Fundamentals of Transport Processes
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Lecture No. # 39
High Peclet Number Transport Heat Transfer from a Gas Bubble

So welcome to lecture number thirty nine in our course on fundamentals of transport processes, where we were going through the final topic in our course and that is transport at high peclet number, where we expect convective transport to be dominant in comparison to diffusion. If you recall, we had obtained a convection diffusion equation for the concentration and temperature fields, and it was of the form shown here.

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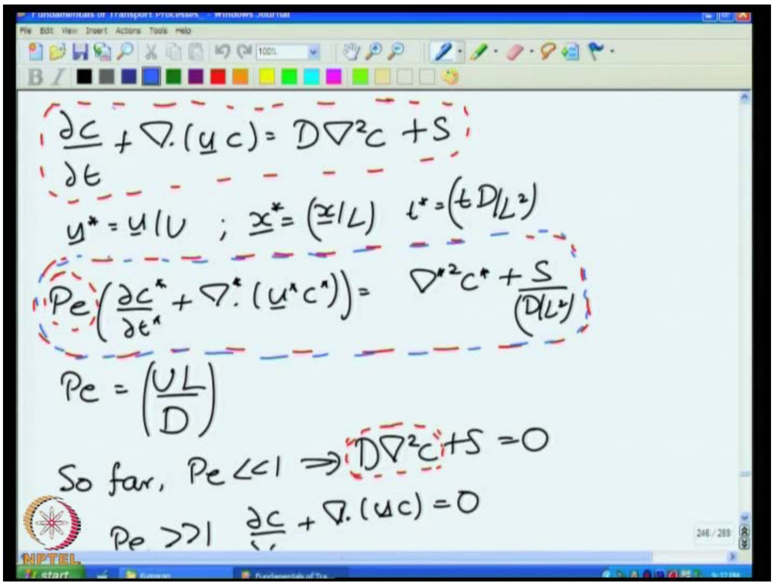

Right on top, it has a time derivative d C by d t plus the divergence of velocity times concentration is equal to d times the Laplacian of concentration plus any sources or sinks. If I scale the velocity by a characteristic velocity capital U and length scale by characteristic length L, then I get a scaled equation, which ends up having a peclet number in it. Ratio of convection and diffusion peclet number is U L by D. D of course, is a diffusion coefficient, which has dimensions of length square per unit time so this is dimensionless. In the limit of small peclet number, the transport is diffusion dominated and we looked at various ways of solving basically, the Laplace equation D del square C is equal to 0 or D del square C plus distributed source is equal to 0.

We looked at various strategies for solving this, including separation of variables as well as the temperature or concentration fields due to point sources and dipoles and so on. And we are on the topic of high peclet number now.

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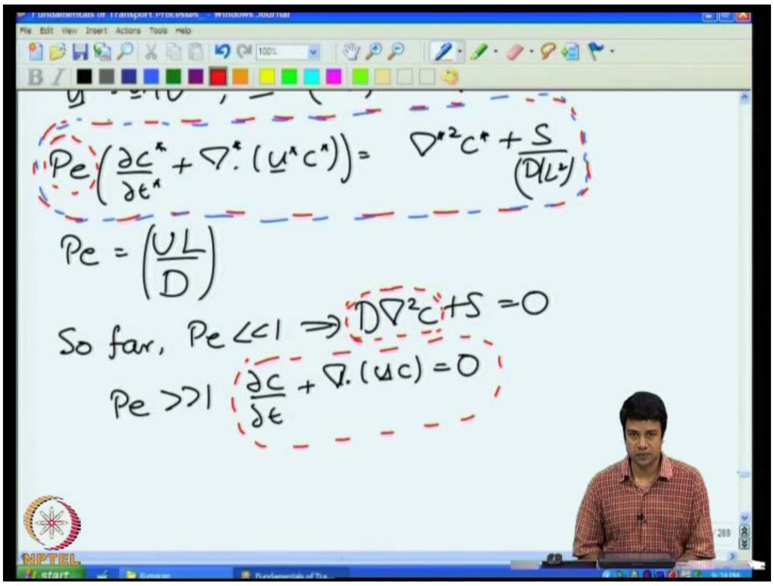

And naively, you might think that in the limit of high peclet number, I could just completely neglect diffusion all together and just solve an equation of the form d C by d t plus divergence of u times C is equal to 0. For incompressible flows, basically what this equation will tell me is that the concentration is unchanged along stream lines.

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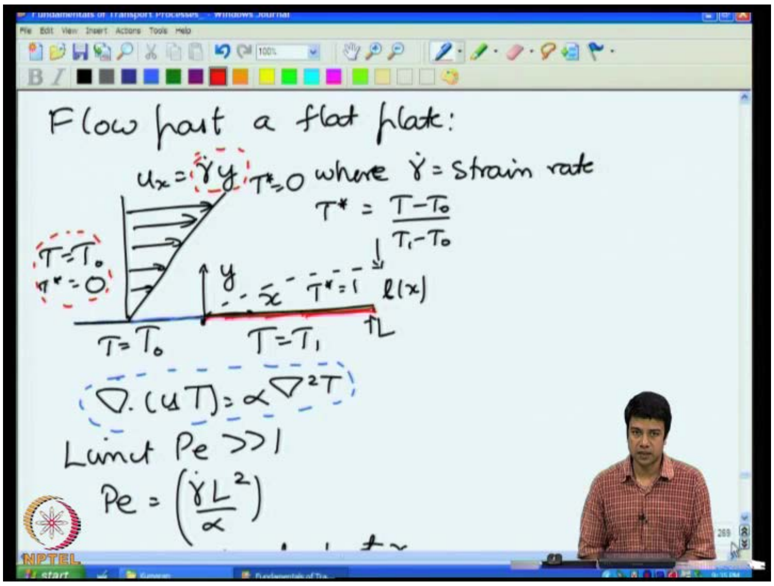

We looked at some special cases first; the flow passed a flat plate at steady state; del dot u T is equal to the thermal diffusibility times del square of the temperature. The flow is assumed to be linear at the surface u x is equal to gamma dot times y. And on this flat surface, there is a heated section of length L from x is equal to 0 to x is equal to L; and due to thermal diffusion, the energy diffuses from the section into the fluid. And our task was to find out the temperature profile in the fluid.

If we just make the assumption that we neglect the diffusion terms altogether, then we just end up with an equation of the form dT divided by dx is equal to 0 that is the temperature is invariant along streamlines and if T is equal to 0 at the inlet, it has to be 0 everywhere. We cannot enforce the boundary conditions at the heated surface in that case, because at the heated surface the temperature has to be equal to 1, the scaled temperature. The problem was that when we neglected the diffusion terms the equation was converted from a second order differential equation to an ordinary equation.

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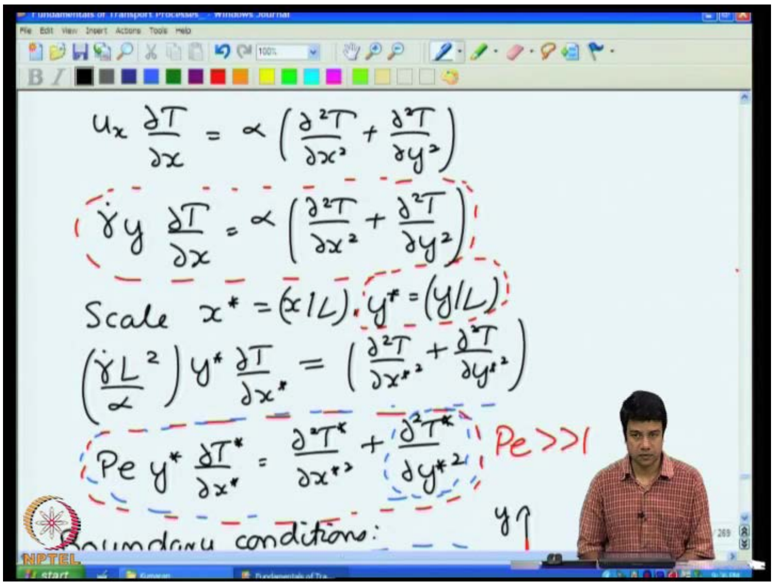

For a second order differential equation in y, there are two boundary conditions. For an ordinary equation, there are none and since we do not have boundary conditions, we do not have constants in the equations with emerge from the integration of the differential equation we were unable to satisfy boundary conditions. Physically, the reason was that when I scaled the equation, I implicitly assumed that the length scale for variation of the temperature field is also capital L, because that is the only length scale in the problem.

Now, if the length scale is capital L, we got the result that convection is large compared to diffusion or gamma dot L square by alpha is large compared to one. That only means that over a length of order capital L, convection is dominant in comparison to diffusion. However there is still going to be diffusion at the surface, because diffusion is due to the fluctuating motion of the molecules. And as convection becomes larger and larger the temperature the energy that is diffusing from the surface gets swept downstream. And therefore, it penetrates only to a small distance within that fluid and if the distance of penetration is small, the gradient is large and one could still have a balance between convection and diffusion.

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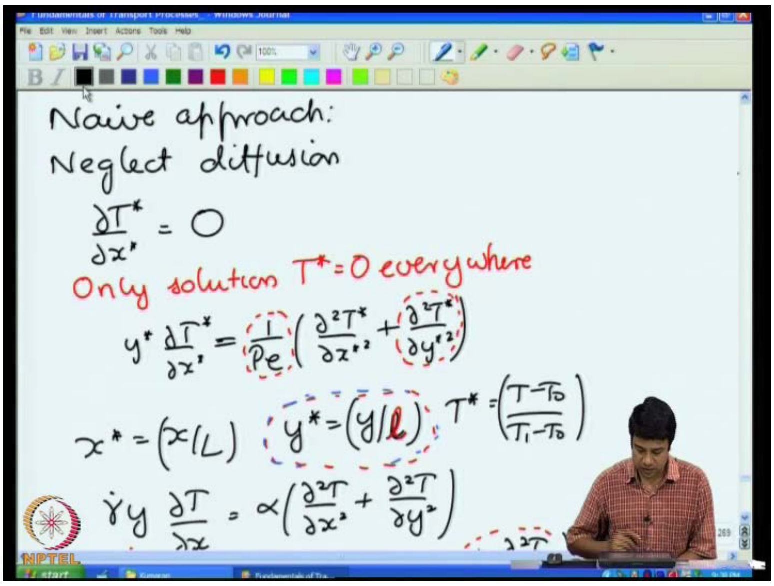

So, the way to solve this problem is to postulate a length scales small L in the cross streamed direction, postulate a length scale small l in the cross streamed direction which is the length scale over which, there is penetration of the energy disturbance due to the heated wall. And we will determine this length scale small L self consistently from the requirement, that convection and diffusion have to be of same magnitude over this length scale small l.

