**Fundamentals of Transport Processes**
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**Lecture No. # 24**
**Unidirectional Transport Spherical Coordinates-II Separation of Variables**

Welcome to lecture number twenty four of this course in fundamentals of transport processes, where we were discussing transport in a spherical coordinate system. And I had defined, for you a spherical coordinate system in the last lecture. This is used for spherically symmetric surfaces most common example is a sphere immersed in a fluid. In, which case, the effect place the origin of my coordinate system at the centre of this sphere and the surface is a surface of constant distance from the origin. So, if I can use the distance from the origin as one of my coordinates, then I apply boundary conditions on a surface of constant radius.

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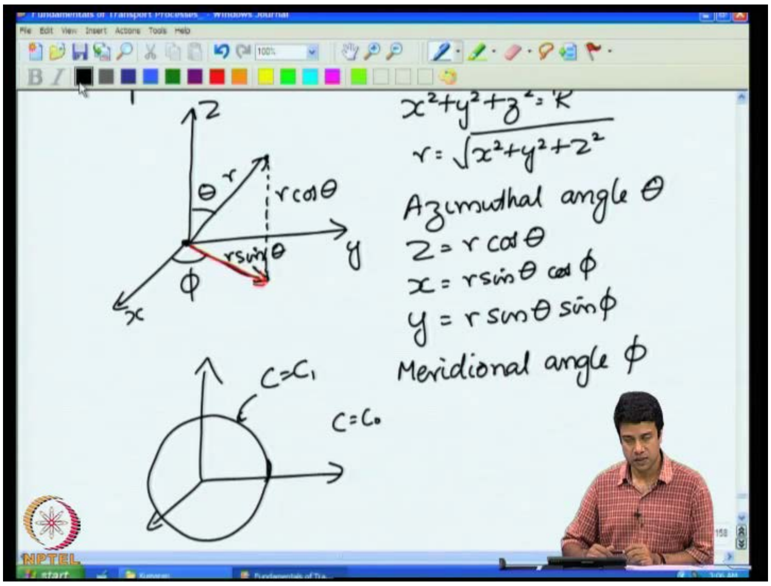


So, I defined, this spherical coordinate system for you in the last lecture r is the distance from the origin. So, it is important to note that we will be placing the centre of our coordinate system at the centre of the particle, if it has spherical symmetry. r is the distance from the origin and I have two other angles theta and phi theta is called the azimuthal angle and I have another angle. So, theta is the angle made by the radius vector with the z axis and I have a third angle called the meridional angle, which is the angle made by the projection of the radius vector on the x y plane with the x axis. So, you can take it with respect to any axis.

When it is conventional to choose the angle with respect to the x axis as the meridional angle and as I told you, this is useful for problems where you have a surface of spherical symmetry such as its spherical particle

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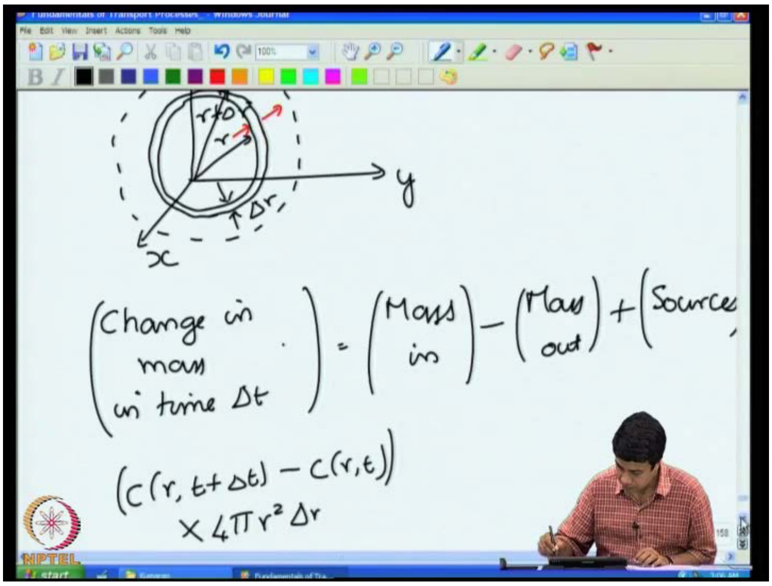


And in this case, we used a balance on a spherical shell, when we write shell balances as I told you, we need to choose two surfaces for the differential volume, which are at constant values of the coordinate. In this particular case, we are considering a coordinate system in which, there is no variation with respect to the theta and phi coordinates. Therefore, we define a shell such that the two surfaces bounding the volume are at radius r and radius r plus delta r. And we write a balance equation for these two surfaces and we got a balance equation in terms of the fluxes.

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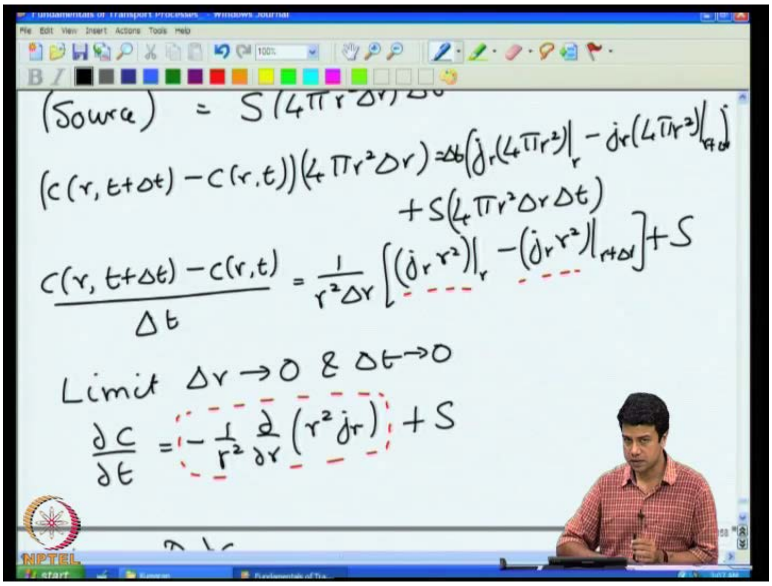


So, first we wrote it in terms of the distances delta r and the time delta t and then divided throughout by the volume in delta t look took the limit delta r going to 0 and delta t going to 0 in order to get the differential equation. In terms, of the flux j r, j r is the flux along the radial direction.

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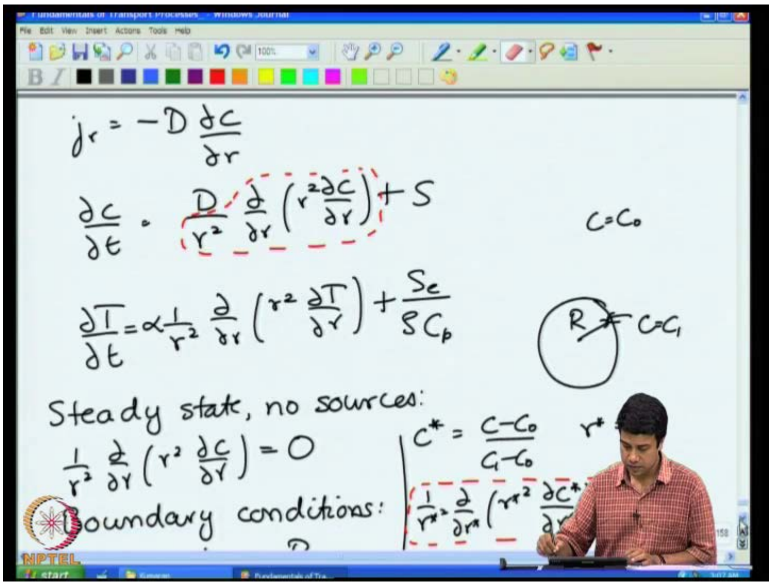


And if I express, j r as minus D times partial C by partial r, then my equation becomes there should be no negative sign there d C by d t is equal to d by r square partial by partial r of r square times partial C by partial r plus the sources or syncs within that differential volume. And equivalent equation for the temperature field in a heat transfer problem replace concentration by temperature and the mass diffusion coefficient by the thermal diffusion coefficient and the mass source by the energy source by rho C p. And you get exactly the same equation; I said we cannot quite do the same thing for a momentum transfer, because there are different components of the velocity. So, we will restrict attention for the present to just mass and heat transfer.

