Fundamentals of Transport Processes
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Lecture No. # 28
Mass and Energy Conservation Cylindrical co-ordinates

Welcome to this lecture 28 of the course on fundamentals of transport processes. And I was deriving for you the conservation equation in a spherical coordinate system. As I said this conservation equation in a spherical coordinate system is important not just for spherical particles and so on. But because it gives us a deeper understanding of the diffusion phenomena and we will see that as we go along a little later.

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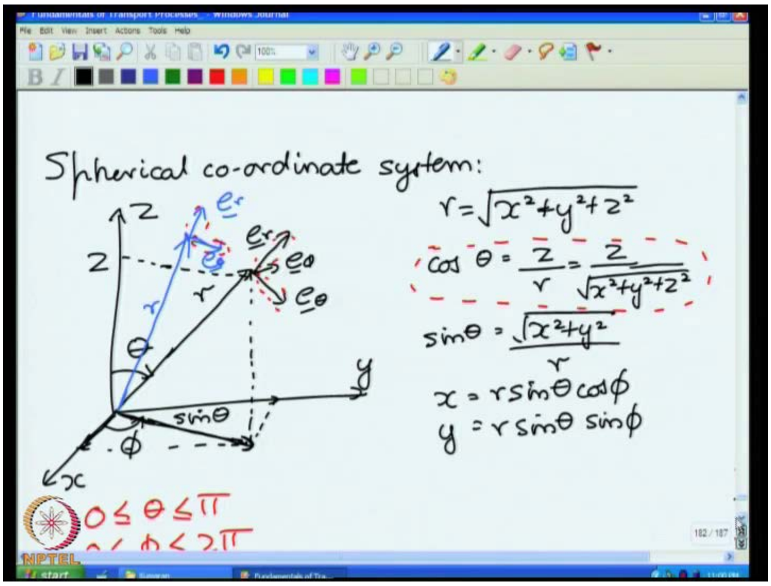

So, the spherical coordinate system,, the coordinate in this coordinate system are theta and phi or it is the distance from the origin. And the unit vector e r is along the direction of increasing r, theta is the angle made by the radius vector with respect to the z coordinate and e theta is in the direction of increasing theta. As I told you cos theta is given by z by r where, r is square root of x square plus y square plus z square. So, the maximum value that cos theta can have varies between minus 1 and plus 1. It is plus 1 if z is along, if the point is along the positive z axis and minus 1, if it is along the negative z axis. So, cos theta varies from 1 to minus 1 which means that theta varies from 0 to pi, theta is the azimuthally angle, phi is the angle around the z axis.

The angle that the projection of the radius vector on to the x y plane makes with the x axis so, phi is the angle around the z axis is the angle, as you go all the way around the z axis and come back to the original location, phi increases by a factor of 2 pi. Therefore, phi varies between 0 and 2 pi. It is in this coordinate system that we started writing down our differential balance equation. When we write a shell balance, we have to write it for a differential volume which is bounded by surfaces of constant coordinate. In this particular case the coordinate are r theta and phi.

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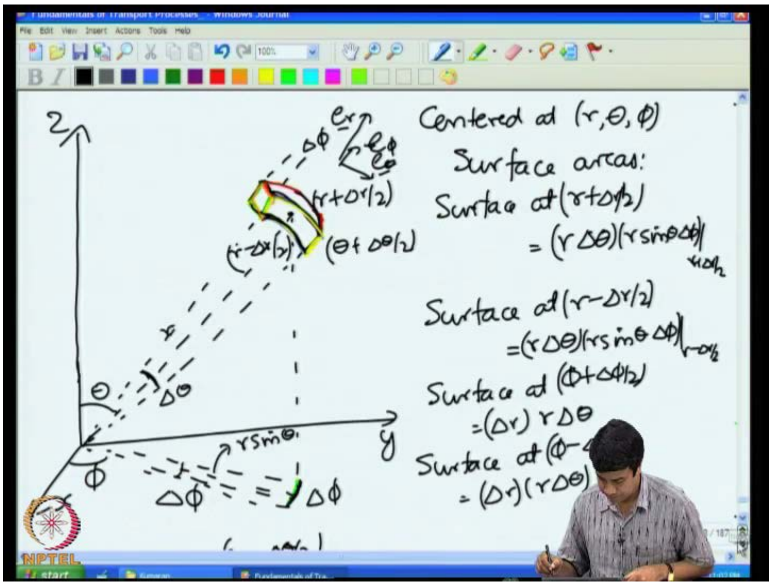

Therefore, the balance equation has to be written for the surface at r minus delta r by 2 and r plus delta r by 2, at theta minus delta theta by 2 and theta plus delta theta by 2 and phi minus delta phi by 2 and phi plus delta phi by 2. And these surface areas do depend upon coordinates. Note that theta and phi are angles; they do not have dimensions of length whereas, in order to get a volume you have to multiply three lengths in order to get a volume and because this angle does not have dimensions of length. You have to actually take the actual length dimension along each of these directions.

So, for the surface at r and r plus delta r, which I showed you in the previous lecture, this is the surface at r plus delta r by 2. You get it by multiplying two lengths; one is this length which is obtained by increasing theta by a value delta theta. So, these angle subtended is delta theta, but the length is actually equal to r times delta theta. Say for surface r plus delta r by 2 is equal to this length r times delta theta times this length.

Now, this length is obtained by varying the phi coordinate from phi minus delta phi by 2 to phi plus delta phi by 2. So, this is the length obtained by varying the phi coordinate.

The radius vector in that case is r times sin theta, the distance from the origin to this location along the x y plane, angles subtended is delta phi therefore, the length is r delta theta times r sin theta delta phi. Similarly, for the surfaces at phi plus delta phi by 2 and phi minus delta phi by 2 which are the front and the back surfaces. One side here shown by the yellow is delta r, the other one is r times delta theta at phi plus delta phi by 2 and phi minus delta phi by 2. Similarly, for the surfaces at theta minus delta theta by 2 and theta plus delta theta by 2, shown here by the orange, surface on top and the surface bellow.

