Fundamentals of Transport Processes
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Module No. # 05
Lecture No. # 27
Mass and Energy Conservation Spherical Coordinates Balance Laws

This is lecture number twenty seven of our course on fundamentals of transport processes. Welcome. We were discussing in the last class conservation equations for mass and energy and I had derived for you in some detail the conservation equations in a Cartesian coordinate system. That is actually the simplest of the coordinate systems that we could work with because in a Cartesian coordinate system the axis are perpendicular to each other, the unit vectors are the same at every location in space and the planes of constant coordinate are parallel to each other and they form flat surfaces.

So, when we write a differential equation for some quantities such as mass or energy we are writing it for a cubic differential volume and a cubic differential volume is the easiest one to deal with.

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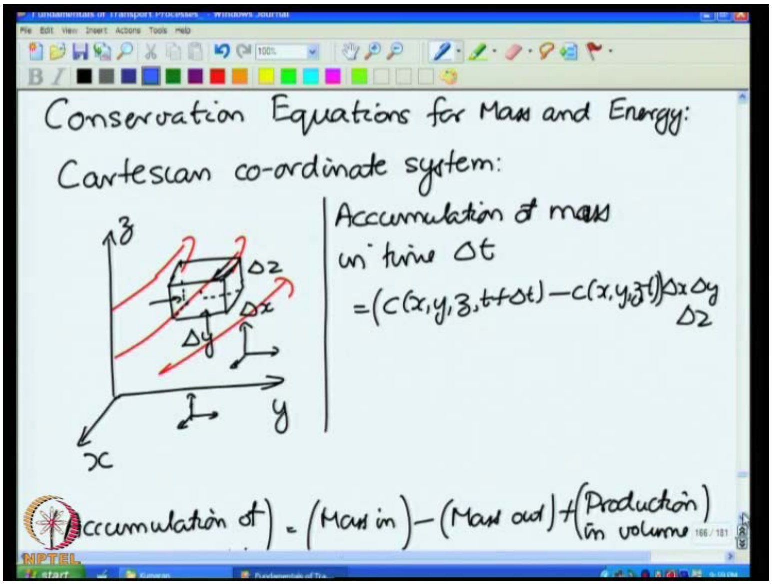

So, when we looked at transfer in a Cartesian coordinate system in the last class I had defined for you a differential volume which was in the form of a cube.

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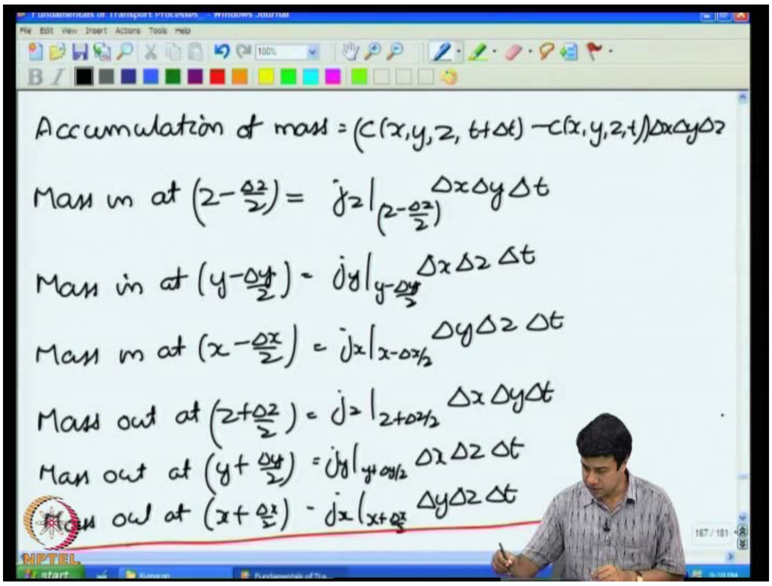

And then, we had written the mass conservation equation; accumulation of mass is equal to mass n minus mass out plus production in the volume and the differential volume is chosen with a certain logic.

You want to choose the differential volume such that the surfaces in the perpendicular to the y axis for example, are at surfaces of constant y that is there at y and y plus delta y or in this case we took y plus delta y by 2 and y minus delta y by 2. Similarly you have two surfaces in the x axis and x plus delta x by 2 and x minus delta x by 2 and 2 in the surfaces perpendicular to the z axis and you write balance equations which contain rate of change. There is a change within the volume, within a time delta t.

The amount of mass coming in and going out of each of these surfaces at the surface at z if there is a flux in the plus z direction the mass increases. Surface at z plus delta z there is flux in the plus z direction leaving that surface. So, at this surface the flux in the plus z direction is leaving the surface. So, the mass decreases.

So, we write down the balance for the mass in and the mass out and from that we get the accumulation and then we divide throughout by volume times time. We divide throughout by volume times time in order to get the differential equation.

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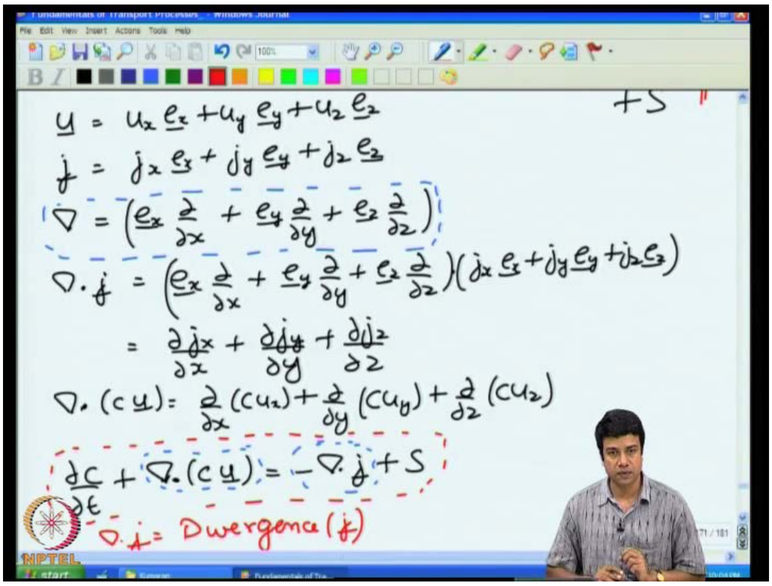

And the equation on top there was the differential equation that we got in the last class for a Cartesian coordinate system. The red terms on the left are the convective terms u x u y and u z are the mean fluid velocities. The terms on the right are the diffusion terms j x j y and j z are the fluxes. Both of these as I explained in the last lecture are vectors. They have three components along with three axis and therefore, I can write this equation as a vector equation and that vector equation is what I had at the bottom of the screen. Del dot C u, the gradient operator is defined as e x d by dx plus e y d by dy plus e z d by d z acts on the flux. So, you take the dot product of this gradient operator and the flux del dot j and you end up with partial of j x with respect x plus partial of j y with respect to y plus partial of j z with respect to z which was exactly the negative of the term that we had on the right hand side. Therefore, the term on the right hand side was minus the divergence. Del dot is called the divergence.