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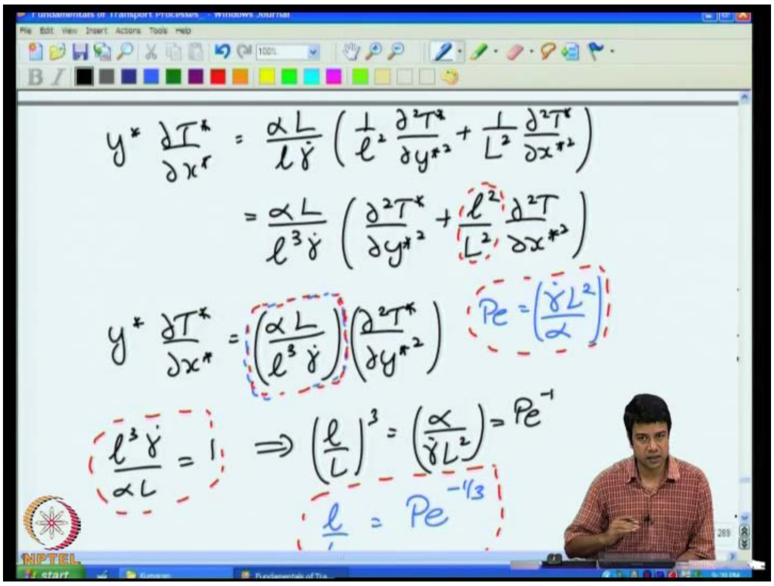

So, you went through that scaling exercise and we finally, found out that for this particular case, where the velocity is 0 at the wall and it increases linearly from the wall. This small l divided by capital L is equal to peclet number per minus one-third, where peclet number is based upon the thermal diffusibility and the length capital L.

So, this scaling analysis gave us a penetration depth for the energy disturbance from the wall, however that did not give us a solution to the problem. The solution was obtained using a similarity transform, which is similar to what we had used for the instantaneous flow past a flat plate in our very first series of lectures on shell balances. So, the procedures that we are using similarity solutions, separation of variables are identical to what we done for the elementary problems in shell balances the same approaches, we using for more complicated problems here.

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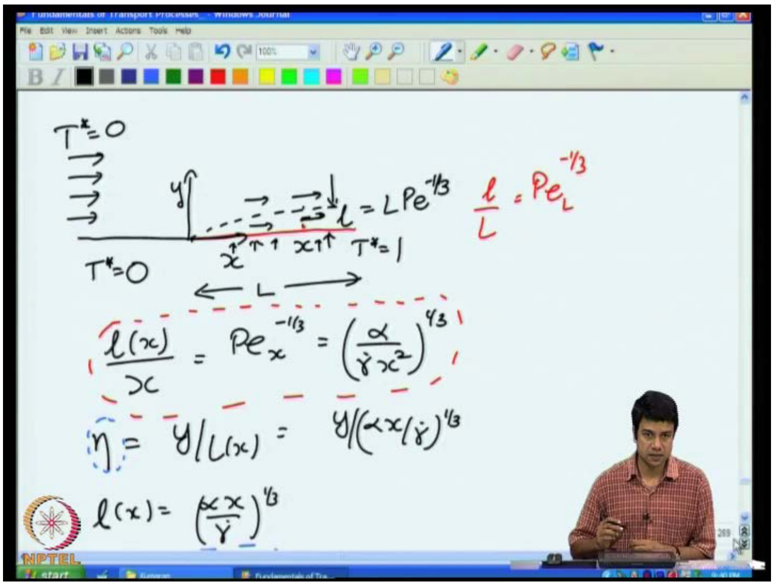

So, the similarity postulate was that, the length scale of penetration small l at a distance x from the start of the heated section does not depend upon the total length capital L, because we have neglected diffusion in the stream wise direction and convection only sweeps temperature the energy disturbances downstream. So, the length scale at a given distance x can depend only upon x itself, and from that we just got a similarity variable y by l of x is y by alpha x by gamma dot power one-third. (( )) means at the boundary layer thickness at a distance x from the upstream section, goes as x power one-third its increasing as one-third power of the distance.

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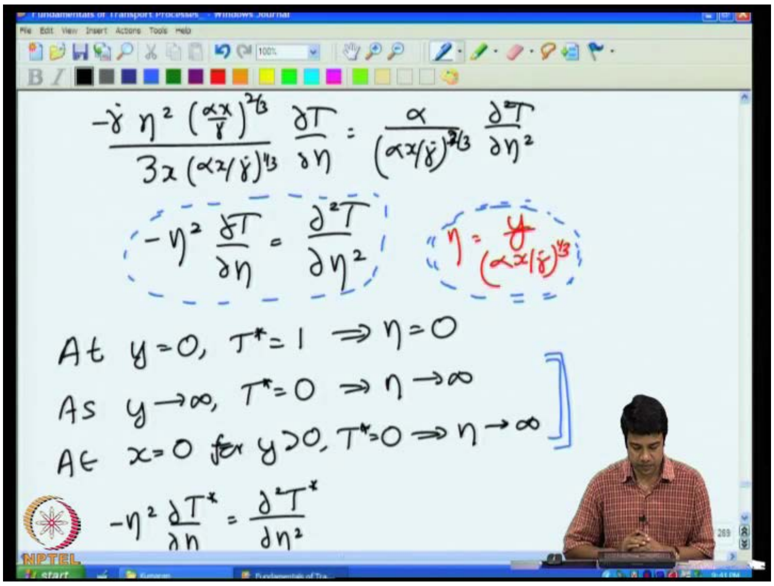

And using that, we had got a similarity solution. The equation that we got was of the form, d square T d eta square is equal to minus eta square time d eta by d T by d eta and that can be solved quiet easily, to get a solution for the temperature field and then a solution for the heat flux, and then a solution for the Nusslet number as a function of the peclet number.

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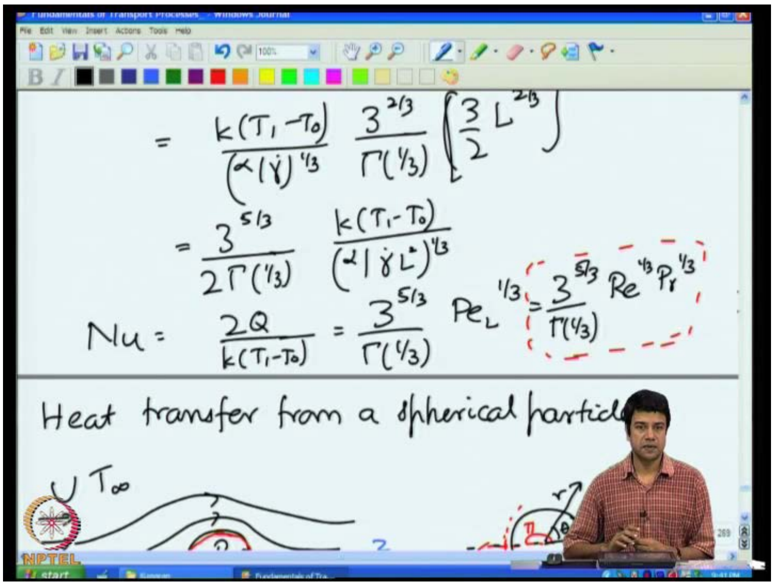

So that is that is the Nusselt number correlation for the flow past a flat plate. We had done the same thing for the flow around a spherical particle. In that case, we have we have given a velocity field.

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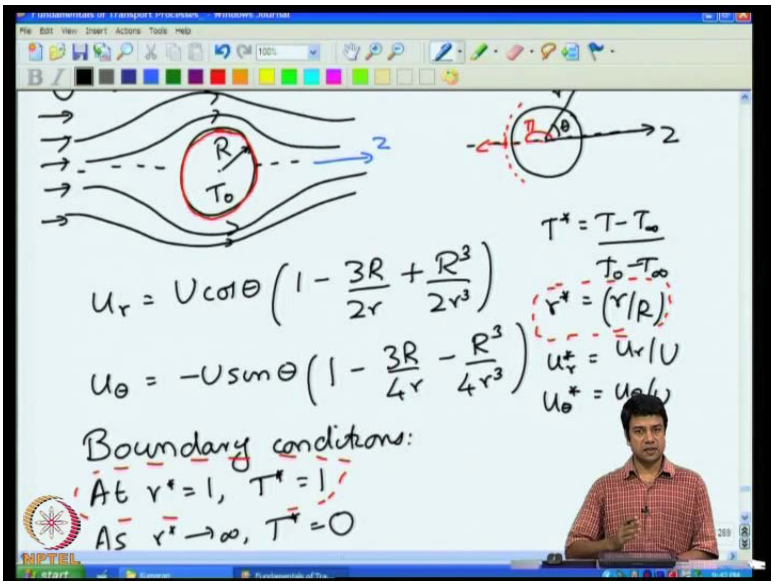

For the viscous flow around a spherical particle, this we did not calculate this is given to us. The calculation of this right now is outside the scope of this curse, but given this velocity field we followed the similar procedure convection is large compared to diffusion. So, if you just simplistically neglect diffusion, the solution you would get is that the temperature is a constant along streamlines. That does not satisfy the boundary conditions at the surface of the sphere therefore, one has to postulate boundary layer of thickness small compared to the radius at the surface.

So therefore we used a similarity first of all, since the thickness of the boundary layer is small compared to the radius of the particle. We can use an expansion for the velocity fields in terms of the distance from the surface.

