Fundamentals of Transport Process
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Module No. # 07
Lecture No. # 37
High Peclet number Transport
High Transfer from a Spherical Particle – 1

This is lecture number 37 of our course on fundamentals of transport processes and we were discussing a general convection-diffusion equation for the concentration or temperature field.

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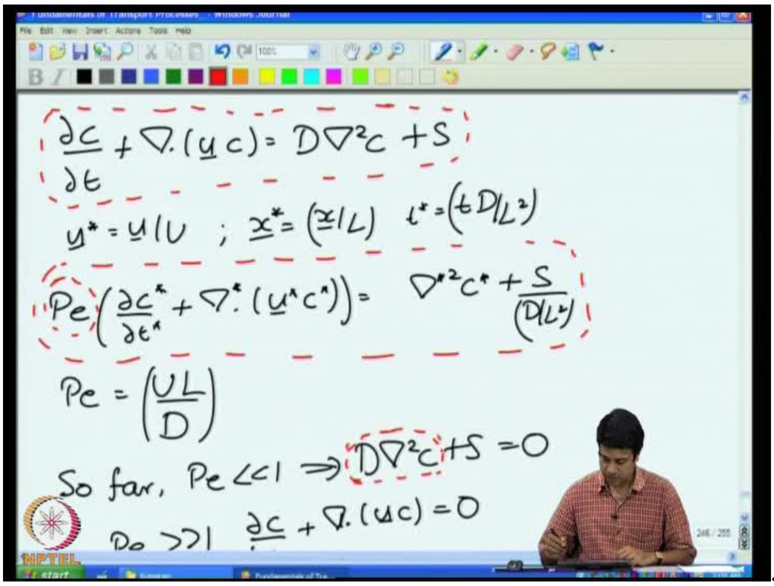

So, the equation has the general form d C by d t plus the divergence of velocity times the concentration is equal to d times the Laplacian of the concentration plus any source or sink of mass or energy within the system. When you scale, the length scale where characteristic length L and the velocity, where characteristic velocity U, you get an equation of the form shown here, where you have Peclet number times the time derivative and the convective term on the left is equal to the Laplacian of the concentration field plus the source or sink of energy or mass, where the Peclet number is U times L by D, U is characteristic velocity, L is length and D is the diffusion coefficient

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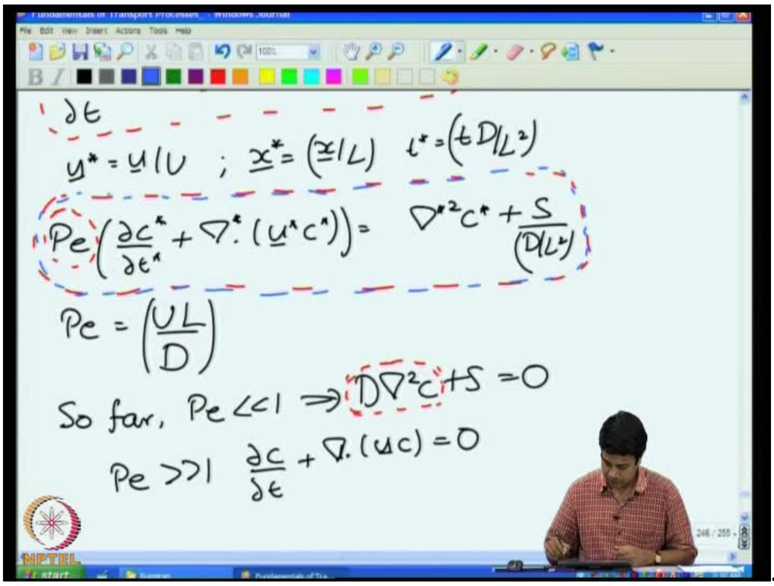

First, we looked at strategies for solving the limit, where the Peclet number is small compared to 1. In that case, we are solving D del square C plus S is equal to 0 and we looked at various way of doing that; basically, ways of solving the Laplace equation.

We did it first by separation of variables in Cartesian coordinates, in cylindrical coordinates as well as in spherical coordinates and in particular, in spherical coordinates we saw that the solution have the form of spherical harmonic solutions.

We also saw another way of solving the same equation using the frame work of point source, a delta function source and that gave as results for a relationship between the spherical harmonic expansion on the one side and the distribution of sources in sinks on the other side.

So, those are the 2 strategies that we have looked at so far for solving the diffusion equation in the limit of Peclet number being small compared to 1, where the diffusion dominated limit. Then, we looked at the case where the Peclet number is large compared to 1; P is much greater than 1 and if you just look at this equation, simplistically, one might think that all you need to do is to neglect the diffusion terms on the right hand side completely and just solve the equations for the convective path alone.

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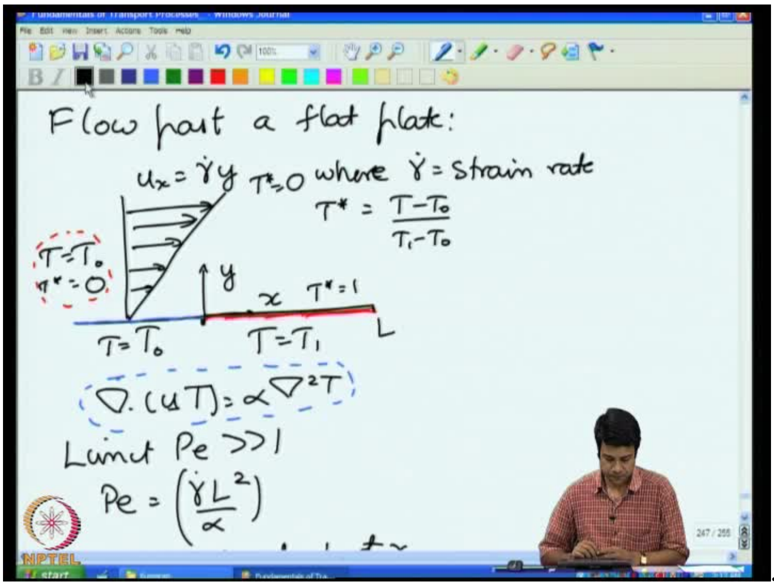

Last class, we looked at the simplest case - the flow past of flat plate, where I have a linear velocity in the flow past the flat plate. The velocity in the x direction, x is the flow direction - the string wise direction, y is the cross stream direction, where there is a variation in velocity.

We assumed a linear velocity profile at the surface. u x is equal to gamma dot times y, where gamma dot is the strain rate and at steady state, we were trying to solve the equation, the divergence of u times T is equal to the thermal diffusivity times the Laplacian of the temperature.

The configuration was as follows. We have a fluid which is incident on this heated surface at a temperature T is equal to T naught. The surface is unheated up to x is equal to 0. So, there is a cold section of the surface up to x is equal to 0, which is at the same temperature T naught as the incoming fluid and at x is equal to 0, the heated section starts - T is equal to T 1 and it continues for some length L.

As I told you, this is the simple approximation for the flows in heat exchangers, for example, where as the fluid in the tube sides flows into the heat exchanger, initially, it is at some cold temperature and at the inlet of the heat exchanger, there begins a heated section and it continues downstream.

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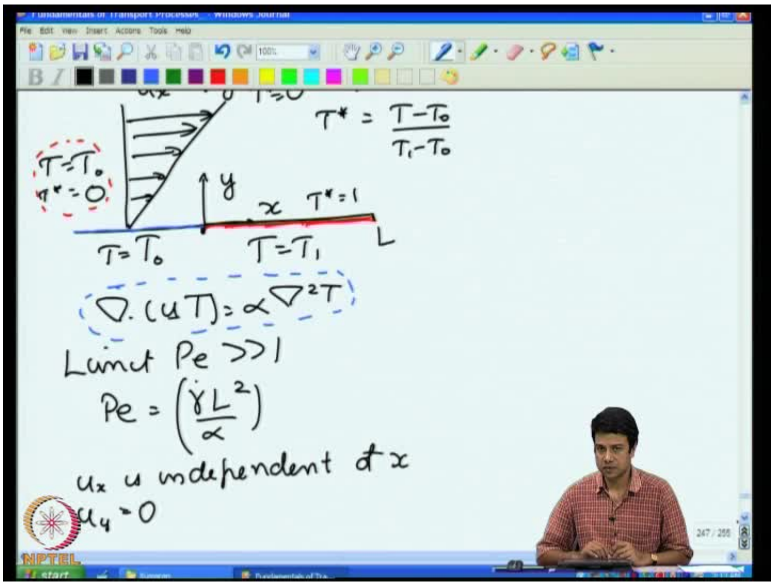

So, that was the configuration that we were trying to solve for in the high Peclet number limit. In this particular case, the velocity itself is not specified; what is specified is the velocity gradient gamma dot. This velocity gradient has dimensions of one over time. The only length scale in the problem is capital L, the length of the plate and therefore, I can define the Peclet number as gamma dot times L square by alpha.

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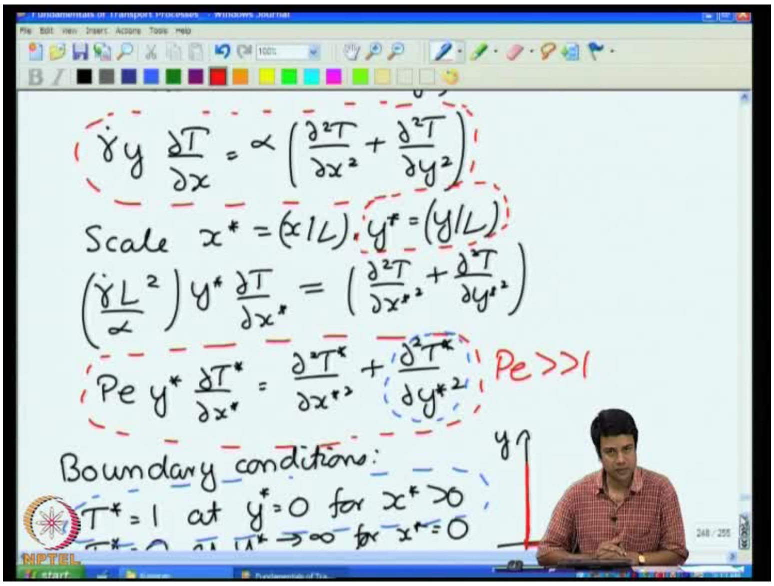

Note also that u x is independent of x is a fully developed flow and u y is equal to 0 everywhere. So, the flow is only in 1 direction. When we put this into the conservation equation, we got an equation of this form and if you scale x and y by L, which is the only length scale in the problem, we get a scaled equation with a Peclet number in it, as expected with the scaled equation of Peclet number in it.