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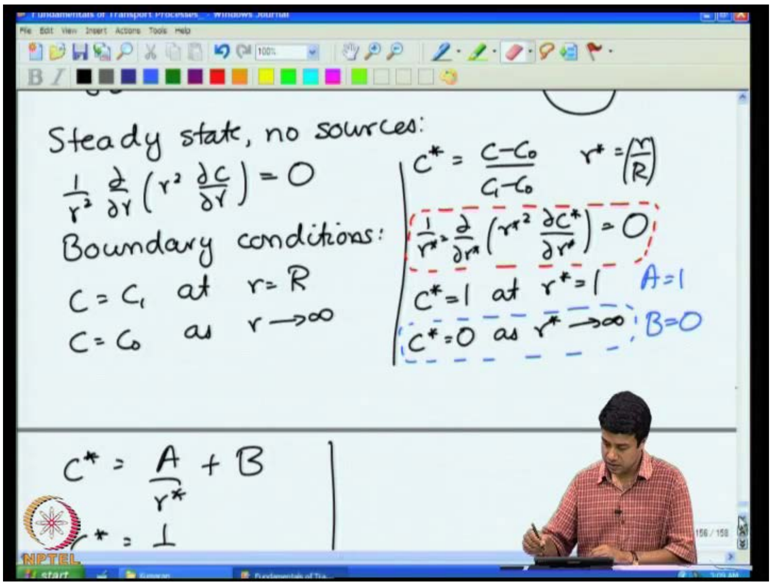


First we looked at the case at steady state, where there are no sources no heat or mass sources. And the equation is quite simple 1 over r square d by d r of r square times d C by d r is equal to 0 with boundary conditions C is equal to C 1 at capital R on the surface of the particle C is equal to C naught as r goes to infinity. So, you have mass being emitted from this particle, because the reaction has taken place on the surface. The surface concentration is C 1 far away it is C naught and one has to find what is the temperature profile. We used this non dimensionalization as we had done for all our problems in the past to simplify the problem ,C star is equal to C minus C naught divided by C 1 minus C naught. So, that its equal to 1 on the surface 0 far away r star is equal to r by capital R defined in such a way that r star is equal to 1 on the surface.

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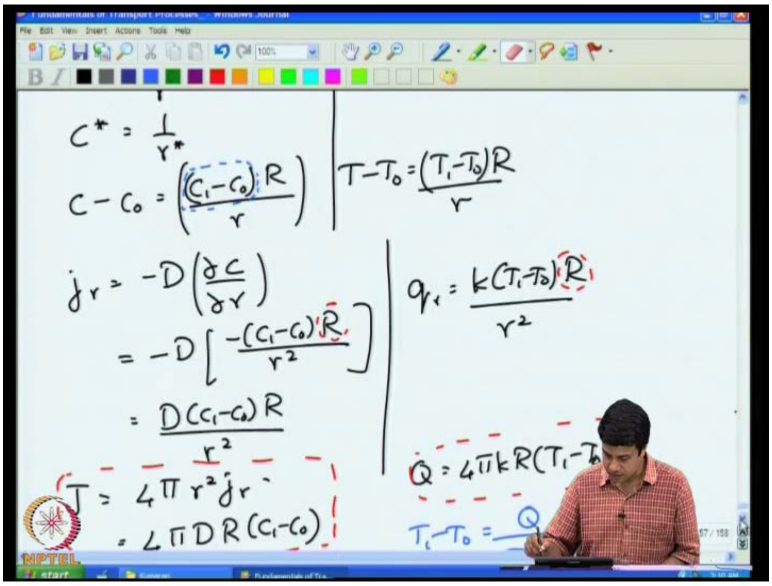


And, if I solve this equation I get C star is equal to just 1 over r star it is decreasingly inversely with the distance from the centre of the particle. If I express this back in dimensional terms I get C minus C naught, which is the difference between the concentration and the far field concentration is equal to C 1 minus C naught times capital R by small r and similar expression can be obtained for the temperature field.

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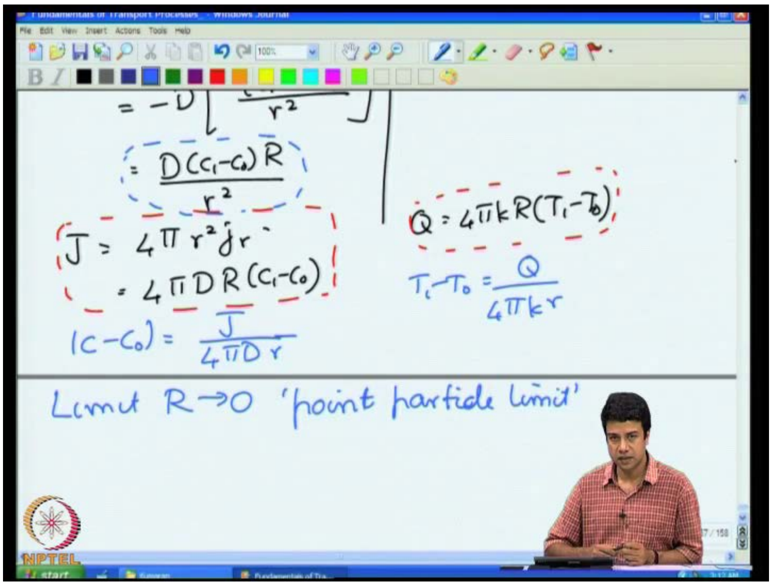


And I had also briefly, told you in the last class in the last lecture rather than express. The concentration in terms the concentration difference between the surface and far away I can also express in terms of the total amount of mass flowing out of the particle per unit time. The total mass is equal to the surface area times the flux. The flux goes as 1 over r square the surface area is equal to 4 pi r square, if I multiply the two I get the total mass coming out per unit time that is independent of the radius as it should be, because there are no sources within the fluid the total and it is at steady state. So, the total mass coming out per unit time of this particle at steady state should be independent of the radius of the surface at which you measure it.

Similarly, the total heat coming out should be independent of the radius at which you measure it. So, I can use this to express C 1 minus C naught in terms of the total mass coming out per unit time. And once I do that I get an expression, which depends only upon the mass coming out per unit time and the diffusion coefficient, it does not depend upon the radius of the particle. Similarly, for the temperature the difference in temperature between the surface and far away depends only upon the total heat coming out per unit time and the thermal conductivity does not depend upon the radius of the particle. So, I expressed in this way the temperature and the concentration fields are exactly the same independent of the radius of the particle. And I told you, if you take the limit of r going to 0 you get what is called the point particle limit, as an aside when we did the cylindrical coordinate system. If you recall the equation at steady state that we had we actually, solved it between two surfaces at radius R I and R o in the previous, when we did the steady state solution.

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There we are.

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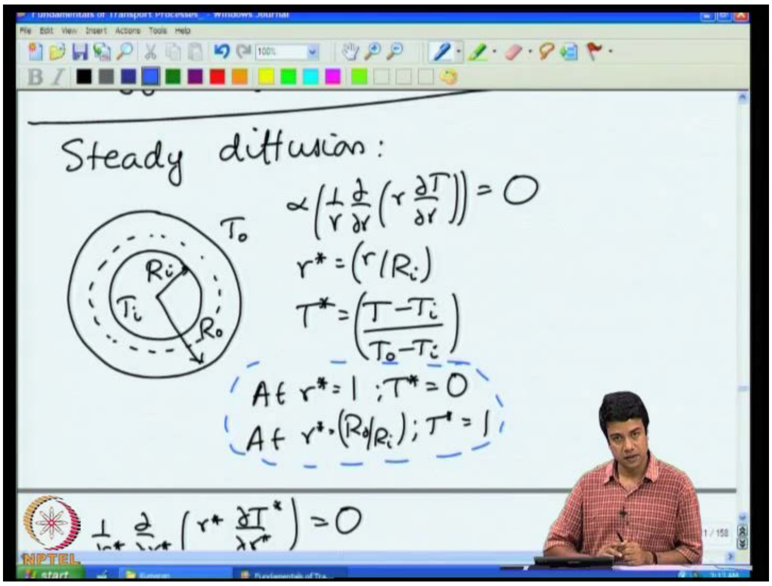


So, we had actually, solved it between two surfaces at radius R I and radius R o at that time I had never solved for you the what, it would be for a heated cylinder in an infinite medium, where T naught was the temperature as r goes to infinity.

