Fundamentals of Transport Process
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Module No. # 07
Lecture No. # 36
High Peclet Number Transport slow Past a Flat Plate

So, welcome to lecture number Thirty Six. So, far discussion of fundamentals of transport processes. The last few lectures, we first derived the convection diffusion equation, in different coordinate systems. That equation had the general form dc by dt plus del dot uc is equal to D del square c plus any sources or sinks that are present, within the flow. And I showed you that, if you scale the velocities is equal to u by capital U and all distance x by, some characteristic distance L, the radius of a particle if a particle is inverse to in the flow, or the pipe diameter, if the flow through the pipe, the time scale becomes is t equal to t D by L square. In that case, you get a dimensionless equation of the form Pe into dc star by dt plus divergence of uc is equal to D del square c plus the source.

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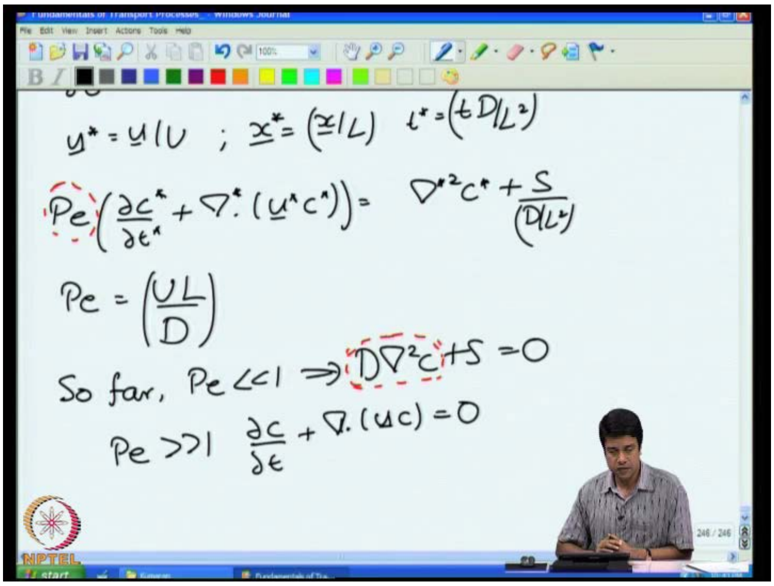

So, this is what you get and there is the peclet number sitting out in front here, which is basically, Pe is equal to UL by D. So for, we have been considering the limit of low peclet number. We have been considering, implies that del square, the equation reduces to D del square c plus source is equal to 0. So, we look that some general strategies to solve this problem. D del square c plus the source or sinks within the flows is equal to 0. Basically, it is general strategies for solving del square c is equal to 0. As the Laplace in of the concentration field is equal to 0. We looked at different ways of solving this separation of variables as well as solution strategies based upon considering a point source within the flow. And then considering the distributed source as the summation of my small point sources. Here, we are going to start looking at the opposite limit. The peclet number is large compared to 1. Simplistically, you would think that we can just go head and solve it by using the equation dc by dt plus divergence of uc is equal to 0. However, that is not so and I will try to explain to you why that is not.

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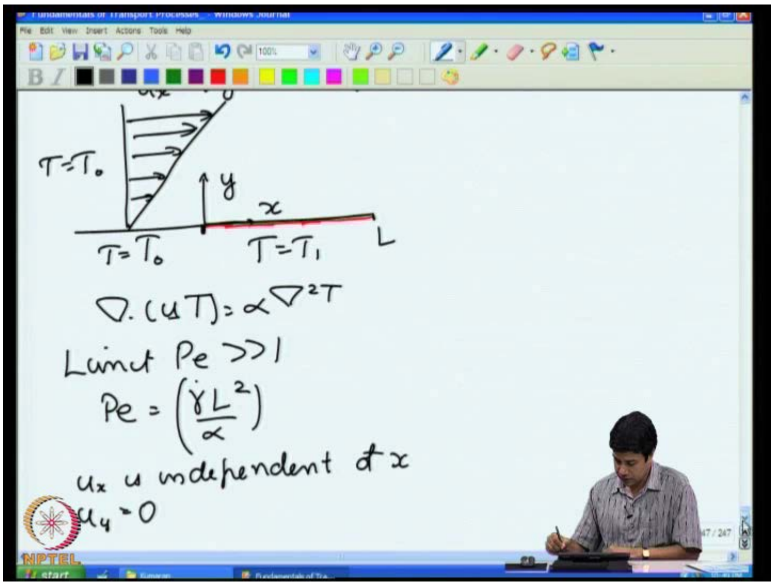

So, by considering first the simplest case, which is the flow past a flat plate. We consider solution of the conduction equation in the flow past a flat plate. So, I have a flat plate here. And T is equal to 0. And at particular location the temperature is instantaneously increased to a high value. So, T is equal to T1. At this particular location, you starting to heat the fluid. And the flow near this fluid is a linear velocity profile. The flow in near the surfaces linear velocity profile. And the temperature of the fluid that is entering is basically, at the same temperature as this unheated section, T is equal to T naught.

Consider for example, a heat exchanger problem for have a heat coming in at the inlet. I have the cold fluid coming at the inlet and at one particular location, it comes into contact with the heated fluid outside. Because, the wall at that particular location is heated. So, I have cold fluid which is initially coming in at temperature T naught. At one particular location, I start heating, I set the temperature is equal T1 at the wall of the tube itself. Rather than considering a cylindrical surfaces, I will just consider a flat plate.

So, Let us first write down our coordinate system. I will use x as the coordinate along the flow, y as the coordinate perpendicular to the flow. The velocity field, the velocities only in the x direction, parallel to the flat plate. So, velocity u x is linear function of y. It increases linear as y increases. I write it as gamma dot times y, where gamma dot is equal to the strain rate, gamma dot is equal to strain rate. And let us consider at that, this process is happening at steady state. So that, there is no time derivative. So, my heat conduction equation just becomes, partial del dot uc is equal to alpha del square, alpha del square T. This is the equation that I am going to try to solve within this domain. In the limit of high peclet number. Now, what does high peclet number mean in this case. So, Let us consider that the heated section has a length L. So, the length L is the heated section, the length of the heated section. What does high peclet number mean? I have a thermal diffusion coefficient alpha, dimensions Length square by unit time.

