that is, partial square u x by partial Z square and this, when I take…, viscosity is the momentum diffusivity for this particular problem.

We used a slightly different methodology for momentum transfer as compared to heat and mass transfer. The reason was that for heat and mass transfer; the flux into a differential volume is considered positive. Therefore, we got equations in the form d c d t is equal to minus d j z by d z, whereas in this case, we have defined the stress to be positive. If it is defined with respect to the outward unit normal, so we would not have the negative sign to construct to the relation. However, when we put that to the balance equation we still get an equation of the same form.

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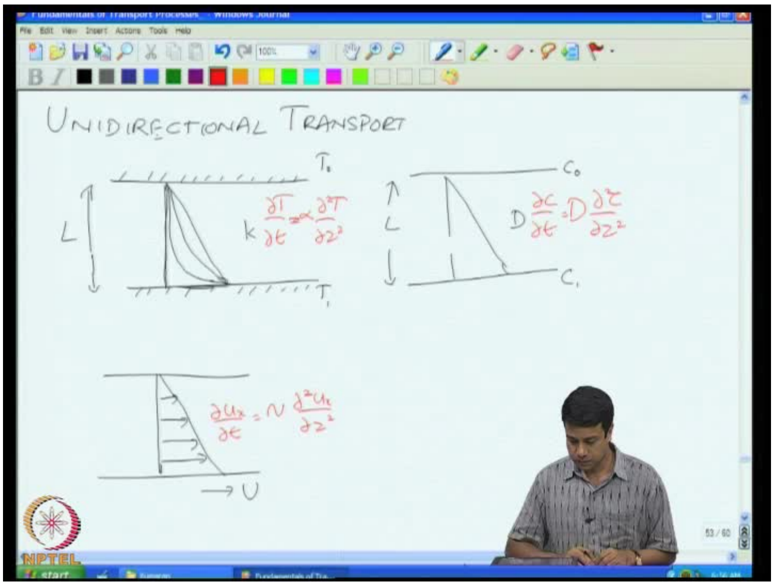

So, for all of my situations, I have common equations. For this case, I have d t by d t is equal to alpha d square t by d z square. In this case, I have d c d t is equal to the diffusion coefficient d square c by d z square. In this case, I have d u x by d t is equal to the (( )) viscosity d square u x by d z square form of the equation is the same in all cases, shows the equivalence of heat, mass and momentum transfer.

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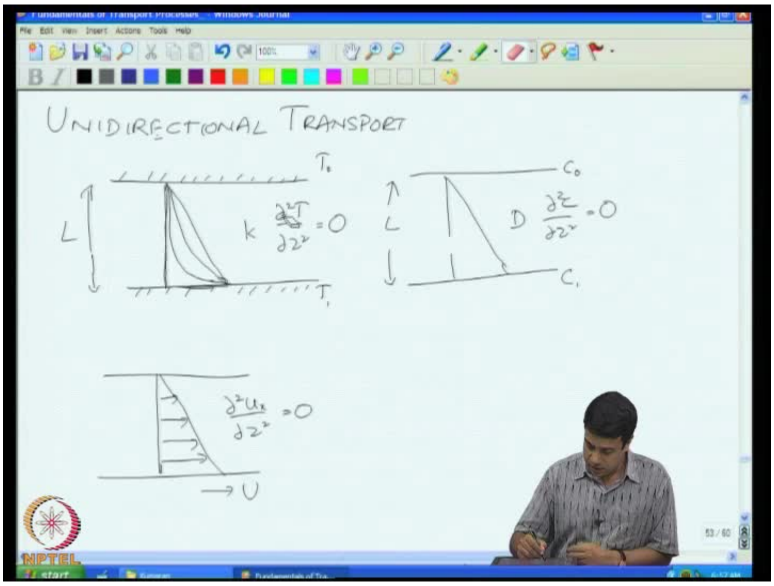

At steady state, the time derivatives are all 0. At steady state, the equations here will become d square t by d z square is equal to 0 d square c by d z square is equal to 0 and d square u x by d z square is equal to 0. So, you can solve any of the equations equivalently in order to get the steady state profiles. However, we will use one thing that will be useful later on when we do other kinds of transfer problems. However, solving these equations directly, we will scale them.

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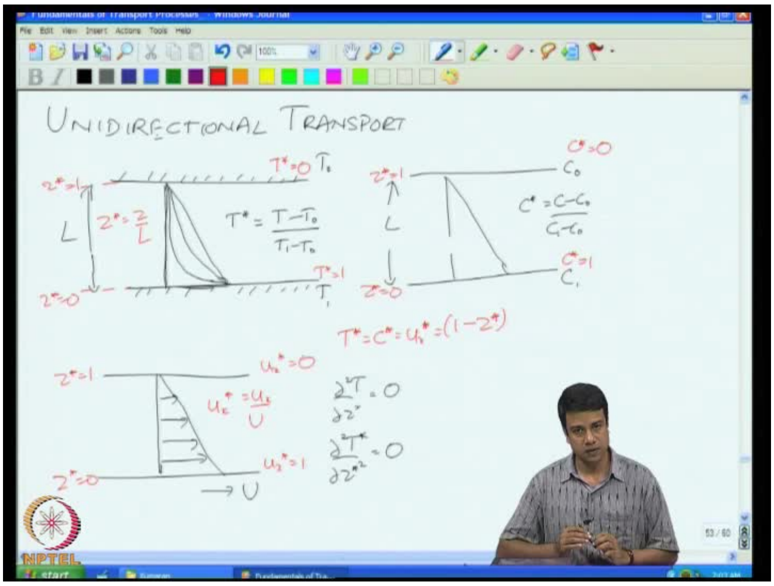