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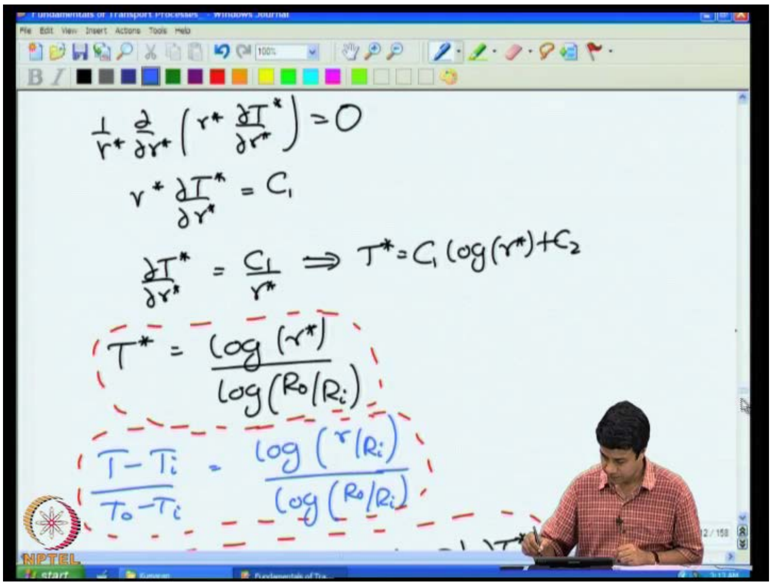


If you recall at that time we got a solution that was logarithmic in r.

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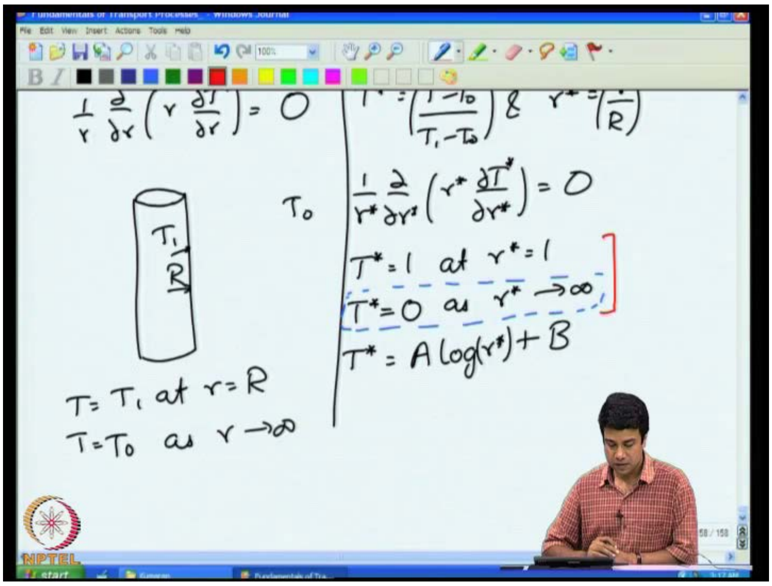


So, for the cylindrical coordinate system the conservation equation is 1 by r d by d r of r times d t by d r is equal to 0. And in this present case I had solved it for a spherical particle in an infinite medium, if I consider the analogous problem for a cylinder an infinite cylinder, of infinite length with temperature T 1 at the surface and T naught far away. Then my boundary conditions would be T equal to T 1 at radius let us say r is equal to R and T is equal to T naught as r goes to infinity.

So, I can solve this problem using the same non-dimensionalization that I had used for the spherical coordinate system. T star is equal to T minus T naught by T 1 minus T naught and r star is equal to r by capital R. In this case, the conservation equation becomes 1 over r star d by d r star of r star d t is equal to 0 with boundary conditions T star is equal to 1 at r star is equal to 1 and T star is equal to 0 as r star goes to infinity. So, this would be the equation at the boundary conditions for that particular case.

Now, if I try to integrate this equation in a cylindrical coordinate system. The temperature field that I get is T star is equal to a log r star plus B, where A and B are the integration constants. So, it is a logarithmic variation of temperature with distance whereas, in the spherical coordinate system I had got 1 by r star. And now, if I try to apply boundary conditions you can see that in this particular case, boundary conditions cannot be applied. Because a logarithmic function always goes to infinity as r goes to infinity a logarithmic function always goes to infinity as r goes to infinity.

So, there is no way that I can apply the boundary conditions one the only way that the log function will go to will the only way that this temperature will be finite in the limit as r goes to infinity is if a is equal to 0 this boundary condition. The only way that the temperature will be finite as r goes to infinity is if a is equal to 0, which means, that T star is equal to a constant, but T star is equal to a constant cannot satisfy both of these boundary conditions simultaneously, because you require a different value at the surface, and far away. So, because of that in two dimensions the equivalent problem actually, does not have a solution.

When we did the two dimensional solution previously we did it between two cylindrical surfaces for a single cylindrical surface in an infinite medium a solution does not exist. So, this is an example, where you cannot get a solution for the diffusion equation around an infinite cylinder in real practical applications there are never infinite cylinders. At some point one has to account for the fact that the cylinder is actually of a finite length. Even, if I had a cylinder for example, if I have the pipe of a heat exchanger, if I am very close to the pipe such that the distance from the surface is small compared to the length then it looks like an infinite cylinder to me, but if I go sufficiently far away. So, that my distance is large compared to the length then it looks like a cylinder of a finite length in which case, you have to solve for the equation for the conduction around a cylinder of finite length.

So, this particular case, of a cylinder of infinite length actually does not have a solution for the steady diffusion into an infinite medium. And that is the reason that we did not solve previously for this particular case, because you cannot apply boundary conditions. Similarly, for the unsteady diffusion into an infinite medium one can once again, not get solutions, because the final steady solution is not defined. We manage to get a solution for a similarity using a similarity transform for a wire of infinitesimal thickness, if the total heat generated per unit time is fixed. But if I have a cylinder of finite thickness the final steady solution is not well defined. So, that is the problem with cylindrical coordinates nothing wrong with either, the formulation or the procedure used to solve it is just that a solution, which satisfies boundary conditions in the limit r going to infinity does not exist in this particular case.

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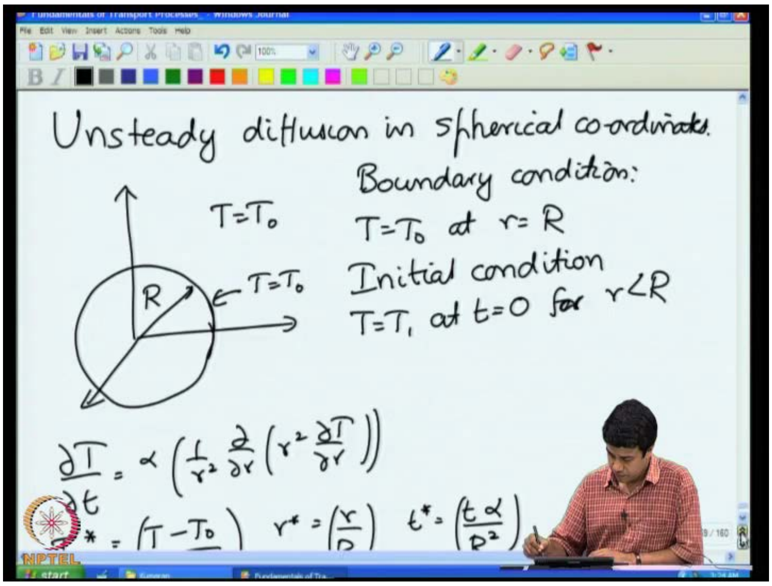


Let us come back to our spherical coordinate system and solve for unsteady diffusion in spherical coordinates. So, what I do is I take a heated sphere and immerse it in a cold fluid I take a heated sphere at temperature T 1 and immerse it in a cold fluid whose ambient temperature is T naught and I want to know how the temperature within the sphere varies with time. So, this is the analogue of the heated cylinder problem in cylindrical coordinates. So, I have a heated sphere of radius capital R the fluid outside is considered to be of infinite extent in such a way that the heat coming out of the sphere does not appreciably change the temperature of the fluid. So, far away from the fluid or everywhere, within the fluid we consider the system to be evolve next just that everywhere, within the fluid T is equal to T naught which means, that at the surface of the sphere itself the temperature is equal to T naught.