So, dot product between two vectors; the gradient is a vector, the flux is a vector and therefore, the divergence of j is a scalar. Similarly, on the left hand side we can write the terms in blue on the in red on the equation on the left as the divergence of the concentration times the velocity. The local concentration times the local velocity. So, that can be written as del dot C u. C is a scalar, u is a vector. Therefore, when I take the dot product of the gradient operator and C times u I get d by dx of C u x plus d by dy of C u y plus d by d z of C times u z. So, that is the mass conservation equation in a Cartesian coordinate system as well as in this more general form. I had told you in the last class that this form is general. It is valid for any coordinate system provided I define the gradient operator and the fluxes and the velocities appropriate to that coordinate system. So, in that sense it is more general. When I write it in the Cartesian coordinate system, then that vector, that equation in terms of the vector gradient flux and velocity expanded out is the equation right on top.

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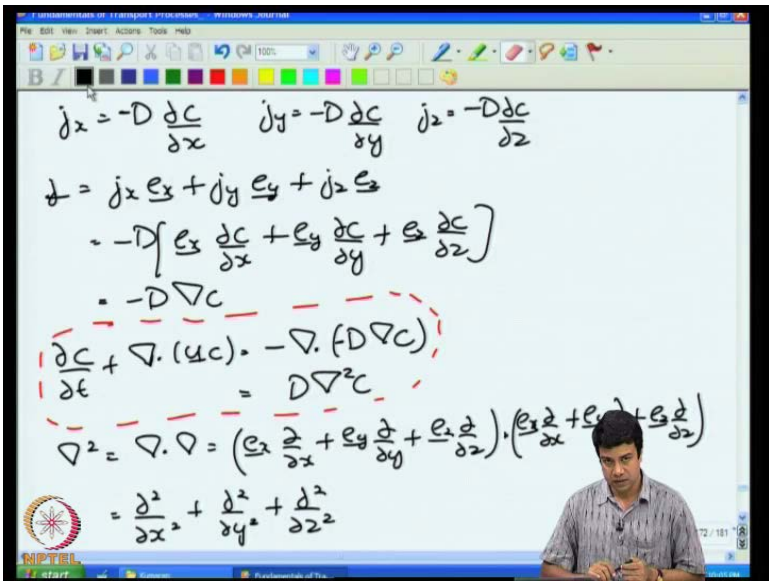

And the fluxes can be written in terms of the concentration variations, j x is equal to minus d times partial C by partial x. j y is equal to minus d partial C by partial y and j z is equal to minus d times partial C by partial z. The vector j vector can be written as minus d times the gradient of C. Same gradient operator that we had defined previously. D times gradient of C. C is a scalar the gradient operator is a vector. So, minus d grad C is a vector. So, if I put this expression for the flux into my conservation equation then, I get the equation as partial C by partial t plus divergence of u time C is d del square C and I told you in the last class that del square is the Laplacian operator. In Cartesian coordinates it is d square by dx square, all partial derivatives plus partial square by partial y square plus partial square by partial z square. This is a scalar it is del dot del. Two vectors dotted with each other give you a scalar.

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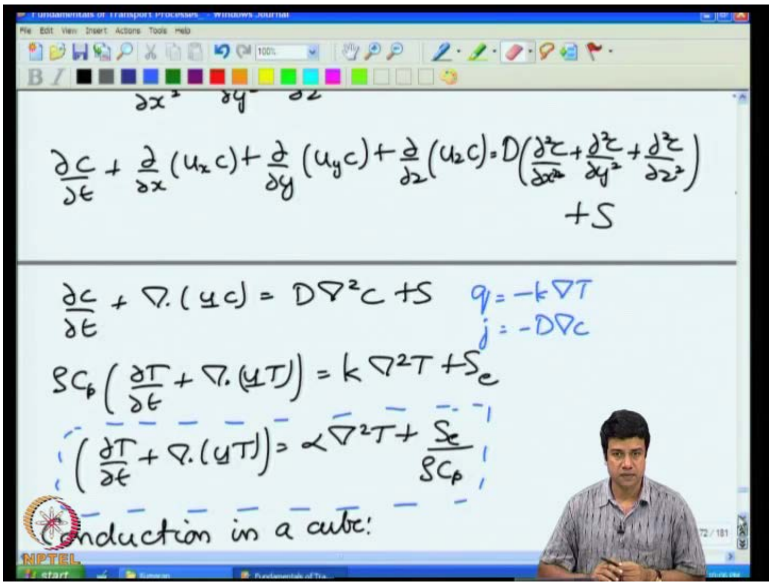

And you can get similar equations for energy as I said the momentum equation is a little more complicated. So, we shall not deal with it at present, but for the energy equation is exactly the same except that temperature is substituted for concentration and the thermal diffusivity for the mass diffusivity.

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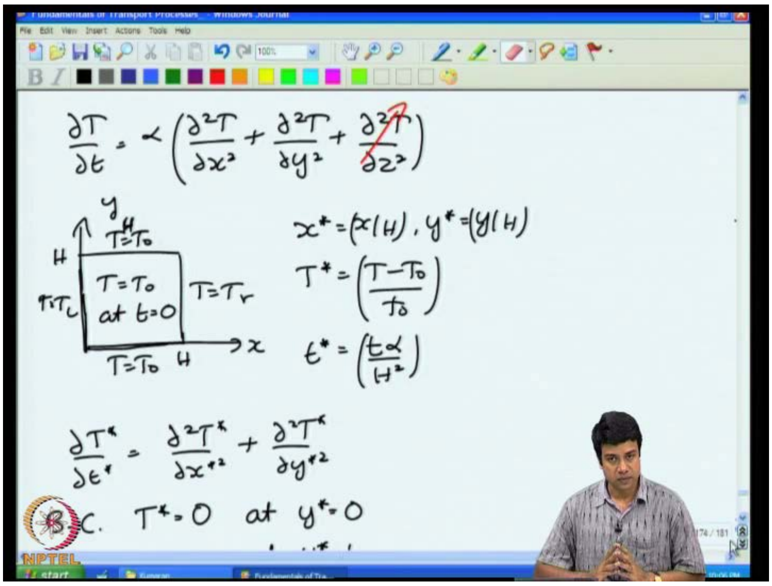

And then we had solved an equation for the conduction in a cube in the previous class; the unsteady heat conduction and I showed you how to do the separation of variables for that particular case. So, when one does separation of variables in three or more variables.

In this particular case it was an unsteady conduction problem with variations in both the x and the y directions.

First one has to solve the steady state problem. The steady solution in the limit as t going to infinity and for that the steady state problem is obtained by from the differential equation for the steady state.

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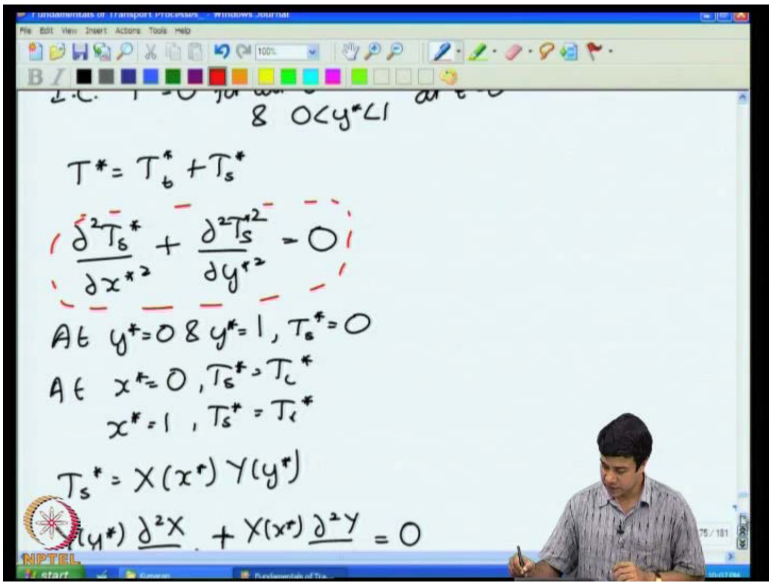

This is solved by separation of variables. Y is the homogeneous direction the boundary conditions are homogeneous in the y direction. X is the inhomogeneous direction. Therefore, the solution will be of the form of sine functions which are the basis functions for the second order derivative operator in the y direction.