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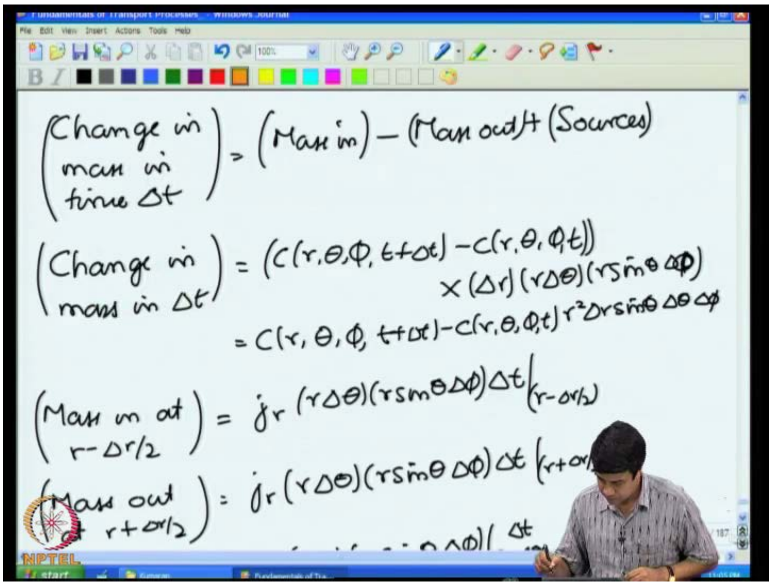

For these surfaces, the surface area is delta r, which is the distance travelled (No audio from 05:35 to 05:41) the surface is delta r, which is this distance. Times the distance travelled perpendicular to the phi direction and that is going to be equal to r times sin theta times delta phi that is this distance. So, the surface areas have to be constructed carefully, you have to take into account the actual length that is traveled in each of these coordinate directions. Once you have that you just write the balance equations as usual change in mass in times delta t is equal to c at r theta phi t plus delta t minus c at r theta phi times t multiplied by the volume itself. Volume is delta r in the r direction, r times delta theta the distance in the theta direction and r sin theta delta phi is the distance in the phi direction. Similarly, the mass is in and out at the surface at r and r plus delta r. Surface at theta, theta plus delta theta. Surface at phi, and phi plus delta phi. All you need to do is multiply the respective fluxes by the surface area. So, you multiply the flux by the surface area to get the mass in and mass out due to diffusion.

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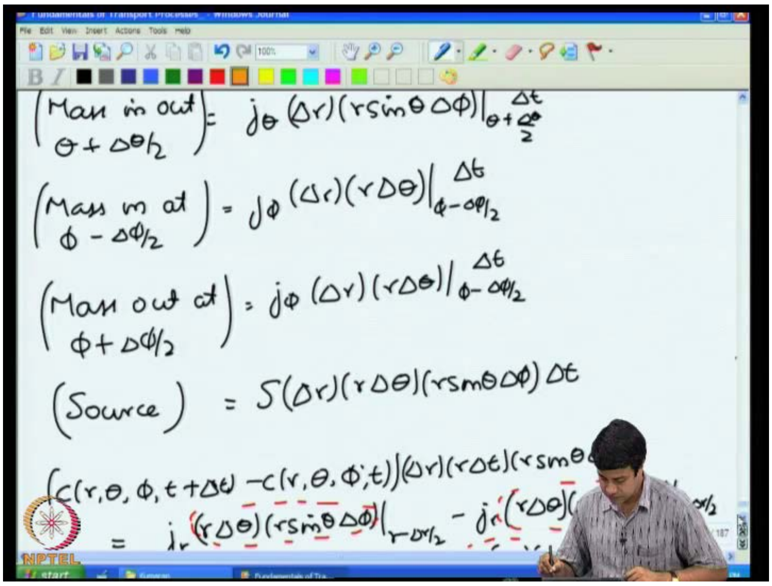

Similarly, for convection due to the mean velocity.

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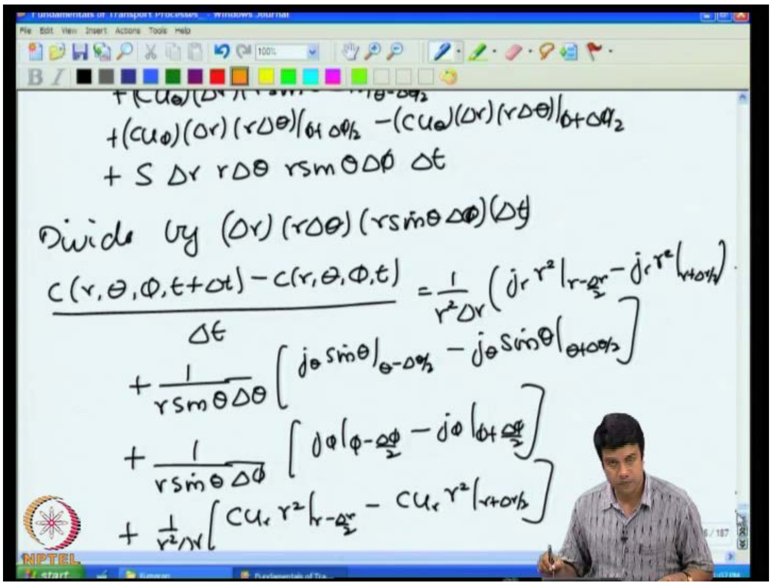

Similarly, for convection due to the mean velocity, the mass in and the mass out is equal to you just substitute c times u r for j r c times u theta for j theta and c times u phi for j phi. Note, j r j theta and j phi are the fluxes along the r theta and phi directions at a given location. However, the r theta and phi directions are also changing in space, they vary from position to position. Therefore, one has to be careful while taking the components u r u theta and u phi at a given location because the unit vectors are changing with position. So, this gives us the final equation and you divide throughout by the volume and time. We divide throughout by volume times time to get a difference equation.

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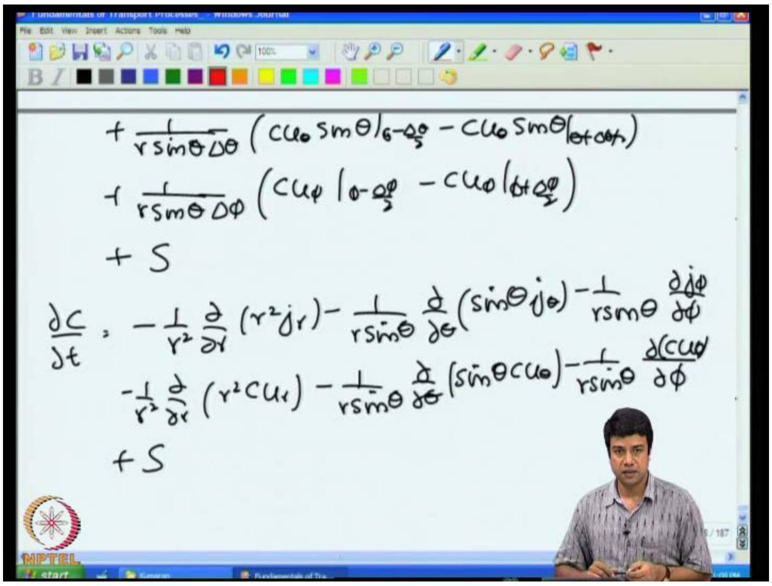

Which basically relates the concentration difference between time t and t plus delta t, to the difference in fluxes on the surfaces of this differential volume? So, not just the difference in fluxes; the surfaces of this differential volume, the surface area is also changing, it changes proportional to r theta and phi. So, in addition to the change in fluxes, the surface area is also changing. And therefore, one has to have the surface area also coming in, when one does this balance. The surface area is changing, it changes in the r as well as in the theta directions, the surface area is changing in both the r and the theta directions. Therefore, that has to become inside the derivative because the surface area is not the same for the two surfaces. And once you take the limit delta r going to 0, delta theta going to 0, delta phi going to 0 and delta t going to 0, you get this equation for the concentration field.

