Fundamentals of Transport Processes

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Module No. # 01

Lecture No. # 03

Dimensional Analysis

Welcome to this third lecture on the fundamentals of transport processes. Last class, we were doing dimensional analysis and we will continue that in this class. Basically to see how dimensional analysis can be used to advantage in reducing the number of variables that are of interest in a problem.

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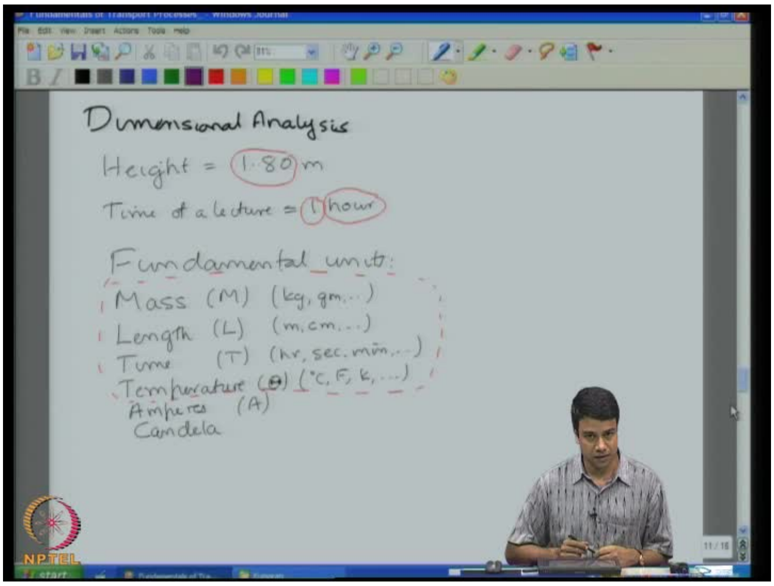

Last class, we had looked at the fundamental dimensions mass, length, time, temperature, amperes and candela and these four mass, length, time and temperature are the fundamental dimensions that we will be using in this course.

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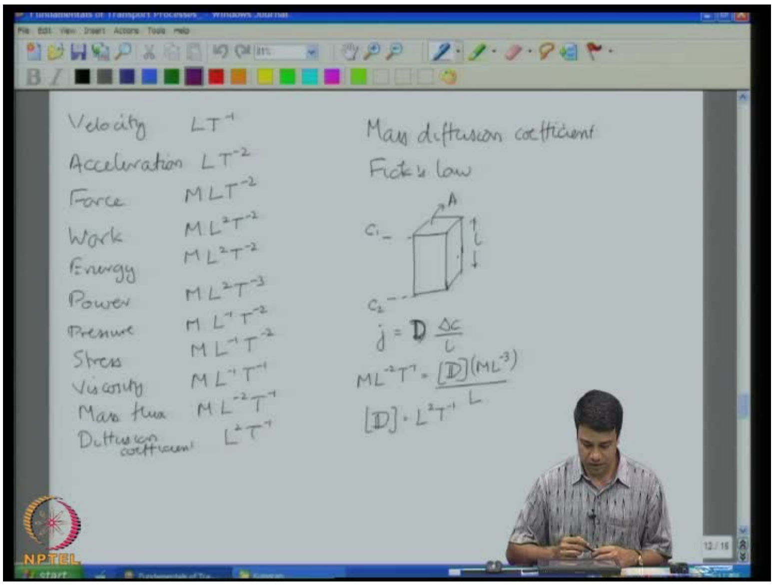

Then we looked at some of the derived dimensions velocity - length per unit time, acceleration - rate of change of velocity, length per time square, force, work, energy, power and some quantities of interest towards in this course.

The stress, the viscosity - viscosity is given from the Newton's law for stress, stress is equal to viscosity times strain rate. So, viscosity has dimensions of stress time's time, mass flux and from Fick's law for diffusion, one can calculate what the dimension of the diffusion coefficient is. Dimension of the diffusion coefficient is the length square per unit time.

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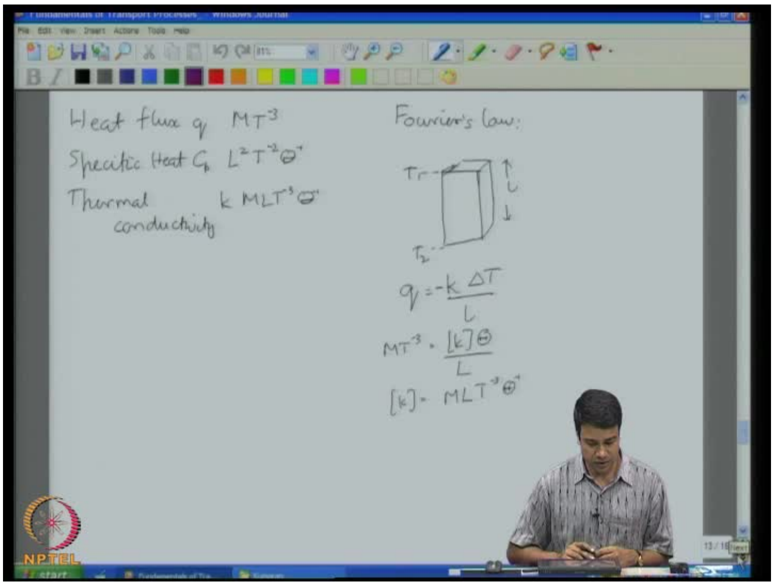

Then, we looked at some quantities of interest in heat transfer. The heat flux, the specific heat, the thermal conductivity and the thermal conductivity comes from Fourier's law for heat transfer.

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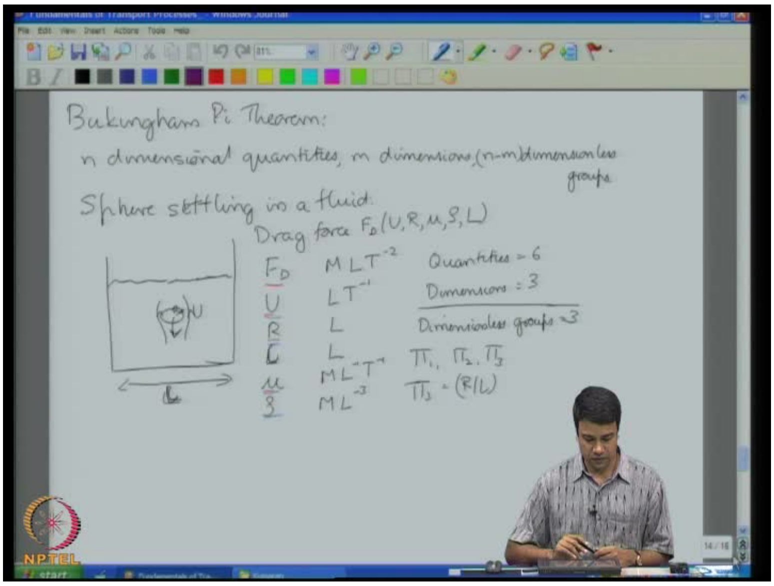

We used dimensional analysis to estimate the drag force on a sphere settling in a fluid. We first listed out the fundamental quantities that are of interest in this problem. The force on this sphere obviously depends upon the speed with which it is moving, it depends upon the radius of the sphere and the viscosity and density of the fluid as well as the length of the holding tank and we got 3 dimensionless groups.

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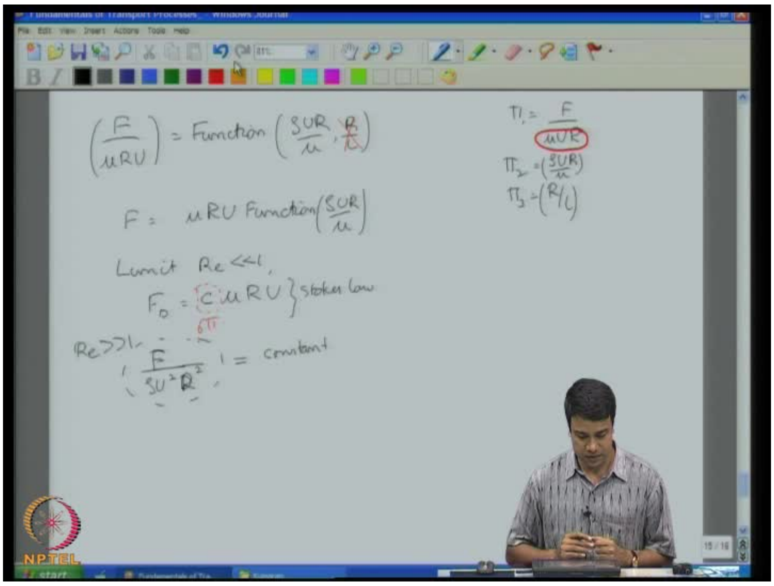

One was just the ratio of lengths; this is a dimensionless group that will recur again and again in this course - some ratio of two characteristic lengths in the problem. The other two are the non- dimensional force. Force, you will notice in this scale has been scaled by the viscous scales. So, it has been scaled by some viscosity times the velocity times the radius.

