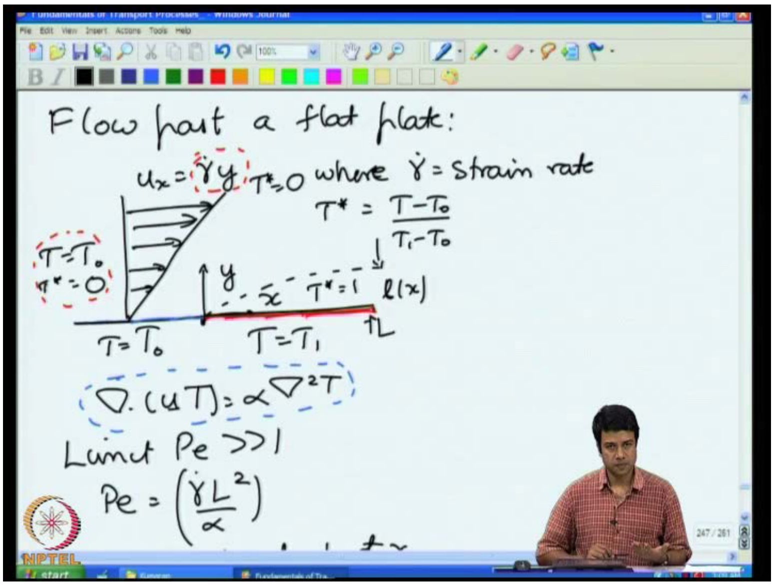
Fundamentals of Transport Processes
Prof. Kumaran
Department of Chemical Engineering
Indian Institute of Science, Bangalore

Lecture No. # 38
High Peclet Number Transport
Heat Transfer from a Spherical Particle - II

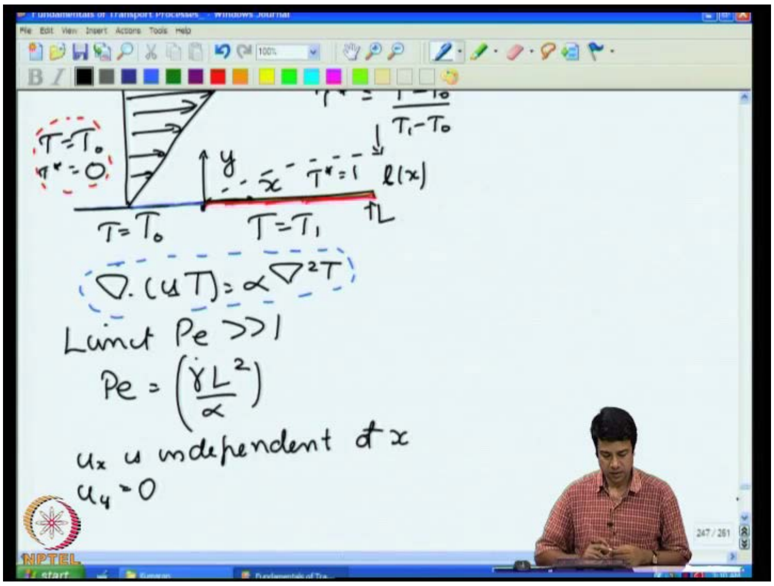
So, welcome to lecture number 38, where we were of the course on fundamentals of transport process, where we were looking at transport in the limit of high Peclet numbers, where we expect convective transport to be dominant compared to diffusive transport. We had first solved the case of transport near a flat plate in the previous lecture.

(Refer Slide Time: 00:45)



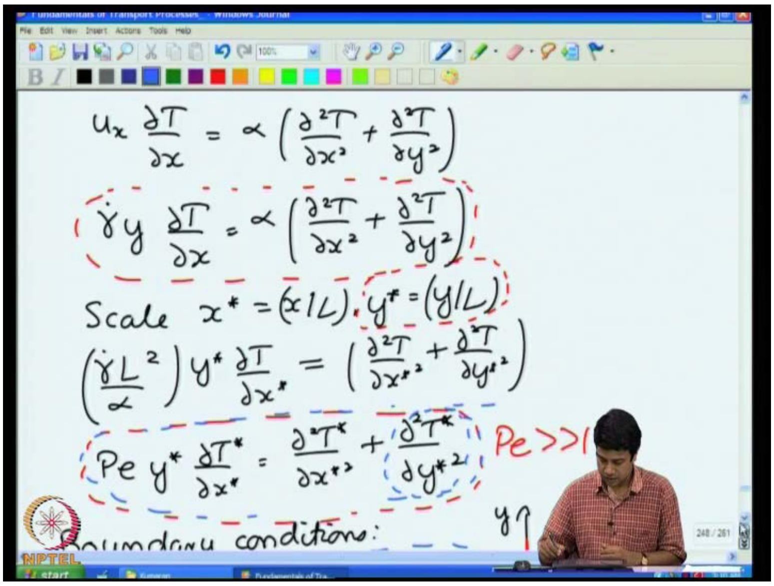
A flat plat with a heated section starting at x is equal to 0, T is equal to T naught for x less than 0 and T is equal to T1 for x greater than 0; the fluid that is incident on that plate has a temperature T naught, and we assume a linear velocity profile at the surface, the velocity u x is non zero, its equal to the strain rate times y, whereas u y is equal to 0.

(Refer Slide Time: 01:21)



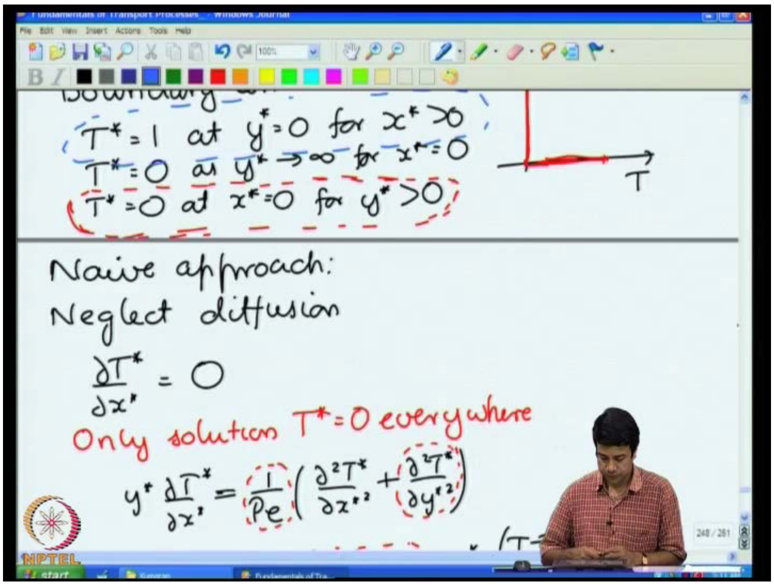
And in that case, if you just scale all length scales by the length of the palate capital L, the Peclet number becomes gamma dot square by alpha, where gamma dot is the strain rate at the surface.

(Refer Slide Time: 01:32)



And the equation ends up becoming of this form P e times y star times d T d x is equal to d square T by d x square plus d square T by d y square simplistically if you neglect the diffusion terms in the limit of Peclet number becoming large.