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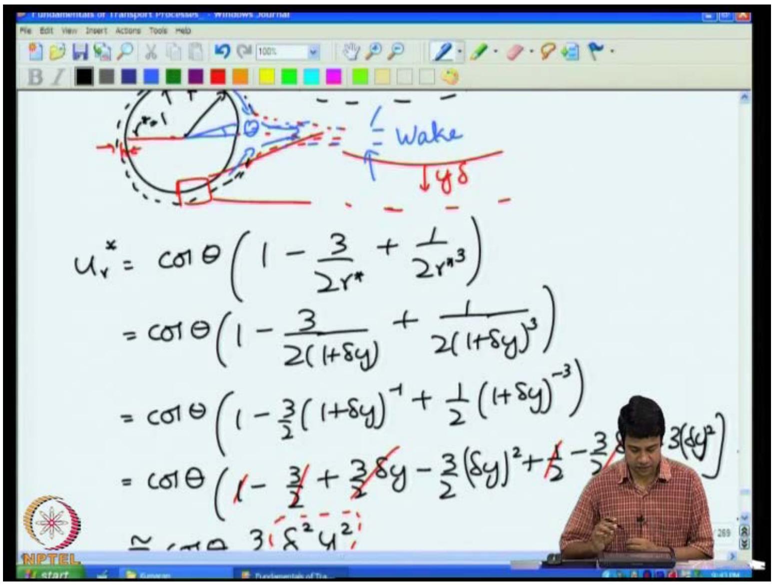

We got an expansion for the velocity fields in terms of the distance from the surface, which was labelled as delta y. Delta was a boundary layer thickness, y was the distance from the surface which is order one in the limit as peclet number goes to infinity. So, in the limit as peclet number goes to infinity y continues to be of order one, while delta scales in some way with peclet number, and that scaling is found out from the requirement that convection and diffusion have to be of the same magnitude. So, we found out that the radial velocity u r went as delta square times y square so, u r was proportional to delta square, while u theta was proportional to delta so, I put this all into the equation for the temperature field.

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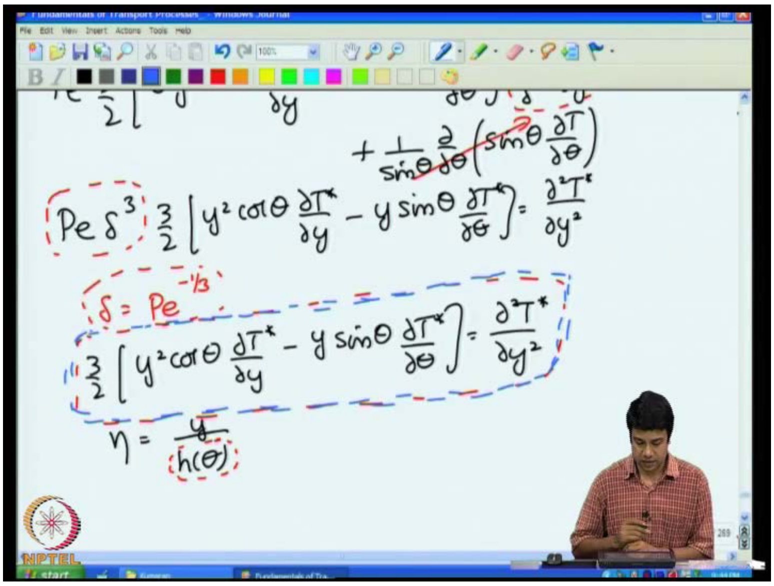

And from the requirement, that convection and diffusion have to be of the same magnitude from the requirement that convection and diffusion have to be of the same magnitude we found that the delta is equal to P e power minus one-third similar to the scaling for the flow past a flat plate. And we got a similarity solution by postulating the similarity variable of the form y by h of theta, where h is some function which when inserted into the equation results in an equation of the similarity form. So that is the basic principle I choose a form for h in such a way that, when I put this into the governing equation, I get an equation, which is of the similarity form. If I put this into the governing equation, I should end up with an equation that is of the similarity form, and I showed you how to do that you express the derivatives with respect to x and y in terms of h. And you will end up with an equation which looks like this at the bottom.

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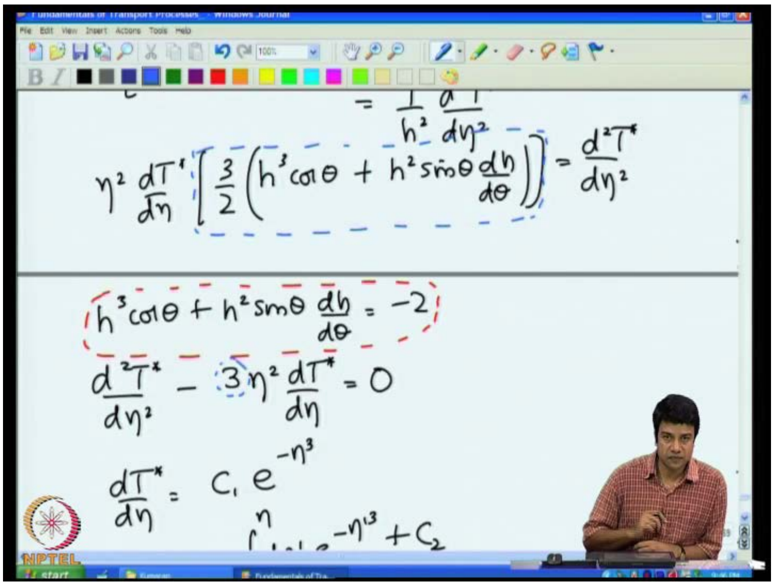

You have eta square d T d eta times something, which is a function of theta is equal to d square T by d eta square. And if we have to get a similarity form, all those terms in blue have to be equal to a constant. These have to be a negative constant if the temperature disturbance is to go to 0 as eta goes to infinity or as y goes to infinity far from the surface.

The specific value of the constant does not really matter we had seen this before that is, because there is h itself is only a scaling factor. If I change h by a factor of two, the solution for the temperature field that I get in terms of eta will change, but the temperature field that again in terms of R will continue to remain the same, because h is only a scaling factor that I have used to scale y. And then you get an equation that is very similar to what we had for the flow past a flat plate. In fact for any geometry this equation will be exactly the same up to a constant sitting in front of the second term on the left.

Only this constant will change nothing else, will change in this equation. Depending upon the scaling factor that I have used for the function eta and the solution ends up being identical to what I get for the flow past flat plate. However we still have to find a solution for h and I showed you that, the solution for h depends upon the specific is obtained as usual by a variation of parameters.

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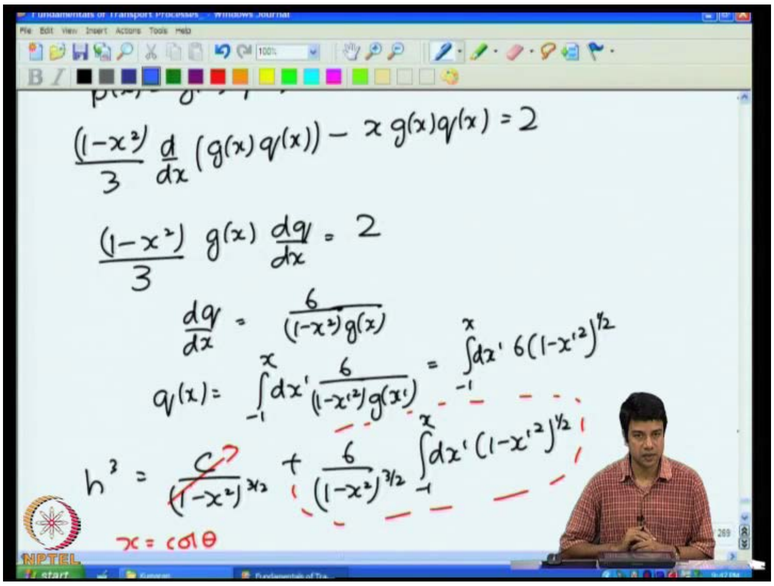

And the final solution for h actually has a constant in it. That constant has to be determined from the boundary conditions in theta one condition that you have for theta is that the upstream stagnation point at theta is equal to pi or x is equal to minus 1. At the upstream stagnation point the boundary layer has to be of finite thickness.

Note that this solution for h cubed actually goes to infinity at both x is equal to plus 1 and x is equal to minus 1. I have only one constant in this I can choose that constant to make h cubed finite either at the upstream or the downstream stagnation point. If I choose the constant to make h finite at the upstream stagnation point, then this constant C has to be equal to 0 and I get a solution for h cubed and this solution as I showed you will tell you that the boundary layer thickness h is going to infinity at the downstream stagnation point.

As I explained in the previous lecture, this boundary layer thickness is finite at the upstream stagnation point. However, as you go downstream h diverges it goes to infinity, h goes to infinity at the downstream stagnation point that means that you have a wake behind the particle. In order to analyse the temperature field in the wake it is a more complicated task, one has to do an expansion in the angle theta about theta is equal to 0 and that is not within the scope of this course, but it turns out that the details of the wake do not really affect the scaling of the Reynolds Nusselt number with the peclet number.

The scaling that I get the dominant contribution to the heat flux from the particle is actually, from the entire surface the entire boundary layer thick region and the contribution from the wake is actually small, because the length scale is becoming large in the wake and therefore, the flux is becoming small. Therefore, the contribution to the total heat coming out of the of the particle is actually small at the downstream wake region. And therefore, it is not necessary to calculate the details of the flow in the wake in order of the temperature field in the wake, in order to get the Nusselt number versus peclet number correlation.

In that obtain the correlation for you in that lecture in the previous lecture by actually calculating the value of h, and then putting that into the heat flux at the surface and integrating that heat flux over the entire surface. And you end up having to integrate a term that goes as one over h of theta times sin theta D theta over the surface.

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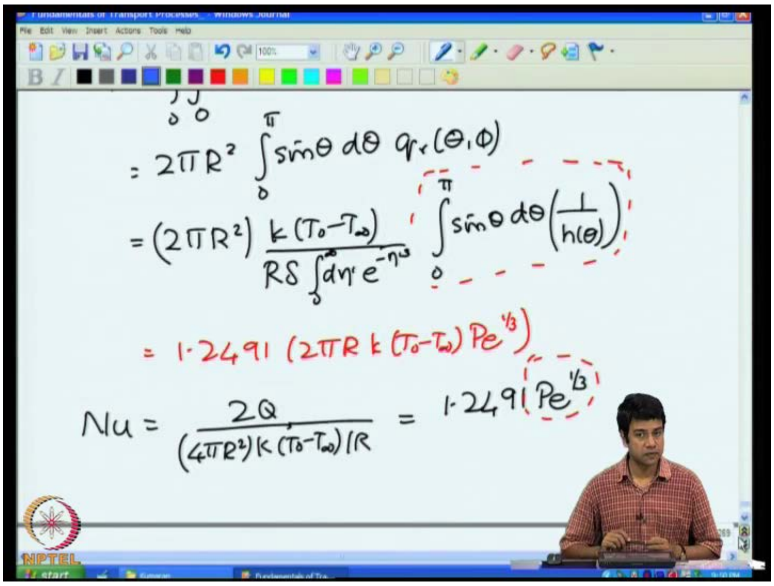

And if you do that, you get that the Nusselt number goes as 1.2491 times P power one-third; exactly the same scaling is the flow past flat plate; in fact as I showed you in the previous lecture for any object, which satisfies the (( )) boundary condition at the surface.