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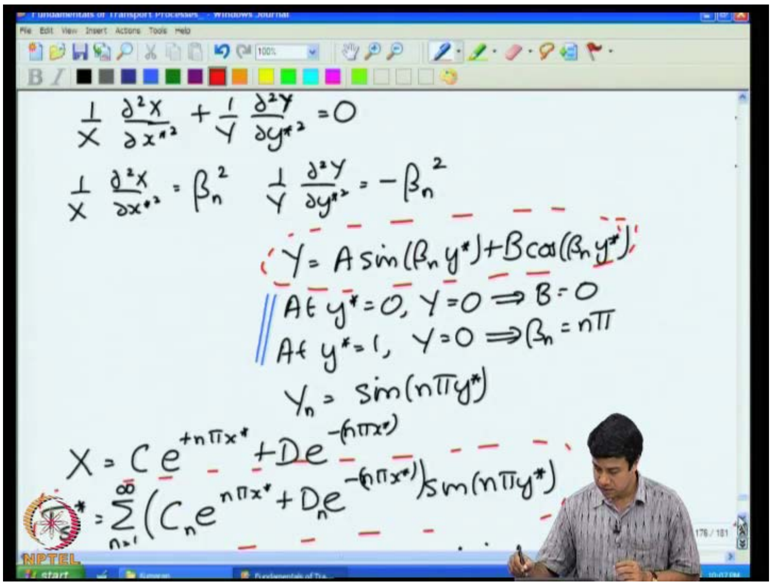

So, in the y direction because the boundary conditions are homogeneous I will get sine and cos functions where the Eigen values beta n are determined from the condition that Y has to be 0 on both surfaces. If Y is homogeneous then x is inhomogeneous you get sine exponentially increasing and decreasing solutions in the x direction.

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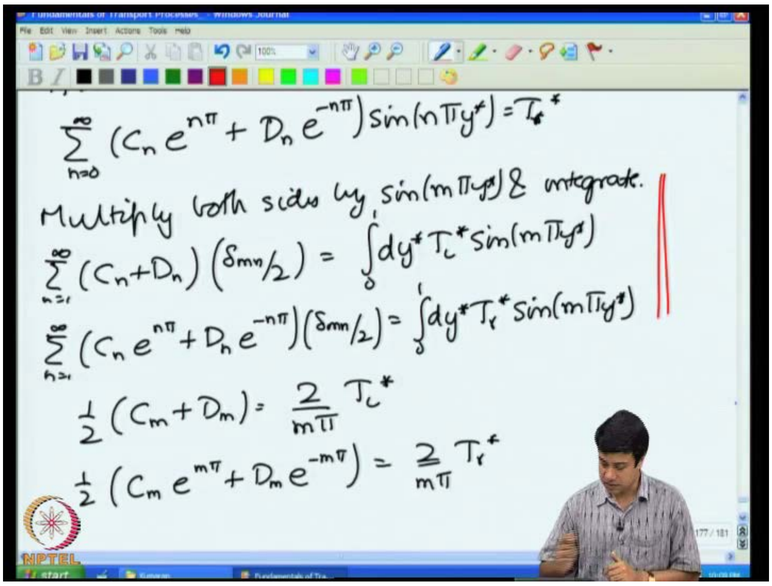

And the coefficients in these are determined using the orthogonality relations for the x direction. So, you get two simultaneous equations if you impose orthogonality relations. From that you can get both the coefficients. That was the steady solution. In order to get the transient solution you subtract out the steady solution from the total solution. When you subtract out the steady solution from the total solution; for the transient problem the boundary conditions are homogeneous in all directions. That we saw in the last class.

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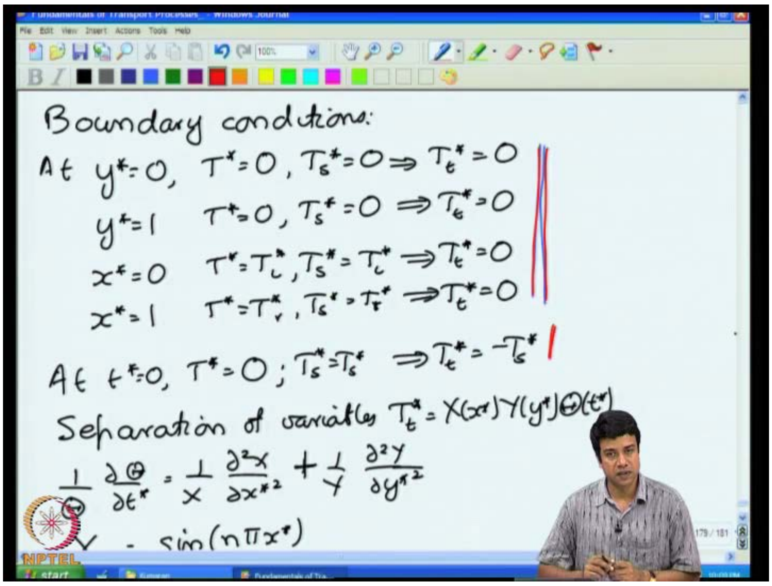

For the transient temperature field, the boundary conditions are actually homogeneous in all directions. Therefore, you get sine and cosine functions in all directions. The in homogeneity is in the initial condition because the steady problem is has has no time dependence whereas, the the the total temperature field is being forced at initial time. Therefore, you get unsteady; you get a non homogeneous initial condition which is now used to determine all of the coefficients in the solution for the total temperature field.

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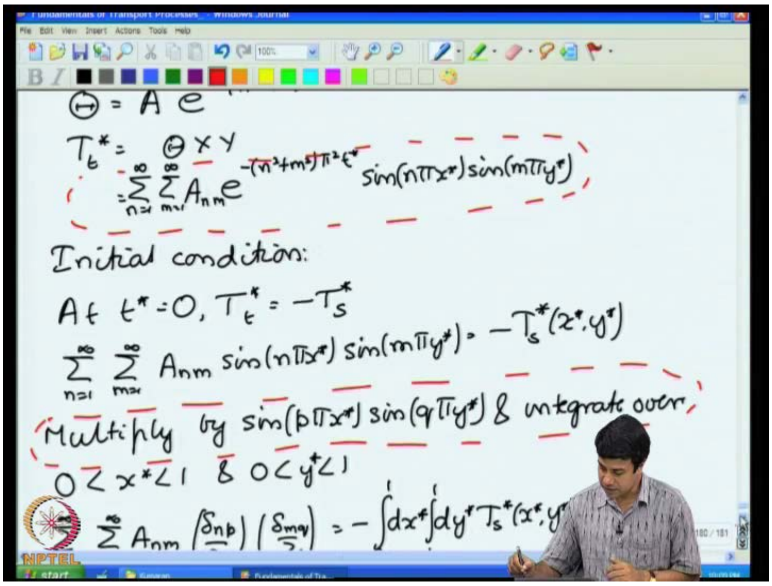

So, this initial condition is determined using orthogonality relations that I have shown you over here. You have to multiply simultaneously by the Eigen functions in both the x and the y directions and integrate to get the coefficients.