So, therefore, the boundary condition is that T is equal to T naught at r is equal to capital R and the initial condition I had immersed a sphere, which was initially at a temperature T 1. Therefore, at initial time T is equal to T 1 at t is equal to 0 for r less than capital R. So, everywhere, within the sphere at time t is equal to 0 the temperature is equal to T 1 everywhere within the sphere. So, now, I need to solve this unsteady problem the diffusion as I said there are no sources or sinks within this sphere of heat therefore, the unsteady diffusion equation is given by d t by d t is equal to alpha into 1 by r square d by d r of r square d t by dr. So, that is the unsteady diffusion equation.

Non dimensional temperature distance in time once again, I can define T star is equal to T minus T naught by T 1 minus T naught, the natural scaling for the radius r is capital R itself. So, that r star is equal to r by capital R the natural scaling for time is the diffusion time required for thermal diffusion over a distance capital R over a distance comparable to the radius of the sphere. So, that t star is equal to t R square by t alpha by R square. So, those are the non dimensional temperature radius and time. The differential equation expressed in terms of these is going to be just d t by d t is equal to 1 by r square d by d r of r square d t by d r.

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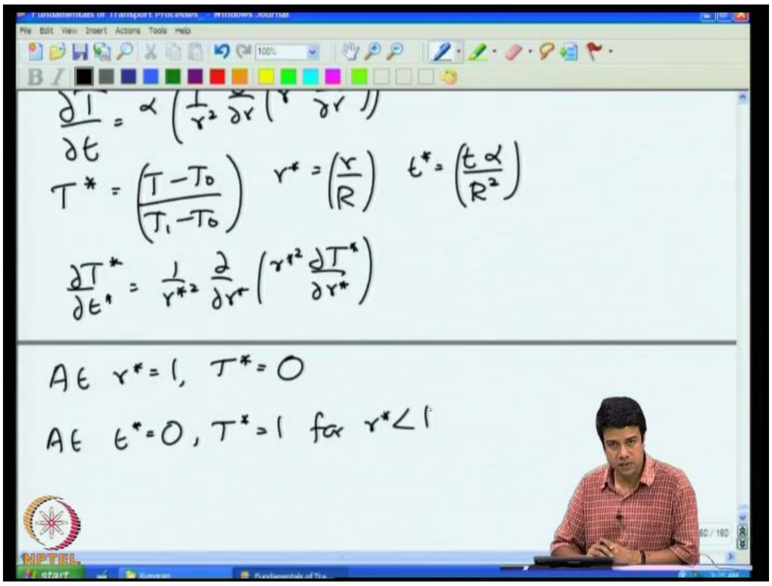


So, that is the differential equation with boundary conditions at r star is equal to 1 T star is equal to 0 and initial condition at T star is equal to 0 the temperature was T 1 at the initial time the temperature was T 1 everywhere. So, therefore, T star is equal to 1 for r star less than 1 everywhere, within the sphere the temperature was equal to T 1; that means, that T star is equal to 1 everywhere within the sphere.

So, this as you can see is a second order differential equation in R. I have only one boundary condition at r star is equal to 1 and yet another boundary condition when we discussed, cylindrical coordinates I had shown you that the other boundary condition. Actually, comes out of a symmetry condition that is just the consideration that the temperature derivative as you approach the origin has to be equal to 0 from all directions.

I said if the cylindrical coordinate system as I approach the axis from the right and the left since, the system is symmetric about as you go around the axis there is no variation in temperature as you go around the axis. That means that the slope has to be the same, if I approach it from left and right; however, if I approach it from the left the slope is positive and approach it from the right it is negative; that means, that the derivative is not well defined at the centre if the derivative is non 0. Because it has different values depending upon, which direction you approach the axis from the derivative will have different values, if you come from different directions for an axis symmetric problem. The only way that the derivative will have the exact same value regardless of which direction you approach from is if the derivative is exactly equal to 0 at the centre. So, if the derivative is exactly equal to 0 so, that regardless of which direction I come from the derivative will go to 0 as I approach the centre. So, if the is equal to 0 then it will be exactly the same regardless of which direction I approach from.

That means, that symmetry itself requires that d t by d r has to be equal to 0 at the origin a similar boundary condition applies for a spherical coordinate system as well.

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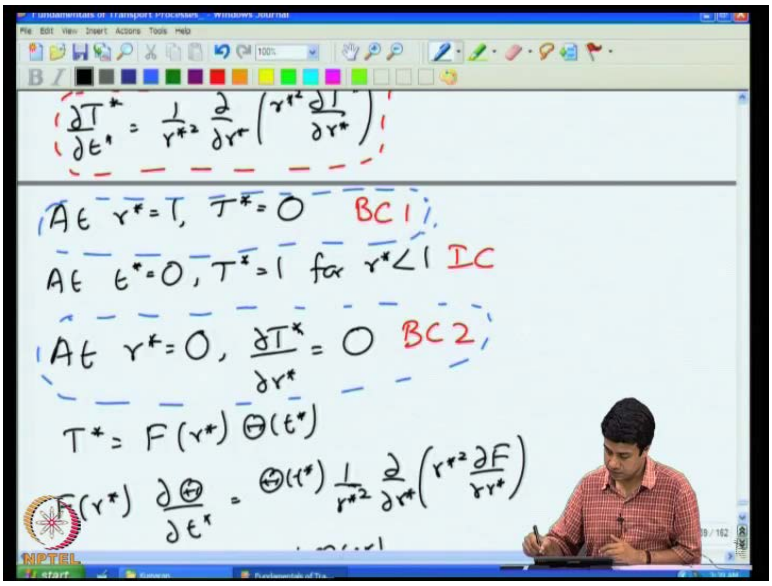


So, that is my second boundary condition. So, this is BC 1 the first boundary condition, this is the initial condition and the second boundary condition is that at r star is equal to 0 d t by d r has got to be equal to 0, that is the second condition. So, with this second symmetric condition I have sufficient number of conditions to solve the unsteady equation so, how do I solve this unsteady equation separation of variables technique.

So, I express T star is equal to some function of r star times some function of time and then I put that into the equation substitute this separation into the equation I will get F of r star times d theta by d t is equal to theta 1 by r 1 by r square and then I divide throughout by F of r star theta of t star to get 1 over theta times partial theta partial t star is equal to 1 over r star square d by d r star 1 by F of r star 1 by r star square d by d r star of r star square d F by d r star. So, that is the final equation that I get for theta and for F the left hand side is a function only of time the right hand side is a function only of r star; that means, that both of them have to be equal to constants.

Before that, if you recall when we did both the diffusion between two flat plates as well as, the unsteady diffusion in cylindrical coordinates into a cylinder. I had told you that it is important to ensure that, the boundary conditions in both the spatial directions are homogenous. It is only on that basis that you will get the Eigen values for the constants and the discrete set of Eigen functions. So, you have to subtract out the final steady temperature to get an get an equation only for the unsteady part of the temperature field. So, you have to define a transient temperature field which is the difference between the actual temperature and the steady temperature.

For this transient temperature field you will find that the temperature is equal to 0 on both boundaries, because both the steady and the actual temperatures satisfy the same boundary conditions on those two boundaries. We did not in this particular case you will find that both of these boundary conditions are actually already homogenous. At r star is equal to 1 temperature is equal to 0 at r star is equal to 0 the temperature derivative is equal to 0. So, the boundary conditions in this case are already homogenous the reason is because the final temperature in the long time limit.

I took a sphere of temperature T 1 immersed it in a fluid of temperature T naught; that means, that if I wait long enough the temperature of the entire sphere is going to become T naught itself. That means, in the long time limit after all the heat is because heat gets transferred from a higher to a lower temperature. If I wait long enough the temperature everywhere within the sphere is equal to T naught itself; that means, t star which is T minus T naught by T 1 minus T naught is identically 0 everywhere within the sphere. That means, that the steady temperature field itself is 0 everywhere within the sphere.