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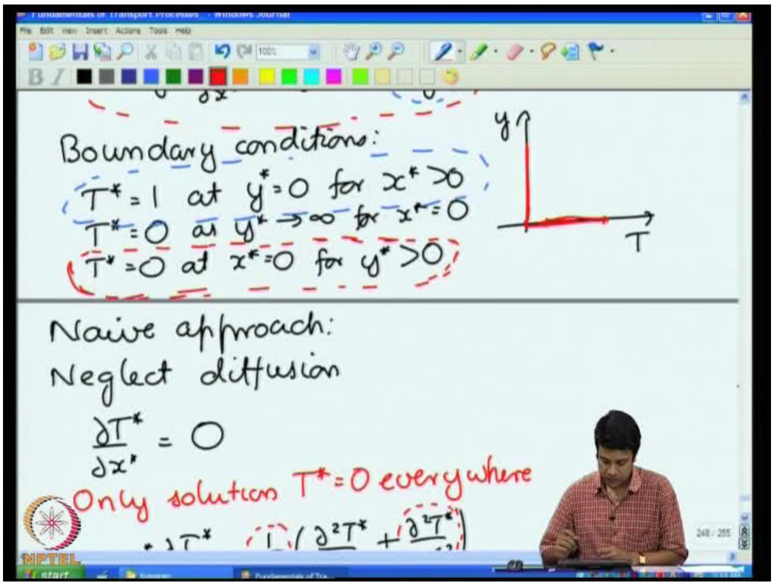

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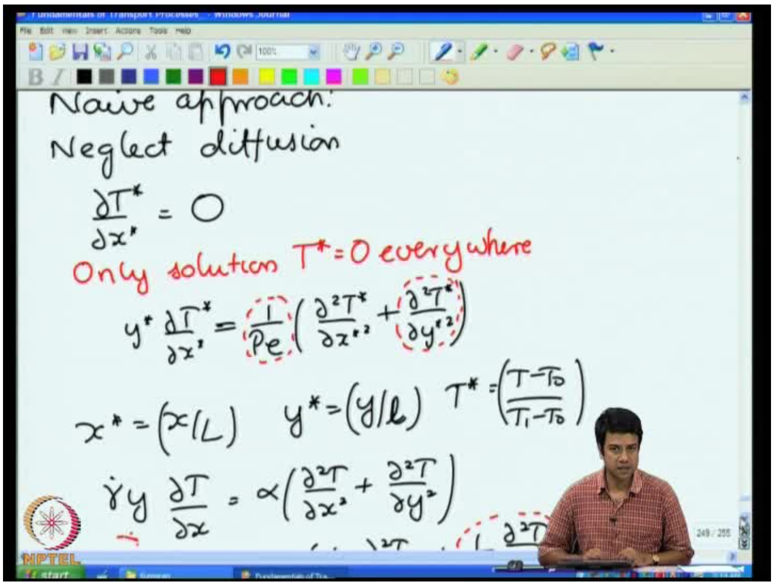

If we just simplistically neglect diffusion, we just get the d T by d x is equal to 0 and the only solution of that is T is equal to 0 everywhere within the flow. Of course that does not satisfy the boundary condition that T star has to be equal to 1 on the plate surface itself and I would explain to you why mathematically that there was this contradiction.

When we neglected the diffusion terms, the diffusion terms contain the highest derivative; they contain the second derivative with respect to the y coordinate and therefore, we have two boundary conditions which can be satisfied in general.

However, when we neglect the diffusion terms, we are neglecting the highest derivative and therefore, we end up with an ordinary equation in the y coordinate, which does not contain any boundary conditions and because of this, we are unable to satisfy boundary conditions.

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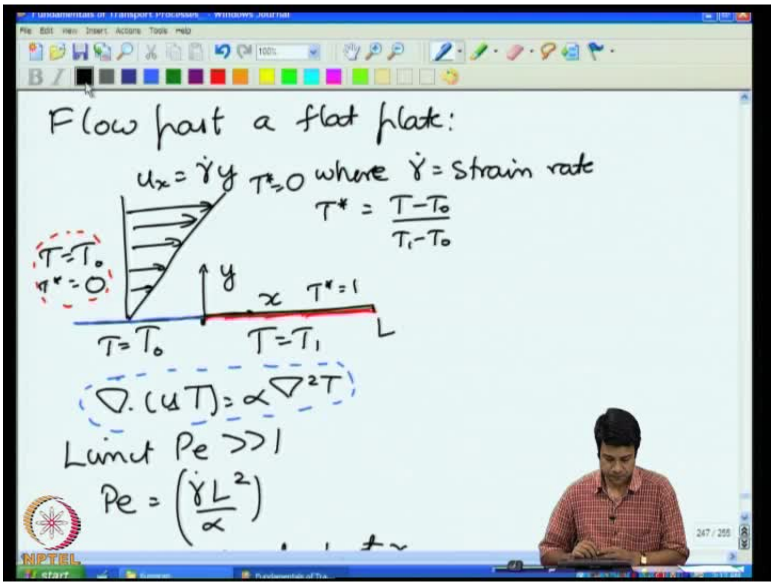

Physically, the reason that we are not able to satisfy boundary condition is because the flow is parallel to the surface - it is tangential to the surface. At the surface itself, there is no velocity perpendicular to the surface. Convection can transport mass, momentum, energy only in the direction of flow.

Since there is no flow perpendicular to the surface, there is no convection transport perpendicular to the surface. Therefore, the only way that heat can be transported from the surface to the fluid is due to diffusion due to the molecular motion of the molecules.

We have neglected the diffusion term in this case and therefore, we are not able to satisfy boundary conditions. Since we have neglected diffusion, there is no way for heat to be transferred from the surface to the fluid and because of that, we are not getting heating anywhere within the fluid.

So, what is the resolution of this problem? As I told you, when we scaled the x and y coordinates by L, we assumed that capital L was the length scale for the variation of the temperature field. However, we are looking at a situation where there is convection - rapid convection along the flow direction and there is the diffusive effect is small.

So, you would expect the flow to sweep any heat that is transported from the surface to sweep that downstream and physically, one might expect the effect of the plate to be restricted to a thin layer near the surface. Diffusion is present, nevertheless and therefore, one cannot have discontinuity in the temperature, but that diffusion effect is small compared to convection over length scale comparable to the length L and because of this, we would expect convection, the heating due to the plate to be restricted to a thin layer near the surface.

If the temperature the length scale for the variation of the temperature field is small compared to capital L, then the gradients are large and because the diffusion term has the second derivative with respect to y, if the length scale is small, the derivative is large and one could still have balance between convection and diffusion. How does one decide what is the length scale? We postulate length scale, put it into the equation and see what it has to be in order that convection and diffusion are of the same magnitude even as the Peclet number goes to infinity.

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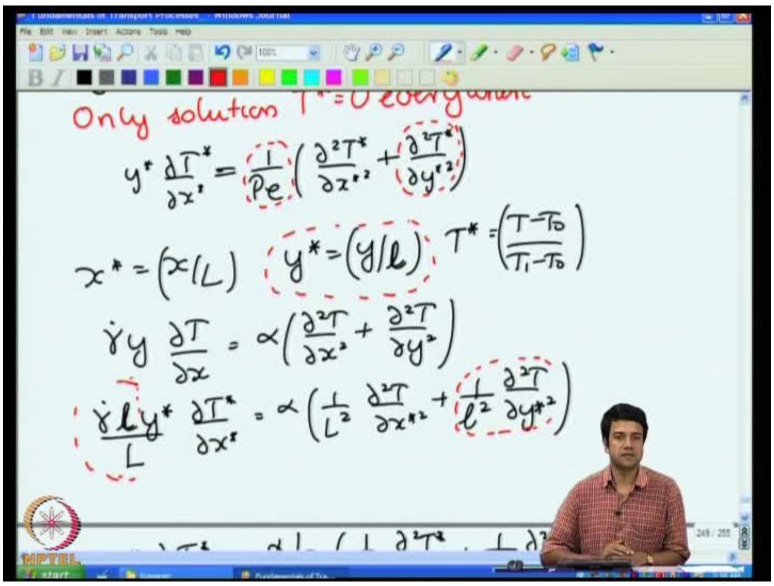

So, that is the strategy that we followed in the last lecture. We postulated a length scale small l such that the effect of diffusion is felt only within that length scale near the surface. So, we scaled x by capital L, which is the total length of the plate because that is the length scale over which there is variation in the x direction and we scaled y by small l, which is the smaller scale over which there is balance between convection and diffusion and we put that into the conservation equation.

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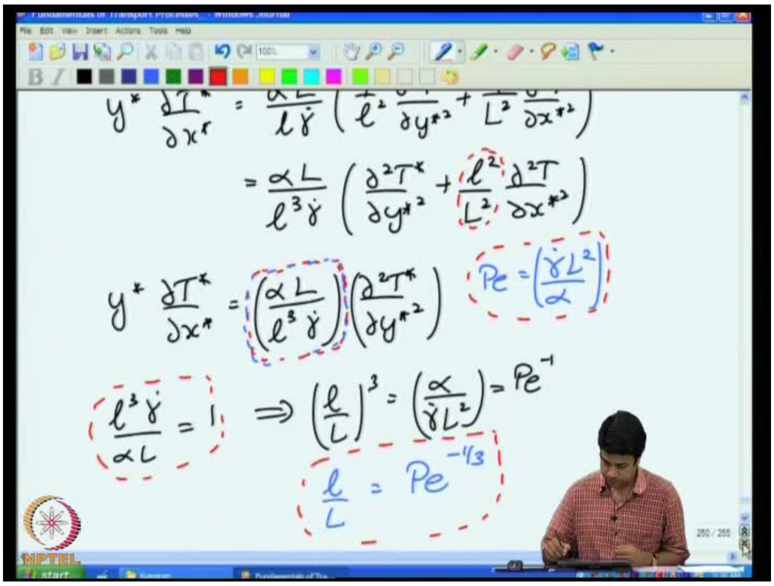

Finally, we found that this dimensionless group had to be finite in the limit as Peclet number goes to infinity, if convection and diffusion are to continue to be of the same magnitude even as the Peclet number goes to infinity. So, if the convection and diffusion are to be comparable, even as the Peclet number goes to infinity, Peclet number based upon the length scale L, which is the length of the plate. If convection and diffusion are to be of same magnitude then we require that this thing has to be order 1 or small l by capital L has to go as P e bar minus one third.