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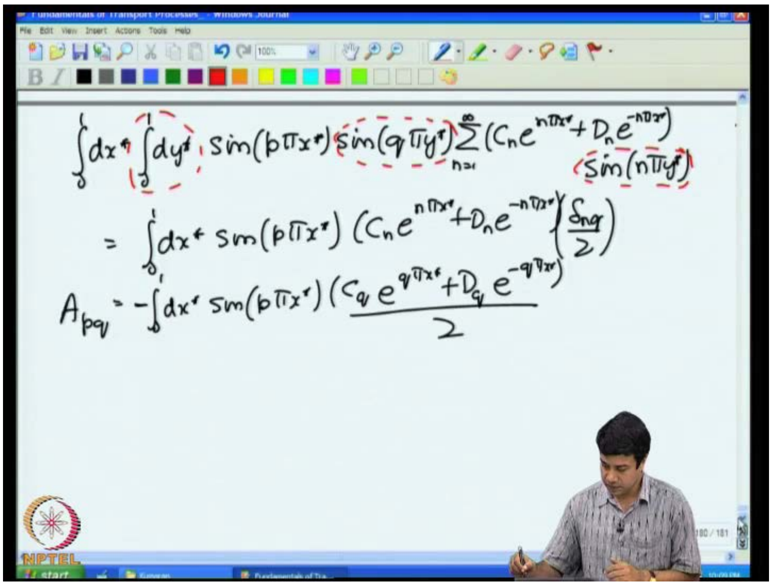

And that gives us the final solution for the coefficients. We have the solution for the transient problem, we have the solution for the steady problem. Put those together and you will get the total temperature field. So, that is how you extend separation of variables to this particular case. As I told you in the previous lecture, we are now going to go and look at a spherical coordinate system. One can do the same thing for a cylindrical coordinate system as well, but a spherical coordinate system gives additional physical interpretation for the solutions of these equations. Therefore, we will now look at the solutions the I am sorry the mass and momentum and energy conservation equation in a spherical coordinate system.

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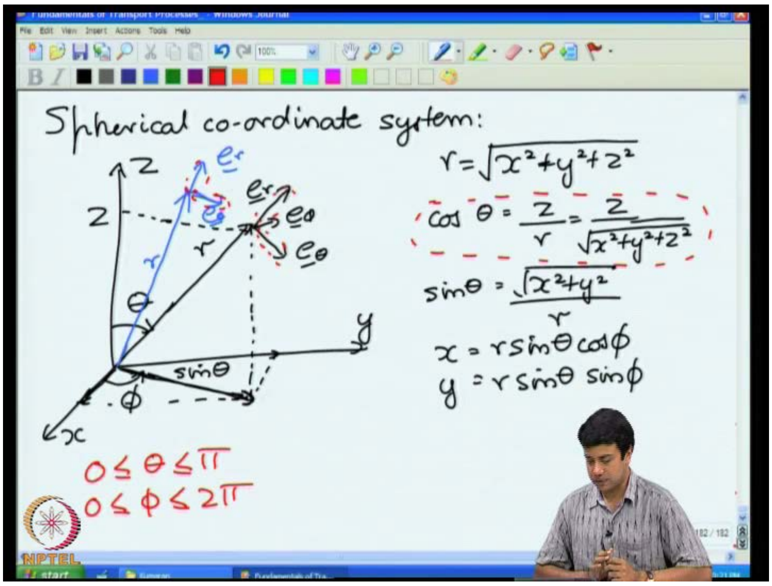

Let us first define the coordinates in this coordinate system. So, x y and z are the coordinates in a Cartesian coordinate system. In a spherical coordinate system as I told you is used for configurations with spherical symmetry. For example, if you are analyzing the diffusion from a spherical catalyst particle; you would prefer a coordinate system where the surface of the particle is the surface of constant coordinate. So, in that case the surface of the particle if you put the coordinate system at the centre of the particle, the surface of the particle is a coordinate of constant r where r is the distance from the centre of the coordinate system.

So, r in this case is one of the coordinates, is the centre from the origin, the centre of the particle if you were working with the spherical catalyst particle. So, r is the is is the vector is the vector from the origin to the location to any particular point. The distance is denoted by the scalar r. Of course, r just gives you the distance. It does not give you the orientation of the vector with respect to some fixed axis. That orientation is provided by two angles in this spherical coordinate system.

The first one is the angle theta. Theta is the angle made by the radius vector with the z axis of the coordinate system. If the system has symmetry about some axis then, you would prefer to put a z axis along the axis of symmetry in a manner similar to what we did in the cylindrical coordinate system. In that case the temperature or concentration field will depend only upon the angle from this axis. So, that is the axis theta, this is a third third coordinate which is also an angle that is what is called the coordinate phi. If I take the projection of the radius vector on to the x y plane, the angle that that makes with the x axis is phi.

So, in terms of the x y and z coordinates of this coordinate system; r is just the distance from the origin which is square root of x square plus y square plus z square. R is just the distance from the origin. Theta is the angle that is made with respect to the z axis. Therefore, the angle theta time theta is equal to z by I am sorry Cos theta. R Cos theta is equal to z by r is equal to z by square root of x square plus y square plus z square. So, that is the angle theta. That defines the angle theta. So, now, if the z coordinate is equal to Cos theta; that means that this coordinate is equal to sine theta. The projection on to the x y plane of this vector has to be equal to sine theta the projection on the x y plane. That means, that sine theta is equal to square root of x square plus y square by.

So, that sine theta the x the x coordinate has to be equal to this distance, this distance times Cos phi the x coordinate here has to be equal to this distance times Cos phi. Therefore, x is equal to r sine theta Cos phi and y is equal to this distance times sine of phi y is equal to r sine theta sine phi. So, these define the coordinate system. r theta and phi; r is called the radius theta is called the Azimuthal angle, the angle from an axis and phi is called the meridional angle, the angle around the axis. Theta z if you look at this expression for Cos theta at x is equal to y is equal to 0 if z is positive along the z axis Cos theta is equal to 1 if z is negative and x and y are 0 then Cos theta is equal to minus 1. That means, that Cos theta can vary only from plus 1 to minus 1; that means, that theta varies from 0 to pi. Phi is the angle around the axis phi is the angle around the axis. So, if I start at phi is equal to 0 go all the way around and come back to the x axis the angle that I would have gone around is 2 pi. So, therefore, phi varies from 0 to 2 pi.

So, within this spherical coordinate system r is the distance from the centre. You can go all the, from 0 at the centre itself to infinity. Theta varies from 0 to 2 pi and phi I am sorry theta varies from 0 to pi and phi varies from 0 to 2 pi. An important point to note about the Cartesian coordinate system; the unit vectors are in the direction of increasing angle.