I have characteristic length which is the length of the heated section L, characteristic length in the problem. And there is no velocity scale but there is a scale for the strain rate, u x is equal to gamma times y. So, the strain rate gamma dot has dimensions of inverse time because velocity is length by unit time is equal to gamma dot times the length. Therefore, the stain rate has dimensions of inverse time. That means, that I can create a dimensionless number, which is the ratio of convection and diffusion has peclet number is equal to gamma dot L square by alpha. So, this gives me the ratio of convection and diffusion, in this particular case. This gives me the ratio of convection diffusion in this particular case. Further, the velocity is not varying as a function of strain vice direction. So, I have u x is independent of y. Sorry, independent of x and u y is equal to 0. There is no flow perpendicular to the plate. So, u x is independent of x and u y, there is no flow perpendicular to the plate.

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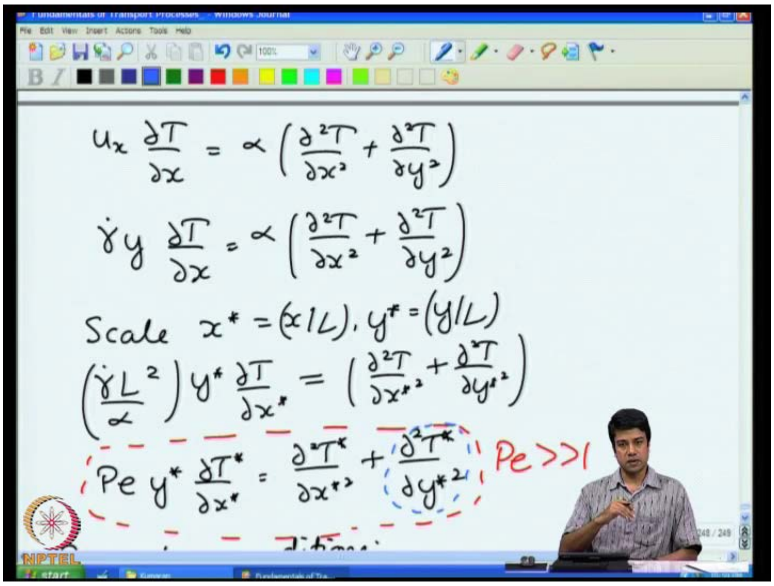

So, with this my conservation equation would become u x partial T by partial x. u x is independent of x. So, I can take it out of the differentiation, plus u y times partial T by partial y. But u y is equal to 0 is equal to alpha into partial square T by partial x square plus partial square T by partial y square. So, this is the convection diffusion equation for the steady case, where there is velocity only in the x direction. So, now, I scale U x is equal to gamma dot times y, partial T by partial x is equal to alpha, partial square T by partial x square plus and I have a length scale. So, I can scale on lengths, scale x star is equal to x by L and y star is equal to y by L. Once, you do that you will get gamma dot L square by alpha, y star dT by dx star is equal to d square T by dx star square plus d square T by dy star square. That is the differential equation in scaled form. Further, I should the temperature as well. I can define a scale temperature T star is equal to T minus T naught by T 1 minus T naught. In each case T star is equal to 1 here and T star is equal to 0. So, in terms of the scaled temperature, since the equation is linear in the anyway, I write in terms of the scaled temperature. I will just get Pe y star dT star by dx star is equal to d square T by dx square plus.

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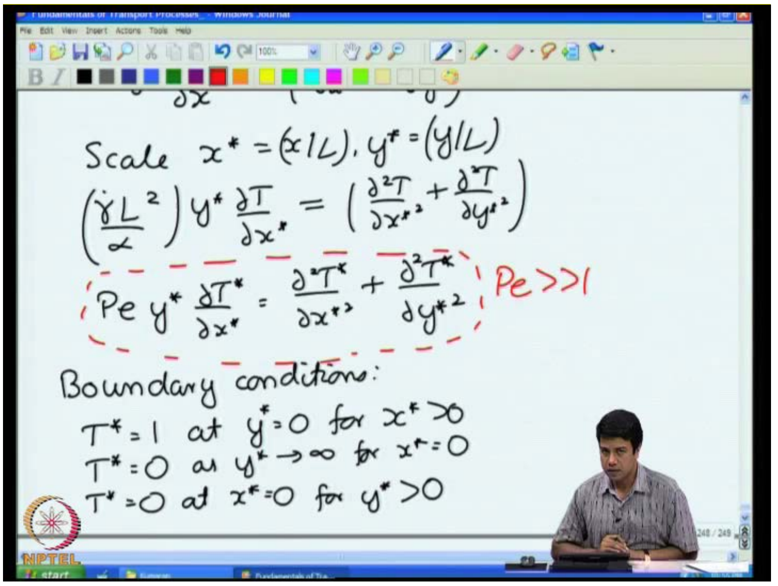

So, this is the equation that I get with boundary conditions, T star is equal to 1 at y is equal to 0 for x greater than 0. So, for x greater than 0, I have the heated section of the plate where T star is equal to 1. So, T star is equal to 1 on this heated section of the plate in the, at y is equal to 0, for x greater than 0 of course. As I go far from the plate the temperature should equal to this free steam temperature. If I go very far from the plate as y goes to infinity, I should recover T star is equal to 0, because the effect of heating from the plate has not reached that far. As y goes to infinity, T star has to decreased to 0. And T star is equal to 0, as y star goes to infinity. In addition, fluid that is incident on the plate for x is less than 0, it has not been heated yet. Fluid that is incident on the plate for the heated section for x less than 0, it is not yet been heated. Therefore, for x less than 0 for any value of y greater than 0, the temperature should be equal to 0. There is the initial condition for the fluid, that is incident on this heated section.

Therefore, I require that T star is equal to 0 at the location x is equal to 0, because it not touch the heated section of the plate, for y greater than 0. y is equal to 0 is heated but anywhere above is not yet heated. So, that the temperature should be 0 for all locations y greater than 0. So, we have to solve this equation subject to these boundary conditions. In the limit, where the peclet number is large so that, we would expect convection to be dominant in comparison to diffusion.

