**Fundamentals of Transport Processes**
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**Module No. # 01**
**Lecture No. # 04**
**Physical Interpretation of Dimensional Groups**

Welcome to this the fourth lecture in our series on fundamentals of transport processes. In the first lecture, I tried to give a motivation about why we need to study transport processes because transport is essential for all the physical or the chemical transformations that take place and in the next 2 lectures, we looked at dimensional analysis, what that can do for us, in terms of defining how to get a flux based upon the concentration difference, how to get a heat flux based upon the temperature difference. I also told you about momentum fluxes and differences in velocities.

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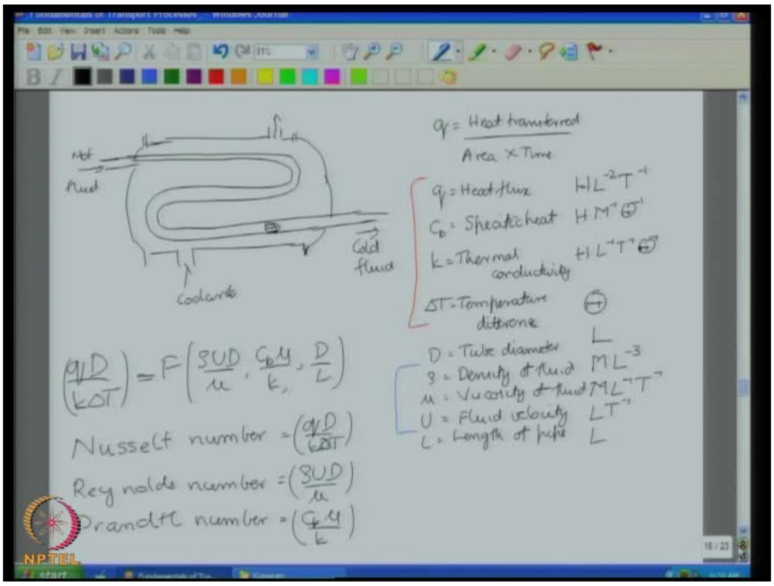


Last class, we specifically did calculation for specific processes. For example, first we looked at the flow in a heat exchanger, where it is of interest to calculate what is the transfer rate of heat from the inside to the outside as the function of the temperature difference, the speed with which the fluid is moving as well as the fluid thermal and mechanical properties and we got a relationship between 4 dimensionless groups. One was a non-dimensional flux for heat, then there was Reynolds number which was the ratio of inertia and viscosity and other dimensionless number involving the specific heat, the conductivity and the viscosity and an aspect ratio which was basically the ratio of length to diameter.

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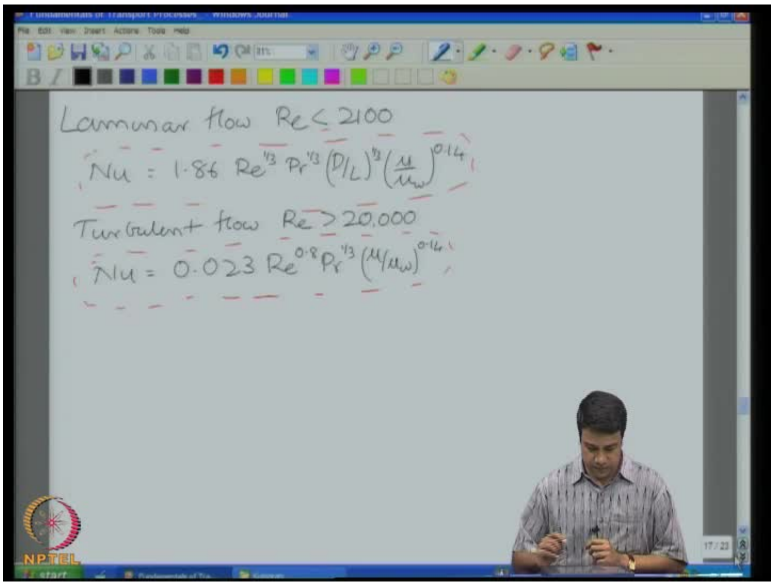


I said you can reduce to this form, but not further. If you want to go further, you have to do experimentation. Change the Reynolds number, change the Prandtl number and see how the Nusselt number changes or the average heat flux changes and we get different correlation depending upon whether the flow is laminar or the flow is turbulent under different conditions and you can actually have diagrams, which actually plot the left hand side verses right hand side.

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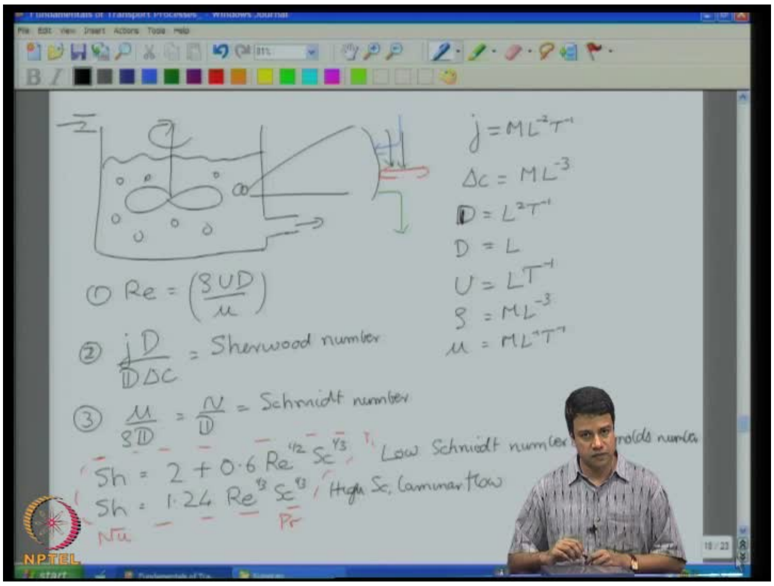


Then we looked at the heat transfer and mass transfer from the surface of a catalyst particle and there, I told you the fundamental numbers, the non-dimensional mass flux Sherwood number, the ratio of a mechanical quantity viscosity and a mass transport quantity - the diffusion coefficient and the Reynolds number, basically because the fluid flow is what is transporting mass to the surface.

We looked at two different correlations for laminar flow and for high and low Schmidt numbers and I briefly mentioned that for the mass transfer problem, nothing changes in these correlations. The numbers are all exactly the same, except you substitute the Nusselt number for the Sherwood number and the Prandtl number for the Schmidt number and those same correlations will work for mass heat transfer as well.

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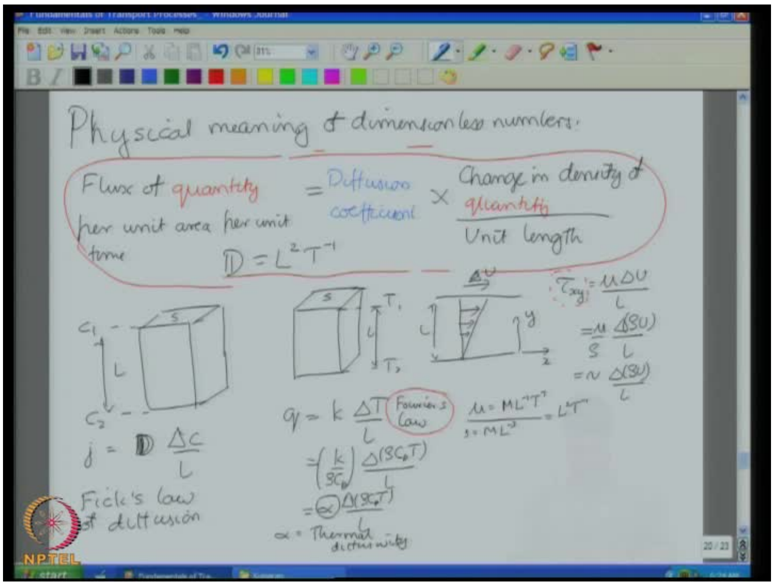


So, there is the fundamental analogy between mass, heat and momentum transfer and we looked at the physical interpretation of how that equivalence comes out. From the physical interpretation of diffusion, this is the general diffusion equation. This equation is a general diffusion equation for any quantity; it can be mass, it can be heat, it can be momentum.

Flux of quantity: This is important. So, we will go through it once again. I have a slab of fluid of some length L and cross sectional area S; S is the area and I apply a concentration difference across the slab of fluid of length L. The flux, the amount that goes through for unit time is going to be directly proportional to the surface area S because if you make the slab wider, there is going to be more mass going through.

The amount is going to be directly proportional to the difference in the concentrations because if I make the 2 concentrations difference higher, I am going to have more material going through, the amount of material is also going to be inversely proportional to the length L because as I make it larger and larger, there is going to be less amount that is going through. Rather than write an equation for the amount of material going through I can write an equation for the flux - amount per unit surface area per unit time.