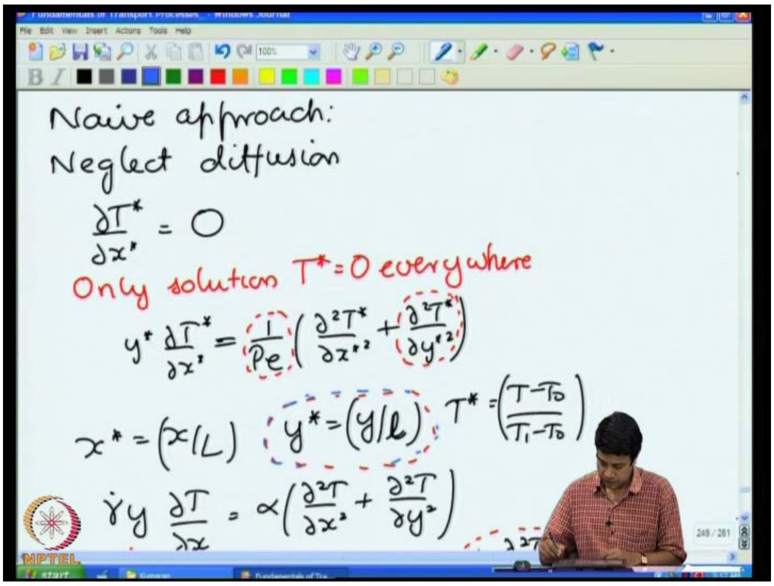
(Refer Slide Time: 01:55)



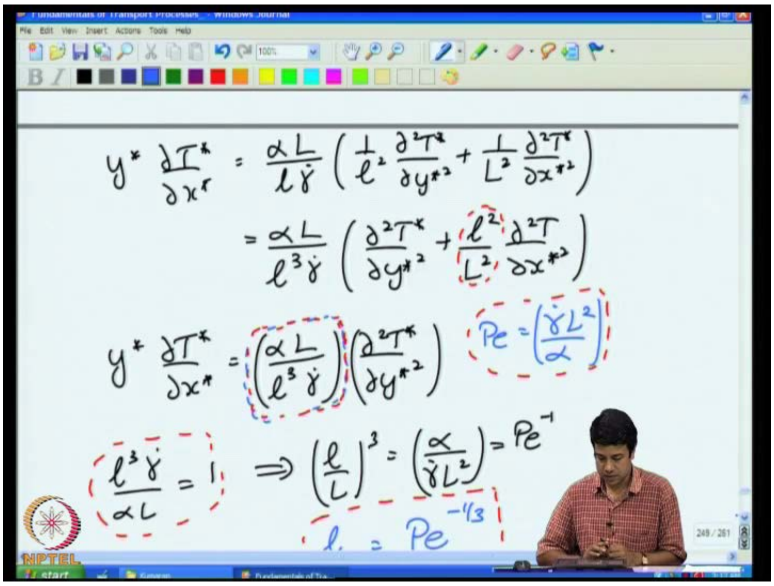
We just get the solution d T d x is equal to 0; that means, T has to be independent of x since T star at the inlet is 0 the only solution is that T star is equal to 0 everywhere. The flow in this argument obviously, was that we had scaled both the x and the y coordinates by capital L. If we convection is dominant, then there is flow sweeping past, the heat generated from the surface and that is the convective effects become larger and larger the whatever diffusion of heat takes place from the surface is going to get swept back faster and faster and therefore, its going penetrate only to a very small distance within the fluid. This penetration depth within the fluid of course, is determined by a balance between convection and diffusion, but this is the penetration depth is the length scale, which I should use for scaling the length in the cross stream direction.

So, the flow in the argument was that the length in the y direction is actually small compared to the length in the x direction, when convective effects are large compared to diffusive effects, and if the length scale is small the derivatives are large and therefore, if I scale my y coordinate by a length scale l, which is small compared to the total length of the plate.

(Refer Slide Time: 03:26)

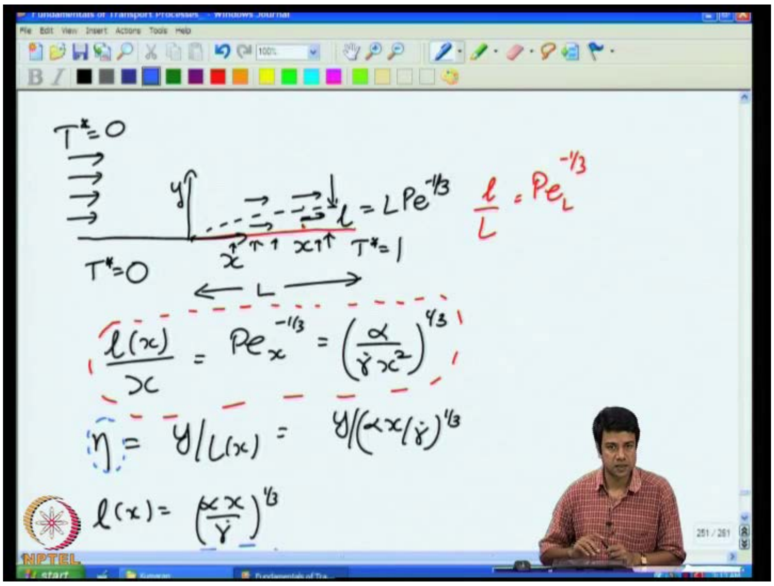


(Refer Slide Time: 03:35)



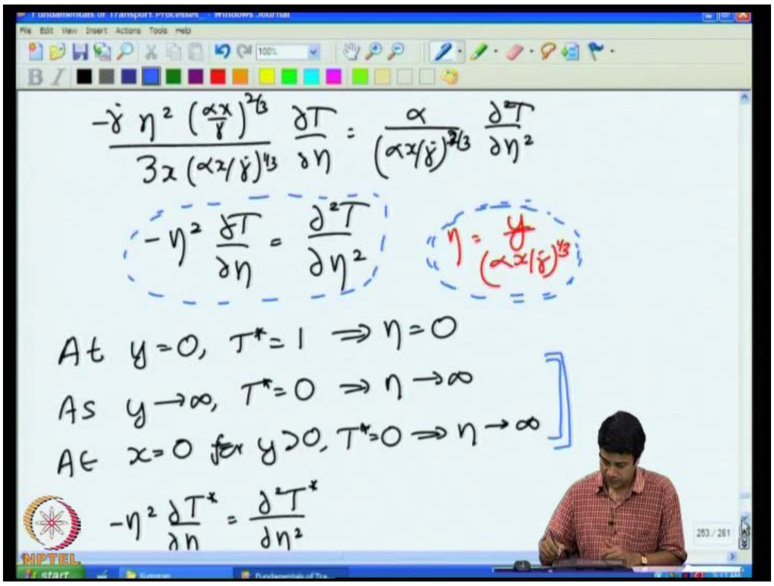
I can then get a balance between convection and diffusion provided this capital L is equal to p power minus one-third times smaller. So, this small l goes to 0 as Peclet number will goes to infinity in such a way that if I scale y by this small l, then the diffusion the cross stream diffusion continues to be of the same magnitude as convection even as the Peclet number becomes large. And then we had used physical arguments to determine, what that length scale should be at a given location x.

