Fundamentals of Transport Processes
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Lecture No. # 23
Unidirectional Transport Spherical Co-ordinates - I Balance Equation

Lecture number 23 in our course on Fundamentals of Transport Processes, we first covered dimensional analysis; and then we looked at diffusion diffusion process is due to the fluctuating motion of molecules. And then, we went on to Unidirectional Transport and I showed you first in cartesian co-ordinates, and then cylindrical co-ordinates; how one does a balance on a small volume element of fluid. And then takes the limit as the size and the time interval go to 0, to get a differential equation. This is a partial differential equation, which has contains derivatives both in position as well as time.

And we looked at different ways of solving it, if you are at steady state; it reduces to an ordinary differential equation, in the spatial co-ordinates alone, a second order ordinary differential equation. If the situation is unsteady such as the impulsive heating of a boundary, then you get as a partial differential equation, and we looked at two ways of solving this.

The first was the similarity solution, where we used dimensional analysis to reduce from two independent co-ordinates to just one independent similarity variable. As well as the procedure of separation of variables, where we separated out the dependence on time and position; in order to get a composite solution. Our procedure basically, involved expressing the solution in terms of a set of basis functions, and then using the orthogonality relations to get the constants in that solution.

So, far we have covered cartesian co-ordinates first, and then cylindrical co-ordinates, as I told you cartesian co-ordinates is the simplest co-ordinate system. Because all axis perpendicular to each other, and planes of constant x, y and z are are are are plane surfaces. On the other hand, we are often required to solve problems, where the boundaries are curved; think of the surface of a pipe for example, where this fluid flowing through the pipe, in that case, it is necessary to apply boundary conditions on a curved surface.

And it is relatively difficult to express the equation of the curved surface, in Cartesian co-ordinates in general and therefore, we look at curvilinear co-ordinate systems, where the surfaces of constant co-ordinate are not flat surfaces, they are curved surfaces. The simplest example, we looked at was the transport and cylindrical co-ordinates, which we looked at in the previous few lectures.

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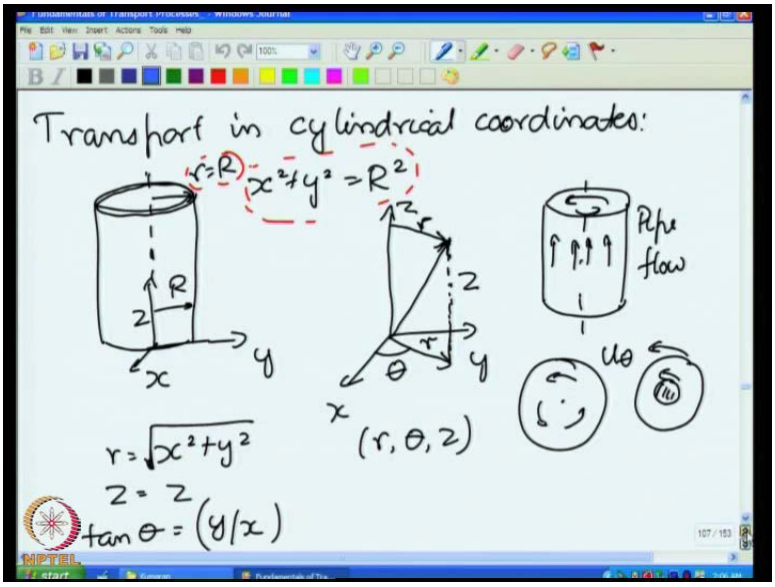

In this case, if we have a cylindrical surface, then the equation of the surface is expressed as x square plus y square is equal to R square, which is in general difficult to express in a cartesian co-ordinate system, it is difficult to enforce boundary conditions, in this co-ordinate system. Therefore, we went to a cylindrical co-ordinate system, where cylindrical surfaces are surfaces of constant r, surfaces along which the radius is a constant. So, for this particular cylindrical co-ordinate system, you have one axis the z axis about which the entire system is symmetric.

So, if for example, I were analysing a pipe flow, the axis would be at the centre of the pipe, the axial co-ordinate would be at the centre of the pipe. The distance from that axis, and the perpendicular distance is the co-ordinate r the radial co-ordinate. And so you have two co-ordinates, one is the z co-ordinate the distance along the axis, the r co-ordinate the perpendicular distance from the axis. And there is another co-ordinate theta in this diagram, which is basically the angle made by the radius vector with the x axis; this is the convention that is adopted in cylindrical co-ordinate system. It is usually taken

as the angle made by the radius vector with the the the x axis, and because the surface are curved, when we wrote down equations for the differential volume.

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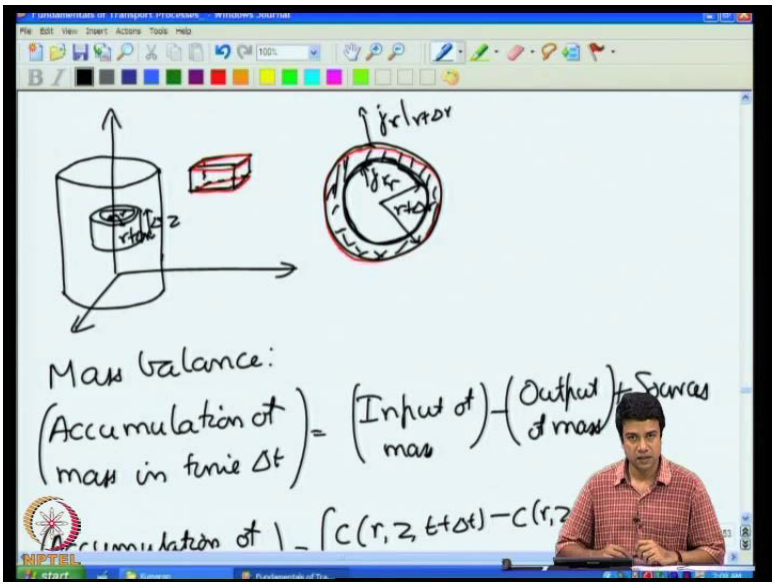

We took a differential volume bounded by surfaces at r and r plus delta r, the surface area on the two surfaces was different, because the surface area was 2 pi r times delta z, where delta z is the height. And the surface area in these two on these two surfaces was different, and that leads to a more complicated form of the diffusion equation (No audio from 05:37 to 05:47).

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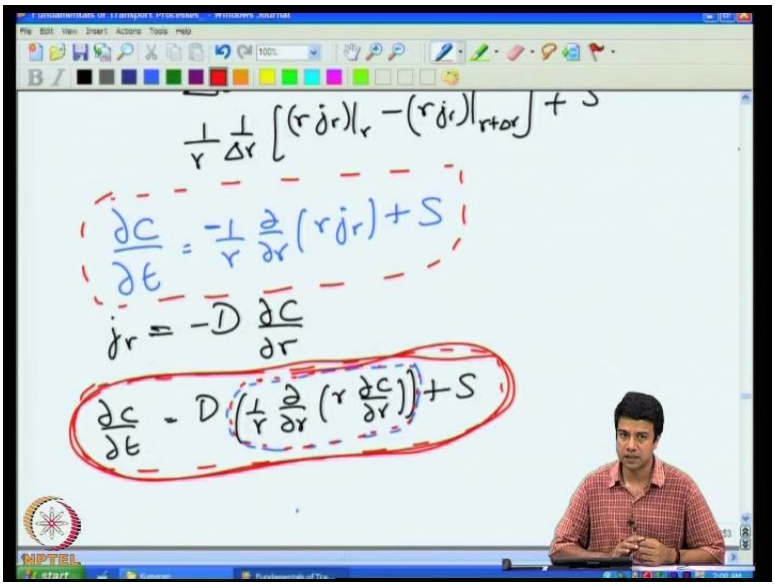

So, this was the concentration diffusion equation whereas, this S is the source, note that it has the operator 1 over r d by dr of r times d c by dr not just a simple second derivative, that we had in a Cartesian co-ordinate system.

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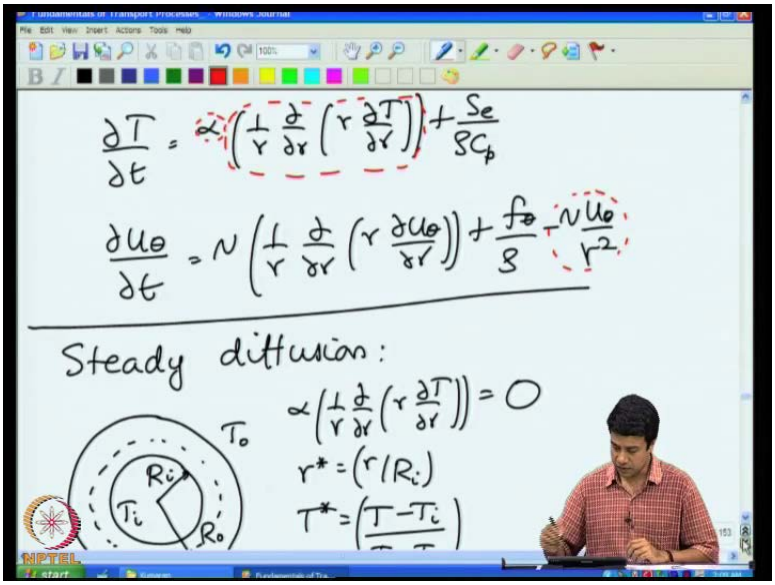

Similarly, one can write down equations for the temperature field, as well as for the angular velocity field, I told you the angular velocity can be in two directions. One is the axial velocity for example, the flow in a pipe, the velocity is in the axial direction along the axis. And the second is a rotational angular velocity for example, if I had the annulus between two cylinders filled with a fluid, and one cylinder was rotating. Then you would expect a velocity along the theta direction; and that case the velocity, in that case is (()) theta. And we solved a few problems. In this one was of the steady diffusion where we got a logarithmic, temperature profile for the case of a steady diffusion.

