Fundamental of Transport Processes
Prof. Kumaran
Department of Chemical Engineering
Indian Institute of Science, Bangalore

Lecture No. # 32
Diffusion Equation Spherical Harmonics

So, welcome to lecture number 32 at the end of the last lecture I told you that we will start looking at a point source in the heat conduction equation. Before that if you recall we had solved the equation for the Laplace equation in a spherical coordinate system the heat conduction equation.

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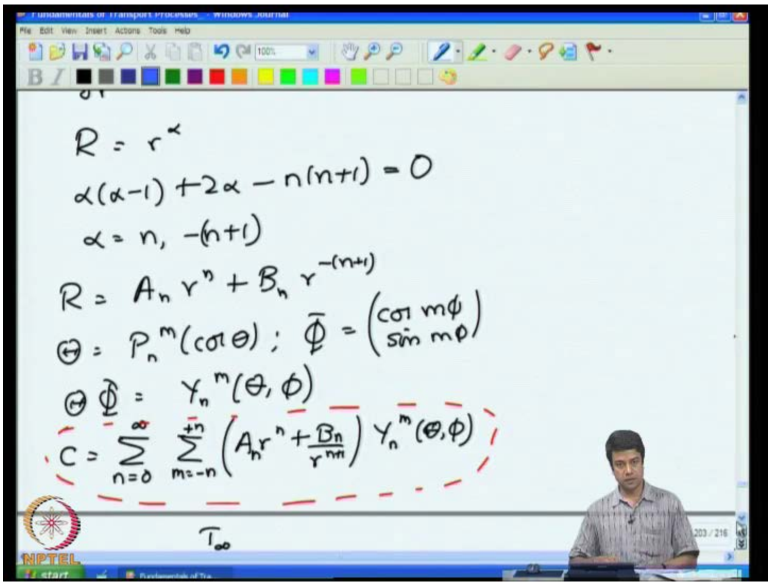

And we had got solutions of the general form shown in the red there a summation over spherical harmonic expansions Y n m of theta and phi. These are products of the Legendre polynomials P n m of cos theta and cos or sine of m phi those are orthogonal functions. They are all orthogonal to each other with the inner product defined as an integral over phi from 0 to phi and over theta from 0 to pi. And then we have these dependencies on the radius r power n plus r power of minus n plus 1.And I had shown you that for n is equal to 0 and m is equal to 0 you get spherically symmetric solutions. And then we had solved for the effective conductivity of a composite where the temperature for this far away from the sphere was proportional to z itself.

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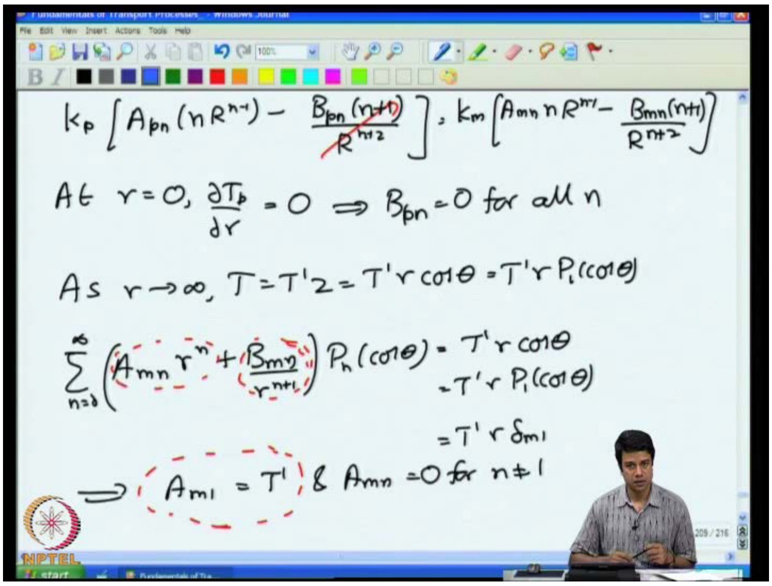

And in that case we got solutions only for n is equal to 1 in that case we got solutions only for n is equal to 1 that was because forcing was of that form. There was a temperature which went proportional to z. As you went far away from the particle that was what was causing all of the temperature variations around the particle. The forcing has that symmetry the one 0 symmetry. Then the solution that you get will also have the exact same 1 0 symmetry and therefore, we can just 0 in on the solution we know that it has to have 1 0 symmetry.

So it has to have the form A n a 1 times R power plus 1 plus B 1 divided by R square times Y 1 0 of theta and phi. And we had used that usefully to actually get the effective conductivity of a composite material in the limit where the composite was dilute in the sense that, the volume fraction of the particles was small.

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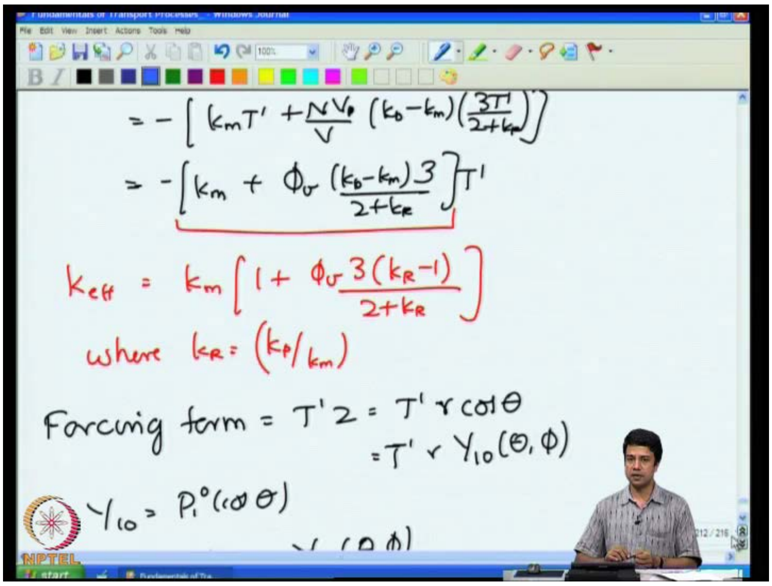

So, that the distance between particles was large compared to the particle radius. In that case the temperature field around one particle is not significantly distorted by the effect of other particles. So, in that case we had actually derived the thermal conductivity in the last lecture.

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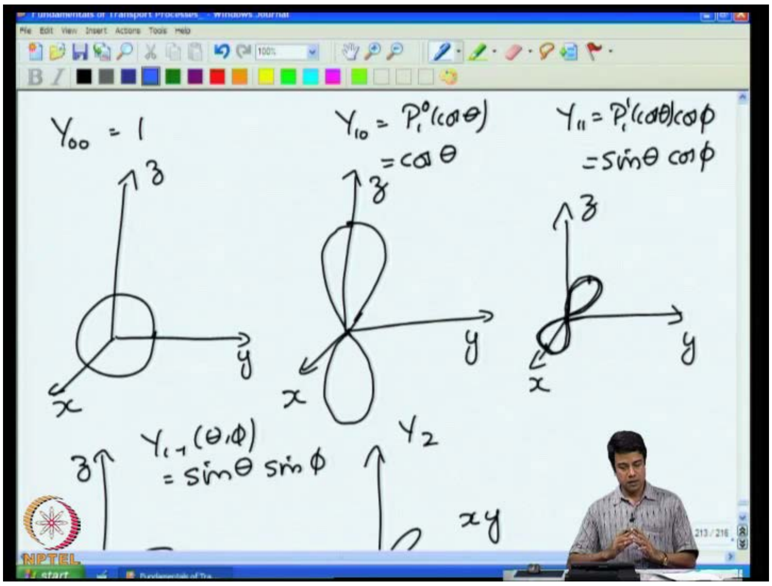

And I had briefly discussed for you the symmetries of these polynomial legendary polynomial expansions the symmetries of this spherical harmonics. Y 0 0 is a constant, it is independent of theta and phi. Therefore, surfaces of constant Y 0 0, there can be a variation in Y 0 0 only in the r direction. So, surfaces of constant Y 0 0 are spherical shells, Y 1 0 is basically looks like a dumbbell along the z axis. Y 1 0 is p 1 0 of cos theta and as cos theta varies as theta varies from 0 to pi cos theta goes from 1 down to 0 and then minus 1. So, it is often plotted as this way similarly Y 1 1 is along the x axis, Y 1 minus 1 is along the y axis and then I plotted for you the symmetries of this n is equal to 2 Legendre polynomials.