So, the angle theta increases in this direction, phi increases in that direction, r increases radially outward. That means, the unit vector e r is in this direction, in the direction of increasing r a unit vector. E theta is in the direction of increasing theta is in the direction of increasing theta perpendicular to r and e phi is in the direction of increasing phi into the plane of the board. We will draw it better perpendicular to both e r and e theta. So, this is an orthogonal coordinate system where the unit vectors are all perpendicular to each other. However, the unit vectors do not remain the same as you vary position. So, to illustrate this, let us take some other position here. This is the radius vector at this new position. In this case e r is in this direction and e theta is in that direction. Note that the direction of e r is not the same over here and over here because the direction of the radius vector is different. Therefore, the direction of the unit vector is also different. Similarly the direction of e theta is not the same. As theta changes the direction of e theta also changes.

Similarly the direction of e phi is also not the same it changes as the angle changes. So, in that sense the unit vectors are not independent of position. When we looked previously at a Cartesian coordinate system, the unit vectors were all independent of position. They were pointing in the same direction at all locations. E x was always along the x axis, e y was always along the y axis and e z was always along the z axis. In this case, the direction of the unit vectors are not the same at all locations. They are dependent upon position they are dependent on position, but they are orthogonal. At each point in space the unit vectors are all perpendicular to each other. So, this is still an orthogonal coordinate system.

So, how do we define, how do we determine balance equations for this coordinate system? As usual we have to construct a differential volume which is bounded by surfaces of constant coordinate. In this case the coordinates are r theta and phi. So, we need to construct a differential volume bounded by surfaces of constant r, theta and phi.

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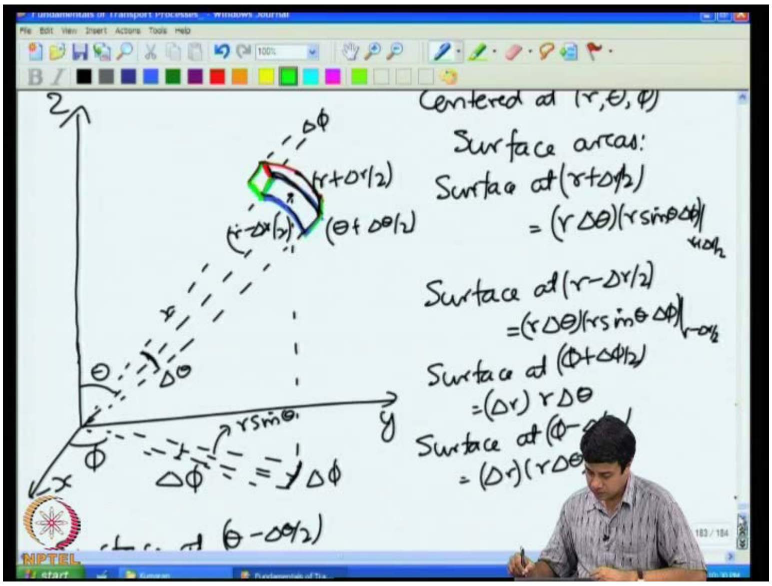

So, I have location here at which I want to construct a differential volume for carrying out my balance equations. So, this total distance is r. Now, my differential volume is centred at the location r theta and phi. That means, that that means, that it has to be bounded by surfaces at r minus delta r by 2 and r plus delta r by 2 theta minus delta theta by 2 and theta plus delta theta by 2 and phi minus delta phi by 2 and phi plus delta phi by 2.

So, let us look at the surfaces at constant r. These are of course, spherical shells. Surfaces of constant r are basically spherical shells. So, at r plus r minus delta r by 2, I will have a spherical shell and at r plus delta r by 2 I will have another spherical shell. So, this is at r minus delta r by 2 and these are two spherical shells at r minus delta r by 2 and r plus delta r by 2 in the theta direction. These are going to be bounded by two surfaces at constant theta minus delta theta by 2 and theta plus delta theta by 2. So, this is theta, this angle is delta theta, this total angle. So, at this is the surface at theta minus delta theta by 2 and theta plus delta theta by 2 and then I have 2 shells in the two surfaces in the phi direction. Phi is this angle, the angle that unit vector makes with this coordinate. So, I have to have two shells in this direction whose difference in angle is delta phi whose difference in angle is delta phi. It takes some effort to visualize this, but if you look at it in this direction it looks something like that It looks something like that where this angle in the direction perpendicular to the board is going to be delta phi. So, these give me the surfaces of this differential volume.

What are the surface area of these surfaces? It is not obvious right now what the surface area is because I have only given you the angles by which these differential volume is bounded. So, surface areas, surface at r plus delta r by 2, it is going to be equal to I have a distance here. So, this distance is going to be equal to r times delta theta because the distance from the centre is r, the angle subtended is delta theta. Therefore, this distance is going to be equal to r times delta theta. So, this surface is bounded by two lines; one is here, the other is here. The other is in the phi direction. In the phi direction this distance this distance here is r sine theta and this angle is delta phi. So, the distance in the phi direction in the phi direction here is going to be equal to r sine theta delta phi at r plus delta r 2.

So, this is the surface for which I have written now the surface area. The surface at r plus delta r by 2 it is going to be equal to r delta theta times r sine theta delta phi at the radius r plus delta r by 2. That is this surface. This is going to be in a similar manner r delta theta into r sine theta delta phi at r minus delta r by 2. So, those are that surfaces the top and bottom surfaces at r and r plus r minus delta r by 2 and r plus delta r by 2. What about the surfaces here? What about the surfaces, this surface and the similar surface at the back? They are bounded by this line which is delta r and this 1 which is r delta theta. So, therefore, this surface at theta plus delta theta by 2 is equal to delta r into r delta theta

I am sorry this should be at phi (( )) plus delta phi by 2 if it is the front surface and the back surface. These two are separated by an angle delta phi and for these two surfaces the front and the back surfaces which are separated by an angle delta phi, they are perpendicular to the phi direction, perpendicular to the unit vector in the phi direction. Therefore, the surface and finally, for this differential volume, I also have these two surfaces. This one this is at theta minus delta theta by 2 and you have a similar one here at theta plus delta theta by 2. Therefore, I will have would be equal to delta r into it is going to be equal to delta r times this distance is delta r and this distance which is travelled along the phi direction. this distance is travelled along the phi direction. I told you that this distance is r sine theta therefore, the distance that you travelled along the phi direction is going to be equal to r sine theta times delta phi.

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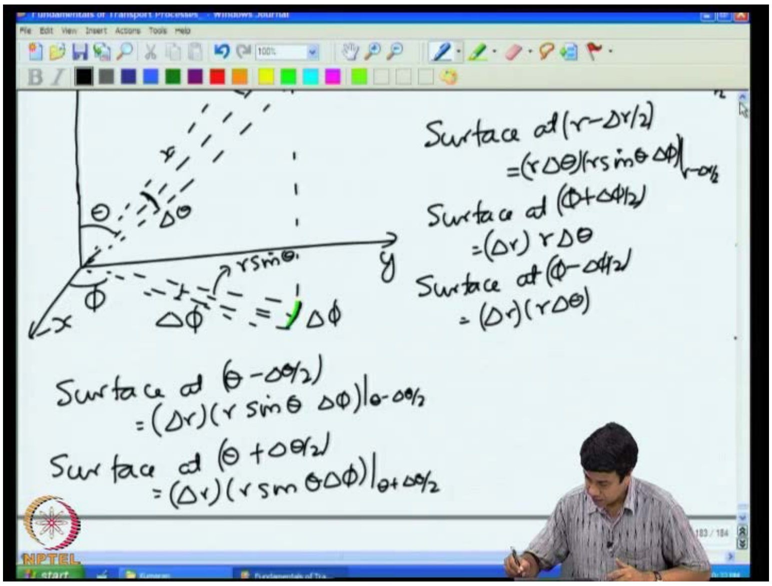