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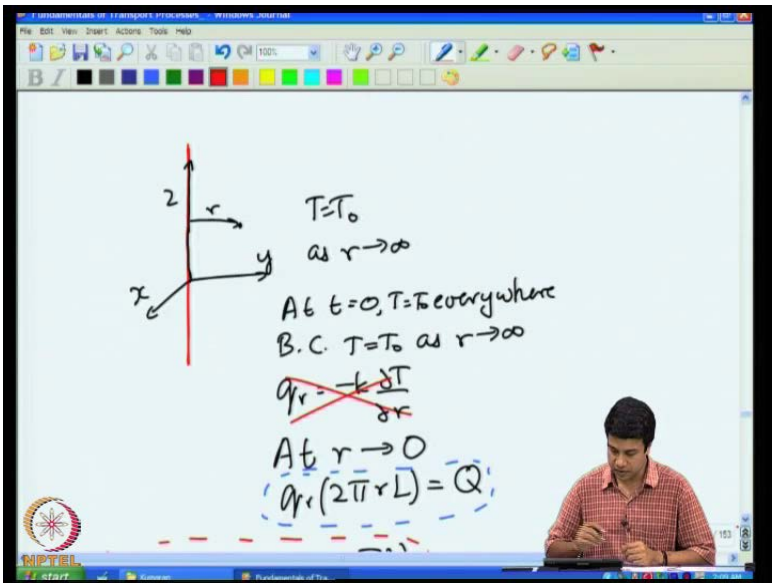

And then we did it solution for similarity variables, where we had to specify not the temperature along the axis of the cylinder, we travel the flux along the axis of the cylinder. The flux itself goes to infinity. But the surface area very close to the cylinder goes to 0, as the cylinder becomes thinner and thinner; and on this basis we manage to get a similarity solution, for the temperature field.

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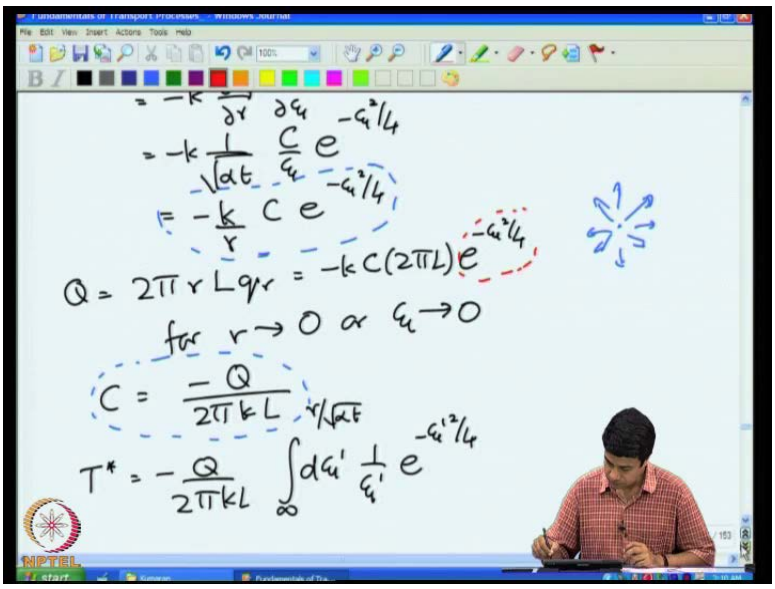

So, in this case, rather than specifying the value of the temperature at the central axis itself, it is necessary to specify the value of the flux.

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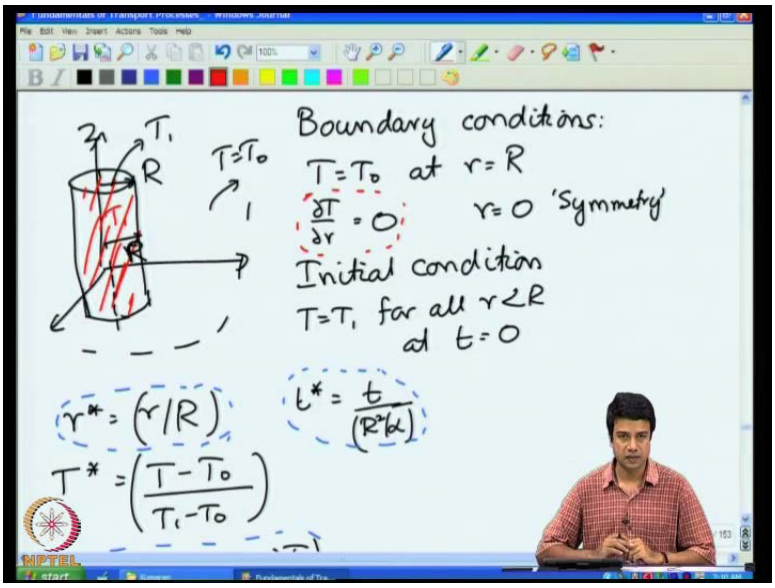

Then we looked at a separation of variables problem, conduction into a cylinder of a finite radius.

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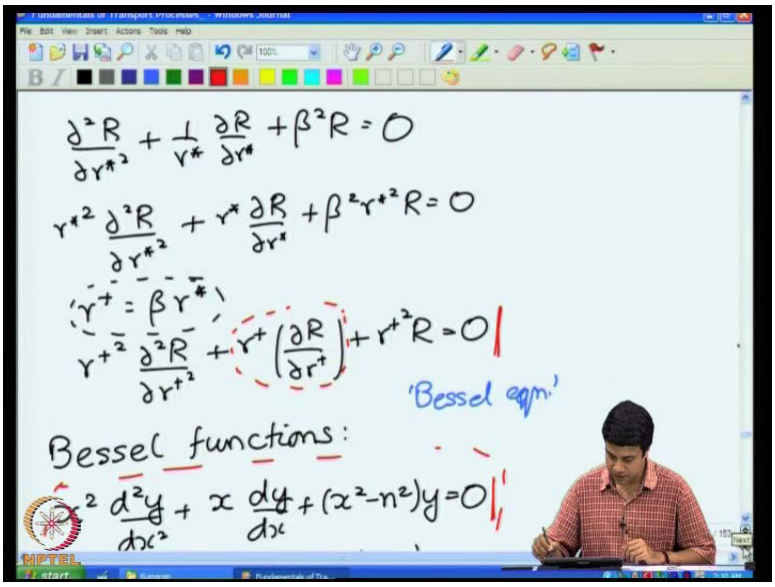

And in that case, the equation that we got for the Eigen function, for the spatial co-ordinate was slightly more complicated, in the case of transport from flat surface, the solutions were just either exponentials or sine and cosine functions. In this case, they are little more complicated, the solutions are what are called Bessel functions.

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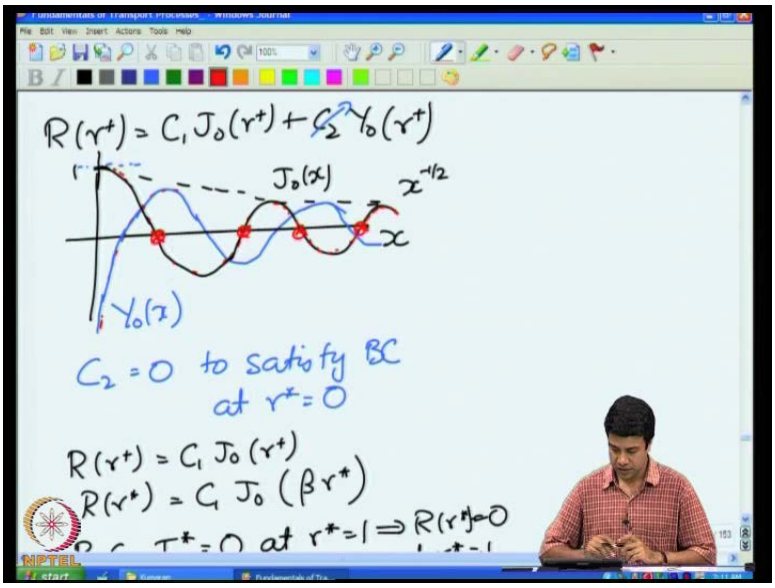

And I plucked out those Bessel functions for you, these functions as well form a complete basis set; and any function in cylindrical co-ordinates can be expressed as the sum of basis functions time times coefficients constants.

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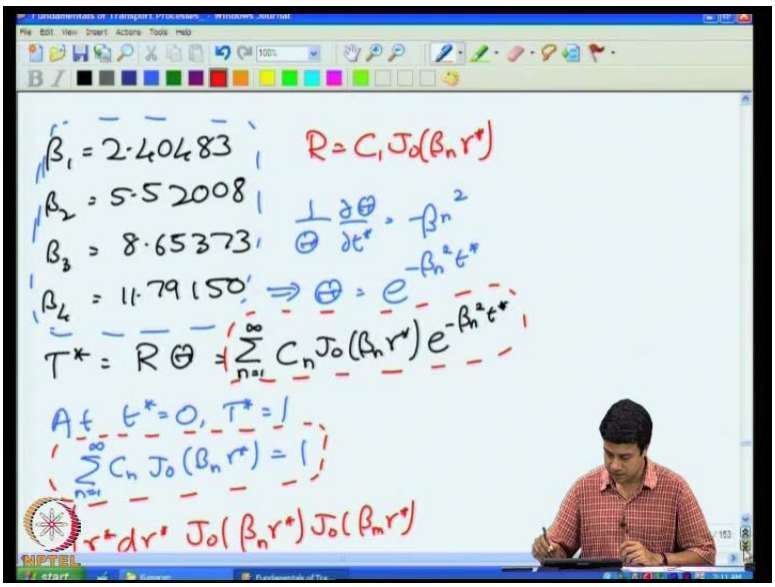

And those constants of course, are determined using orthogonality relations, for the basis functions.

