**Fundamentals of Transport Processes**
**Prof. Kumaran**
**Department of Chemical Engineering**
**Indian Institute of Technology, Bangalore**

**Module No. # 04**
**Lecture No. # 21**
**Unidirectional Transport Cylindrical co-ordinates - VI**
**(Oscillatory flow in a Pipe regular perturbation expansion)**

Welcome to lecture number 21 in our course on Fundamentals of Transport Processes and we had got down to the business of an actually analyzing flow of importance in practical applications. We look at transport and cylindrical co-ordinates and I told you that, in cylindrical co-ordinates is an example of curve linear co-ordinates, where the co-ordinates constant are is not a straight line and due to that, the form of the differential equation that we get for unsteady transport is slightly different. From that what we had for the transport from a plain surface and we had looked at a couple of problems one over heat conduction from wire using the similarity transform and second problem that, we looked at was the unsteady state conduction into a cylinder into a cylindrical volume.

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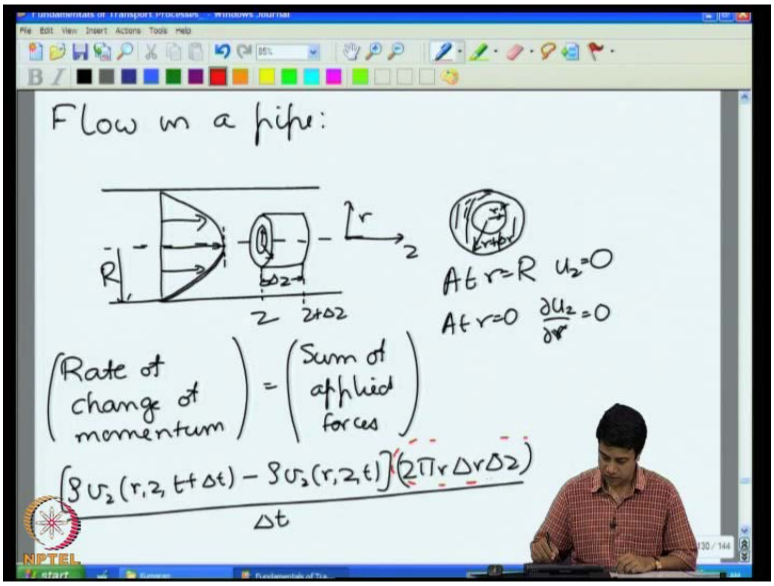


And then, we had started looking at the flow in a pipe and doubtless needless to say this is commercially important flow. And the way we solve this problem was to first consider a cylindrical shell and apply the momentum balance condition; rate of change of momentum is equal to sum of applied forces on the surfaces.

The rate of change of momentum of course, the change in the momentum for a unit time, which is the density times change in velocity divided by the time interval and of course, the density times the velocity is the momentum density for unit volume, so it multiplied that by volume of the shell over which you doing the balance.

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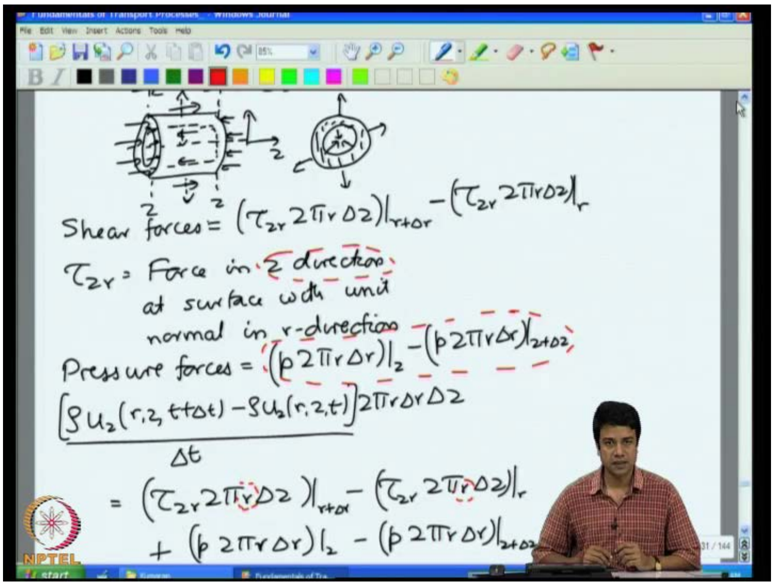