You may note that as in any curvilinear coordinate system; these surface areas depend upon coordinate in particular the surface area at r plus delta r proportional to r square sine theta delta theta delta phi at phi plus delta phi by 2 and phi minus delta phi by 2 is equal to r times r delta theta the third 1 is equal to delta r times r sine theta delta phi. So, these coordinates actually do depend upon the I am sorry these surface areas do depend upon the coordinates. We saw that when we did the cylindrical coordinate system earlier. The surface area for a curvilinear coordinate system thus depend upon coordinate because of that you get more complicated expressions for the balance equations than just a simple expressions in the Cartesian coordinate system where the surface areas delta x delta y, delta y delta z and delta x delta z were all independent of positions in the Cartesian coordinate system. So, now, that we have defined this differential volume we can now go ahead and do the balance equation.

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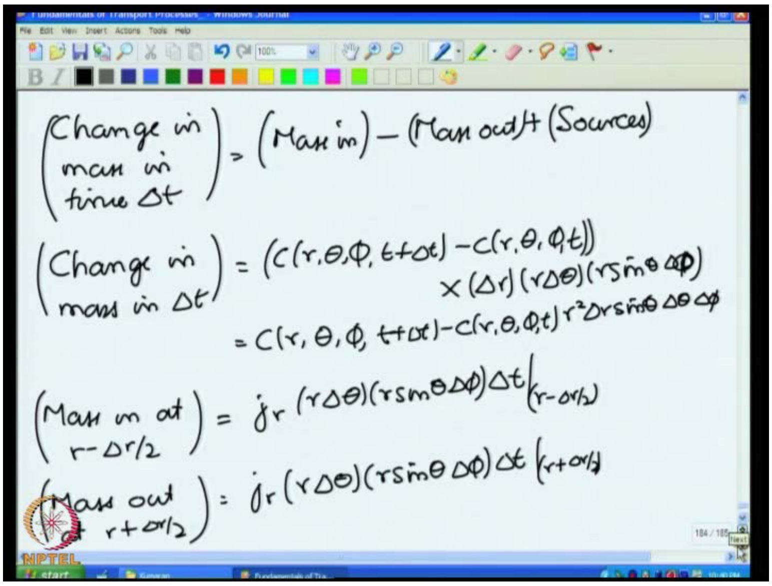

So, first is the change, the balance equation basically is that change in concentration, in the mass, in time delta t is equal to mass n minus mass out plus any sources or sinks that are there. So, what is the change in mass in time delta t? This is equal to C at r theta phi t plus delta t minus t at r theta phi t concentration times the volume. What is the volume of this differential volume? This is a surface which has sides delta r in this direction, the sides of the surface are delta r in this direction. In this direction the side is r delta theta in this direction the side is r delta theta and in the direction the phi direction this side is r sine theta delta phi.

Therefore, the change in mass is going to be this times times the volume which is delta r r delta theta into r sine theta delta phi which is equal to C at r theta phi t plus delta t minus C at r, r square delta r sine theta. So, this is the change in mass within that differential volume. This is going to be equal to the sum of mass coming in from the surfaces, bounding surfaces and the mass going out leaving at the bounding surfaces. There are six surfaces in this particular case one at r minus delta r by 2 and 1 at r plus delta r by 2 theta minus delta theta by 2 and theta plus delta theta by 2 and phi minus delta phi by 2 and phi plus delta phi by 2 and the mass coming in and going out is due to two reasons; one is due to convection and the other is due to diffusion.

When we solved the, obtained the balance equations for the coordinate system, for the Cartesian coordinate system if you recall, the flux due to diffusion had the form j z at z minus delta z by 2 times delta x delta y by t and similarly j y j x and similarly at z plus delta z by 2. That was the flux due to diffusion. Due to convection you had very similar terms except that instead of the flux j you had the concentration times the velocity. C times u z C times u I and C times u x. Because of that I can write down that expressions for just the fluxes alone and the terms due to convection, I just need to substitute C u x for j x C u y for j y and C u z for j z. In this particular case, I will substitute C times u r for j r C times u theta for j theta and C times u phi for j phi.

So, I will first derive the expressions for the fluxes and just write down the expressions for the convection, convective parts just by analogy. So, first mass in at r minus delta r by 2, mass in at this particular surface, mass in at this surface the bottom surface. Mass is coming in if the flux is positive in this particular case in the coordinate system that I have chosen e r is in the plus r direction. Therefore, at the bottom surface there is mass coming in because the even the flux is along the e r direction.

Therefore, this mass in can be written as the flux j r. j r is the component of the flux along the e r direction times the area. The area of that surface is equal to r delta theta times r sine theta delta phi. Therefore, this is equal to j r times r delta theta, r sine theta delta phi times the time fluxes per unit area, per unit time. Therefore, in order to give you in order to determine the total mass coming in I have got to multiply this by the surface area and multiply it by time this whole thing is at the surface r minus delta r by 2. Similarly, there is mass out at r plus delta r by 2. This is going to be equal to j r times r delta theta into r sine theta, delta phi times delta t at r plus delta r by 2.

So, those are the masses at the surface at r and at the surface at r plus delta, the amount of mass coming in going out. Similarly I have two surfaces in the theta direction; theta minus delta theta by 2 . So, theta minus delta theta by 2 is the top surface here.

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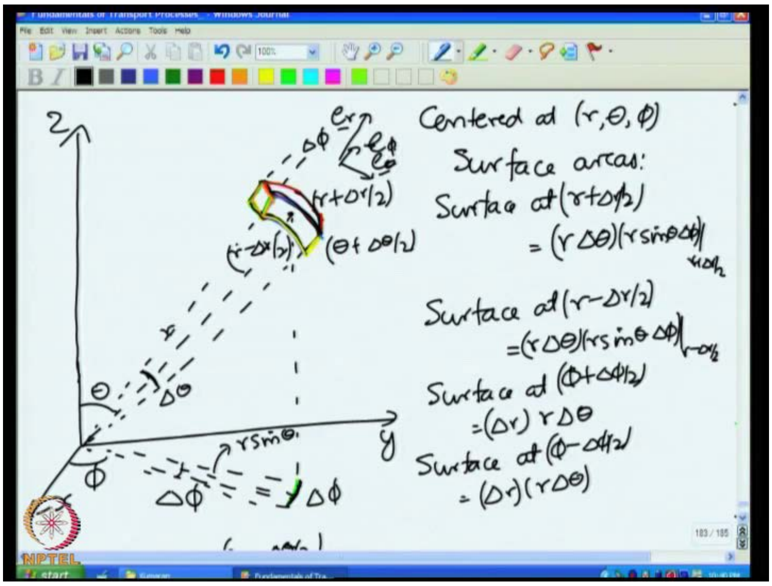

So, it is equal it is effectively the top surface over here and as I said the coordinate system that I am using it has unit vector e r in the r direction, e theta in the theta direction and e phi is in the direction of variation of phi perpendicular to the plane of the board. Therefore, in the theta direction the mass in is going to be equal to the component of the flux in the theta direction.