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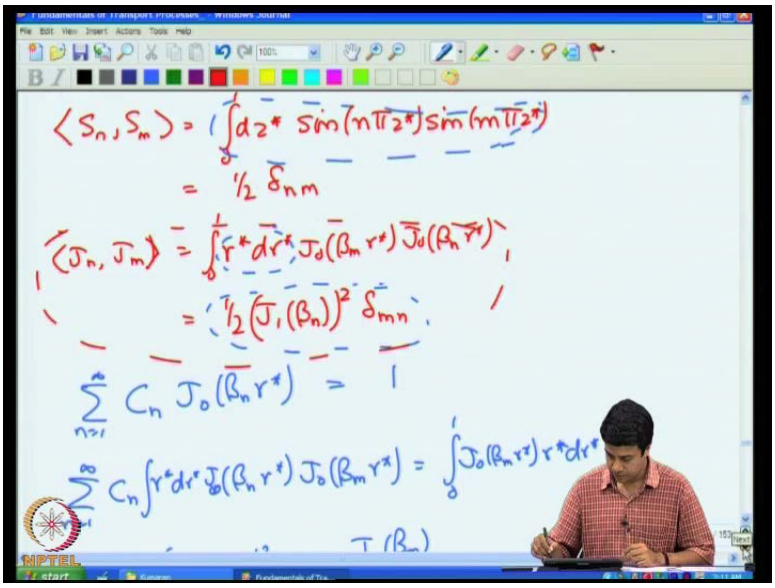

So, I showed you, how to get orthogonality relations for these basis functions.

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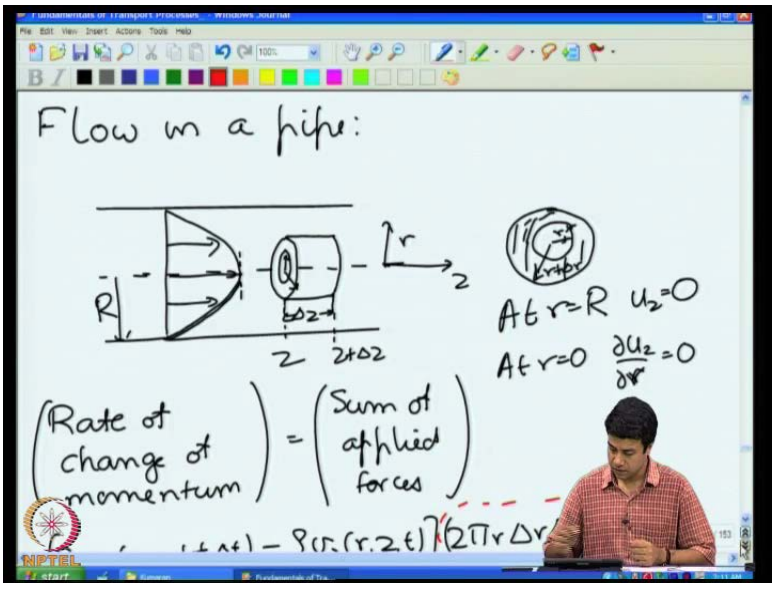

And then, we had solved the problem of the flow in a pipe.

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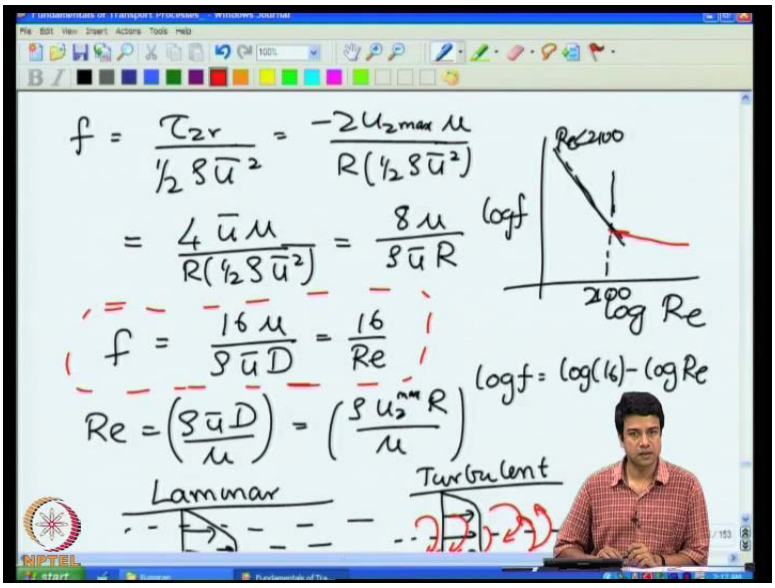

An oscillatory is first a steady flow for the parabolic profile, where we manage to get out the friction factor versus Reynolds number relationship f is equal to 16 by R e, this is the friction factor for the laminar flow in a pipe, and this much we can get just based upon simple shell balances. The laminar flow was valid only until a Reynolds number of about 2100, beyond that point there is a transition from a laminar to a turbulent velocity profile. And once the velocity profile becomes turbulent, this analysis no longer holds, because momentum diffusion is not due to the molecular mechanism alone, it also takes place, because of eddy diffusion. Eddies are correlated motion of parcels of fluid, which fluctuate in a turbulent flow, and the cross stream transport of momentum due to these eddies results in a far more efficient transport of momentum as compared to a laminar flow. And therefore, the friction factor is much higher than, what you would expect for a laminar flow.

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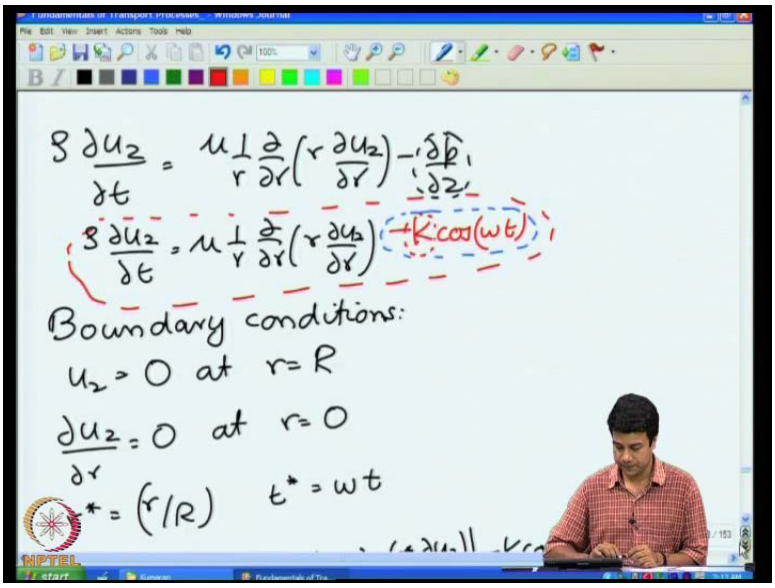

And then, we had looked at the oscillatory flow in a pipe, once again a laminar oscillatory flow, and I have illustrated two techniques of solution for you, the regular perturbation solution, and the singular perturbation solution, at low and high Reynolds numbers. We managed in this particular case, to actually get an analytical solution for the velocity profile, we just get that, we managed in this particular case to get an analytical solution for the velocity profile, in terms of Bessel functions.

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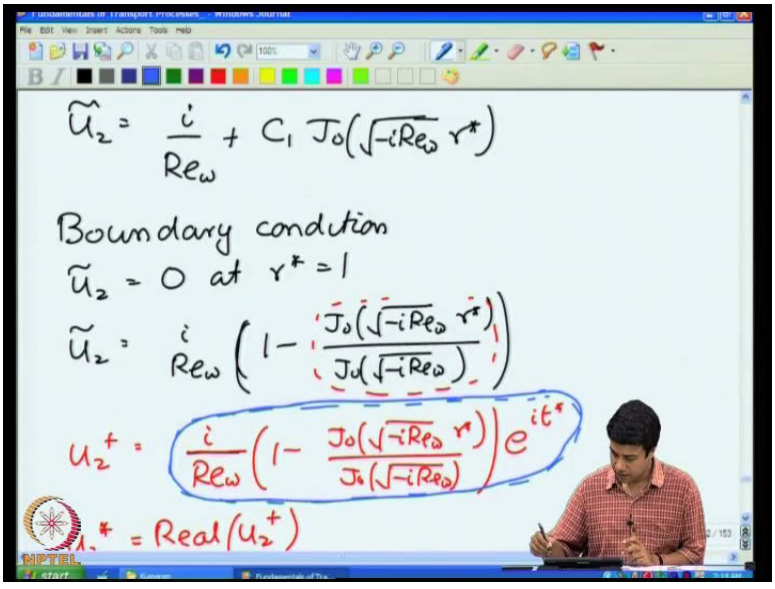

And even though you manage to get this solution, it is a complicated solution, you end up having to take the real part of this times e power i t, and then plot it for different values of Reynolds number, and t in order to get the velocity profiles, the actual velocity profiles. However, one can get a better physical understanding by looking at the limits of high and lower Reynolds numbers.

As I showed you in the limit of lower Reynolds number, the inertial term in the equation can be neglected in comparison to the viscous term; in this particular case once again there is a balance between the pressure and gradient, and the viscous term in the momentum equation.