(Refer Slide Time: 04:16)



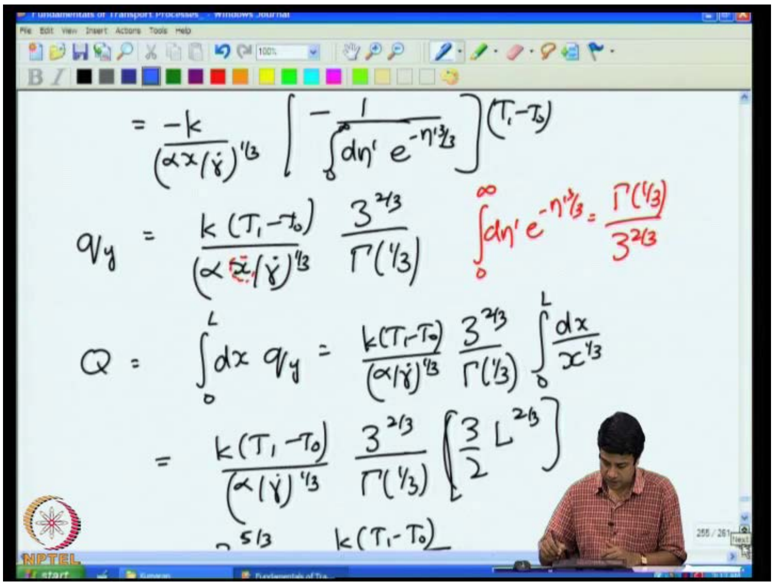
The velocity and the temperature fields should not depend upon the total length of the plate, but only upon the length from the beginning of the heated section. So, it is only from the length from the the the start of the heated section. The distance from there that should effect the temperature fields, because what happens at downstream will not really effect, what happens at a give location because convection is going downstream and we have neglected diffusion in the streamwise direction.

(Refer Slide Time: 04:514)



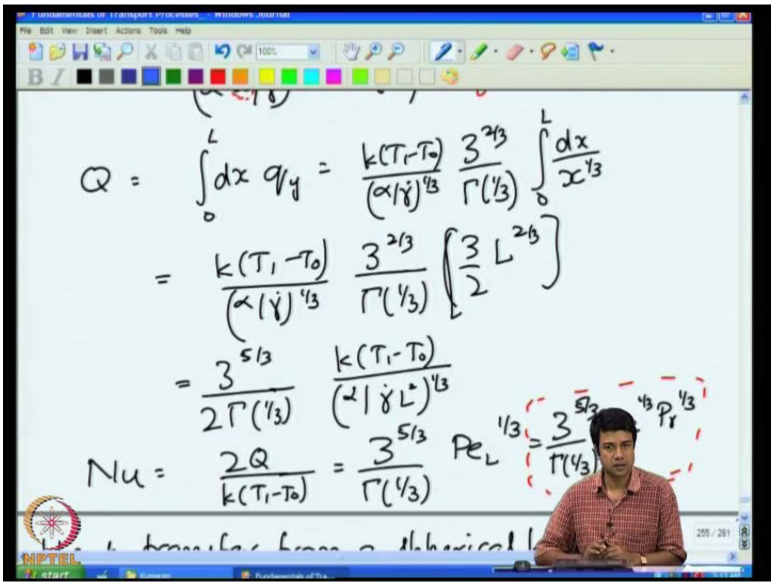
And using that argument, we manage to get a solution in terms of a similarity variable eta. The similarity variable eta was defined as y by alpha x by gamma dot power one-third in terms of this similarity variable, I got a solution for the temperature field.

(Refer Slide Time: 05:10)



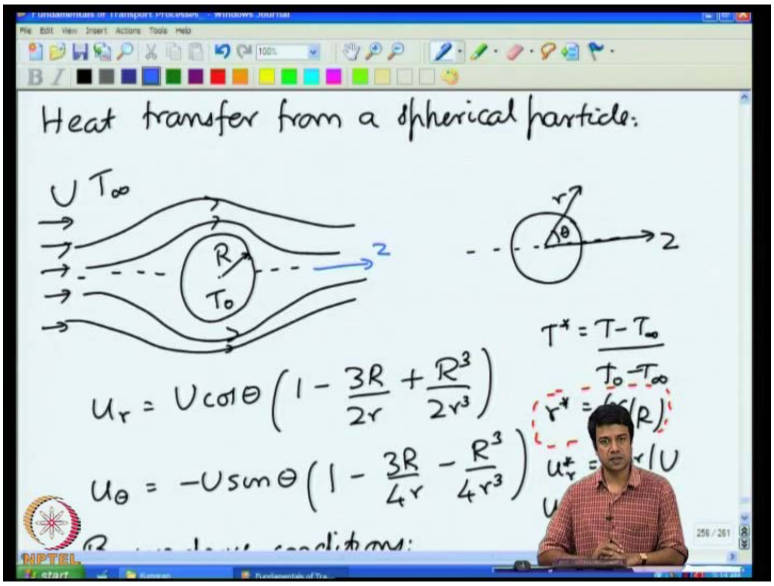
And from that I got the heat flux.

(Refer Slide Time: 05:13)



And finally, at correlation between the Nusselt number, the fractal number and the Reynolds number.

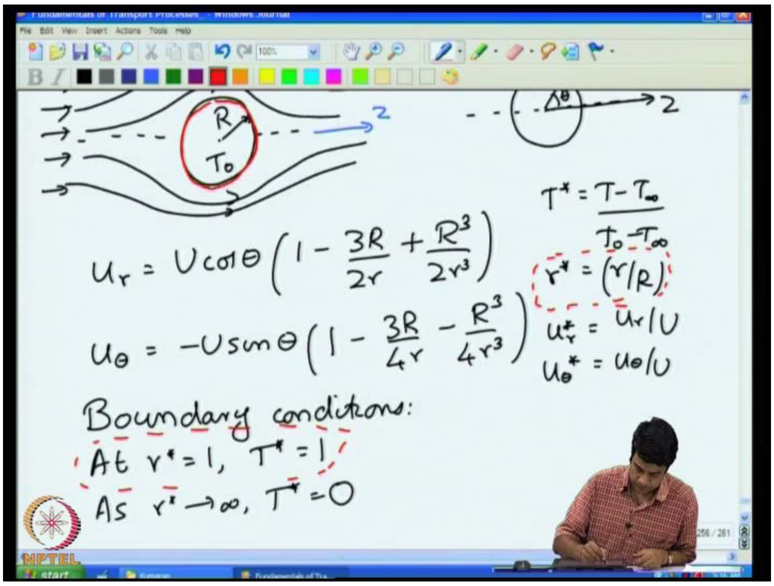
(Refer Slide Time: 05:20)



In last lecture, we were looking at the slightly more realistic case of the diffusion from a spherical particle, we have a particle at temperature T naught in a fluid, whose temperature is T infinity faraway and there is flow past this particle this flow is laminar flow low Reynolds number flow, the flow was of curse uniform is equal to capital U in the stream wise direction. So, in this direction in this z direction the velocity is capital u for upstream of the sphere however, the streamlines gets distorted as it approaches the sphere because the flow it has to go around this spherical particle which is at rest and if we uses cylindrical coordinate system r theta coordinate system. There is an axis for this flow there is z axis as shown in such a way that there is no dependence on the angle phi around this axis.