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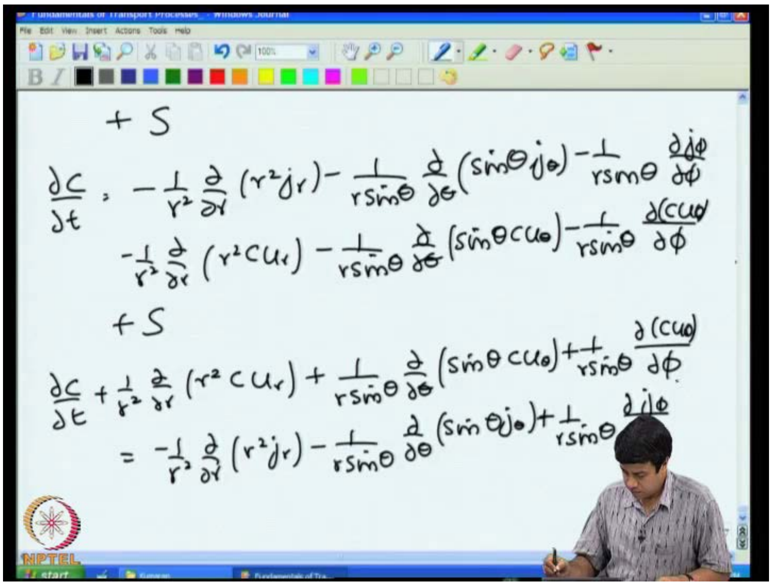

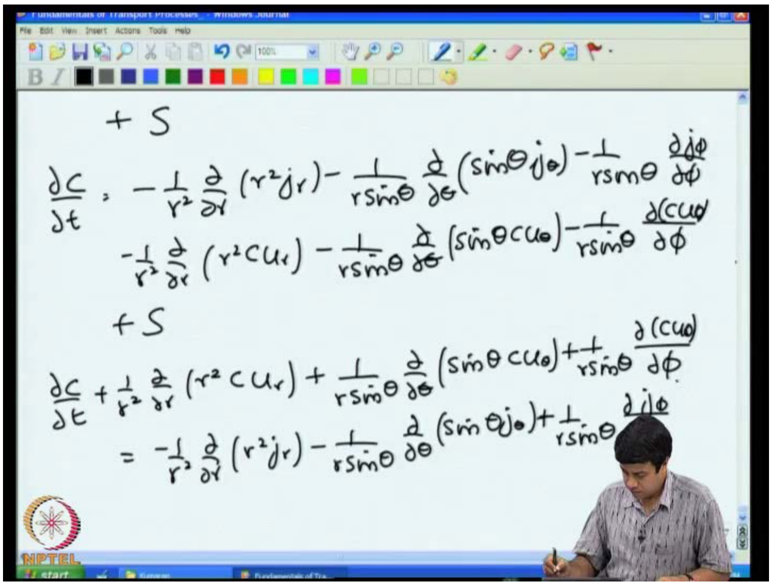

I can rewrite this equation a little bit. I take the convective terms to the left hand side as usual and I will get partial c by partial t plus (No audio from 09:43 to 09:55) r square.

(No audio from 09:57 to 10:25)

is equal to.

(No audio from 10:27 to 10:54)

So, this is the differential equation for the concentration field. And now of course, I have to express the flux in terms of the concentration gradient, using fixed law for diffusion. I need to express the flux in terms of concentration gradient using fixed law for diffusion.

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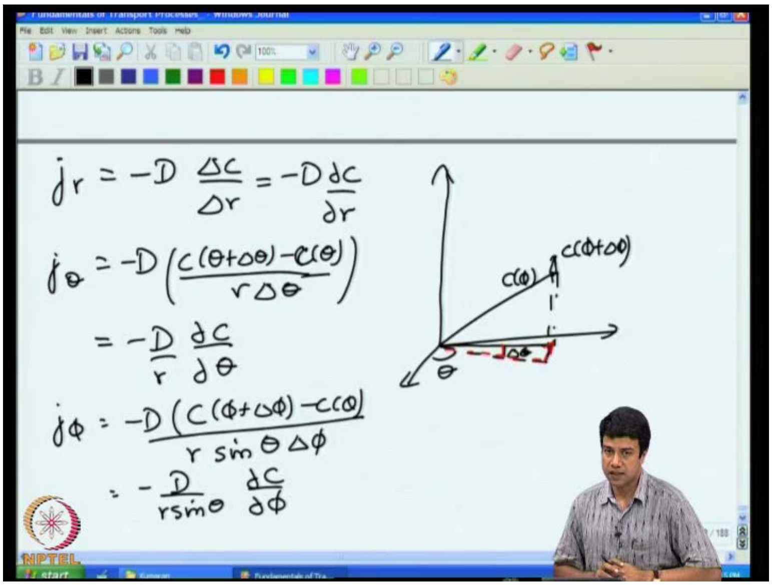

So, what is the flux? In the radial direction, j r is equal to let us first write it in differential form, minus D times delta c by delta r that is in my Cartesian in my spherical coordinate system. (No audio from 11:40 to 11:46) If I am at a location r and I go to a new location r plus delta r, I take the value of the flux j r at r plus delta r minus j r at r and divided I am sorry. (No audio from 12:03 to 12:17) delta r. So, from that difference at r plus delta r minus c at r divided by delta r, that is delta c by delta r. And if I take the limit of delta r going to 0, this is effectively minus d partial c by partial r, this is t times partial c by partial r. (No audio from 12:40 to 12:52)

What is j theta? So, I start at some value of theta, then I go to some other location theta plus delta theta such that this angle is delta theta. So, I take c at theta plus delta theta c at theta, this j theta is equal to minus D times c at theta plus delta theta minus c at theta divided by the distance traveled, the flux is equal to minus D times difference in concentration divided by distance. The distance that you travel in going from theta to theta plus delta theta is r times delta theta. This distance is equal to r times delta theta, that is the distance you travel in going from theta to theta plus delta theta is equal to r times delta theta.

Therefore, this is equal to minus D partial c by partial theta minus D by r times partial c by partial theta. (No audio from 14:13 to 14:23) Similarly, if I have had a location phi and I go to some other location delta phi away so, I have c at phi c at phi plus delta phi, j phi is going to be equal to minus D times c at phi minus I am c at plus delta phi minus c at phi by the distance traveled, what is this distance travelled? This distance travelled is equal to the angle delta phi times this radius. That distance, that radius is equal to r times sin theta because r cos theta was equal to the z distance, square root of x square plus y square was equal to r times sin theta. Therefore, the distance travelled is r sin theta delta phi, which if I take the limit of delta phi going to 0 is equal to minus d by r sin theta partial c by partial phi.