The other dimensionless group was the Reynolds number - ratio of the inertial and the viscous forces. I said in the limit when the Reynolds number is small, the inertia should cease to be the parameter in the problem and therefore, it should get the force just in terms of the viscous stresses and from that, we got Stokes' law for the force in the sphere.

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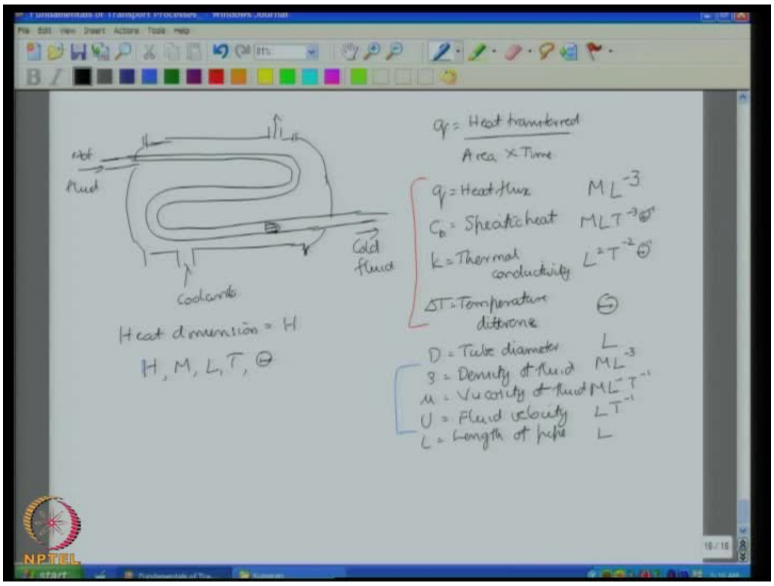

Then we were looking at this problem of heat transfer in a heat exchanger - slightly more complicated problem. You have a tube; hot fluid is injected into the tube side of this heat exchanger and the cold fluid comes out and it is cooled by contact with a coolant and if you were to analyze the system as a whole, then what we would look for is the average heat flux across the tube surface because the total heat transfer rate divided by the area of the surface gives you the average heat flux.

So, rather than work with the total heat that is transferred per unit time, we could look at the average flux, which is the flux per which is the heat transferred per unit area per unit time. That has to be a function of the average temperature difference between the inside and the outside fluid - the temperature difference delta t that I have written down here.

Of course, it depends upon the thermal properties the specific heat, the thermal conductivity, the diameter of the tube, the length of the pipe. The diameter and length are going to play an important problem because they determine the surface area and the length of contact between the two faces and you will see immediately here, the ratio D by L appears as a dimensionless parameter similar to D by L in the previous problem.

Even though the transfer across the surface is taking place by heat conduction, the transport of heat away from the tube from the inner fluid to the wall of the tube is taking place due to both conduction and convection. We had looked at the 2 fundamental processes in the last class conduction and convention. Convection brings in heat at the inlet and takes out the cold fluid at the outlet. So, heat is brought in to the heat exchanger at the inlet due to convection and then it is transported across the surface due to conduction.

Convection is a transfer due to the mean flow of the fluid and that is going to be affected by the flow properties, specifically the fluid velocity as well as the density and viscosity. Density determines the inertial stresses in the flow and viscosity determines the viscous stresses.

Now, if I were to write the dimensional variables in terms of the fundamental units as I did in the last class, what I would write is that q is equal to M L power minus 3, the specific heat is equal to M L T power minus 3 theta inverse, the thermal conductivity is L square T power minus 2 theta inverse.

The temperature difference has units of theta, tube diameter is L, density is M L power minus 3, the viscosity is M L inverse T inverse and the fluid velocity is L T inverse and the length of the pipe is L. 1, 2, 3, 4, 5, 6, 7, 8, 9 - 9 dimensional groups and how many dimensions are there. There are 4 – mass, length, time and theta. So, you have to get 5 dimensionless groups.

At the end of the last class, I also told you that one can simplify the problem further. If there is no transport of energy from thermal to mechanical energy, that is, if energy is not converted from fluid flow to heat by fluid viscosity or vice versa, if there is no convection of energy from mechanical to thermal energy or vice versa, then thermal energy and mechanical energy are separately concerned. Therefore, I can consider thermal energy to be a separate fundamental dimension in this problem because the thermal energy is conserved by itself; it is not being converted into mechanical energy.

Therefore, the heat dimension can be return as H, in which case now I have 5 fundamental dimensions, 9 dimensional groups which means I should have 4 dimensionless groups. So, let us see what those are.

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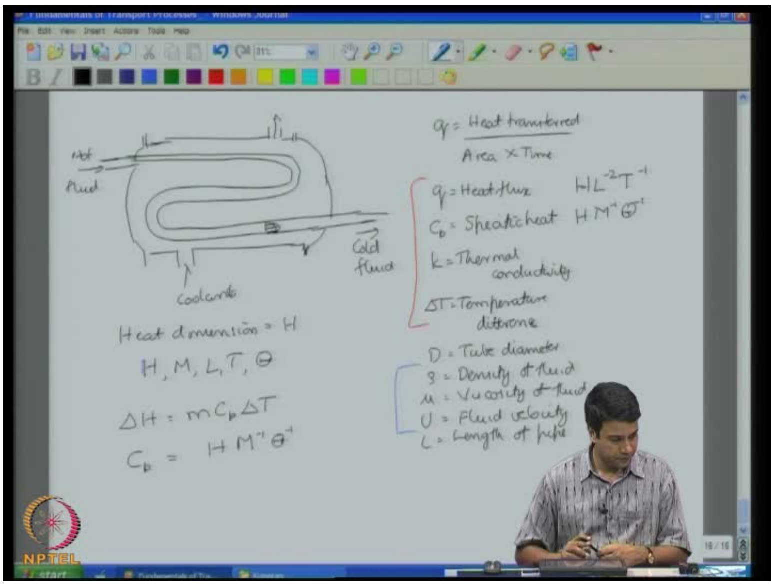

So, if I consider H also as another fundamental dimension then I should write the thermal quantities in terms of H. Heat flux is heat transported per unit area per unit time. Heat flux is heat transported per unit area per unit time, specific heat is the heat contained per unit volume of fluid.

So, I can write it on the basis of delta H is equal to M C p delta T, which means that the C p has dimensions of H M inverse theta inverse. So, specific heat has dimensions of H M inverse theta inverse.

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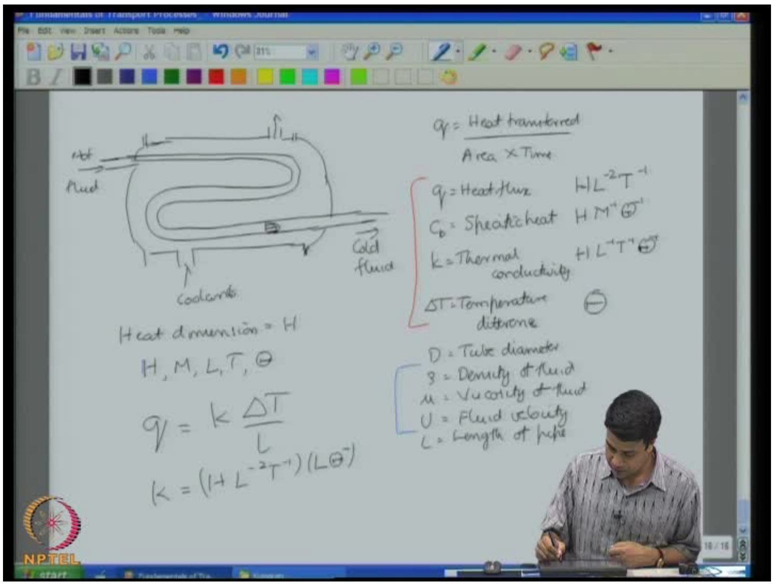

Thermal conductivity comes from Fourier's law for heat transfer. q is equal to k delta t by L. So, therefore, k has dimensions of H L power minus 2 T inverse into L theta inverse. So, this has dimensions of H L inverse T inverse theta inverse and temperature has dimensions of theta as usual.