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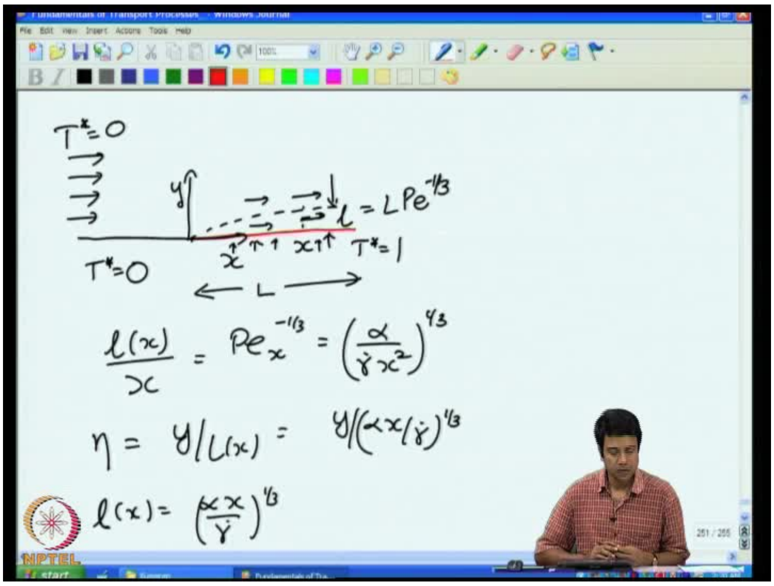

So, as Peclet number becomes larger and larger, this boundary layer length scale, small l becomes smaller and smaller and it decreases as P e power minus one third as the length scale becomes large. So, that give us a scaling for boundary layer thickness for a plate of length capital L. The microscopic length - the boundary layer thickness has to go as P e bar minus one third times capital L.

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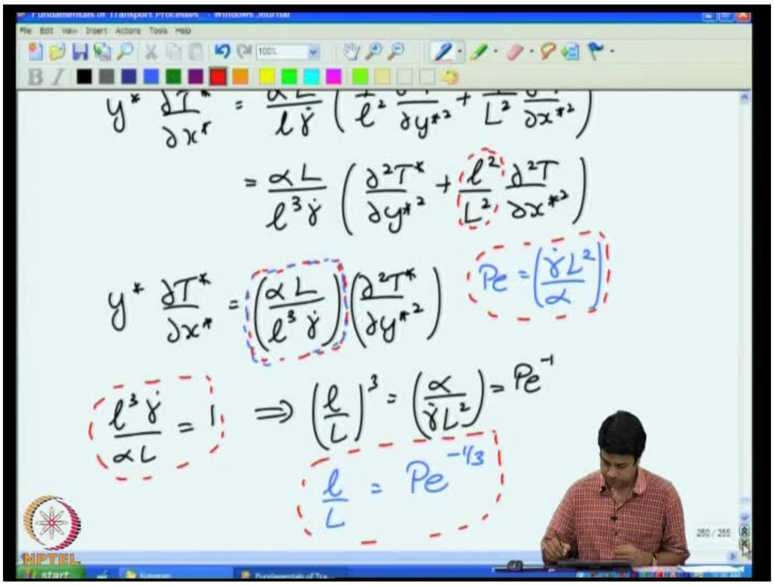

But how do we proceed and actually solve the problem. We still have, in this case, a second order differential equation in the y coordinate and a first order in the x coordinate. So, it is a second order differential partial differential equation, second order in y and first order in x.

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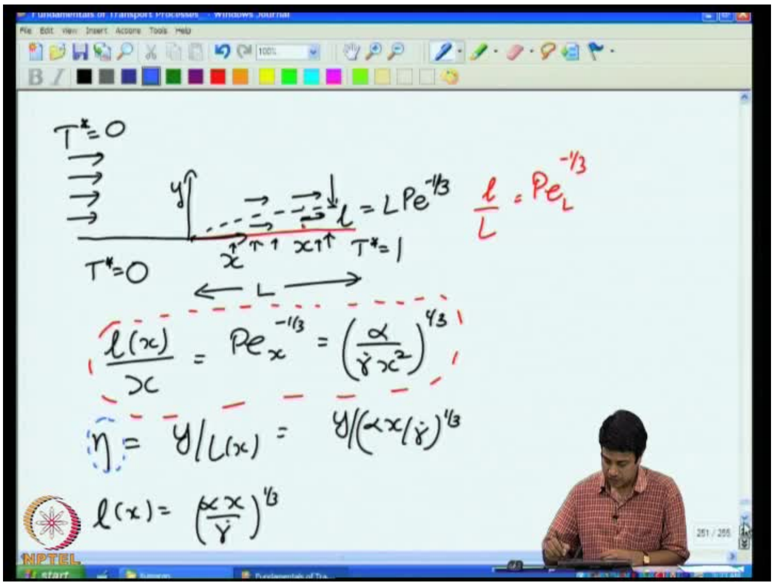

So, how do we solve this? The solution was based upon little bit of physical insight. At any given location x - at a given location x, there is convection, there is diffusion from the surface and there is convection downstream from that location. Therefore, the temperature field at that location should depend only upon the distance x from the upstream and of the heated section and not on the total length L.

I have neglected in this case, stream wise diffusion. Therefore, whether the plate is of length L or of length 2L, that is downstream of the location x and therefore, the boundary layer thickness, the temperature at that location cannot depend upon the total length L of the heated section. It should only depend upon the distance x from the beginning of the heated section.

So, rather than writing small l by capital L is equal to Pe based upon L to the minus one third, what I said based upon physical insight was, at a given location x, the boundary layer thickness can depend only upon the distance x itself. In that case, the only form for the relation is of this type small l by x is equal to Pe based upon x to the minus one third power, where the Peclet number based upon x is gamma dot x square by alpha; we substitute x instead of L

So, now, x is the downstream distance - the coordinate along the stream in x direction and since the boundary layer thickness depends only upon x, it already depends upon x. When I expressed the equation in terms of scaled coordinate, which is y divided by the boundary layer thickness, then I should end up with an equation which depends upon this similarity variable alone.

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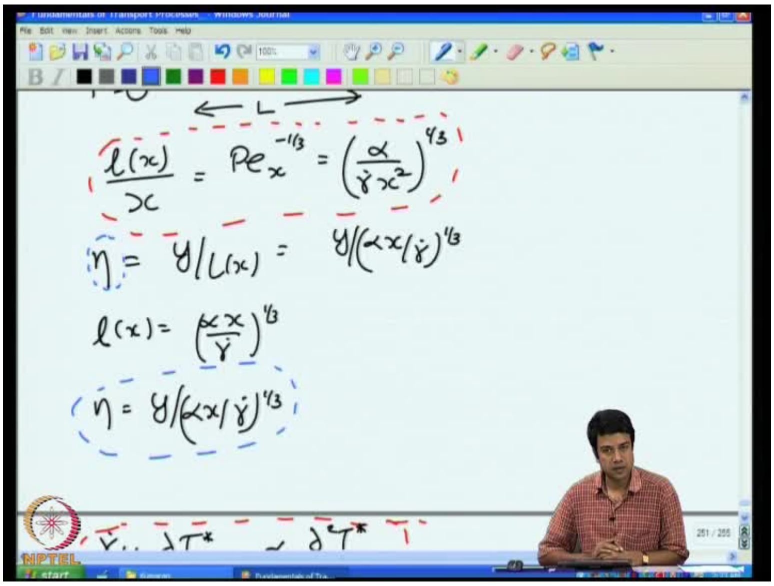

It should not depend individually on x and y and it should also not depend upon the stream rate or the diffusion coefficient. It should only depend upon x alone I am sorry It should only depend upon eta alone. So, this was the similarity variable that we had defined based upon our argument that at a given location x, the temperature field depends only upon x.

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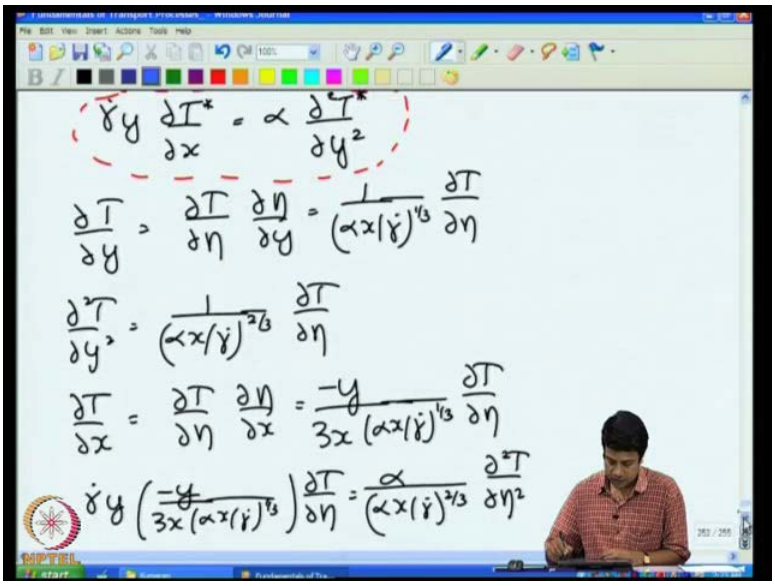

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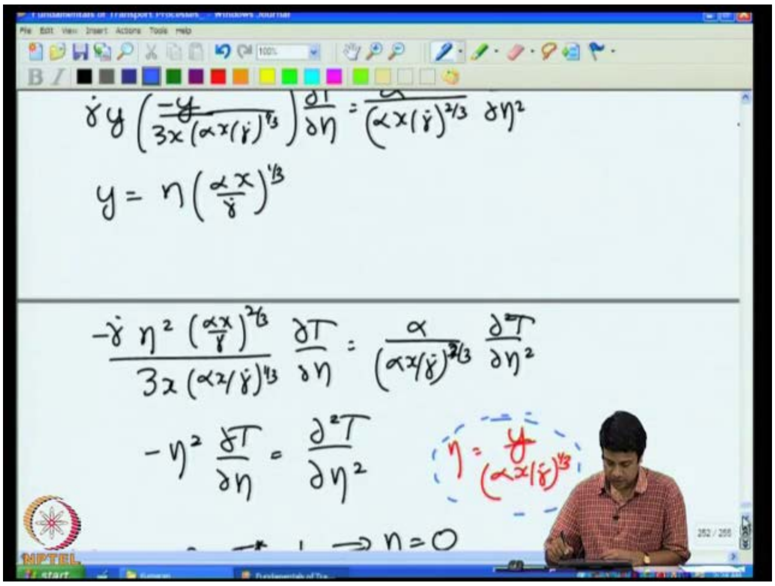

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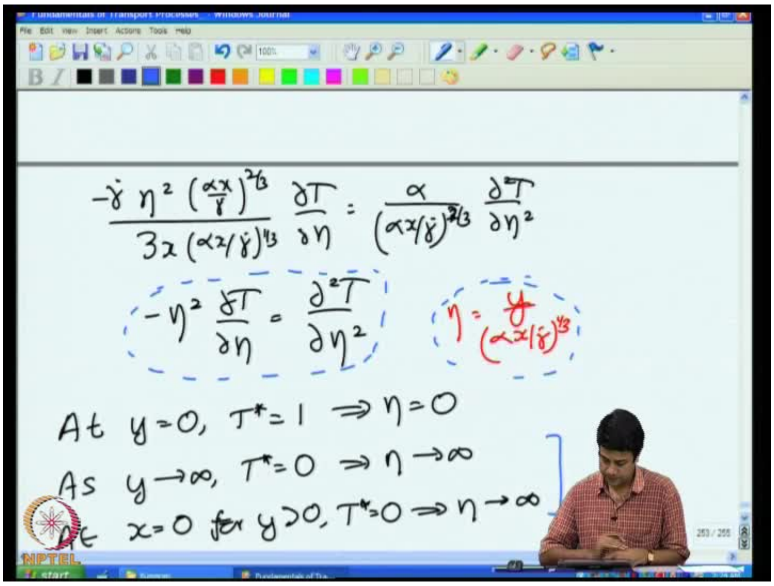

So, this was inserted into the conservation equation using chain rule for differentiation and as expected, we did in fact, end up with an equation, which depends only upon the similarity variable eta alone. It does not depend individually on x and y, but it depends only upon the similarity variable eta alone.