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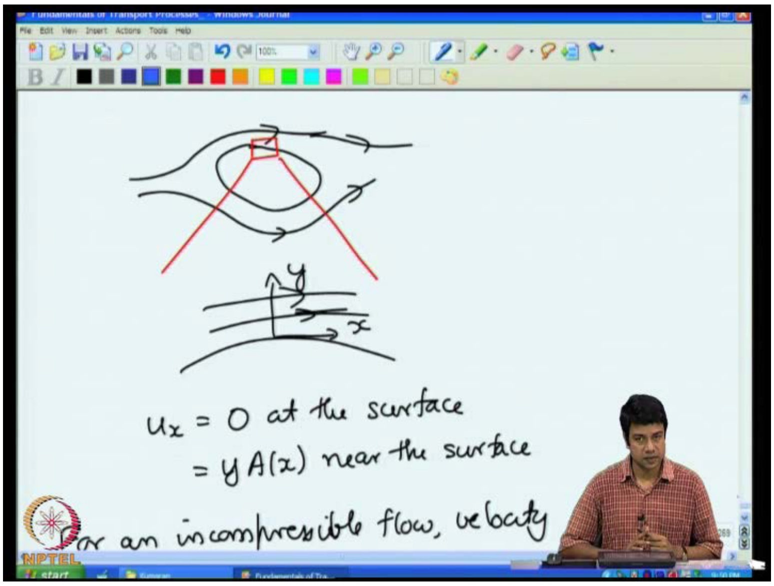

The tangential velocity has to go to 0 at the surface if there is a net sheer stress, so that the velocity gradient is non 0 that means that the tangential velocity has to go as y where y is the distance from the surface. From the mass conservation equation, the normal velocity has to go as y square and if you put in these dependences of y and y square, you will end up with a boundary layer thickness that goes as P power minus one-third if the boundary layer thickness goes as P power minus one-third. Then it is inevitable that the nusselt number will go as P power plus one-third, only the constant changes the scaling of the nusselt number with the peclet number remains the same.

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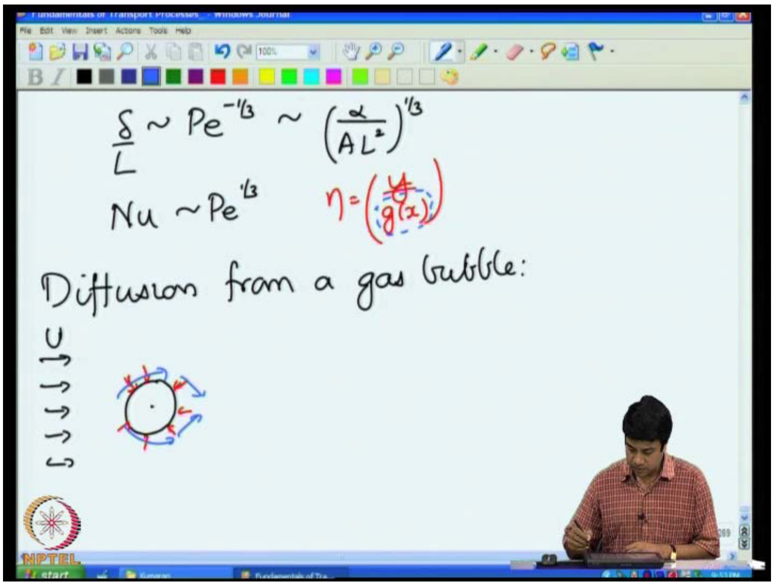

Now, this is one particular case where the velocity goes to 0 at the surface. Let us consider another case, where the velocity itself the tangential velocity itself is not 0 at the surface. If I have a gas bubble that is moving with a velocity relative to the fluid, then the tangential velocity at the surface of the bubble is not 0, because we do not apply no slip conditions at the surface. Rather we apply 0 sheer stress conditions at the surface so for example, if I had a bubble and there was a fluid that was incident on this bubble with a velocity U, then at the surface of the bubble provided the bubble is spherical and remains spherical as the flow goes faster.

The normal velocity at the surface has to be equal to 0, because the fluid cannot penetrate the bubble. However, the tangential velocity along the surface of the bubble is not 0, because the fluid can have a non 0 tangential velocity along the surface. So, in general there is this 0 sheer stress condition, which basically means that the gradient of the velocity has to be 0 at the surface, but the velocity itself does not have to be 0 at the surface. The question is in this case, what is the relation between the peclet number and the nusselt number? We have already solved one such problem, where the tangential velocity is non zero way back when we looked at unidirectional transport.

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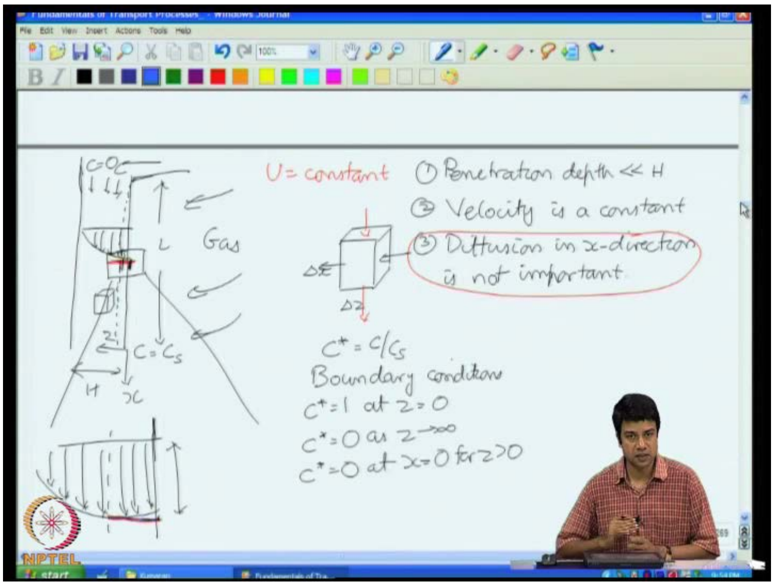

If you recall this following film the following gas film, in this case as well the tangential velocity is non zero at the surface and we did a differential balance. We assumed that the velocity was a constant at the surface the assumption basically is that over length scale over which diffusion of mass takes place, the velocity is not changing. And we went back and found out conditions for that case why the under what conditions, can we assume that the velocity is a constant.

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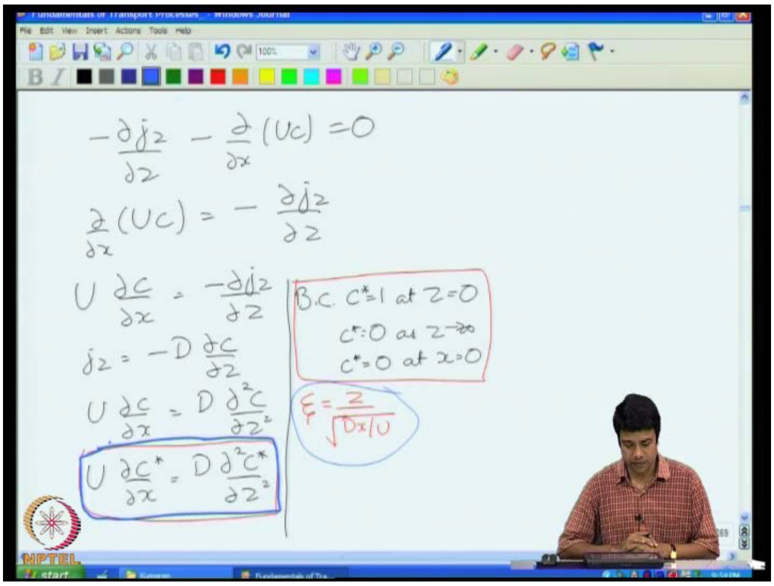

And using the mass balance condition, we got an equation of this form U d C divided by d x is equal to D d square C by d Z square and we solved this got a solution that went as e power minus i square divided by 4.

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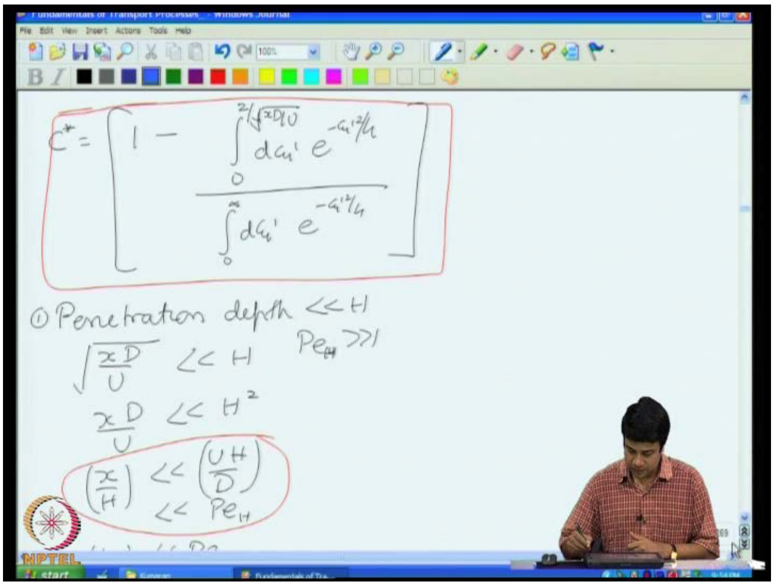

And from that, we had got a correlation between the nusselt number and the peclet number.

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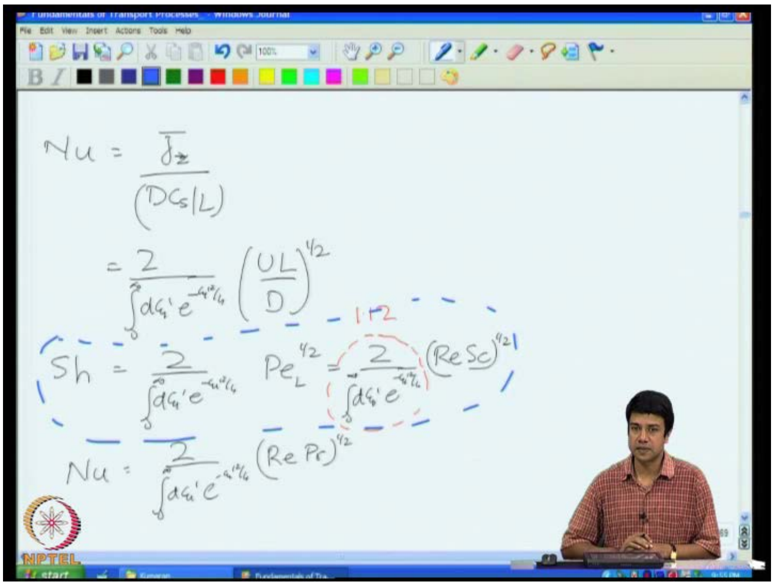

And this was the correlation this was the correlation for the Sherwood number or the nusselt number. As a function of the peclet number, it goes as peclet number to the half power in contrast to the one-third that we had for flow past solid surfaces.