So, since the amount of material going through is proportional to surface area, the flux which is the amount per unit area per unit time is independent of the area. It only, is proportional to the difference in concentration; it will increase, if you make the concentration difference larger, it is inversely proportional to the total length. As you make this slab thicker and thicker, the amount of material going through is going to become less and less; this is Fick's law of diffusion and this coefficient here, the diffusion coefficient, D is the diffusion coefficient. Heat conduction: I take a slab of material. This heat conduction argument actually works for both fluids as well as solids. Solids also follow the same conduction equation as fluids do.

So, heat conduction equation - the heat flux across the material: I have a material with area S and I apply a temperature difference across the material, the thickness of that material is L. So, the heat flux is the heat going. The heat going through is proportional to the surface area of the material, proportional to the difference in temperature, inversely proportional to the length of the slag of that material.

So, the heat flux which is the heat per unit area is going to be independent of area because you have already taken the heat transfer per unit area per unit time. It is proportional to the difference in temperature. As you make the temperature difference larger, more heat is going from the region of high temperature to region of low temperature and it is inversely proportional to the length of material. So, it is k delta T by L and this is Fourier's law for heat conduction.

Now, in my diffusion equation, I said that the flux of the quantity per unit area per unit time is equal to a diffusion coefficient times the change in density of that same quantity. Density of that quantity is quantity per unit volume, the change in that density of that quantity per unit length. So, in this case the density is the energy density. So, I have got k times delta T by L, but I have to express it in terms of the change of the density of that quantity which is energy itself.

Energy has to be written as rho C p times T. So, the change in rho C p times T divided by L and obviously, if I write it in this form the coefficient that appears in front is k divided by rho C p. So, I get this thermal diffusion coefficient alpha times the change in that quantity per unit length. So, alpha is the thermal diffusivity and this diffusion coefficient as well contains the dimensions of length square per unit time.

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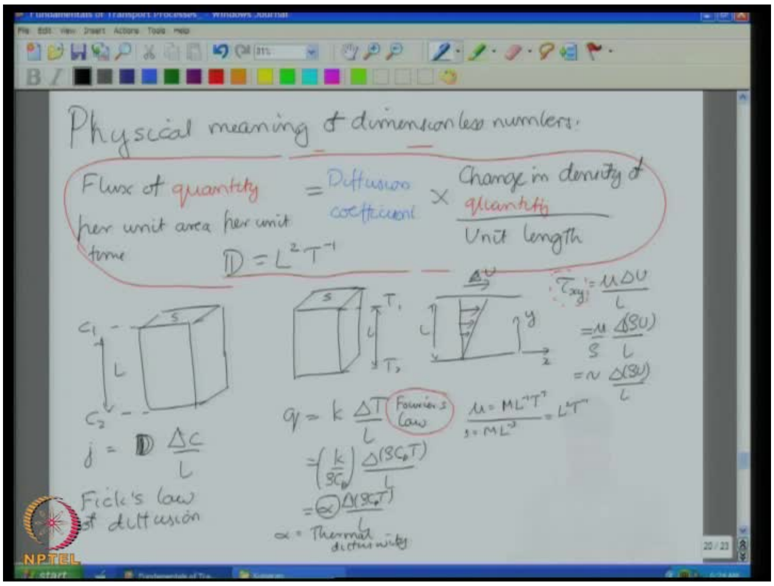


For momentum, the sheer stress acting at a surface. I take a slab of fluid of length L and apply a velocity difference across that. Within the slab itself, I have a linear variation of velocity with position. The sheer stress, which is the force per unit area and force is rate of change of momentum. So, it is the change in momentum per unit area per unit time. It is a flux of momentum and is equal to some coefficient times the change in density of that same quantity; that quantity is momentum. So, I have to have a change in density of momentum per unit length.

Momentum is mass times the velocity. That means that the momentum density is the mass density times velocity. So, change in momentum density rho times U divided by L and the coefficient that appears in front is mu by rho, which is the kinematic viscosity. Therefore, these are the 3 diffusivities for mass, momentum and energy. For mass, the diffusion coefficient is usual diffusion coefficient in Fick's law; for energy, the diffusion coefficient is k by rho C p; alpha the thermal diffusivity, where k is the conductivity and rho and C p are the specific heat and the density respectively.

For momentum transfer, the diffusivity is the viscosity divided by the density, which is referred to as the kinematic viscosity and you can easily verify that both of these have length square T inverse. For example, the viscosity is mass length inverse T inverse; the density is mass L power minus 3. So, if we take the ratio of these two, we will get L square T inverse as the ratio of the viscosity and the density and therefore, all diffusion coefficients will have dimensions of length square T inverse. This k was a physical interpretation for what all those dimensionless numbers mean.

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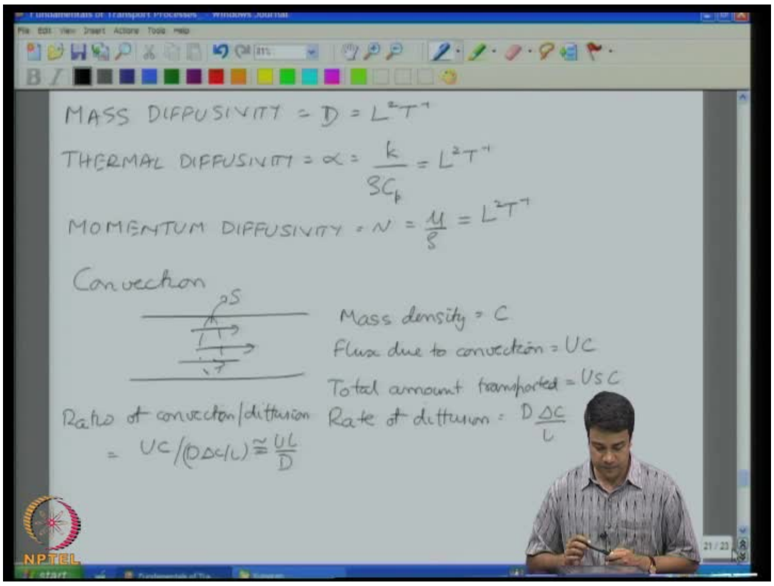


The mass diffusivity d as dimensions of length square T inverse, thermal diffusivity alpha k by rho C p has dimensions of del square T inverse and momentum diffusivity nu has dimensions of del square times T inverse.

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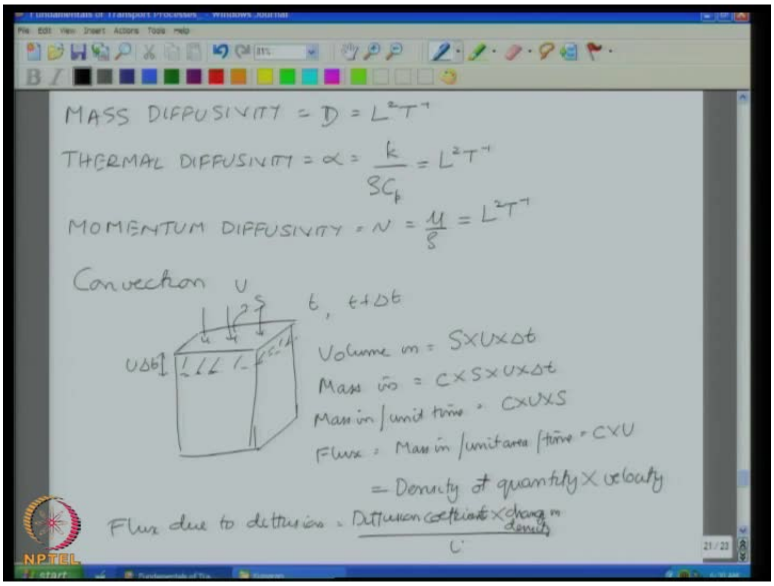


Now, let us look at the rate of convective transport. Convective transport is the transport due to the convection of material. So, if I have some slab of material of surface area S and it has some velocity field here, it is moving with some velocity U, then material is transported because there is a velocity for this material. Because material has the velocity, it is going to get transported even though there is no difference in concentration across two different positions. Even when there is no concentration difference, there is still going to be material transported because of mean convection. So, for example, in the reactor problem there was convection, which was bringing in mass at the inlet and convection taking the product out of the outlet.

In the case of the heat transfer problem, heat was coming in at the inlet due to convection. So, convections transport, even when there is no change in the concentration or the temperature or the momentum. What is the amount of material that is transported? Let us look at two times; at time t and time t plus delta t. So, at time t, there is certain amount of material that has not yet entered, at time t plus delta t, some of the material that is here has come down into this differential volume. The distance it has moved is equal to U times delta t within a time delta t.