So, in this particular case the temperature that I am calculating is the transient part itself because the steady temperature t star it is identically equal to 0 everywhere. That is the reason for the transient part I already have homogenous boundary conditions and there is no problem in going ahead and doing the separation of variables solution. So, I have separated the variables left side depends only upon the time the right side depends only upon r. And therefore, both have to be constants because if they are not constants I could keep time of constant and change r and only the right side will change the left side will not and the equality will no longer be valid.

Therefore the left side depends only upon time, the right side depends only upon r. That constant as we know has to be a negative constant it is only if it is negative that theta will be exponentially decreasing in time. And therefore, in the long time limit the temperature will decrease to 0 only if theta is exponentially decreasing in time.

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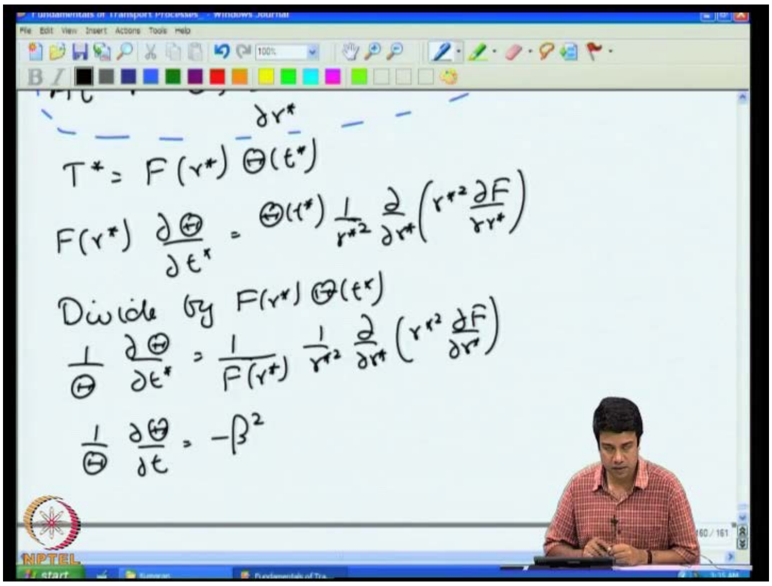


Therefore, I need to set d theta by dt is equal to minus beta square a negative value beta is some constant and it is equal to a negative value.

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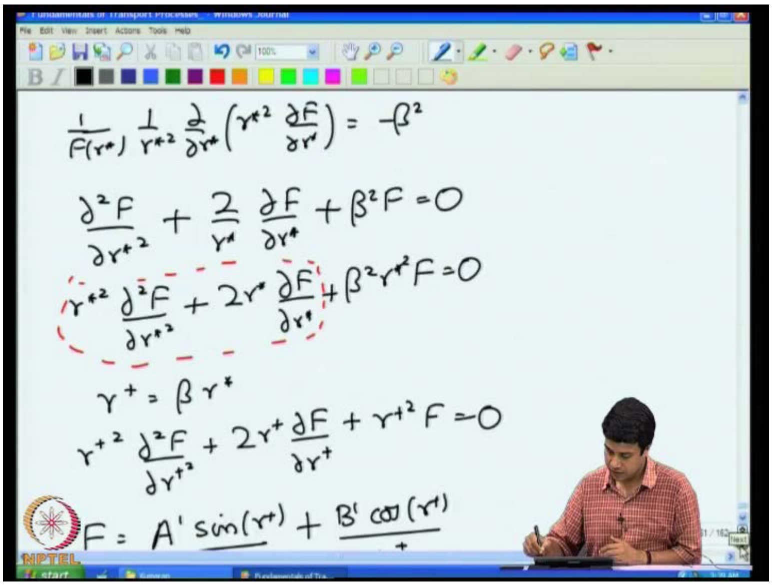

And 1 over F 1 by r square d by dr of r square d F by dr is equal to minus beta square. So, I have two equations one for the temporal part of the temperature variation the other for the radial part along the r direction. So, this radial part now I can solve so therefore, I will get if I solve this I will get 1 over r square I am sorry d square F by dr square plus two by r d F by dr star plus beta square F is equal to 0 So, this is the equation for F. Alternatively I can write this multiply throughout by r square to get r square d square F by dr square plus 2 r d F by dr plus beta square r square F is equal to 0. So, this is the equation in terms of beta and r star note that the first two terms have 0 net dimension in r I have r square d square F by dr square.

So, if I change F by a factor beta this will not change second term is 2 r d F by dr if I change this by a factor of beta this will not change. Therefore, I could define a new coordinate r plus is equal to beta times r star and if I change r by a factor beta the first two terms will not change. So, the final equation that I get in terms of r plus will be equal to r plus square d square F by dr plus square plus 2 r plus d F by dr plus plus r plus square F is equal to 0. Now, this equation does have an analytical solution, the analytical solution is of the form F is equal to A prime sine of r plus by r plus plus B prime cos of r plus by r plus.

You can verify by substituting this into the equation that this actually satisfies the equation. Alternatively if I write it in terms of r star I can write this as A sin of beta r star by r star plus B cos beta r star by r star. So, this is the solution for F where A and B are constants which have to be determined from the boundary conditions. The boundary conditions were one was dt dr has to be equal to 0 at r is equal to 0 that is B C two dt dr is equal to 0 at r is equal to 0. If you look at this equation here cos of beta r star goes to 1 as r star goes to 0 and therefore, the contribution due to that cos function goes as 1 over r star the derivative of that is not equal to 0.

Whereas sin of beta r star goes as r star in the limit as r star goes to 0; that means, that the ratio goes to a constant value the derivative is equal to 0.

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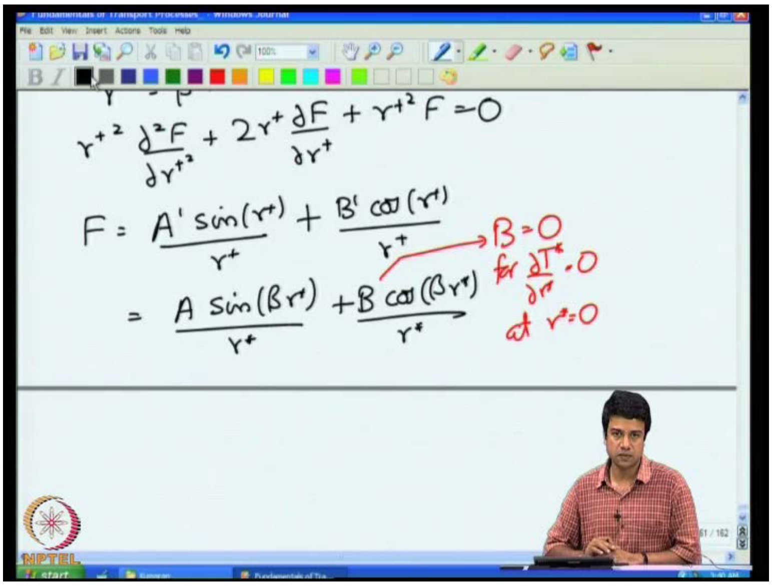


So, just the requirement that the derivative is equal to 0 implies that this coefficient B is equal to 0 for dt by dr is equal to 0 at r star is equal to 0. So, therefore, if I have to if the symmetric condition is valid at the centre of the sphere that is dt by dr is equal to 0 at r is equal to 0; that means, that d is equal to 0. Because b is multiplying a Cos function which is going to one in the limit is as r goes to 0 therefore, Cos beta r by r actually goes to infinity as r goes to 0.