This once again is a common feature of all flows, where there is slip at the surface where the tangential velocity at the surface is non 0 and we will see that that is the solution that you get even for the case (No audio from 23:00 to 23:08) where you we have flow past a gas bubble.

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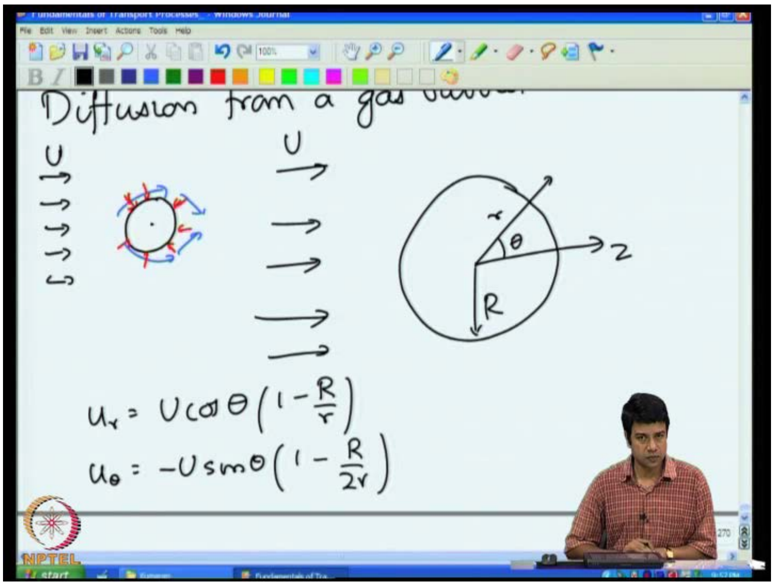

So, let us look at this problem (No audio from 23:14 to 23:25) so, I have a gas bubble here and I will use this gas bubble has radius R and I will use a coordinate system, where this is the Z coordinate so, the distance is R and this is the angle theta and the fluid is incident with a velocity u on this gas bubble. Now, the velocity components u r and u theta for this case this once gain is an axis symmetric configuration, there is no dependence on the angle phi around the axis and the angle and the velocity in that direction in the phi direction is identically equal to 0 so, there is no dependence on the phi coordinate in this problem.

One can obtain the velocity components for this particular case once again right now that is outside the scope of this course, because we have not gone through the details of fluid mechanics, but I can write the velocities for you u r is equal to u cos theta into 1 minus R by r and u theta is equal to minus u sin theta 1 minus R by 2r I should change this. So, these are the two velocity components u r and u theta.

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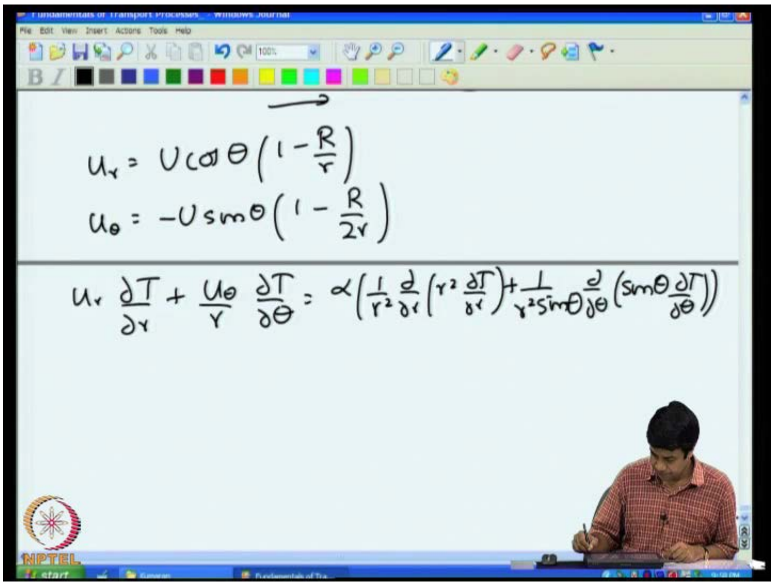

And my convection diffusion equation is U r d T divided by d r plus U theta by r partial T by partial theta is equal to alpha into ...

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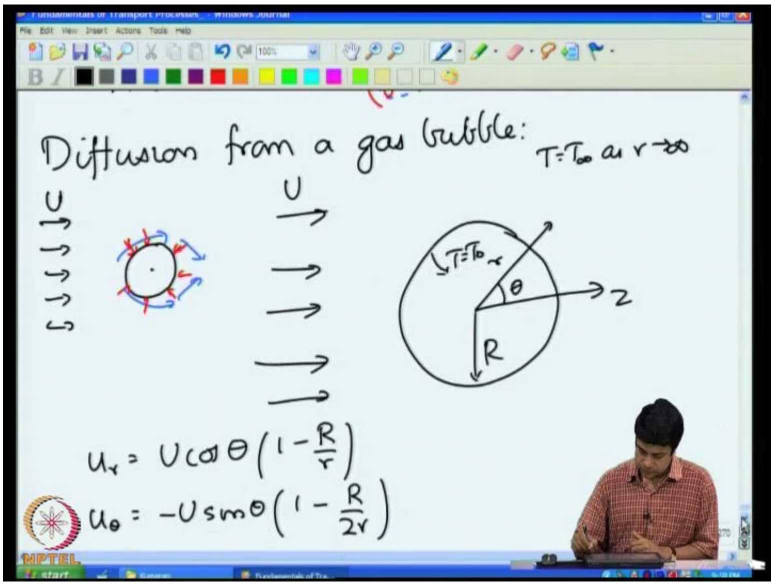

And the boundary conditions that I impose on the surface are as usual T is equal to T naught on the surface and T is equal to T infinity as R goes to infinity. So, the temperatures T infinity far from the surface and is equal to T naught at the surface of the bubble itself and one would like to find out, what is the variation of the temperature field and from that the heat flux and the nusselt number.

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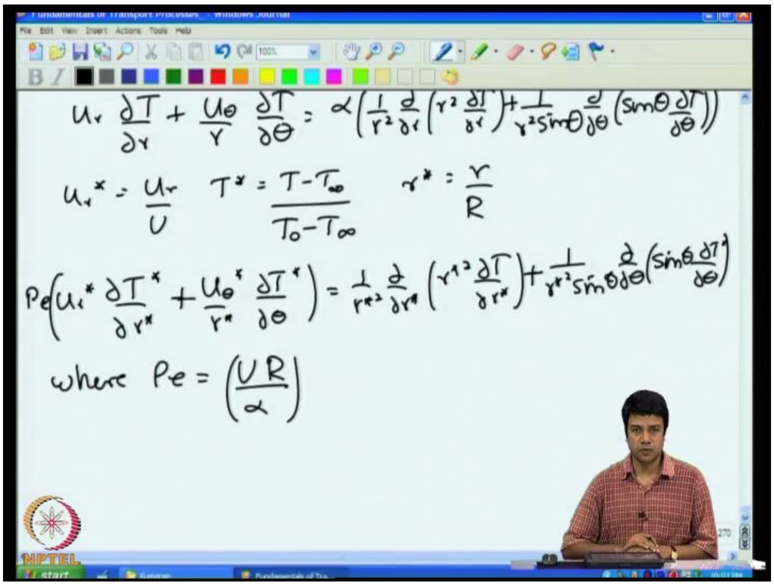

Define scaled variables once again u r star is equal to u r divided by U, T star is equal to T minus T infinity divided by T naught minus T infinity and r star is equal to r by capital R. And my conservation equation becomes u r star d T divided by d r plus ... The whole thing into a peclet number the whole thing into a peclet number is equal to 1 by r square d divided by d r ... where P e is equal to U R by alpha.

Once again, if you take the limit of high peclet number if you were just to neglect the diffusion terms altogether, then the solution you would get is that the temperature is a constant along streamlines. That clearly does not satisfy the boundary condition at the surface of the particle of the bubble therefore, one has to postulate a boundary layer of small thickness at the surface, in which there is a balance between convection and diffusion.

As the thickness of the boundary layer becomes small, the gradient becomes large and the diffusion in the direction perpendicular to the surface becomes large and if the thickness is sufficiently small, such that the diffusion term balances the convection term then one has a balance between convection and diffusion; diffusion perpendicular to the surface and convection in both the flow as well as the gradient directions.

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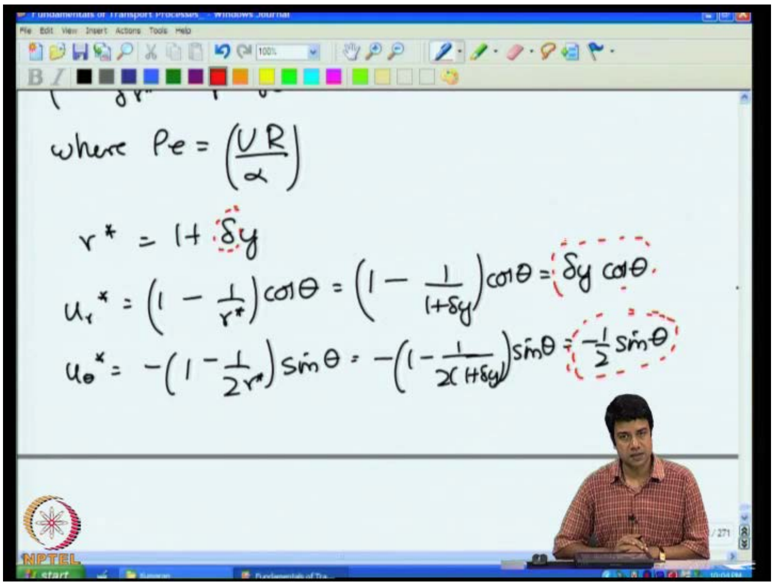

So I will postulate a boundary layer r star is equal to 1 plus delta y r star is equal to 1 is the surface R is equal to capital R, r star is equal to 1 is the surface therefore, I am focussing on a region close to the surface 1 plus delta y region close to the surface. In this region close to the surface, the assumption here is that y is order one in the limit as peclet number goes to infinity y continues to be a number that is order one. Whereas, delta is the one that is varying with peclet number you would expect delta to become smaller and smaller, as the peclet number becomes larger and larger. So, the region over which diffusion takes place becomes smaller as the peclet number becomes larger.