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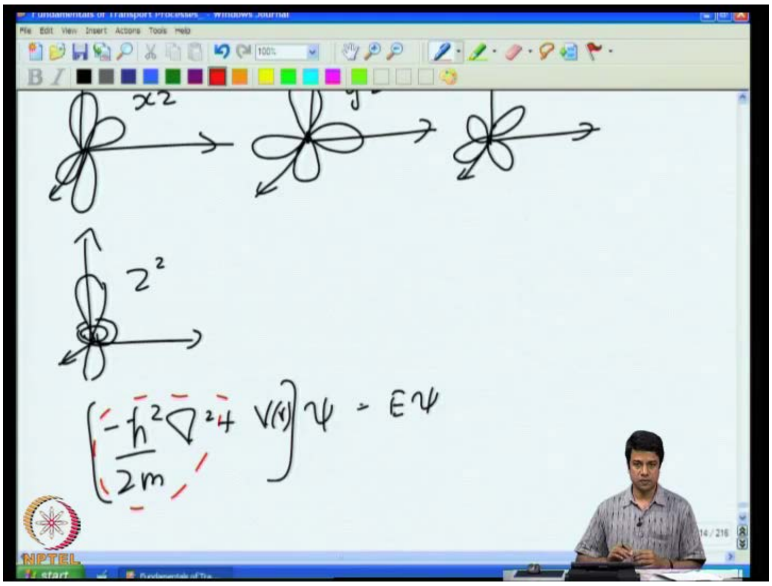

Similarly, there are solutions for 3, 4, 5 etcetera. For a value of n there is 2 n plus 1 solution because for a given value of n, n can vary from minus n to plus n. And I briefly remarked in the end of the last lecture that these equations are identical to the solutions Schrodinger equation. Because in that case as well you are solving the Laplace equation for the theta and phi components of the wave function. And that is the reason that the solutions the symmetries of the solutions are exactly the same where there you solve the diffusion equation the Laplace equation, equation for the potential in electrostatics. The only thing that changes is the dependence on r because in this case I have a potential which depends upon r.

Therefore, the solutions in that I get in this case are what are called Laguerre polynomials in the radial direction, for the unsteady heat conduction you get spherical vessel functions. In this particular case for the steady diffusion equation I get two sets of functions, one of which increases proportional to r power n. The other decreases proportional to 1 over r power n plus 1 called the increasing and the decreasing harmonics respectively. If you are solving the problem within an object of finite size, then the solution would only contain the increasing harmonics because the decreasing harmonics actually go to infinity at the origin.

So, therefore at the temperature is finite throughout the domain we can have only the increasing harmonic solution. On the other hand if you are solving a problem outside an object such as a spherical particle, we can have only decreasing harmonic solutions. Because the increasing harmonics will go to infinity as r goes to infinity.

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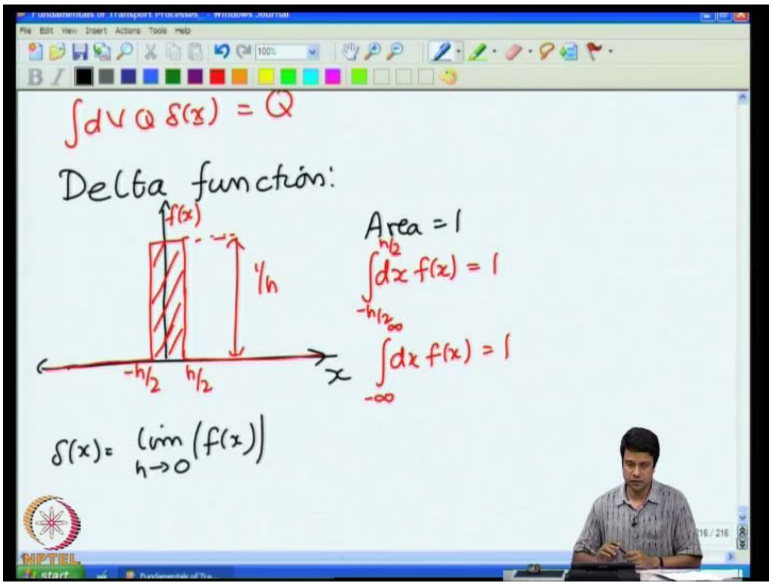

And then I briefly started the discussion of source dipole quadrupole and so on let us just continue with that delta function in one dimension this is x. if I take a function(No Audio From: 06:49 to 06:56) that looks like this(No Audio From: 07:00 to 07:10). The function that looks like this, it goes from along the x axis, it goes from minus h by 2 to h by 2 and the height of this function is 1 over h. The total area under the curve is going to be equal to 1, the reason is because the width of this function is h the height is 1 over h. I multiply the two therefore I will get just one that means that this function f of x satisfies the integral condition integral dx f of x is equal to 1 from minus h by 2 to h by 2.

However this function is 0 everywhere else, it is non 0 only in the interval between minus h by 2 and h by 2 it is 0 everywhere else. That means that I could very easily write the same thing as minus infinity to infinity dx f of x is equal to 1. So, this is non 0 between minus h by 2 and plus h by 2 the height of this is 1 over h therefore, the integral is 1. What is the delta function? delta of x is equal to limit as h goes to 0 of f of x, that is if I take the limit as h goes to 0 that is the two points minus h by 2 and plus h by 2 as the approach the origin. In that limiting case this function is the delta function, limit h is going to 0 the height is 1 over h.

So, as h goes to 0 the width of this function along the x axis goes to 0 the height goes to infinity therefore, in the limit as the width goes to 0 and the height goes to infinity I get the delta function. So, sharply peaked function it is non zero only at x is equal to 0, 0 everywhere else the total integral or the area under that function is equal to 1. We have to (( )) the limit as the thickness goes to 0 the height goes to infinity such that the product of the thickness and the height is exactly equal to 1. That is one formal definition of the delta function the one that we will use here for our present purposes. There are other functions also that you can define we will not bother with that right here we will use this particular formula definition.

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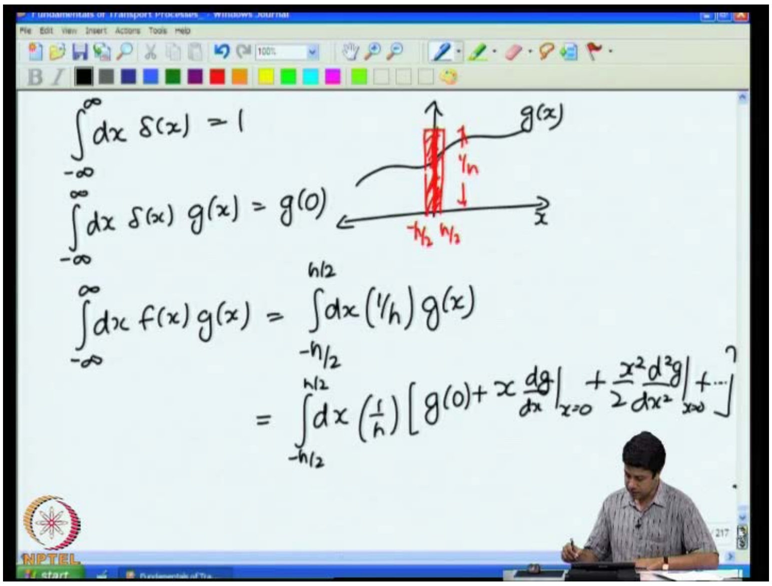

What are the properties of the delta function, this automatically implies that delta of x is equal to 0, for x is not equal to 0. The delta function has the property like whenever x is not equal to 0, the delta function is identically equal to 0, it is non zero only at x is equal to 0. At x is equal to 0 the value of the delta function is strictly speaking undefined. However I have an integral condition integral dx delta of x from minus infinity to infinity this has to be exactly equal to 1. So, there is an integral condition stating that the integral of delta of x dx has to be equal to 1. In addition there is another property that is that integral minus infinity to infinity dx delta of x times any function g of x is equal to g at 0.