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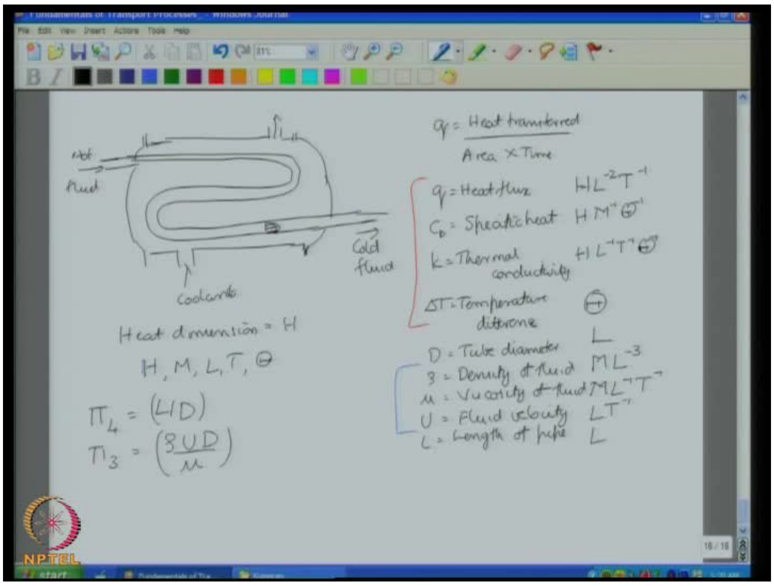

Tube diameter has dimension of L as usual, density is a mechanical quantity and so it has dimensions of M L power minus 3, viscosity has dimensions of M L inverse T inverse, fluid velocity is L T inverse and the pipe diameter dimensions of L.

So, now I should have 4 dimensionless variables. One of these is pretty easy to see. The fourth one is just L by D, one of these is pretty easy to see - L by D and the third one is what we had determine before, in the previous problem, the Reynolds number. We do not have to do it all over can. Now, the 2 others involve the thermal quantities. One of these has to involve the heat flux, which is what it is that we are after. We want to know what the flux, average flux through the surface is.

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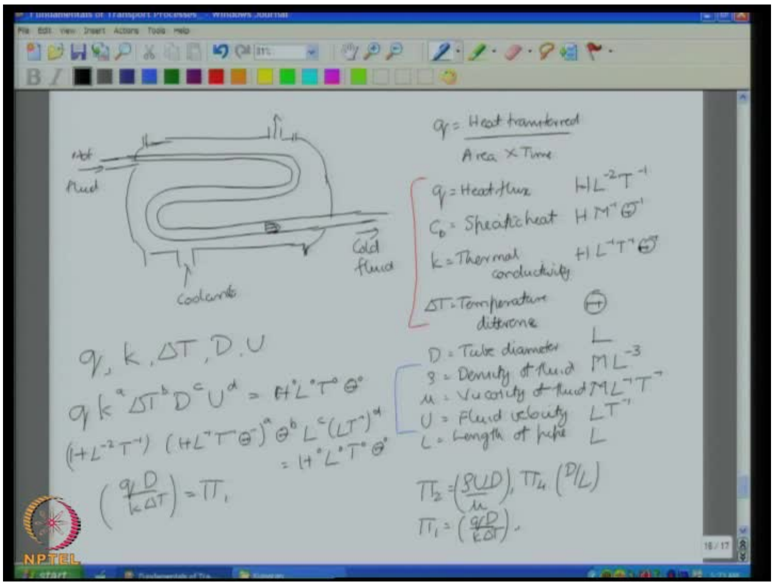

So, we can get one other dimensionless group, which involves the heat flux q. I will just rub this off. So, the dimensional groups that we will involve at the heat flux q and I can choose 3 others because q has dimensions of H, L and T. So I have 4 groups with dimensions of H, L and T; then, I can create non-dimensional group.

Now, actually you cannot find 4 groups with dimensions of H, L and T. The heat flux can depend upon the thermal conductivity k with dimensions of H, L inverse, T inverse theta inverse as well as the temperature difference theta. Then I have to have 2 other groups of having length and time. I can choose those as D and U. Now I have 1, 2, 3, 4, 5 dimensional groups and 4 dimensions H, L, T and theta and from these, I can construct a dimensionless group.

Let as look at what that dimensionless group should be. So, I have to have q times k power a delta T power b D power c and U power d has to dimensionless. so it has be H power 0 L power 0 T power 0 theta power 0. We do it the same way that we did it for the problem of the force on this sphere.

So, this q is equal to H L power minus 2 T inverse power a into H L inverse T inverse theta inverse power b theta power c, I will just rub this, power a theta power b L power c and L T inverse power d is equal to H power 0 L power 0 T power 0 and theta power 0.

So, now I have 4 simultaneous equations. I can solve them for the 4 unknowns and what I will get if I solve them is a dimensionless group which goes as q D by k delta T, which is the first dimensionless group. One can solve it and see.

This is the first dimensionless group. Now, I said there are four. The second one was the Reynolds number. There was one, which was the just the ratio of lengths. We got the first one which is q D by k delta T and there should be a fourth dimensionless group.

Now, that fourth dimensionless group has to involve this specific heat because the specific heat has not yet appeared in any of the dimensionless groups that we have used so far because the specific heat is an important quantity. It has not appeared so far in any of the dimensionless groups and that means that the fourth dimensionless group has to involve the specific heat in some way.

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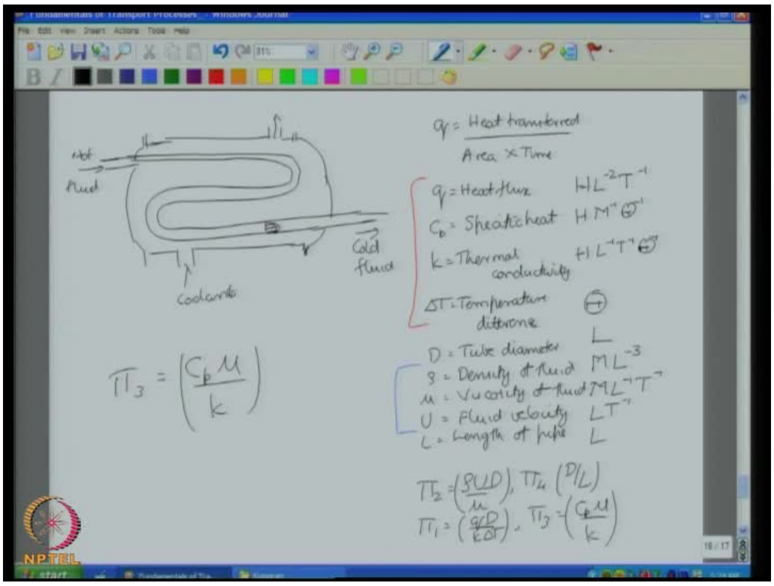

Specific heat involves both thermal quantities; it has an H in it and it has mass in it. Therefore, you should be able to get the dimensionless group involved in the specific heat, one thermal quantity and one mechanical quantity. In this case, I can choose the viscosity without loss of generality as a mechanical quantity and then I will get the third dimensionless group as C p times mu by k. Just by dimensional analysis, you can check that this is correct. So, pi 3 is equal to C p mu by k.

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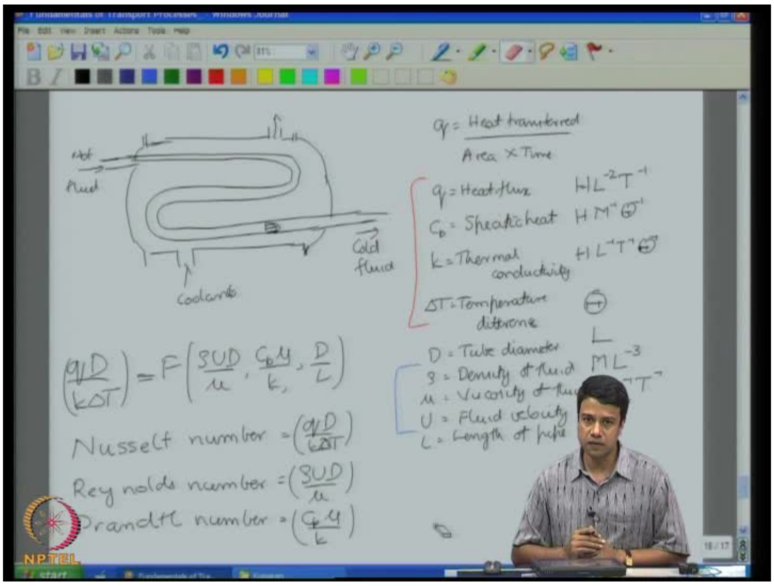

So, now for this problem, I have got 4 dimensionless groups and I can write the average heat flux simply as a relationship between these 4 dimensionless groups. So, I can write this pi 1, q D by k delta T is equal to some function of rho U D by mu C p mu by k and D by L.

Now, I have only 4 dimensionless groups to vary and by varying this 4 dimensionless groups suitably, I can find out what the relation is between the heat flux and these other quantities. So, I have reduced it to a parameter space involving just 4 parameters. In the initial case, I had a parameter space that involved a total of 9 parameters and if I wanted to find a relationship, then I had to vary each of these 9 parameters independently in order to get at a relationship.

Now I have reduced it to this is 4 parameters and I can vary these 4 parameters independently to get a relationship. These numbers have names; this q D by k is called the Nusselt number.

The Reynolds number, rho U D by mu and then I have something called a Prandtl number, C p mu by k and then there is D by L, which is the ratio of 2 lengths.