So, that means that the amount of material that has come in, is equal to this volume that has come in times the concentration of that volume. The volume that has come in is equal to the surface area times this distance U delta t, volume that has come in is surface area times U times delta t. So, the mass that has come in is going to be equal to the volume times the concentration, which is equal to C into S into U into delta t. So, this is the volume that has come in - S into U into delta t and the mass in - C into S into U into delta t.

Therefore, the total mass in per unit time is equal to C into U into S divided by time and the flux is equal to mass in per unit area per time and is equal to this mass in per time divided by surface area S, which is just going to be equal to C into U.

So, the flux is equal to concentration times the velocity - flux of mass. Flux of energy is equal to the energy density - the energy per unit volume into velocity. Flux of momentum is the momentum density - that is, the momentum per unit volume into velocity. Therefore, the conductive transport, the flux will be equal to just the concentration of equal to density of quantity into velocity.

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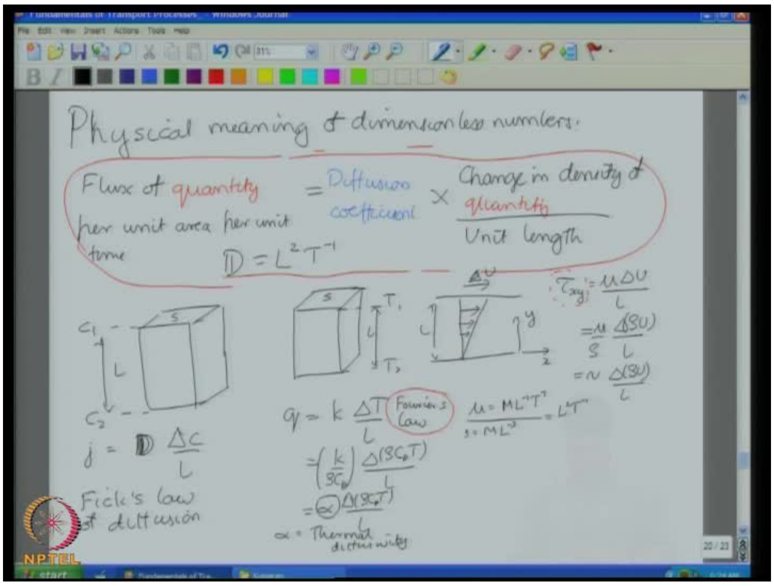


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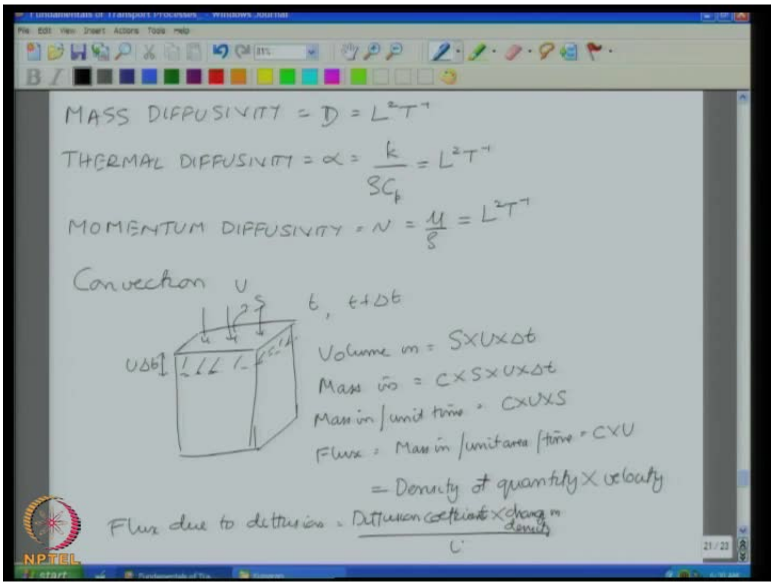


Now, the flux due to diffusion is equal to diffusion coefficient into the change in density divided by length L. The flux of quantity is equal to diffusion coefficient times change in density of quantity divided by unit length L; that is the flux due to diffusion. The flux due to convection is just equal to the concentration times the velocity.

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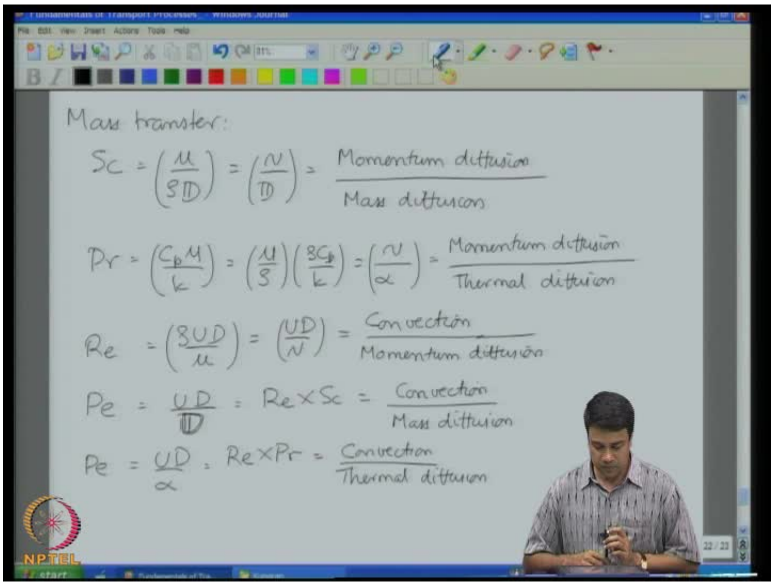


Therefore, the ratio of those two will give you, the ratio of those two will give you, the ratio of flux due to diffusion and flux due convection. So, in this case, the Reynolds number is the ratio of the flux due to convection U times length divided by convective viscosity.

The Schmidt number that we had seen earlier was the ratio of momentum diffusion and mass diffusion. Momentum diffusion coefficient, kinematic viscosity has dimensions of L square time inverse; mass diffusion coefficient has dimensions of L square time inverse. The Prandtl number, we saw that that is equal to the ratio of momentum diffusion and thermal diffusion, both of which have dimensions of L square T inverse. The Reynolds number was ratio of convection and diffusion. The kinematic viscosity has dimensions of L square T inverse; velocity is length per unit time.

So, if you take U into D by kinematic viscosity, it will be a dimensionless number - ratio of convection by momentum diffusion. One can also write the ratio of convection by mass diffusion and convection by thermal diffusion. So, these dimensionless numbers - the Reynolds number, the Prandtl number and the Schmidt number give you either the two different diffusivities or the ratios of convection and diffusion.

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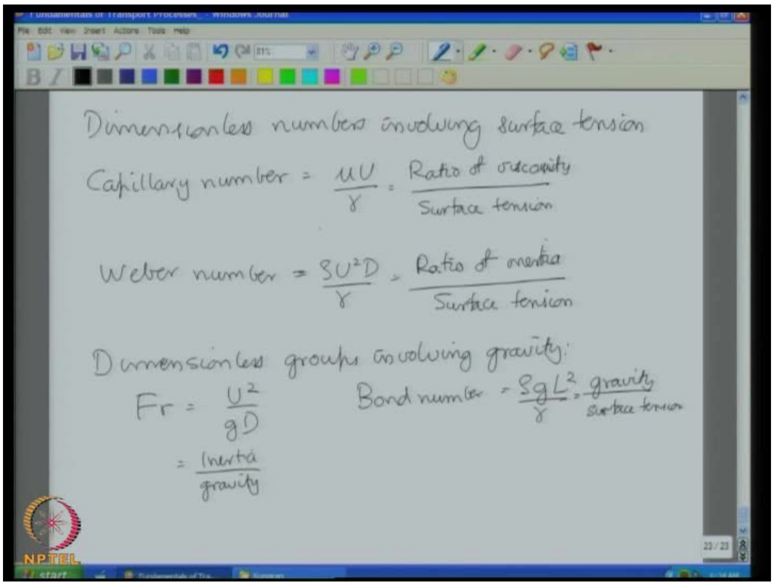


So, this will be a recurring theme in the course the ratio of convection and diffusion or the ratio of 2 different diffusivities. There are other dimensionless numbers that are also important for our course. We have seen already some of those. One could for example, have dimensionless numbers involving surface tension. The dimensionless numbers that involve surface tension are called the capillary number and the Weber number. The capillary number is written as mu U by gamma, where gamma is the surface tension, mu is the viscosity and U is the characteristic velocity. So, this is the ratio of viscosity divided by surface tension.

One can also have ratio of inertia and surface tension; that is what is called as the Weber number. This is written as rho U square D by gamma, which is the ratio of inertia by surface tension. You can have dimensionless groups involving gravity. For example, we already see one which is Froude number, which was U D by g, where D is the characteristic length. I am sorry, this should go as U square by g D.