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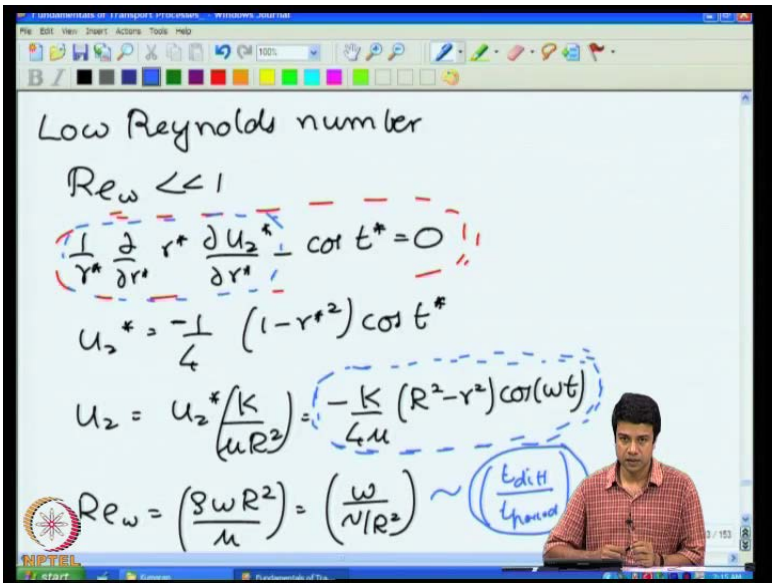

And so, the velocity profile that you get is identical to the velocity profile that you would have got, for a steady flow except that now, instead of the steady pressure gradient d p by d x, I have instantaneous, pressure difference at a time t. And I had explained this to you on the basis of interpreting the Reynolds number, as the ratio of two times (()); one is the diffusion time, the time it takes for momentum diffusion over a thickness of order r, that is R square divided by the kinematic viscosity.

The second is the time period of oscillation, if the diffusion time is small compared to the time period of oscillation, then diffusion is instantaneous, and so the velocity field responds instantaneously to the applied pressure gradient. So, in the limit of very low

Reynolds numbers, if we neglect the inertial terms completely, we just get back a parabolic velocity profile, but we can do better than that.

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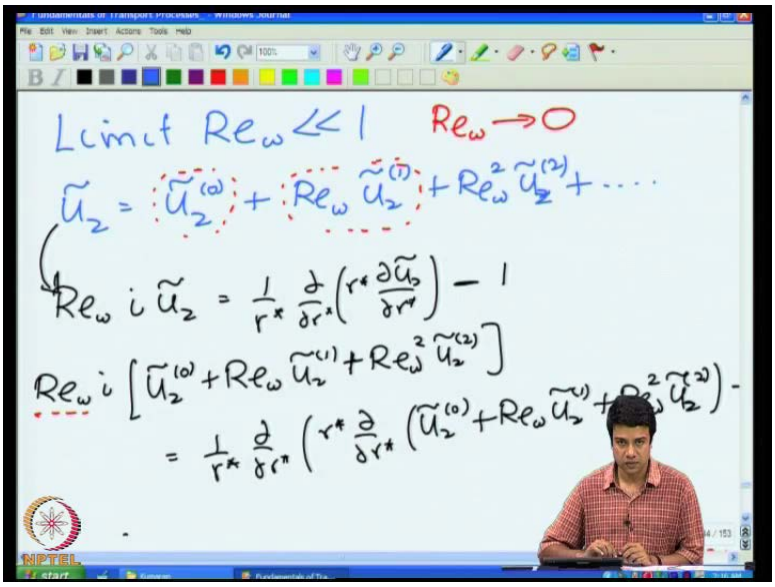

We can for example, consider the correction to this parabolic velocity profile due to inertial effects, using an expansion; we use the Reynolds number as a small parameter, and expand the velocity field in the Reynolds number, substitute this into the equation.

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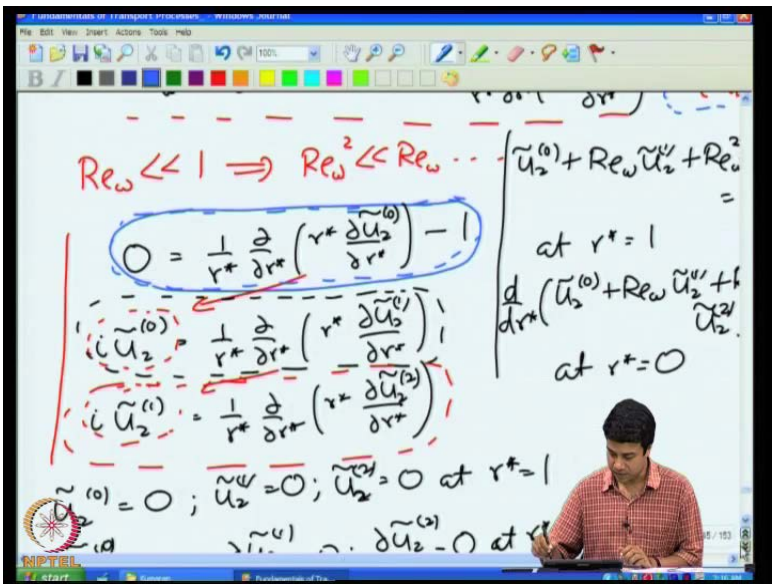

And end up with the series of equations, one which is the order one implying that it remains finite in the limit as R e goes to 0, then I have the order R e equations,

proportional to R e omega in the limit as R e omega goes to 0; the next higher is the equation proportional to R e omega square and so on. And in the limit R e omega going to 0, each of these individually has to be balanced, and from that I get a series, and I can solve at each order in R e omega. So, this was called a regular perturbation expansion.

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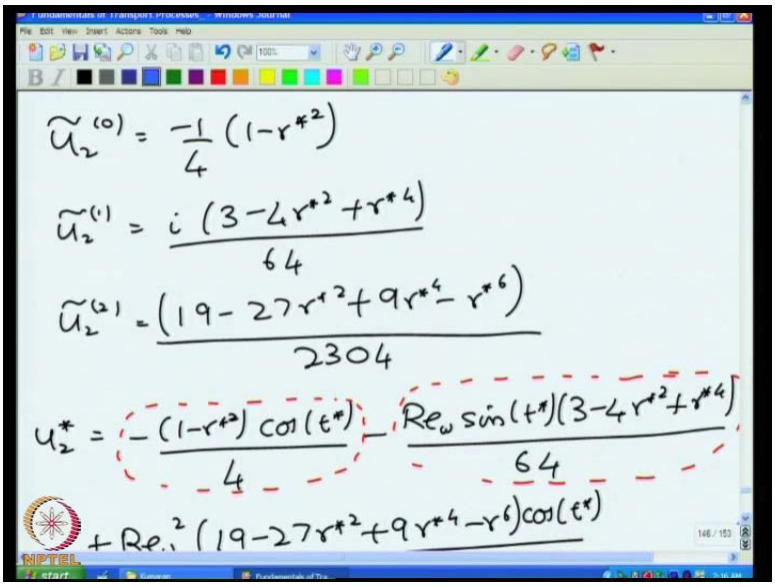

And I showed you how to get the solutions for at each order in the expansion, and put those together to get the final solution; it is called the regular perturbation expansion.

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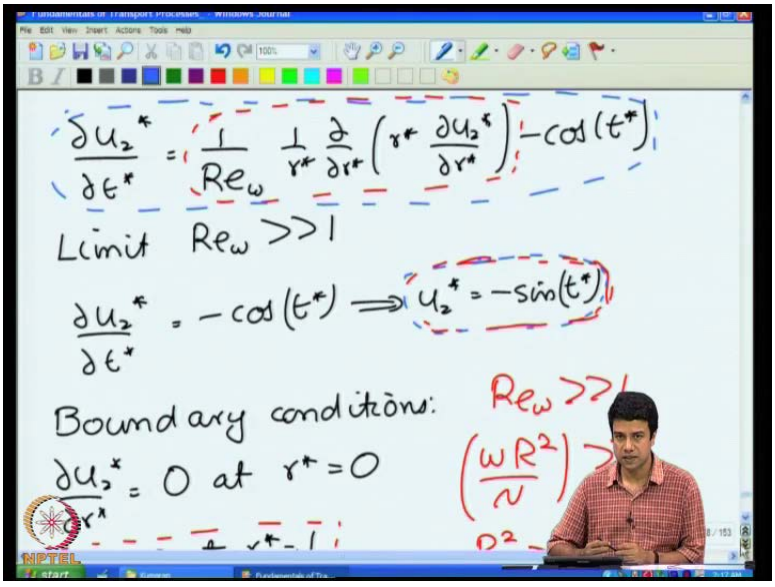

How about the limit where R e omega is large compared to 1? So, this is the limits where inertia is dominant, in that case I should scale the velocity by the inertial scales, in the equation. And I have 1 over R e omega multiplying the viscous term, naively you would think you can just neglect this term, and proceed and get a solution. Solution turns out to be simple, u z is equal to minus sin t, and problem is it does not satisfy the boundary condition. Because, when you are neglected the viscous term, I reduce the equation from a second order differential equation in r to an ordinary equation; an ordinary equation has no boundary conditions to it, so I cannot satisfy the boundary conditions.

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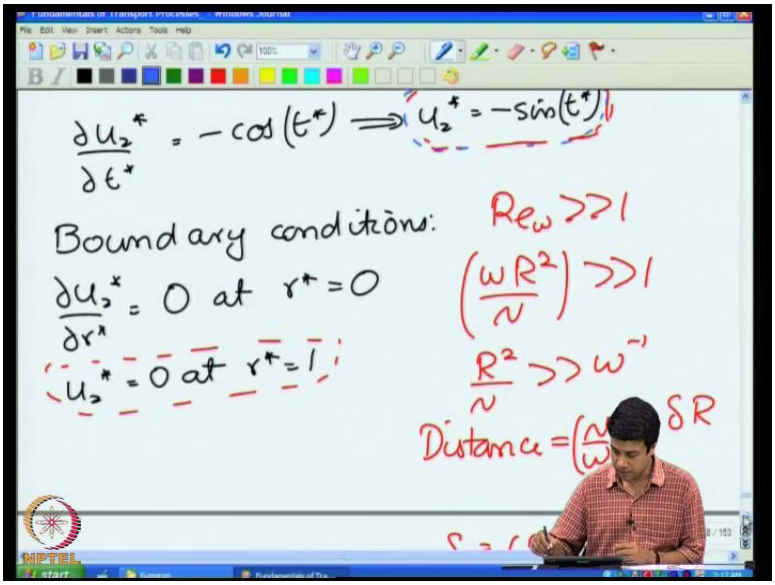

And once again physically I had explained this, in terms of a competition between momentum diffusion and the the the the period of oscillation, R e omega large compared to 1, implies that the time it takes for momentum diffusion is large, compared to the time period of oscillation. That means that momentum from the surface does not diffuse very far over one time period, so on that basis I had neglected the viscous term; however, when I took scaled the radius pi capital R, I was assuming that, the viscous term we have considering the time for the time for diffusion all the way to the centre.