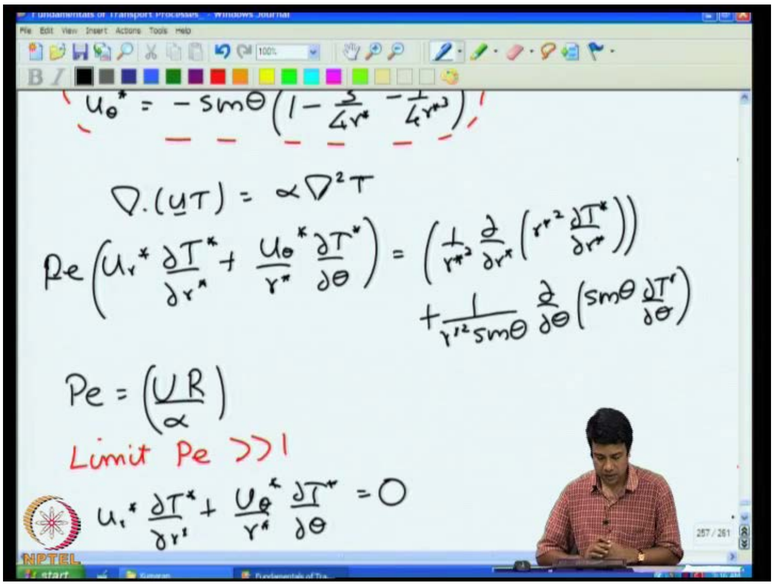
So, this is a two dimensional problem, it depends only upon r the distance from the origin and theta, which is the angle from the z axis; there is no dependence, there is no variation in either the velocity or the temperature as you go around this axis at constant r and theta. So, the velocity fields that you get by solving the Navier-stokes equation for this particular case, I have written them down for you here.

(Refer Slide Time: 06:55)



These are given to us the velocity fields the solution is outside the scope of this course, we will assume that these velocity fields are known, and then try to determine what is the temperature fields due to these velocity fields. We had defined the scaled temperature, distance as well as u r and u theta scaled by the obvious scales, the radius by capital R and the velocity is by capital U and the boundary conditions are T star is equal to 1 at the surface itself. So, you have a surface that is at a higher temperature and T star is equal to 0 far from the surface in the limit as r goes to infinity. So, those are the two conditions and I had written down the scaled velocities for you.

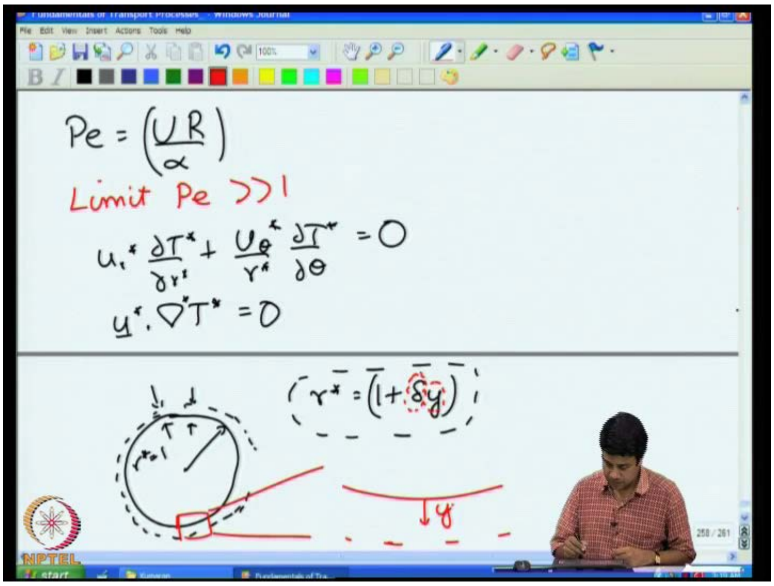
(Refer Slide Time: 07:49)



The convection diffusion equation at steady state del u T is equal to alpha del square T, when expressed in terms of the r theta coordinates scaled coordinates, I have the Peclet number times u dot grad u is equal to del square u, where the Peclet number is u times capital R by alpha in this case the velocity scale is u the length scale is the radius capital R.

If I consider the limit of very high Peclet number and neglect the diffusion terms all together, then I get an equation of the type u dot grad T is equal to 0 or u r d T by d r plus u theta by r d T by d theta is equal to 0 u dot grad is equal to 0 means, there is no variation of the temperature along the streamlines. Since there is no variation in temperature along the streamlines, the temperature far upstream is T infinity and there is no variation in temperature along streamlines; that means that the temperature everywhere along the streamlines has to be T infinity. So, there is no variation anywhere in the temperature. So, that is the result that you get just by neglecting the diffusion terms all together in the limit of high Peclet number.

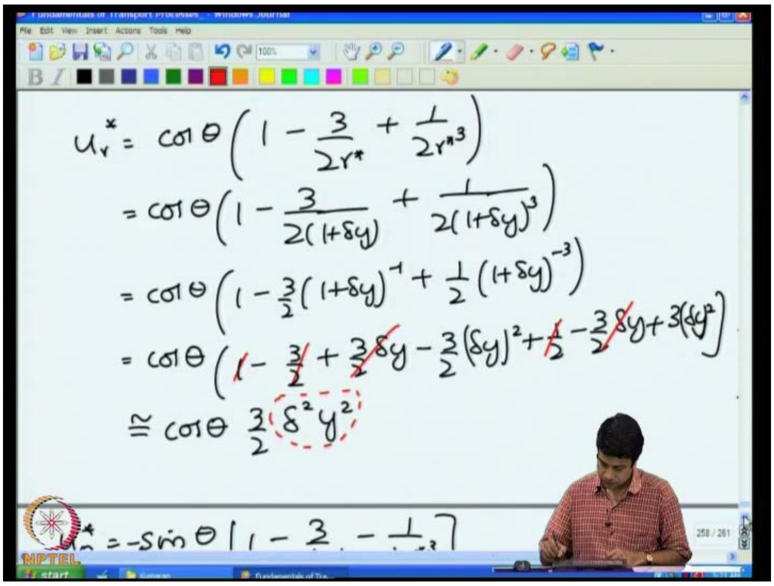
(Refer Slide Time: 09:37)



And as usual, the problem is that when I neglect diffusion, since there is convection only along the flow, there is no diffusion perpendicular to the flow at the surface itself the velocity is equal to 0. That means, when we neglect diffusion there is no heat transfer from the sphere surface to the flow, physically in the limit of high Peclet number there is heat being convicted downstream there is diffusion from the surface as the convection velocity increases, their downstream transport gets larger and larger. And therefore, the temperature the heat that is diffusing from the surface gets restricted to a smaller and smaller distance from the surface the boundary layer thickness.