So, note that the flux is they are not just I cannot just write the flux as minus d times partial c by partial theta or minus d times partial c by partial phi. Flux has to have dimensions of a diffusion coefficient times concentration divided by distance, theta and phi are angles, they are dimensionless. Therefore, one has to have the flux has to have dimensions of the diffusion coefficient times a concentration difference divided by distance. And therefore, you get fluxes that are of this form, in the theta and phi directions. So, it is important to note because in we have a coordinate system here, in which the coordinates are not necessarily, do not necessarily have dimensions of length, the coordinates in this coordinate system do not have dimensions of length. Therefore, the expressions for the flux are a little more complicated. (No audio from 16:55 to 17:02)

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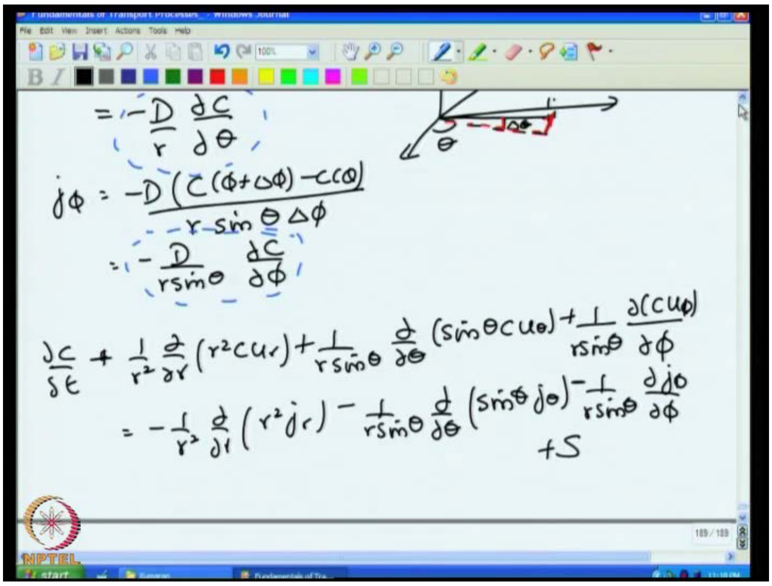

Now, if I insert these expressions into my original concentration equation, d c by d t plus one by r square d by d r r square c u r.

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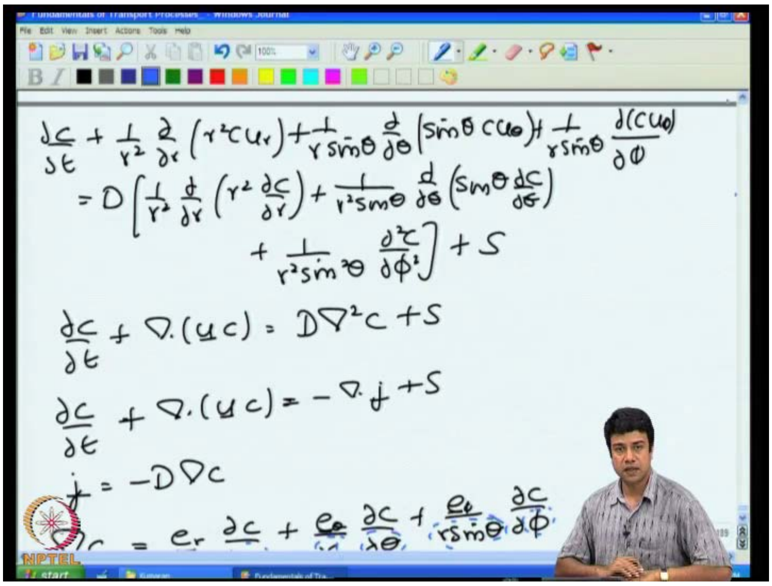

Substitute the expressions for j r j theta and j phi into this. I will end up with a differential equation of the form d c by d t plus one by r square t by d r of r square c u r plus one by r sin theta.

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Minus.

(No audio from 19:25 to 20:02)

So, this is the final differential equation that you will get, if you substitute the expression for the flux into the concentration equation that I had just above. Physically this equation can be written as we had done in Cartesian coordinate systems as d c by d t plus del dot u c is equal to D del square c plus s. Only thing is that the operators now are slightly different. So, this is the concentration equation in terms of the concentration field. If I write it in terms of the fluxes, I will get d c by d t plus del dot u c is equal to minus del dot j plus s where j is the flux vector, j vector is equal to minus D grad c as usual. Where the gradient operator is now defined as grad c is equal to e r times partial c by r partial r plus e theta by r partial c by partial theta plus e phi by r sin theta partial c by partial phi.

This is the gradient operator. It is basically I can write if I write j is equal to j r e r plus j theta e theta plus j phi e phi j vector is written as the sum of its components in the r theta and phi directions so that j is equal to j r e r plus j theta e theta plus j phi e phi. Gradient operator is written as e r times partial by partial r plus e theta by r partial by partial theta plus e phi by r sin theta partial by partial phi. Note this, these additional terms that come in here because theta and phi do not have dimensions of length. Therefore, I get these additional terms here that is the gradient operator, the gradient acting on a scalar, in this case the gradient acts on a scalar. I have defined these terms in terms of a divergence operator acting on a vector.

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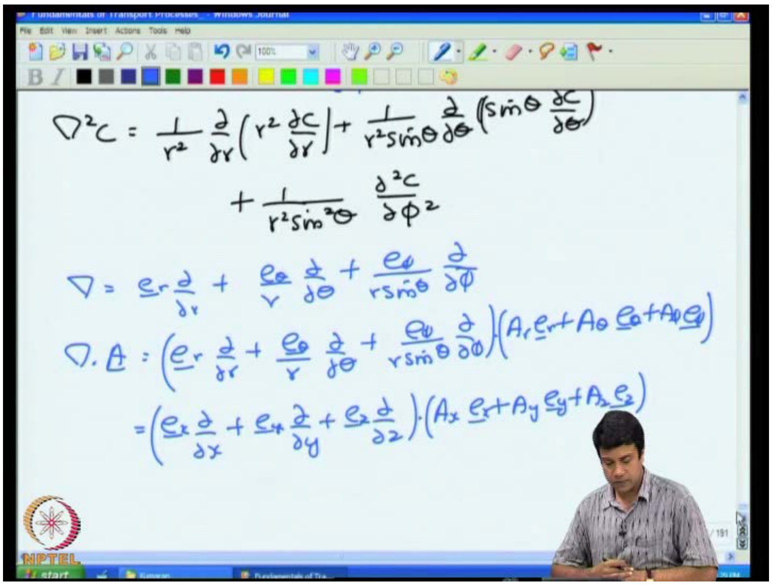

Therefore, the divergence operator acting on a vector del dot c u is equal to 1by r square d by d r of r square c u r plus 1 by r sin theta d by d theta.