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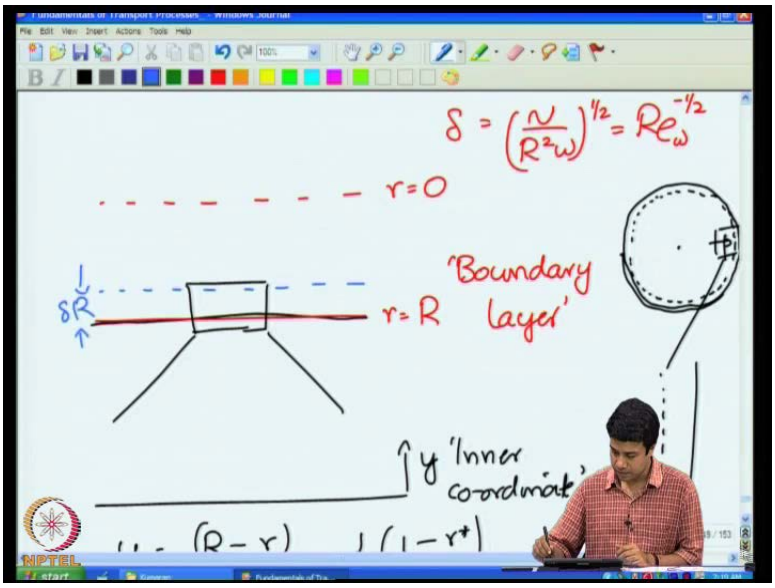

Even though the diffusion of momentum to the center, does not take place within a time period, proportional to the time of oscillation, there is still momentum diffusion. And it is confined to a thin layer near the wall; the thickness of that layer was obtained just simply on the basis of just on the basis of the kinetic viscosity and the period of oscillation. Just by dimensional analysis, the thickness of this layer over which diffusion takes place has to be the kinematic viscosity times, the period of oscillation to the half power or if I scale this by R then the thickness goes as the Reynolds number power minus half.

And once I know that I can rescale my co-ordinate in the inner layer, you can define a scaled co-ordinate y, which represents the thickness over which there is a balance between the inertial and viscous terms in the limit Reynolds number becomes large. And once I have defined it that way, I do get a balance between inertial and viscous terms within this layer, and I can get an analytical solution for the velocity field (No audio from 17:18 to 17:29).

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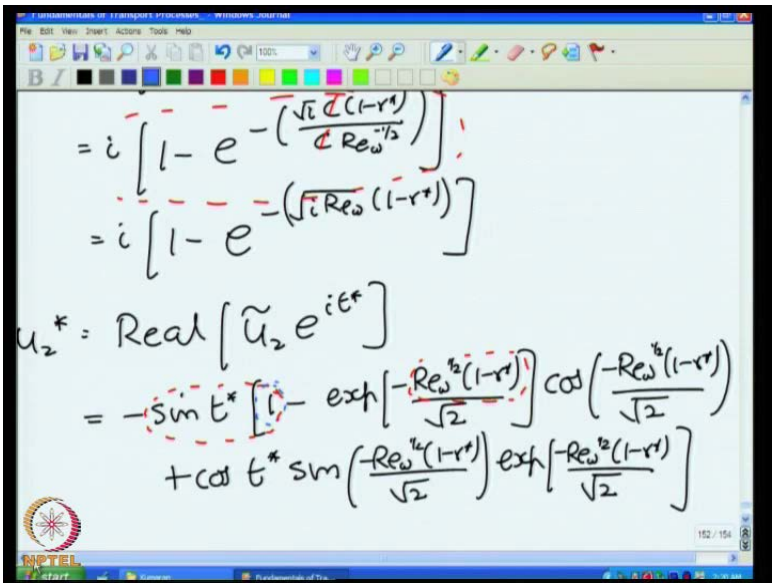

I have derived for you in the last class, the exact analytical solution for this velocity field, it looks very similar to the analytical solution for diffusion from an oscillating flat plate; that we had done in cartesian co-ordinate system. The reason is because, just as in cartesian co-ordinate system, in this case as well if the thickness of that boundary layer is small compared to the radius of curvature of the tube, very close to the surface, it appears to be diffusion from a flat surface.

Because, the curvature of the surface or the radius of curvature of that surface is very small, compared to the thickness I am sorry, the radius of curvature of that surface is very large, compared to the thickness of the boundary layer.

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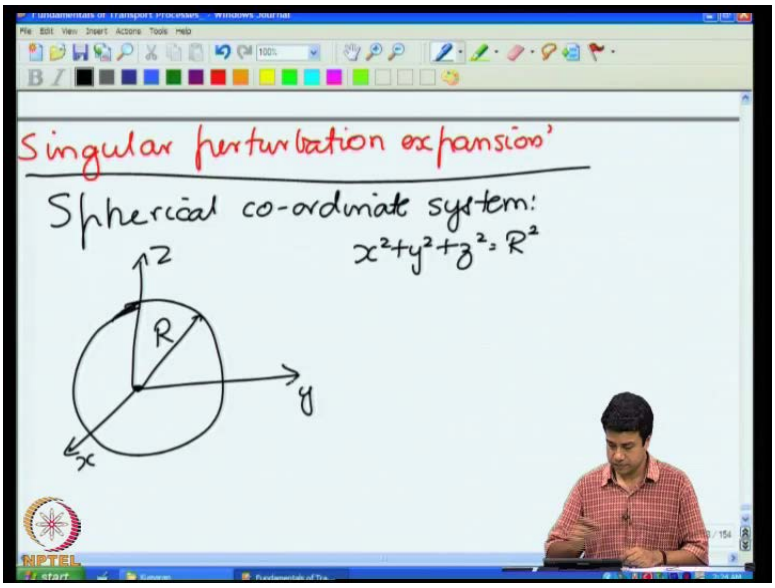

This I showed you as an example of singular perturbation expansions, when the ratio of convection to diffusion or inertia to viscosity is large, simplistically one would think, one can just neglect diffusion all together in proceed. However, even though convection is large compared to diffusion within the bulk of the flow, when one come comes very close to the surface, convection takes place only parallel to the surface, there is no convection perpendicular to the surface, because the normal velocity at the surface has to be identically equal to 0.

So, the velocity has to come to 0 at the surface that means that the convection perpendicular to the surface, there cannot be convective transport perpendicular to the surface. Ultimately transport perpendicular to the surface, has to take place only due to diffusion, and therefore, one has to postulate a layer of small thickness at the surface, where the thickness is small therefore, the gradients, the derivatives with respect to position or the gradients are large.

In such a way, that there is still a balance between convection and diffusion, within this layer, and that thickness of that layer is basically determined by this very balance requirement that in the limit Reynolds number goes to infinity. They should continue to be a balance between the convection and diffusion, within this layer of thickness delta which is small compared to the microscopic size.

So, that is why we are right now, next I will go into another example of say curvilinear co-ordinates, which is a Spherical Co-ordinate System, spherical co-ordinate system is useful for considering. For example, diffusion from a catalyst surface, a spherical catalyst particle or heating of of of spherical object and situations like that where one has spherical symmetry.

So, if I have a sphere without loss of generality, I can put my origin at the center of this sphere, and this sphere radius is R. So, if I want to try to express the equation of the surface in Cartesian co-ordinate system, I would have to save that x square plus y square plus z square is equal to R square which is rather cumbersome surface to deal with. So, I would prefer, if I could deal with situation, where the surface of this sphere itself where surface of constant co-ordinate.

Since, the surface is a spherical surface is curved surface, and then I have to have a curved surface on which there is a constant co-ordinate. And the co-ordinate system that is used for this is what is called as spherical co-ordinate system? In this particular co-ordinate system I would put the origin at the center of this sphere, in the origin would be right at the center of the sphere.

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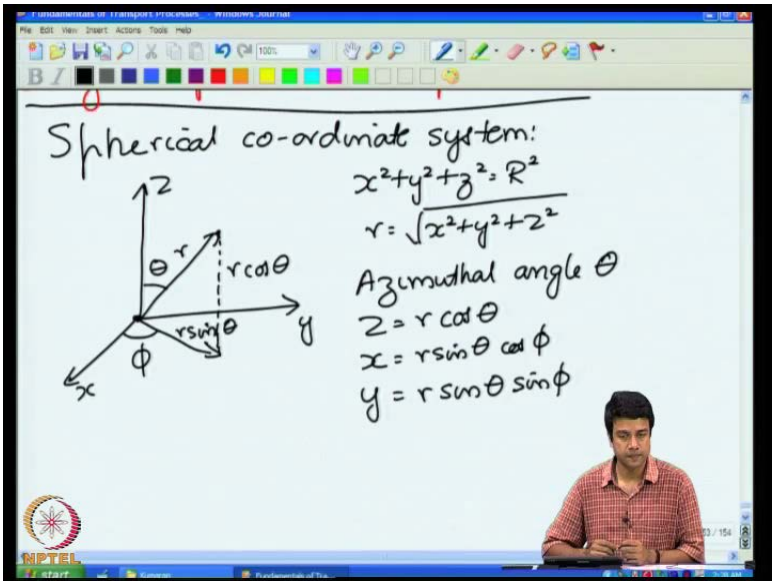

The distance of a point from the origin is the distance r, that is one co-ordinate in this (()), so r is equal to square root of x square plus y square plus z square, so I had x, y and z in the Cartesian co-ordinate system. I similarly need three co-ordinates in the spherical

co-ordinate system; traditionally r is defined as the distance from the center. Then I have two other co-ordinates, one is azimuthal angle theta, this is the angle made by the radius vector with the z axis; in general the z axis will be a along some line of symmetry and therefore, theta will be the angle made with respect to the z axis.