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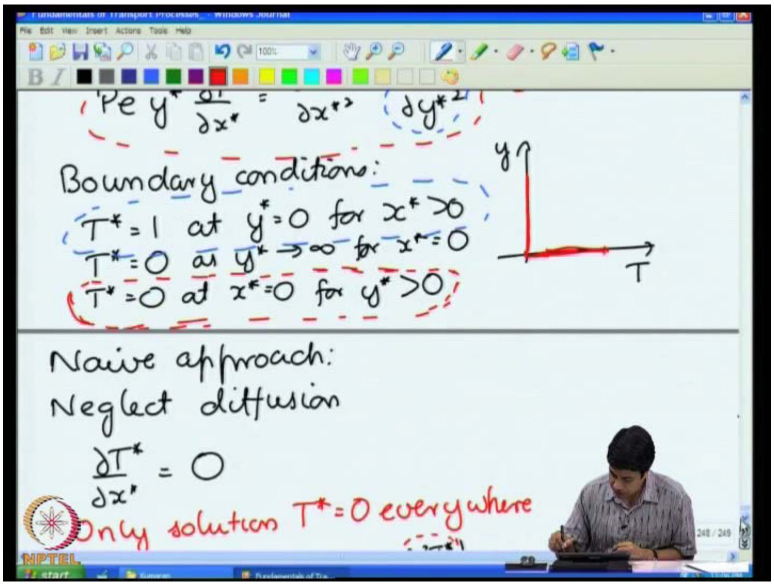

So, simplistically, we just neglect diffusion. And we will end up with the equation of the form dT star by dx star is equal to 0. Because, if I neglect diffusion then I just get Pe y star times dT by dx is equal to 0. And if y is nonzero, the peclet number non zero, then dT by dx has to be equal to 0. That means, the temperature variant in the x direction. The temperature is independent of x. The temperature is independent of x. It depends only upon the y coordinate. However, that means, the dT star by dx star is equal to 0. This condition, tells us that T is equal to 0 for all y, at x is equal to 0. Basically, what dT dx is equal to 0 tells you, there is no variation along the flow stream lines of the temperature. However, at the inlet the temperature was equal to 0 independent of position. That means, that the only solution is T star is equal to 0 everywhere.

Since, T is independent of x, at the inlet T was equal to 0 everywhere. Therefore, the value has to be constant at all values of x. At the inlet it was equal to 0. Therefore, at all values of x constant as we go downstream, the temperature has to be identically equal to 0. So, that is the solution that you get in the high peclet number limit. There is no diffusion. So, because of that there is no transport across stream lines. No transport across stream lines and because of that, whatever the temperature was there at the inlet continuous to be the same temperature everywhere, within the flow at the inlet the temperature was 0. So, everywhere within the flow the temperature continuous to be 0. Clearly, this solution in not compatible with this boundary condition, that in the heated section T is equal to 1. Clearly, it is not compatible with this boundary condition. What this solution is saying, if we neglect diffusion all together the temperature continuous to be a constant and there is no effect of heating, in that heated section.

So, it is incompitable with one of the boundary conditions. The mathematical reason for this incompatibility is quite obvious. If I neglect the diffusion terms all together, the diffusion terms had the second derivative with respect to the y coordinate, the diffusion term at the second derivative with respect to the y coordinate. That second derivation because it was second order differential equation in y, I had 2 boundary conditions in y. When I neglected that diffusion terms, the second order term in y, I converted this from a second order differential equation to an ordinary equation in y.

So, the ordinary one cannot satisfy any boundary conditions. That is the reason, that we are not able to get, satisfy the boundary conditions at the surface of the plate. Because, when we neglect the diffusion terms, we neglect to the highest derivatives. And once I neglect the highest derivative, it can no longer satisfy all the quite boundary conditions. So, that is the mathematical reason, why we are not able to satisfy the boundary conditions. The physical reason is, as I have been telling you numerous times during the course of these lectures, convections transports mass, momentum and energy only along the flow direction. Transport across stream lines can takes place due to diffusion. Because convection, convective transport takes place only along the flow direction. Diffusion has to take place along perpendicular of flow direction. When I neglect diffusion in comparison to convection, there is no transport across stream lines. However, if you look at the mean velocity itself at a bounding surface the mean velocity has to be 0, from the no slip condition and from the no penetration. There can be no flow perpendicular to the surfaces. At a surface because at the surface itself, the velocity of the surface has to be equal to the velocity of the fluid. There is no relative velocity between the fluid and the surfaces at the surface itself.

So, there is no relative motion between the fluid and the surfaces and because and of that can be no net transport perpendicular to the surface due to convection. Any transport perpendicular to the surfaces has to ultimately, takes place only due to diffusion. So, because of this physical reason, because we are neglected diffusion perpendicular to the surface, there is no transport of heat. And consequently, there is no heating up, change in temperature due to the heated section, due to when we neglect the diffusion terms. However, diffusion is to present. It is a molecular phenomenon and it is going to exist. It may be small but it is still going to exist. So, how does one bring into this problem? And the key basically lies here. Note, that this condition says that T star is equal to 0 at x star is equal to 0 for y greater than 0. Note this conditions, it says for y greater than 0. Because, at the surface itself there is 0 velocity, the velocity is equal to 0 exactly at the surfaces, there is transport along the surfaces. So, for y greater than 0, T has to be equal to 0.

These are the boundary conditions that says that T star is equal to 1 exactly at y is equal to 0. So, if I plotted the temperature field due to this, if I plot it as the function of y, if I plotted the temperature at the surfaces itself the temperature has to be 1, but everywhere above it has to be equal to 0. That is what the diffusion equation, the convection diffusion equation is saying without the diffusion terms. However, a profile like this is step profile. If we have a step profile, the derivative of a step function is a delta function which goes to infinity. So, if I have a step profile the derivatives are actually very large. So, even though the peclet number may be a large. And even though, I have an equation of that kind, gamma dot y dt by dx is equal to 1 by Pe times d square T by dx square plus d square T by dy square. Even though this prefecture, may be small because the peclet number is large, if the derivatives is large, the product of these two could still be comparable to the convection term, the product of these two could still be comparable to the convection term.

