Fundamentals of Transport Processes
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Lecture No. # 26
Mass and Energy conservation Cartesian
Co-ordinates Heat conduction cube

Welcome to lecture number 26, in our course on fundamentals of transport processes, where we had started deriving general conservation equations for mass and energy. I told you that we will defer discussion of the momentum conservation equation, because momentum itself is a vector and it can diffuse in three different directions and it makes things complicated.

So, first we consider a Cartesian coordinate system and we derive the mass conservation equation for this Cartesian coordinate system. So, the surfaces, we consider volume whose surfaces are bounded by surfaces of constant coordinate.

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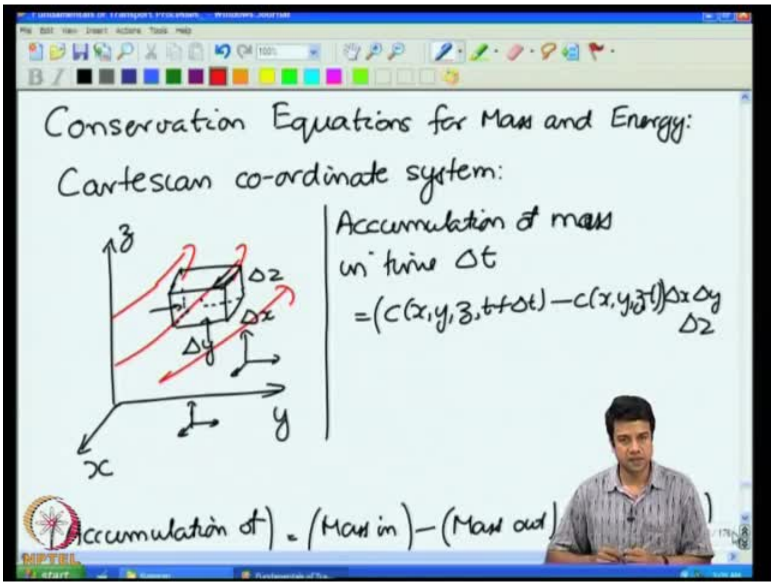

So, in this particular cubic volume, the bottom and top surfaces are at constant values of z, the left and right surfaces are at constant values of y and the front and the back surfaces are the constant values of x. We write a conservation equation, which basically states that the change in mass within a time delta t is equal to what comes in minus, what goes out plus, any production of mass, due to a reaction within this differential volume.

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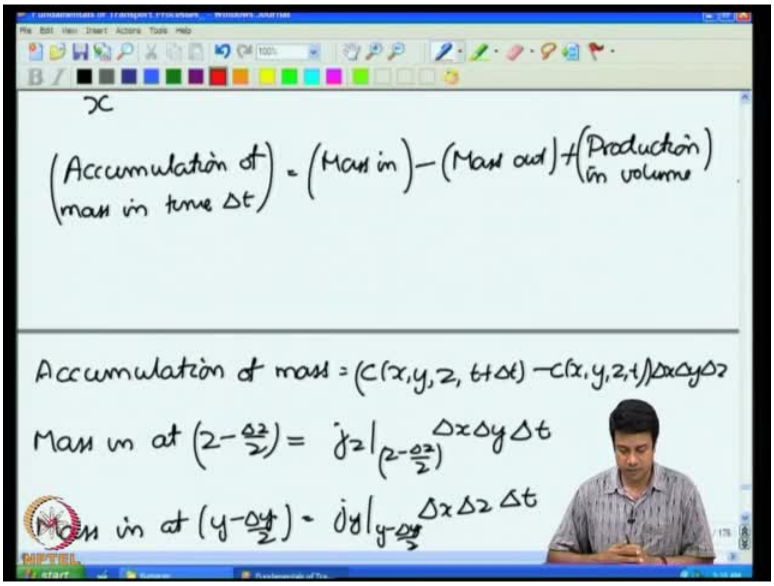

So, this accumulation of mass within this differential volume in a time delta t, we wrote that as the concentration note that the center of this differential volume is at the location x, y and z. Therefore, the accumulation of mass is equal to the concentration at x, y, z at time t plus delta t minus at the concentration at x, y, z, t multiplied by delta x delta y delta z. Note that, when we take the accumulation term, t is changing between t and t plus delta t, but x, y and z are remaining a constant, that is, we are setting at one particular location and space and finding out how the concentration changes in time.

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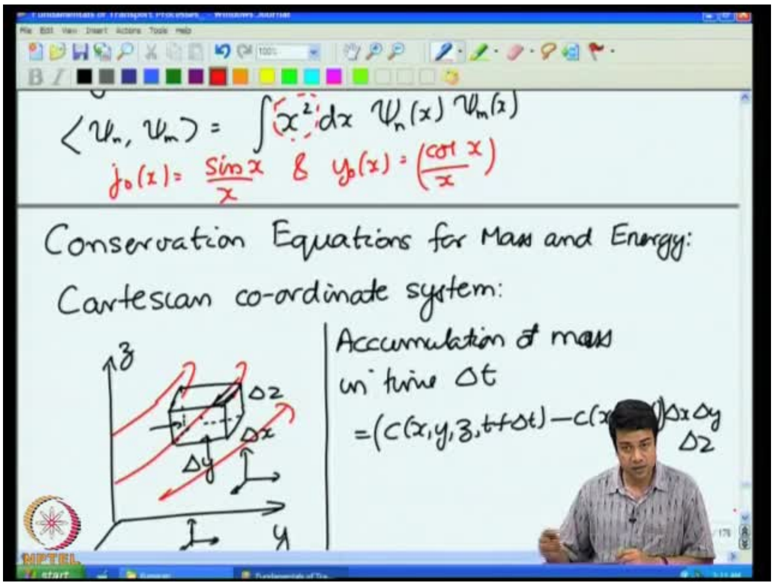

The flux is defined at bounding surfaces. The flux at the surface at z. We considered surfaces at z minus delta z by 2 and plus z delta z by 2. The flux at the surface z minus delta z by 2 is equal to the flux j z, because at the surface z minus delta z by 2, the flux is positive and results in accumulation. If material enters the differential volume, the flux j z is positive in the plus z direction.

Therefore, a flux j z at the surface z minus delta z by 2, results in an accumulation within that volume. Flux j z at z plus delta z by 2 is leaving the differential volume and that result in mass decreasing within this differential volume.

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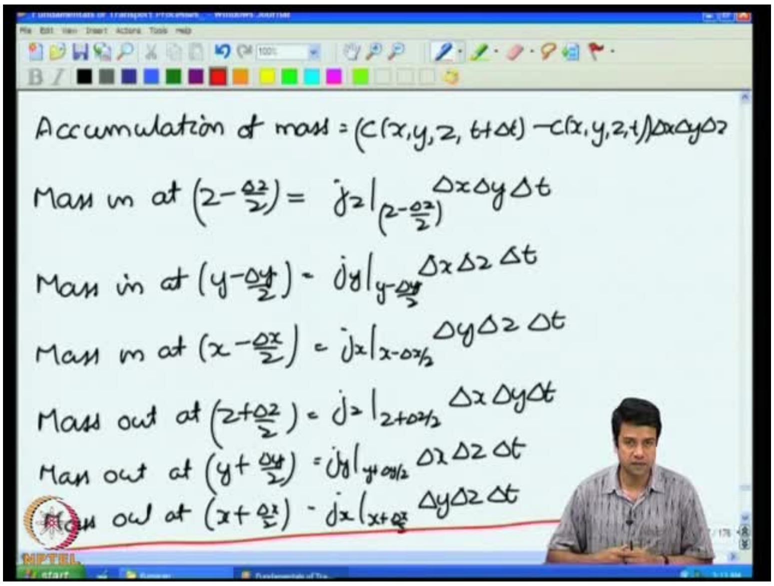

Therefore, the mass in at z minus delta z by 2 is equal to j z times. The area delta x delta y of this bottom surface time is delta t. Similarly, the mass n at y minus delta y by 2 on the left face and x minus delta x by 2 on the rear face is this cubic volume.

What about the mass out? At the surface at z plus delta z by 2, if there is a flux j z in the positive set direction? That results in mass leaving this differential volume. Therefore, that is the mass going out, resulting in a decrease in the mass within the volume. Therefore, the mass out at z delta z by 2 is equal to j z at z plus delta z by 2 times delta x delta y delta t and similarly at y plus delta y by 2 and x plus delta x by 2.