So, this basically gives you this varies from z is equal to from theta is equal to 0, at theta is equal to 0 the radius vector is along the z axis, at theta equals to pi the radius vector is along the minus z axis, so that z is equal to r times cos theta. So, z goes from positive to negative values along the positive z axis theta is equal to 0, along the negative z axis theta is equal to pi, so theta varies from 0 to pi.

And there is a third angle which is the angle made by the radius vector around the z axis, so if I take a projection of this radius vector on to the x y plane I take a projection of this radius vector along to the x perpendicular to the x y plane, then the angle that makes with phi with with the x axis is referred to as phi; the angle that the projection on to the x y plane makes with the x axis is called phi.

So, because the angle that r makes with the z axis is is is theta therefore, this is equal to r cos theta and this projection is r sin theta, the projection on to the x y plane is r sin theta, which means that x is equal to r sin theta cos phi, and y is equal to r sin theta sin phi. So, that gives me the relationship between the co-ordinates in a Cartesian and a spherical co-ordinate system.

In this lecture, we will we will consider the simplest case, where the temperature concentration, we would not consider velocity fields for the present spherical co-ordinate system, because it becomes a little complicated. But, for the concentration or temperature fields for mass or heat transfer, we will consider the simplest case where there is a variation only in the r direction, and there is no variation in the theta and phi directions. That means that the temperature and concentration fields depend only upon the distance from the origin, not upon the angle around the origin or the angle with respect to the z axis.

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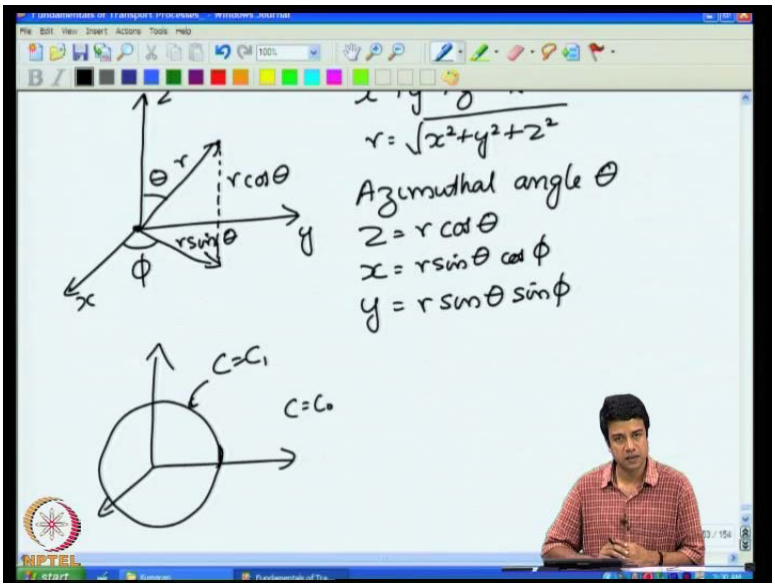

That is of course, valid if I had the simplest case that is if I had for example, a spherical particle with concentration C is equal to C 1 at the surface, and C is equal to C naught far away from the surface; so, catalyst particle for example, where there is some reaction happening at the surface, approximate that catalyst particle as a spherical particle. And if I put my origin at the center of this catalyst particle, then the concentration everywhere on the surface is C 1 far away is C naught.

Therefore, the concentration should vary only depending on the distance from the origin or the r co-ordinate, it should not vary as I go around the spherical surface or as I go around any spherical surface present the fluid, because there is diffusion taking place only radially outwards. So, we will consider this simplest case first, and write a shell balance for this particular case. The shell I told you have to be between two surfaces of constant co-ordinate, in this case we are considering a spherical surface.

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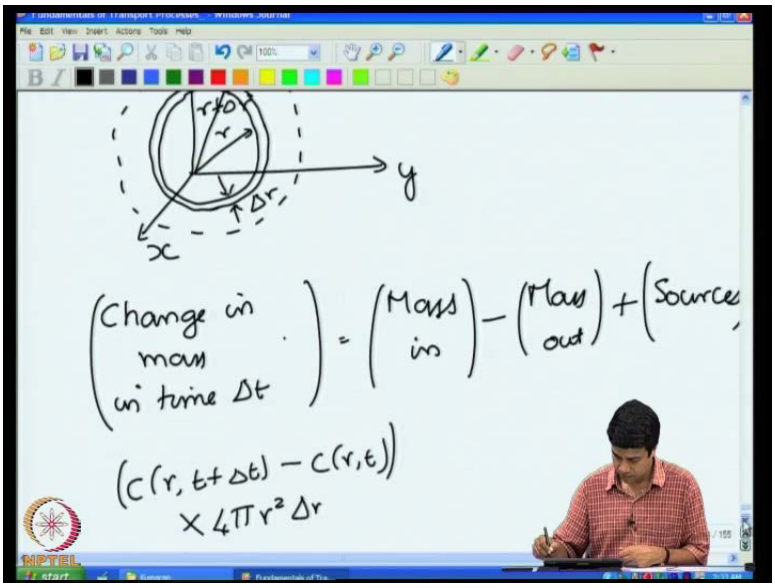

And therefore (No audio from 27:34 to 27:56), if I have a particle that look something like this, I consider a shell between two surfaces, one of radius r and another spherical surface at radius, r plus delta r, there are two surfaces; one surface is at radius r and the other is at radius r plus delta r, the thickness of this shell is delta r. And let us derive the concentration equation first, since that is the simplest, if the concentration equation would basically state that, change in concentration in time delta t over at least say change in mass is equal to mass in minus mass out plus any sources that are present within the time delta t.

So, what is the change in mass in time delta t, the change in mass will be equal to the concentration at r t plus delta t minus concentration at r times t; concentration is mass per unit volume therefore, I have to multiply this by the volume of the shell. The volume of the shell is equal to the surface area times the thickness, surface area 4 pi r square thickness delta r therefore, the volume of the shell is 4 pi r square delta r (No audio from 30:17 to 30:38).

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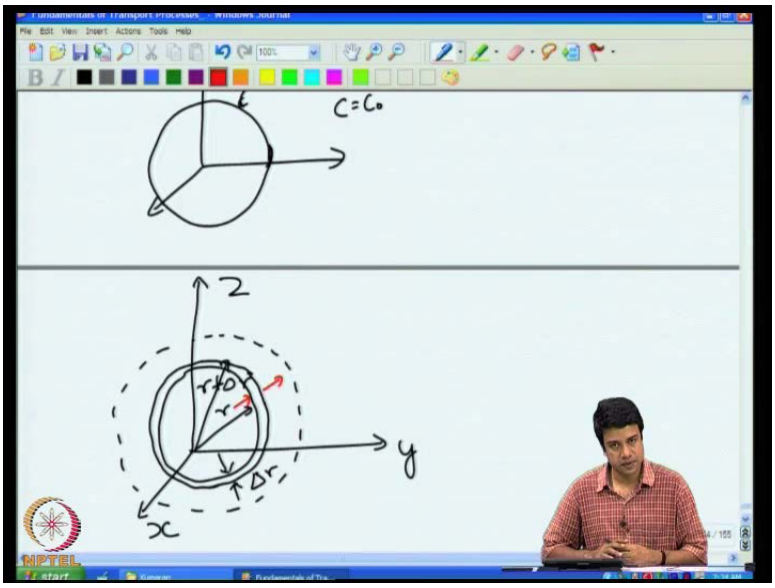

What is mass in? Mass in is going to be equal to the flux times the surface area, in this particular case, I have defined the radius vector as the distance, the vector from the origin to the surface therefore, there is a mass in at the surface at r. So, at the surface at r that is going to be a mass in, at the surface at r plus delta r the mass is going out of this volume.

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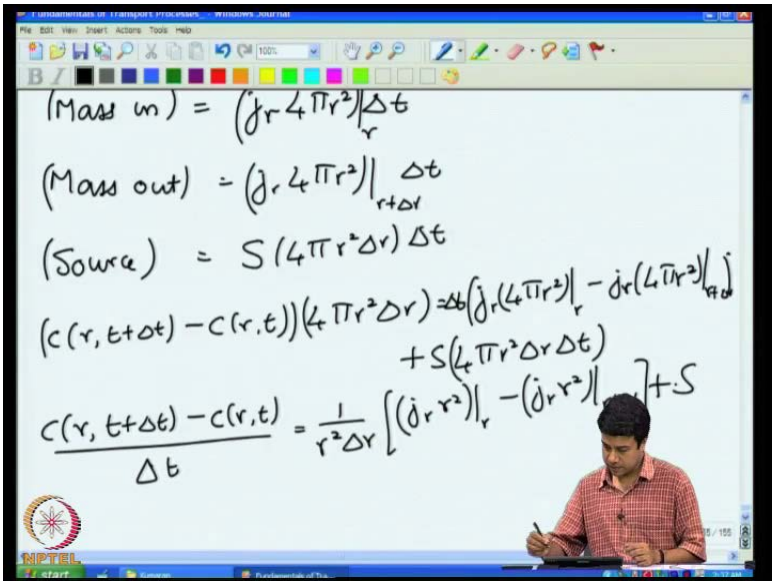

The mass in at the surface at r is going to be equal to the flux times the surface area the mass in at the surface at r is going to be equal to the flux times the surface area, so it is going to be the flux j r times the surface area which is 4 pi r square, flux times surface

area is mass per unit time, so I have to multiply this by delta t to get the total mass in within the time delta t. And this is at the location r, this mass in as I said is at the surface at distance r from the origin, mass out is at the surface at distance r plus delta r from the origin. So, therefore, the mass out to be equal to j r times 4 pi r square at r plus delta r times delta t (No audio from 32:22 to 32:39).