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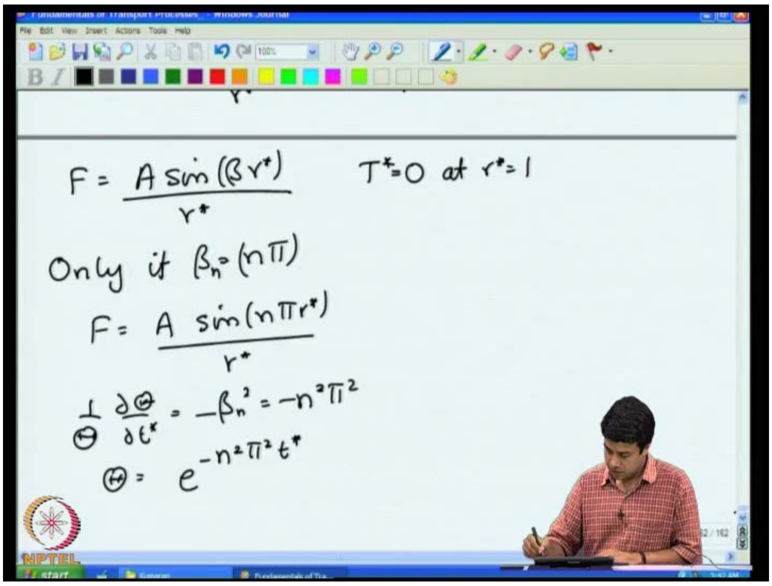


So, therefore, F is equal to a sin of beta r star by r star and I have to satisfy the other boundary condition. T star is equal to 0 at r star is equal to 1 that is the second boundary condition that has to be satisfied that T star is equal to 0 at r star is equal to and you can easily see that this is satisfied only if beta is equal to n times pi, where n is an integer. So, this second condition gives me discrete set of values for beta, it is valid for any n. So, long as it is have an integer. So therefore, this is valid for any value of n. So, long as n is an integer therefore, the solution for F which satisfies this boundary condition is F is equal to a sin of n pi r by r star. So that is the solution for F the equation for theta now becomes 1 by theta d theta by d t is equal to minus beta n square is equal to minus n square pi square. That means that theta got to be equal to e power minus n square pi square t star.

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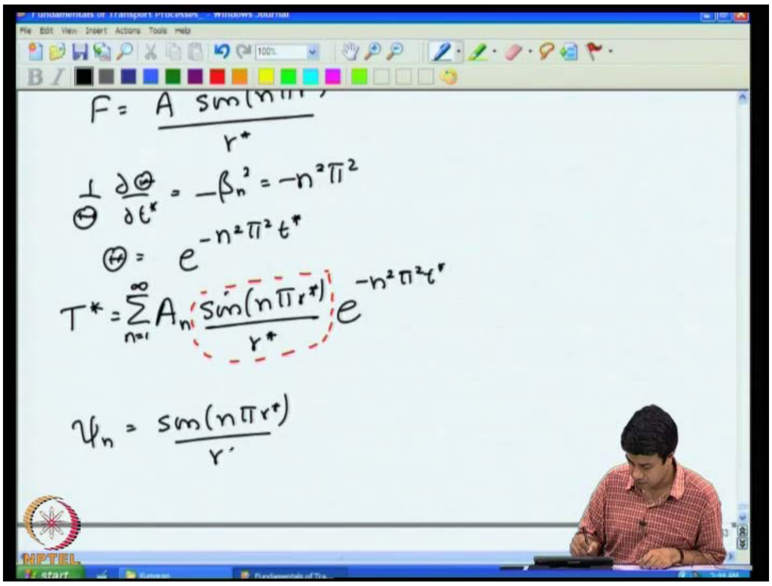


So, the most general solution for the temperature is equal to a sin of n pi r star by r star e power minus n square pi square t star. So, that is the most general solution for the temperature field. This solution is valid for any value of n therefore; the most general solution will be a summation over all possible values of n. It will be a summation over all possible values of n where the n's are the coefficients in the expansion. So, for the spherical coordinate system the bases functions are these ones, these are the Basal functions for the spherical coordinate system similar to sine of n pi z star for a Cartesian coordinate system or the Bessel function for the cylindrical coordinate system. How do I find out the coefficients A n by using orthogonality relations. These functions satisfy the orthogonality relation that is if I use the bases function psi n is equal to sine of n pi r star by r star.

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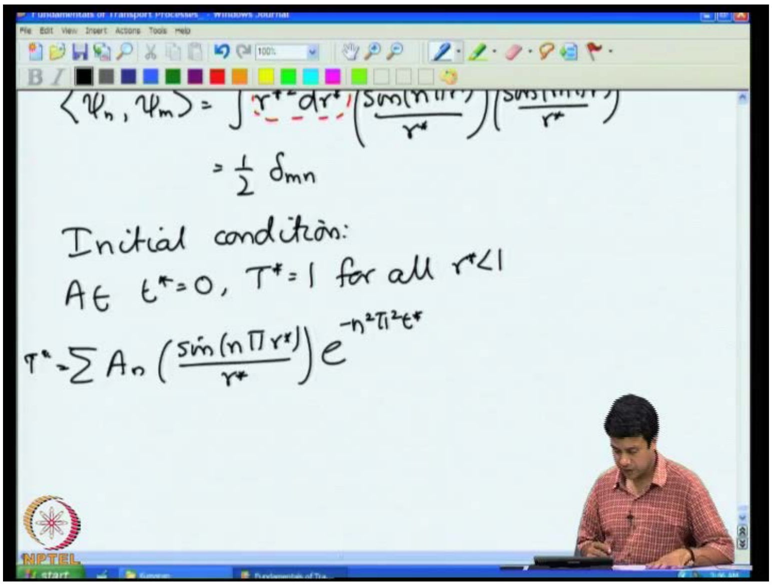


Then I have to define the orthogonality the the inner product as psi n comma psi m is equal to integral r star square d r star it is not just a simple integral or radius it is a integral of r star square times d r star times sin of n pi r star by r star 2 sin. And when I define it in this manner I get the orthogonality relation as this is equal to half delta m n, where delta m n as I explained to you earlier is one if m is equal to n and is equal to 0 if m is not equal to n. So, in this particular case I have to define the orthogonality relation with an r star square here .When we when we did the transport unidirectional transport in a Cartesian coordinate system, the orthogonality relation was just d z star times sin n pi z star times sin of n pi z star.

You get exactly the same orthogonality relation here as well except that my bases functions have 1 over r in them. So, I have to put r square d r in the orthogonality relation. So, this is the orthogonality relation for the bases functions for this spherical coordinate system. So, how do I use this I know the initial condition. At T star is equal to 0 T star is equal to 1 for all r star less than 1. So, t star that I know the solution is summation of A n sin of n pi r star by r star e power minus n square pi square t star, this is the solution for t star, that is the temperature field.

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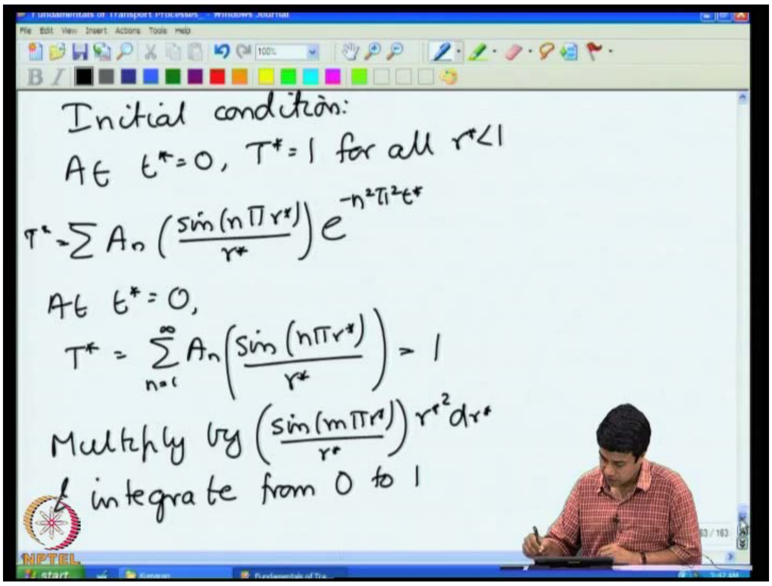


At T star is equal to 0 T star is equal to summation of A n sine pi r star this has to be equal to 1, and the summation has n going from one to infinity. So, how do I determine the constants? I use the orthogonality relation that is I multiply both sides by sin of m pi r by r times r square d r. And integrate from 0 to 1 multiply both sides by sine of m pi r by r and integrate both sides from 0 to 1.