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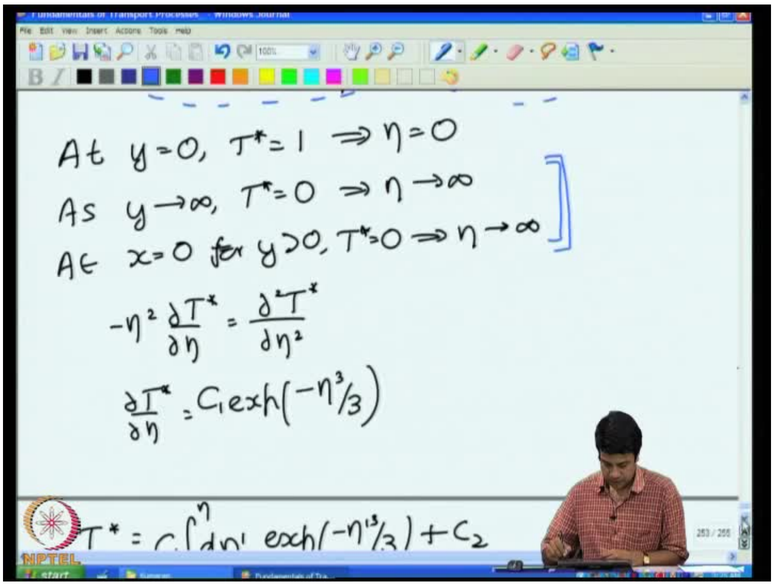

This is the second order differential equation in eta. We have to prescribe boundary conditions for this. Those boundary conditions emerge from the initial boundary condition that we had for the x coordinate. At y is equal to 0, T star is equal to 1 implies that eta is equal to 0 T star is equal to 1. As y goes to infinity or as eta goes to infinity, T star is equal to 0 and at x is equal to 0, for all y greater than 0, T star is equal to 0. x is equal to 0; finite y implies that eta goes to infinity.

As I showed you in the last lecture, these 2 boundary conditions - 1 in the y coordinate and the other in the x coordinate reduce to the same condition when expressed in terms of eta. That is consistent because I had reduced an initial first order equation in x and the second order equation y to just a second order equation in eta.

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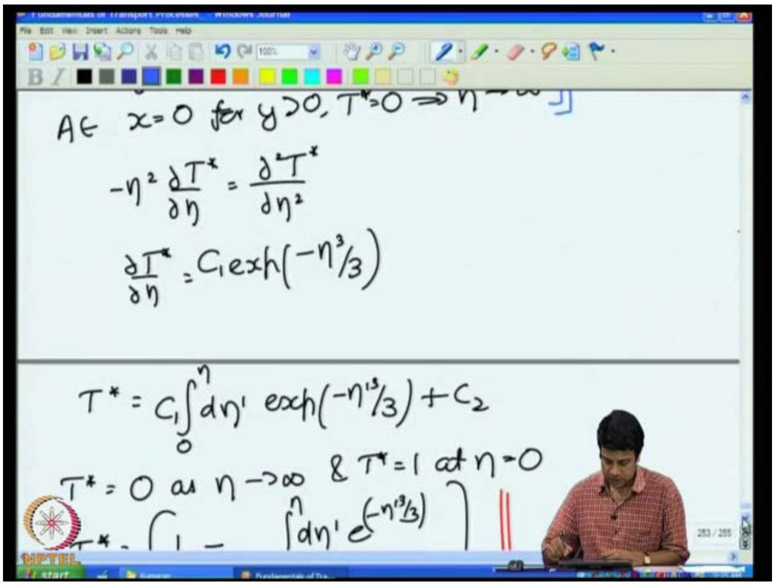

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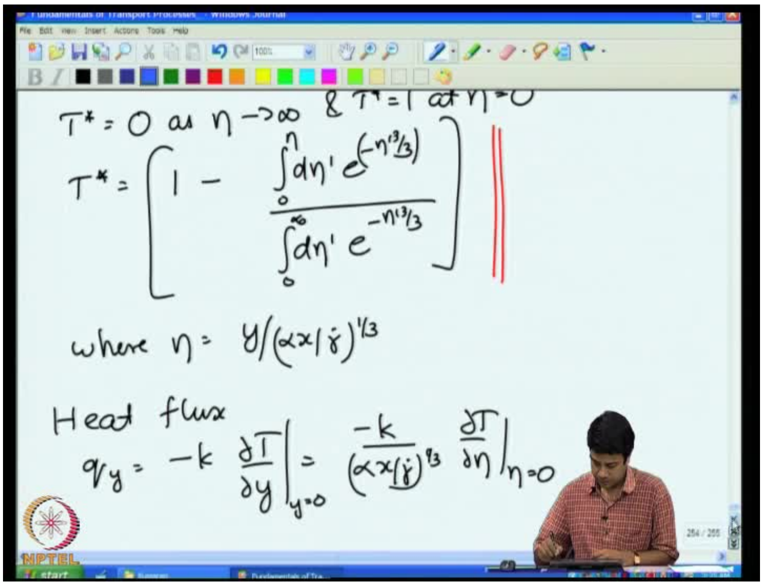

The original equation had 2 conditions in x and 2 conditions in y and 1 in x and whereas, the equation that results in terms of the similarity variable can have only 2 conditions and these were the 2 conditions. You had solved this equation explicitly and got the temperature field.

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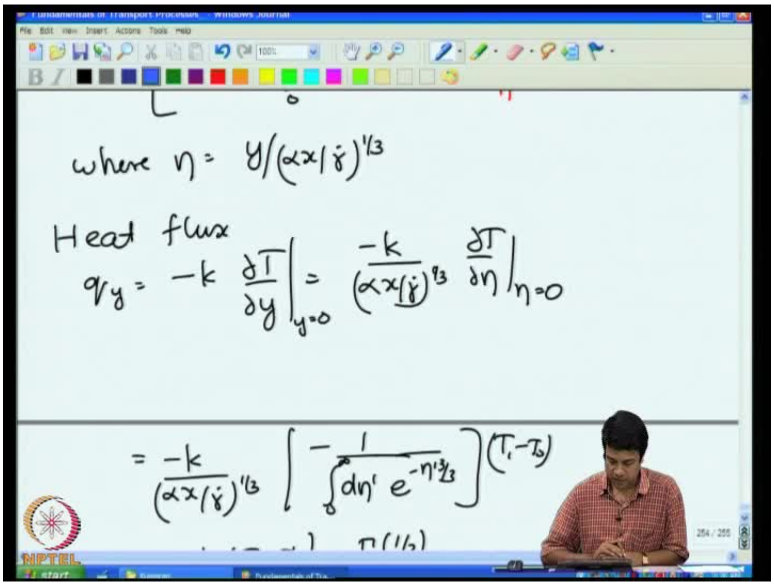

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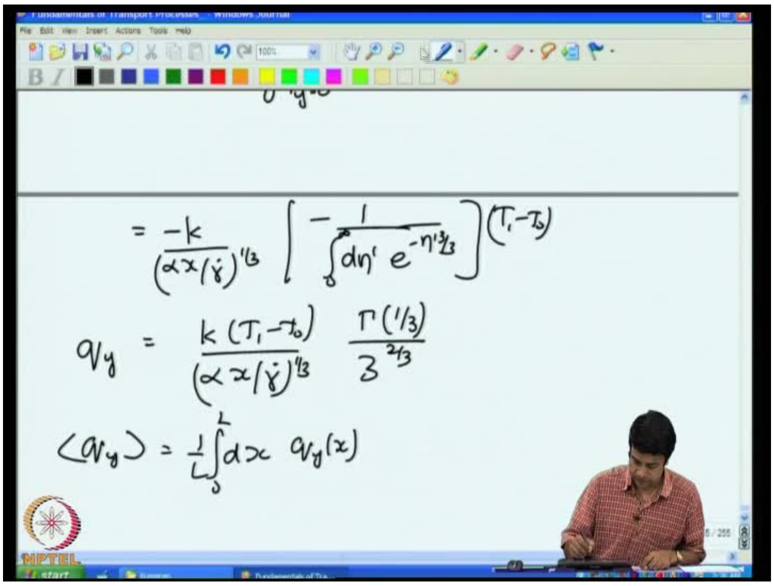

The next task is to get the heat flux. I told you that the heat flux is equal to minus k d T by d y at y is equal to 0. That is, the flux that is emerging from the surface is equal to minus k times d T by d y at y is equal to 0. I can express that in terms of eta because y is related to eta, as eta is equal to y by the boundary layer thickness.

So I can express that in terms of eta and once I do that, I get an expression for the heat flux at the surface which goes as x power minus one third. So, the heat flux of the surface decreases as you go downstream. That is because the boundary layer thickness is increasing. The flux goes as the temperature difference divided by the length scale. In this particular case that length scale is the boundary layer thickness. As the boundary layer thickness increases, the heat flux is going to decrease

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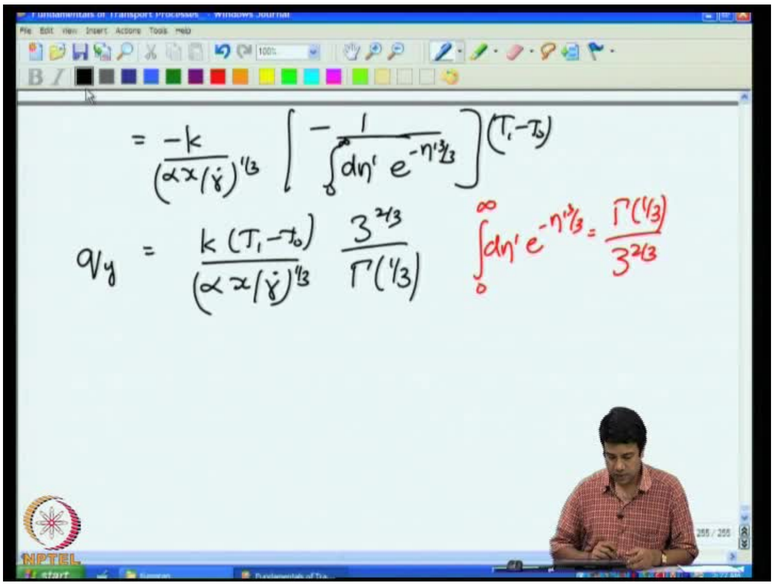