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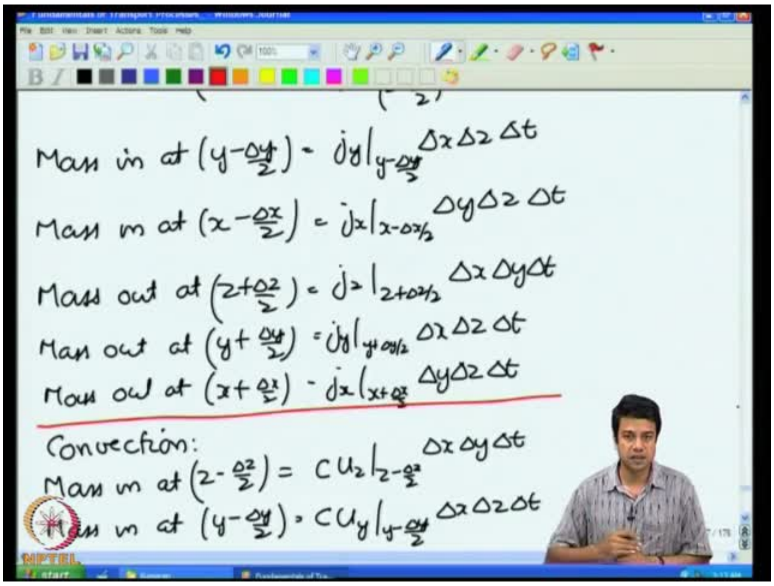

In addition to that, there is also mass coming in and leaving because there is fluid flow. A fluid velocity carries mass with it and therefore, fluid comes into the differential volume. There is going to be a net mass coming into the differential volume and that is the convective transport.

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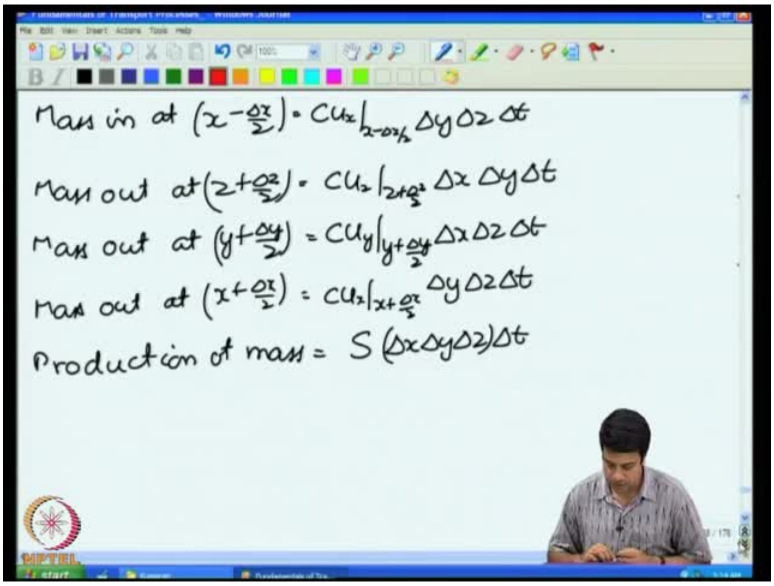

The flux is that I have just calculated are the diffusive parts and are due to diffusion. There is also a convective part, due to the mean convection and the flux, due to convection is the concentration times the velocity. Therefore, the total mass n is going to be concentration times, velocity times. The area and the time interval and we have exactly analogous contributions, due to convection surfaces at z minus delta z by 2 x minus delta x by 2 and y minus delta y by 2. There is mass coming in, due to convection and at the other surfaces, at x plus delta x by 2 y plus delta y by 2 and z plus delta z by 2, there is mass leaving due to diffusion.

In addition, there could be some production of mass that is going to be equal to S, which is the production per unit volume per unit time times delta t. For example, the production of mass due to a reaction d c a by d t is equal to minus k c a, if it is first order. That production is the production rate of increase of concentration with time. So that is the production.

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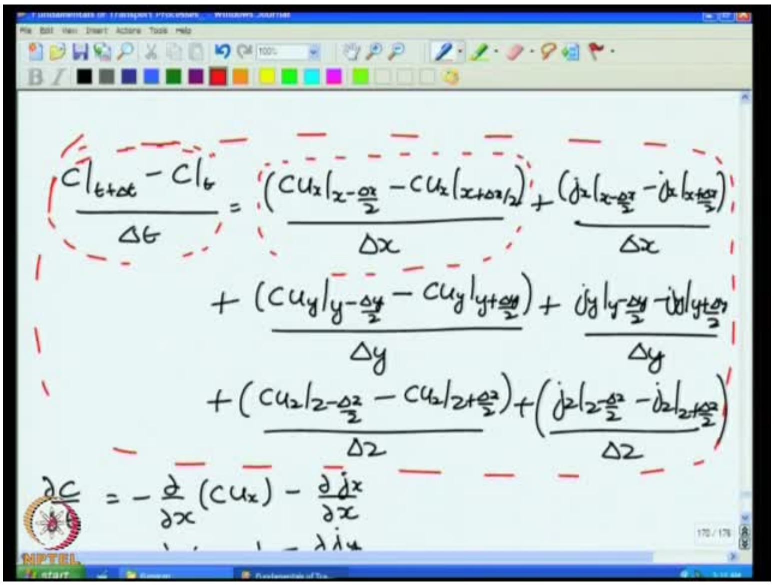

Put all of these together and divide by volume and time in order to get, what is called a difference equation? A difference equation, this entire equation here for you is the difference equation basically and contains, first of all, change in concentration between two time intervals at the same location.

The difference in concentration times u x at x plus delta x by 2 minus, the difference in concentration times u x at x minus delta x by 2 minus x plus delta x by 2 divided by delta x, that is, when I take the concentration at x minus delta x by 2 at the rear face and finding out the average concentration times of velocity; that is I am taking it at the location y and z, because that is the center point of the rear face times x minus delta x z by 2.

So, in this difference term, the y and z coordinates have been kept a constant. Only x is varying and so on, with the other terms, so when you take the limit of delta x, delta y, delta z going to 0, you get a partial differential equation, I have written out that partial differential equation for you here.

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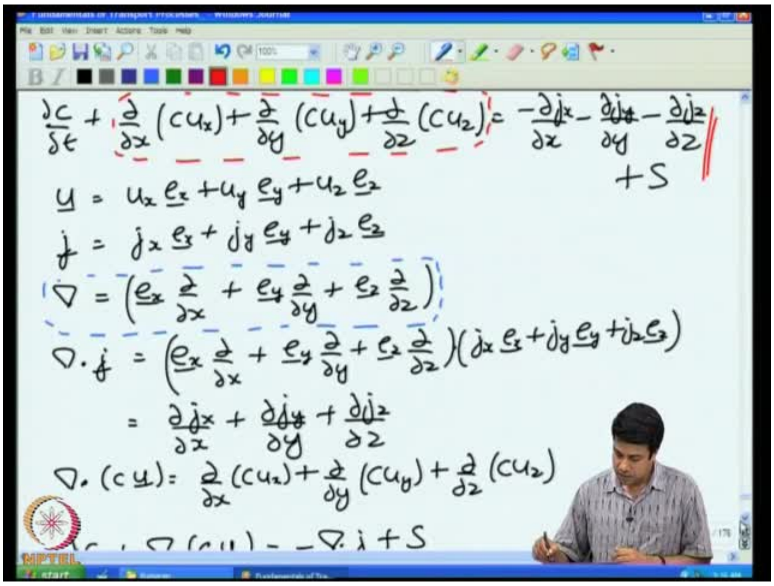

The partial derivative with respect to x implies that y z and t are constant partial derivative with respect to y implies that x z and t are constant and the partial derivative with respect to z implies that x y and t are constant the partial time derivative implies that x y and z are constants same location.

So, this is the partial differential equation I showed you. How to write that in a more compact form using vector notation? Velocity is of course a vector. It has 3 components and 3 unit vectors. The flux is also a vector. The flux in the x direction is equal to d times the derivative of x of concentration in the x direction. So, from the Fick’s law along the x direction, one can also have concentration variations in the y and z directions. Therefore, the flux is also a vector. It has contributions in the x, y and z directions and the terms in the equation I showed you can be written as del dot c u, where we use the vector and minus del dot j, where j is a flux vector, where the gradient operator is e x times d by d x plus e y d by d y plus e z times d by d z.

So, I can write it compactly in this form. The equations are actually independent of coordinate systems. One can find equivalent descriptions in both the cylindrical and spherical coordinate systems, in which these terms produce to exactly the same form. So, one can find alternative descriptions in which these terms are written this way that they reduce exactly to the same form. We will look at how to get these derivations, so that we can get these terms into exactly this form.

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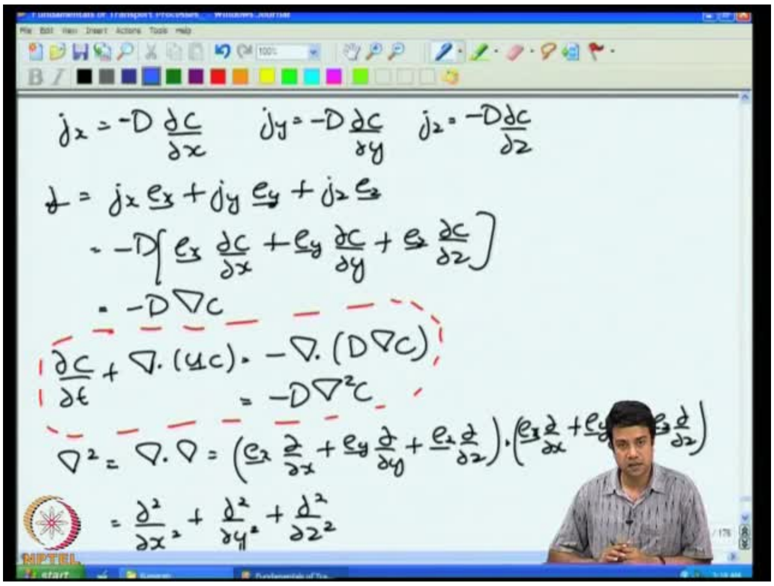

The components of the flux, I wrote in terms of the variation of concentration with respect to the 3 coordinates using Fick's law of diffusion and once you do that, you get an equation shown in red there which contains the Laplacian operator del square. The Laplacian operator del square is d square by d x square plus d square by d y square plus d square by d z square. So, it contains the Laplacian operator.