So, within this region I now need to find out, what is the velocities I do not need the entire velocity field in terms of R I just need to know what are the velocities within the small region of order delta so, u r star is equal to 1 minus 1 by r star cos theta this I can write it as 1 minus 1 by 1 plus delta y cos theta and if I use an expansion in the small parameter delta this just becomes equal to delta y cos theta. So, u r is going as delta times y. u theta is equal to minus of 1 minus 1 by 2 r sin theta is equal to minus of 1 minus 1 by 2 into 1 plus delta y sin theta. So, this is actually just equal to minus half sin theta.

So, note that in the previous problem for the flow past a particle u r was proportional to delta times y the whole square and u theta was equal to delta times y whereas, in this case u theta is a constant that is a slip at the surface, there is a relative motion between the fluid and the bubble at the surface. And u r goes as delta times y and these this is a common feature of all problems, where you have a slip at the interface.

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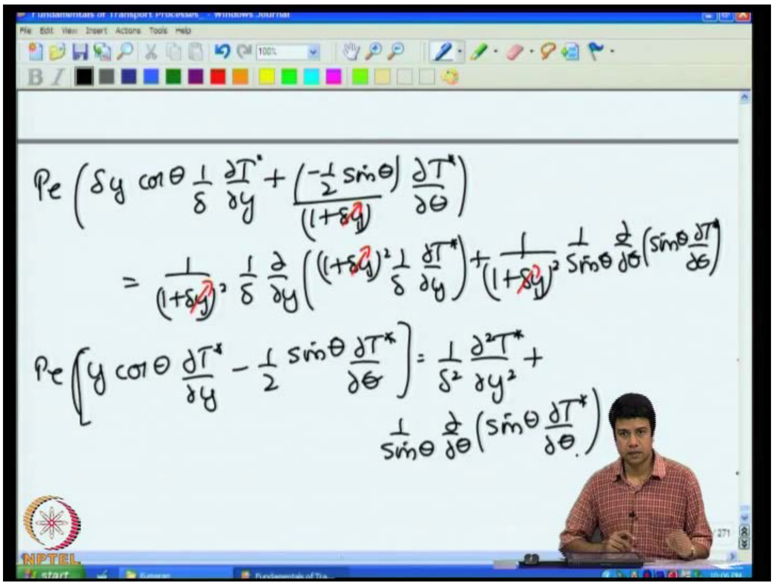

So, I insert these two into my equation for the temperature field ... into delta y cos theta d T by d R is 1 over delta partial T by partial y plus minus 1 by 2 sin theta divided by R, which is 1 plus delta y partial T by partial theta is equal to 1 divided by 1 plus delta y the whole square 1 divided by delta d divided by d y of 1 plus delta y the whole square 1 divided by delta partial T by partial y plus ...

So, this is just the convection diffusion equation that I had earlier that is, this particular equation in which I have substituted R is equal to 1 plus delta y and taken the approximate expressions for u r and u theta and of course, in this case I can neglect delta y in comparison to 1, because delta is a small parameter and once you do that you will get peclet number into y cos theta d T divided by d y minus 1 divided by 2 sin theta d T by d theta is equal to 1 by delta square partial square T by partial y square plus 1 by sin theta ...

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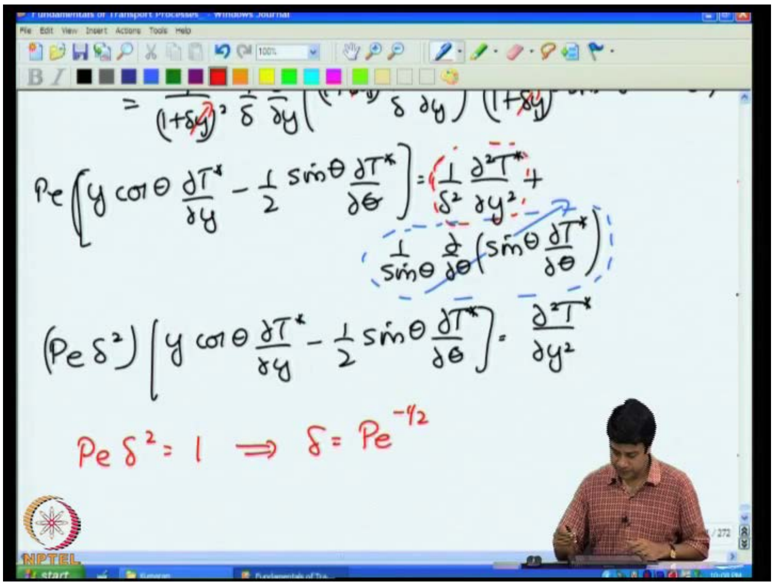

So, note that both terms in the left hand side of equal magnitude they both scale they are both order one in the limit as delta goes to 0 so, they scale as delta to the power 0 in the limit as delta goes to 0. On the right hand side I have one term which goes as one over delta square and the other term which has no delta is in front of it. The cross stream diffusion term goes as one over delta square the cross stream diffusion term as one over delta square this is diffusion in the R coordinate which is across the flow.

The stream wise diffusion term has no delta is in it therefore, one can neglect the stream wise diffusion in comparison to the cross stream diffusion. And once I do that I will get P e times delta square into y cos theta d T d y minus 1 divided by 2 sin theta d T divided by d theta is equal to partial square T divided by partial y square.

Obviously convection and diffusion are of the same magnitude only if P delta square is 1, which implies that delta is equal to P power minus 1 divided by 2, so that is how you get the peclet number power minus half scaling for flow past surfaces, which have a slip condition a 0 stress condition or a slip condition at the surface.

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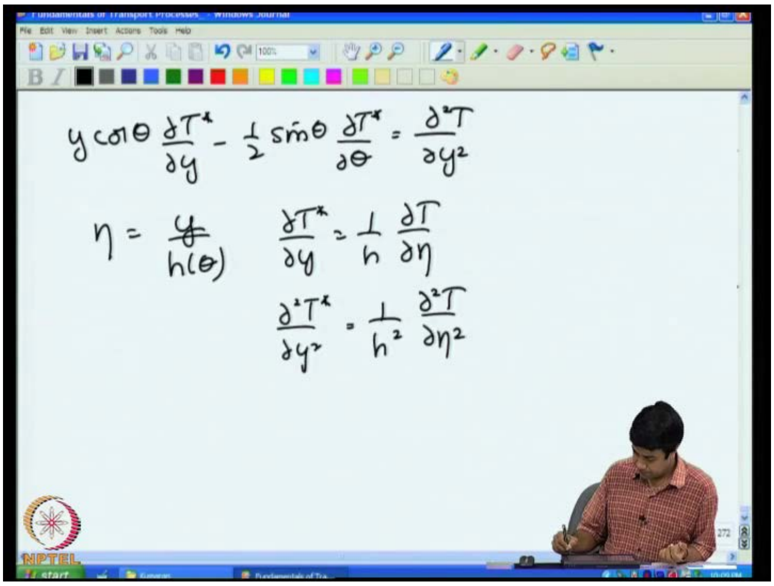

And once delta is equal to P power minus half the rest of the equation becomes y cos theta d T divided by d y minus half sin theta d T divided by d theta is equal to d square T by d y square. And now, I postulate a similarity solution of the form identical to what we had previously. I postulate a similarity solution of the form identical to what we had previously (no audio from 36:45 to 37:05). So, I postulate a similarity solution of the form eta is equal to y by h of theta, which means that partial T by partial y is equal to 1 divided by h partial T by partial eta by differentiating by chain rule partial square T by partial y square is equal to 1 divided by h square partial square T by partial eta square.

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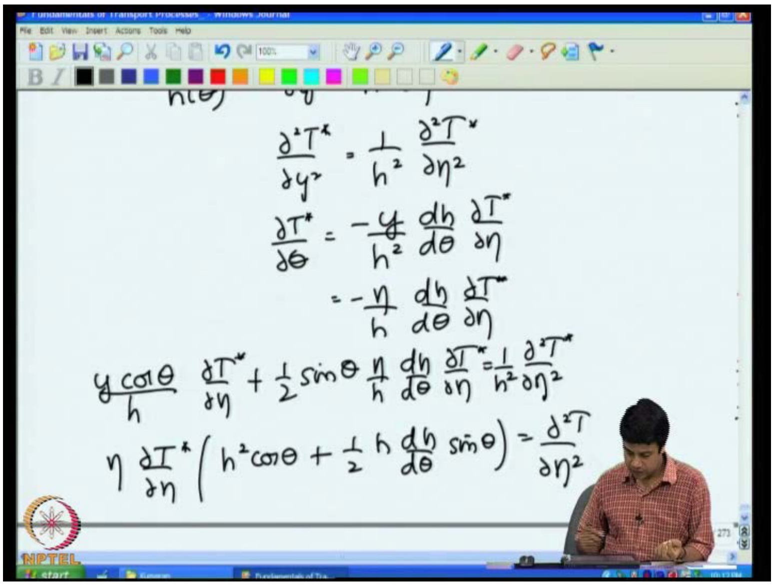

And partial T by partial theta is equal to minus y by h square d h by d theta partial T by partial eta is equal to minus eta by h d h by d theta partial T by partial. So, we substitute this into the equation, you substitute this into the equation and we will get into this equation and we end up with an equation of the form y cos theta by h partial T by partial eta minus I should a plus here half sin theta eta by h d h by d theta partial T by partial eta is equal to partial square T by partial eta square.

And after some simplifications, I can reduce this to an equation of the form eta times partial T by partial eta into h square cos theta it should have one over h square here plus half h d h by d theta times sin theta is equal to partial square T by partial eta square. So, clearly in this problem to be able to get a solution of the similarity form this entire term has to be equal to a constant to get a similarity form this entire term has to be a constant. It also has to be a negative constant, because I require the temperature to decay exponentially as distance goes to infinity it has to be a negative constant. So, the temperature decays exponentially as eta goes to infinity or as y goes to infinity the exact value does not really matter. Because if I change h by some factor the solution in terms of eta is going to change of course, but the solution that I get in terms of R cannot change because h is a scaling factor I have used to scale the coordinate.