Now, beyond this, it is difficult to proceed just based upon dimensional analysis. You cannot further reduce, you have to do experimentation and experiments have been done and they produce what are called correlations.

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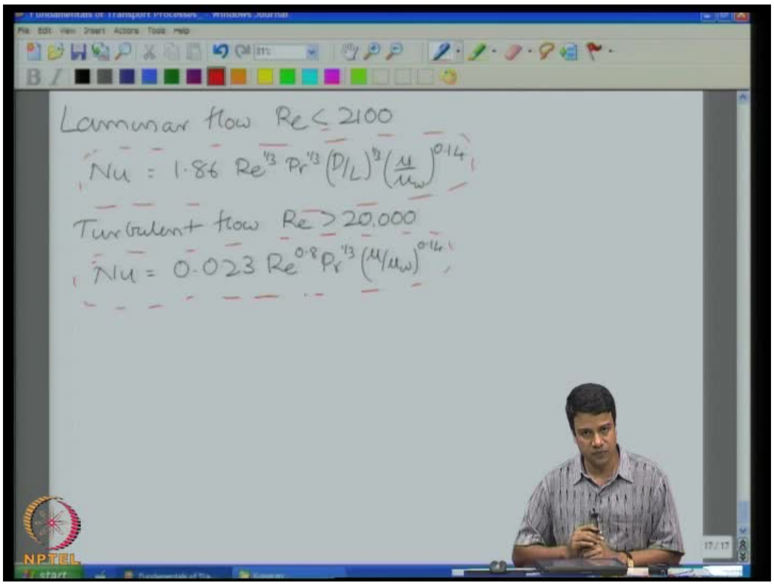

Correlations which tell you how the Nusselt number varies with the Reynolds number and the Prandtl number and the aspect ratio. Now, these correlations are based upon extensive experimentation and they depend upon the flow regime within the tube. There are 2 regimes. One is for laminar flow and the reynolds number is less than about 2100.

So, when the Reynolds number is less than about 2100, you have one set of correlations that apply and these correlation are of the form Nusselt number is equal to 1.86 Reynolds number power 1 third, Prandtl number power 1 third, D by L power 1 third.

So, this is a correlation that applies for a laminar flow when the Reynolds number is small and this once again is based upon extensive experimentation and heat transfer problems very often, the viscosity of the liquid, depends upon the temperature.

So, if we have heat transfer from the fluid inside to the fluid outside, then the viscosity at the wall will be different from the bulk viscosity because the temperature at the wall is different from the temperature in the bulk and to take that into account, there is usually another correction factor that is put in and that is the ratio of viscosity at the wall to the average viscosity - 0.14.

So, this is an empirical correlation that is derived. For turbulent flow, the Reynolds number is greater than about 20000. The relation between the Nusselt number and the Reynolds number in the Prandtl number is given by what is called Sieder tate correlation and that is of the form Nusselt number is equal to 0.023 R e power 0.8 P r power 1 third and mu by mu tau mu power 0.14 this should be mu by mu w 0.14.

So, these are empirically derived correlations. We got to the stage of finding out what are the parameters the Nusselt number, the Reynolds number, the Prandtl number and the aspect ratio; up to that stage, we got just by dimensional analysis. To go further, either you have to do experimentation or you have to look at these correlations.

In this course, we will try to see how these correlations can be derived from a fundamental analysis of the combined heat transfer in fluid flow; can be derived this. In some cases, we can; in laminar flow, for example, we can show why this power 1 third is coming in all of these correlations. Turbulent flows are much more difficult; in laminar flows you have nice parallel stream lines, the velocity profile is parabolic. In turbulent flow, on the other hand is highly chaotic with that these of all dimensions and because there is much more difficult to analyze. But we would like to get some physical understanding for why relations of this kind arise in these systems.

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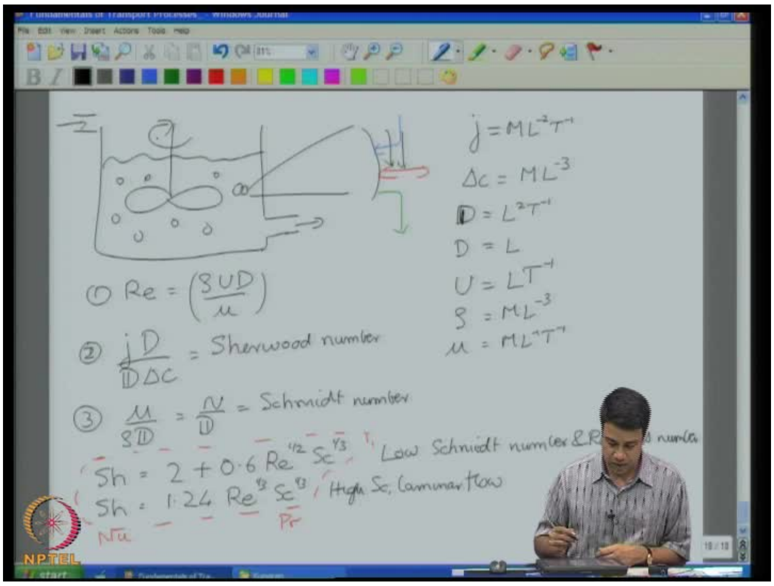

So, let us look at dimensional analysis of another system, that is, the reactor system that we have talked about in the very first lecture. If we recall, we had reactor in which had catalyst pellets and the reaction was taking place on the surface of the catalyst pellet.

Now, we said it is important to focus on a small area very near the catalyst surface, where the transport is actually taking place and it is taking place by a combination of the convection and diffusion. The reactant has to come to the surface, the product has to leave the surface and there has to be transfer of heat either to or from the surface depending upon whether the reaction is exothermic or endothermic.

Let us look at the diffusion problem - the reactant going onto the surface. In this case the fundamental quantity is the mass transported per unit time to the surface because unless you transport that much mass per unit time to the surface, it is not going to get reacted and it is not going to come out. So, fundamental quantity is the mass transported by unit time to the surface.

However, that depends upon things like the number of catalyst particles and so on. So, if you want to get an intensive quantity something that is not depended upon the total size of the system, then it is better to work with the mass flux. j is the mass flux; the mass flux has dimensions of mass transported per unit area per unit time.

The mass flux to this catalyst surface in the mass transfer problem is going to depend upon the concentration difference between the catalyst surface and far away - the difference in concentration between the concentration on the surface of the catalyst and the concentration far away. That difference in concentration average has dimensions of mass per unit volume and in all mass transfer problems ultimately, the rate of transport is going to depend upon the diffusion coefficient.

The diffusion coefficient: last class we had actually derived the diffusion coefficient from the flux and the concentration difference and the diffusion coefficient has dimensions of L square T inverse.

If you recall, we said that the rate of transport of mass per unit area per unit time is equal to a coefficient times concentration - mass per unit volume; from that we got the diffusion coefficient and it is of course, going to depend upon the diameter of the catalyst particle which has dimensions of L.

In addition, the flow of the fluid very near the surface is of course, going to depend upon the average velocity with which the fluid is being swirled around. So, it is also going to depend upon the fluid flow properties which has an average velocity of the swirling of the fluid around it and those fluid properties the velocity U with the dimensions L T inverse as well as the density and the viscosity.

Now, there are 1, 2, 3, 4, 5, 6, 7 dimensional groups and how many dimensions do they contain? They contain 3 dimensions. Therefore, there should be 4 dimensionless groups. One dimensionless group is quite easy to see - the Reynolds number.

Then there is one dimensionless group, which involves the mass flux suitably non-dimensionalised. So, the mass flux suitably by non-dimensionlalised. In this case, we can write down the non-dimensional mass flux as the mass flux times the diameter divided by the diffusion coefficient and delta C; this is called the Sherwood number.

The third one involves diffusion coefficient and some combinations of the density and the viscosity and you can easily see that this combination is mu by rho times the diffusion coefficient and is equal to the kinematic viscosity divided by D; this is the Schmidt number.

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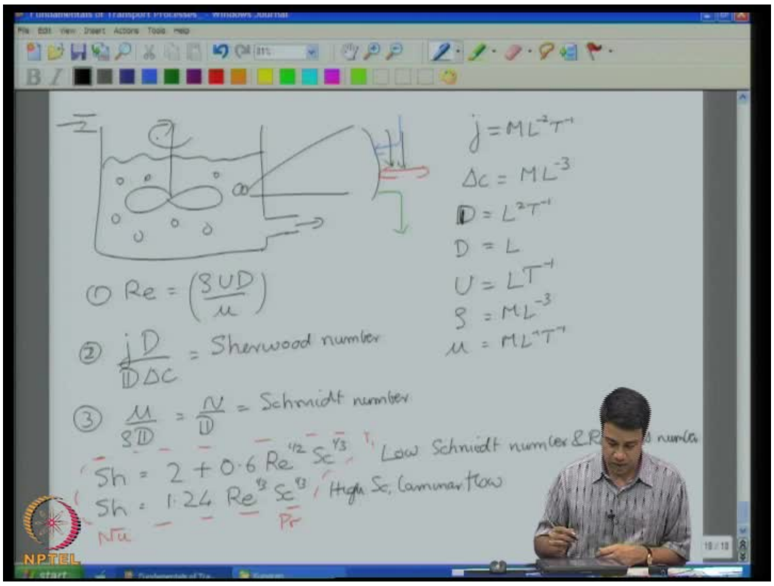

So, these are the different dimensionless groups and once I know these 3 dimensionless groups, then I can find out the average flux to the surface and from that, I can calculate all of the quantities that are desired. Of course, beyond these one cannot go just by dimensional analysis alone one has to go to correlations in order to find out what the diffusion coefficient is.