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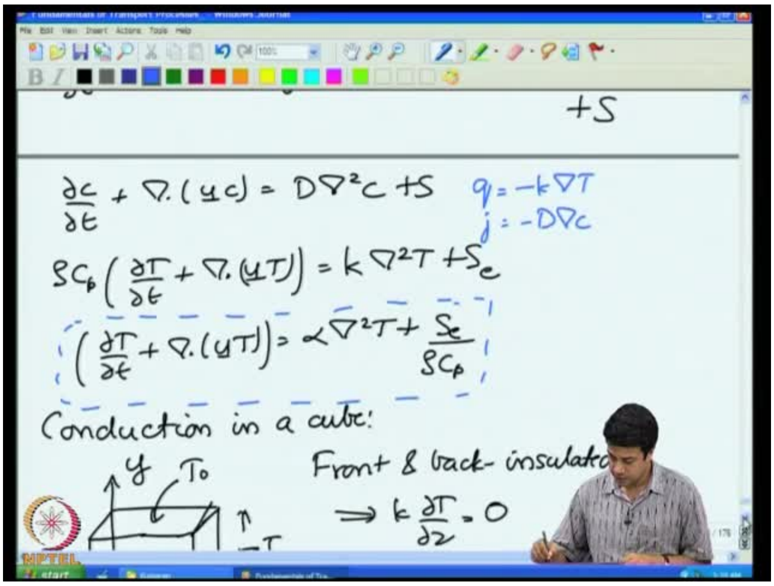

Finally, I can write the concentration equation in this form in a Cartesian coordinate system. This is valid for any geometry in Cartesian coordinates and the energy conservation equation can be written in a similar form. The heat flux, in Cartesian coordinates can be written as q is equal to minus k grad t analogous to j is equal to minus d grad c for the mass flux, where k is ethanol conductivity, q is the energy flux energy per unit area per unit time and using that is the Fourier's law for heat diffusion; using that I can get this temperature equation. I just substitute the temperature instead of concentration and the thermal diffusivity instead of the mass diffusivity.

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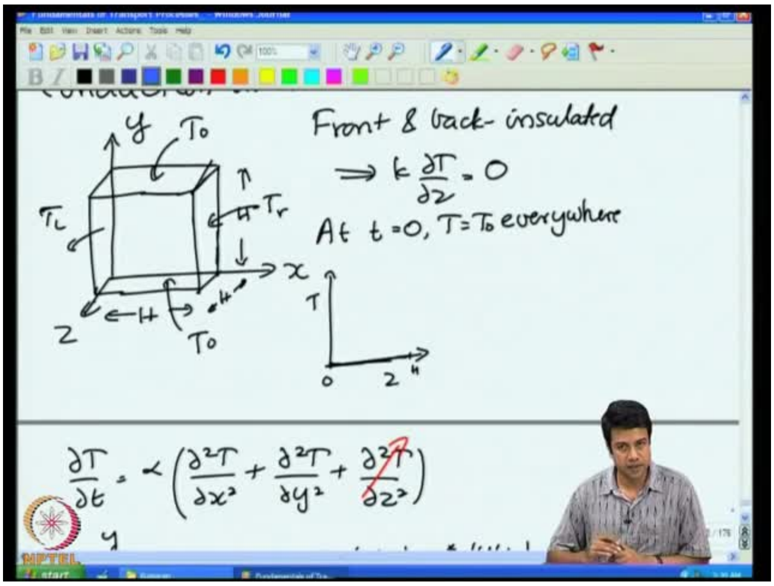

So, these are general equations and we started applying them for a specific case of the conduction in a cubic volume. So, I have a cubic volume in which the front and back faces have 0 flux, the top and bottom faces are temperature T naught, the left faces at temperature T L, the right faces at temperature T r. Exactly at t is equal to 0, the cube was prepared in such a way that the temperature within the cube everywhere was equal to T naught and at time t is equal to 0, I imposed the temperature T L at the left face, the temperature T r at the right face and I want to find out, how the temperature varies both with position and with time.

I told you that since, there is insulation in the front and the back, there is no temperature variation in the front and the back. Therefore, you would expect that there is no variations along the z coordinate, because the temperature that is independent of z, will satisfy the equation. It will also satisfy the boundary condition, that is, d t d z has to be equal to 0 at the front and the back.

Note that this equation is a linear partial differential equation. For a linear equation, as provided, you have consistent boundary conditions. It is guaranteed that a solution exists and it is unique. So, if I can find a solution, which satisfies the boundary conditions for this linear equation, then I know that is the only solution.

In this particular case, if I postulate that the temperature is independent of z, that solution identically satisfies the flux conditions at the front and the back. Therefore, a temperature there is independent of z, satisfies the differential equation. It satisfies the boundary conditions as well. Therefore, it is a valid solution and since the equation is linear, I know that is the only solution. I have 0 flux conditions at the front and the back; you will find that there is no variation in that direction.

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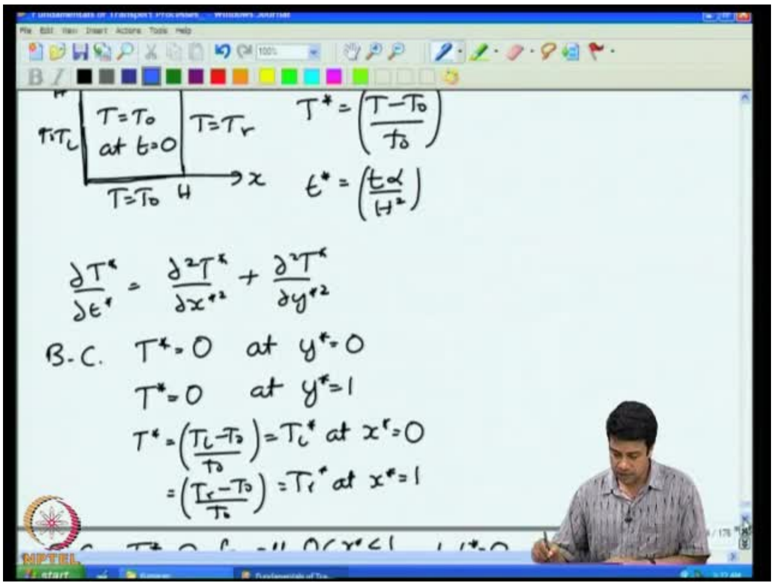

So, this reduces to a 2 dimensional problem in the x and y coordinates and a non-steady problem in time and we started looking at the procedure to solve this problem. First things first, we scaled the x and y coordinates by H, because that was the length scale. We defined a scale temperature T minus T naught by T naught and a scale time and we got the differential equation. Second order in x and y, first order in time requires 2 boundary conditions and 1 initial condition. The boundary conditions were t is equal to 0 at both the top and bottom faces because T star is equal to T minus T naught by T naught.

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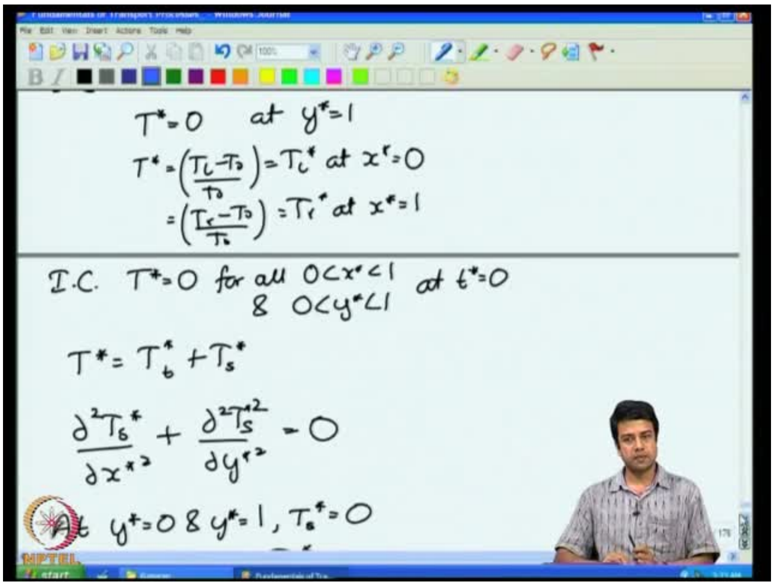

So, that has to be equal to 0 at the bottom and the top faces on the left and the right faces, it is not 0; you have a non-zero temperature at both the left and the right faces. And at initial time T star is equal to 0. Everywhere throughout the domain at the initial time you just applied the 2 temperatures on the left and the right. Initially the temperature everywhere was equal to 0. So, this is the problem that we would like to solve.