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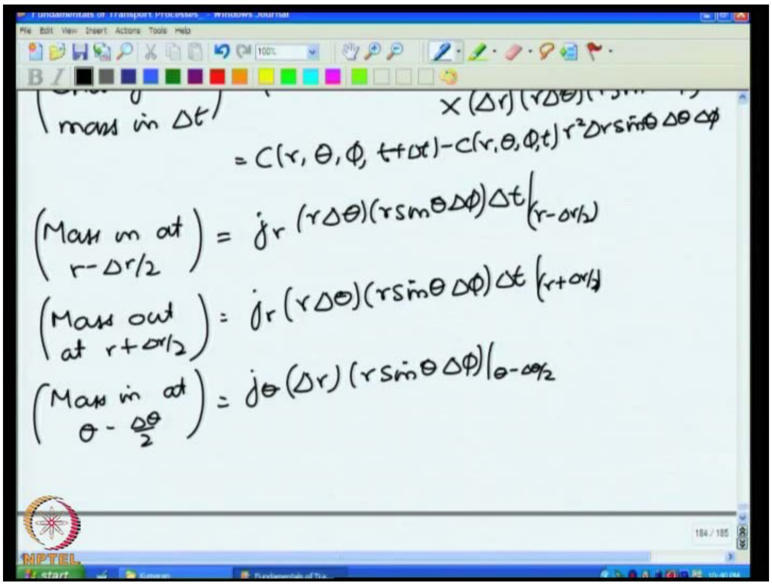

J theta times the surface area. The surface area for that surface on top is equal to delta r times r sine theta delta phi at theta minus delta theta by 2.

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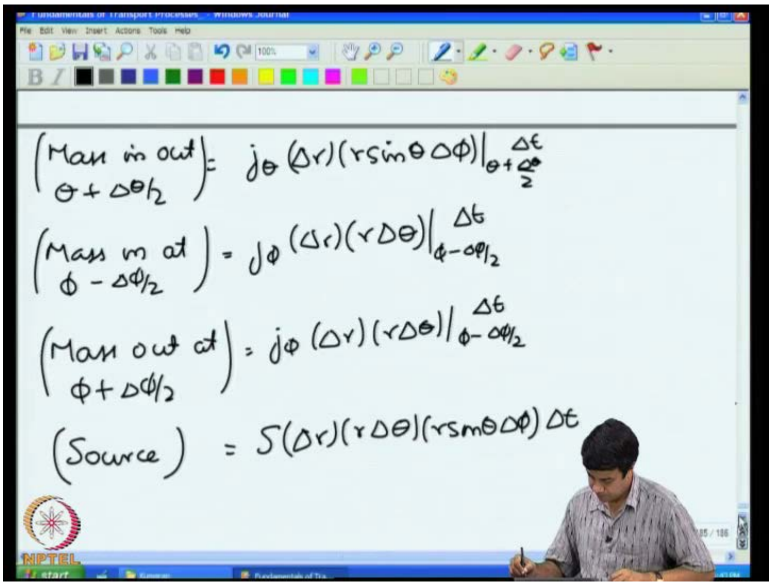

Then I have a mass in at the surface at theta plus delta theta by 2 is equal to I am sorry mass out j theta times delta r times r sine theta delta phi at theta plus delta theta by 2 and then I have two surfaces in the phi direction. That is the front and the back surfaces here; the two surfaces in the phi direction, this surface and the surface at the rear. So, for this the flux is given by at phi minus delta phi by 2 is equal to j phi times delta r times r delta theta and similarly mass out at phi plus delta phi by 2 is equal to j phi times delta r times r delta theta at phi minus delta phi by 2.

So, these are the masses in and out in all the surfaces due to diffusion. As I said the mass due to convection is obtained by just substituting C u r C u theta and C u phi in these expressions. Sources and sinks is equal to, is of the form s times the volume. Delta r r delta theta into r sine theta delta phi times delta t. All of these have to be multiplied by delta t because we are considering it over a differential time interval. So, we put all these together to get the final differential difference equation.

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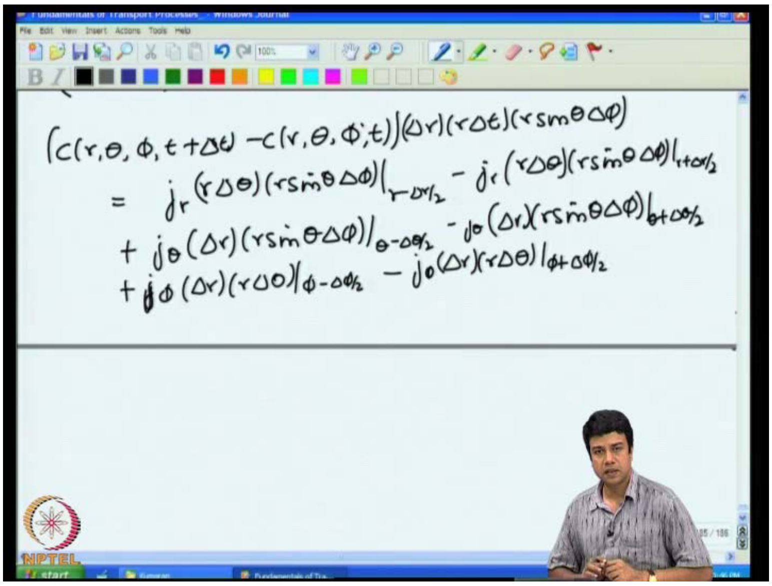

So, this gives me the concentration first, C at r theta phi t plus delta t minus C at r theta phi t into the volume. This is going to be equal to the sum of all the mass in and out due to fluxes.

So, this is going to be equal to the mass in at the surface at r minus delta r by 2 is equal to j r into r delta theta into r sine theta delta phi at r plus r minus minus j r into r delta theta sine theta delta phi at r plus delta r by 2 plus j theta into delta r r sine theta delta phi at theta minus delta theta by 2 minus j theta at delta r into r sine theta delta phi at theta plus delta theta by 2 plus j phi at delta r into r delta theta at phi minus delta phi by 2.

So, these are the terms due to diffusion. I had told you there are additional terms due to convection, the terms due to convection are obtained just by substituting C u r for j r, C u theta for j theta and C u phi for j phi.

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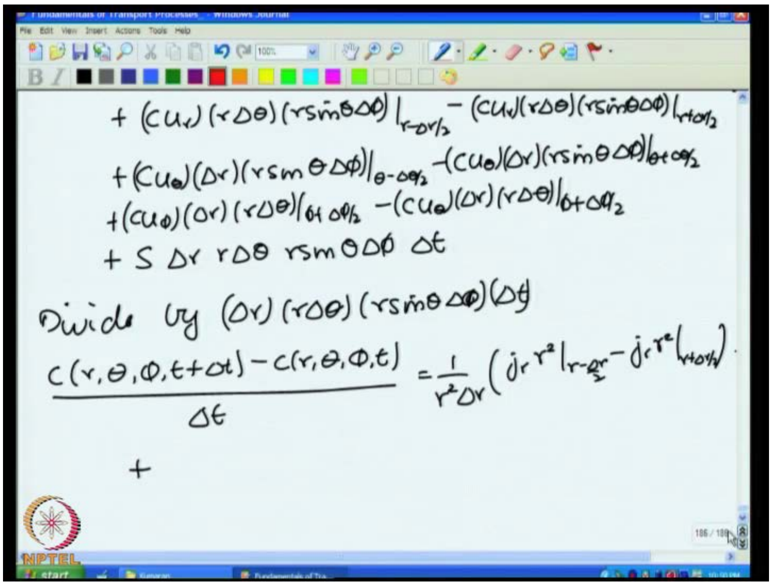

So, I will have convection terms C u r times r delta theta into r sine theta delta phi at r (no audio 48:39 to 49:40)). So, those the terms due to convection and finally, I am going to get a source term plus s delta r r delta theta r sine theta delta phi and delta t. So, those are there is the final expression for the difference equation and in order to get the differential equation as usual I divide by delta t and I divide by volume.