Let me explain this a little bit detail this is x, this is the function g of x the delta function is non zero only at the origin so, this is the delta function everywhere else. Apart from x is equal to 0, g of x is being multiplied by 0. So, it is identically equal to 0 only exactly at the origin g of x is non zero and I am sorry delta of x is non zero. So only at the origin is g of x being multiplied by a non zero number therefore, this result is non zero only at x is equal to 0. Let us do this limiting a little bit more carefully let us take a finite delta of x with height 1 over h and going from minus h by 2 to h by 2 multiply g by this function.

And then take the limit as h goes to 0 so, this integral dx f of x g of x where f of x was this function that I had there. I multiply that by delta of x and take the limit this is going to be equal to since f of x is non zero only between minus h by 2 to plus h by 2. I get minus h by 2 h by 2 dx between minus h by 2 and plus h by 2 this is equal to 1 over h times g of x. Now we are taking the limit as h goes to 0 therefore, in this limit I can write this as dx minus h by 2 to h by 2 times 1 by h, expand g of x in a Taylor series about x is equal to 0 that is equal to g of 0 plus x times dg by dx, at x is equal to 0 plus x square by 2, d square g by dx square plus dot dot dot.

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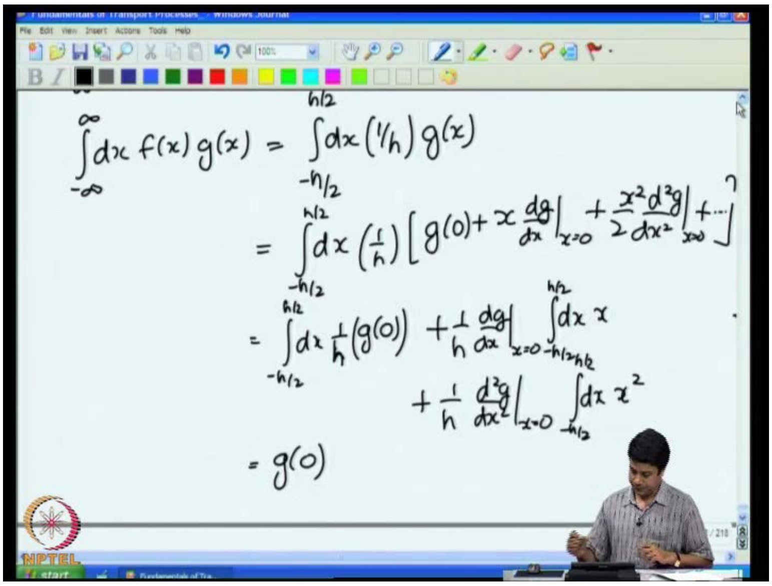

So, this is the Taylor series expansion of the function g of x and then I can integrate out the terms one by one. The first term that is equal to integral minus h by 2 to h by 2 dx 1 over h g of 0, within the integral the only there is g of 0 which is a constant and 1 over h which is a constant. So, I am just integrating a constant from minus h by 2 to plus h by 2 plus the second term 1 over h is a constant dg by dx evaluated at x equal to 0 is a constant integral dx times x plus the third term will be 1 over h d square g by dx square at x is equal to 0, dx times x square. So, this first term is just an integral of a constant so, minus h by 2 to plus h by 2 integral dx 1 over h this is just equal to g of 0.

The second term if you see I have an integral of x times dx so, that is going to as h square when I integrate it from minus h by 2 to plus h by 2. This particular case it is just equal to 0 but, in general it is not equal to 0, the third term I have an integral of x square dx from minus h by 2 to plus h by 2. That will give me a contribution which goes as h cubed in the limit as h goes to 0, each of these contributions goes to 0 and therefore, I am just left to the function g of 0. So, this was when the delta function was at the origin the delta function can be at any position.

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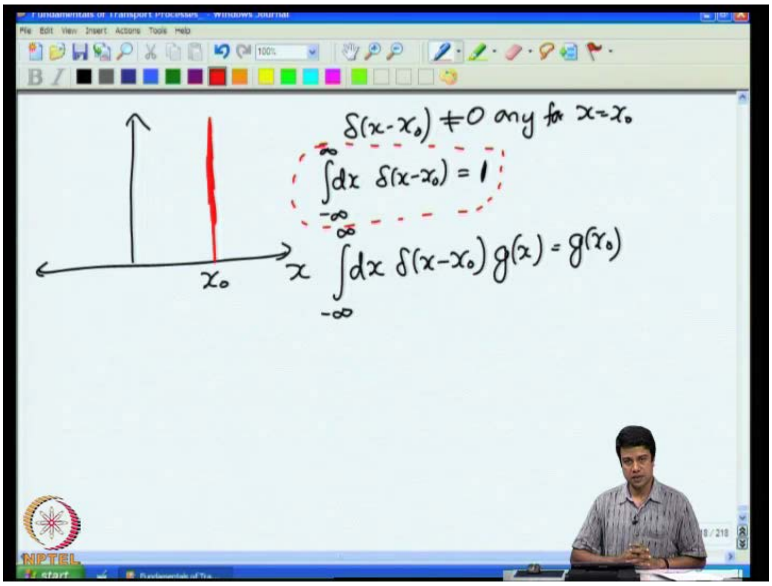

So, if there is a delta function at some position x 0 the delta function will be defined (No Audio From: 16:48 to 17:08) at the location x 0, delta of x minus x 0. So, I have just shifted the origin of my coordinate system to x 0 and I defined delta the delta function at the location x 0 as delta of x minus x 0. So this has exactly the same properties as delta of x except that in the case of delta of x x 0 was exactly equal to 0. So the same properties carryover here as well, this is not equal to 0 only for x is equal to x naught. Integral dx from minus infinity to plus infinity is equal to 1 area under the curve of this delta function is independent of where it is located it is always equal to 1. And integral minus infinity to infinity dx delta of x minus x 0, g of x is equal to g of x 0.

So, these are the properties of third delta function. Note that this condition integral dx times delta function is equal to 1. That means that the delta function has dimensions of one over length in one dimension the delta function has dimensions of 1 over length. Because integral dx times delta x is equal to 1, that means that delta goes as inverse of length because x is a length in one dimension. One can similarly, define delta functions in two and three dimensions.

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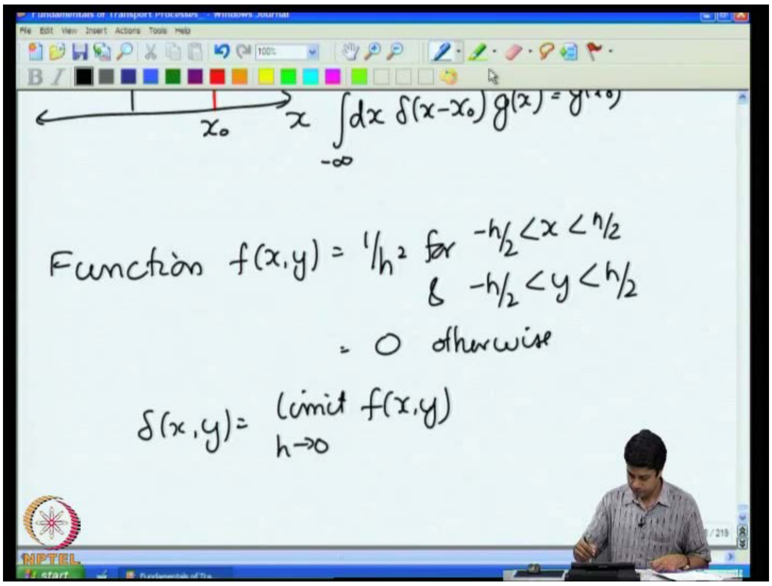

So, let me just briefly take you through the definition in two dimensions it is exactly analogous. Delta of (x, y) is equal to 0 for so let us first work with the finite delta function. Function f of (x, y) is equal to 1 over h square for minus h by 2 less than x less than h by 2 and minus h by 2 less than y less than h by 2 is equal to 0 otherwise this is a two dimensional delta function. And the limit h goes to 0, delta of(x, y) is equal to limit h going to 0 of f of (x, y).