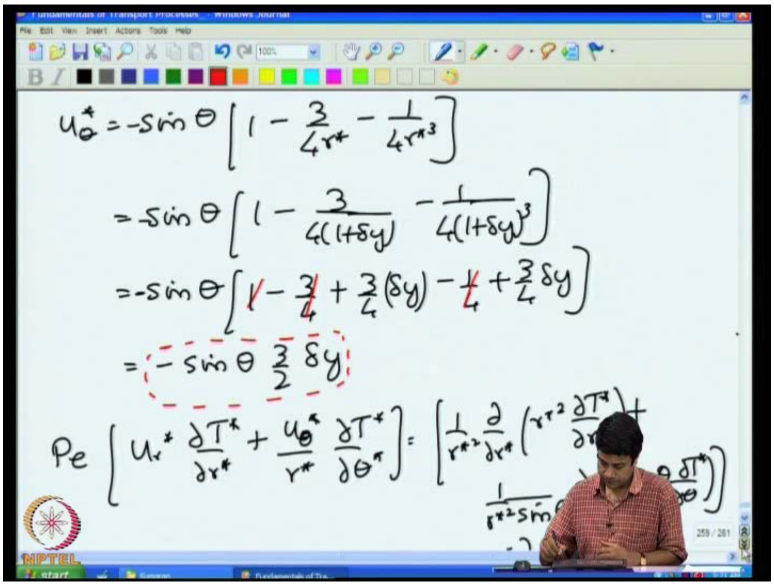
The boundary layer thickness becomes smaller and smaller and as the boundary layer thickness becomes small the gradients the temperature field become large and at some point, there will be a balance between convection and diffusion, when the boundary layer thickness is sufficiently large sufficiently small. To capture this we expand it we focused our attention on a thin region near the surface of the sphere on a thin region near the surface of the sphere, and define the distance y times delta as the distance from the surface y times delta is the distance from the surface delta is a constant a small number which decreases as the Peclet number increases, y is a coordinate a scaled coordinate which is order 1 it continues to be order 1 in the region, where the temperature disturbance is significant even as the Peclet number becomes large. So, therefore, I define r star is equal to 1 plus delta y and substitute that into the differential equation for temperature field.

(Refer Slide Time: 12:03)



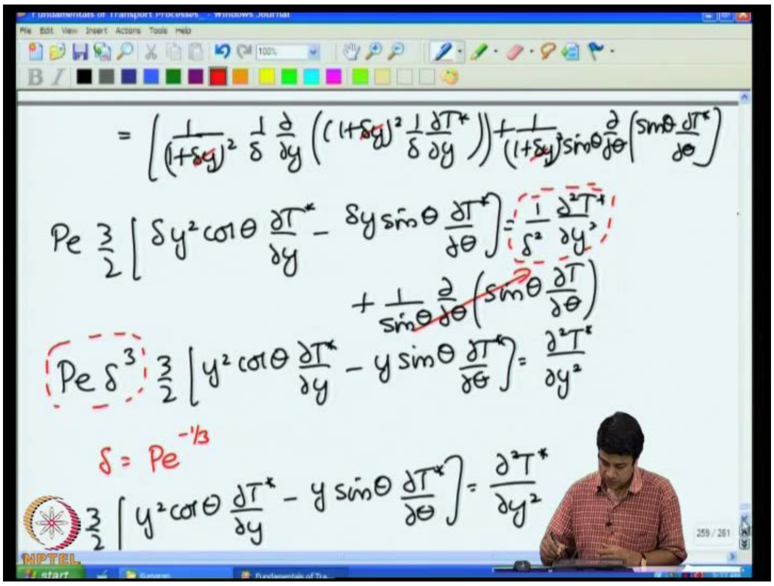
Since I am focusing attention on a thin layer near the surface, I do not need the complicated expression for the velocity components, I can use an expression for the velocity components, which is accurate near the surface and that velocity expression you get by writing down r is equal to 1 plus delta y and then expanding in a series in delta that series expansion tells us that u r star the dimensionless velocity in the r direction is equal to cos theta times 3 by 2 delta square y square. So, this velocity goes as delta square this is the radial velocity the velocity perpendicular to the surface. The velocity in the theta direction is obtained by a similar expansion.

(Refer Slide Time: 12:36)



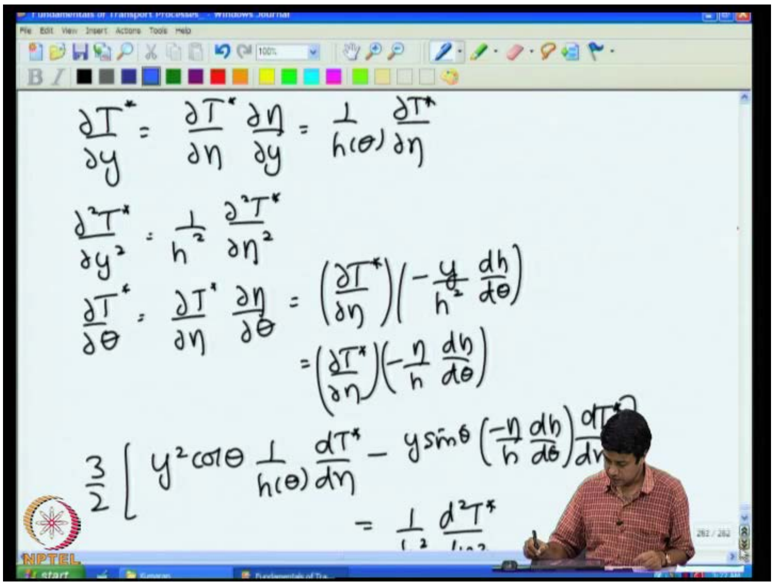
This velocity goes as minus sin theta into 3 by 2 delta y. So, this velocity goes as delta y in the limit as delta goes to 0, whereas if you recall u r was proportional to delta square times y square in the limit as delta goes to 0.

(Refer Slide Time: 13:087)



This was inserted into the conservation equation, this was inserted into the conservation equation and we did an expansion in delta y wherever we had 1 plus delta y, we neglected delta y in comparison to 1.

(Refer Slide Time: 15:03)