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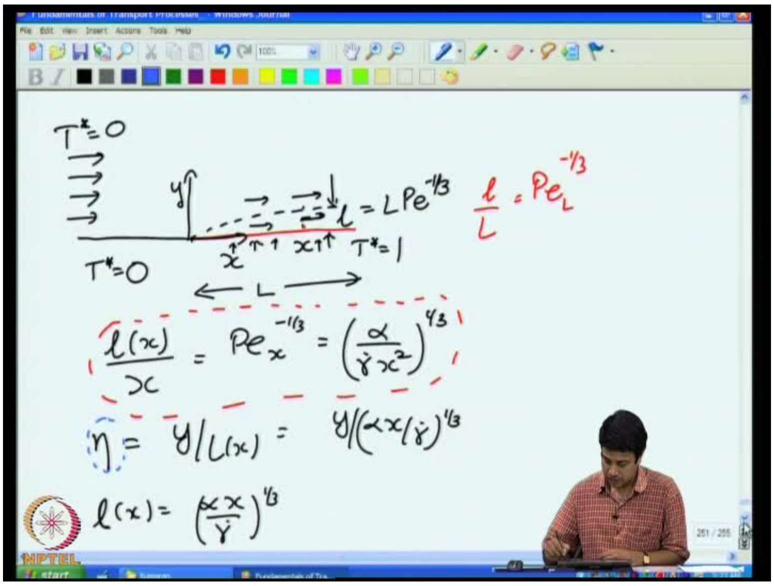

What is the total heat coming out of the surface? I should change this; this is actually and the rest of the formula as shown in the slide because I have and the rest of the formula as shown in the slide. So, what is the total heat flux coming out of the surface? This is the total amount of heat coming out per unit length perpendicular to the plane because as you recall, in this case, we are considering a two dimensional problem, where there is the plain surface and the fluid is incident on the surface and this is a two dimensional problem. Therefore, we are considering two dimensional problem per unit length in the third direction perpendicular to the plane of the board.

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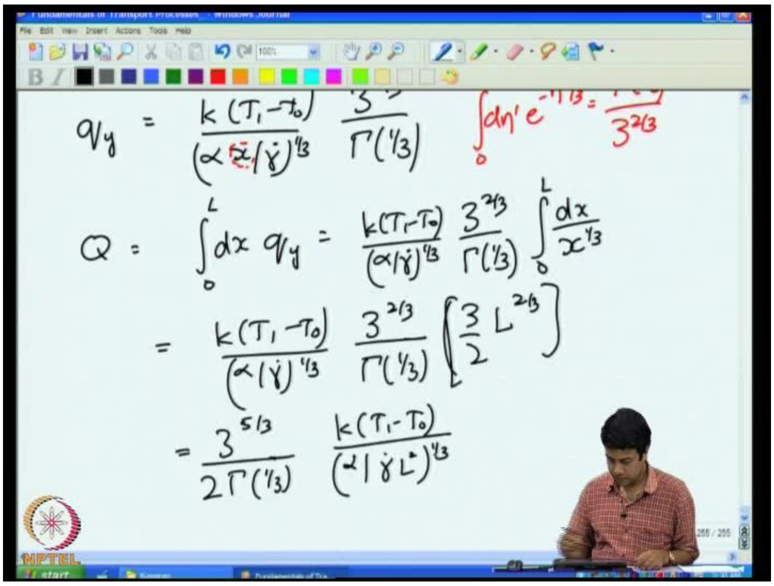

So, the total amount of heat coming out per unit length perpendicular to the plane is going to be equal to integral 0 to L dx qy; that is, you integrate the heat coming out per unit area at each differential area along the x direction, then at the molar. So, this is equal to the total heat that is coming out. This is equal to k into T 1 minus T naught integral 0 to L dx by x power one third because I have this dependence on x in the equation for the heat flux. So, this is equal to k into T 1 minus T naught by alpha by gamma dot power one third into 3 power 2 by 3 by gamma 1 by 3 into 3 by 2 times L power 2 by 3.

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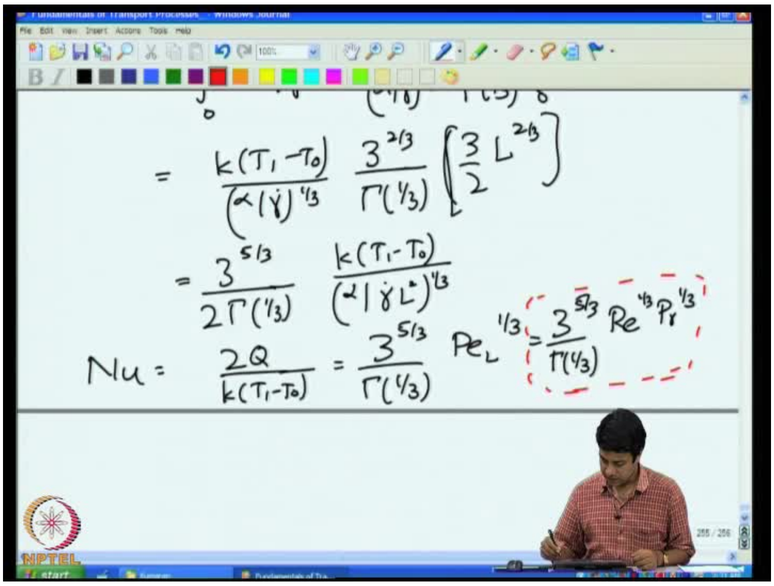

So, this is equal to 3 power five third by 2 gamma 1 by 3 into k T 1 minus T naught by alpha by gamma dot L square power one-third. So, this is the final equation for the heat transfer from the plate. The Nusselt number traditionally is defined as 2Q by k into T 1 minus T naught and you can see that this would be given by 3 power five-third by gamma of one-third into Peclet number based upon L to the minus one third power. I am sorry, this is Peclet number based upon L to the plus one third power.

So, this gives us the Nusselt number verses Peclet number correlation for the heat transfer from a flat plate. When I discussed this initially, when we looked at correlations for different situations, I told you that Nusselt number goes as Peclet number goes to the one-third power. This is basically 3 power five-thirds by gamma of one-third Re power one-third Prandtl power one third because Peclet number is the product of the Reynolds number and the Prandtl number.

So, this is the correlation for the heat transfer from a flat plate. In the limit, where the flow is laminar that the velocity is linear at the surface, in that case, the correlation for heat transfer goes as a constant times Peclet number to the one third power and the reason it goes as Peclet number to the one third power is physically is because the heat flux from the surface goes as the temperature difference divided by length scale; that length scale goes as P power minus one third from the scaling that we had just derived. Therefore, the Nusselt number goes P power plus one third as expected.

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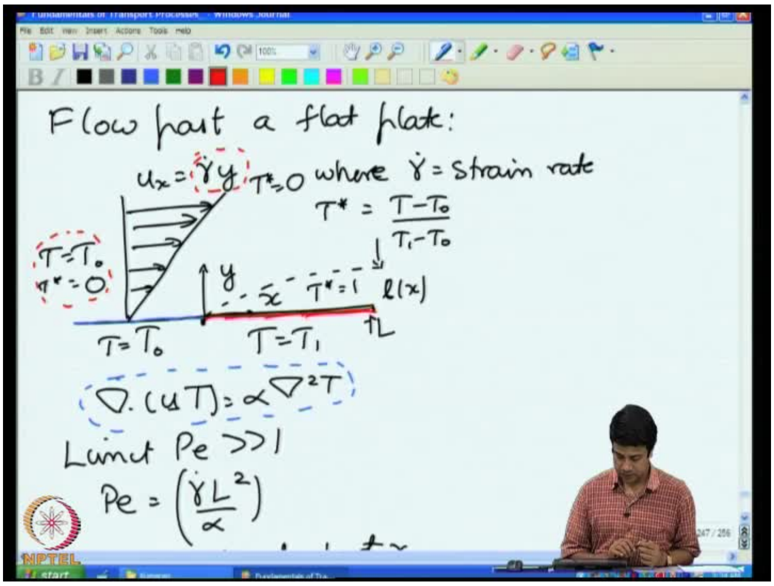

This we solved for the particular case where the velocity is linear - this the linear velocity profile where there is a linear velocity profile at the surface. However, the solution is actually more general than that. Regardless of the voltage profile that I have, the boundary layer thickness is actually small compared to the length scale.

So, for the solution to be valid, the only requirement is that the velocity profile has to be linear over a length scale comparable to the boundary layer thickness. The velocity profile has to be linear over a length scale comparable to the boundary layer thickness.

So long as the boundary layer thickness is small compared to the macro scale, the velocity has to come down to 0 at the surface anyway. So, near the surface, the velocity has to increase linearly from 0 and if you are expanding in a thin region, in a Taylor series expansion, the first term in that expansion will be the linear term.

So, this will be the first term in the expansion for any velocity profile, provided the region over which I am expanding is thin, small compared to the macroscopic scale. In that case, the velocity profile near the surface is going to be linear anyway and therefore, I can use an expansion in this velocity profile of this form u x is equal to gamma dot y at the surface. This is valid, whenever there is a no slip condition at the surface so that the fluid velocity has to come down to 0 at the surface. It is not valid in general, if you had a non-zero velocity at the surface. We will see a little later, how to deal with non-zero velocities at the surface.

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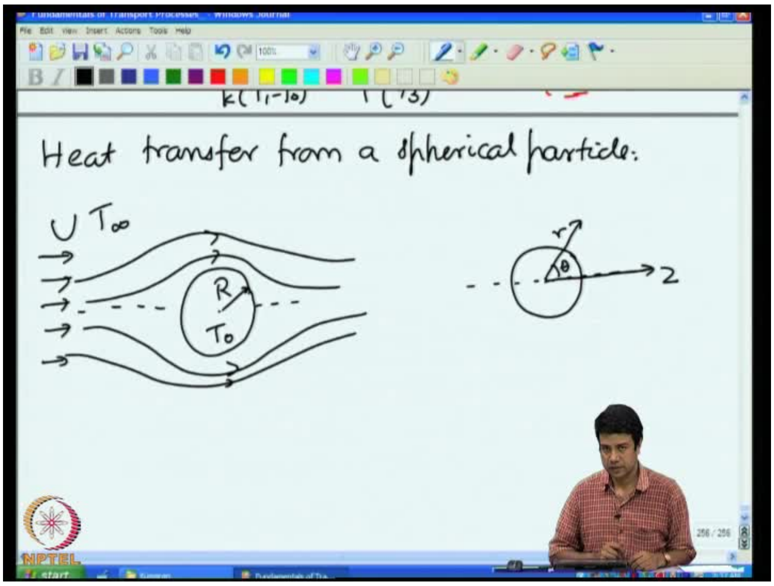

So, this is the Nusselt number correlation, for this simple case of the flow past a flat plate. Let us now consider a more complicated case - heat transfer from a spherical particle. So, this is the case that is actually of practical interest. One might very often have situations, where there is transport from a spherical particle or some other shaped particle within the flow. So, first, I will consider the case, where there is heat transfer from a spherical particle and then we will look at some general results that can be obtained for any shape of particle.