So, first things first, we need to find out what is the steady solution for the temperature field in this geometry, because in the long time limit, we would expect that the temperature converges to final steady value and it is that steady value that we should try to find out first. After that, we can go and find out, what is the transient part. At initial time, the difference between the actual temperature and the steady temperature is the transient part of the temperature.

So, further steady temperature as set that time to equal to 0 and I just get a partial differential equation in x and y coordinates. The boundary conditions for the steady temperature profile are identical to the boundary conditions for the original temperature profile, because I am keeping the temperature at constant 0 on the top and bottom, T L on the left and T r on the right. Therefore, the temperature even the final steady state has to have exactly the same boundary conditions.

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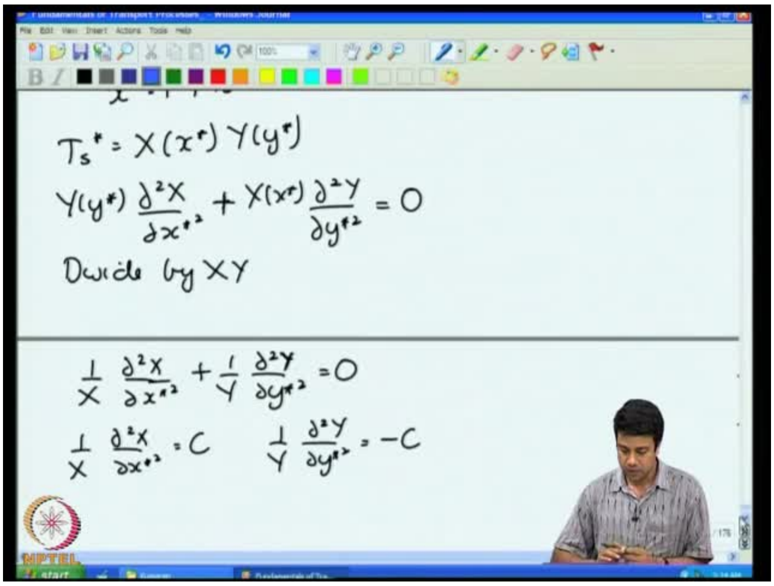

We started off by doing separation of variables T s is equal to a function of x times of function of y, substituted that into the equation, divided by x y and we got an equation, which contain 2 terms; one is only a function of x the other is only a function of y. That means that each of these functions individually has to be a constant.

And some of these two constants has to be equal to 0 to satisfy this differential equation. Therefore, if one over X d square x by d x square is equal to c, then one over Y times d square y by d y star square has to be equal to minus c, Should this constant be positive or negative, that is, where we had left of in the last lecture.

How does one decide, whether this constant has to be positive or negative? If you recall, when we discussed separation of variables for the transient problem, we had homogenous boundary conditions in the 2 spatial directions and there was an initial condition for the transient problem, which was not homogenous. There was a first thing at the initial time. Since the boundary conditions were homogeneous, the spatial coordinate we took the constant to be negative for that spatial operator. So, only if you take a negative constant that you get sin and cosine functions and you can satisfy homogenous boundary conditions on 2 surfaces, if you take the constant to be positive, then you get exponential solutions and you cannot satisfy boundary conditions on the 2 surfaces. So therefore, we have to take the constant to be negative in the direction were the boundary condition is homogenous.

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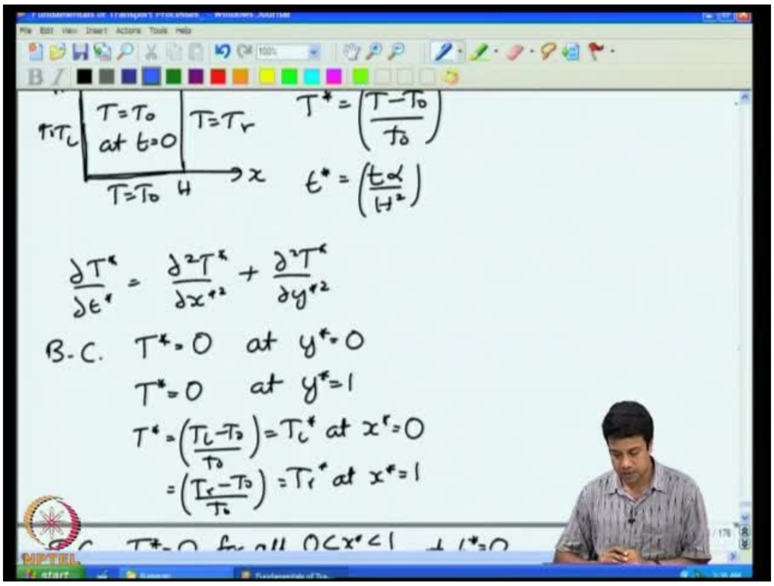

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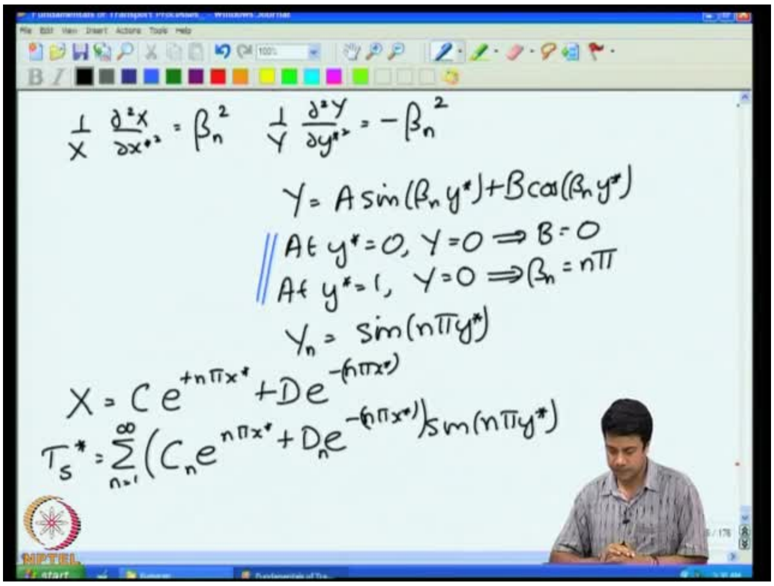

Coming back to the boundary conditions, here you can see that the boundary conditions are homogenous in the y direction, T star is equal to 0 at y equal to 0 and T star is equal to 0 at y is equal to 1; this is the homogenous direction. Therefore, you have to get sin and cosine solutions in this direction and therefore, this constant c has to be a positive constant, so that you can get homogenous boundary conditions in that direction.

So, let us call this c as some beta n square. This constant c is equal to some beta n square, some and the solution of this Y is equal to A sin of beta n y star plus B cos of beta n star y star, if beta n is if one over Y d square y by d y star square is equal to minus beta n square; then this is the solution.

I require that at y star is equal to 0, capital Y is equal to 0, which means that B has to be equal to 0. So, it is only a sin function and at y star is equal to 1 capital Y is equal to 0 capital Y will be 0 either. If A is equal to 0 or if beta n is equal to n times pi A cannot be 0 because A 0 and B is equal to 0. The solution is y is equal to 0 everywhere. So, therefore, A cannot be 0; therefore, beta n has got to be equal to n times pi.

So, therefore, the solution for Y out of the form Y n is equal to sin n pi y star exactly the same that we got for the separation of variable for the unsteady unidirectional transport problem, that is, because this solution is a property of this operator of this particular operator. So, this is the solution for Y, what about the solution for X?

In this case, d square x by d x square is equal to beta n square, which means that capital X is equal to some constant C e power plus n pi x plus D e power minus n pi x, where C and D are constants and therefore the steady solution T s star is equal to k times sin of n pi y.

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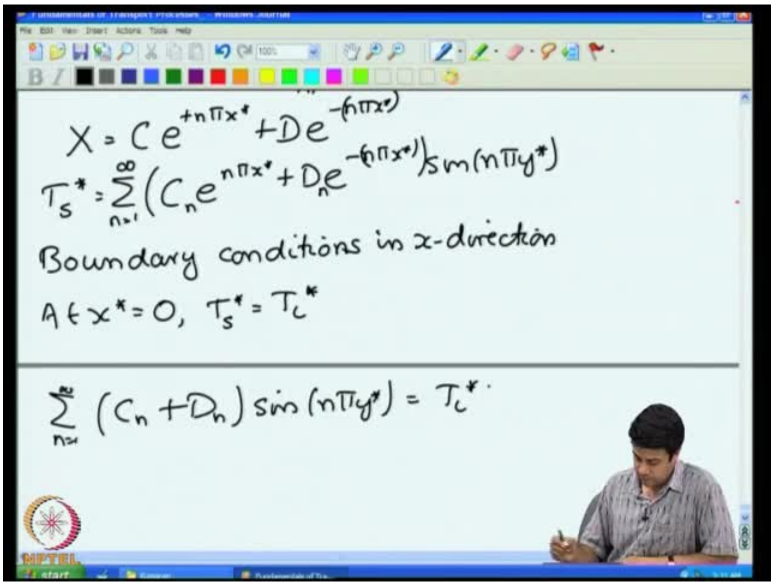

So, if any integer value of n satisfies the boundary conditions in the y direction, it means that the most general solution is the solution is the summation. Over all possible values of n, where C n and D n are unknown coefficients, we have enforced boundary conditions in the y direction; we not yet enforced boundary condition in y direction, what are the boundary conditions in the x directions.