You can also have something called the Bond number, which is rho g L square by gamma, which is the ratio of gravity by surface tension. Froude number is the ratio of inertia with respect to gravity. So, these are important dimensionless number for example, which involves surface tension gravity and so on.

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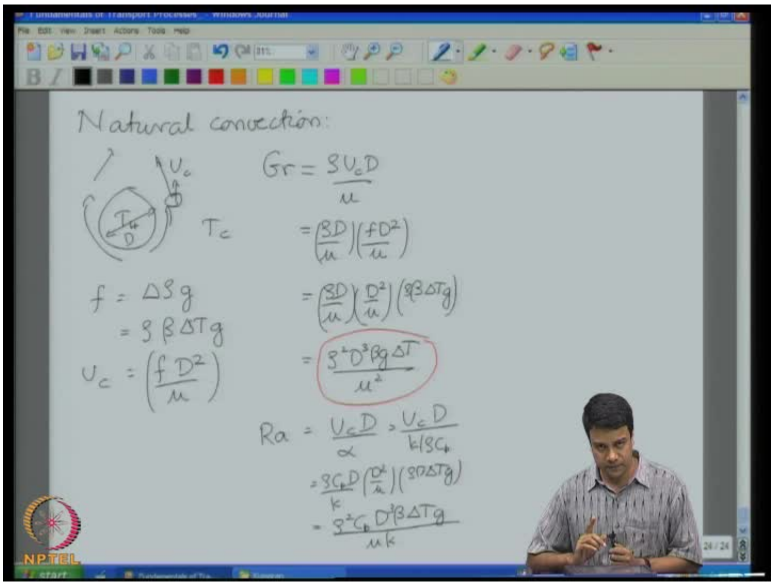


There are other problems, where it is not very clear what the ratio they exactly represent is. One such important case is actually what is called natural convection and I will take a little bit of time to explain what exactly this is because it is important especially in heat transfer problem because as you probably know there are 2 main mechanisms of heat transfer.

One is forced convection and the other is natural convection. Forced convection heat transfer is where the transfer of heat, the velocity field is basically determined by the external action by the pumping, for example, of a fluid through a tube or around an object and that velocity field is what convects the heat away from that object.

Natural convection is the other way. The surface is heated and because the fluid is hot near the surface, the density is lower and because the density is lower it tends to rise because of buoyancy effects. So, the velocity field itself is created by heating of the surface itself and so, in that sense, the natural convection is little more complicated.

So, let us say that you have a buoyancy force. It is the force acting on the surface. So, for example, we had some heated object, you would have fluid rising upwards. If this was the hot temperature and this is cold, you would have fluid rising upwards because near the surface, the fluid is heated and the density of that fluid is lower and because of that it rises upwards.

If you look at any parcel of fluid here, there is an upward force due to buoyancy that acts on this parcel. What is the force due to buoyancy? That force is equal to the change in density - the difference in density between this and the outside and the gravitational acceleration. So, this is the force per unit volume acting on a parcel of fluid because its temperature is higher than the temperature outside.

What is delta rho, the temperature of that parcel of fluid? This is equal to the density times the coefficient of expansion times delta T, where beta is equal to the change in density due to unit change in temperature; beta is the coefficient of thermal expansion. So, the force is equal to rho beta delta T times the gravitational acceleration. This force causes a velocity in the fluid and what is that velocity going to be?

One can consider 2 different cases: one is where the force exactly due to this is balanced by the inertial stresses in the fluid; the second, where it is balanced by viscous stresses fluid. It will not make very much difference from the point of view of dimensional analysis, which one you consider, but let us for the moment consider that this force is balanced by the viscous stresses in the fluid. This object has a diameter D and if this force is balanced by viscous stresses in the fluid, then the characteristic velocity with which the fluid is moving upwards U c, just from the dimensional analysis will go as f times D square by mu, where f refers the force per unit volume. You can very easily verify that this ratio as dimensions of velocity itself. So, that is the characteristic velocity with which the fluid is raising.

With this characteristic velocity, I can now write down a Reynolds number because the Reynolds number is the ratio of inertia and viscosity and if I write down the Reynolds number with that characteristic velocity, it goes by the name of the Grashof number is equal to rho U c D by mu and I substitute for you U c here, rho D by mu into U c was f D square by mu, which is rho D by mu into D square by mu into rho beta delta T times g. So, you assemble all terms together and you will get rho square D cube beta g delta T by mu square.

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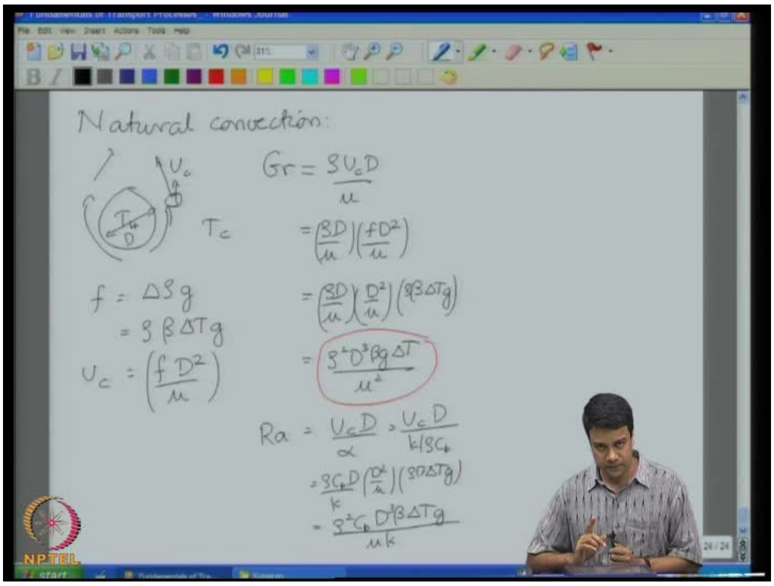


So, this is the Grashof number. It is a complicated number; it looks like it has many different factors in it, but if you break it down to its basics, it is basically the ratio of inertia and diffusivity, where in inertia include the characteristic velocity which is obtained by a balance between the applied force, which is the force due to difference in density and the viscous stresses.

So, this is the Grashof number and one can also get a different number called the Rayleigh number. The Rayleigh number, you are not balancing taking the ratio of inertia and viscosity, but rather you are taking the ratio of convection and thermal diffusion. So, the rally number is basically equal to U c times D by thermal diffusion coefficient alpha and this is equal to U c times D by thermal diffusion coefficient is k by rho C p. So, this is equal to rho C p D by k into U c, which was D square by mu into rho beta delta T times g. So, this works out to rho square C p D cubed beta delta T g by mu times k.

Once again a complicated number, but if you break it down to the ratio of different forces, it is quite easy. It is basically ratio of convection and thermal diffusion, where in convection, I have taken the velocity that is caused due to the difference in density and scaled that by the and use the balance between the force that is caused due to the difference in density and the viscous forces in the fluid,

So, the fundamental numbers for convection are the Grashof number and Rayleigh number. So, if I had a heat transfer, a natural convection heat transfer instead of having the Reynolds number - the Reynolds number is ratio of inertia and viscosity, where the velocity that you take is the applied velocity in forced convection. Natural convection, you have to take the velocity is obtained due to the density difference due to the buoyancy effects. So, the Nusselt number in that case will be a function of the Grashof number and the Prandtl number.

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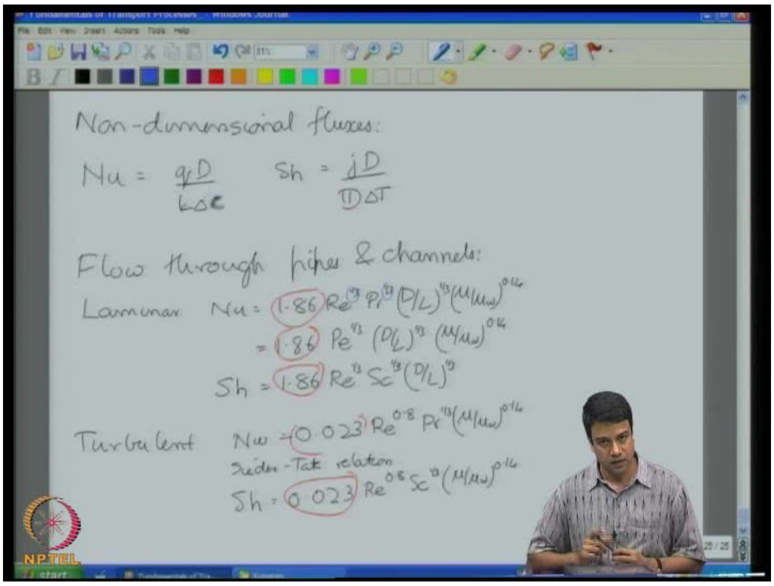


Then we have non-dimensional fluxes. You have already seen those - the Nusselt number and the Sherwood number. So, for heat and mass transfer problems, the Nusselt number and the Sherwood number would be written it in terms of ratio of all other forces the Reynolds number, the Prandtl number or the Schmidt number as the case may be and surface tension forces, if they are important, gravitational forces, if they are important. We will know surface tension is important, if the Weber number or the capillary number is small, then surface tension is important and similarly, gravity will be important, if the Froude number is small.