So, we have a particle of radius R, which is at temperature T naught and there is fluid flow around the particle and for upstream, the fluid has some velocity U; the fluid velocity is U for upstream of the particle. The temperature far away from the particle is T infinity and one would like to know what is the heat flux due to the flow as well as the diffusion - thermal diffusion from the particles. Let us fix our coordinate system. We have a spherical particle and therefore, it is most convenient to consider spherical coordinate system.

This spherical coordinate system has an axis of symmetry. That is, as you rotate around this axis, there is no change in the velocity field. So, it is most convenient to consider this as the z axis and the angle theta is measured from this z axis, where r is the distance from the origin in this coordinate system and in this coordinate system, I would prescribe u r and u theta and so on.

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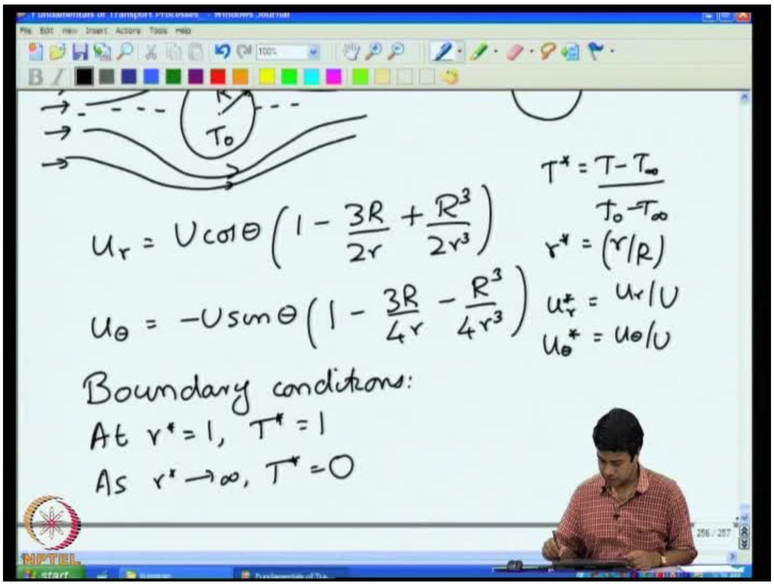

We assume that the flow is viscous in the low Reynolds number limit and in that case, you can get analytical results for u r and u theta. We would not derive here, I will just use these results to calculate the heat flux. The analytical results are u r is equal to U cos theta 1 minus 3R by 2r plus R cubed by 2 r cubed and u theta is equal to minus U sin theta and the rest of the formula as shown in the slide.

So, these are the expressions for the velocity field everywhere within the flow. u phi is equal to 0 because there is no flow in that direction and there is no dependence of either the velocity or the temperature on the phi coordinate, which represents rotation around the z axis.

So, this is heated sphere at temperature T naught in a fluid at temperature is equal to infinity at T infinity. So, I can define my scaled temperature as usual T star is equal to T minus T infinity by T naught minus T infinity and I will define scaled radius as r by capital R and u star and the rest of the formula as shown in the slide. So, scaling by the obvious scales, the velocity scale is u, the radial distance is capital R and T is scaled as T minus T infinity by T naught minus T infinity. So, the boundary conditions are at r star is equal to 1, T star is equal to 1 and as r star goes to infinity and the rest of the formula as shown in the slide.

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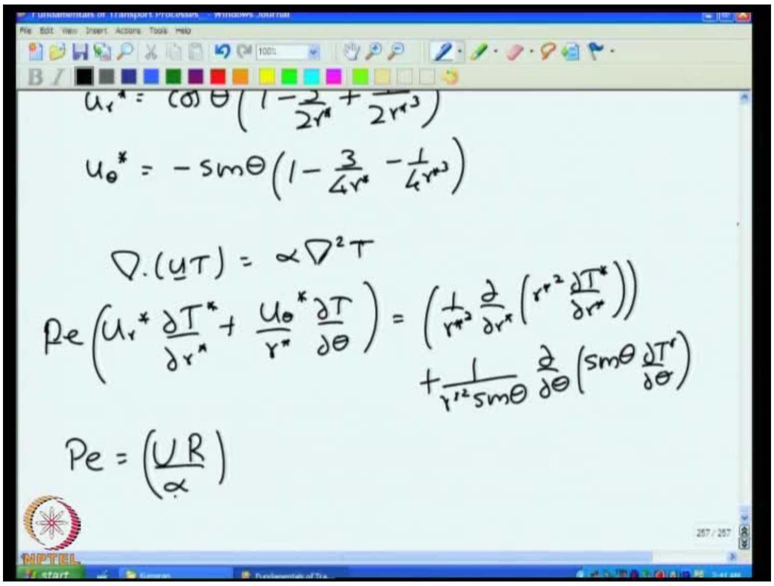

So, the temperature is 1 at the surface and it goes to 0 as the distance goes to infinity and the scaled velocity is expressed in this form. u r star is equal to cos theta 1 minus 3 by 2r star plus 1 by 2r star cubed and u theta star is equal to minus sin theta 1 minus 3 by 4r star minus 1 by 4r star cubed.

If you insert these into the steady state convection diffusion equation, u dot grade is equal to alpha del square T. What you will get is u r partial T by partial r plus u theta by r partial T by partial theta into the Peclet number is equal to 1 by r square d by d r of r square d T by d r plus 1 by r square sin theta d by d theta of sin theta. So, this will end up being the scaled equation. I just scaled the velocity by capital U the length by capital R and the Peclet number is equal to U R by alpha.

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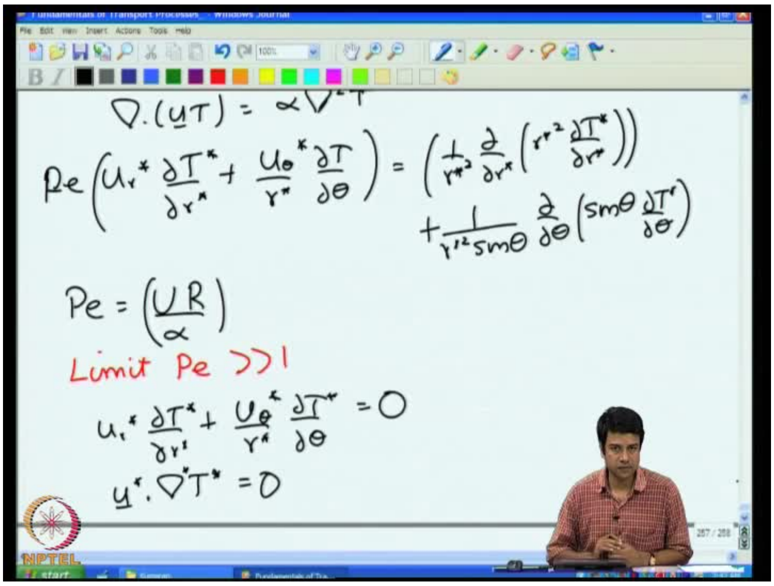

Now, in this equation, if we take the limit, Peclet number becoming large compared to 1, the equation just becomes u r d T by d r plus u theta by r d T by d theta is equal to 0. One can also write this alternatively as u dot grade T is equal to 0. So, what this says is that u dot grade T is equal to 0; that means, the gradient of T along the velocity is equal to 0.

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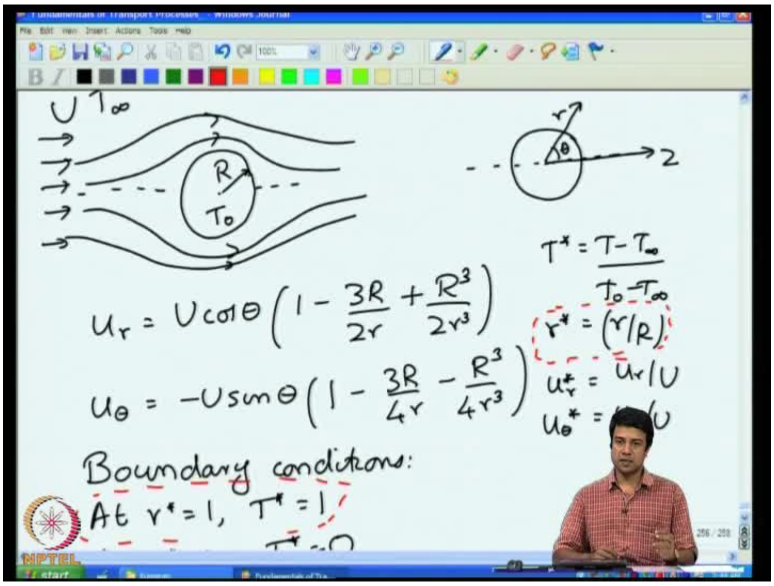

Temperature does not vary along stream lines. Temperature does not vary along stream lines, the temperature at the upstream entrance is equal to T infinity. Therefore, the temperature upstream is T infinity and it does not vary along stream line; the temperature has to be T infinity everywhere.

So, that is the only solution that you can get - the temperature has to be T infinity everywhere. It cannot satisfy this boundary condition. In order to satisfy this boundary condition as in the problem of the flow past a flat plate, we have to postulate a boundary layer of thickness small compared to r at the surface.

Previously, we had scaled r by capital R; the assumption there was that the length scale for the variation of the temperature field is capital R itself, but when the length scale is capital R, the Peclet number is large and diffusion is small compared to convection. Diffusion and convection can be of the same magnitude, only if there is a smaller length scale, which is small compared to capital R such that the temperature field varies only over that small length scale. In that case, the gradients of temperature field are large and one could have a balance between convection and diffusion.

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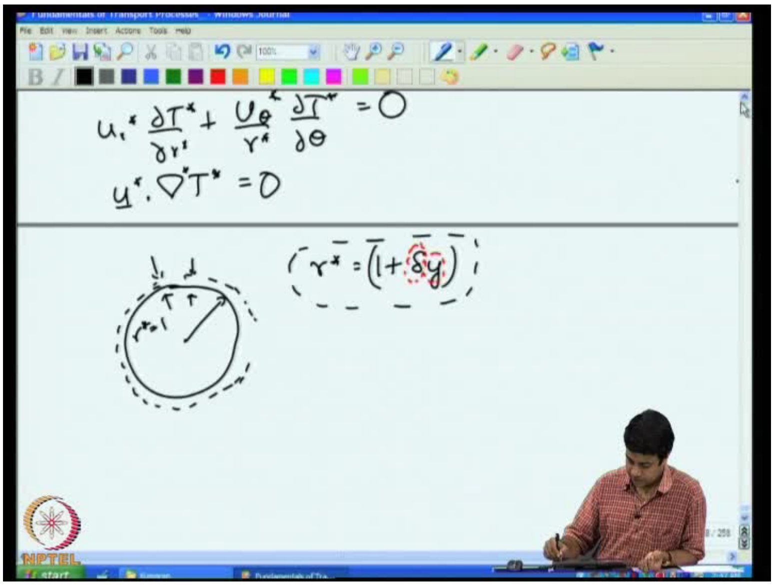

So, what we have to do now is to postulate the presence of a layer near the surface. The presence of a layer near the surface, where there is the effect of heating from the sphere penetrates into the fluid. What should be this distance? The length scale, we will obtain that from the consideration that convection and diffusion are of equal magnitude within this layer.