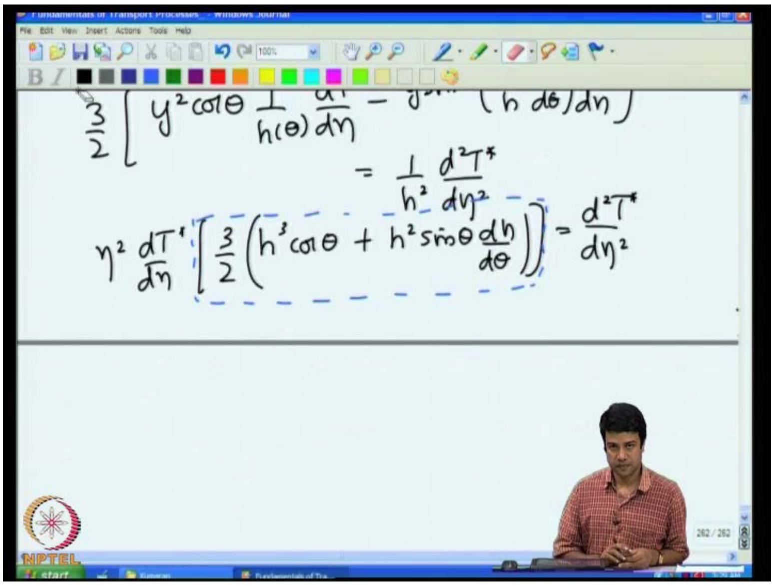
In this case as well, we found that the cross stream diffusion is actually large compared to the streamwise diffusion; in this case, r is the cross stream direction theta is the direction tangential to the surface as the boundary layer thickness becomes small. The boundary layer thickness proportional to delta the cross stream diffusion term goes as 1 over delta square, whereas delta, whereas the length scale in the streamwise direction is still capital R. So, the streamwise diffusion remains the same and as delta goes to 0 the cross stream diffusion becomes large compared to the streamwise diffusion and in this case we get a simplified equation is P e times delta (( )) times 3 by 2 y square cos theta d T by d y minus y sin theta d T by d theta is equal to d square T by d y square. So, this is the simplified equation that we have to solve in order to find out what is the temperature field very near the surface.

So, last lecture we started solving this problem by defining a similarity variable eta is equal to y by h of theta, where h is right now some unknown function. Now I can substitute for y and theta in terms of eta. So, I can substitute partial T by partial y is equal to partial T is equal to 1 by h of theta partial T by partial eta in similar manner partial square T by partial y square is equal to 1 by h square partial square T by partial eta square. So, these are the two derivatives with respect to y in the conservation equation in this conservation equation.

Now, I also have a derivative with respect to theta, I also have a derivative with respect to theta partial T by partial theta is equal to partial T by partial eta times partial eta by partial theta is equal to partial T by partial eta into minus y by h square d h by d theta differentiating by chain rule is equal to partial T by partial eta into minus eta by h d h by d theta. So, we put this all into this original differential equation 3 by 2 y square cos theta d T by d y minus y sin theta d T by d theta is equal to the second derivative. So, I have 3 by 2 into y square cos theta into d T by d y which is 1 by h of theta d. (No audio 16:59 to 17:43).

So, this is the equation that I get and I can multiply throughout by h square I can multiply throughout by h square, and substitute in for y in terms of eta, and I will get eta square 2 this is equal to d square t by d eta square . So, I just substituted in terms of the similarity variable, recall that I had used the similarity variable eta is equal to y by h of theta and use this in this original equation, I just substituted within this original equation and I ended up with this equation.

(Refer Slide Time: 18:56)

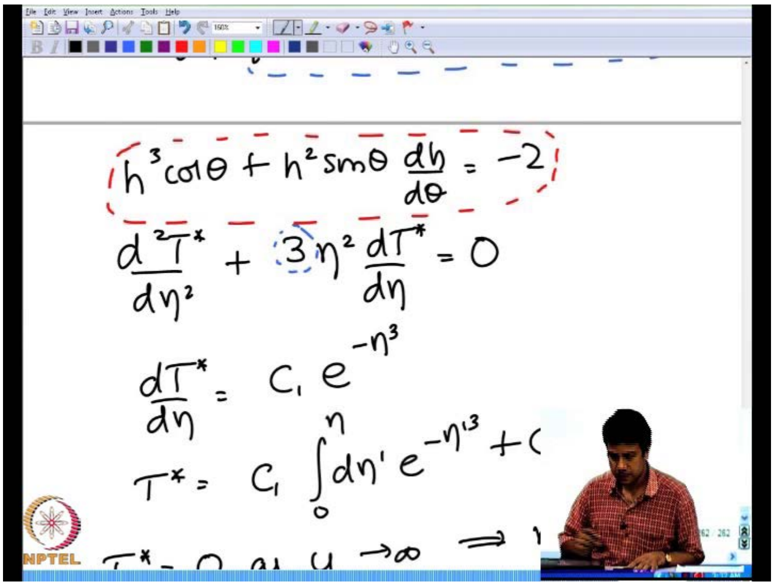


Now this equation has a this equation admits a similarity solution only if this term here is a constant, it admits a similarity solution only if this term here is a constant; if it is not a constant, it does not admit a similarity solution; if this a constant, then there is a the then the equation that results the equation that results is an equation, which depends only upon eta it does not depend individually on y and theta it depends only upon eta. So, this gives us the form of the function h this is the equation, which this function h has to satisfy if we are to get a similarity solution for this equation. What should be the value of this function? First of all it should of course, be negative, because if it were positive for d t by d eta, it get an exponentially increasing function temperature cannot increase exponentially as you go faraway therefore, this has to be a negative function.

Further note that this is a solution a function for this is a function h in the similarity variable, this is a function which is scaling the variable y which is the distance from the surface. As we have discussed, before if I change this is of course, this this is this is a boundary layer thickness this function here is some kind of a boundary layer thickness, if I change this function by a constant factor, then the result for the temperature field in terms of eta will change, where the temperature field in terms of the original variable r will not change, because this is a scaling that I am using in order to get reduced variable.

If I change that the solution that I get in terms of eta will change with the solution that I finally, get in terms of r will not change . So, I can choose any constant value provided it is negative for this function. So, long as I choose a negative constant value for this function, I~~I~~ will always get the same solution which is a positive value of course, I get an exponentially increasing temperature and so, I cannot choose a positive value, but any negative value that I choose, I will end up getting the same value for this function.

(Refer Slide Time: 22:26)



Now, the simplest negative value that I choose in this case the most convenient one is to choose h cube cos theta plus h square sin theta d h by d theta is equal to minus 2 if I choose that the equation for the temperature in terms of eta becomes d square T star by d eta square minus 3 eta square d T by d eta is equal to 0. This gives a solution for the temperature T star is equal to d T star by d eta is equal to some constant e power minus eta cubed I~~i~~ just integrate this once and I will get d T by d eta is equal to constant e power minus eta cubed therefore, T star is equal to c 1 e power minus eta cubed I am sorry plus c 2.

And now I have satisfy the boundary conditions, the boundary conditions are T star is equal to 0 as y goes to infinity, which corresponds to eta goes to infinity T star is equal to 1 at y is equal to 0, which corresponds to eta equal to 0. So, these are the two boundary conditions and in terms of these I can get a solution T star is equal to 1 minus integral 0 to eta d eta prime e power minus eta prime cubed by integral 0 to infinity. So, this is the final equation that I get for the scaled temperature field. So, this is a final scaled temperature equation.

In terms of the variable eta, which is y by h of theta, and you can see that this equation looks remarkably similar to the equation for that we got for the solution past a flat plate, this equation here for the flow past a flat plate, if I just scale eta as eta by 3 power one-third, I will end up with exactly the same equation for the past a sphere. And in fact, this will be the equation for the flow past an object of any shape provided, you have no slip conditions at the surface provided; you have no slip conditions at the surface. This is the equation that you will get for the flow past an object of any shape what changes is this 1, what changes is the form of the function h in the similarity variable. This will of course, depend upon the shape of the object as well as the velocity field close to the object.