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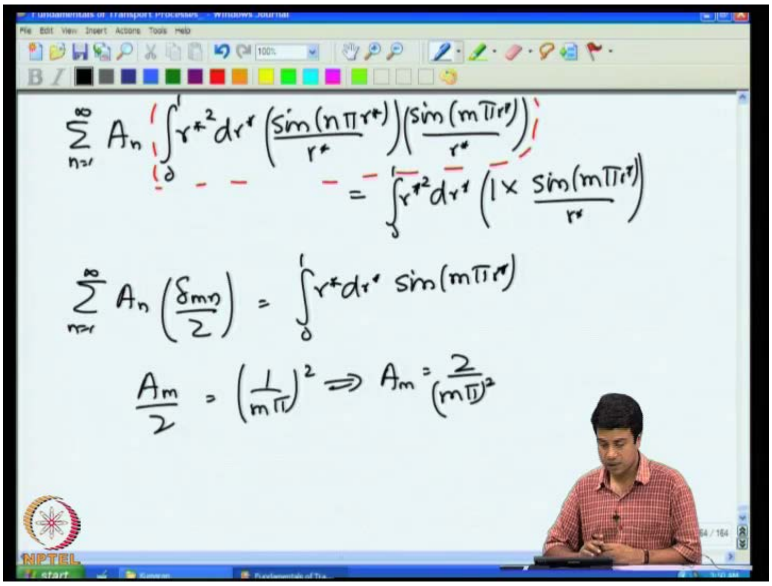


So I will have summation m is equal to 1 to infinity of A n integral 0 to one r square d r sin of n pi r by r sin of m pi r by r will be equal to integral 0 to one of r square d r the right hand side just has one in it. So, this is one into sin of m pi r by r. So, this basically gives me summation n is equal to 1 to infinity of A n times delta m n by two that is what I get from the orthogonality relation for this first term here A n times n by two, that has to be equal to integral 0 to one of r d r sine of n pi r. A n times delta m n summed from n is equal to 1 to infinity delta m n is equal to 1 only when n is equal to m delta m n is equal to 1 only if n is equal to m.

Therefore, I will get A n by 2 is equal to this integral if you work it out on the right hand side this turns out to be equal to 1 over m pi the whole square which implies if the coefficient A m is equal to 2 by m pi by whole square . So, this gives me the coefficients in the expansion and if I put these coefficients into the equation for the temperature field.

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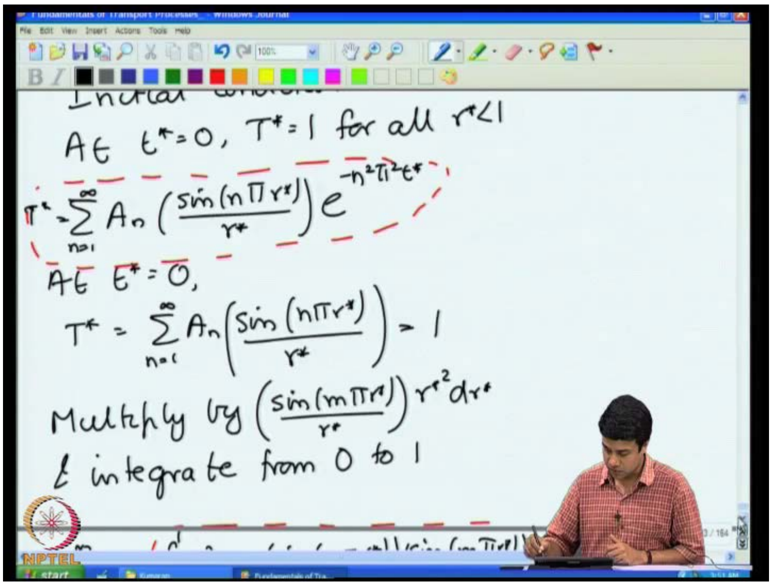


I will get the final solution for the temperature field, couple of things to note about this solution, first thing is it is the series expansion one to infinity. So, it is if it is essentially an infinite series; however, if I am you can see that the higher and higher terms in the series decrease as e power minus n square pi square times t. So, for any finite value of t the higher terms in the series will decay exponentially as e power minus n square. So, the higher terms in the series become smaller and smaller as n increases. Therefore, if I am interested only in a numerical approximation to the solution I can truncate this series at some finite value, in order to the approximate solution and the number of the solution becomes more and more accurate as the number of terms increases. So, that is the first point

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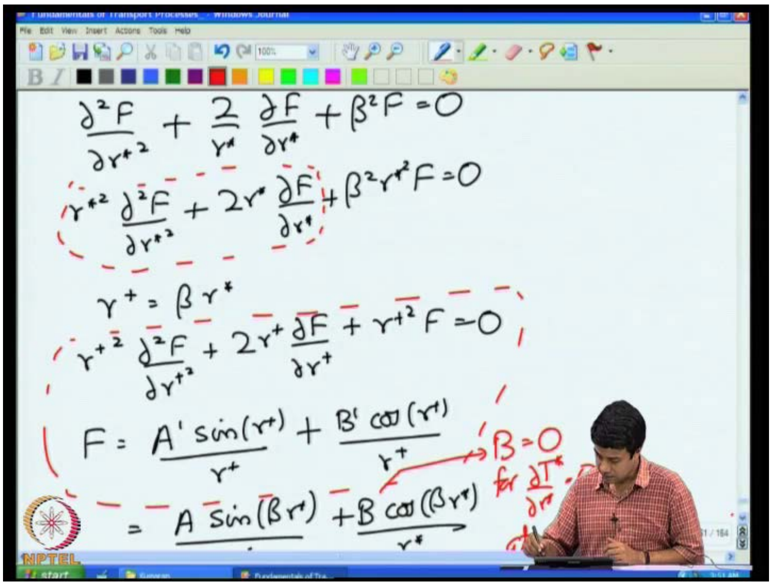

The second point is that I manage to get a solution here for this particular equation ,this solution is actually one of a class of solutions, the previous example of cylindrical coordinates, the solution turned out to be Bessel functions.

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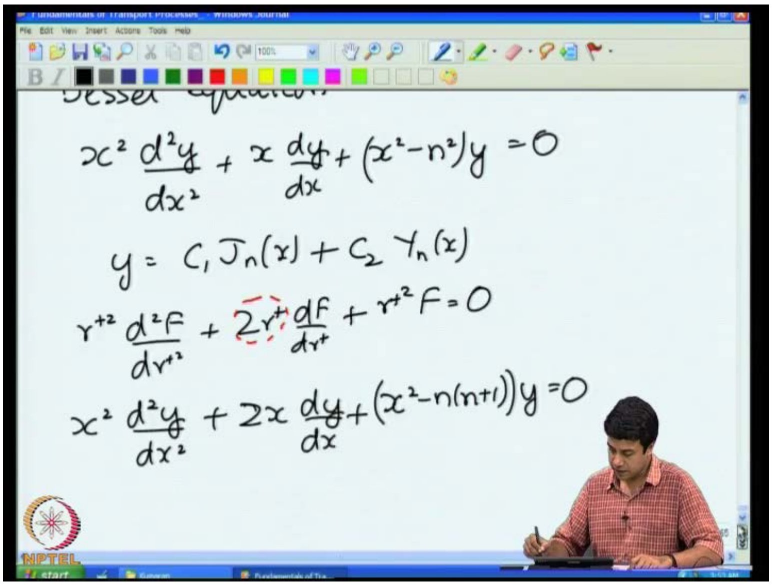

If you recall the Bessel equation was x square d square y by d x square plus x d y by d x plus x square minus n square y is equal to 0 and that had a solution y is equal to C 1 J n of x plus C plus C 2 times y n x. Where J n and y n are the Bessel functions, the equation that we had here was slightly different, it was r plus square d square F by d r plus square plus 2 r d F by d r plus r square F is equal to 0. This coefficient two was present here in the second term. This is one of a general class of equations of the form x square d square y by d x square plus 2 x d y by d x plus x square minus n into n plus 1, y is equal to 0 this is called the spherical Bessel equation.