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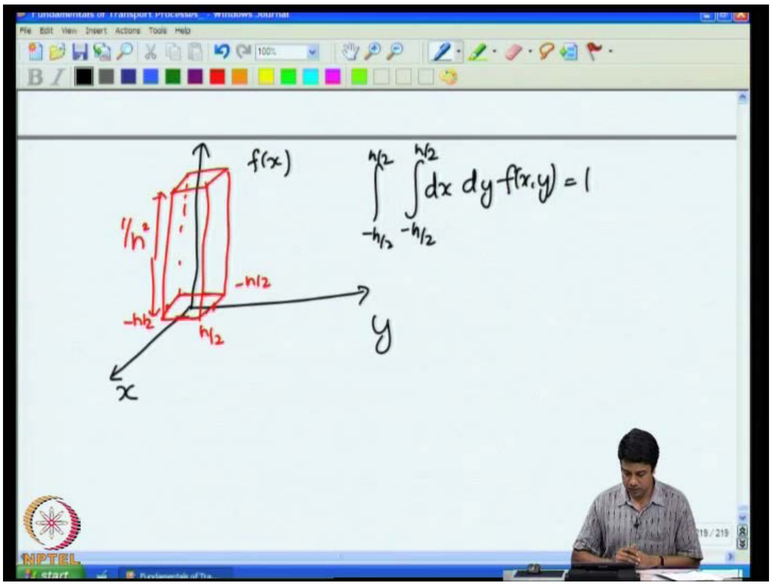

So, to visualise this we can visualise it in three dimensions. So, let us say that I have x y this is x, this is y and this is f of x, this f is non zero only when x is between h by 2 and minus h by 2. As well as y is between plus h by 2 and minus h by 2 that means that in this rectangle. Only within this rectangle between minus h by 2, h by 2 both in x and in y this is minus h by 2 and x and h by 2 and x as well as in y it is only within this rectangle around the origin that this is non zero. And within this rectangle this function has a constant value and that constant value is 1 over h square. This constant value is 1 over h square over this rectangle, this rectangle has length which actually a square with length h in the x axis and length h in the y axis.

Therefore, integral dx dy f of (x, y) from minus h by 2 to h by 2, this has to be equal to 1 so, this is the total volume under the curve in this case. And since this is non zero only between minus h by 2 and plus h by 2 the integral condition could well be written as integral minus infinity to infinity dx integral minus infinity to infinity, dy f of (x, y) is equal to 1 .

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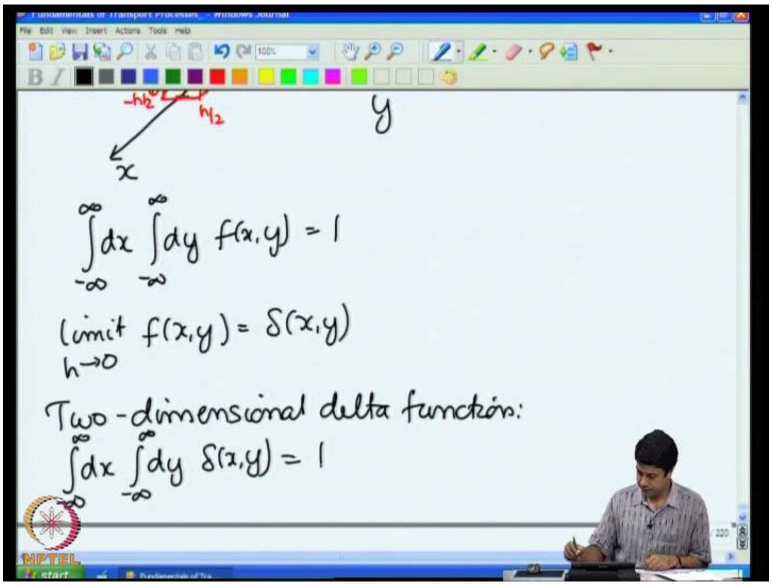

So, this is the property of the two dimensional delta function in the limit h going to 0 (No Audio From: 23:05 to 23:15) the limit as h goes to 0, f of x y is equal to delta of (x, y). Therefore, the two dimensional delta function has the properties (No Audio From: 23:27 to 23:37) integral minus infinity to infinity, dx this has to be equal to 1.

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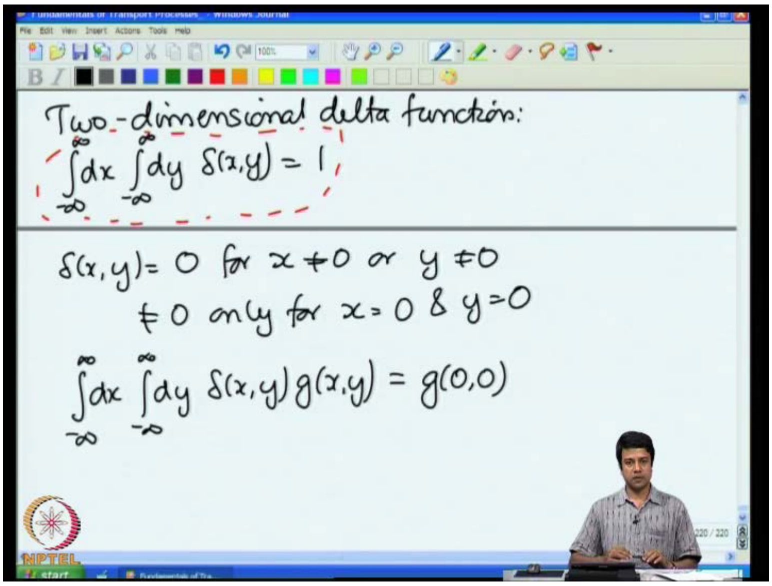

Delta of (x, y) is equal to 0 for x not equal to 0 or y not equal to 0. That is everywhere except at both x and y equal to 0 it has to be 0, there is not equal to 0 only for x is equal to 0 and y is equal to 0. So, it is not 0 only when both x and y are 0 otherwise it is 0 and the area under the curve has to be equal to 1. And we had previously shown you in one dimension that integral delta of x times g of x is equal to g of 0. Similar condition can be easily derived here integral minus infinity to infinity dx integral delta of (x, y), g of (x, y) where g is any function this is equal to g of (0, 0).

So, this is the equivalent condition for the function g delta times g integrated from minus infinity to plus infinity is equal to g of (0, 0). Now the three dimensional delta function is the extension of the same thing two three dimensions I cannot draw it for you in three dimensions, but I can define it for you.