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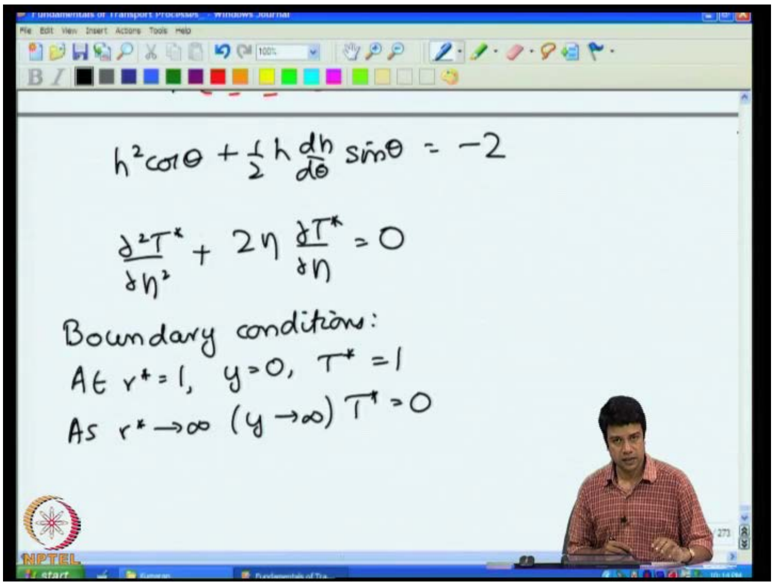

So, the solution that I get in terms of the radius R will not change even though the solution in terms of eta will change. So, without loss of generality I can assume for example, that h square cos theta plus half h d h by d theta sin theta is equal to minus 2 with this my equation for the temperature now becomes d square T divided by d eta square plus 2 eta times d T divided by d eta is equal to 0. And I can solve this subject to boundary conditions at r star is equal to 1 which means that y is equal to 0, T star has to be equal to 1.

That is the boundary condition at the surface of the bubble and as r goes to infinity, y goes to infinity, T star has to be equal to 0 so far away T is equal to T infinity that means that T star is equal to 0 at the surface itself, T star is equal to 1.

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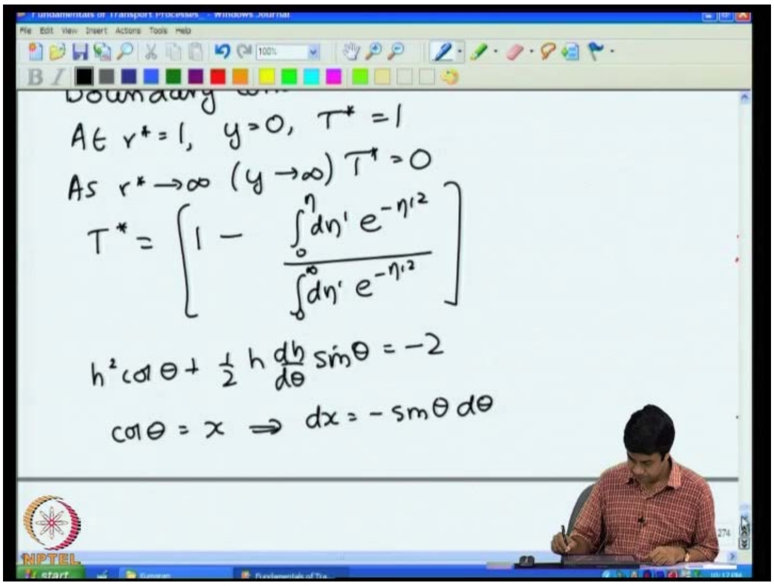

And the solution for this equation that satisfies these boundary conditions, you can verify that for yourself it turns out that this is equal to 1 minus integral 0 to eta e eta prime e power minus eta prime square divided by integral 0 to infinity.

Note that this is different from the solution for the flow past solid surface in that case we had eta cubed e power minus eta cubed in this case we have e power minus eta square and in fact this exponential solution will turn out to be the same for any surface at which you have the velocity being non 0 in fact, if you go back to our earlier example of the flow past of the of the transport to a falling film if you go back to our earlier example of transport to a falling film and look at the solution that we got there the solution is once again identical the similarity variable in that case was psi and we got e power minus psi square by four.

Therefore for any problem, where there is a non 0 velocity at the surface you going to get a solution that goes as e power minus eta square. So that is for the temperature field in terms of eta we still have not solved this equation to get h, because eta is equal to y by h and unless we know what is h we will not be able to find the solution for the temperature field.

The solution for h is obtained the same way as it was for the flow past a spherical particle the equation that I have is h square cos theta plus half h d h by d theta sin theta is equal to minus 2. I will use the substitution cos theta is equal to x, which implies that d x is equal to sin theta minus sin theta.

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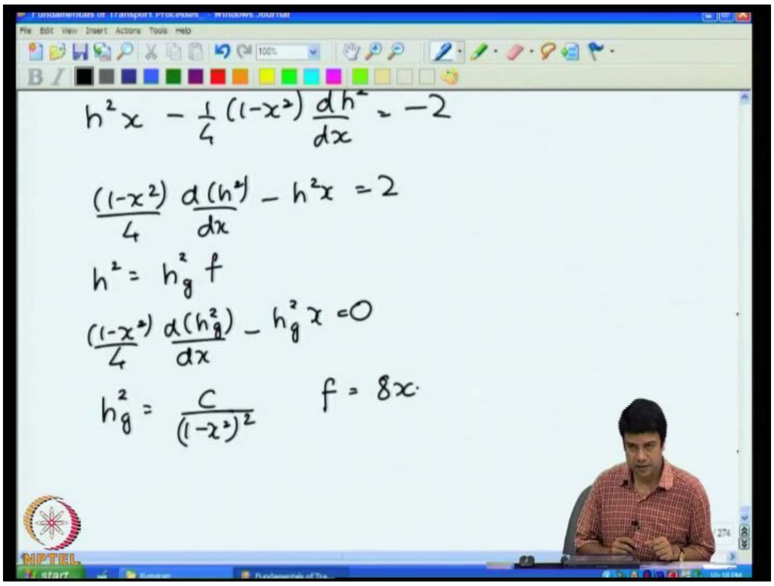

With this, I will get h square times x minus one-fourth into 1 minus x square d h square divided by d x is equal to 2 minus, minus one-fourth into 1 minus x square, because the d x is equal to minus sin theta d theta. Moreover I have to solve an equation of the form 1 minus x square by four d of h square by d x minus h square x is equal to 2. And this equation one can solve quiet easily by variation of parameters the solution h square has two parts, which is equal to a general solution times a function that is obtained by variation of parameters, where the general solutions satisfies the equation 1 minus x square by four d of h square g by d x minus h square g times x is equal to 0.

And the solution for this can be obtained quiet easily h square the general solution is equal to some constant by 1 minus x square the whole square. The particular function f in this particular case is obtained by substituting h square g into the original equation and you can do that and finally, you will get the solution for f is equal to eight times x.

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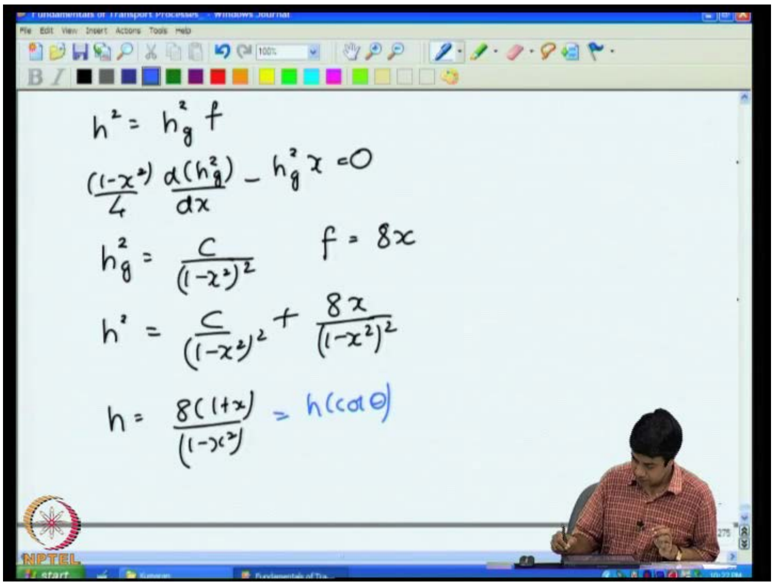

So, the final solution for h square g for h square is of the form C by 1 minus x square the whole square plus 8 x by 1 minus x square. Now, this function once again h square goes to infinity at x is equal to plus or minus 1, x is plus 1 is the downstream stagnation point. x is plus 1 corresponds to theta is equal to 0 that is the downstream stagnation point x is equal to plus 1, x is equal to minus 1 is the upstream stagnation point x is equal to minus 1 is the upstream stagnation point and x is equal to plus 1 is the downstream stagnation point.

The solution diverges at both x is equal to minus 1 and x is equal to plus 1. However I have an unknown constant here I have a constant C and I can choose the constant C in such a way that the solution does not diverge at one of the two stagnation points. If I choose my constant C such that the solution is of the form h square h is equal to 8 into 1 plus x by 1 minus x square by an appropriate choice of the constant C then the solution does not diverge at the upstream stagnation point, it does diverge at the downstream stagnation point .

So by appropriate choice of this constant I can make sure that the solution is finite that the boundary layer thickness is finite at the upstream stagnation point, which means that it goes something like this. It does diverge at the downstream stagnation point resulting in the formation of a wake as in the case of flow past spherical particle. In this case as well h goes to infinity at x is equal to plus 1, if I choose my constant in such a way that h is finite at x is equal to minus 1. And in the downstream stagnation point h goes to infinity of course, our analysis is valid only when delta is small so as delta becomes large our analysis is no longer valid.

Therefore, we need to do a different analysis in the limit of small theta in order to capture the details of what is happening in the wake region, but as in the case of heat transfer from a solid particle the heat flux in this wake region is actually small. The distance is large therefore, the gradient is small and the heat flux is small. So, I do not really need to evaluate the details of the temperature field in the downstream wake in order to find out what is the heat flux, because the contribution there is actually small. So, this basically determines the solution for h, with h of theta which is equal to h of cos theta.