So, the surface of the sphere itself is at r star is equal to 1. Therefore I will postulate that there is another coordinate r star is equal to 1 plus delta y, where y is a scaled distance within the fluid away from the surface. Basically, delta times y is the distance of a point from the surface because r star is equal to 1 is the surface. So, r star is equal to 1 plus delta y gives you basically, the distance of a point in the fluid from the surface of the sphere and that distance has to be small for there to be a balance between convection and diffusion.

So, this y is the variable that I am using. y is the scaled variable that I will use for differentiation and delta is the boundary layer thickness. y, I will assume is order 1. It remains finite in the limit as Peclet number becomes large; delta becomes smaller and smaller as Peclet number become larger and larger in such a way that delta times y gives me a region near the surface of the sphere, where there is a balance between convection and diffusion.

So, it is important to note that I will assume that y is a scaled coordinate which continues to be of order one. it varies from the temperature The effect of heating from this sphere is non-zero, when y is of order 1. Delta is a length scale that is going to 0 as the Peclet number becomes large. So, y is the scaled coordinate in our case. First of all, I have to substitute this in the equations for the velocity field.

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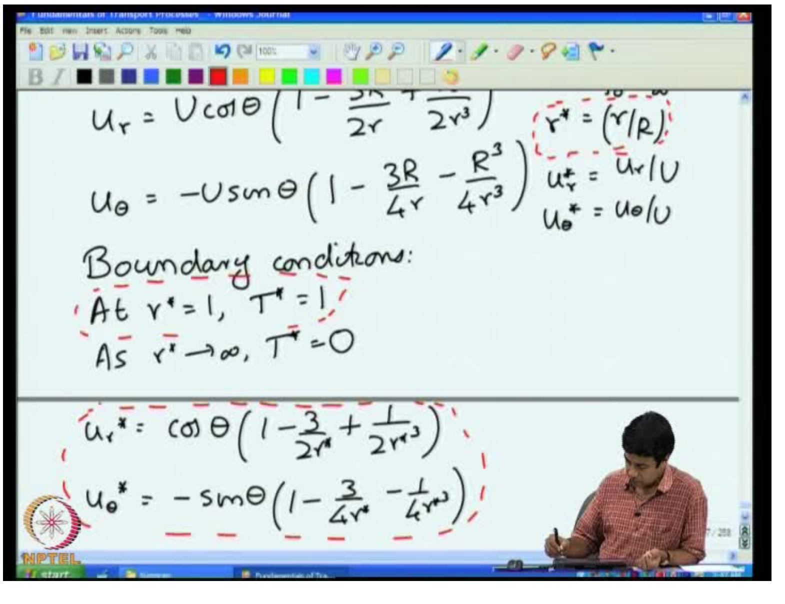

Since I am interested in the region near the surface, it will be sufficient for me to find out approximate expression for the velocities in the region very close to the surface. It will be sufficient to find out approximate velocities in the region very close to the surface.

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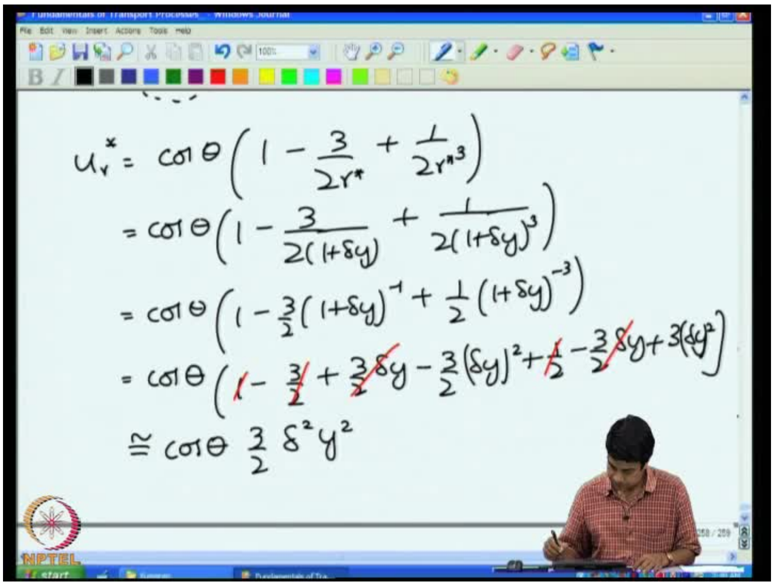

So, velocity field u r is equal to cos theta 1 minus 3 by 2r plus 1 by 2r cubed. I substitute r in terms of 1 plus delta y. So, this becomes cos theta 1 minus 3 by 2 into 1 plus delta y plus 1 by 2 by whole cubed. Now, I can use a binomial expansion in the small parameter delta. So, this is going to be equal to cos theta 1 minus 3 by 2 into 1 plus delta y power minus 1 plus 1 by 2 and the rest of the formula as shown in the slide.

If I expand this out, I will get cos theta 1 minus 3 by 2 plus 3 by 2 delta y minus 3 by 2 delta y the whole squared plus 1 by 2 minus 3 by 2 delta y and plus 3 delta y the whole squared using the binomial expansion and you can see that in this, the leading order terms actually cancel out. I have 1 minus 3 by 2 plus 1 by 2; those 3 cancel out. The terms proportional to delta also cancel out. I have 3 by 2 delta y minus 3 by 2 delta y and the first correction that I get is of order delta y the whole squared. So, this in this limit is approximately equal to cos theta times 3 by 2 delta squared y squared. So, the radial velocity goes as delta square times y square very near the surface.

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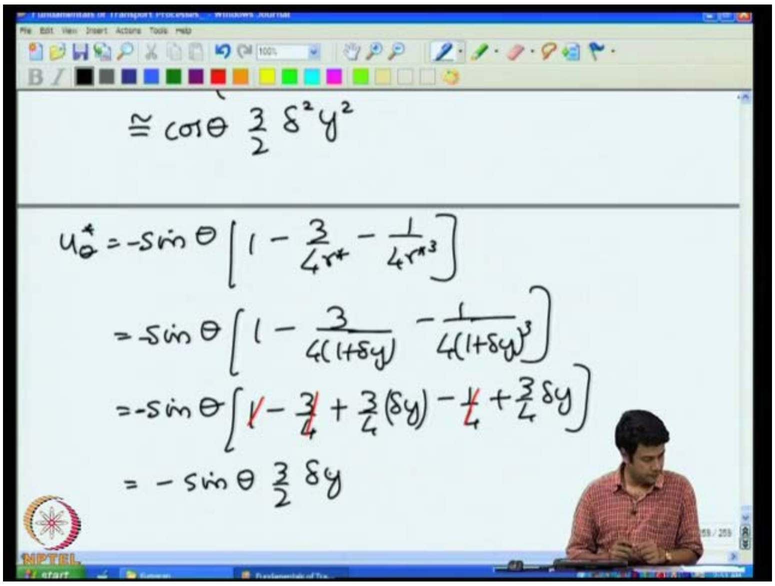

What happens to the other velocity u theta? u theta is equal to sin theta into 1 minus 3 by 4r minus 1 by 4r cubed. This can once again be expanded out sin theta into 1 minus 3 by 4 1 plus delta y minus 1 by 4 into 1 plus delta y whole cubed and if I use a binomial series, I will get 1 minus 3 by 4 minus 3 by 4 plus 3 by 4 delta y minus 1 by 4 plus 3 by 4 delta y. In this case, the terms that are order one, actually cancel out - 1 minus 3 by 4 minus 1 by 4.

What I end up with is equal to minus sin theta into 3 by 2 delta y. So, these are the velocities u r and u theta. The approximation for the velocity is very close to the surface, when delta is small compared to 1 So, these are the approximations for the velocity fields. The radial velocity goes as delta square, the velocity in the theta direction goes as delta. It is no surprise because at y is equal to 0 itself, both component of the velocity have to be equal to 0.

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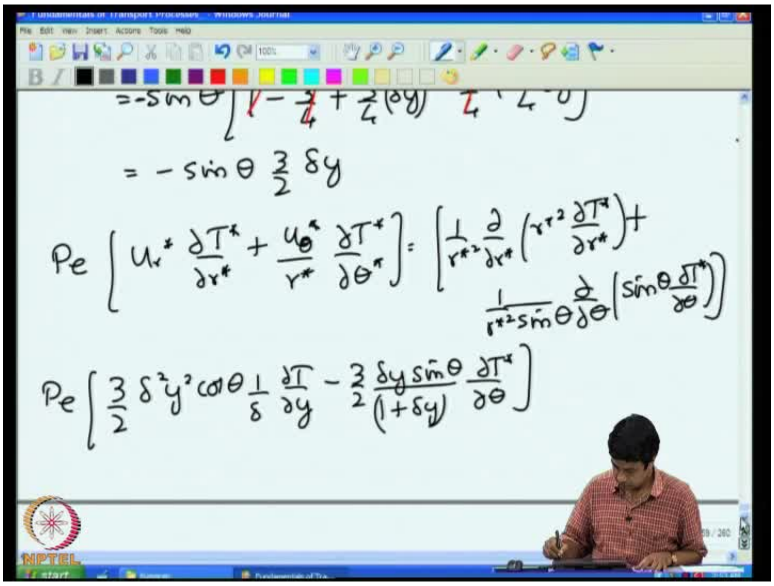

So, I now put these into my differential equation. Pe into and the rest of the formula as shown in the slide. So, this is the differential equation and I substitute in terms of r in this equation and also substitute the velocities that I got.