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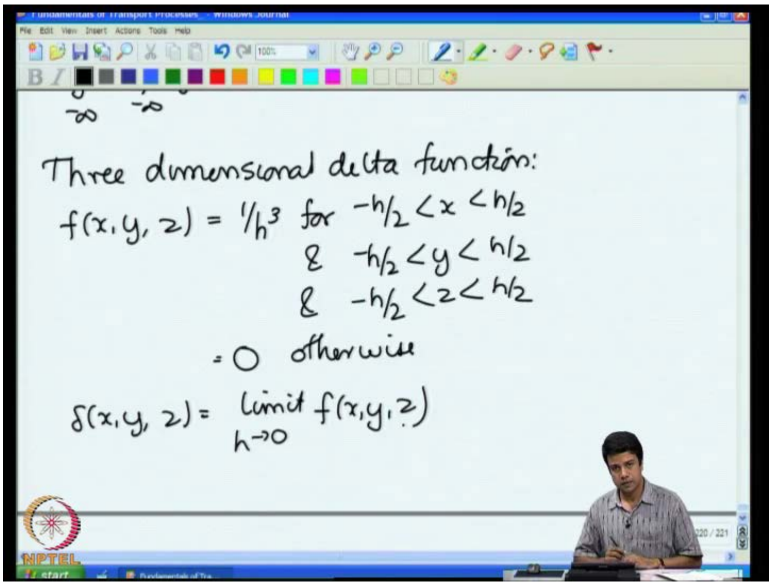

(No Audio From: 25:34 to 25:51) Note before that that in two dimensions the delta function integral dx dy delta of (x, y) is dimensionless it is just equal to one. That means in two dimensions the delta function has dimensions of 1 over length square, as I said this function is equal to 1 over h square between minus h by 2 and plus h by 2 h is a length. Therefore, the delta function has to have dimensions of 1 over length square in two dimensions. Similarly, I can do it in three dimensions in that case the formal definition is of this function is f of (x, y, z) is equal to 1 over h cubed for 0 less than x less sorry (No Audio From: 26:44 to 26:51) and and (No Audio From: 26:56 to 27:03) is equal to 0 otherwise. So, this is the formal definition of the function f and delta function delta of (x, y, z) is equal to limit as h goes to 0 of f of x.

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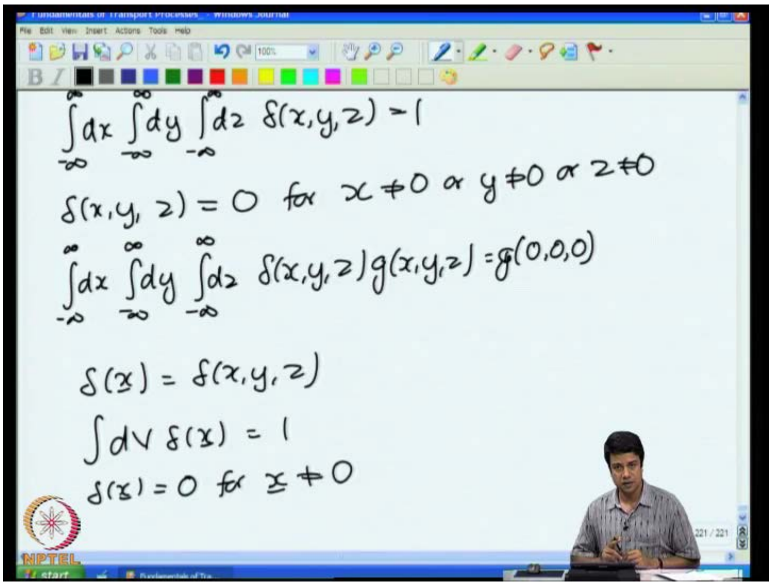

And it has the following properties integral minus infinity to infinity (No Audio From: 27:35 to 27:45) is equal to one (No Audio From: 27:47 to 27:54) is equal to 0 for x not equal to 0 or (No Audio From: 28:01 to 28:28) just the analogous conditions from one, two down to three dimensions. Now this delta function in three dimensions has dimensions of 1 over length cubed because integral of the delta function times a volume is equal to 1. Rather than writing it separately in terms of x y and z, I will use a shorthand notation I will represent integral dx dy dz as just a volume integral. So, what I am going to do is to define the delta function delta of x, this is understood to be delta of (x, y, z).

Since x is a vector with three components this delta function has dimensions of one over length cubed. And the properties of this delta function are integral dv delta of x is equal to 1, where v is the entire volume which includes the location of the delta function. So, the entire volume includes the location of the delta function, then integral of the delta function over that volume has to be equal to 1. And delta of x is equal to 0 for x not equal to 0 because the vector will be 0 only if all of these it is three components are equal to 0.

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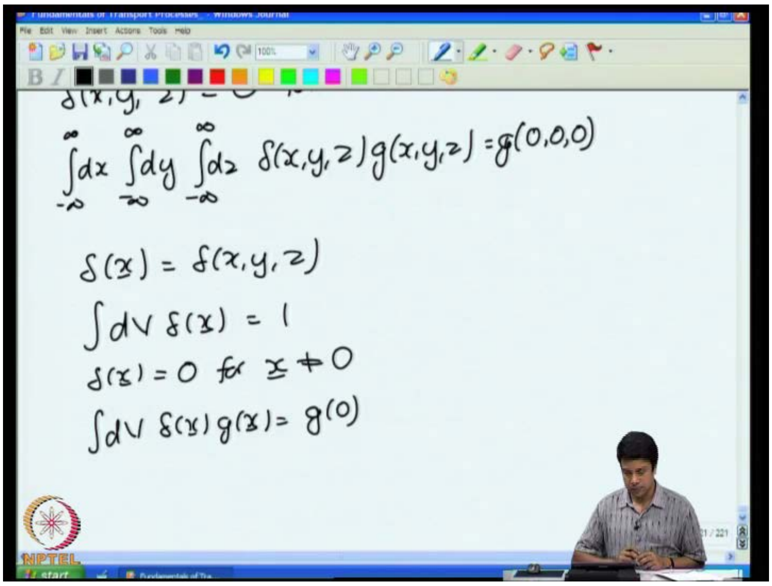

So, for delta of x is equal to 0 for x not equal to 0, integral of volume of delta of x g of x where g is some function of the vector is equal to g of 0. So, these are the properties of the delta function that I will use.

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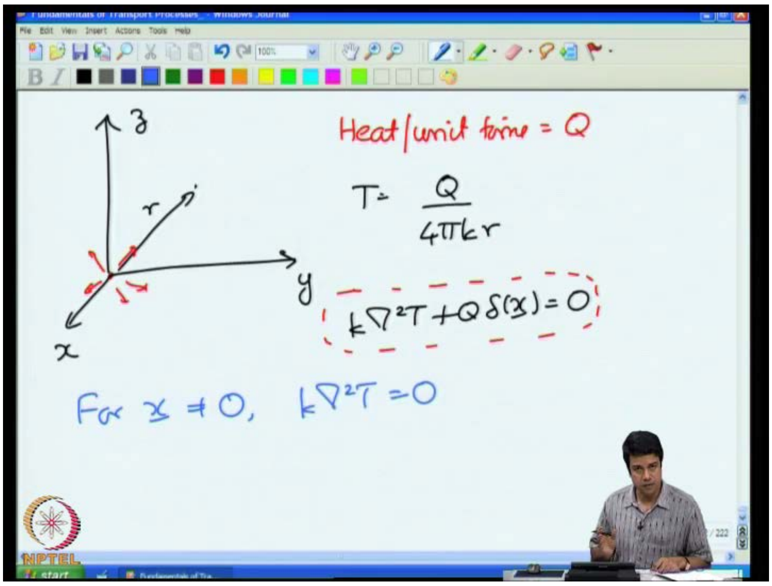

So, back to our heat conduction problem (No Audio From: 30:19 to 30:30) I told you that if there is a point source at the origin. We have discussed this in the last lecture which is emitting heat, heat per unit time is equal to Q. Then the solution for the temperature field is T is equal to Q by 4 pi k r where r is the distance of the observation point from the origin in a spherical coordinate system. Turns out that this solution is also a solution of the equation k del square T plus Q, delta of x is equal to 0 how does that come about. So, first of all if I try to solve this equation note that k del square T plus Q times delta function is equal to 0, delta function is non zero only when x is equal to 0. Therefore, for x not equal to 0, the equation is k del square t is equal to 0 the solution in spherical coordinates. Note that this is a spherically symmetric system there is no variation in the theta or phi coordinates.