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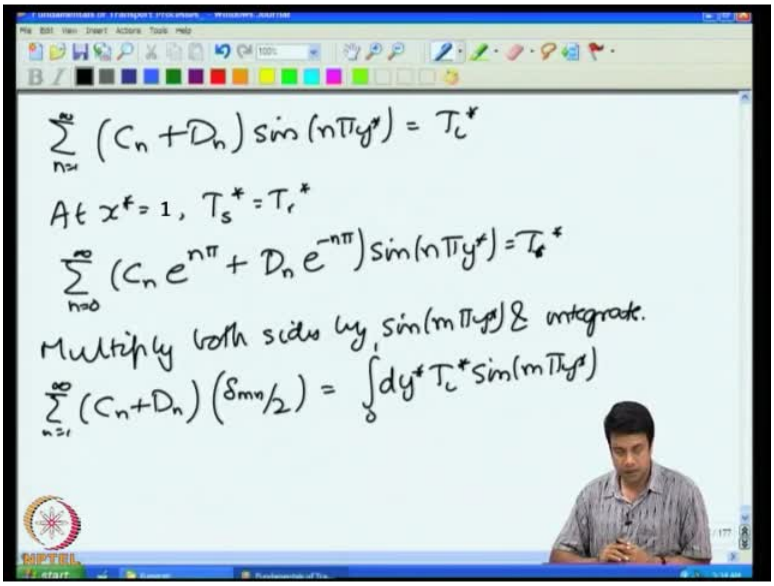

The boundary conditions at x is equal to 0, t is equal to T L. This implies that summation n is equal to 1 to infinity of C n plus D n sin n pi y. The other boundary condition is that at x star is equal to L T s, which implies summation. So, from these two boundary conditions, we have to evaluate the constants C n and D n. How do we do that? We use as before, the orthogonality relations, as we have done earlier. We will use the orthogonality relations; multiply both sides by sin m pi by star and integrate from 0 to 1.

If I will get summation n is equal to 1 to infinity C n plus D n times integral of sin n pi y times sin n pi y, which is basically going to end up being delta m n by 2 is equal to integral d y T L sin m pi. Note that, if I multiply by sin n pi y and integrate from 0 to 1 on the left hand side, orthogonality relation, I will get m n by 2.

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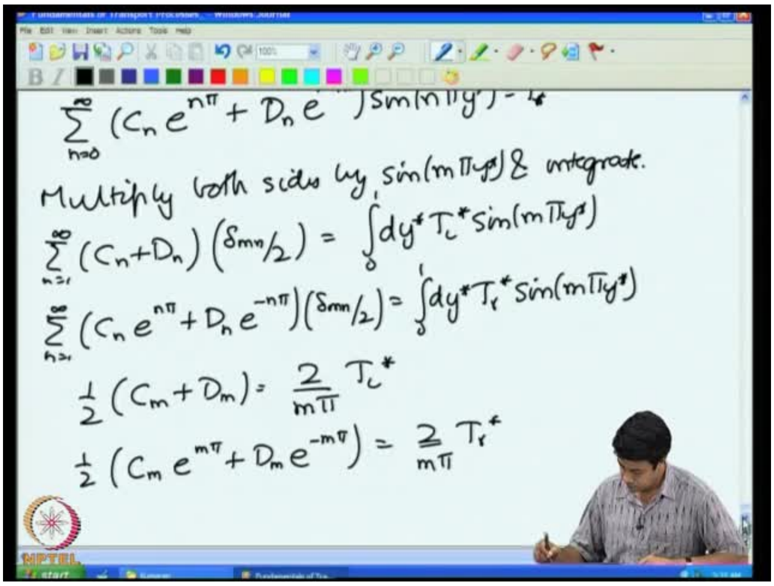

I can do that for the second equation. Summation n is equal to 1 to infinity of C n e power n pi plus D n e power minus n pi into delta m n by 2 is equal to integral 0 to 1 times T r sin m pi y star.

Now, delta m n is 1, if m is equal to n and it is 0 otherwise. So, summation n is equal to 1 to infinity of delta m n times C n plus D n is going to be equal to half Cm plus D m is equal to 2 by m pi times T L and half of C m e power m pi plus D m e power minus m pi is equal to 2 by m pi into T r. And these are two simultaneous equation, which I can now solve in order to get the constant C n and D n and the values of the constant C n and D n turn out to be equal to..

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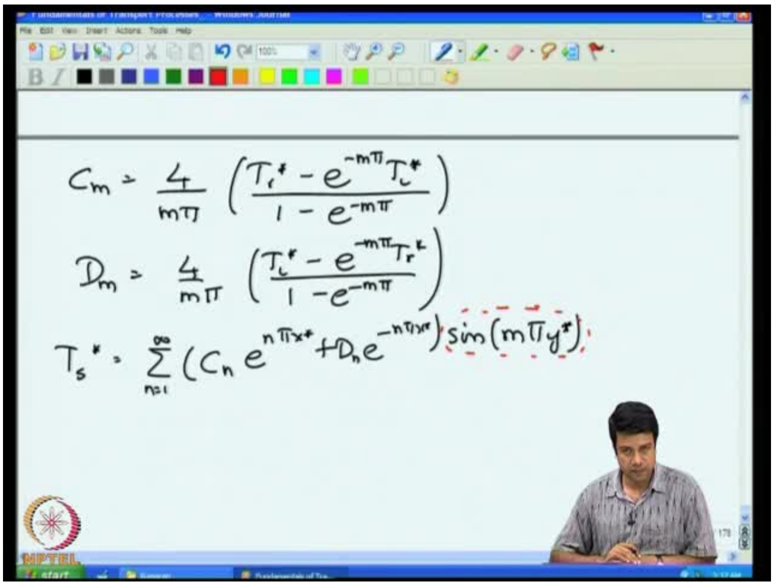

C m is equal to 4 by m pi T r minus e power minus m pi T L write 1 minus e power minus m pi and D m T L minus. So, with these, we get the final solution of the steady equation, T s is equal to summation of n is equal to 1 to infinity of C n e power n pi x plus D n e power minus n pi x sin.

So, crucial to the solution was actually identifying the region, where direction in which we have homogenous boundary conditions. Once we have identified the direction in which have homogenous boundary conditions, we know that the solution has to be sin and cosine functions in that particular direction. The other direction of course, it will be exponentially increasing or decreasing and the constants can be determined from the inhomogenous terms for the exponentially increasing and decreasing functions.

So, that is the basic idea of how we extend the solution for separation of variables to 2 dimensions from 1 dimension. So far, we have got only this steady part of the solution how about the unsteady part of the solution. So, for the unsteady part, we will be using separation of variables once again, but in this particular case, whenever we do separation of variables, we have to ensure that there is only 1 inhomogenous direction that all other directions are homogenous, then we will get eigen values in basis functions in all of those directions, then 1 inhomogenous directions where the initial conditions or the boundary condition will end of forcing the profile.

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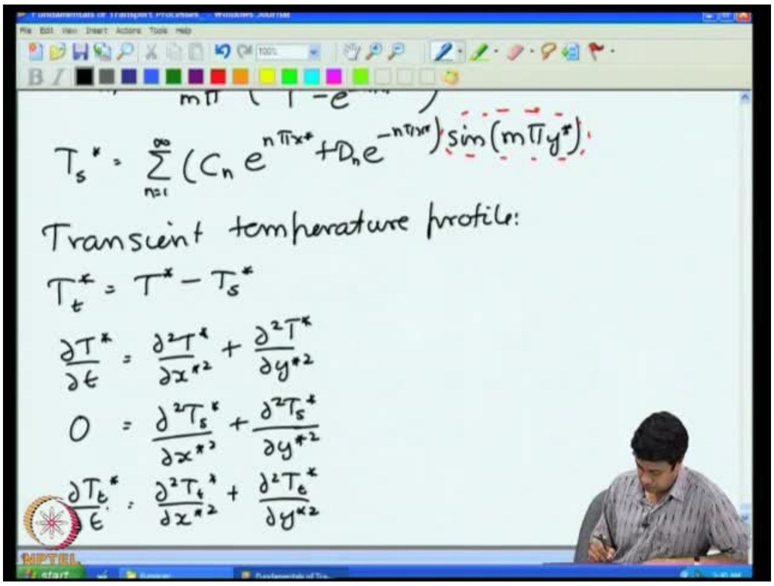

So, the transient temperature was defined as the actual temperature minus the steady temperature. The actual temperature satisfies the equation. This was the equation that was satisfied by the actual temperature, the steady temperature satisfied the equation d square T steady. So, we subtract the two and use the fact that the steady temperature has no time dependence. Again, you will get the equation for the transient temperature. So, this is the equation for the transient temperature profile.