And finally, I have the source, if I had some reaction taking place and the reaction rate was given by S the reaction rate was given by S that is the rate of reaction per unit time, per unit volume. Then the total amount of material that is generated within the time delta t is going to be equal to S times the area times delta t. I am sorry S times the volume times delta t, because S is the rate of reaction per unit volume per unit time, so S times volume 4 pi r square delta r times delta t.

So, for the spherical co-ordinate system putting all this together, we get C at r t plus delta t minus C at r t times 4 pi r square delta r is equal to j r times 4 pi r square at r minus j r times 4 pi r square at r plus delta r, the whole thing multiplied by delta t plus the source into 4 pi r square delta r delta t, so that is the total equation. And then we divide throughout by volume times time, we divide throughout by 4 pi r square delta r delta t, in order to get C at r t plus delta t minus C at r t divided by delta t is equal to 1 by r square delta r into j r times r square at r minus j r times r square at r plus delta r plus any source that is present.

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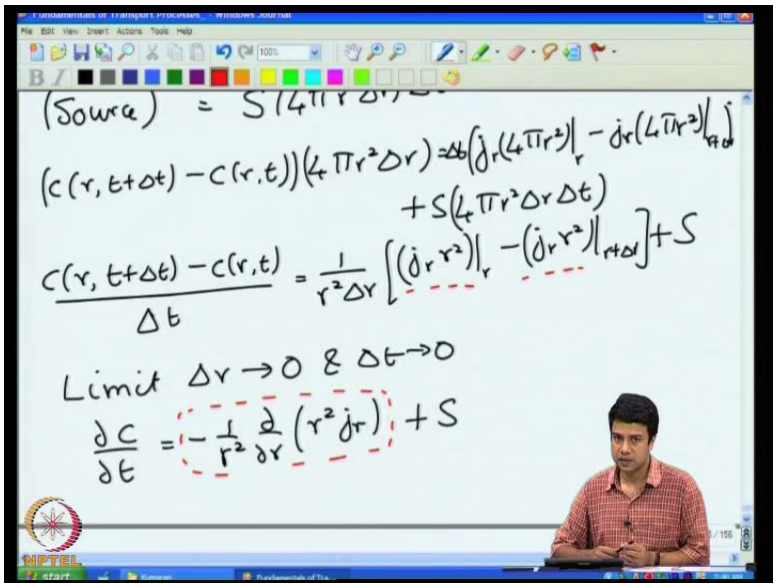

Note that in these two terms, just as in the cylindrical co-ordinates, I cannot cancel the r square in the numerator and the denominator, because the surface area at r, the surface area at r plus delta r are different in general. So, I cannot just go ahead and cancel out, r square between the numerator and the denominator. Now, taking the limit (No audio from 35:38 to 35:48), delta r going to 0 and delta t going to 0, I get a differential equation for the variation of concentration, in the spherical co-ordinate system.

The left hand side limit as delta t goes to 0 at C of t plus delta t minus C of t is just equal to d c by d t, on the right hand side I have j r times r square at r minus j r times r square at r plus delta r divided by delta r. Now, j r times r square at r plus delta r minus j r times r square at r would be d by dr of j r times r square, but I have the negative of that, so this is equal to minus 1 by r square d by dr of r square times j r, where j r is the flux plus any source or sink that is present, so this is the concentration equation.

Note that this term here is the diffusion equation, when we did it in Cartesian co-ordinates, we had d square, we had d j z by d z, when you did it in cylindrical co-ordinates we had 1 over r d by dr of r times j r; now we are doing it in a spherical co-ordinate system. I have minus 1 over r square d by dr of r square times j r, because in a spherical co-ordinate system surface area is proportional to r square where r is distance from the origin.

Cylindrical co-ordinate system surface area is proportional to r that is why, I had 1 over r d by dr of r times j r. So, to finally, get the equation for the concentration field, I need to express the flux, in terms of the gradient of the concentration, j r is the flux along the r direction.

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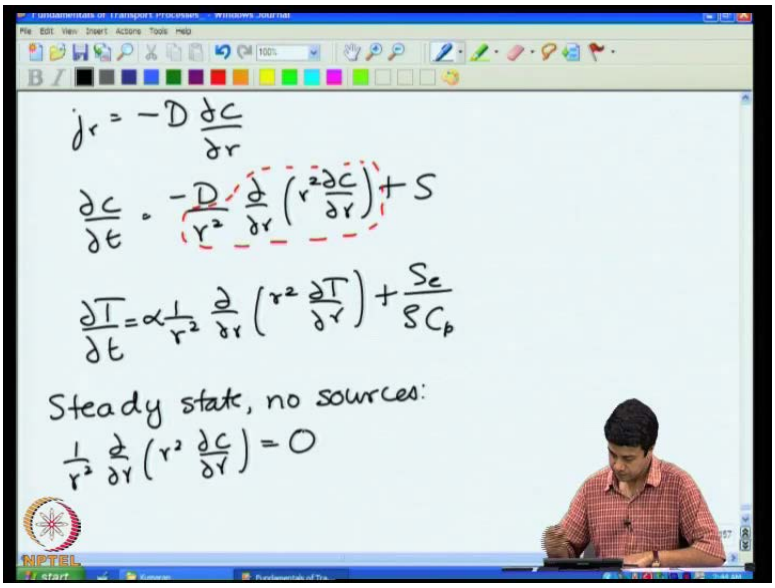

So, I need to express j r in terms of the concentration gradient, and as we know this is minus D times d c by dr minus of the diffusion coefficient times the change in concentration with distance. So, once I put this in, I will get d c by d t is equal to I will assume here that the diffusion coefficient is independent of position, then I will get D by r square d by dr of r d c r square d c by dr plus may sources or sinks.

Note this operator here, acting on the concentration is equal to diffusivity times 1 over r square D by dr of r square times d c by d r, so this is the equivalent of the diffusion of operator in a spherical co-ordinate system. I could do this quite easily for a heat transfer problem the only difference is that instead of C, I will have the temperature instead of D I will have the thermal diffusivity alpha (No audio from 39:12 to 39:40), I will have the thermal diffusivity alpha instead of the mass diffusivity D, this is not as easy to do for momentum transfer.

So, because you have velocities velocity could be along the theta as well as the phi direction, and momentum itself is a vector, so this equation is in general difficult to solve in for momentum transfer problem. So, we shall not try to write down the extension of this equation to momentum transfer problems, so these are the basic governing equations in a spherical co-ordinate system.

Now, back to the original problem that we had, let us say there is a catalyst particle on which C is equal to C 1 at the surface, and C is equal to C naught far away from the

surface, and we want to calculate what is the concentration profile around this particle at steady state (Refer Slide Time: 40:22). So, the first case is steady state, and no sources for the steady state and you if have no sources, then this equation reduces to 1 by r square d by dr of r square d c by dr is equal to 0.

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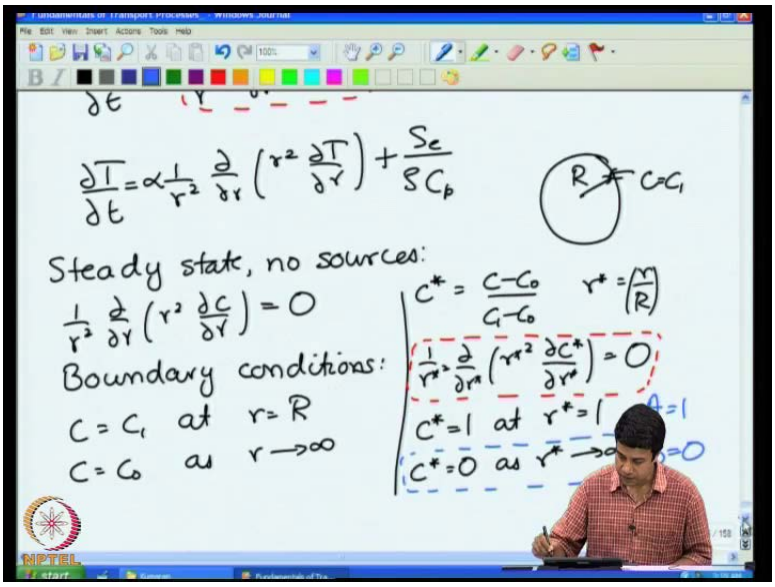

And as I said we have a catalyst particle of radius R, C is equal to C 1 on the surface and C is equal to C naught far away from the surface, as the distance becomes large C becomes equal to C naught. So, therefore, the boundary conditions (No audio from 41:42 to 41:55) C is equal to C 1 at r is equal to capital R, and C is equal to C naught as r goes to infinity far away from the surface, C becomes equal to the concentration C naught.

We can do scaling as usual, I could define a scaled concentration, C star is equal to C minus C naught by C 1 minus C naught, and r star is equal to r by capital R, so we are non-dimensionalizing the concentration and the distances. Then the equation remains the same, but the boundary conditions change, the equation remains the same 1 by r square d by dr of r square d c by dr is equal to 0.

Boundary conditions are C star is equal to 1 at r star is equal to 1 and C star is equal to 0 as r star goes to infinity for, so for the scaled concentration and distance fields, these are the boundary conditions. Now, if I solve this equation (No audio from 43:31 to 43:38), the solution of this equation is quite easy.