There are various correlations that I have used in this problem. I will just briefly give you 2 correlations. One is that the Sherwood number is equal to 2 plus 0.6 Reynolds number power half times Schmidt number power 1 third; this is in the limit of very low Schmidt number and Reynolds number, where the flow was basically in the laminar stage. Now, if the Schmidt number is large then I have a correlation of the kind, Sherwood number is equal to 1.24 Re power 1 third Sc power 1 third; this is in the limit of high Sc and laminar flow.

You have other correlations in the limit of turbulent flows. There are various correlations that are proposed for turbulent flows around a particle. We will see that a little later, but the point is that in all these cases, you can get correlation within the Sherwood number Reynolds number and the Schmidt number and these correlations are different forms in different regimes. In the laminar regime, they have one form; in the turbulent regime, they have another form and the question that you will ask in this course is why do these kinds of correlations come about in the flow past particles.

I could do the same analysis for the heat transfer problem and you will get correlations that are exactly of the same form except that these correlations instead of the Sherwood number, they would have the Nusselt number and instead of the Schmidt number, they would have the Prandtl number. This is the fundamental reason why correlations are of this form. The reason is because in all cases, whether its heat transfer or mass transfer, the mechanism is the same. The Nusselt number and the Sherwood number are scaled fluxes; Sherwood numbers are scaled mass flux, the Nusselt number is scaled heat flux, the Reynolds number is ratio of inertia to viscosity, the Schmidt number and the Prandtl number are the ratio of the diffusivities of momentum to diffusivities of mass and heat respectively and that is the reason that they all have the same form.

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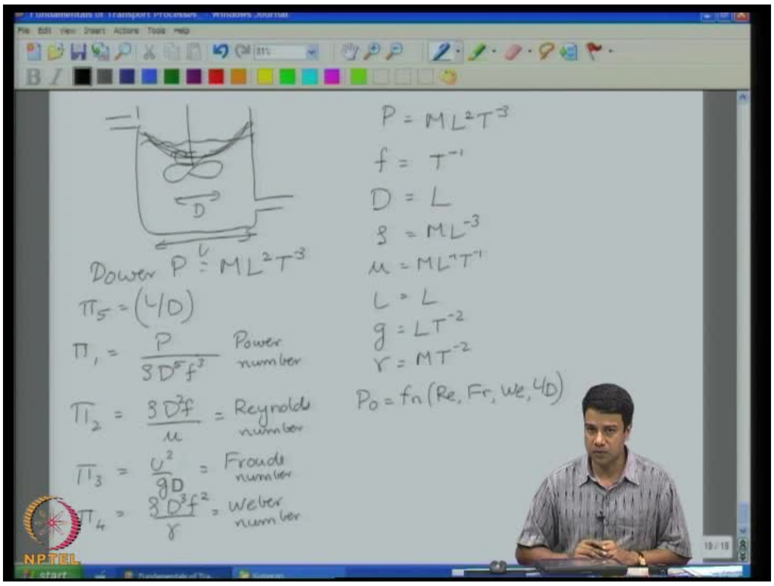

We will come back to that in the next class and see why that is so. That is as far as the mass and heat transfer to the particle is concerned. How about the power requirement for the impeller? So, let us look at the power requirement for the impeller. I have an impeller that is moving in it is rotating a fluid and I want to estimate what is the power required for this impeller. So, what is the power? Power P is equal to rate of change of energy, rate of input of energy. Energy is M L square T power minus 2; the rate of energy is energy power unit time. So, that is equal to M L square T power minus 3.

So, what does this depend upon? Of course, it depends upon the frequency with which the impeller is rotating - the frequency f, frequency is the number of rotations per unit time; so, it has dimensions of T inverse. It will depend upon the impeller diameter; it is going to depend upon the diameter of the impeller T; the larger the impeller, the more power is required and so, it will have D as dimensions of length and then it depends upon density and viscosity.

There are two other things that it could depend upon. First of all it could depend upon the total length of the tank, which I will call small l; so, L has dimensions of the length. In addition as this impeller is rotating, this is going to cause surface deform; the surface will cover upwards at the walls and the downwards in the middle.

So, this is determined by a balance between gravity which tends to make the surface flat, the inertia which tends to push the fluid away - the centrifugal forces which tend to push away from the surface as well as the surface tension. So, it depends upon gravity, which is acceleration - rate of change of velocity, L T power minus 2 and surface tension gamma which is M times T power minus 2.

So, there are 8 dimensional groups out of which there are 3 dimensions. Therefore, they should be 5 dimensionless groups and one of these is quite to see that is the ratio of lengths. So, pi 5 is equal to the ratio of length L by D. So, as long as I keep the aspect ratio, what I said was scale up of this reactor, if I keep all dimensions proportions to each other then this will not change.

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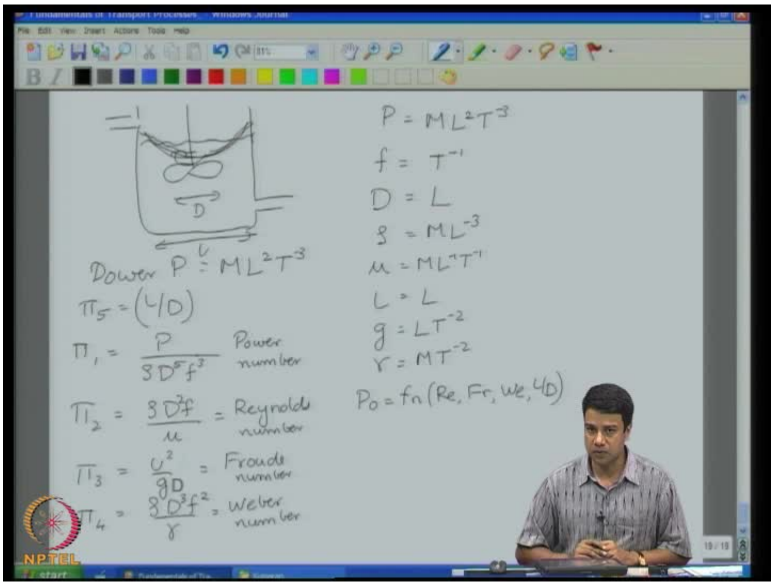

There are three other dimensionless groups. One of them is the power itself and the power, I can scale it by the frequency, the diameter and either the density or the viscosity. If I were to choose the density to scale it then my dimensionless group for the power is going to be equal to P by rho D power 5 f cubed; this is called the power number.

So, this is the power number, which is the dimensionless power input to the system and then there have to be two other dimensionless groups. One of them involves the density and viscosity that is our familiar Reynolds number except that this now has to be expressed in terms of the frequency. So, the Reynolds number express the in terms of frequency will be rho D f square by mu. I am sorry this is going to be rho D square f by mu; this is Reynolds number.

Then I have gravity and surface tension; there have to be dimensionless groups which involve gravity and surface tension. So, the third dimensionless group if we assume involves gravity is quite easy to frame. This is equal to U square by mu and by gravity this is called the Froude number and there has to be one that involves surface tension. In that once again, dimensional basis is quite easy to see what that is. This has to be rho D cube f square by gamma; this is called the Weber number.

So, I will finally get the power number is equal to some function of Reynolds number, Froude number, Weber number and L by D; there is the final relationship that I will get. Then further scaling will depend upon what these parameters are. So, for example, if I am considering the limit of very small Weber number then I will consider the Weber number is not parameter in the problem. So surface tension is dominant force whereas, the Weber number is very large, surface tension is not important. So, I can neglect the surface tension.

Froude number: if U D by g is large, that means that the inertial forces are large compared to the gravitational forces; that means that gravity is not being a factor whereas, U D by g is small or order 1 that means that gravity is also important.

Similarly, the Reynolds number I can set whether it is large or small and then I can get the correlations different regimes. These correlation have to be obtained for each different configuration. In other words, correlation between the power number and Reynolds number, Froude number, the Weber number will be valid only for that particular geometry. If I change the shape of impeller then I have to do the calculation once again.