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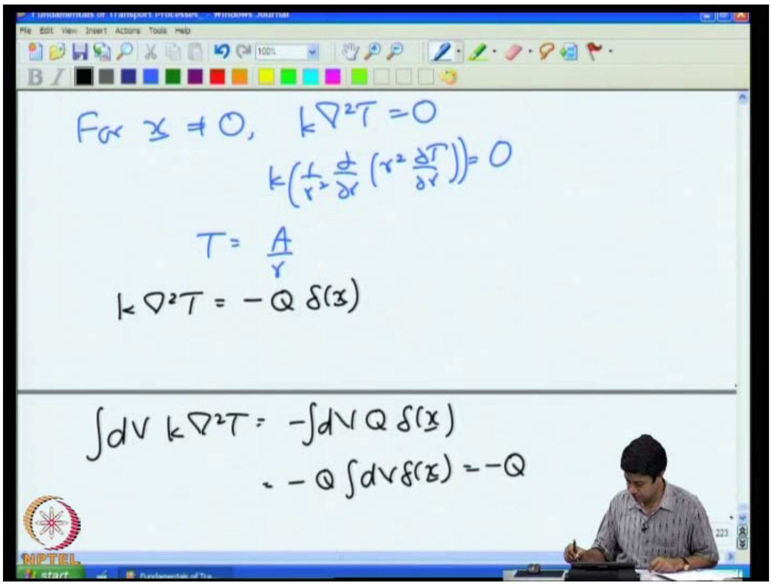

So, for a spherically symmetric system this can be written as k into 1 by r square d by dr of r square dT by dr is equal to 0, which means that the solution is of form T is equal to a by r plus b. And if I define my temperature in such a way that the temperature goes to 0 at a large distance from the origin then the constant b has to be equal to 0. Therefore, the solution is simply of the form T is equal to some constant divided by r, how do I relate this constant to Q? it can be done in the following way. I have k del square T is equal to minus Q delta of x. (No Audio From: 33:28 to 33:42) Now delta of x is non zero only when x is equal to 0 so, I can usefully obtain a condition for this by taking a volume integral over the entire volume.

I know that delta is equal to 0 for x naught equal to 0, but however instead of equating to each and every point I can do it as a volume integral. So, we take integral over the volume dV k del square T is equal to integral over the volume of Q times delta of x so, it is minus Q integral dV delta of x this is minus Q.

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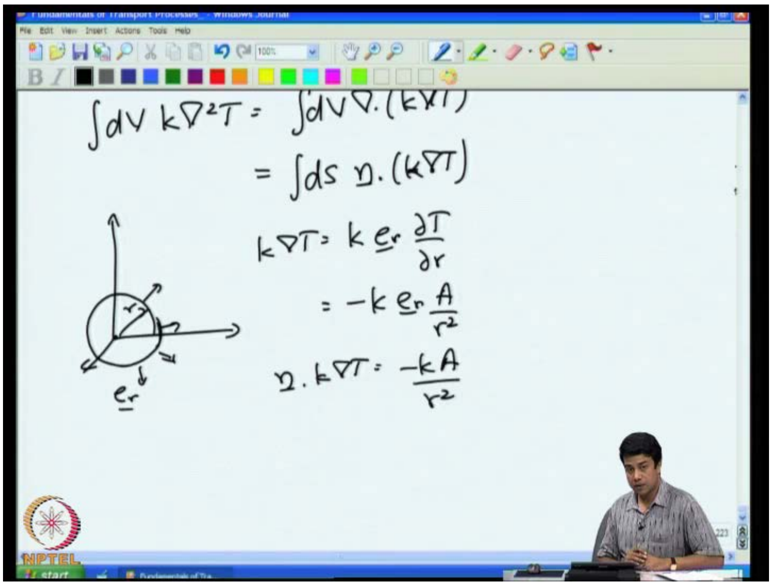

Now on the left hand side this volume integral k del square T I can write it as integral over the volume dV of the divergence of k grad T. The divergence theorem tells me that this can be simply written as integral of the surface of n dot k grad T where n is with the surface s is any surface that includes the origin. So, what is n dot k grad T ? the simplest thing to solve is for a spherical surface of some radius r. In that case k grad T is equal to k times, in this case there is only a variation of the temperature with respect to the radial direction. T is equal to A divided by r so, there is a variation of the temperature only with respect to the radial coordinate.

Therefore, this k grad T I can write it as k times the unit vector in the radial direction times dT by dr is equal to k was equal to k by r. So, there I will get minus k e r A by r square, the unit normal to the surface is also in the radial direction. Therefore n dot k grad T is equal to minus k A by r square.

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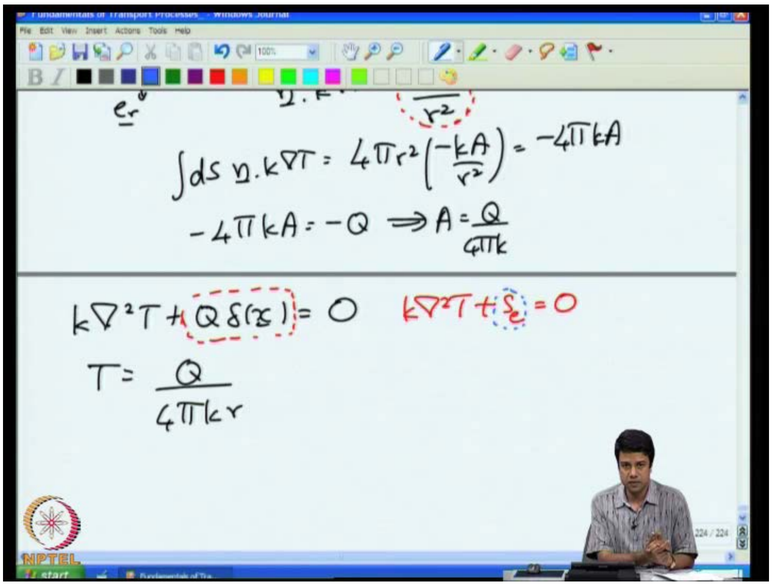

Now what is the integral over the surface? Integral ds n dot k grad t is equal to the surface area this is equal to 4 pi r square. So the surfaces at constant radius this function here is a constant because the surface is at a fixed radius r so, this function is a constant. So, I get the surface area times minus k A by r square and this just become equals to minus four pi k A. Recall that integral of the volume of k del square T was equal to minus Q therefore I will get minus four pi k A is equal to minus Q which implies that A is equal to Q by 4 pi k. So, what I have just shown you is that the solution to the equation k del square T plus Q delta of x is equal to 0 if this is the equation the solution is T is equal to Q by 4 pi k r.

A by r, A in this case is equal to Q by 4 pi k therefore this is the solution for the temperature field from a point source, point source defined as Q times delta of x. Recall that in the heat conduction equation when we defined the delta function we noted that it had dimensions of one over length cubed or one over volume. In the heat conduction equation we previously had an equation of the form plus s e is equal to 0, s e was defined as the heat generated per unit volume. In this case Q is the total heat generated delta has dimensions of one over volume. Therefore Q times delta has dimensions of heat generated per unit volume per unit time, which is a same as the dimension of s e there is a source of heat. Heat generated per unit volume per unit time.

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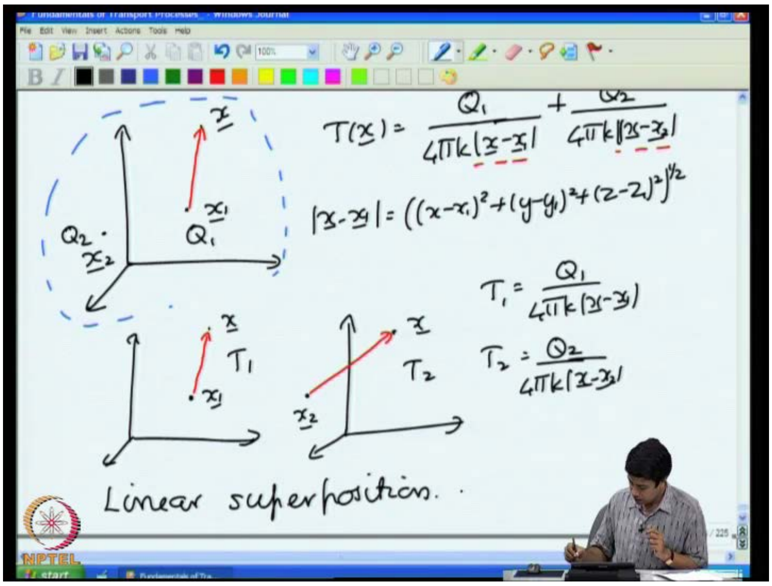

So, with this definition I get a solution for the point source. This definition can be extended quite easily if I have two sources, if I have let us say a source that is not located at the origin, but at some position x 1. And if I am observing it from some position some x prime is the position from which I am observing I am measuring temperature at x prime there is a source of strength Q 1 at x 1. That means that the temperature field at the location x prime I am sorry means this is x, that means the temperature field at the location x is equal to one by four pi k into r. If you recall r was the distance between the position x and the origin in the previous case that was because the source was at the origin in the previous case.