There is a temperature scale, because the temperature varies between t one at the bottom and t naught at the top. So, if I define a scale temperature t star is equal to t minus t naught by t one minus t naught, if I define t star is equal to t minus t naught by t one minus t naught, this is a scale temperature.

In that case, at the bottom surface, I will have t star is equal to one because t is equal to t one. So, t star is equal to one at the top surface; I will have t star is equal to 0. So, the scale temperature varies approximately between 1 and 0 in all 3 cases. I could define a scale at length z star is equal to z by L in that case z star varies from 0 at the bottom to 1 at the top.

In the case, of concentration field, I could define c star is equal to c minus c naught by c 1 minus c naught. That means at the bottom, c star is equal to 1 and at the top c star equal to 0. Once again, at the bottom z star is equal to 0 and z star is equal to 1. In the case, the momentum conservation problem, I could define a scaled velocity u x star is equal to u x by U in which case u x star is equal to 1 at the bottom and is equal to 0 on top.

When defined in terms of scaled quantities, the temperature concentration and velocity in scaled form have identical values on the top and bottom plates. The scale distance also has identical values. Therefore, the solutions will be identical in scale form - the solutions will be identical. The solution for all 3 cases will be T star is equal to C star is equal to u x star is equal to …

It has to vary between 0 and 1, 0 at z equal to 1, z equal to 0 and it has to be linear, because the second derivative was 0. The differential equation in all cases were d square T by d z square is equal to 0. Even, if I expressed it in scaled form, I will still get d square T star by d z square is equal to 0. In scaled form, you divide t by t one minus t naught and z by L and you will get the same differential equations. In scaled form that means, the t star c star and u x star have to be linear functions of position and you can easily verify that the linear function that satisfies this is 1 minus Z star. So, that the solution of the equation is exactly the same as in the steady state for all three cases, once I have expressed in terms of scaled variables.

This scaling is a useful concept, because for example, I could have a real temperature between these two plates varying between say 20 degrees and 40 degrees centigrade, but rather than represented on the scale going all the way from 0 to 100. I could focus on the raise that I am interested in because that the temperature raise that I am interested is only between 20 and 40. So, I do not have to look at it on an absolute scale I look at it in a scale variable where 20 is equal to 0 and 40 is equal to 1, so it varies between 0 and 1. That takes into account the scaling for the variation of the temperature once it scales the temperature fields, concentration fields and velocity fields. In this way, it takes into account the real variation across this distance.

Similarly, z could vary from 1 meter; 1 centimeter and so on, but what really matters is the length scale of the plates itself. So if I scale it by the distance, then I will just get 1 boundary to be at 0; the other boundary to be at 1 and that scaling will turn out to be very important concept in the course, that as we call it.

So, I derived for you the conservation equations for unidirectional transport mass momentum and energy they all have identical forms. In the case, of mass conservation, we have the mass diffusion coefficient, momentum conservation that is thermal diffusivity alpha. In the case, of energy conservation there is a thermal diffusivity of alpha momentum. Conservation there is the kinematic viscosity or the momentum diffusivity, which is the ratio of viscosity and density.

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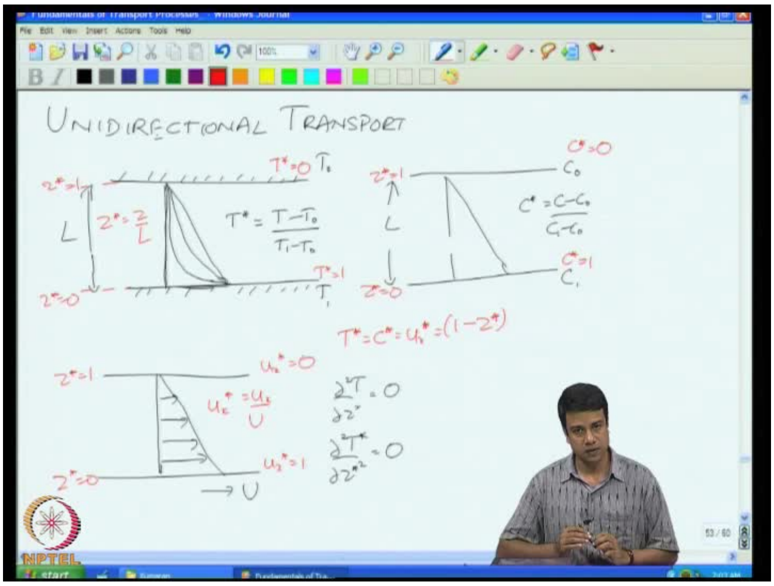

Once expressed, in this form, all 3 equations have identical forms. One could work with the form of the differential equation that is of the form d c by d t is equal to some diffusivity than d square c by d z square, and one has to solve that problems subject to the conditions on the 2 surfaces, as well as the conditions at the initial time.