If I change the aspect ratio that is the thickness of the impeller to the total tank size, then I have to do the calculation all over again. So, even though we can get up to this stage, we cannot go further and the question is how do we go further in our analysis. In particular, firstly, what do these dimensionless numbers mean? I manage to get dimensionless numbers just in terms of dimensional variables, but however there is the question of what is the physical interpretation of these dimensionless numbers. Is there are meaning to them or could I just choose it at random. Could I take the product of two different numbers and call that a new dimensionless number and if I did that then what would that mean?

So, I will spend the rest of this lecture trying to give you a physical understanding of what is the meaning of dimensionless numbers? How do these correlations come about and then we will proceed to what is the procedure that is going to be used in this course for obtaining many of the same results that we have got here.

In this course, we will focus in detail at a microscopic level on the surface of the particles; actually calculate the variation in velocity concentration everywhere around the particle and from that whack out what is the relationship between the dimensionless numbers for the average flux for that particle.

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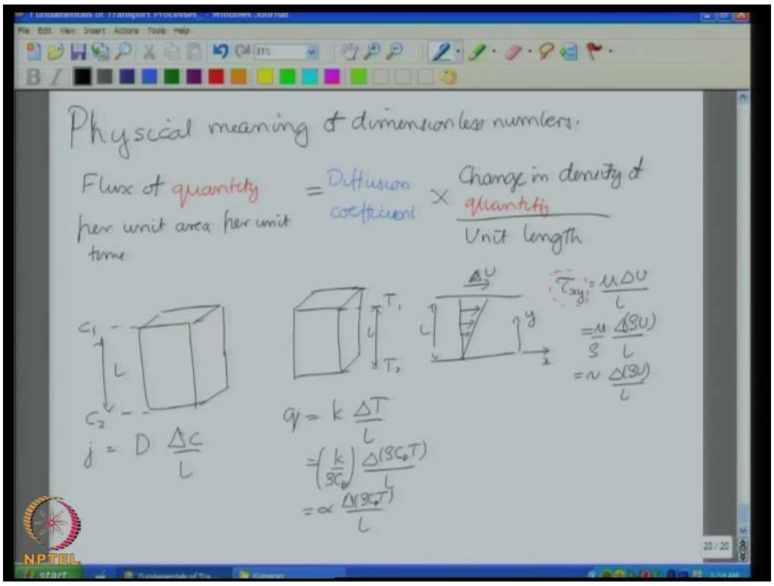

Physical meaning of dimensionless numbers: let us first start of with just the physical meaning of diffusion. In this case, we will talking about the transport of mass, heat, momentum, diffusion of mass, diffusion of heat conduction, momentum transfer due to stresses, but one can write down a diffusion equation for a general quantity and the diffusion equation for a general quantity can be written as flux of quantity per unit area per unit time is equal to the diffusion coefficient into the change in density of the quantity per unit length.

So, for example, if I had a slab of fluid across which I was applying the difference in concentration C 1, C 2 and the difference in concentration across a length L, then the flux j is equal to D into delta C by L.

Flux of quantity per unit area per unit time, flux of mass per unit area per unit time j is equal to diffusion coefficient D times change in the density of that quantity; density is quantity per unit volume that is the concentration; change in concentration divided by the length across which we have applied the concentration gradient. So, this is quite obvious for mass transport.

Heat transport takes place by conduction and if I have 2 temperatures T 1and T 2 across a length L and the heat flux q per unit area per unit time is equal to k into delta T by L. The heat flux is equal to k times delta T by L. Flux of quantity per unit area per unit time, flux of energy per unit area per unit time, the quantity is energy; flux of energy per unit area per unit time is equal to k delta t by L. This is not in the form of difference in the energy density. The energy density, energy per unit volume is going to equal be rho C p times T, the density mass per unit volume times specific heat times the temperature.

So, the density, I have to write this as something times delta of rho C p T by L change in the energy across energy per unit volume between the two faces. So, obviously this has to have k by rho C p. So, energy flux is equal to the diffusion coefficient times the change in energy density - change in density of the quantity that is energy density per unit length between two points.

So, this change in energy density is equal to the thermal diffusion coefficient. How about for momentum transfer? For momentum transfer, I take 2 faces; this is moving with the velocity u and this is stationary and I have linear velocity perfect. Let us take a coordinate system x and y and the Newton’s law viscosity states that tau x y is equal to mu times delta U by the total length L. So, the delta U is the difference in velocity. Flux of quantity in this case, flux of momentum is momentum transferred per unit area per unit time, which is rate of change of momentum is a force. So, flux is the force per unit area - that is the stress.

So, this thing is the flux of momentum. That has to be written as something times the change in the momentum density between these two points, something times the change in the momentum density.

Now, momentum is mass times velocity; momentum density will be mass density times velocity. So, this will be delta rho U, the difference in the momentum density times of velocity divide by L and obviously the pre-factor has go to be equal to mu by rho.

So, the diffusivity for momentum is the kinematic viscosity. This is equal to mu times of delta rho U by L. So, the diffusivity of momentum is the kinematic viscosity - mu by the viscosity. Expressed in this general form, I have a diffusion coefficient for mass which is just the mass diffusivity itself. I have a diffusion coefficient for heat which is the thermal diffusivity it is k by rho C p.

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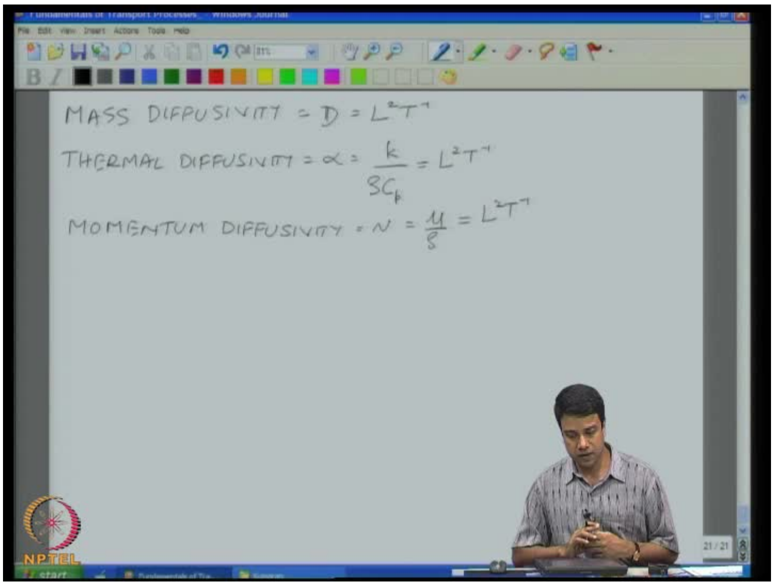

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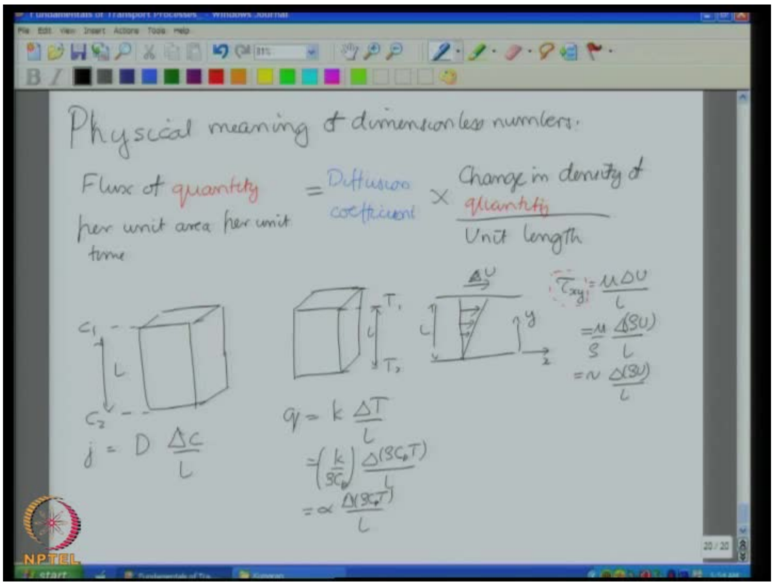

I have a diffusion coefficient for momentum, which is viscosity divided by rho - the kinematic viscosity and it is easy to see with all 3 coefficients. Mass diffusivity which is D and has dimensions of L square T inverse; thermal diffusivity which is alpha which is k by rho C p. You can work it out for yourself and you will find the this is also equal to L square times T inverse and then there is momentum diffusivity, which is new kinematic viscosity which is mu by rho. That will also have dimensions of the L square T inverse.

So, all diffusion coefficient of all quantity have dimensions of length square by time inverse provided I express the flux of that quantity in terms of the change in the density of that quantity per unit length. We will see later that this change in density per unit length can be written as a gradient. I should note that flux is a vector; it has a direction to it.

We will see that later on the course. I have been dealing with it right now as if, it was just a scalar, but we will see later on in the course that flux is a vector, which has a direction to it and that is the direction. So, flux will go in the direction where there is a change in the concentration.