(No audio from 23:36 to 23:52)

So, this is the divergence operator in a spherical coordinate system. Previously, in a Cartesian coordinate system our gradient was just equal to e x d by d x plus e y d by d y plus e z d by d z. This case it is more complicated because the theta and phi coordinates do not have dimensions of length. Similarly, in the case of Cartesian coordinates the divergence of c times u was simply defined as d by d x of c u x plus d by d y of c u y plus d by d z of c u z. This case you are getting more complicated forms and I showed you that is because the surface area of surfaces of constant r is changing as r changes. Surface area of surfaces of constant theta also changes as theta changes and because of that you get more complicated forms. And finally, we have defined the Laplacian in a Cartesian coordinate system as one over r square d by d r of r square c u r plus one over r square sorry, in the Cartesian coordinate system as d square by c by d x square plus d square c by d y square plus d square c by d z square.

So, in the Cartesian coordinate system, we had defined that simply in terms of the second derivatives with respect to the coordinates. As you can see here if the right hand side is equal to d del square c, this is a more complicated form of the Laplaican in a spherical coordinate system. So, in the spherical coordinate system I will define the Laplacian del square c is equal to one over r square d by d r of r square d c by d r plus one over r square sin theta. (No audio from 25:55 to 26:10) So, this is the Laplacian in spherical coordinate system, once again a slightly more complicated form.

And once again I just said the reason is because first of all the angles do not have dimensions of length. And secondly, the surface areas are varying in curvilinear coordinate systems, if the surface area vary with position in a curvilinear coordinate system. But once it is defined in this manner the mass conservation equation in vector form is exactly of the same form for both Cartesian and spherical coordinate systems, when it is written in terms of vectors. One doubt that naturally occurs when you write the gradient and divergence in this form, it is that the gradient of c as expressed here is just equal to e r times t by c by d r plus e theta by r d c by theta plus e phi by r sin theta d c by d phi.

So, why is it that I get a more complicated form when I express the gradient the divergence which is basically the dot product of the gradient and a vector? This is basically just the dot product of a gradient operator and a vector. In that case why do I get a more complicated form for the divergence? Similarly, del square is just del dot del. So, why is this, not this y does this not appear to be just del dot del? I will briefly go through the reason here, I would not give you a detailed explanation, I will just give you briefly the reason why these assume more complicated forms. I defined the gradient as e r d by d r plus e theta by r d by d theta plus e phi by r sin theta d by d phi. If I have to take the divergence of a vector, this is equal to e r d by d r plus e theta by r d by d theta plus e phi by r sin theta d by d phi of the vector. The vector is now in spherical coordinates therefore, I will have A r the three components A r e r plus A theta e theta plus A phi e phi.

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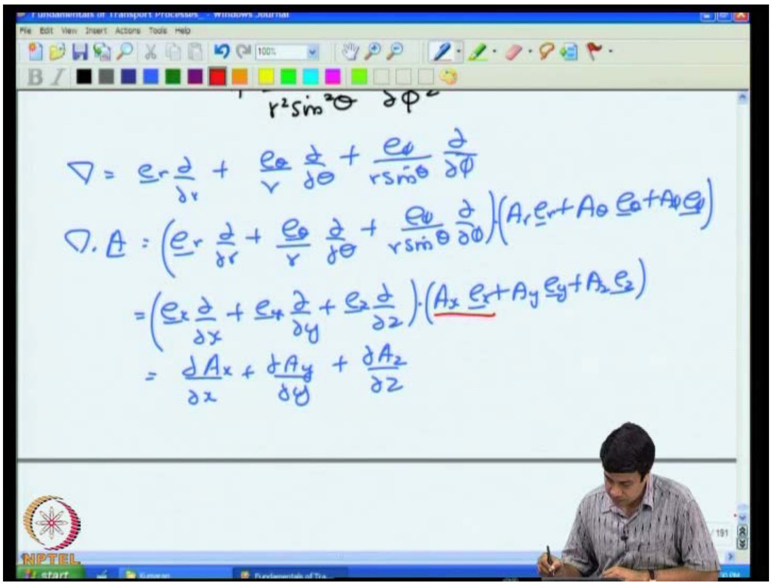

So, if you are doing this just simplistically the way that we did it in a Cartesian coordinate system, what you would say is that we will just take the derivatives of the components and take the dot product of the unit vectors, that is what we did in a Cartesian coordinate system. If you recall in a Cartesian coordinate system this was e x d by d x plus e y d by d y plus e z d by d z of A x e x plus A y e y plus A z e z, this was in a Cartesian coordinate system. (No audio from 29:55 to 30:04) And the equivalent in the spherical coordinates system. In the Cartesian coordinate system what we did was we just took the derivatives of the components and dotted the unit vectors to finally get d a x by d x plus d a y by d y plus d a z by d z. So, that was the simple solution in a Cartesian coordinate system.

Note that strictly speaking, both the component and the unit vector are within the differentiation are within are quantities that are being differentiated. And strictly speaking you should differentiate by chain rule; first take the derivative of the component times the unit vector and then the component times the derivative of the unit vector. In the Cartesian coordinate system we had the advantage that the unit vectors are independent of position. Unit vectors do not vary with location. And therefore, I was just able to take the unit vectors out of the derivative sign and take the derivatives only of the components.

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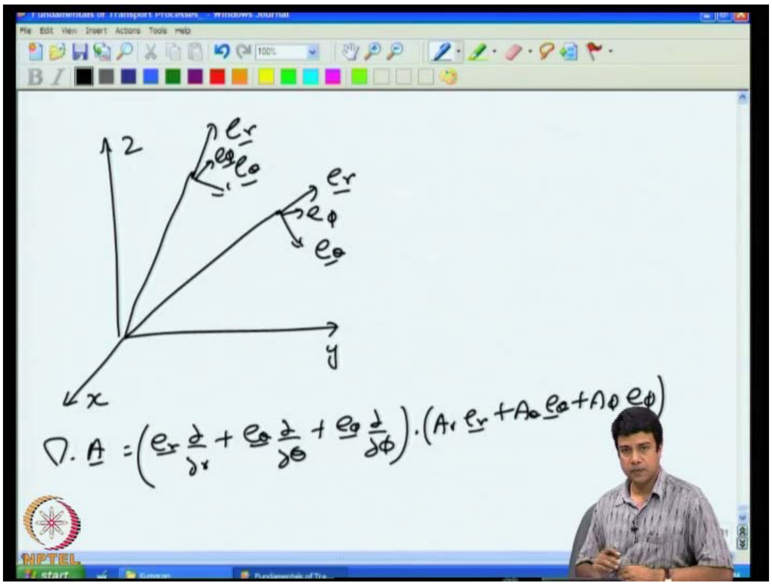

In the spherical coordinate system that we are considering here, (No audio from 31:21 to 31:32) the unit vectors do change with position. (No audio from 31:36 to 31:51) This is at one location. If I go to some other location (No audio from 31:56 to 32:02) this is e r, this is e theta and this is e phi. So, the unit vectors themselves are changing with position in the spherical coordinate system.