Here the source is at a location x 1 that means that the temperature has to be proportional to the distance between these two points. In other words the temperature has to be equal to 1 by 4 pi k into x minus x 1. So, this is the temperature field for a source that is not located at the origin. I could have multiple sources, I could have an another source at x 2 simultaneously have two sources, simultaneously one of strength Q 1 at x 1 another of strength Q 2 at x 2. The temperature field due to the source of strength Q 1 at x 1 is equal to Q 1 divided by 4 pi k into x minus x 1. I have another source of strength Q 2 at x 2 that means that the temperature field due to that is just Q 2 by 4 pi k into x minus x 2.

Note the modulus here in these two cases this is the distance between the two locations that means that this x minus x 1 is equal to x minus x 1 the whole square plus y minus y

1 the whole square plus z minus z 1 the whole square to the half power. Just as r was equal to square root of x 1 square plus x 2 square plus x 3 square, this is the distance between these two locations the length of the vector x minus x 1. So, it is 4 pi k into x minus x 1 the whole square plus y minus y 1 the whole square plus z minus z 1 the whole square whole under root. So, I could have two sources in this case what I have implicitly done is to separate out the problem into two parts.

(No Audio From: 42:52 to 43:01) One is to have a source here (No Audio From: 43:17 to 43:27) and then evaluate the temperature field due to this and the other is to have a source here and evaluate the temperature at this point due to this source. The sum of these two gives you this original problem this original problem is given by the sum of these two problems that there is one source in one problem the other source in the other problem. I add up the two I get the problem with two sources. So, I can solve for the temperature T 1 in this case and T 2 in this case.

T 1 in the first problem is going to be equal to Q 1 by 4 pi k into x minus x 1 (No Audio From: 44:03 to 44:12) and I just add up the two the differential equations are the same for both del square T 1 is equal to 0, del square T 2 is equal to 0. The solution for del square T 1 is equal to 0 with the point source at x 1 is T 1 solution for del square T 2 is equal to 0, with the point source of strength Q 2 at T 2 is of Q 2 by 4 pi k into x minus x 2. I add up the two and I get this the final temperature field due to these sources I could very well do exactly the same thing for any number of sources. Note that this procedure will not apply if you have sources of finite size this procedure will not apply.

This procedure applies only for point sources and that is the big advantage of point sources. That is the reason that we defined our delta functions gave a definition of the solution for the delta function source. Because this linear super position principle will apply only for point sources this principle is called (No Audio From: 46:33 to 46:40) and this applies only for point sources if I had a problem.

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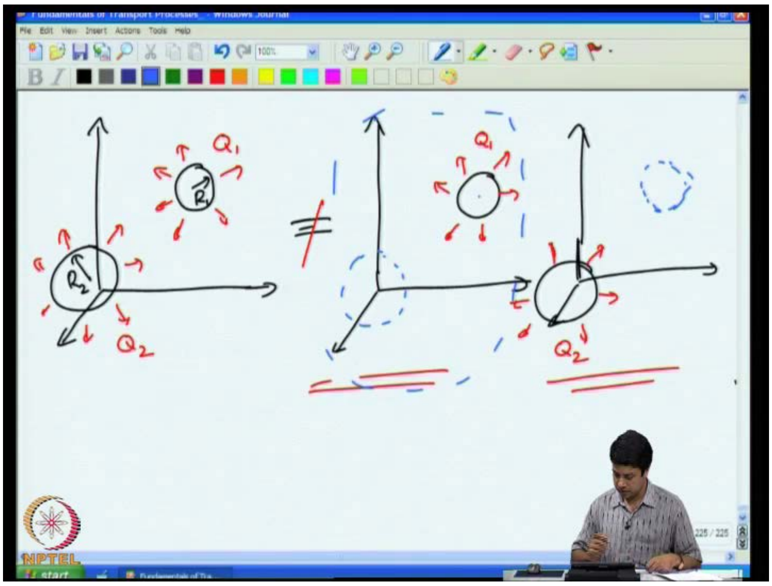

That I had particle of radius R 1 with heat coming out heat generated Q 1, and another particle somewhere else of radius R 2 with the amount of heat coming out as Q 2. In this case I cannot separate it out into two parts, one consisting of Q 1 and the other consisting of heat coming out as Q 2. This is not possible, the reason is because if I add the temperature due to this one, due to if I add the temperature due to this one and the temperature due to this one I will not get the temperature field here because in this particular geometry in this particular geometry this second particle was not present. So, I will have some other temperature at the location of the second particle.

Similarly, in this geometry the first particle was not present therefore I will have some other temperature at the location of the first particle. If I add up the two I will not get the result that I have the configuration with two particles simultaneously within the field. The heat flux on the surface of the second particle here will not be same as the actual flux that is coming out because there is some flux due to the field generated by the first particle and therefore I will not get the same. You can superpose temperatures on boundaries of fluxes on boundaries, but you cannot change boundaries when you do linear super position. And the original problem there were two boundaries the two surfaces of two particles. I cannot separate that in out into two problems each of which contains one surface each the boundary surfaces have to be exactly the same when you do linear super position.

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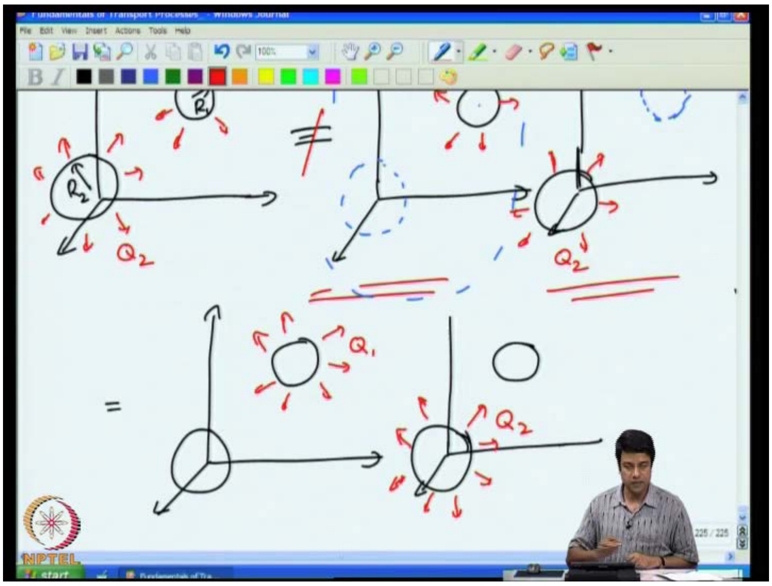

What I can do is to write this as sum of two problems one with both particles, but with Q 1 coming out of the first particle, and nothing coming out of the second particle. And the second being a problem once again with both particles present with nothing coming out of the first particle and (No Audio From: 49:35 to 49:43) Q 2 coming out of the second particle. This is a valid way of doing linear super position because the boundaries are the same in both problems and if I add up the fluxes coming out from the two configurations I get the flux for the original problem. So, long as I do not change boundaries I can always do linear super position. Why does it work for the case of point sources? Why did this linear super position work for the case of point sources, the reason is because the point source has no sides it does not constitute a surface within the flow. It has no sides so since the point source has 0 size anyway when I add up the two I will still get the correct solution.