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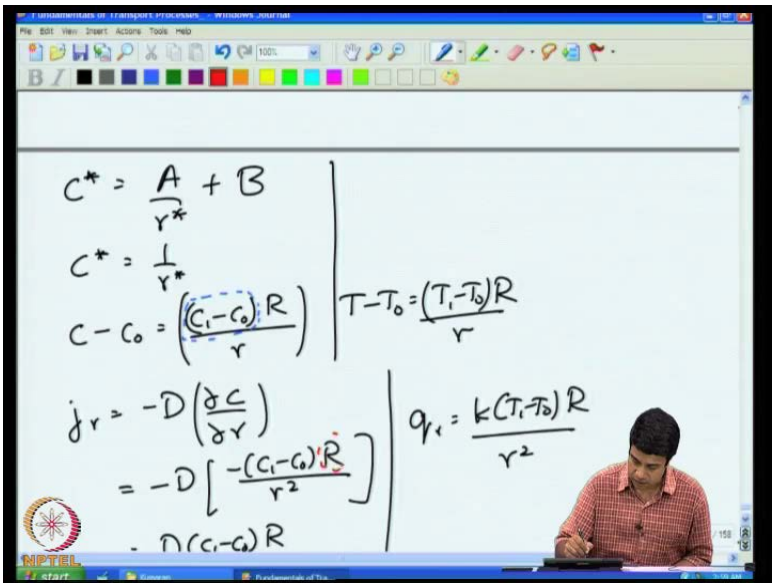

The solution is basically C star C star is equal to some constant divided by r star plus some constant B, A by r plus B, this is second order differential equation, it is quite easy to solve it. The boundary condition C is equal to 0 as r goes to infinity C is equal to 0 as r goes to infinity that means, that as r goes to infinity A by r star goes to 0 therefore, C is equal to B and if C has to be equal to 0 as r goes to infinity, that implies that B is equal to 0. And C is equal to 1 at r star is equal to 1 implies that A is equal to 1.

So, I gave the simple solution for this particular case, for the temperature field for the concentration field outside of a cylinder outside of a spherical particle, C star is equal to 1 over r star, alternatively if I have to express it back in terms of my dimensional concentration and distance, I would get C minus C naught is equal to C 1 minus C naught into R by r. So, this is the concentration field, it goes as 1 over r, as you go far away from the center of the spherical particle, the concentration decreases as 1 over r.

That is the difference between the concentration, in the concentration faraway decreases as 1 over r, equivalent temperature if I did the same problem for heat conduction, where the temperature was T 1 at the surface, and T naught faraway I would get the exact same result, except that I would have temperature, instead of concentration T minus T naught is equal to T 1 minus T naught into R I am sorry by r.

So, this is all very well it depends, so the temperature field depends upon the radius of the particle and the distance, at the surface itself capital R by small r is 1 and therefore, I

just get C 1, far away capital R by small r is 0 and it goes to C is equal to C naught. So, it decreases inversely with distance from the center of the co-ordinate system. What about the flux of mass, the flux j r is equal to minus D times partial c by partial r which is equal to minus D times minus of C 1 minus C naught R by r square. So, the flux is equal to D C 1 minus C naught R by r square.

So, the flux decreases as 1 over r square the flux decreases as 1 over r square, the surface area increases proportional to r square, the surface area is 4 pi r square therefore, the surface area increases proportional to r square. Therefore, the mass coming out of any surface is exactly the same, it has to be we are at steady state, there are no sources or sinks anywhere, mass coming out of any spherical surface all the way from the particle surface, as you keep increasing the surface.

The mass that is coming out has to be the same, because it is it is a at steady state and there are no sources anywhere, surface area increases proportional to r square that means, at the flux has to be decrease proportional to 1 over r square, so just a mass conservation condition, same thing for temperature; q r is equal to k into T 1 minus T naught R by r square. Now, the total heat or the total mass coming out of this particle the total mass coming out of this particle capital J will be equal to 4 pi r square times j r the total mass coming out of this particle is going to be equal to 4 pi r square times j r.

And since, j r goes as 1 over r square, this is just equal to 4 pi R 4 pi dr into C 1 minus C naught, good so this is the total mass coming out. So, for example, if I had a catalyst particle with the reaction happening at the surface, this would tell me how much reaction is happening at the surface, the amount of material produced at the surface is equal to the total mass that is coming out. Similarly, for a heat transfer problem, if there were reaction at the surface the latent heat of the at that surface times the reaction rate gives me the total amount of heat coming out of the surface.

So, in that case Q will be equal to 4 pi K R times T 1 minus T naught, now if I wanted to express the temperature field or the concentration field, I had previously expressed concentration and temperature, in terms of the temperature difference or the concentration difference between the surface and faraway. I could also express it, as the a function of the total amount of mass coming out, and the total amount of heat coming out, for that I need to use this expression, and write C 1 minus C naught in terms of

18capital J (No audio from 50:24 to 50:35) (Refer Slide Time: 50:18), alternatively to use this expression and write T 1 minus T naught in terms of capital Q. So, since C 1 minus C naught is J by 4 pi D times r I find that C minus C naught is equal to J by 4 pi D times r.

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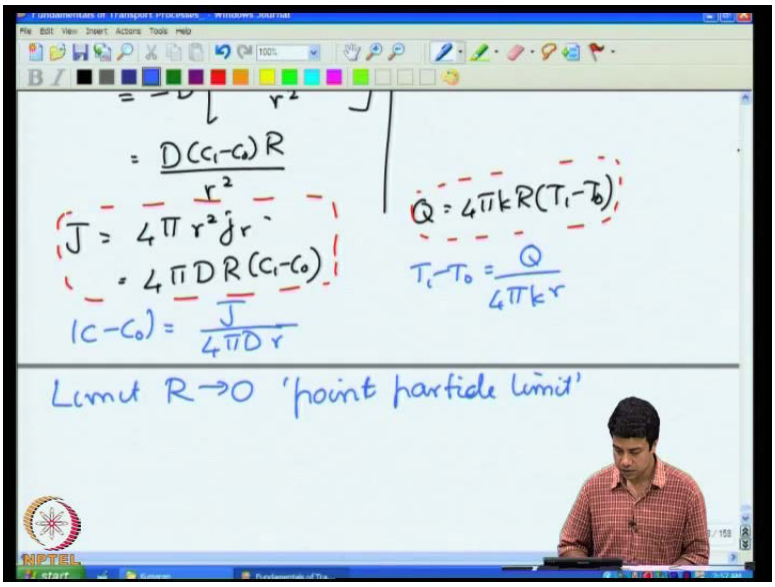

So, what I basically done is taken this expression, and substituted for C 1 minus C naught, in terms of capital J in this expression; you can see that since C 1 C naught is j by 4 pi D times capital R, that capital R will cancel out. And I will just get an expression for C 1 minus C naught as J divided by 4 pi dr or T 1 minus T naught is equal to Q by 4 pi k R. So, why I am emphasizing on this, because written in this manner, the temperature in the concentration fields are independent of the radius capital R.

Written in terms of the total amount of energy coming out or the total amount of mass coming out, the temperature in the concentration fields are independent of the (()) capital R. So, regardless of what capital R is, the temperature outside when expressed in terms of Q or the concentration, when expressed in terms of j is independent of capital R. In fact, this is solution even in the limit capital R going to 0, as a (()) particle smaller and smaller, even in the limit of the capital R going to 0, these solutions are still valid.

The limit are going to 0 is called the point particle (No audio from 52:37 to 52:47) (Refer Slide Time: 52:37), is called the point particle limit, in the limit as the particle shrinks to a point; in that limit even is R goes to 0 provided the mass flux or heat flux from the

particle remains a constant I am sorry, provided the total amount of mass or the total amount heat coming out of the particle remains a constant, you will still get this temperature or concentration field. So, this is the point particle limit, as I make the particle size smaller and smaller, while keeping the total heat generated by unit time or the total mass generated by unit time or constant, if I make the particle smaller and smaller, I still get the same expression.

In the limit as the particle radius goes to 0, the concentration goes as 1 over r or the temperature goes as 1 over r, and when expressed in terms of the total heat coming out per unit time or the total mass coming out per unit time, I get exactly the same solution independent of the radius of the particle. So, this is what is called the point particle approximation; and this forms the basis for solutions called greens functions solution, that we will see a little later, when we deal with diffusion equation, as I said this equation is the diffusion equation at steady state.

So, just to briefly recap, we started looking at a spherical co-ordinate system, I had define the co-ordinates for you the important thing is that in this co-ordinate system the radial co-ordinate r if the center, is the distance from the origin of the co-ordinate system (Refer Slide Time: 54:18). So, this is useful for spherical surfaces, provided I place the origin at the center of this sphere. So, if I place the origin at the center of this sphere, then I have a solution, and then I have a radial spherical co-ordinate system, in which the distance from the center is the co-ordinate r.

And then I have two other angles, the theta and phi angles, we shall call the azimuthal and meridian angles theta and phi, for the present we considered only transport in the r direction, where I have spherical surfaces at r and r plus delta r; and I did a shell balance for these two spherical surfaces. And you end up with and equation of this kind d c by d t is equal to minus d into this operator 1 over r square t by dr of r square d c by d r, in the case of a Cartesian co-ordinate system, this was just d square c by d z square.

That is because, the surface area was independent of the z co-ordinate, in the cylindrical co-ordinate system this was 1 over r d by dr of r d c by d r, that is because the surface area of a cylindrical surface increases proportional to r, this present spherical co-ordinate system it is 1 over r square d by dr of r square d c by dr. That is because, in a spherical ordinate system, the surface area increases proportional to r square as the distance from

the center increases. we solve the simplest problem study state, no source or sinks and in that case we got a solution, that goes as where the temperature or the concentration goes as 1 over r, this equal to C 1 minus C naught times capital R by small r, if C 1 is the concentration of the surface, C naught is the concentration faraway (Refer Slide Time: 56:24). However, if I express this in terms of the total mass generated per unit time or the total heat generated per unit time, it turns out to be independent of this radius R of the particle; I just get Q by 4 pi K R for the temperature field or C by 4 pi dr for the concentration field.

So, I just have to substitute the conductivity for the diffusivity, and the heat flux for the mass flux, and in this case it is independent of the radius, so it is works only even for a point particle, in the limit as the radius goes to 0. Next lecture, we will look at unsteady solutions of this diffusion equation, in spherical co-ordinates, continue this in the next lecture, will you see then.