So, let us try to solve this equation. The equation that we get is h cubed cos theta plus h square sin theta d h by d theta is equal to minus 2. I can write this as sin theta by 3 d of h cubed by d theta plus h cubed cos theta is equal to minus 2, just writing in terms of the derivative of h cubed with respect to theta. And we can use the usual substitution that we have been using before x is equal to cos theta, which means that d x is equal to minus sin theta d theta. So, with this substitution, this will become minus sin square theta by 3 d of h cubed by d x is equal to plus h cubed x is equal to minus 2 or minus of 1 minus x square by 3 d of h cubed by d x plus h cubed x is equal to minus 2.

So, this is an inhomogeneous equation that we have to solve for h and for h cubed, I can separate out this function h cubed in to a general solution plus a particular integral, I can separate out the function h cubed in to a general solution plus a particular integral, the general solution satisfies the equation 1 minus x square by 3 d g by d x minus x g is equal to 0. So, this is the general solution and I can integrate this out quite easily d g by d x is equal to 3 x g by 1 minus x square, which will give me the general solution as g is equal to 1 by 1 minus x square the general solution is 1 by 1 minus x power 3 half. So, this is the general solution.

Now, the particular integral I can actually write it by integrating by parts I can write the particular integral p of x is equal to g of x times other some other function q of x and this satisfies 1 minus x square by 3 d by d x of g of x q of x minus x is equal to 2 and I can expand out this integral to get 1 minus x square by 3 d q by d x is equal to 2 and this will give me d q by d x is equal to 6 by 1 minus x square g of x.

Therefore, q of x is equal to integral from x from minus 1 to x d x prime 6 by 1 minus x square and since g was equal to 1 over 1 minus square power 3 half, this basically becomes equal to is equal to integral minus 1 to x d x prime 6 into 1 minus x prime square power half therefore, the total solution for h cubed is going to be equal to some constant divided by 1 minus x square power 3 by 2 plus 6 by 1 minus square power 3 half integral minus 1 to x d x prime 1 minus x prime square power half. So, that is the final expression for h cubed it has 1 constant in it, it has one constant in it c what should the value of that constant be.

h is a function of theta note that we had taken theta to be 0 along the z axis we had taken theta to be 0 along the z axis; that means, that along the minus z axis theta is equal to pi. So, the upstream stagnation point for the sphere is at theta is equal to pi the downstream stagnation point is at theta equal to 0, the flow is incident on the sphere at theta is equal to pi the flow is incident on the sphere at theta is equal to pi; that means, that at theta is equal to pi 1 would expect the boundary layer thickness to be its minimum value at theta is equal to pi you would expect the boundary layer thickness to be its minimum value.

Note that here we have used x is equal to cos theta x is equal to cos theta. So, theta is equal to pi means x is equal to minus 1. So, at the value x is equal to minus 1 the boundary layer thickness should be finite that would require that this constant has to be equal to 0 because if this constant is non 0 note that the second integral has an integral from minus 1 to x . So, this second term here is finite at x is equal to minus 1 therefore, if the boundary layer thickness has to be finite at x is equal to minus 1 or theta is equal to pi this constant has to be equal to 0 and therefore, this boundary layer thickness h cubed has to be equal to 6 by 1 minus x square power 3 by 2 integral minus 1 to x d x prime 1 minus x prime square power half.

So, this is the boundary layer thickness, note that this boundary layer thickness, we fix the constant in such a way that the thickness was did not go to infinity at theta at x is equal to minus 1 or cos theta is equal to minus 1 or theta is equal to pi, however the boundary layer thickness dose go to infinity at x is equal to plus 1, because the denominator of this h cubed term goes to 0 the numerator is actually finite and therefore, the thickness goes to infinity.

What that means, physically is that there is a boundary layer around the sphere, but the part that I did not continue for you here this is the upstream side here, you have some finite value of h on the upstream side on downstream side h actually goes to infinity. That means, that the boundary layer looks something like this the value of h actually goes to infinity as you go downstream this region here is what is called the wake downstream of the sphere there is convection downstream. Therefore, the effect of the velocity the the velocity for the stagnation streamline both upstream and downstream are identically equal to 0, and because of that there is a diffusive effect downstream of the sphere, which gets reflected in wake in a long wake as you go downstream; our analysis of the boundary layer thickness has only told us.

So far that the wake exists it does not tell us any details about the temperature within the wake and that is, because I have explicitly assumed that delta is small compared to 1 when I did the boundary layer analysis. And therefore I require that eta, which was y by h of theta has to be order 1 if h of theta becomes large then that condition is no longer satisfied and I cannot use the analysis that I have been using. So, far if I really wanted to study what was the temperature field in the wake. What I would need to do is to use a small angle expand in the angle theta away from this and relax the assumption that the radius has to be closed to capital R. So, I will do an expansion in this angle theta about 0 not in the radius, and if I expand in the angle theta, then I will be able to capture the temperature field within this thin wake region downstream of the sphere.

So, in all flows past bluff bodies, there is going to be wake at the downstream side upstream we make, we enforce the condition that the boundary thickness has to be finite because the flows is incident on the upstream side of the sphere and it was sweeping the heat that is generated from the surface is being swept backwards on the upstream side is being swept backwards on the downstream side as well. And that backward sweeping on the downstream side is causing a wake on the downstream side of the sphere and the boundary layer analysis is able to pick up the fact that the wake exits it has not told us the details of the temperature field within the wake, but it at least picks up the fact that there is a wake where h is going to infinity and within that region I have to use a different scaling to get the details of temperature field within that wake.

However, as far as the heat flux is concerned this wake is not really important the heat flux coming out of the the the surface of the sphere is large compared to the heat flux within the wake region. And therefore, I can get the details the the the the the expression for the heat flux without worrying about what the temperature field in the wake is. So, we can proceed now that we have a solution for both the function h, which satisfies the similarity variable as well the temperature field. I can go ahead and calculate what is the heat flux the heat flux at the surface of the sphere is given by q r at r is equal to 1 is equal to k times make this at r is equal to 1 is equal to minus k into T naught minus T infinity by r partial T star by partial r star at r star 1. Now I express r star in terms of y r star is equal to 1 plus delta y. So, this is equal to minus k into T naught minus T infinity by r 1 by delta partial T by partial y at y is equal to 0.

Note that r star is equal to 1 corresponds to y is equal to 0 and then I express y in terms of eta y is equal I am sorry eta is equal to y by h of theta. So, therefore, I can write this as minus k into T naught minus T infinity by r delta h of theta partial T by partial eta and then I have an expression for T in terms of eta and therefore, in this expression if I take 1 derivative d T by d eta is equal to minus 1 by integral 0 to infinity and this can be substituted there.

To minus 1 by. So, this is the final expression for the heat flux from the surface I can write this as k into T naught minus T infinity by r delta h of theta into 1 by integral 0 to infinity d eta prime and eta prime cubed now. So, this is the local heat flux at every position on the sphere this is a local heat flux at every position to get the total heat flux total heat being emited by the sphere I need to integrate this over the entire surface area. So, the total heat coming out is equal to integral of r square sin theta of q r at q r at given theta and phi r square because the surface area of the sphere the surface area of the sphere is capital r square times the angle sin theta times d theta which is the differential element.