So, the change in concentration upwards, the flux will go upwards; the change in concentration right to left, the flux will go from right to left. That is true for mass flux, for heat flux. Momentum flux is a little more complicated because momentum itself is a vector. We will see how to deal with that later.

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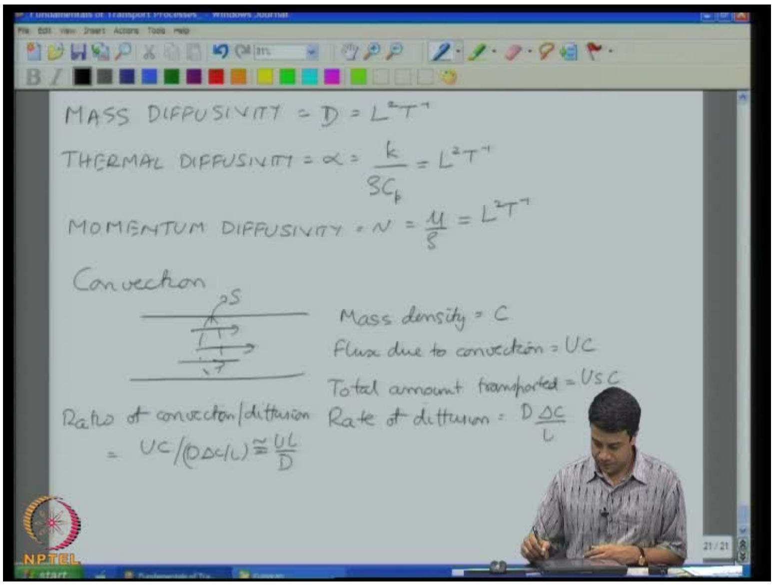

So, I have explained to you what the rate of diffusion of different quantities is. Now, what is the rate of convection? If I have some pipe with some cross section and there is the velocity field going across this pipe, convection gives you the rate at which the material is transported from left to right, due to the mean fluid flow.

So, if I have a density some quantity, let us say the mass density as concentration. So, mass density - this is basically the concentration locally. The flux of material transported across a surface area per unit time is going to be equal to the velocity times the concentration.

That is because the total amount of transported, if I have some surface area let us say S, the total amount transported is going to be equal to the velocity times S times the concentration. Therefore, if I divide by the surface area to get the flux, I will get just the velocity times concentration.

The rate of diffusion is equal to D times delta C by L, where L is the characteristic distance; it is the particle diameter in the case of the flow past a sphere; it will be the cube diameter for the diffusion happening across the flow.

Therefore, the ratio of convection by diffusion will be equal to U C by D delta C by L, which will be approximately equal to U L by D. So, this is the ratio of convection to diffusion in all of these problems.

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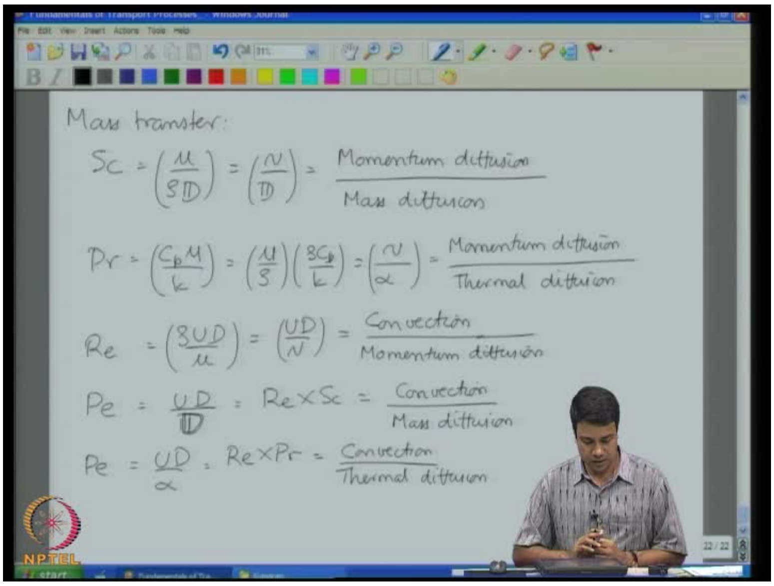

What are these dimensionless numbers? Previously, for mass transfer, I had said that the dimensionless number was the Schmidt number, which was mu by rho diffusion coefficient.

This is equal to the kinematic viscosity divided by the momentum diffusivity. So, this is equal to momentum diffusion by mass diffusion. The Prandtl number - C p mu by k. I can write this as mu by rho times rho C p by k; mu by rho is the kinematic viscosity, mu the momentum diffusivity and k by rho C p is the thermal diffusivity alpha. So, this is equal to momentum diffusion by thermal diffusion. So, that is the Prandtl number.

Reynolds number that I had calculated earlier was rho U D by mu. This can be recast as U times D by mu because mu by rho is the kinematic viscosity. So, this is going to be equal to convection by momentum diffusion. So, the ratio of inertia and viscosity can also be thought of as the ratio of convection and momentum diffusion. From these, I can get the other dimensionless numbers.

For example, I can get the Peclet number is equal to U D by the mass diffusion coefficient and you can easily verify that this is equal to Reynolds number into Schmidt number. This is the Peclet number. So, convection by momentum diffusion times momentum diffusion by mass diffusion gives you convection by mass diffusion. So, this is the Peclet number for mass transfer.

One can write a similar Peclet number for heat transfer. This is the Peclet number for heat transfer will have the form U D by alpha; this will be Reynolds number times the Prandtl number. Reynolds number - convection by momentum diffusion. Prandtl number - momentum diffusion by thermal diffusion. So, this is equal to convection by thermal diffusion.

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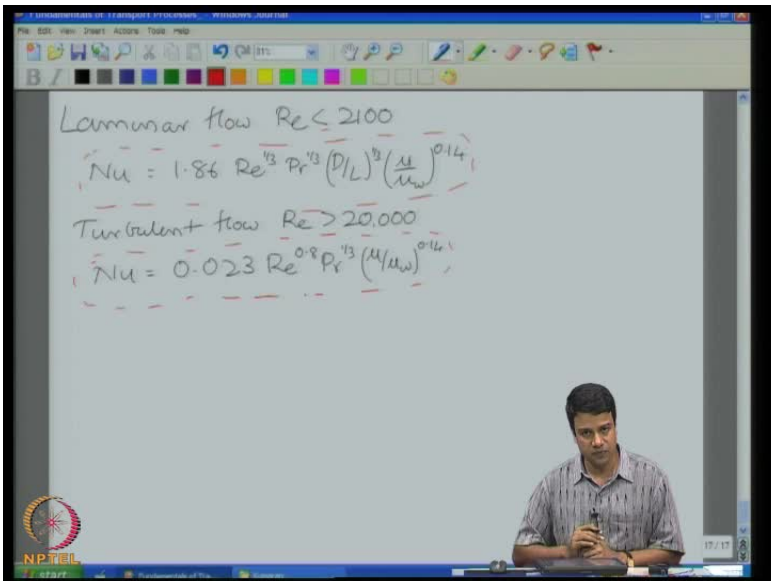

So, the fundamental numbers can be written in this way as either as the ratio of convection and diffusion or a ratio of 2 diffusion coefficients. All of the numbers on the right hand side that I had got here the Reynolds number, the Schmidt number for mass transfer as well as the Prandtl number for heat transfer can all be written as the ratios of convection and diffusion or the ratio of two diffusivities.

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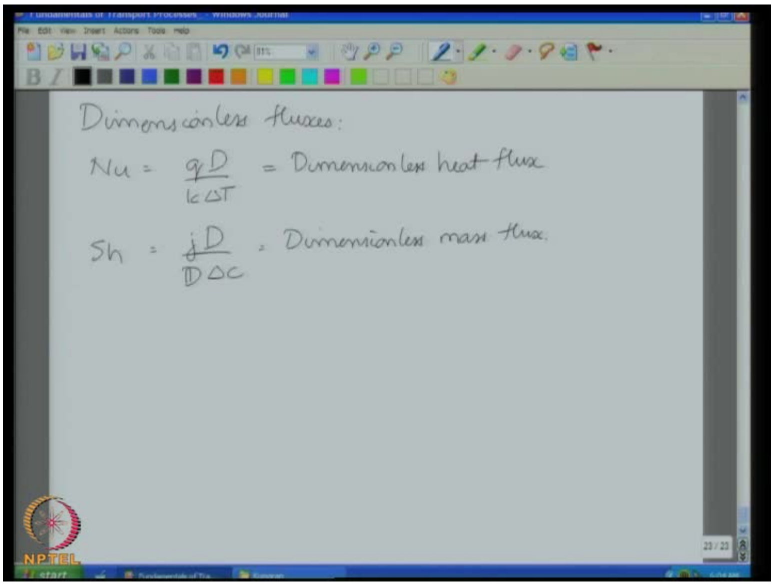

There are other numbers, which are dimensionless fluxes. You already have seen two of those. The first was the Nusselt number, which was q D by delta T, which is the dimensionless heat flux and then, we saw the Sherwood number, which was j D by diffusion coefficient times delta C, which was the dimensionless mass flux.