Note, that I when did the scaling here, when I did the scaling here, I implicitly assume that y star is equal to y divided by L. That means, the line scale for variation of temperature in the y direction is equal to L. So, when I did the scaling in this manner, I implicitly assume that L is the line scale of which the temperature varies in the y direction. On that basis, I did my scaling and then, I found out that convection was large compared to diffusion. And therefore, I try to neglect the diffusion term. However, if the length scale for diffusion is smaller than L, the length scale for diffusion is smaller than L then the gradient will be larger, because the derivative goes as temperature difference divided by the line scale of variation. So, the length scale is smaller. The gradient could be much larger and if the length scale is sufficiently small, I could still get a balance between convection and diffusion, even though the peclet number is based on (()) large. I could have a smaller length scale of which their diffusion such that the gradient sufficiently large, such that the balance between convection and diffusion.

That is what is actually happening in high peclet number flows. Ready near the surface diffusion has to exist. Because, that is the only way the flow is to get heated. But however, if I go line scale comparable to L from the surface, my balance between convection and diffusion tells me convection has to be dominant. If I go a distance L away from the surface, diffusion could still be important within much smaller length scale, because convection is so fast, the heat can penetrate very far into the fluid. But, diffusion was still be important over a much smaller length scale. And that length scale is determinant in such a way or has a sufficient value that, over a distance comparable to that, there is balance between convection and diffusion. So, how I am going to solve this problem? I said physically, the reason that we are not able to get balance is because, we have assumed that the length scale for variation in the y direction is capital L. If the length scale is variation in the y direction smaller than capital L, there might be balance between convection and diffusion. What should that length scale be?

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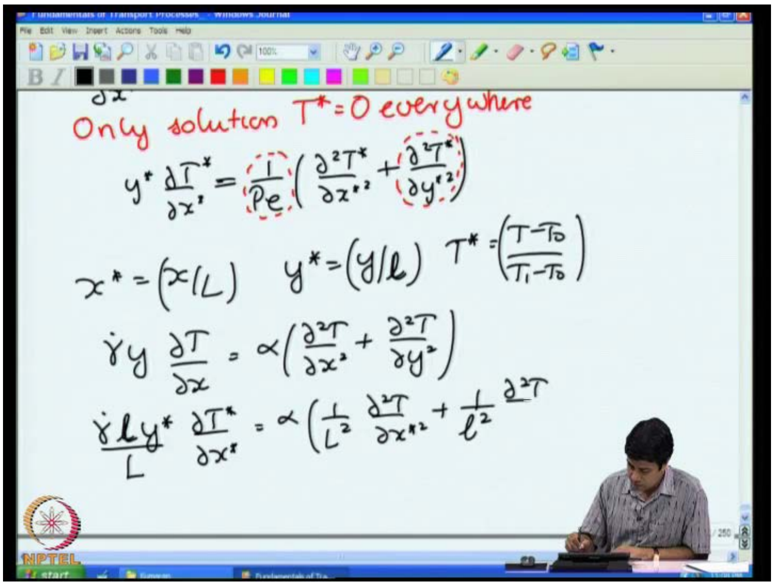

So, what I will do now is to just write down, scaling x star is equal to x by L. This is usual, this is the length scale for variation of temperature, along the x direction, because the heated section has length capital L. However, in the y direction I will postulate smaller, in the length scale. A smaller length scale, smaller. And then go back and scale my equations once again. So, my original equation was gamma dot y partial T by partial x is equal to D into partial square T by partial x square plus partial square T by partial y square. There should be thermal diffusibility alpha. And with this scaling as well as T star is equal to T minus T naught by T1 minus T naught. This becomes gamma dot L y star by capital L partial T by partial x is equal to alpha into 1 over L square.

So, this now is my convection diffusion equation with y scaled by, as unknown scale length smaller. Note that, within this equation in the diffusion terms, this one contains prefactor, 1 over small l square. The first terms contains the prefactor 1 over capital L square. It postulated that small l, was small compared to capital L. That means, that this diffusion term in the y direction is large, compared to the diffusion term x direction. Because, the diffusion term in the y direction goes has small l square, the diffusion term in the x direction goes as over capital L square. That means, that this second coefficient term, in the y direction, the cross stain direction is large compared to the steam vise direction. So, I divide throughout by alpha by L the whole square, if I divide throughout whole alpha by L whole square, rather in the limit of high peclet number I should divide by this term, divide throughout.

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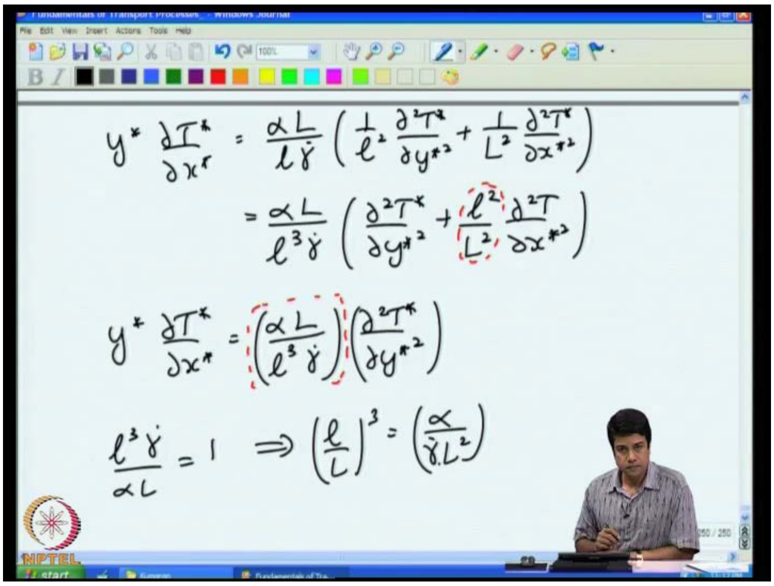

The equation becomes y star dT by dx is equal to alpha L by l gamma dot into 1 over l partial square T by partial y square plus 1 over L square. Alternatively, I could write this as alpha L by l cube gamma dot into. So, this then is my convection diffusion equation. Note, that this is small number, this is a small number, because I postulated that small l compared to capital L. Therefore, I can neglect stream vise diffusion comparison to cross strain diffusion. If I do that, what I get is that y star partial T by partial x is equal to alpha

L by l cube gamma dot partial square T by partial y square. Now, this is the convection diffusion equation. And if convection and diffusion have to be comparable in the high peclet number limit, it means that this term here, has to be of order 1 in the high peclet number limit. If convection and diffusion are to be comparable in the limit of high peclet number, I require that this term has to be order 1. That means, that l cube gamma dot by alpha L is 1, which means that l by L the whole cube is equal to alpha by gamma dot L square.