Please note down this equation. So, that when we divide it is obvious to you what you get because it is a long and complicated equation. But once we divide it we will see that it becomes much simpler. Divide by delta r r delta theta into r sine theta delta phi into delta t and what I will get is C at r theta phi t plus delta t minus C at r theta phi t whole thing divided by delta t is equal to. Now, when I divide I have to be careful because the surface area is here are now dependent upon r. The surface area’s here in this case for example, these this area and this area is not evaluated at constant value of r. Both of the surface areas are not evaluated at constant values of r. Therefore, you cannot just divide r on the left hand side and the right hand side.

So, for the j r you can of course, cancel out theta and phi, sine theta can be cancelled out because you are only varying r, you are not varying theta. When you do, the two surfaces at r and r plus delta r. So, the sine theta factor will cancel out delta theta and delta phi will cancel out, but within r I will have 1 by r square delta r into j r times r square at r minus delta r by 2 minus j r times r square at r plus delta r by 2 plus. In the theta direction I will have the surface area in the theta direction goes as delta r times r sine theta delta phi. So, the surface area in the theta direction is going as delta r times r sine theta delta phi.

So, when I divide throughout by delta r or delta theta and r sine theta delta phi, the sine theta factor has to remain because that you taking the the the fluxes at two different values of theta. Therefore, sine theta changes, r does not change and phi does not change. So, I can cancel out delta r and delta phi in these two equations, but sine theta has to be included within the derivative.

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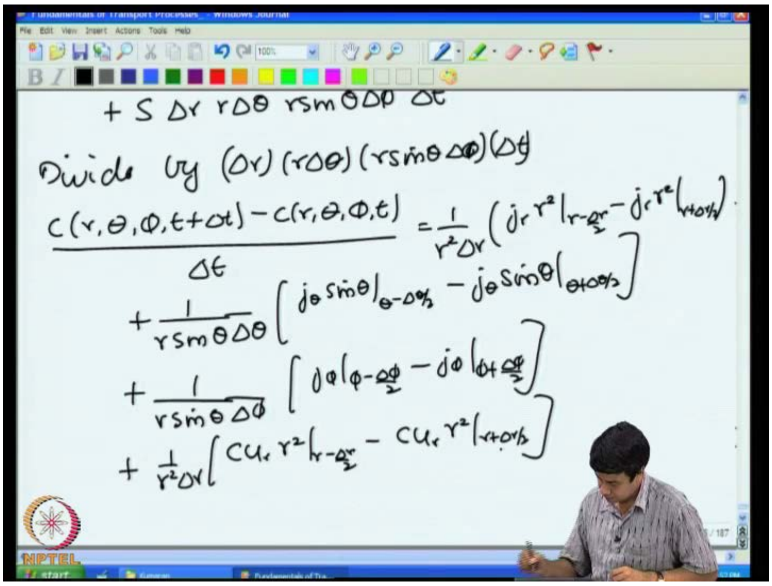

So, that will give me 1 by 1 by r sine theta delta theta into j theta times sine theta at theta minus delta theta by 2 minus j theta times sine theta at theta plus delta theta by 2. So, that was for the variation in the theta direction because the surface area is changing.

Similarly, I will have in the phi direction the areas delta r times r delta theta and therefore, when I divide throughout I will get 1 over r sine theta delta phi into j phi at phi minus phi minus delta phi by 2 minus j phi at phi plus delta phi by 2 and then there are the convective terms. I get the convective terms by just substituting C u r C u theta and C u phi within these equations. So, I will get plus 1 by r square delta r times C u r times r square at r minus delta r by 2 minus C u r times r square at r plus delta r by 2.

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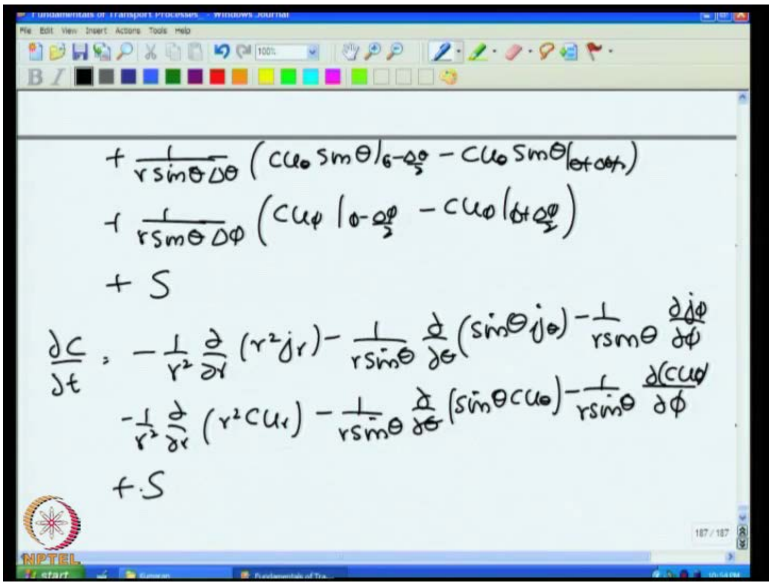

Plus 1 by r sine theta delta theta into C u theta times sine theta at theta minus delta theta by 2 minus C u theta sine theta at theta plus delta theta by 2 plus 1 by r sine theta delta phi of C u phi at phi minus delta phi by 2 minus C u phi at phi plus and finally, I will have just the source term. So, that is the final complete expression for the difference equation.

To get to the differential equation, I have to take the limit delta t delta r delta theta and delta phi going to 0 and if I take that limit it is easy to see that the form of the differential equation that I get is d C by d t is equal to minus 1 by r square d by d r of r square j r minus 1 by r sine theta d by d theta of sine theta j theta minus 1 over r sine theta partial j phi by partial phi minus 1 by r square d by d r of r square C u r minus 1 by r sine theta d by d theta of sine theta C u theta minus 1 by r sine theta partial of C u phi by partial phi plus any sources or sinks that are there.

This is the differential equation in the Cartesian, in the spherical coordinate system so, little complicated as you can see. It is not as simple as the one that we had got in the Cartesian coordinate system. We still have to get our diffusion equation in terms of after writing the fluxes in terms of the concentration gradients. We will continue that in the next lecture, how do we get the diffusion equation from this and then we will look at how we will make use of this in order to get solutions in a spherical coordinate system. Kindly go through this once again. It looks a little complicated at first, but if you go through it one or two times you just recognize what are the, what is the differential volume that I am considering what are the surfaces, the surface areas and then you write the equation as you usually do for a Cartesian coordinate system, flux time surface area. You have to keep in mind that surface area for a curvilinear coordinate system changes as the coordinates change.