So, all of the correlations that we had seen so far are correlations which will relate these dimensionless fluxes: the dimensionless heat flux, dimensionless mass flux to the ratios of convection and diffusion of momentum, the ratios of diffusion of mass momentum. ratio of diffusion of heat and momentum

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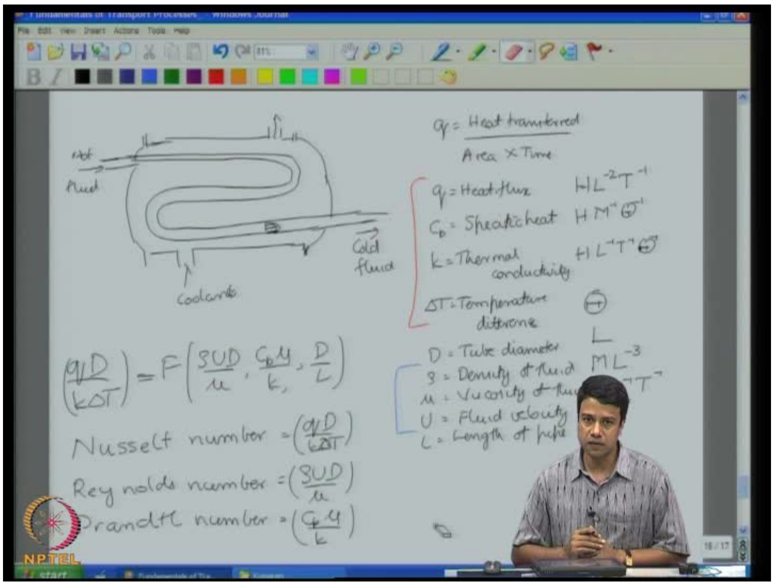

So, to summarize what we have done so far. We first looked at the heat transfer problem. The heat transfer in a heat exchanger and we first started doing it simplistically, the way that we would normally do it, if you are just given the dimensional analysis problem. We listed out all the dependent variables the heat flux, the specific heat, thermal conductivity, temperature difference as well as the various mechanical parameters that can affect this. The tube diameter, the density, viscosity of fluid, fluid velocity and the length of the pipe and we made one simplification. We said that since the thermal energy and mechanical energy cannot be interchanged, we can consider the thermal energy as one fundamental dimensional group.

So, that has increased the number of dimensionless groups that we have and reduce the number of dimensionless groups down to four and we got q D by delta T as the one dimensionless group, the Nusselt number as the function of the Reynolds number and the Prandtl number and the ratio of D by L and correlations are available in literature in various regimes, which tells you what this relationship is.

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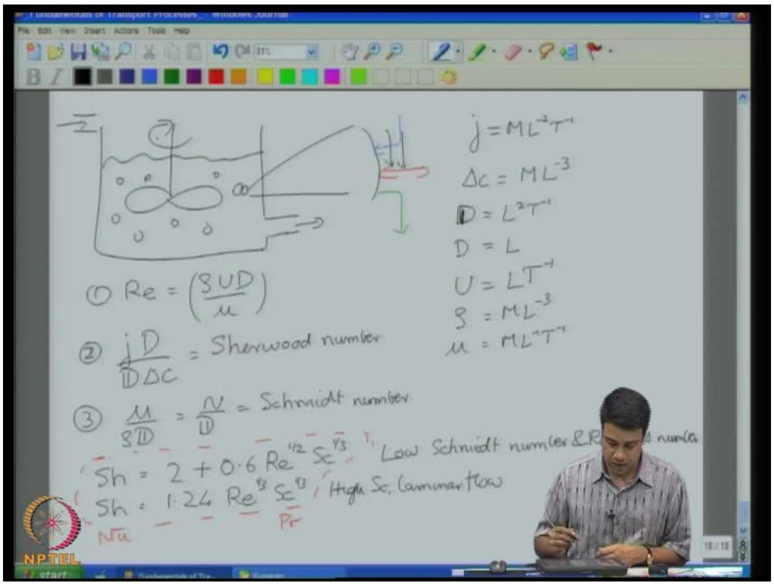

In a similar manner, we looked at the heat transfer and mass transfer from a catalyst surface in the reactor problem. In that case, the mass flux is related to the difference in concentration, the diffusion coefficient as well as the particle diameter, the mass the velocity density and viscosity

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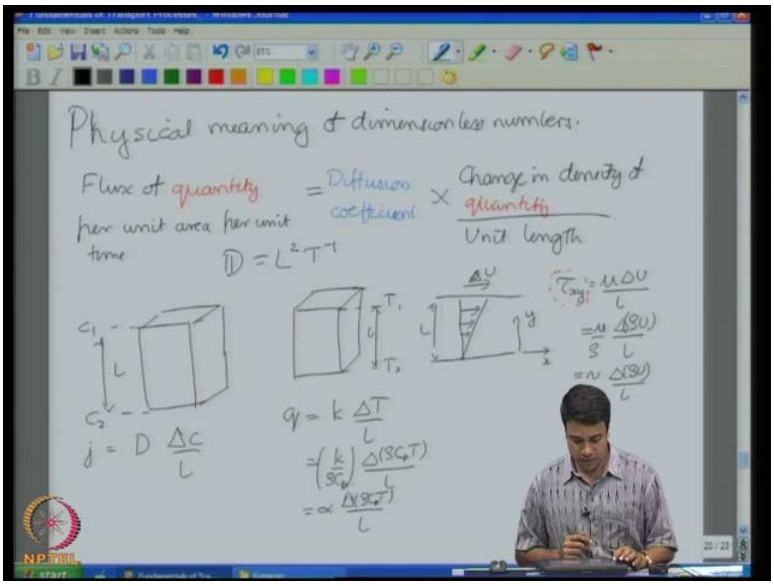

On that basis, we got a relationship between the dimensionless mass flux, which is the Sherwood number and 2 other parameter, the Schmidt number and the Reynolds number and I said these correlations are available in literature. If one were to solve the same problem for the heat transfer, one would get the exact same correlations except that one would replace the Nusselt number instead of the Sherwood number as the dimensionless flux and the Prandtl number as the Schmidt number. We saw little later based upon what are the physical interpretation of the different dimensionless numbers, why that is so? Because I can write the flux of any quantity per unit area per unit time as a diffusion coefficient times the change in the density of that quantity per unit length and straight away, since the quantity has the same dimensions on both sides, the diffusion coefficient will have dimension of length square T inverse.

I have per length square, per time on left hand side and I have 1 over length cube for the density and 1 over length. So, I will get length square times T inverse and we calculated the diffusion coefficients for mass heat and momentum.

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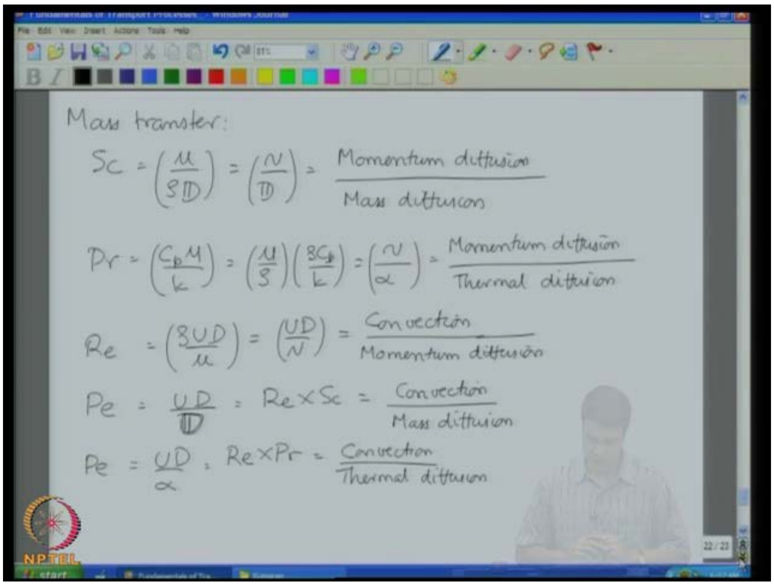

Similarly, I told you that you can get diffusivities for mass heat and momentum. The connective part just goes as U times the quantity; the flux due to convection just goes as U times quantity. On that basis, we have got the dimensionless groups as the ratios of the convection and diffusion are the ratios of two different diffusivities.

We will use this frame work in order to broadly classify all dimensionless groups and all the correlations that are obtained for transport processes in the next class, before we proceed with how we are going to start doing the analysis in the next class. So, kindly keep in mind the different dimensionless groups and their interpretations and we will start from there in the next class. Thank you and we will see you next time.