Now, alpha by gamma dot by L square was the original peclet number that I had inverse. Because, if we recall, we defined peclet number has equal to gamma dot L square by alpha for this problem. So, therefore, I require that l by L is equal to Pe power minus one-third. So, what this is telling me is that, in the limit has the peclet number becomes large, there is a length scale, length; very near the surface small l over which diffusion is comparable to convection and that value of small l decreases Pe power minus one third in the limit has peclet number becomes large. I said that this term has to be just order 1. But, however, I can set it equal to 1, without loss of geniality, because it is just a length scale, which I obtaining by scaling. I can set equal to any constant value, it will change the equations in terms of the length scale L, what I get a final solution for the physical problem, there will be no dependence on this constant.

So, without loss of gentility, I said just the constant is equal to1. This is what we called boundary layer thickness, the thickness of the region over which there is balance between convection and diffusion. Even, when the peclet number is large for this reason that I said even, when the peclet number is large, even though it appears the convection is dominant, is large compared to diffusion, there is still going to be the region very close to a surface where diffusion is present. And it is comparable to convection. That is because the transport from a surface cannot takes place due to convection, because there is no velocity normal to the surface. Diffusion on the other hand is an isotopic process. It is due to the fluctuating velocity of these molecules which have equal magnitudes in all directions. And therefore, they transport mass, momentum, energy equally in all directions. Therefore, very close to the surfaces even though convection is not present, there is still diffusion, and that diffusion is what causes transfer from the surfaces itself.

And the variation of concentration very close to the surface has a length scale L, which is determined from the requirement, that convection and diffusion have to be comparable very close to the surfaces. This requirement itself, what is the value of the length scale. In the limit as Pe goes to infinity, the peclet number becomes large, this length scale goes as Pe power minus one-third, in the limit has peclet number becomes large. And as we saw just now in this equation, stream vise diffusion is always small compared to the cross stream diffusion, very close to the surface. Because, the length scale for cross strain diffusion is the small l, which goes Pe power minus one-third. The length scale for steam vise diffusion is the macroscopic scale capital L itself.

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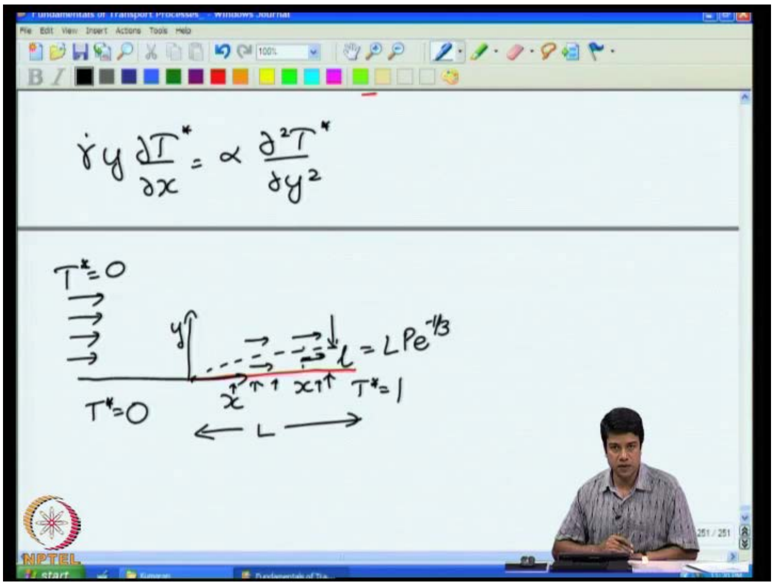

Therefore, for this particular problem the convection diffusion equation reduces to gamma dot y dT by dx is equal to alpha d square T by dy square. For this particular problem, this is the equation. We can neglect the steam vise diffusion in comparison to the cross vise diffusion. Now, we obtained based upon scaling, this as the simplified equation. How do we solve this? This is still a partial differential equation contains variations in both x and y. The solution procedure for this simplified equations is, we can formulate a solution procedure based upon are physical understanding of this problem.

So, I have the section and then I have a heated section here. So, this is y and this is x. And this heated section has some length L. The flow is incident on this heated section. So, T is equal to T naught, or T star is equal to 0. T star is equal to 0 and T star is equal to 1. And we are considering the limit, where convection is dominant comparison to diffusion. And on that basis, that there will be some boundary layer length here, length of small l, which goes as L times Pe power minus one-third, over which there is balance between convection and diffusion.

However, if you consider for example, any particular location, if you consider any particular location x, we consider any particular location x, I have diffusion coming out of the surface, diffusion of heat is coming out of the surface, and there is convection. Now, convection is taking the energy downstream. convection is taking the energy downstream. We are considering, in this case convection is large compared to diffusion based upon the length L. So, convection is taking energy downstream, the diffusion from the surface. That means, at a given position x the temperature should not depend upon the total length L because, the total length L is downstream of the position x. The temperature at given location x cannot depend upon the total length L downstream of that location. It should depend only upon the length of the heated section up to that particular location x. It should only depend upon the length of the heated section up to that particular location x, that means, that at the only length scale which the temperature at given location x should depend is on the distance x from the start of the heated section itself not the total length of the heated section, because the total the remainder of these heated section downstream of these position and flow is carrying the heat downstream. So, that can be no diffusion in the stream vise direction which basically, enables the temperature at this location x to feel the downstream locations.