So, that is the reason that in the dimensional analysis that we did earlier, we always got these numbers coming out because these numbers are dimensionless fluxes and they are written in terms of the ratios of convection and diffusion of different kinds. So, for example, we can broadly classify the correlations as flow through pipes and channels.

We already saw those for the flow through the pipe in the heat exchanger problem. The Nusselt number for laminar - the Nusselt number is equal to 1 point 86 R e power one third P r power one third D by L power one third mu by mu w power 0 point 14. Two things here, first thing is I can write this whole thing 1 point 86, the Reynolds number times the Prandtl number is the Peclet number, P e power one third, power 0 point 14.

So, the Nusselt number depending only upon the ratio of convection and diffusion of mass, the ratio of convection and diffusion of momentum is not appearing in this correlation, apart from this correction factor that is there. If instead of a heat exchanger problem, I had the equivalent mass transfer problem that is the solid flowing through the tube and it is diffusing across the surface through a porous surface to the fluid outside, I would get exactly the same correlation for the laminar flow, that is, the Sherwood number is equal to 1 point 86 R e power one-third Schmidt number power one-third D by L power one-third.

In this case, we have seen the diffusion coefficient does not change across the cross section of the tube and the reason is because these transport problems are determined by the two effects; one is diffusion and the other is convection. In one case, it is the diffusion of mass, the other case is diffusion of heat, but both of those follow the same relations for the flux as the function of difference in concentration or the difference in energy density.

For a turbulent flow, the Nusselt number is equal to 0 point naught 2 3 into R e power 0 point 8 P r power one third. In this case, I cannot reduce it to the Peclet number alone because the powers on R e and P r are not the same. So, there is a dependence both on mass diffusion and momentum diffusion in this case. So, this is the sieder-tate and if I were doing the equivalent mass transfer problem, I get exactly the same. So, that was the flows through pipes and channels.

So, in this case, these correlations are for flow through cylindrical pipes. For flow through other kinds of geometries, these coefficients here will change, but the general relationships will not change for other kinds of geometries. These powers will remain the same; the powers that I have here will end up remaining the same. We will look in this course why powers come about.

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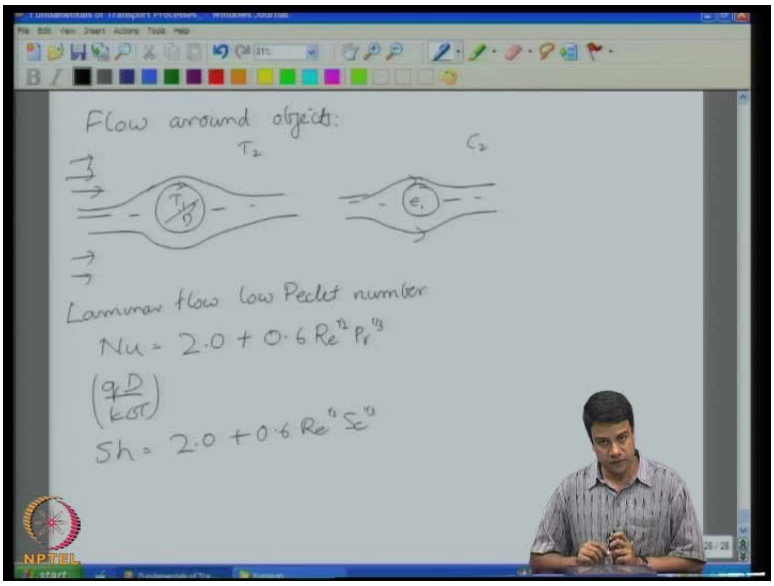


The other broad class of problems is flow around objects and in this case, this is the object and this has let us say, temperature T 1 and T 2 far away and there is a flow around this object and you want to know what is the average flux either going into this object or coming out of this object as a function of temperature difference and the velocity, we usually use the velocity far away from the object because obviously velocity always changes as it comes near flows around the object.

The equivalent mass transfer problem C 1 and C 2 and there is a flow around this object and in this case, the non-dimensional flux is based upon the diameter of this object and for example, for laminar flow and low Peclet number, I will have a correlation of the form Nusselt number is equal to 2 plus 0 point 6 R e power half P r power one third. So, in the limit of low Peclet number, that is, the Peclet number is the Reynolds number times Prandtl number. If both of them are low, that means, the diffusion is dominant. Peclet number - ratio of convection and diffusion and so if it is small that means, the diffusion is dominant.

This Nusselt number is going to a constant and that is because the diffusion is large there is no convective effect and therefore, if I remove the velocity from my dimensional analysis, I can get only one dimensionless group that is q D by k delta T. If velocity is no longer affected, this only the dimensionless group that is left. Therefore, it has go to a constant value. Similar for mass transfer, the equivalent is 2 point 0 plus point 6 R e power half Schmidt number power one third.

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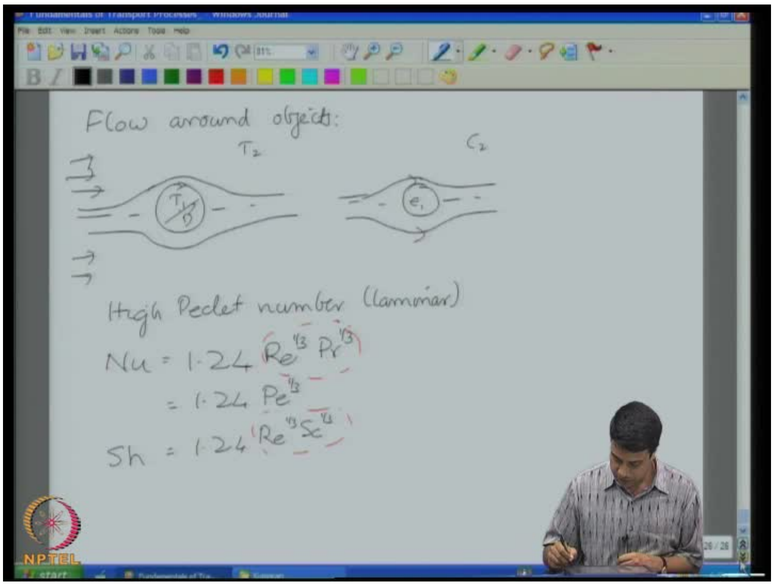


So these are the correlations for low Peclet number, for laminar flow. In the limit of high Peclet number, convection is becoming important. In the limit of high Peclet number, convection is dominant and because of that I will have different forms of correlations. The Nusselt number is equal to 1 point 24 R e power one third P r power one third.

You can see that this is equal to 1 point 24 times Peclet number power one third; so, this is still in laminar flow. So, Nusselt number goes as the Peclet number to the limit of to the power of one third. Peclet number is ratio of convection and diffusions. So, effect of convection is large, but still the Nusselt number is depending upon the diffusivity depending upon the Peclet number and equivalent for mass transfer Sherwood number is equal to 1 point 24 times R e power one third Schmidt number.

So, these are the kinds of correlation that are available and we will see during this course why these kinds of powers come out, why do we get these kinds of power for mass and heat transfer. As I already told you, the diffusion mechanisms are the same. So, the correlations end up being the same.