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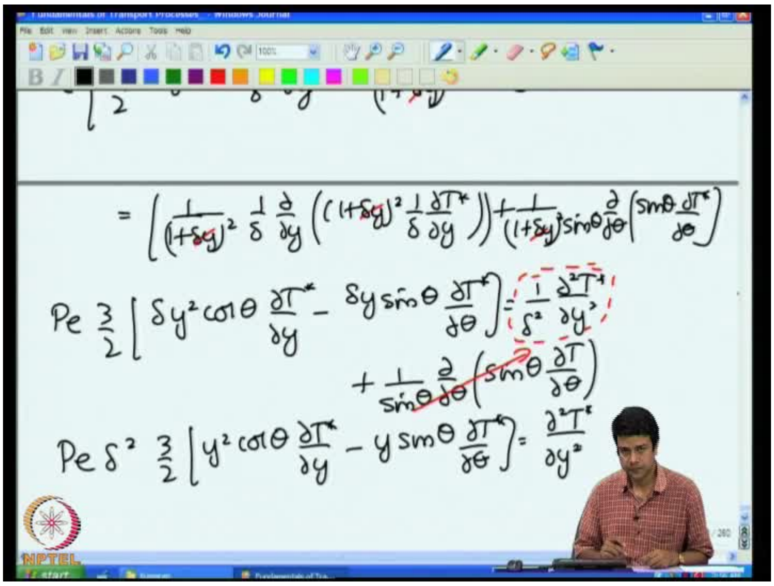

So I put this in and I will get Pe into 3 by 2 delta square y square cos theta into partial T by partial r is one over delta partial T by partial y plus u theta is minus 3 by 2 delta y sin theta by 1 plus delta y r is 1 plus delta y times partial T partial theta. On the right hand side, I have 1 by 1 plus delta y the whole square d by dy is 1 by delta of r square is 1 plus delta y the whole square times 1 by delta partial T by partial y plus 1 by 1 plus delta y the whole square sin theta d by d theta of sin theta d T by d theta.

Now delta is small compared to 1. Therefore, I can neglect delta in comparison to 1 - here and here, as well as here; I can neglect delta in comparison to 1 and once I do that, I get Pe into 3 by 2 delta square y square I am sorry delta y square cos theta partial T by partial y minus delta y sin theta partial T by partial theta. The first term here is 1 by delta square partial square T by partial y square and the second term is one by sin theta d by d theta of sin theta partial T by partial theta, after I have neglected all the terms proportional to delta and retaining only the leading order terms.

Now, in this equation, it is easy to see this first term goes as order 1 over delta square whereas, the second term does not contain any deltas. In the limit as delta goes to 0, this second term is small compared to the first because we except delta to become a small number. Therefore, 1 by delta square will become a large number. Therefore, in the limit as delta goes to 0, you would except this to be small compared to the first term. On the left hand side of course, both terms are proportional to delta.

So, both of them are of equal magnitude in the limit as delta goes to 0. So, if I neglect the second term and multiply throughout by delta square, what I will get is Pe delta square into 3 by 2 into y square cos theta partial T by partial y minus y sin theta partial T by partial theta is equal to partial square T by partial y square.

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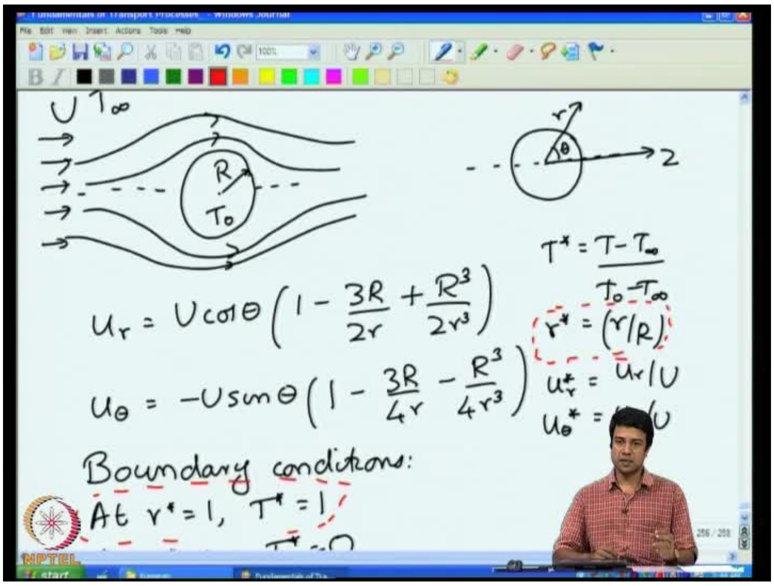

Note that at the surface of this sphere or that the flow at the surface of the sphere is in the theta direction. So, theta is the stream wise direction, r is the cross stream direction - r is the direction perpendicular to the flow at the surface of this sphere and as in the case of a flat plate, what we are finding is that the diffusion in the cross stream direction, the first term there is large compared to the diffusion in the flow direction - in the theta direction along the surface.

So, diffusion perpendicular to the surface is large compared to diffusion along the surface, when the boundary layer thickness is small. So, that is consistent with what we have in flow past a flat plate. What is different here - flow past a flat plate is that we have convection both in the r and theta directions and both of these are of equal magnitude.

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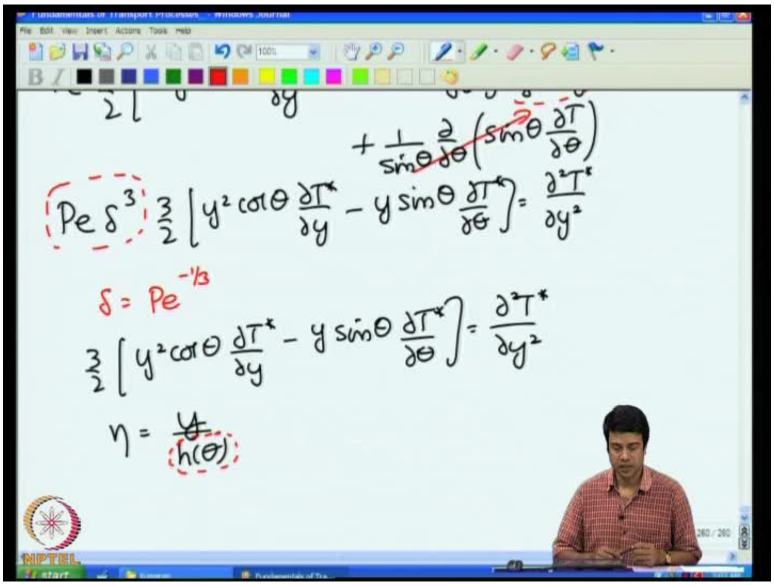

So, as I said, within the boundary layer y is order 1, delta goes to 0. In the limit as delta goes to 0, if y is finite, this number has to be order 1, if there is to be a balance between convection and diffusion and once again, we get delta is equal to Pe power minus 1 by 3. So, when delta is equal to minus Pe power minus 1 by 3, we have a balance between convection and diffusion in the limit as the Peclet number goes to infinity.

So, this provides us with the boundary layer thickness. As in the case of a flat plate, the boundary layer thickness goes as the Peclet number to the minus one third power - once again no surprise. Very close to the surface, if we can approximate the velocity profile as the linear function away from the surface, then the velocity increases proportional to the cross stream distance. The diffusion coefficient goes as one over the length scale square. So, ultimately, I will get Pe times L cubed being order 1. So, in that sense, it is no longer a surprise.

So, if we use delta is equal to Pe power minus one third, then our equation becomes 3 by 2 into y square cos theta partial T by partial y minus y sin theta partial T by partial theta is equal to partial square T by partial y square. So, this is a partial differential equation, contains 2 variables: one is theta and the other is y. In the previous case, we had only one component of the velocity.

We assume that the similarity variable does exist. Assume a similarity variable of the form y by h of theta, where h is some function of theta. I will put this form of the similarity variable into the differential equation, solve it and then try to see whether I can get the functional form of this similarity variable in such a way that the resulting equation does not depend individually on theta and y but depends upon the similarity variable eta alone.

So, instead of trying to use some physical insight in the case of the flow past a flat plate, in order to derive what the similarity variable will be, I will go head and assume a form for the similarity variable, I will put that into the equation and the resulting equation that I get one can then check whether there is some form of this function h of theta, which actually satisfies the condition that the resulting equation is a function of eta alone. So, this part, we will continue in the next lecture.

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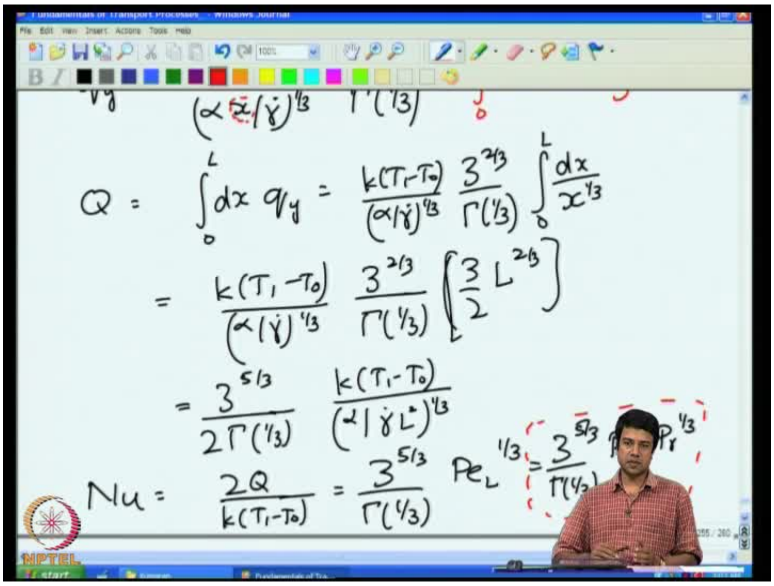

To briefly recap what we did in this lecture, first we looked at the flow past a flat plate and explained to you how to get a Nusselt number correlation. In this case, the heat transfer from the surface, there is a boundary layer of the thickness Peclet number power minus one third at the surface because the boundary layer thickness decreases as Peclet number to the minus one third power. The heat flux which is the temperature difference divided by the length scale is proportional to the temperature difference divided by that length scale. There are some constants of course, but in the limit as the Peclet number becomes large, the heat flux has to be proportional to the difference in temperature divided by the boundary layer thickness. That heat flux goes as Pe power plus one third because the length scale is going as Pe power minus one third.

Once I use that condition, I can then derive an expression for the Nusselt number in terms of the Peclet number or the Reynolds number and Prandtl number and then we looked at the heat transfer from a spherical surface. This was for a laminar flow. The convection-diffusion equation for the laminar flow in the heat transfer past a spherical particle. The velocity components are prescribed for us in this case. we will see It is outside the scope of this course to derive these velocities themselves, but we will assume that these velocities are known and once these velocities are known, we can then insert them into the convection and diffusion equation. If you scaled the radius by capital R and the Peclet number is large, then you end up with an equation of the form, u dot grade T is equal to 0; that means, temperature does not change along stream line and therefore, whatever temperature in the inlet continues to be there throughout the domain.

With this, we cannot satisfy the boundary condition T star is equal to 1 on the surface of this sphere. In order to satisfy the boundary condition, we had postulated boundary layer thickness and we had calculated the value of that boundary layer thickness. We have not yet done the similarity solution for the temperature field. That we will continue in the next lecture. So, we will stop here and I will briefly summarize this in the next lecture before we continue to determine what is the form of the similarity solution. So, we will see you then.