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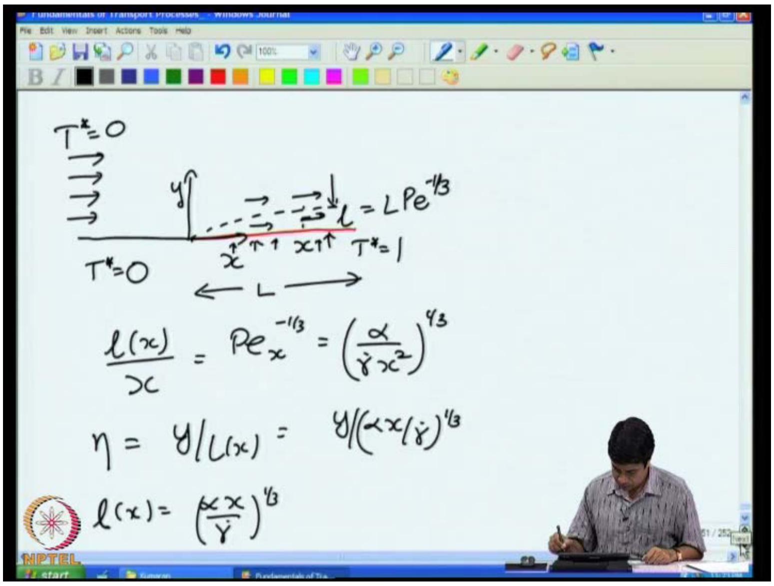

So, if x is the only length scale in the problem and the boundary layer thickness l at a given location x divided by x has to be equal to the peclet number based upon x to the minus one-third. Which is equal to alpha by gamma dot x square power one-third. So, the only length scale in the problem is x. So, I will just substituted x for L in this particular case assumed that the boundary layer thickness L is the function of x. This will enable us to get a similarity solution because of the length scale, the y direction is L, then I can define a scaled similarity variable has y by L of x. And L of x is equal to alpha x by gamma dot power one-third, if I can define this as y by alpha x by gamma dot power one-third.

This was obtained just basically from the consideration that at a given location x the temperature should depend only upon the distance from the start of the heating section, not the total length of the plate itself. So, I cannot scale my x, my y coordinate the boundary layer thickness by capital L. Because, at a given location x, it should not depend upon the total length L. It should depend only upon distance x from the upstream section. So, my boundary layer thickness L of x has to be alpha x by gamma dot power one-third. If this is the boundary layer thickness, then I just divide y by that scaled variable because the characteristic length scale in the y direction is this small l. So, I can define eta is equal to y by alpha x by gamma dot power one-third. This is the similarity variable, which reduce the problem from two dimensions into one dimension. If you recall the impulsively impulsively started plate, there we had obtained the similarity solution based upon dimensional analysis the fact that, there is no length scale or time scale in the problem. Therefore, I can get only one similarity variable, here we are doing based upon physical insect.

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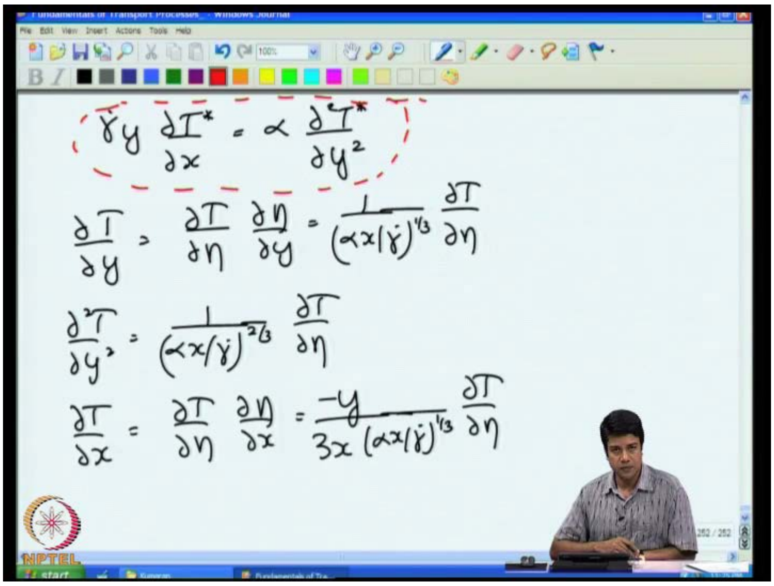

So, now, I have to substitute in this convection diffusion equation. I have the convection diffusion equation gamma dot y partial T by partial x is equal to alpha partial square T by partial y square. So, in this equation I have to substitute y and x in terms of eta. So, I will get partial T by partial y is equal to partial T by partial eta times partial eta by partial y is equal to 1 over alpha x by gamma dot power one-third partial T by partial eta d square T by d y square is in similar manner 1 over alpha x by gamma dot power two-thirds. And dT by dx is equal to dT by d eta times d eta by dx is equal to y by power one-third, x power minus one-third. The derivatives this will give you minus 1 by 3 times x power minus 4 by 3. This finally, has to be substituted into this differential equation. And then we have to get solutions for this differential equations. If what we done here is correct, then we substitute this into that differential equation, I will end up with this equation in terms of eta alone. It should not depend separate x and y. The final equation should depend upon eta alone.