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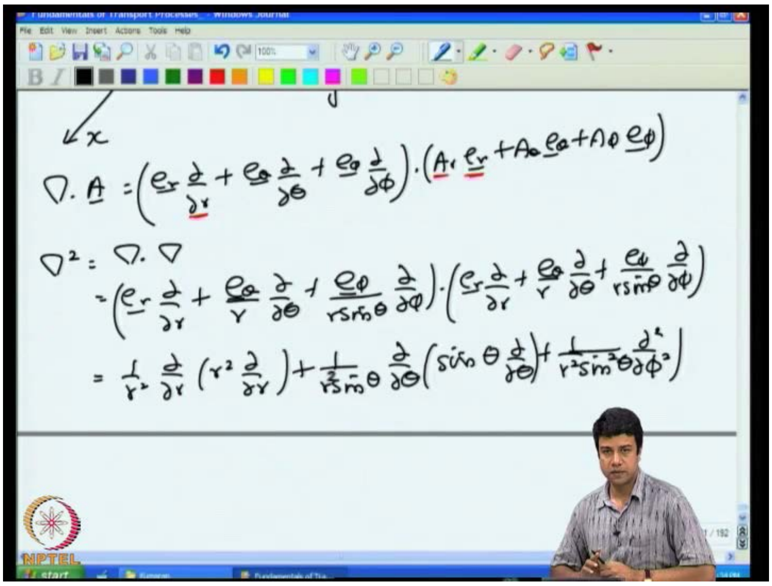

That means, that when I define del dot A is equal to e r d by d r plus (No audio from 32:26 to 32:33) dotted with A r e r plus A theta. When I define it in this fashion, I have to take the derivative, the derivatives here, both with respect to the component as well as the unit vector and then take the dot product. And because the unit vectors are changing with position, I will get additional terms in the conservation equation due to the change of the unit vector with respect to position. And those additional terms in the equation for del dot A can effectively be shown to be equal to give these additional terms of this kind. One over r square d by d r of r square c u r plus one over r sin theta d by d theta of sin theta c u theta plus one over r sin theta d c d of c u phi by d phi. So, that is why the divergence has a form that is different from the gradient. Similarly, when I take the Laplacian del square is equal to del dot del which will be equal to e r d by d r plus e theta (No audio from 34:02 to 34:12) dotted with

(No audio from 34:14 to 34:31)

I have to take the derivatives of not just the components with respect to the r theta and phi, but also the unit vectors. And if you take the derivatives of the unit vectors you will end up with one over r square d by d r of r square.

(No audio from 34:49 to 35:18)

So, this is the Laplacian operator in a spherical coordinate system. And you could do it in two ways; one way that I have just shown you by actually constructing a volume at surfaces of constant r minus delta r by 2 and r plus delta r by 2, theta minus delta theta by 2, theta plus delta theta by 2 and phi minus delta phi by 2 and phi plus delta phi by 2. Write down the balance equation for this differential volume and from that you end up with an equation the concentration equation right on top there. And identify the terms on the left hand side and the right hand side as del dot u c and d del square c and once you do that, you get the definitions of gradient divergence and the Laplacian operator.

The other way to do it is to actually (No audio from 36:12 to 36:25) use this expression for the divergence. And the Laplacian and actually take the derivatives of the unit vectors with respect to r theta and phi, that can be done and there are there is a well defined procedure for taking the derivatives of unit vectors with respect to coordinates. And when you do that, you get exactly the same expression that I had got earlier by doing a shell balance. So, both of these procedures gives you exactly the same result, whether you do it by shell balance and then take the limit of delta r delta theta delta phi going to 0.

You get a differential equation; identify the terms in that differential equation as the divergence and the Laplacian. Or you take directly the derivatives of the unit vector both of these give you the same result. We will not do the second way by taking the derivatives of the unit vector because we have not yet studied how to look at derivatives of vectors and tensors in vector calculus. But if you do it that way you will get exactly the same result.

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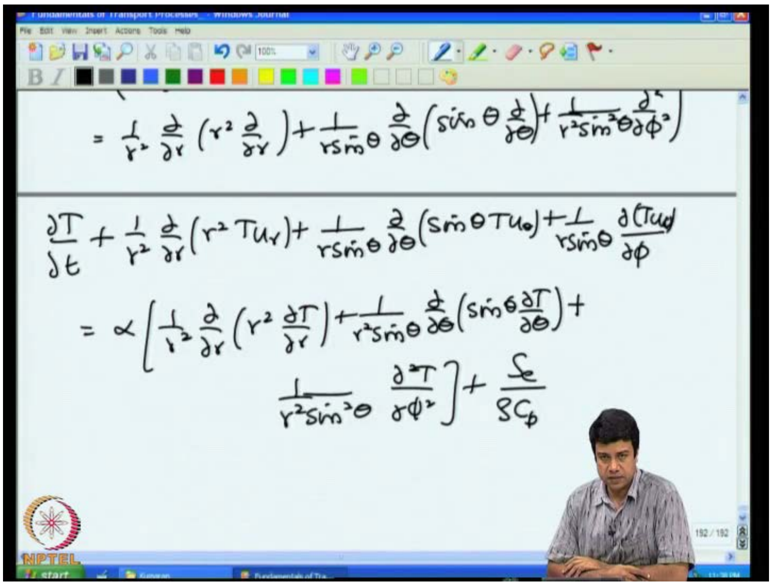

So, that is the equation for mass conservation. The equation for energy conservation is quite easy to write, substitute temperature for concentration.

(No audio from 37:47 to 38:33)

I should substitute the thermal diffusivity for the mass diffusivity.

(No audio from 38:38 to 39:04)

Plus s e by rho c. So, that is the conservation equation for energy in a spherical coordinate system. So, this completes the derivation of mass and energy conservation equations in a spherical coordinate system, momentum conservation equation we will deal with a little later the reason is because as I told you in the last lecture. Momentum itself has three components and it for each of those components there can be variations in three different directions. So, it is better to look at momentum conservation equation after we have looked at after we have developed some basics in vector calculus. So, this is the mass and energy conservation equation in a spherical coordinate system.

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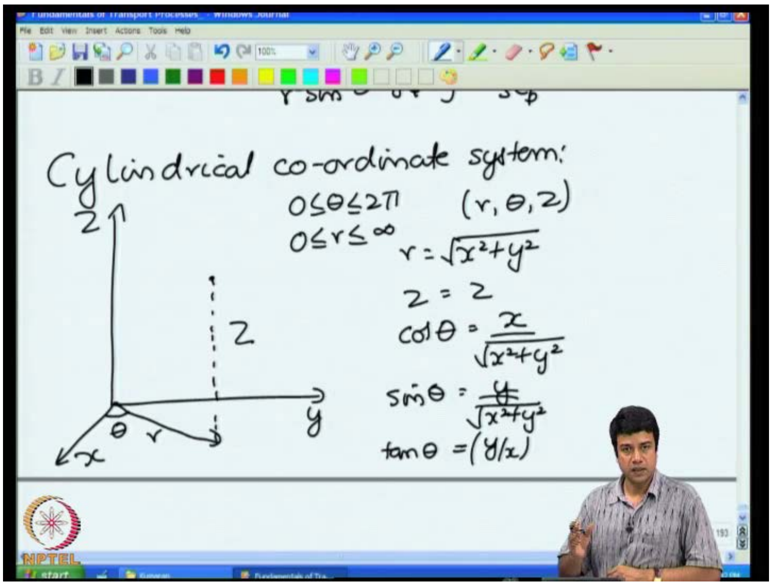

One can do a similar thing for a cylindrical coordinate system.