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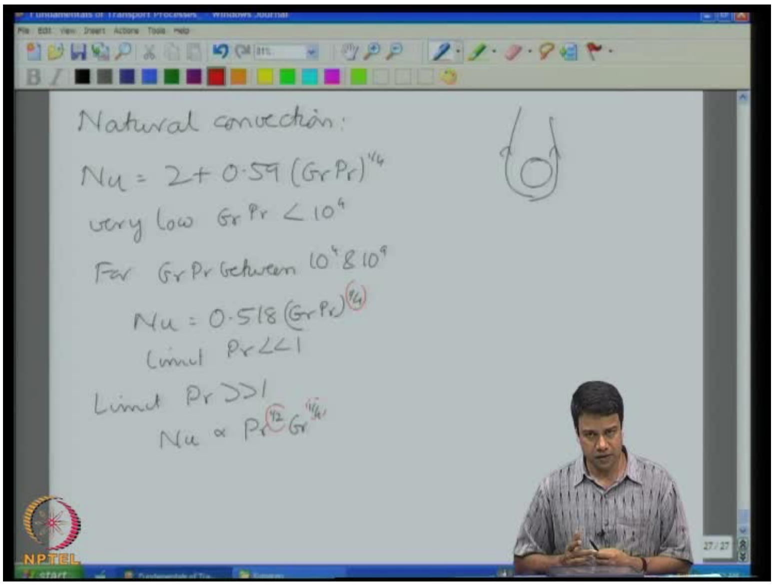


The other kinds of correlations are for natural convection and in this case, the natural convection depends upon the kind of configuration you have. If you have sphere heated in a fluid, you will get one kind of correlation; if you have a cylinder the long cylinder heated in a fluid, you will get another type of correlation; if you just had a heated flat plate, you will get another type of correlation, but they all follow some basic relationships. So, for example, if have a sphere in a large body of fluid around which there is natural convection going on, I will get Nusselt number is equal to 2 plus 0 point 59 Grashof number times Prandtl number power one-fourth.

This is for very low G r times P r - less than about 10 power 4. For Grashof times P r between 10 power 4 and 10 power 9, I will get relationships of the form Nusselt number is equal to 0 point 518 G r times P r power one-fourth. This is for the limit of P r small compared to 1. In the limit of P r large compared to 1, I will get the Nusselt number is proportional to P r power half times G r power one-fourth and once again, you can see all these power coming - half, one fourth, one fourth here and so on. We will try to derive why these relations are of this form in the remainder of this course.

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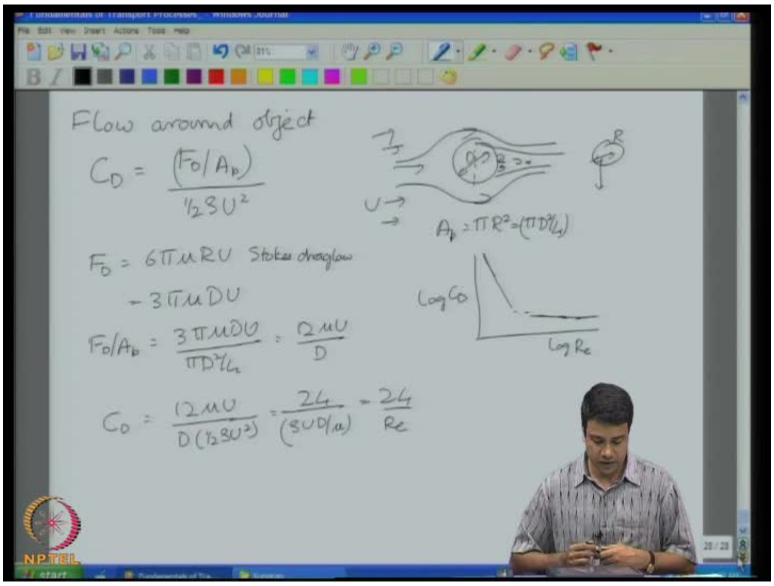

So far, I have talked about non-dimensional fluxes for mass and heat. What about the non-dimensional fluxes for momentum? Historically, the non-dimensional fluxes for momentum have been defined differently. For the flow around on object, the non-dimensional flux is defined as the drag coefficient. Drag coefficient is the drag force by the area of projection. So, if I have a sphere and there is a velocity that is going around this sphere with free steam velocity u, sphere diameter D, the projected area is the area perpendicular to the velocity vector.

So, in this case, the projected area is equal to pi times R square, where R is the radius of the sphere; so, it is pi D square by 4. So, this is the measure of the stress - the force divided by the projected area is the measure of the stress and this is divided by half rho U square. So, that is how the drag coefficient is defined. If you recall, we solved the problem of the flow past a sphere settling of fluid with diameter R. The drag force was equal to 6 pi mu R U. In that problem I set a low Reynolds number, the Stokes drag law is equal to 3 pi mu times the diameter times the velocity.

So, therefore, F D by A b will be equal to 3 pi mu D U by pi D square by 4, which will be equal to 12 mu U by D. That is the stress force by unit area. So, C d is equal to 12 mu U by D into half rho U square, which is equal to 24 by rho U D by mu, which is equal to 24 by R e. So, you must have studied the correlation for the drag coefficient, for the force on a sphere settling in a fluid.

In the limit of low Reynolds number, where the forces are primarily viscous, the drag coefficient is given by 24 by R e. So, if I plot log C D verses log R e, I get something that goes like this, but at some points as the Reynolds number increases, there is the transition from the flow around the sphere to a more complicated velocity profile, where I have the separation of the boundary layer and the formation of wakes at the back and once that happens, this drag coefficient goes to some other constant value; it goes towards 2 and we will come back and see why that is a case.

You can see that if the force is due to the inertial force of the fluid, then the force acting is just going to be equal to half rho square acting on the surface. So, the drag coefficient has to go to a constant value. So, this relationship, this C D, this drag coefficient is a scaled momentum flux, F D by A p is an average momentum - average force per unit area acting on the surface perpendicular to the direction of the flow. So, F D by A p is an average stress or a pressure acting on the surface. I said stress is nothing, but flux of moment; that scaled by half rho U square. Flux of momentum is force per unit area, half rho U square; half mu square is kinetic energy; half rho U square is kinetic energy per unit volume. So, I am scaling the force per unit area with the energy per unit volume and we have the same dimensions because energy is force times length.

So, force per unit area, the stress being scaled by energy per unit volume and that is how you get the drag coefficient. I could very well have scaled this with the viscous scales and in mass and heat transfer, the non-dimensional stresses are defined with respect to viscous scales with respect to the diffusion coefficients.

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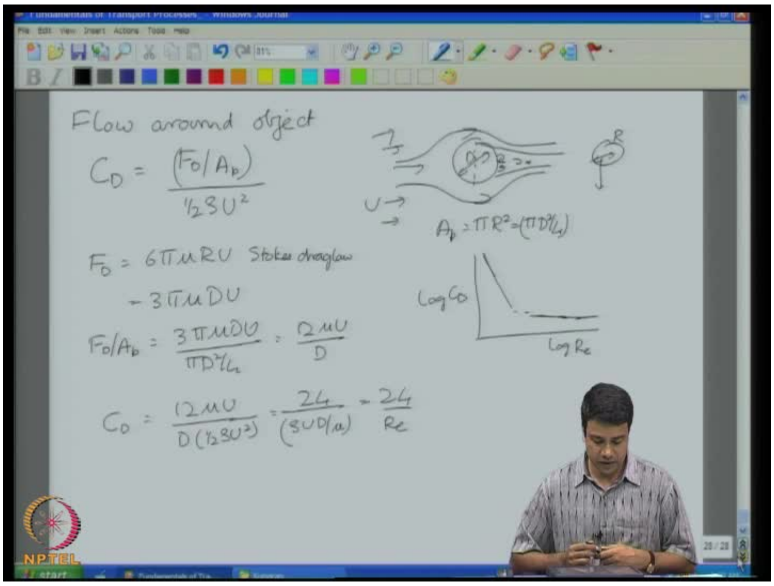


So, in this case, mass flux, non-dimensional mass flux is defined with respect to a diffusion coefficient; heat flux, the non dimension flux is defined with respect to the diffusivity of heat and that is traditional. Diffusion is a mechanism, obvious mechanism of transport; it is traditional to do that whereas, in the case of momentum flux alone, historically, the flux has always been scaled by convective scale.

I could have scaled it with respect to the viscosity. I could have used viscosity for non dimensionalising it, in which case I would have got a diffusion scale, I would have scaled the stress by diffusion scale. Traditionally, it is done by a convection scale and that is the reason that C D is effectively a non-dimensional momentum flux and goes as 1 over a Reynolds number in the limit of very low Reynolds number.