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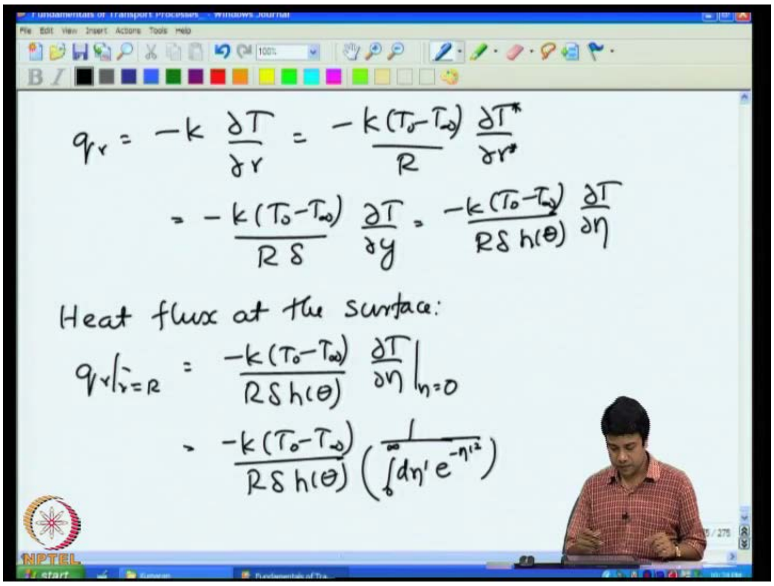

Now, I can determine the nusselt number relation as usual the heat flux radically outwards q R is equal to minus k d T by d R which is equal to minus k into T naught minus T infinity by R partial T star by partial R star write R star in terms of delta and y. So, this becomes minus k into T naught minus T infinity by R delta partial T by partial y. Change variables from y to eta so, this is equal to minus k T naught minus T infinity by R delta times h of theta into partial T by partial eta.

The heat flux at the surface q r at r is equal to capital R will be equal to minus k into T naught minus T infinity by R delta h of theta times d T by d eta at eta is equal to 0 at the surface itself. And this we have evaluated if you go back to our earlier expression for T star in terms of eta this we have an expression for T star in terms of eta so, I can use that expression to write this as minus k T naught minus T infinity by R delta h of theta times 1 divided by integral 0 to infinity D eta prime e power minus eta prime square, this is the derivative of T star with respect to eta.

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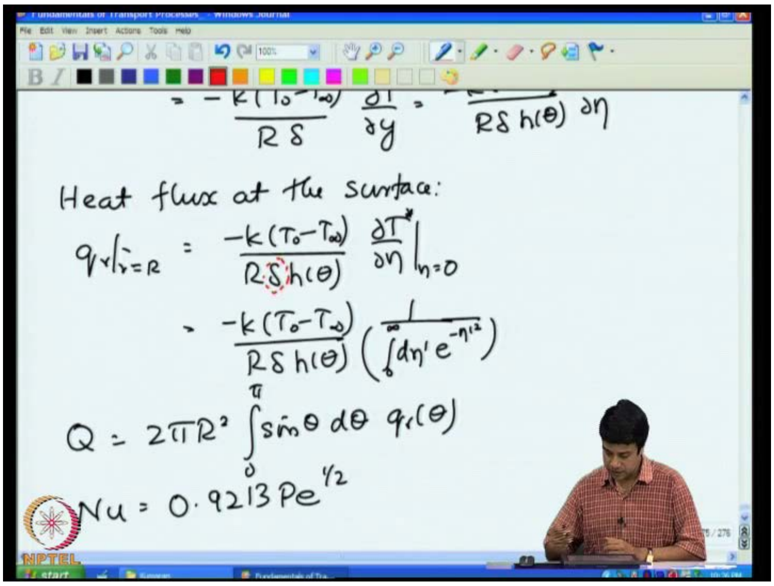

The total heat Q is equal to 2 pi R square integral from 0 to pi sin theta d theta times q r of theta. So, if I know what the form of h of theta is, then I can evaluate this total heat coming out exactly if I know what the form of h of theta is, I can evaluate the total heat that is coming out exactly. And from that I can find out the nusselt number correlation (no audio from 52:53 to 53:05) from that I can find out the nusselt number correlation 2 Q divided by 4 pi R square T naught minus T infinity by R.

So, I would not go through the details but, if you actually do the entire calculation what you will get in the end is that the nusselt number is equal to a constant peclet number power plus half. That peclet number power plus half scaling emerges, because I have a delta in the denominator here delta is equal to P power minus half and I have delta in the denominator the rest of the terms are all constants so, because of that I get nusselt number going as P in peclet number power plus half.

So, this is the relation between the nusselt number and the peclet number for the flow past objects, where there is no there is a slip conditions, where there is non zero tangential velocity at the surface.

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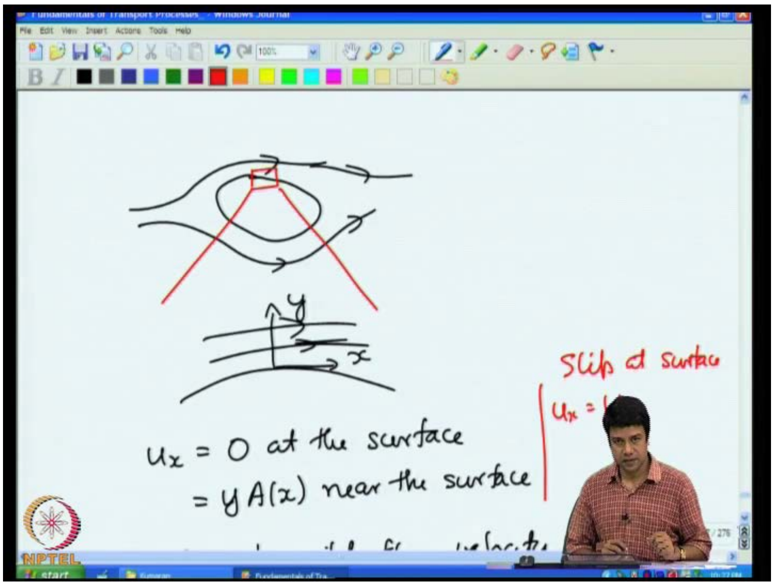

And this once again is a relation that is valid for any shapes of object I just briefly go through the analogy for the case where there is a slip at the surface. If there is a slip at the surface then u x is non 0 at the surface. So, I will just write down the relation for the case where there is a slip at the surface slip at surface u x will be equal to capital U at the surface there is a slip velocity so, u x will be non zero.

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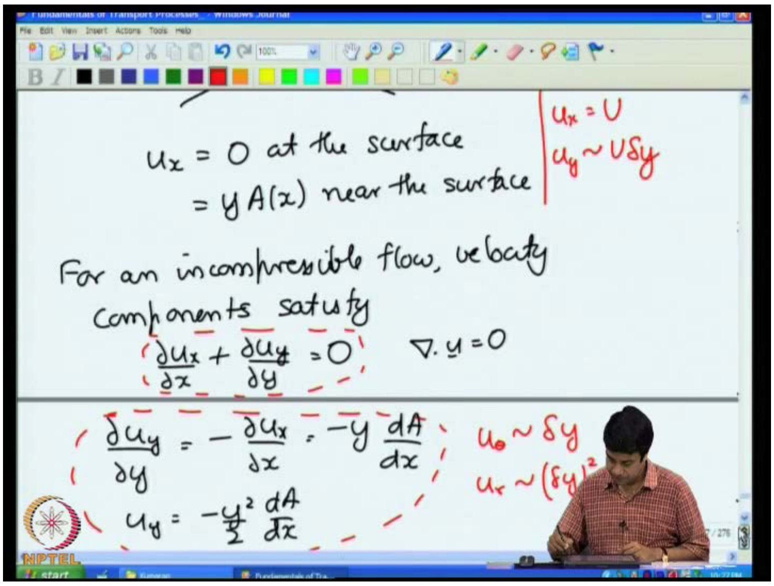

And what that would imply from the incompressibility condition is that u y will be equal to y times delta at the surface. The perpendicular velocity will go as u times delta times y in contrast to the delta y and delta y square scaling here you will get 1 and delta y here and if you put this into this equation.

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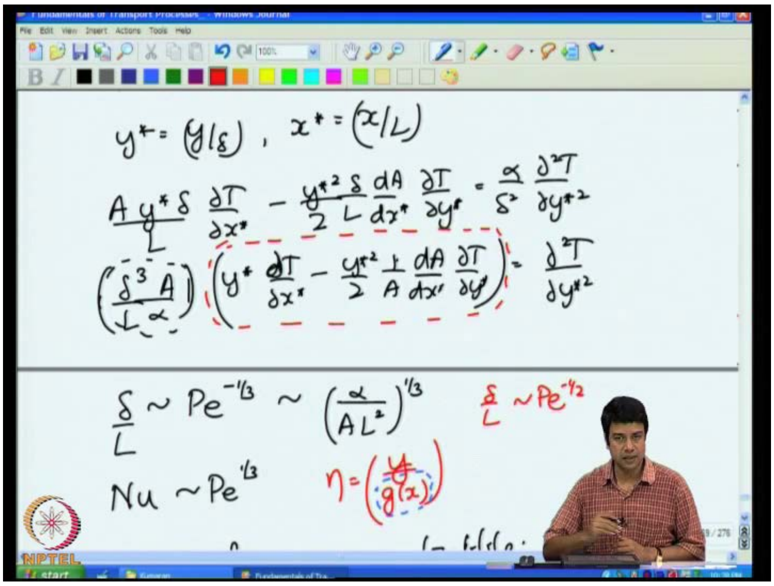

In this case I have y delta and y square delta square here whereas, when there is a slip there will be 1 and delta y here. And with that I will straight away get delta by L goes as P power minus 1 divided by 2.

So, the form of the solution the form of the similarity variable h of theta in terms of theta or delta of g of x in terms of x will change, but the scaling ends up remaining the same. And that is why you get a scaling of order of the type nusselt number goes as peclet number power half for all problems, where there is convection dominant and there is a non 0 velocity at the surface.

So, to summarize everything that we have done in this series of lectures on high peclet number transport, when the peclet number is large convection is large compared to diffusion. However very near the surface there is a region over which diffusion is still important, because convection cannot transport material from the surface to the fluid, because there is no velocity perpendicular to the surface. Thickness of that region is determined by balance between convection and diffusion, by postulating a length scale within that region over which diffusion is comparable to convection. The thickness of that region goes as P power minus one-third, when there is a no slip condition at the surface goes as P power plus minus half when the velocity is non zero at the surface.

From that straight away the nusselt number goes as P power plus one-third, when there is no slip condition at the surface goes as plus half, when there is a slip at the surface. The coefficients in the nusselt number versus peclet number correlations can be determined by doing boundary layer theory and similarity solutions. In case of flat plate, the similarity solution is quiet straight forward for parallel flows for flows past objects, there is actually a boundary layer wake at the back, where the boundary layer thickness goes to infinity.

We did not go through the details of the flow in the wake surface to say that the transport in the wake does not really affect the scaling, because the heat flux in the wake region is small compared to the heat flux everywhere else. And that is a brief summary of high peclet number transfer. So, we will briefly summarize everything we have done so far in the next lecture and we will conclude; we will see you then.