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One can do a similar thing in cylindrical coordinate system, I do not want to go into the details I will just give you a brief overview of how one derives balance equation cylindrical coordinate system and I will give you the final result.

(No audio from 40:32 to 40:48)

In a cylindrical coordinate system, the coordinates are r theta and phi I am sorry, the coordinate are r theta and z. Theta the meridional angle is the same as phi for the spherical coordinate system. So, for any position the first coordinate is the distance from the x y plane that is the z coordinate which is the distance from the x y plane that is z, r is the distance from the origin of the projection of this onto the x y plane. So, r is equal to square root of x square plus y square, z is the same as the z coordinate in a Cartesian coordinate system, r is equal to square root of x square plus y square, theta is the angle made by this projection with respect to the x y plane (No audio from 41:56 to 42:06)

Theta is the angle made with respect to the x y plane; that means that cos theta is equal to x by square root of x square plus y square sin theta is equal to y by root of and finally, tan theta is equal to y by x. Therefore, the angle theta varies from 0 to 2 pi, r is the distance along the x y plane so, it always positive. So, therefore, 0 less than or equal to theta less than or equal to 2 pi and r can vary from 0 to infinity, r is the distance so, the distance always has to be positive.

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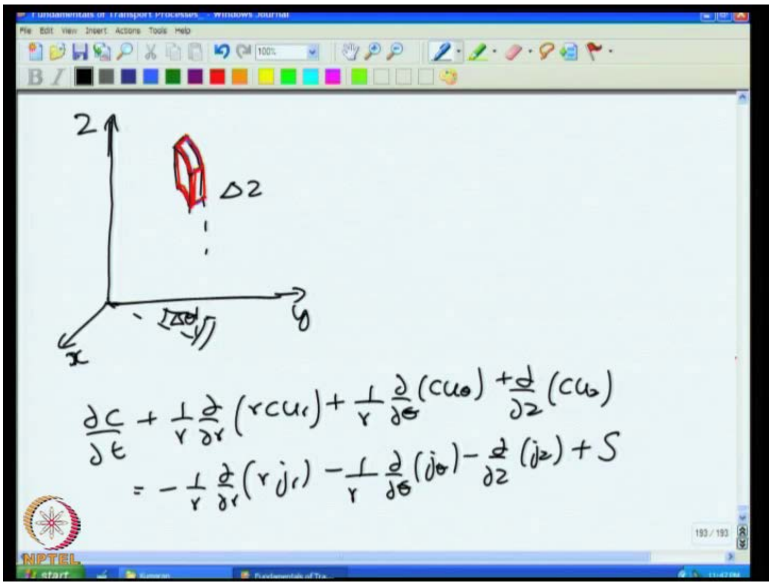


And z is the distance along the z axis so; this varies from minus infinity to plus infinity. We had solved using a cylindrical control volume for shell balances for a cylindrical coordinate system. If you want to derive in general a three dimensional conservation equation in cylindrical coordinates, then the surfaces of the control volume have to be bounded by surfaces of constant r, surfaces of constant theta and surfaces of constant z in a cylindrical coordinate system. If you recall previously when we did cylindrical coordinate system, it actually taken a cylindrical shell for doing the shell balance.

This cylindrical shell had height delta z in the z axis and the thickness was delta r, height delta z and thickness delta r. This is valid only for systems which have ax symmetric that is their symmetric as you go around the z axis. In general problems do not have axisymmetric so, we have to take a cylindrical shell of bounded by surfaces of constant r, at r minus delta r by 2 and r plus delta r by 2. Surfaces of constant z at z minus delta z by 2 and z plus delta z by 2 and surfaces of constant theta at theta minus delta theta by 2 and theta plus delta theta by 2. That means, that the differential volume that I consider has to be.

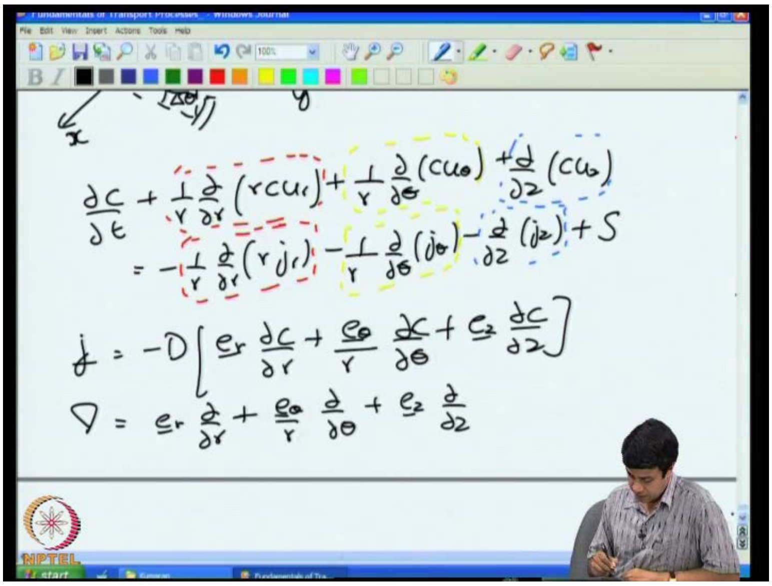
(No audio from 45:01 to 45:17)

Has to be of this form. (No audio from 45:19 to 45:27) This will be the differential volume, such that the angle if I take the angle made by the projection on to the x y plane, this angle subtended is delta theta. So, I have two surfaces constant r that is these ones this is at r minus delta r by 2, there is another one on the other side at r plus delta r by 2. Two surfaces at constant z the top and the bottom, this one is at z plus delta z by 2 and the bottom surface is at z minus delta z by 2. And then I have two surfaces at constant theta, theta minus delta theta by 2 and a surface at the back at theta plus delta theta by 2. And the surface areas for the surface at constant r is going to be equal to r delta theta times delta z because r delta theta is this distance and delta z is the distance perpendicular. At constant z, at z and z plus delta z is going to be equal to delta r which is this distance times r delta theta. Let theta and theta plus delta theta by 2 is going to be equal to delta r times delta z.

So, if I put all these together, put it into the conservation equation divide by the volume I will finally, get an equation of the form; partial c by partial t plus one by r d by d r of r c u r plus 1 by r d by d theta of c u theta plus d by d z of c u z is equal to minus (No audio from 47:30 to 47:44) plus the source term. If you recall the terms that look like this are terms that we are already got when we did unidirectional transport problems, these terms it already recovered when we did unidirectional transport problems.