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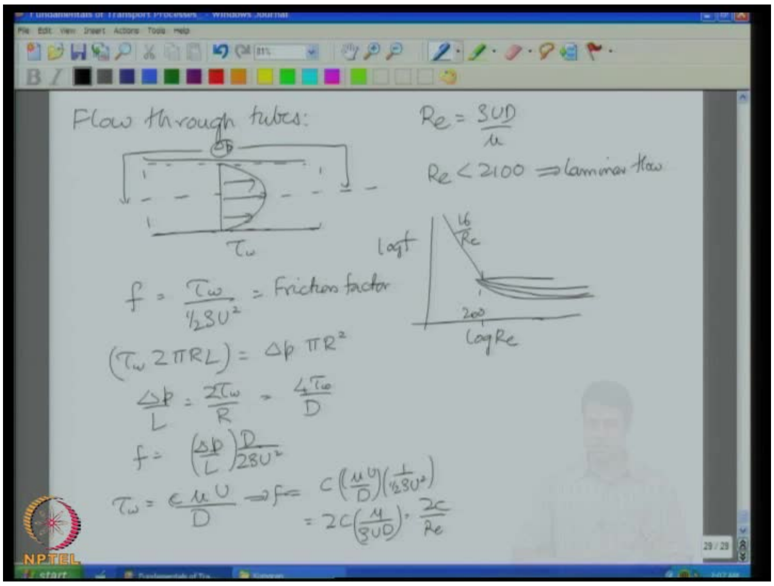


When it goes 1 over Reynolds number, the force is due to viscosity alone and then at some point there is a transition. So, this was the flow around an object; the non-dimensional momentum flux for the flow around an object is the drag coefficient with the difference that the drag coefficient is traditionally scaled by the inertial scales, the kinetic energy per unit volume and not the diffusion scales which would have been the kinematic viscosity times of velocity. How about for the flow through tubes? For the flow through tubes, the non-dimensional momentum flux is defined with respect to the sheer stress acting on the wall of the tube.

So, the wall stress tau w is the sheer stress acting on the wall of the tube. It is the force per unit area acting on the wall of the tube and the non-dimension flux is defined as the friction factor, which is equal to tau w by half rho U square; tau w is the viscous stress, the force per unit area acting on the wall of the tube, force per unit area, half rho U square is the kinetic energy density, the energy per unit volume and both have the same dimensions. So, this f is called as frictional factor.

You can also write it in terms of the pressure gradient because whatever the force is exerted on the walls of the tube due to the fluid flow, the wall exerts a force backward as a fluid is flowing forward, the walls are exerting the backward force and that has to be balanced by difference in pressure between the 2 ends. So, you apply a pressure gradient across the 2 ends of the tube to balance the force that is exerted due to the stress backwards.

Force exerted due to the stress is equal to stress times the cylindrical area. This is equal to tau w into 2 pi R times L; this has got to be equal to the pressure times the cross sectional area; that is, equal to the difference in pressure times pi R square. Therefore, delta p by L is equal to tau w by R.

From that, I will get the friction factor is equal to delta p by L D by 2 rho U square. So, this is equal to 2 tau w R, which is equal to 4 tau w by D. So, this is the friction factor expressed in terms of pressure gradient. Now, what would you expect to happen? There are 2 limits: one is the limit of very low inertia; other one is the very low limit of viscosity. The ratio of inertia and viscosity is the Reynolds number - R e is equal to rho U D by mu.

When this R e is less than 2100, the flow is in the laminar region; that means that there is the balance between the viscous stresses in the fluid and the applied fluid gradient. Therefore, the friction factor, the sheer stress should be only a function of the viscosity, the velocity and the diameter; that means that tau w is equal to some constant times viscosity times velocity divided by the diameter, which means that friction factor is equal to this constant into mu U by D into 1 by half rho U square, which will be equal to 2 C into mu by rho U D, which is equal to 2 C by Reynolds number.

So, in the limit of very low Reynolds number, where you have only viscous stresses, you do not have inertial stresses, the stress has to depend only upon the viscosity. Therefore, the friction factor has to go as one over the Reynolds number and the friction factor versus Reynolds number plot log f verses log R e and in the limit of very low Reynolds number, this friction factor is the constant divided by R e and this constant is 16. At a Reynolds number of about 2100, there is a transition that takes place to a more turbulent velocity profile and after that the friction factor depends upon various things like the reference of the wall and so on.

So, the friction factor is the non-dimensional momentum flux in the flow through pipes and channels. Just from dimensional analysis, the friction factor in the case of flow through a tube for the heat transfer problem, we had scaled the heat flux divided by the thermal diffusivity. Here, rather than scaling it by the diffusion coefficient, it is traditional to scale it by the kinetic energy density, which is energy per unit volume and that is the reason that in the limit of low Reynolds number, the friction factor goes as 16 over Reynolds number up to a Reynolds number of 2100. After the Reynolds number 2100, there is the transition to a turbulent flow and the friction factor has a much more complicated profile.

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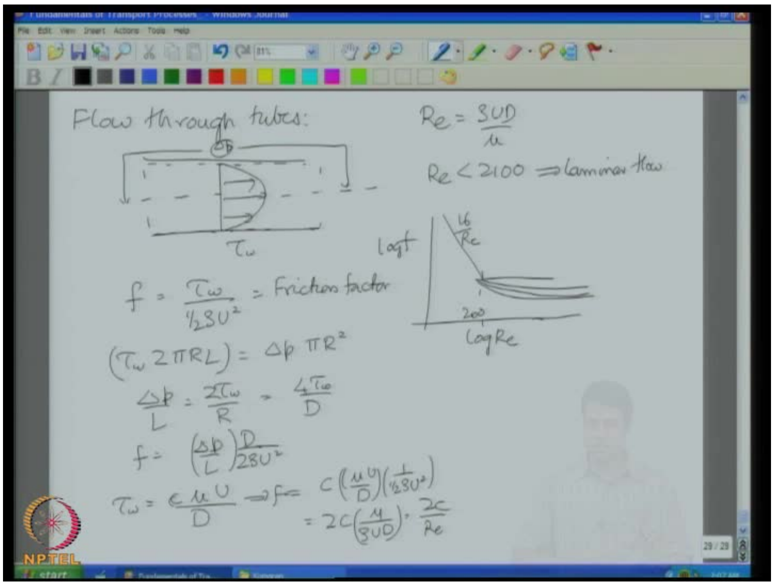


We have completed our dimensional analysis part of the problem, basically, the take away from this section. You can do simple dimensional analysis or you can do more sophisticated dimensional analysis based upon what are the dominant forces in the system. In all cases in heat mass and momentum transfer, you are interested in predicting what is the average transport rate or which depends upon it is an extensive quantity and it depends on the system size or the average flux, which is the transport of energy and transport of that quantity per unit area per unit time and we can define non-dimensional fluxes by using the diffusion coefficient to non-dimensionalise the flux.

That gives me the Nusselt number, the Sherwood number and these are to be functions of other dimensionless group. Those dimensional groups have interpretations. They are mostly either the ratio of convection and diffusion - Reynolds number, Peclet number, the ratio of 2 different diffusivities - for example, the Schmidt number is the ratio of the momentum diffusion and mass diffusion, the Prandtl number is the ratio of momentum diffusion and thermal diffusion.

In the case of natural convection, you have dimensionless group several ratio of once again convection and diffusion; expect that convection is costly to buoyancy forces. In the case of momentum transfer flow through tubes and flow past objects, it is more traditional to define these dimensionless groups in terms of the drag coefficient or the friction coefficient; both of these are momentum transport rates scaled by the kinetic energy per unit volume.

In the case of momentum transfer, these dimensionless transport rates are functions only of the Reynolds number because there is only convection and diffusion. In the case of heat and mass transfer, they can be ratios of both convection and diffusion of heat and mass as well as inertia and viscosity because the velocity profile that basically determines the rate of transport is determined for the fluid by the ratio of inertia and viscosity. So, all of these dimensional groups have this common form.

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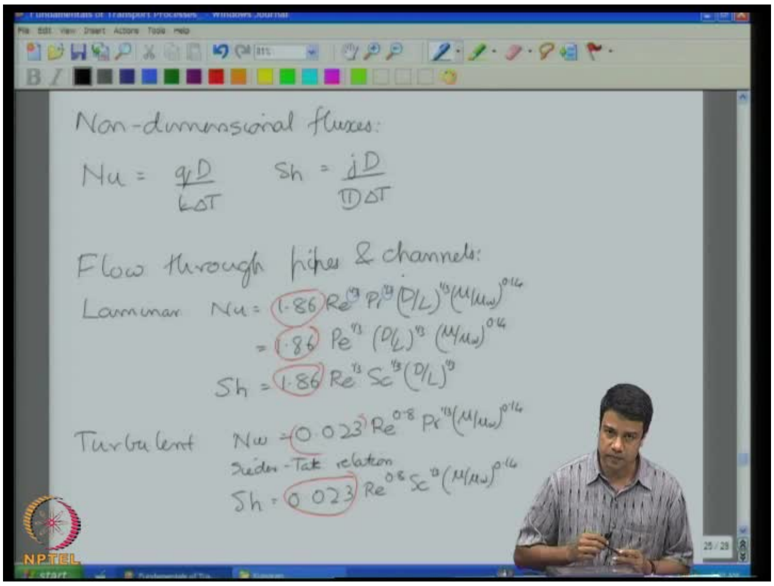