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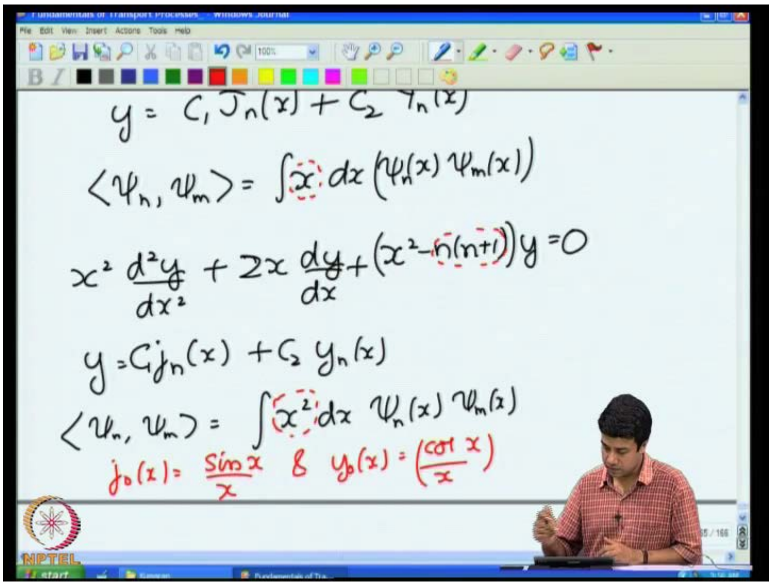


And the solution of these are the spherical Bessel functions J n C 1 J n of x plus C two times y n of x, where J n and y n are the spherical Bessel functions. These are actually related to the cylindrical Bessel functions as well, but they are the separate class of solutions called the spherical Bessel functions for which the orthogonality relations are defined as psi n comma psi. n is defined with respect to x square d x psi n of x psi n of x. Note this factor here, this is for the spherical Bessel function when I had done the same thing for the cylindrical Bessel function I had defined the orthogonality relation as psi n comma psi m equal to integral x d x psi n x psi n of x, this is your cylindrical Bessel function this has the waiting function x.

So, depending upon the function space to choose the waiting function appropriately. So, that the integral becomes the orthogonality relation is satisfied for that particular case for our particular problem we were solving it with n is equal to 0 the particular case that we were solving was the case where n is equal to 0 and for that particular case J n turns out to be sine x by x. And I am sorry j naught of x is equal to sine x by x and y naught of x is equal to Cos x by x, y naught goes to infinity as x goes to 0.

So, this is a particular class of solutions called as spherical Bessel functions for which we did the, we got the solution in this case in terms of sin and cosin functions.

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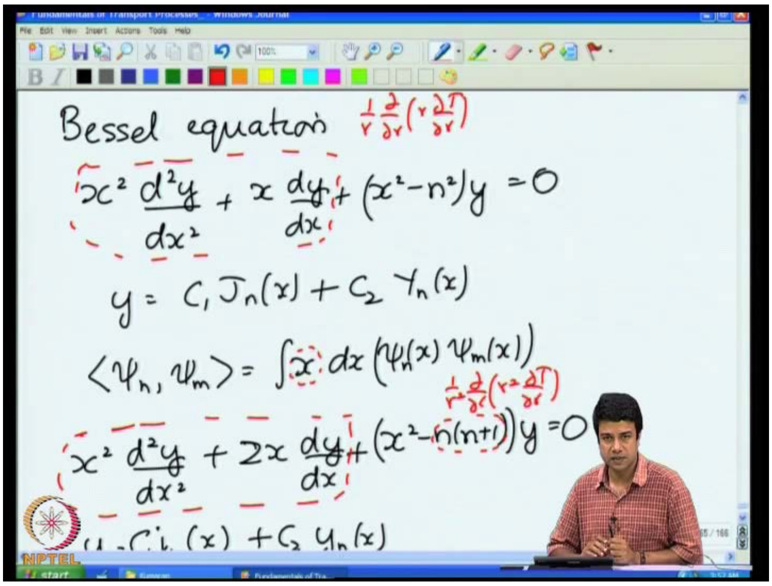


In general when you solve an equation in cylindrical coordinates, you always have an equation of the type 1 by r d by d r of r d t by d r is equal to 0. That is going to give you d square by d a d r square plus 1 over r d by d r is equal to 0 and so, you are going to get here.

Whereas if you solve the same thing in spherical coordinates you are going to get 1 over r square d by d r of r square d t by d r and for that particular case you will get r square you will get d square by d r square plus 2 r d t by dr. So, you will get this operator and that is why Bessel functions always end up being solutions for cylindrical coordinates. Spherical Bessel functions end up being solutions for unsteady problems in spherical coordinates in both cases in their respective coordinate systems; these functions form a complete and orthogonal set on bases functions any function can be expressed as a linear combination of these functions. And that is why they are used to solving problems in cylindrical and spherical coordinates.

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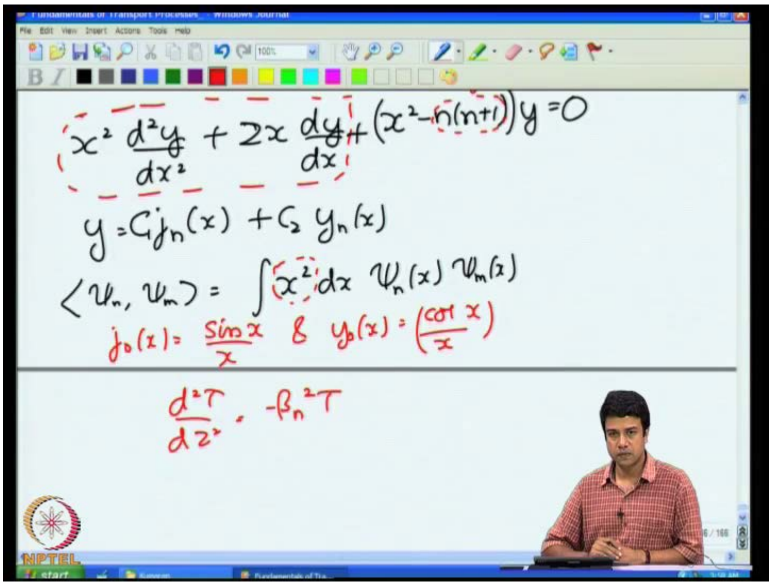


The same thing when we did it earlier for Cartesian coordinates you will end up with d square T by d z square is equal to some alpha times T. And therefore, you will get sin and cosin functions. So, the equivalent of sine and cosine functions for a Cartesian coordinate system are the Bessel functions for a cylindrical coordinate system and the spherical Bessel functions for a spherical coordinate system. So, with this we have completed one more chapter in our journey through the fundamentals of transport processes and that is unidirectional flows.I showed you how to do shell balances shells are defined to be volumes which are bounded by surfaces of constant coordinate, write a shell balance, unsteady, take the limit as the volume goes to 0 and the time goes to 0 to get a differential equation, different ways of solving that differential equation.

We first started off with steady problems where just the there is no time dependence and then the equation reduces to a ordinary differential equation, next we looked at similarity solutions, when we can reduce the variables based just upon dimensional analysis. Separation of variables in order to write the solution in terms of a set of bases functions for that particular differential operator the bases functions were sine and cosine for the differential operator in Cartesian coordinates, Bessel functions for the differential operator in cylindrical coordinates and the spherical Bessel function for the differential operator in spherical coordinates. And use the orthogonality relation to determine the constants in the expansions. So, that was the separation of variables procedure.

So, now we, but we are still limited because we looked only at transport only in one direction. Now we will extend our analysis to look at cases where transport happens in all directions. There is both convection as well as diffusion for the present I will restrict attention only to convection and diffusion for heat and mass transfer, momentum transfer is a little different, because when I am looking at simultaneous variation in all three directions. There is variation in three directions, but momentum is also a vector. So, the momentum has three components to it and that makes it more difficult.

So, we will look at transport of scalars, heat and mass. We will look at momentum later on the way to look at momentum transport is to consider not components of vectors, but rather vectors themselves and that requires some background in vector calculus. So, first we will look at heat and mass transport in all three dimensions simultaneously. And we will derive equations and then we will look at ways to solve them. So, this completes unidirectional transport. Now, we go into a more general case of transport in all three directions. We will look at diffusion dominated transport and convection dominated transport and we will see you next class.