Since, the derivative in the z since the z axis is identical to the z axis for a Cartesian coordinate system. The terms proportional to z are identical to this; these two terms are identical to what you would have in a Cartesian coordinate system. In addition, you have these two terms proportional to theta which comes out about because of the transport through the surfaces at theta and theta plus delta theta.

(Refer Slide Time: 43:19)



The expression for the flux will be equal to e r minus (No audio from 48:38 to 48:45) D into e r d c by d r plus e theta by r d c by d theta plus e z.

(No audio from 48:59 to 49:16)

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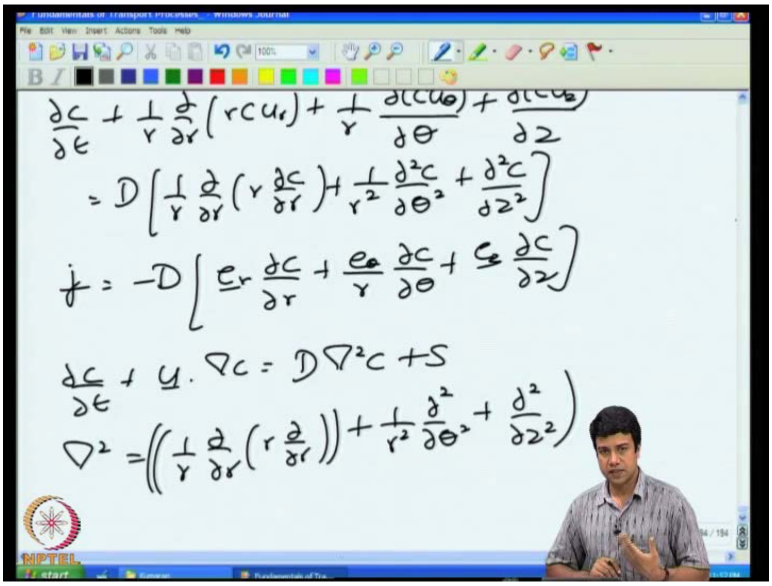

So, this is the gradient operator in a cylindrical coordinate system. (No audio from 49:22 to 49:32) That is the gradient operator and the divergence del dot j can be written as 1 by r d by d r of r j r plus 1 over r partial j theta plus partial j z by partial z. If I substitute the expression for the flux into the conservation equation, then I get an equation of the form partial c by partial t plus.

(No audio from 50:15 to 50:48)

So, that is the equation that I will get for the conservation equation. if I substitute Fick's law of diffusion which is basically that j vectors equal to e r partial c by partial r plus e theta by r d c by d theta (( )). (No audio from 51:09 to 51:27) So, therefore, my Laplace in operator if I write this equation as d c by d t plus u dot grad c is equal to d del square c plus s and the Laplace in operator in cylindrical coordinates is equal to 1 over r d by d r of r. (No audio from 51:49 to 52:03)

So that will be my Laplace in operator in a cylindrical coordinate system, similar to the equations that we had got in a spherical coordinate system. And of course, the equation of motion for I am sorry, the equation for the temperature for energy transport problems is of exactly the same form, except I substitute T instead of c and the thermal diffusivity instead of the mass diffusivity. So, this completes the derivation of conservations equations in both spherical and cylindrical coordinates, these have more complicated forms. These are easily available you can refer to standard text books for the formula, for the divergence curve and gradient, as well as for the diffusion equation in these spherical and a cylindrical coordinate systems.

My objective here was to show you how these expressions actually come about. They come about exactly the same way that they come about in a Cartesian coordinate system. Choose differential volume bounded by surfaces at constant coordinate, the change in mass when the differential volume is equal to the sum of mass in and mass out. And the masses coming in and out are the flux times the surface area. In cylindrical coordinate systems the surface area changes as the coordinate changes. And if you do all of those correctly you get the correct balance equations for both the cylindrical as well as the spherical coordinate system. No doubt these are more complicated than the equations that we had got the simple equations that we would got for a Cartesian coordinate system.

However, as we will see they have some important physical interpretations. Especially, this spherical coordinate system has important physical interpretations, which gives you much deeper understanding than the equations in a Cartesian coordinate system.

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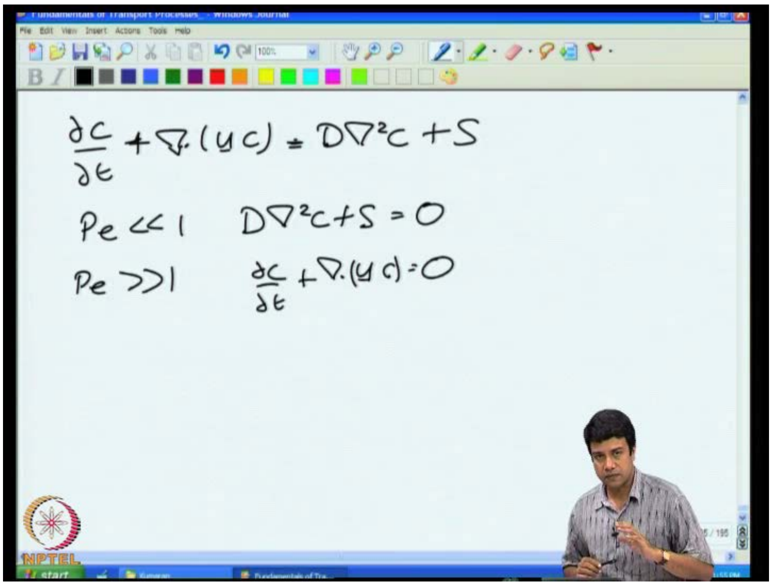

So, this is the diffusion equation. I can write it simply as d c by d t plus divergence of u time c is equal to D del square c plus any sources or sinks. When we started this course we talked about the relative magnitudes of convection and diffusion. Written this way the left hand side is the time derivative the variation with respect to time plus the convective term, the right hand side is the diffusive term plus any sources or sinks (( )). So, therefore, if diffusion is large compared to convection when the peclet number a small compared to 1, you would expect D del square c plus any sources or sinks to be equal to 0. When convection is dominant, you would expect partial c by partial t plus del dot u c c equal to 0. We will now look at how to derive equations in these two limits; first we look at the limit of low peclet number where the diffusion is dominant and we will be solving the equation D del square c is equal to 0 subject to boundary conditions.

So, next lecture we will start are solution of the diffusion equation. And we will come back to the lecture on the convection on the high peclet number limit a little later. Turns out we cannot just simply use this high peclet number approximation in that case, we need to do something more complicated. And we will see that when we do boundary layer theory for high peclet number transport. So, we will start the diffusion equation in the next lecture, how to solve it, we will start with the spherical coordinate system. Solution is this procedure is the same, we have to implement a separation of variables; what does that give us and how can that be used for real problems that will be started in the next lecture so, will see you then.