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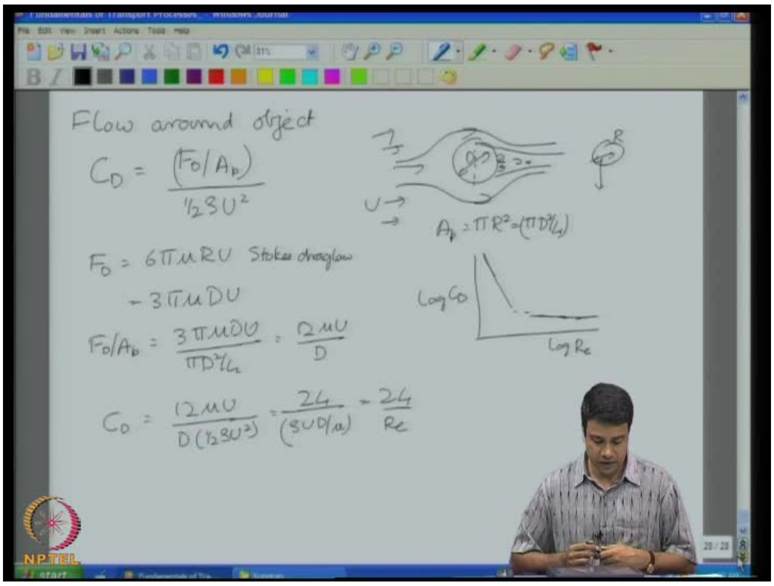


Now, the remaining course: so, I have told you what can be done so for with dimensional analysis. In the remaining courses, we are going to try and see, give some motivation for why these kinds of things come up. So, for example, for the drag force, the flow around this sphere for example, I know that the drag force just from the dimensional analysis has to got to be equal to the mu R U times some constant. I do not know what the constant is.

If I want find out that constant, I have to solve velocity around the object; from the velocity solution around the object I can get the stress on the surface; integrate the stress around the entire surface in order to get the total force.

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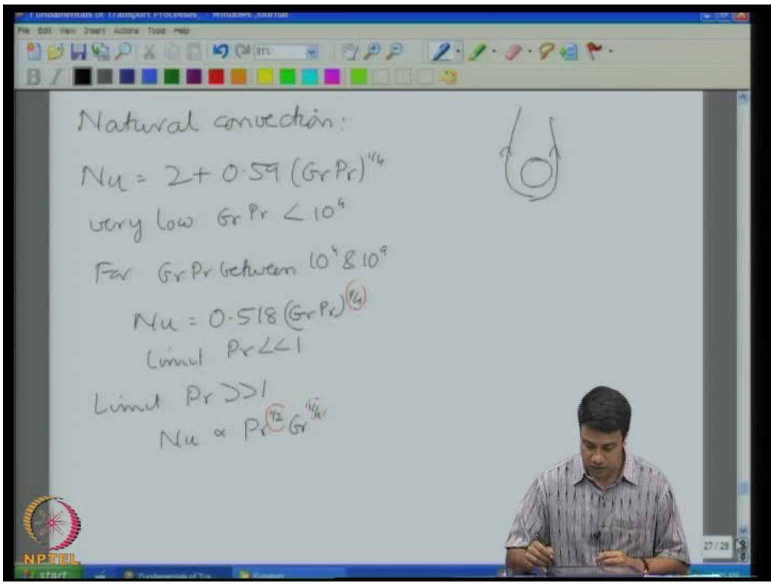


Similarly, for these flux problems, for example, the Nusselt number is equal to 2 plus point 59 times this. I have to solve for the momentum transfer near the surface and from that, I have to find out the total flux.

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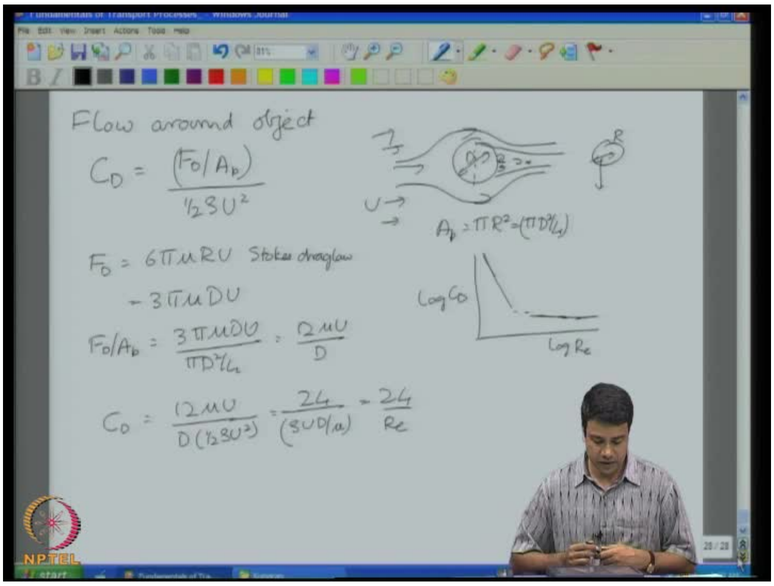


So, the entire temperature field around this object as well as the entire velocity field around this object. For the flow around objects, why do these kinds of relations come about in the limit of high Peclet number for laminar force?

I have to solve. In the case of forced convection, I am given what is the velocity around the object, but I do not what the temperature is. The temperature is transferred by convection along with the velocity. It is also the diffusing across the object. I have to solve for those 2 independently to find out what the temperature field is. From that, the gradient of temperature of the surface times conductivity will give me the flux.

So, the remaining course will focus on these different forces. We actually try to solve for the entire problem around the object; difficult to solve for the entire problem, but can we make approximations in regions, where we expect either convection or diffusion to be dominant. If the Reynolds number is small, momentum diffusion is large compared to momentum convection, viscosity is large compared to inertia; can I neglect inertia and solve for the problem by just balancing the viscous forces?

Similarly, when the Peclet number is small, convection is small compared to diffusion. You can solve for the diffusion problem alone, neglect convection and just solve for diffusion problem. From that find out what is the temperature profile everywhere. Opposite limit: Peclet number is large; convection is large compared to diffusion.

Simplistically, one would expect that one can neglect diffusion and solve for convection alone. It is important to remember however that convection is only flowing material past an object; there is no convective transport to the object perpendicular to the surface. ultimately convective transport perpendicular to the surface has to happen. Ultimately, transport perpendicular to the surface, fluid velocity at the surface is 0; there is no transport perpendicular to the surface. Therefore, transport perpendicular to the surface ultimately, has to happen due to diffusion.

So, even if you have the ratio of convection and diffusion to be large, ultimately, very near the surface, diffusion has to become important, if material is transported to the surface. So, in that limit, we will see, there will be a boundary layer very near to the surface, where convection and diffusion is same, even though in the bulk of flow convection is larger compared to diffusion. How do we analysis these different problems? So, that will be the subject of this course.

Next class, I will tell you the basic framework of what we are going to do in order to analyse these problems and then I will look at a more microscopic description of diffusion. How does diffusion raise from molecular scale? How do I calculate the diffusion coefficients knowing what the molecular properties are? We will look at the class after next.

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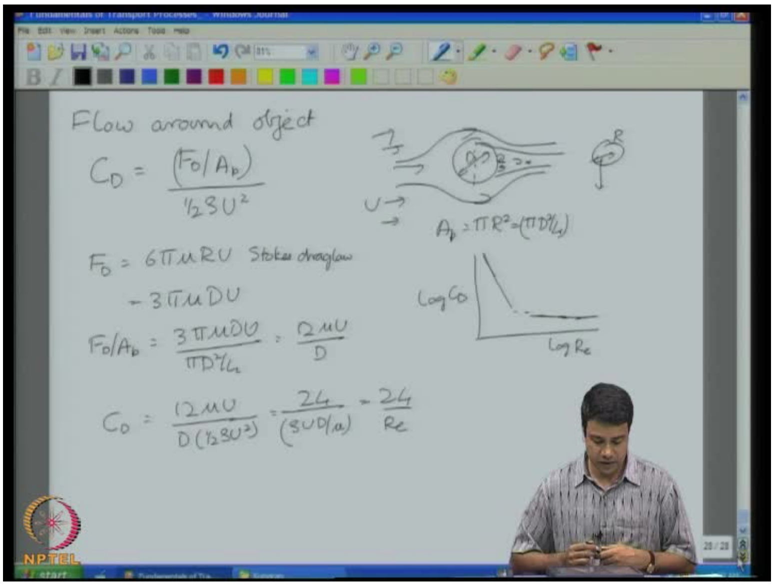


So, next class I am going to tell you briefly, in a brief summary, how we solve this problems. Then we will go on to diffusion. So, with that we have completed the basic introduction to the course and I will tell you something about the kind of description that we will use in the next class. Contrast this with the present description, where primarily we are dealing only with the dimensionless numbers for the entire system, average flux to the entire system, average temperature gradient for the entire system. In next class, we will look at more fundamental quantities.