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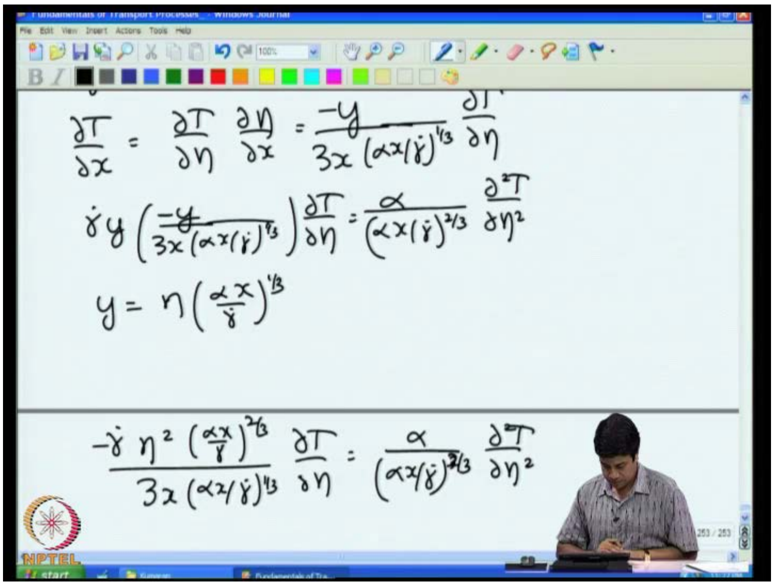

So, what we get substitute this? Get gamma dot y into minus y by 3 x alpha x by gamma dot power one-third dT by d eta is equal to alpha into divided by alpha x gamma dot power two-thirds d square T by d eta square. So, I can simplify this a little bit by using the simplification y equal to eta into alpha x by gamma dot power one-third. So, I will get gamma dot eta square into alpha x by gamma dot power one-third by 3 x into alpha x by gamma dot power one-third dT by d eta is equal to alpha by power two-thirds d square T by d eta square. And you can easily verify that all of these terms will cancel out.

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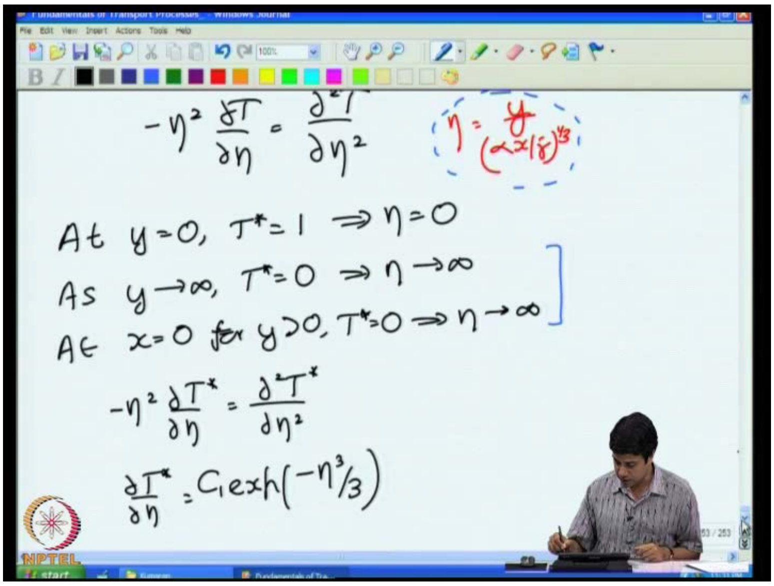

Finally, to give you minus eta square dT by d eta is equal to d square T by d eta square. So, this then is the final solution for the equation in terms of eta. As we expected the scaled the y coordinate by that length L, which is based upon the x, x distance. The distance from the upstream edge, then you will finally end up the solution in which, it does not individually depend upon x, y, gamma dot and alpha. It depends only upon the similarity variable eta. So, this gives us the solution the temperature field in terms of the similarity variable alone and does not upon separately on x and alpha, y and x, but only on the similarity variable alone.

So, now we have equations in terms of the similarity variable eta. We have to reformulate the boundary condition as well in terms of similarity variable eta. The original boundary conditions, at y is equal to 0, T star is equal to 1 T star is equal to 1, at y is equal to 0 implies that eta is equal to 0, because eta is equal y by alpha x by gamma dot by alpha power one-third. As y goes to infinity T star is equal to 0, y going to infinity implies that, this implies that eta is going to infinity. And at x is equal to 0 for y greater than 0, T star is equal to 0. And you can see from this, that x is equal to 0, basically means that eta goes to infinity. x is equal to 0 basically means that eta goes to infinity. So, at x is equal to 0. So, you can both of these boundary conditions one boundary condition in the y coordinate as y goes to infinity and the other, x coordinate at x is equal to 0 reduced the same boundary condition, expressed in terms of eta. These two reduces exactly the same, when expressed in terms of eta. That is expected.

Originally, I have the first order differential equation in x and second order differential equation in y, shown in on red, right on top there. The first order in x and second order in y, the two conditions in y and x. When I reduce it transform, I got only one second order differential equation in eta x L there is only two boundary conditions. That means, that one of the boundary conditions y as well as the initial condition in x has to reduce to same when expressed in terms of eta. So, I can solve this equation minus eta square partial T by partial eta is equal to partial square T by partial eta square, integrated once to get partial T by partial eta is equal to e power minus eta cube by 3. I integrated once to get partial T by partial eta is equal to C. I am sorry, some constant C1 minus eta cube by 3.

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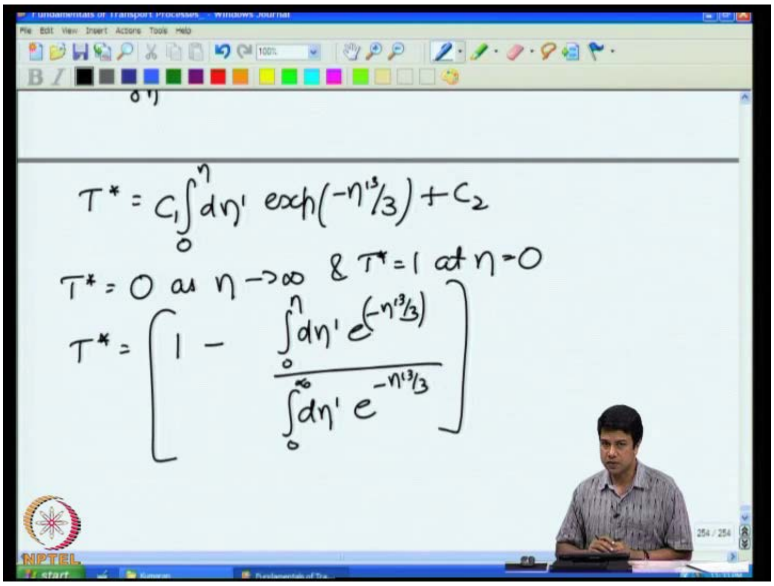

And I can integrate this as second time, to get T star is equal to integral from 0 to some value of eta. eta prime exponent of minus eta prime cube by 3 plus some other constant C2. So, this is the final solution. This integral cannot be evaluated exactly because of the, because, we have definite integral here. Because, we cannot integrate minus e power cube exactly.