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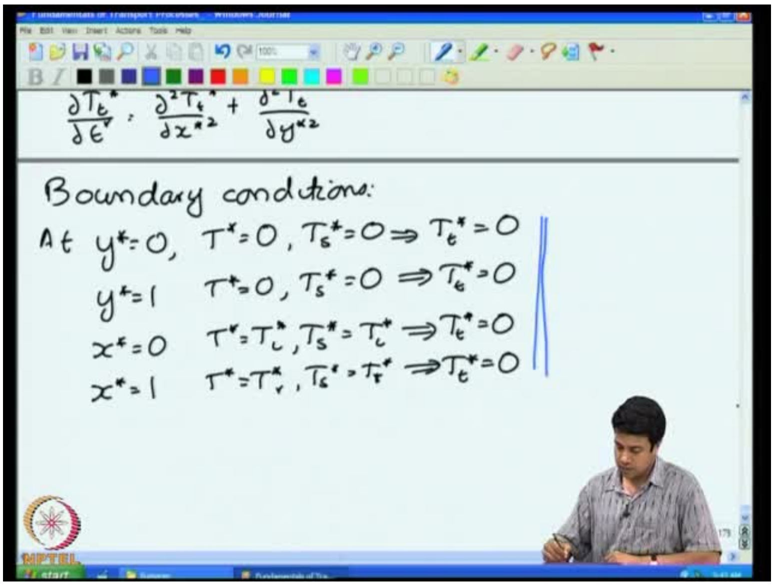

How about the boundary condition? These are the important boundary conditions. Y star is equal to 0, T star is equal to 0, and the steady temperature was also equal to 0, which means that the transient temperature, the difference between these two is equal to 0, y star is equal to 1. The temperature is once again equal to 0, the steady temperature is also equal to 0, which means that the transient temperature is equal to 0, differences between these two. At x star is equal to 0, T star was equal to the temperature on the left. The steady temperature was also equal to the temperature on the left. Note that - we apply those same boundary conditions for the steady part as for the total temperature field, which means that the transient temperature is equal to 0.

Similarly, on the right side, and the steady part is also equal to the temperature on the right hand side. It is implied that the transient part is equal to 0. So, for the transient part alone in both the spatial coordinates, I am getting homogenous boundary conditions for both the spatial coordinates, you end up with homogenous boundary conditions.

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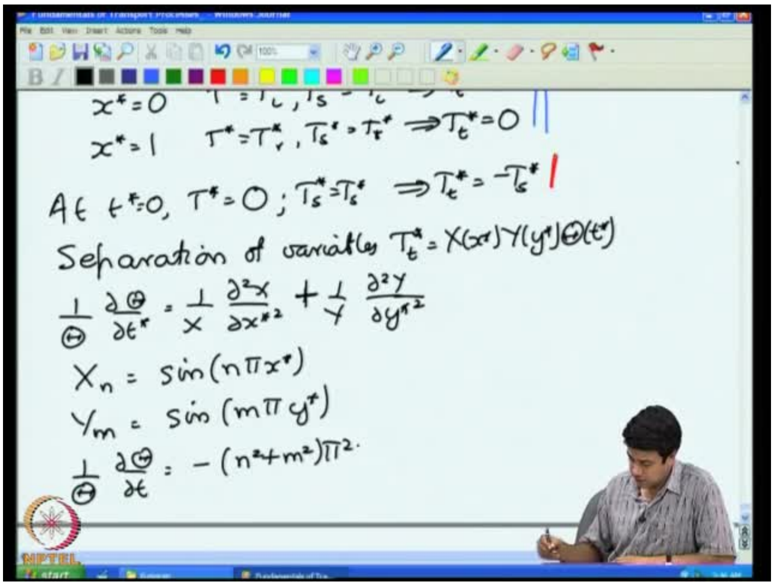

What is not homogenous is the initial condition, at t star is equal to 0. This temperature was equal to 0 initially because the dimensional temperature t was equal to T naught. The steady temperature is independent of time. Therefore, the transient temperature is a difference between these two. This is going to be equal to minus T s. So, the forcing is coming in at the initial time for this particular equation.

So, separation of variables T star transient is equal to some function of X, some function of Y, some function of time. It is an initial function of time and we shall call as theta of t. Put that into the equation, simplify, divide throughout by x, theta and you end up with an equation of the form 1 over theta d theta by d t is equal to one over X d square x by d x square plus one by Y d square y by d y square. So, I have an equation in which the left hand side is only a function of time. The right hand contains 2 terms, one of which is only a function of X, the other is only a function of Y.

So, have the equation whether left hand side is only a function of time, in the right hand side. The first term is only a function of X and the second term is only a function of Y. That means that each of these individually have to be constants. What should those constants be? Clearly, in both the x and y directions, I have homogenous boundary conditions, d star for the transient part is identically equal to 0 left, right, top and bottom. Therefore, the constants for both of these directions have to be negative.

So, constants for both of these directions have to be negative. We also know what these constants should be. If the constant is negative, the solution is in the form of sin and cosine functions. Clearly, the cos function does not satisfy the condition that the temperature is 0 at x is equal to 0 or y is equal to 0. Therefore, the only solution is the sin function in order to satisfy the boundary conditions at x is equal to 1 and y is equal to 1. I have to have a sin of n pi times x star. So therefore, the solutions that I get for x and y are of the form X n is equal to sin n pi x and Y n is equal to sin. Note that - the integers that I use for x and y, could in general be different. I should have Y m is equal to sin of m pi y.

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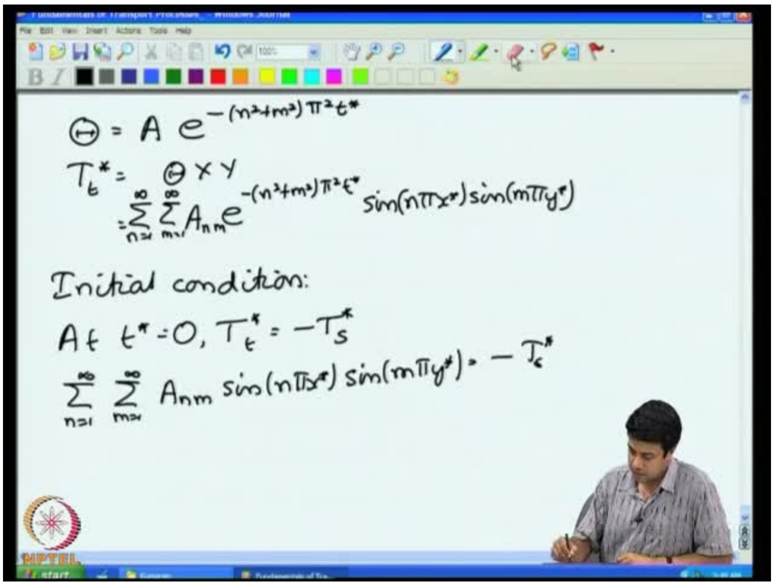

So, if X n is of the form sin of n pi x and Y m is of the form m pi y, that means, my solution for theta has to satisfy 1 over theta d theta by d t is equal to minus of n square plus m square times pi square, which means that theta is equal to some constant. Let us, call it as A e power minus n square plus m square pi square t star. So, that is the final solution for theta. So, finally the transient part of the temperature is equal to theta times X times Y is equal to A e power minus n square plus m square pi square t star sin of n pi x sin of m pi y. So, this solution for any value of n and m satisfies the equation. That means, that the most general solution is the summation over all values of n as well as over all values of m. So, the summation n is equal 1 to infinity summation m is equal to 1 to infinity of A n m, A n m is the coefficient and then I have an exponential term and 2 functions, one basis function for the x direction and other, basis function for the y directions. So, I have two basic functions here, one along the x direction and other along the y direction, with two eigen values, n pi for the x direction, and m pi for the y direction.

How do I determine the constants? You determine that from the initial condition. You determine the constant from the initial condition, that is, that at t star is equal to 0 the temperature should be minus the steady state temperature. Therefore, that means that summation n is equal to 1 to infinity m is equal to 1 to infinity at t star is equal to 0 the exponential is just 1. So, the exponential part is just 1. So, I just have A n m sin n pi x star sin m pi y star is equal to minus the steady state temperature. Note that - the steady state temperature is the function of both x and y until the steady state temperature is the function of both x and y.