Because there are no boundaries to this point sources it is just a source within the fluid therefore, if I add up the two for point sources I can remove one source and leave one end because the source that I removed did not have a boundary any way. So, because of that I can superpose the solutions due to point sources I could in this problem for example, put in a point source which is not generating any heat and similarly, in this one and that does not change the temperature field anyway. Whereas, a finite size particles will change the temperature field around it depending upon it is thermal conductivity.

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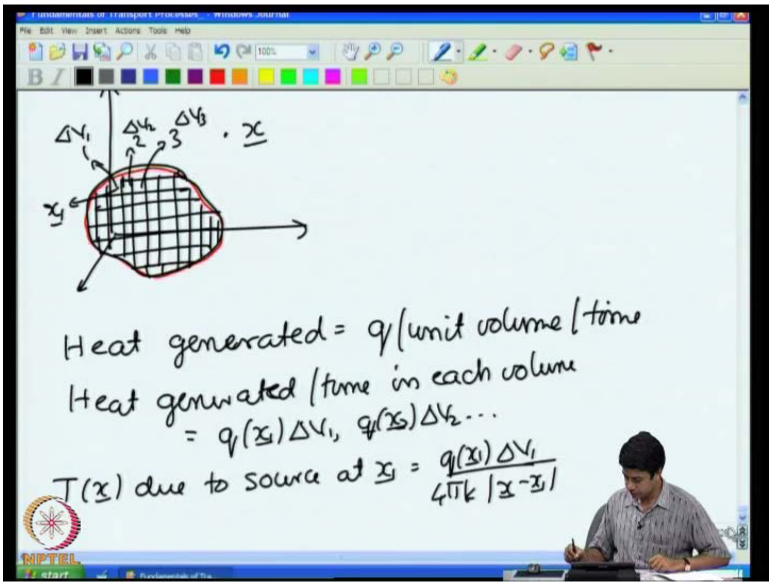

So, therefore I can use linear super position for point sources. I could also use the concept of point sources to solve a problem where I have some distributed source let us say I have a source which is generating heat q volume. Within the source heat generated is equal to q per unit volume per time, this is what you will have. For example if you have an exist thermic or an endo thermic reaction, in that case you will have a certain amount of heat generated depending upon the reaction rate and the heat of reaction. You will have a certain amount of heat generated per unit volume per unit time.

And then one can ask the question, what is the temperature field at a location x, due to the heat generated from this source in the absence of convection at steady state by solving the diffusion equation?. So, how do I use the principle of delta functions in order to determine the temperature as follows? I divide this into a large number of volumes (No Audio From: 52:52 to 53:00) I will number those volumes as 1 2 3 etcetera. So, the volumes have the volume initial these is delta V 1, delta V 2, delta V 3 and so on up to delta V n. I have divided into each one into each small differential volumes and delta V 1 is located at the location x 1, delta V 2 at x 2, delta V 3 at x 3 and so on.

And I can consider within each of these differential volumes, the source (No Audio From: 53:37 to 53:47) heat generated per time in each volume is equal to q at the location x 1 times delta V 1, q at the location x 2 times delta V 2 etcetera. The respective the heat generated at that particular location times the respective the local volume. that I have divided in two what is the temperature at due to source at x 1 will be equal to q at x 1 delta V 1 by 4 pi k times the distance x minus x 1.

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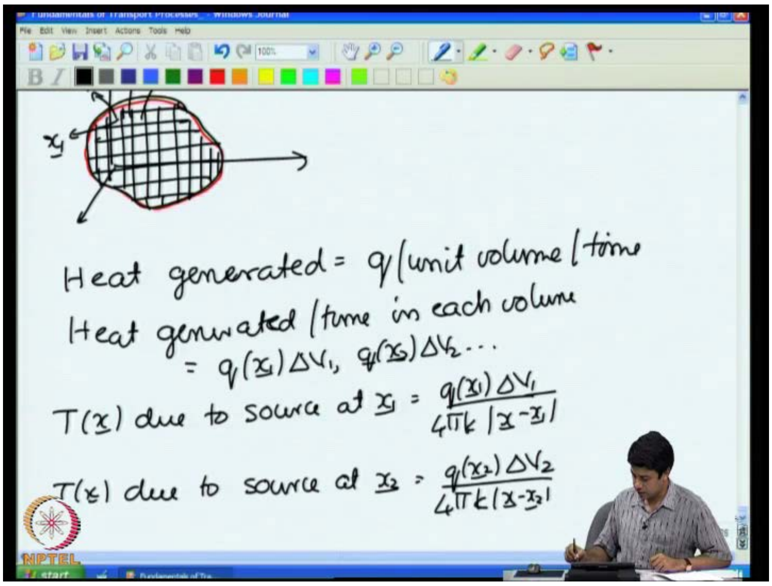

Similarly, T at x due to source at x 2 is equal to q of x 2 delta V 2 by 4 pi k x minus x 2 and so on for each of those differential volumes. That means that the total temperature at x is equal to summation overall i is equal to 1 to n, q at x I, delta V i by x minus x i.

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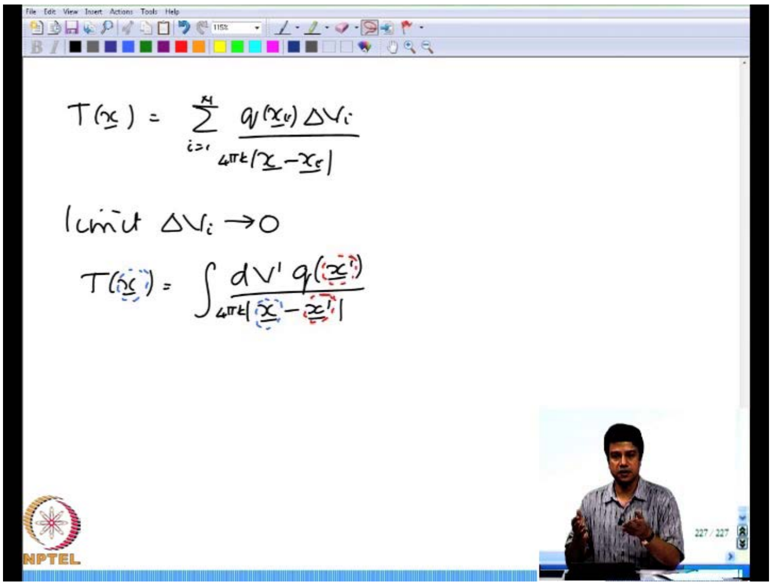

If you take the limit delta V i going to 0 this T of x is equal to an integral over the volume of the amount of heat coming out per unit volume per unit time at the location x prime divided by x minus x prime. So, that is the total temperature due to all of these small volumes super position principle again. So, I have distributed source I take the integral of the temperature field at the observation point. Note that x is the observation point, the point at which you are measuring temperature, x prime is the source point, the point at which the source is located. When you integrating over all the positions where the source is located of their heat flux per unit volume at that source point multiplied by the differential volume divided by the distance between the source and observation point.

If I integrate that out, I will get the temperature field once again we using linear super position except that this is for a distributed source q is per unit volume. So, in the previous case I had capital Q which was the heat generated per unit time temperature was just capital Q 1 by x minus x 1. In this case small q is heat generated per unit volume per unit time and the temperature is integral dV times q at that location divided by x minus x prime super position principle once again. So, this was the temperature field due to of point under distributed source. In the next lecture first we will briefly discuss the relationship between these point sources and the spherical harmonic expansions that we had derived in the previous lecture.

And then we will go on to study some properties of these point sources so, I have solved for you the heat conduction equation with a point source. I got the solution as q by 4 pi k r and then I told you that linear super position can be used to get the temperature field even when there are multiple sources as well as when there are is the distributed source in that case it is just an integral equation. Next lecture we will look at the relationship between that and the spherical harmonic expansions we did previously so, we will continue this in the next lecture and we will see you the next time.

Thank you.