And then we apply the boundary conditions. T star is equal to 0 as eta goes to infinity and T star is equal to 1 at eta is equal to 0, in order to determine, specify constants C1 and C2. So, the constants turnout to be this final solution, after imposing the boundary conditions turns out to be eta prime power eta prime minus power 2 by 3 divided by integral 0 to infinity. You can easily verify that this solution is 1 at eta is equal to 0 and it is 0 as eta goes to infinity. This is the final solution for the temperature field. In terms of the variable eta where eta is equal to y by alpha x by gamma dot power one-third. There is the final expression in terms of this eta.

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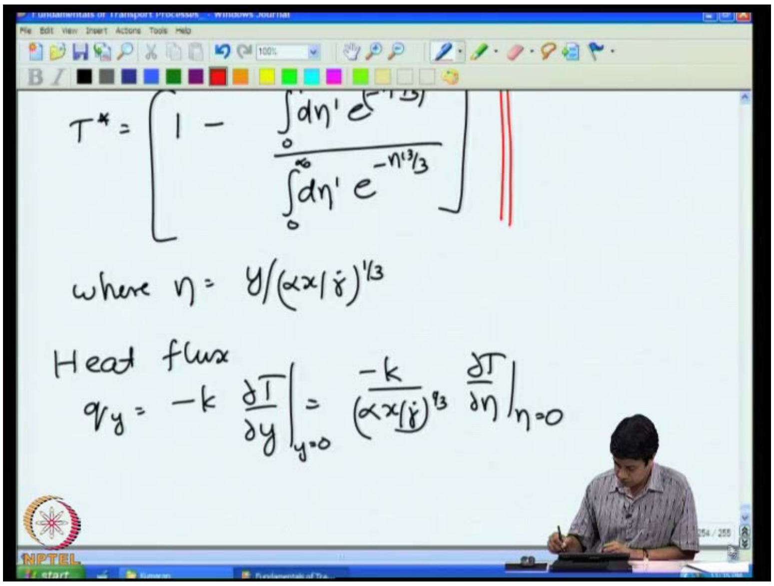

How can we use this to calculate the heat flux? The total amount of heat coming out of the heated surface? Locally the heat flux q y is equal to minus k dT by d y. I have to express this in terms of eta, if you recall dT by d y is equal to 1 by alpha x gamma dot to power one-third times dT by eta. The heat flux coming out of the surface y is equal to 0. So, this will be equal to minus k by alpha x by gamma dot power one-third dT by d eta at eta is equal to 0. And dT by d eta at eta is equal to 0 can be quite easily calculated from this solution here.

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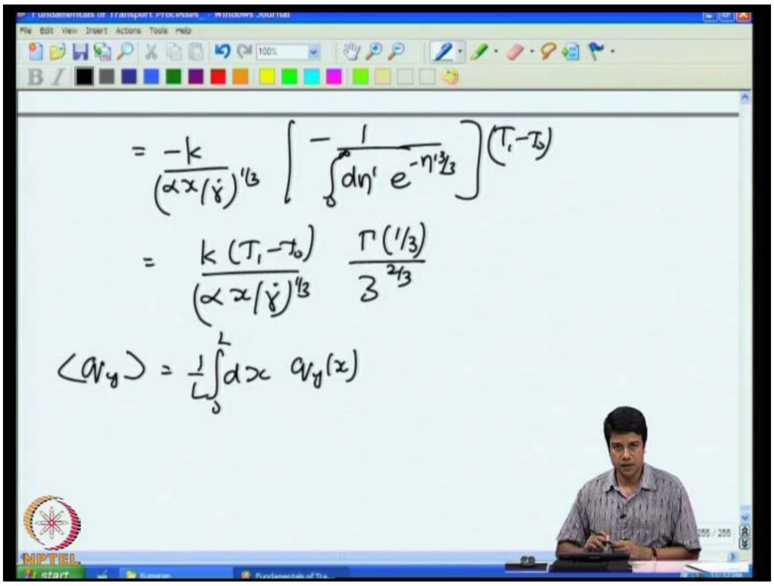

And d T by d eta is just equal to minus, so this q y will be equal to minus k by alpha x by gamma dot power one-third into minus 1 by integral 0 to infinity d eta prime, e power minus eta prime cube by 3 into T1 minus T naught. Because T is equal to T1 minus T naught, T minus T naught minus T 1 minus T naught times T star. This is the final expression, we get for the heat flux. So, this is equal to k into T 1 minus T naught by alpha x by gamma dot power one-third times. This function can be integrated exactly gamma 1 by 3 by 3 power two-thirds. That is the final solution heated flux from the surfaces.

The average heat coming out of the surface, the average heat coming out from the surface is the average over an entire length 0 to L dx times q y at this location x. And this can be evaluated analytically, in order to get the average heat that is coming out from the surfaces. From this, we will now go calculate the nusselt number coordination for the heat transfer from the surface in the limit of high peclet number. When we did the initial lectures, I showed you that the nusselt number, in the case of convicted dominant flows at high peclet number goes Pe power plus one-third. You have done here an analytical solution for the flow past a flat surface peclet numbers. And we got a solution in terms of similarity variable. I will now use this, how we get the high nusselt number coordination. So, I will first complete the solution of flow past a flat surface peclet numbers and then got a solution in terms of similarity variable. I will now use this to show you, how we get a high peclet number coordination, complete the solution of flow past the flat plate in the next lecture. And then, I will discuss the flow past the spherical particle and evaluate the nusselt number coordination. So, slowly we are recovering all original results that we had as a empirical correlation is the same thing deriving here, by exact calculation. Using certain approximations the low nusselt number limit, I am sorry the peclet number limit and the high peclet number limit. We will continue this in next lecture. Will see you then.