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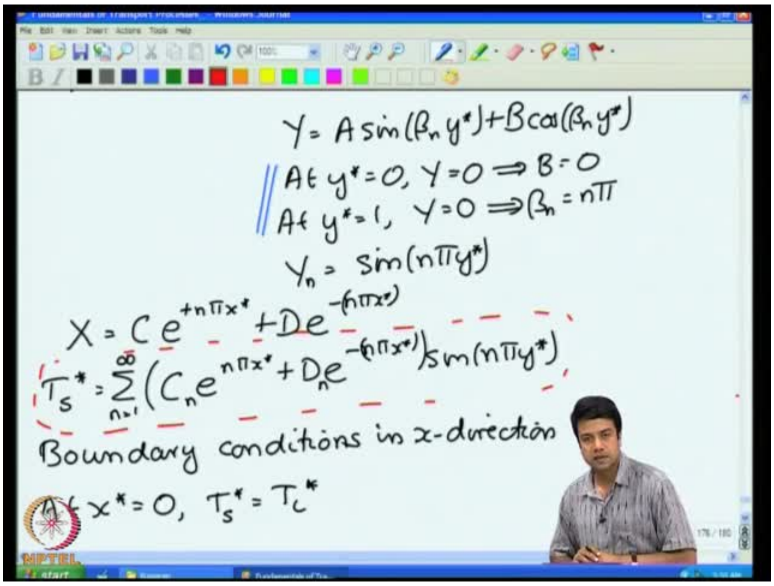

I had got for you the solution for steady state temperature earlier. So, this was the steady state temperature that we had got earlier. It is the functions both of x and y. It has that is the sin function in the y coordinate and exponential in the x direction.

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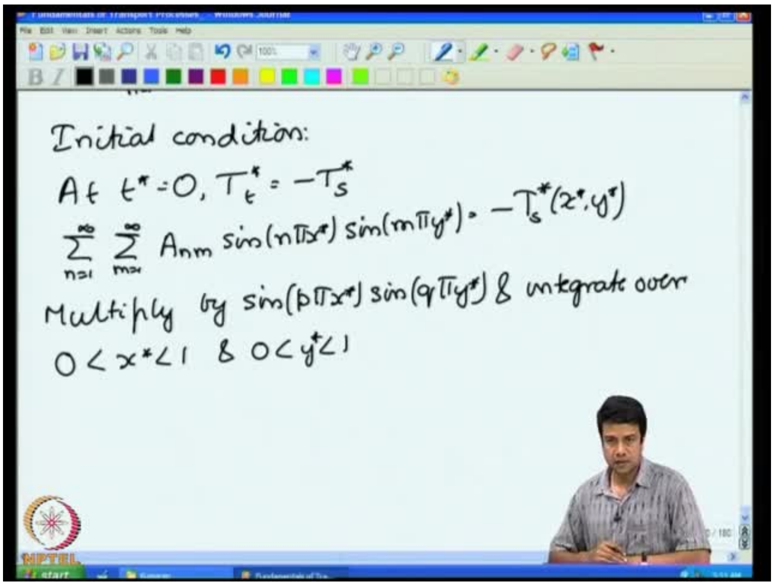

So, this is the function of both x and y. So, this is the equation I have to solve and the way you solve that is to use orthogonality conditions. Simultaneously in both the x and the y directions, you see orthogonality conditions. Simultaneously, in both the x and y directions, I multiply by sin p pi x times sin q pi y, where p and q are both integers multiplied by one sin function in the x direction, one sin function in the y direction and integrate over 0 less than x less than 1 and 0 less than y less than 1, that is, I multiply by sin p pi x sin q pi y, integrate over both the x and the y coordinates from 0 to 1.

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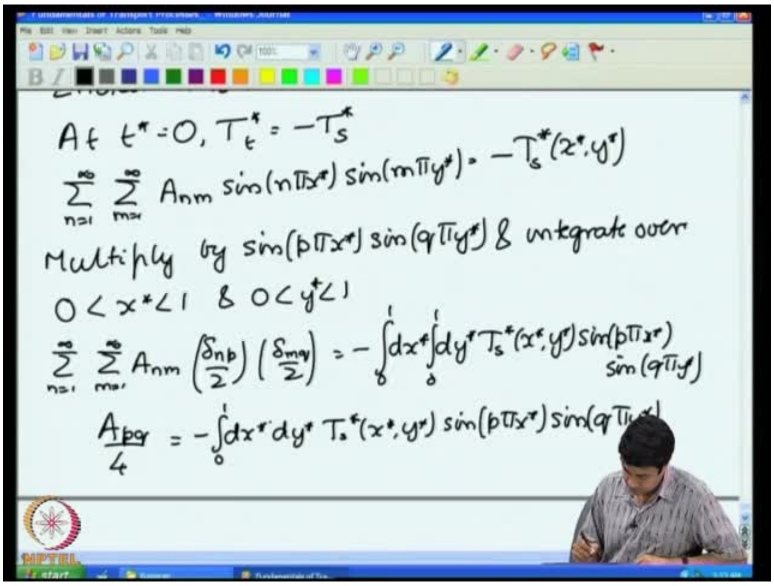

If I do that on the left hand side, integral of sin n pi x times sin p pi x from x is equal to 0 to 1 will basically gave me times sin n pi x times sin p pi x from 0 to one, give me delta n p by 2 sin of m pi y times sin of q pi y will give me delta n q by 2 is equal to minus integral d x integral d y T s of x and y sin p pi x sin q pi y have summation from n is equal to 1 to infinity of A n m delta n pi by 2 summation 1 is equal to infinity of A n m times delta of m q by two.

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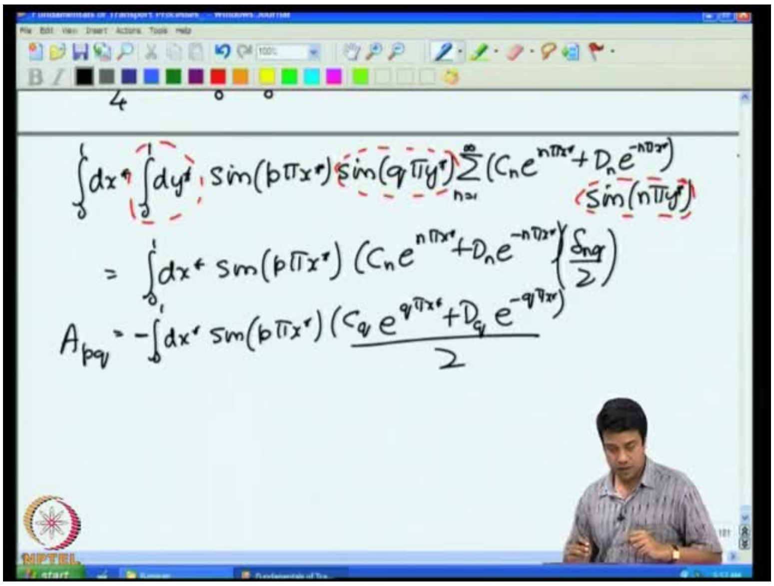

So, this is just going to be equal to A p q by 4, because delta n p is non-zero only when n is equal to p and delta m q is non-zero only when m is equal to q. So, i just get A p q by 4 is equal to minus integral d x d y T s of x y sin p pi x sin of q pi y and both of these are from 0 to 1 and if I look at this integral 0 to 1 d x integral 0 to 1 d y of sin of p pi x times sin q pi y times the temperature solution. If you recall the temperature solution was summation n is equal to 1 to infinity of C n e power n pi x plus D n e power minus n pi x sin into sin of n pi y.

Now, sin n pi y times sin q pi y will be equal to delta n q by 2. So, this in the product of these two, this one times this one integrated over this is just the whole orthogonality relation that I had. So, this is just going to be equal to integral 0 to 1 d x sin p pi x into C n e power n pi x plus D n e power minus n pi x into delta n q by 2.

So, this is going to be equal to integral 0 to 1 d x sin p pi x into C q e power q pi x plus D q e power minus q pi x whole divided by 2. So, i will just need to carry out this integration in order to evaluate the actual value of the constant. So, this A p q is equal to minus of this and I can evaluate the value of each constant these. Once I have evaluated this constants I, now have the solution for the transient part of the temperature profile.

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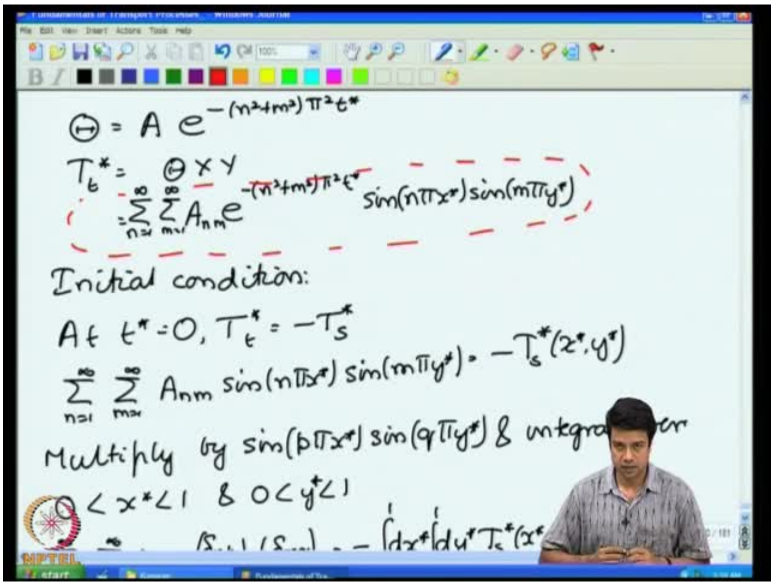

So, this the solution for the transient part of the temperature profile in which I have evaluated the constants using separation of variables. I have the steady temperature profile as well and in that case as well, I have evaluated constant using separation of variables and therefore, I have an analytical solution for the entire temperature profile.