So, r sin theta d theta the differential element of theta direction and r sin theta d phi in the phi direction theta goes from 0 to pi and phi goes from 0 to 2 pi there is no variation in the phi coordinate of course, and. So, I will just get 2 pi r square integral 0 to pi sin theta d theta times q r at theta phi. So, I substitute this expression here to into this to get the total heat flux k into T naught minus T infinity by r delta 1 by times integral over theta 0 to pi sin theta d theta to 1 by h of theta.

So, I have a functional form for this h of theta it is a complicated form expressed in terms of an integral here h cubed is given by this. So, its expressed in terms of integral, but I do have an analytical form and I can solve for this in order to get solution for the heat flux and if I solve for the heat flux what you get is, this is equal to 0.2491 into 2 pi r k T naught minus T infinity divided by delta delta is actually P e power one-third this is equal to P e power one-third and so, this gives me the total heat that is coming out of the sphere per unit time from this I can get a Nusselt number correlation n u is equal to 2 q by 4 pi r square into k into t naught minus t infinity by r and this will be equal to 0.2491 P e power one-third.

So, this is the correlation for the Nusselt number as a function of Peclet number for heat transfer from a spherical particle once again its equal to some constant times p e power one-third. So, it differs from the result that I got from a flat plate only in as far as there is a difference in that constant in all cases laminar flow near the solid surface with no slip condition at the surface the Nusselt number is always proportional to the Peclet number power one-third and this is, because we are we have heat transfer at a solid surface the velocity field in the fluid at that surface has to decrease to 0 at the surface itself. So, it has to increase linearly with distance away from the surface; and if the boundary layer thickness is small compared to the macroscopic scale, I can use a linear approximation for the velocity field near the surface. So, this result is a general result which holds for any form of of an object with flow past it provided the velocity is 0 at the surface itself and it increases from 0 very close to the surface.

So, let me give you a little bit more of physical insight into why this Peclet number power one-third scaling comes there is of course, a constant we evaluated the constant exactly for a flat plate and for a sphere where we knew what the velocity profile was that constant will change if the object is of different shape, but the scaling remains the same. So, let us say I have an odd shaped object with flow past that object and if I look at a region which is very close to the surface if I look at a region which is very close to the surface and I am focusing on a thin region therefore, locally I can take x as the streamwise coordinate, y as the cross stream coordinate, locally at a point because I am focusing on a very thin region.

So, long as the extent of the region is small compared to the radius of curvature that region will look like a flat surface to me. So, I focus on this thin region and look at the velocity field as the flow goes past this region, the velocity field u x is equal to 0 at the surface therefore, this has got to be equal to y times some function of x near the surface it has to be increased linearly away from the surface unless for some strange reason the strain rate is also 0 at the surface. So, the strain rate is not 0 at the surface then the velocity field at the surface has to decrease to 0. Therefore it has to go as a of x times y as you go away from the surface very near the surface this is only when the length scale for the boundary layer thickness is small compared to the macroscopic scale over, which the velocity varies in the case of flow past a particle the velocity was varying over length scale capital R. So, long as your boundary thickness is small compared to capital R closed to the surface you can make this approximation.

Now, for an incompressible flow the velocity components satisfy the equation of the form d u x by d x plus d u y by d y is equal to 0 this once again is outside the scope of the present lectures, but we will assume that this is given this is equal to saying del dot u is equal to 0 at the surface itself d u x by d x plus d u y by d y has to be equal to 0; that means, that d u y by d y it is got be equal to minus d u x by d x is equal to minus y d a by d x where a as I said is some function of x would be the theta coordinate for example, the flow past a sphere y would be the radial coordinate. Therefore u y is got to be equal to minus y square by 2 d a by d x note that u y also has to go to 0 at the surface. So, if I integrate once I will get u y going as y square recall that we got u theta going as delta y and u r going as delta y the whole square in the flow past a sphere the reason for that is this incompressibility condition u y goes as minus goes as y square and u x goes as y from the surface because u x has to be 0 no slip condition at the surface.

Substitute that into the differential equation u x d T by d x plus u y d T by d y. Note that in this diffusion term, I already know that the streamwise diffusion has to be small compared to the cross stream diffusion since the streamwise diffusion has to be small compared to the cross stream diffusion, I can write this as alpha times partial square T by partial y square. So, I will have d T by d x into a y minus y square by 2 d a by d x d T by d y is equal to alpha d square T by d y square and now if I scale y star is equal to y by delta and x star is equal to x by l where l is the macroscopic scale and delta is the boundary layer thickness then in this equation, I get a y star times delta d T by d x star minus y star square by 2 delta d a by d x star just scaling the equations and then if I multiply throughout by delta square by alpha, if I multiply throughout by delta square by alpha, I will get delta cubed by l a by alpha into y star partial T by partial x star minus y star square by 2 1 over a d a by d x star is equal to partial T by partial y star square and this will straightaway give you because, I have delta cubed there this will give me delta goes as p power minus 1 by 3.

So, that is a common feature of any flow past a surface with no slip condition at the surface if a is a velocity scale no a is a a velocity scale for the strain rate if a is the scale for the strain rate then straightaway from this equation, I will get this going as alpha by a l square delta by l power one-third. So therefore, this boundary layer thickness of order Peclet number power minus one-third is a common feature of all flows past solid surfaces where the velocity has to decrease to 0 at the surface itself in all such cases the boundary layer thickness goes as p power minus one-third and for that reason the Nusselt number goes as p e power plus one-third.

The coefficient in this correlation of course, depends upon the specific problem that you are solving it depends upon the specific form of the velocity profile here it depends upon the specific form of this velocity profile you have to put in the specific form of the velocity profile into this equation and then reduce this 1 using a similarity transform you have to reduce this 1 using a similarity transform where use a similarity variable eta is equal to y by some function g of x use this similarity transform in the equation and then find out the condition on the boundary layer thickness g of x, which satisfies which reduces the conservation equation towards similarity equation and then from that you calculate the heat flux the way we done for the flow past a sphere, but independent of what the configuration is you will always get Nusselt number going as p e power one-third whenever there is a flow past a solid surface . So, this is one class of boundary layer problems transport from a solid surface with no slip boundary conditions at the surface.

We had already done another problem for the transport from a liquid transport to a liquid gas interface we had already done this problem of transport to a liquid gas interface in the limit of high Peclet number. If you recall we had solved the problem where there was a a liquid film flowing down a channel and we had calculated the velocity profile the the the temperature profile in a manner similar to the temperature profile for the flow past a flat surface and in that case as well we had got something that look like a similarity solution however, in that case if you had gone ahead and calculated the Peclet number versus Nusselt number correlation we actually got a result Nusselt number goes as p e power half different from the one-third that we got in this case that is another class of high Reynolds number high Peclet number boundary layer problems we will see why that scaling of P e power plus half comes in that particular case that is because that was a film and that was a (( )) between a gas and a liquid and for a gas liquid interface the velocity is non 0 at the surface and because of that relations of this kind actually do not apply for example, a gas liquid interface and because of that the scaling is different we will see that in the next lecture before we close this series of lectures we will see you next time thank you.