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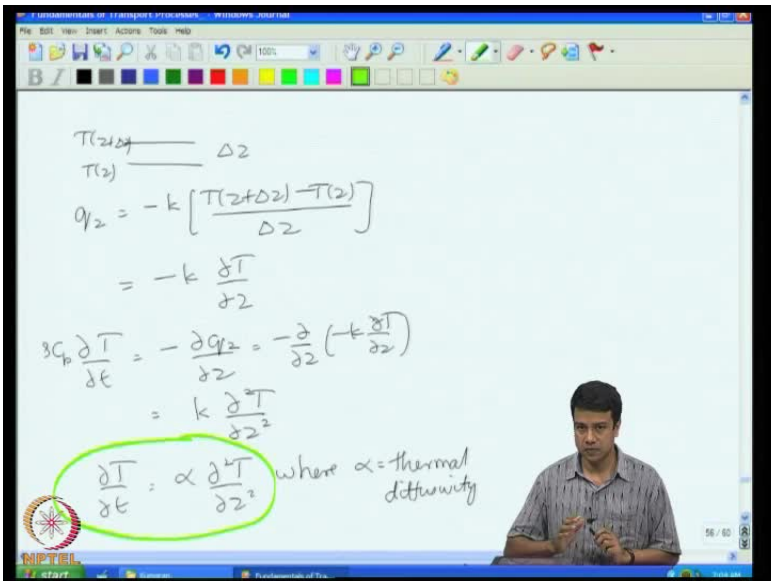

So these, all three equations which I got here, this equation for example, is the first order differential. In time differential equation, in time second order differential equation, in z you need two boundary conditions on z. For example, in the problem that we took, the two boundary conditions are at the two plates, z is equal to 0 and z is equal to L. But, it needs two boundary conditions in z and since it is first order in time you also need an initial condition.

We will first consider the simplest case, where it is at steady state - so there is no variation in time. Then d square t by d z square is equal to 0. Temperature is just a linear function of z, easy to solve. I introduce the concept of scaling. just to so that you can be aware of it, at this point. Next class, we will look at other unsteady problems.

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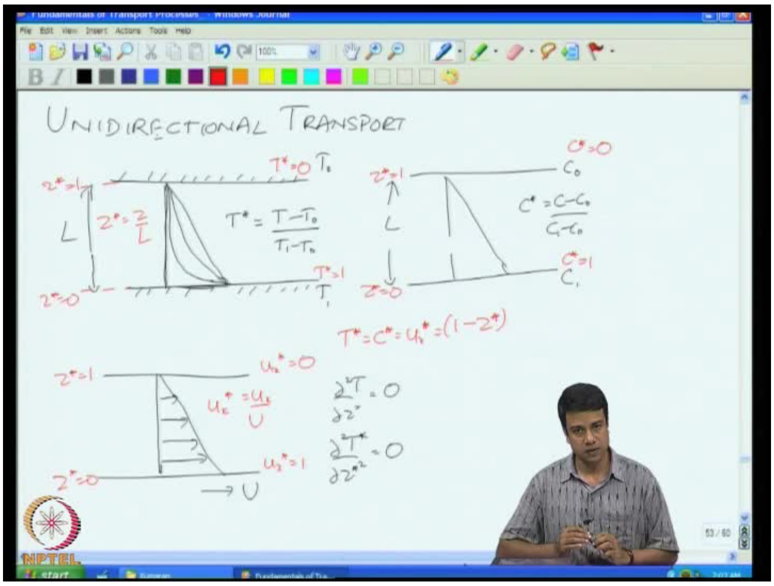

It looks specifically at this case, we start-off from the whole, and fluid is at constant temperature. Instantaneously, you heat up the bottom to a higher temperature, how does the temperature field evolve within the fluid? Equivalent concentration field at time t is equal to 0. You introduce some solid on the bottom plate and that diffuses across, how? Does it vary at time? Momentum conservation equation, you instantaneously impart the velocity in the x direction to the bottom plate, how does the fluid velocity vary as a function of time?

so, initially it will look something like this, later times look something like this and finally it will converge to a linear profile. Those time dependent problems we will consider in the next class. We have spent some time on this unsteady diffusion problems and we will also look at oscillatory diffusion problems. After that, we will go on looking at more problems that are complicated. In other types of geometry, cylindrical, spherical geometries and so on.

So, I have derived for you, the unsteady conservation equations here. Then out be of the same form for mass momentum and energy. I showed how to solve it at steady state, turned out to be an ordinary differential equation because you should not have any variations in time.

We will take up next class with more difficult task of solving the partial differential equations where there is a variation in time in. We will look at different ways doing that.

See you next time.