So, this is how one solves separation of variables problems involving more than one spatial coordinate, as well as unsteady separation of variables first things first. We have to find out, what is the steady solution that is important because we have to recover the steady solution in the limit as time goes to infinity. That means that I have to separate out the temperature into a steady and a transient part that transient part has to decrease to 0 in the long time limit that so that I recover that steady solution in the long time level.

So first, I have to find the steady solution that steady solution is obtained by solving the second order differential equation in 2 coordinates. In this particular case, we had homogenous boundary conditions in 1 coordinate, which is in the y direction. We had homogenous boundary conditions for the steady problem and therefore, we got an Eigen value problem in that y direction in which the Eigen values for n pi the basic functions for sin n pi y.

There was an inhomogenous direction; the x direction whether the temperature on the left face was not 0 the temperature on the right face was not 0 either for that transient solution for that inhomogenous direction. We manage to obtain a solution basically in the form of exponentials one exponentially decreasing the other, exponentially increasing and the final solution was a series expansion with the basics functions in the y direction as well as the exponentially increasing and decreasing functions in the x directions and then I showed you how to use similarity variable, how to use the orthogonality relation in order to obtain the value of that solution in that direction.

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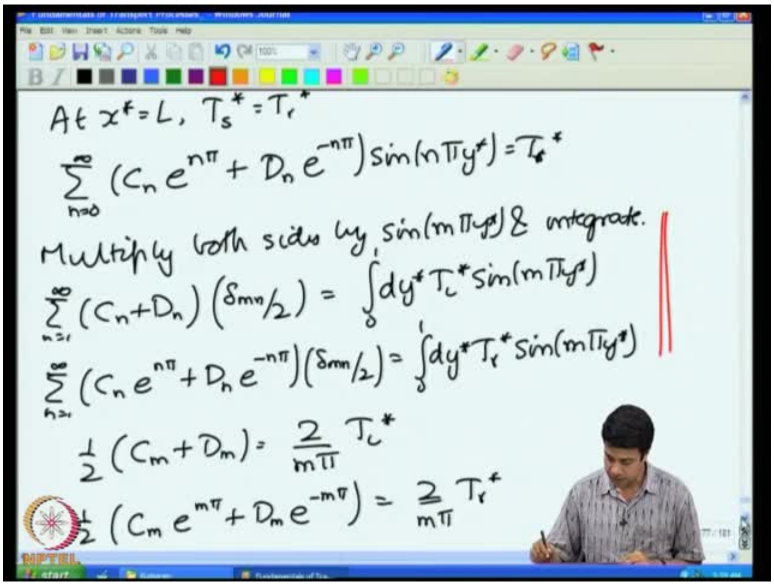

So, we used orthogonality relations for the steady problem in order to obtain the values of each of these constants, use orthogonality relations in order to obtain the value for each of these constants. Basically, I have to solve a simultaneous equation in two variables in order to determine the value of these constants. So that was for the steady part in the limit of T going to infinity and then, we went on to the transient part of the solution. You subtract the steady part from the total solution in order to get the transient part, because the steady solutions as well as the total solution have exactly the same boundary conditions.

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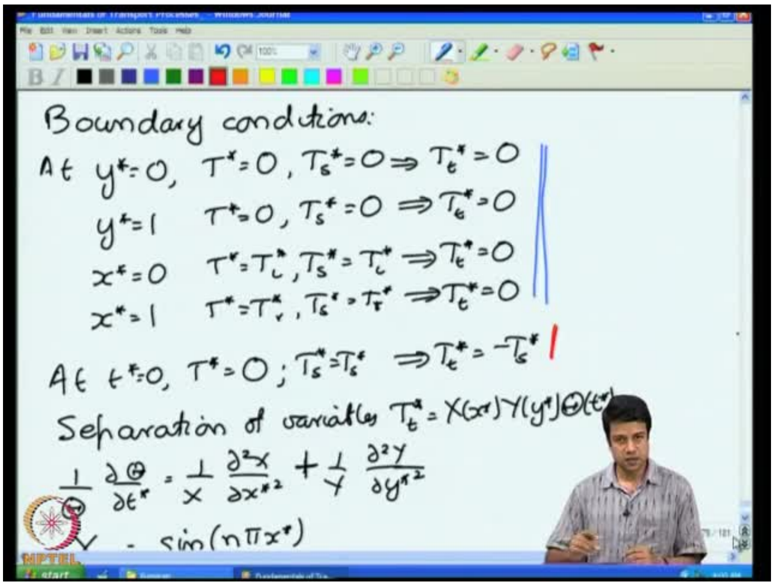

The transient part has homogenous boundary conditions in all spatial directions. There is an inhomogenous boundary condition only at initial time t is equal to 0. This is important when we solve a separation of variables problem. We have to make sure that there is homogenous boundary condition in all directions except one that is the inhomogenous direction, where we will find what the solutions and that was done by a two-step procedure. Here the initial problem was that we had homogenous boundary condition the y direction inhomogenous in x and inhomogeneous in time. We first separated that into a steady problem, where we had one homogenous and one inhomogeneous direction and a transient problem, which was homogenous in all directions, but inhomogenous in the initial condition at time t is equal to 0 for the transient problem, because we had homogenous boundary conditions in all spatial directions.

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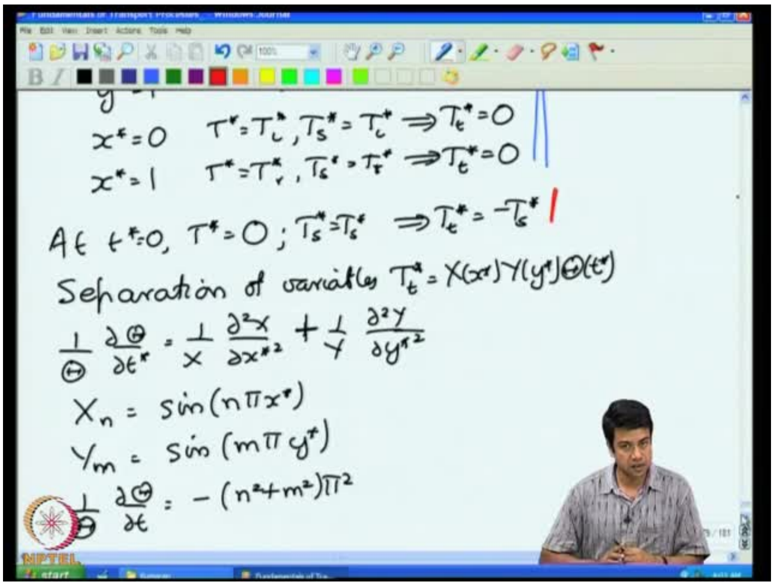

We were able to get Eigen value problems in both those directions, sin function in both directions the time the solution for the function of time turn out to be exponential. In that case and exponentially decreasing function in the time coordinate and putting all of those together, we were able to obtain a solution and we simultaneously used orthogonality in both the x and y coordinates in order to find out what is the value of that solution. So here we simultaneously used orthogonality in both the x and the y coordinates in order to find out these coefficients in that equation and that completes the final solutions.

So, just to recap, whenever we use separation of variables in multiple directions we have to make sure that it is steady, that it is homogenous in all directions except one. That one direction is the forcing direction and we will get an Eigen value solution for all directions except one and that a solution for that direction which is not homogenous is calculated using the orthogonality relations for the basis functions.

When we do not have homogenous boundary conditions in multiple directions, we have to reduce the problem to sequence of problems. Each of which has inhomogenous boundary conditions in only one direction and homogenous in all other directions and how do you reduce it to that sequence of problems? I showed you a first example here how introduce into a sequence of problems in this particular case.

We will see later on that one can reduce it to a sequence of problems in other cases as well. As we proceed, we will get more experience and how to reduce it to a sequence of problems. So, this was derivation of the conservation equation for mass and energy in the Cartesian coordinate system and how does one solve problems using that Cartesian coordinate system? Next time, we will derive exactly the same thing for a spherical coordinate system; and I will show you how it is done for a spherical coordinate system. I do not want to do it in detail for a cylindrical coordinate system, because it is just a previous exercise. I will leave it to you to do it for a cylindrical coordinate system. I will just give you the final results for this spherical coordinate system

In the next lecture, I will show you how to derive exactly the same conservation equation and after that we will look at situations where diffusion is dominant and situations where convection is dominant, how one does solve equations for each of these cases.

So, next class we will start obtaining a differential equation, balance equation for a spherical coordinate system and then I will show you how to use that in order to solve problems.

So, we will see you in the next lecture.

Thank you.