And once we do that, we get an equation for the momentum balance as a function of the forces acting on the surfaces of the shell; there are two cylindrical surfaces at r and r plus delta r that are bounding this shell and there are two plain surfaces at z and z plus delta z. We are writing a balance equation for the momentum in the z direction along the axis of the pipe; therefore, for the cylindrical surfaces the force exerted is due is to the shears stress, the viscous shear stress, that acts tangential to the surface along the z direction. For the two flat surfaces at z and z plus delta z, the forces are due to the pressure, which is normal to the surfaces as you know, pressure always acts perpendicular to the surface and is directed inward.

In this pipe flow, there is variation in pressure along the length of the pipe, the flow happens, because of you have a high pressure at the inlet and a low pressure at the outlet, this pressure difference causes the flow and there is pressure gradient at every point within the fluid, the pressure is gradually decreasing linearly with length as you go along the pipe and due to that, there is a pressure gradient therefore, if I take a small section of this pipe the cylindrical shell that have been analyzing all this wire, there is a difference in pressure between the surface at the left the upstream surface and the downstream surface and that pressure difference also enters into the momentum balance equation.

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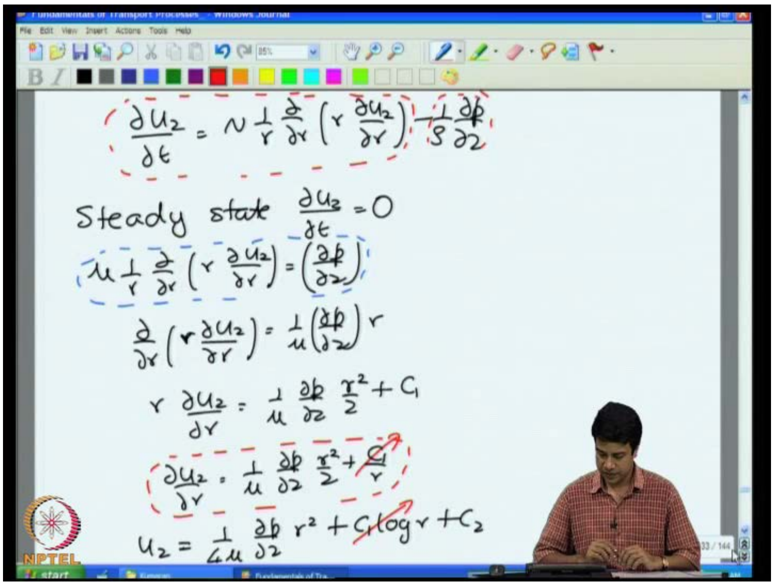


And once we put all of that in we got an equation for the unsteady fully developed momentum balance condition, fully developed means that, the velocity is invariant along the axis of the pipe, it does vary from the centre to the wall, but as you travel along the axis at any radial location, the velocity is independent of axial location. And within this momentum balance equation, we had put in an expression for the shears stress as the viscosity time the velocity gradient and from that, that we got momentum balance equation.

At steady state is of course, you can solve it quite easily, it is an ordinary differential equation in r, because the pressure itself is independent of r. I had discussed with you in the last class, why pressure is independent of r is basically because, if I write a momentum balance equation for the radial direction as well, it would contain various terms, the inertia in the radial direction which is proportional to the radial velocity, the viscous stresses, which are once again proportional to the derivatives of the radial velocity and there is also a pressure gradient in the radial direction.

The radial velocity is identically equal to 0 and therefore, the pressure gradient, the variation pressure in the radial direction has to be equal to 0 therefore, since the pressure is in front of the radial co-ordinate, it is a function only of the axial co-ordinate; so because the radial velocity is identically equal to 0, all terms in the radial momentum equation which depend upon the radial velocity are equal to 0. Therefore, the pressure variation in the radial direction also has to be equal to 0, therefore p is only a function of the axial co-ordinates.

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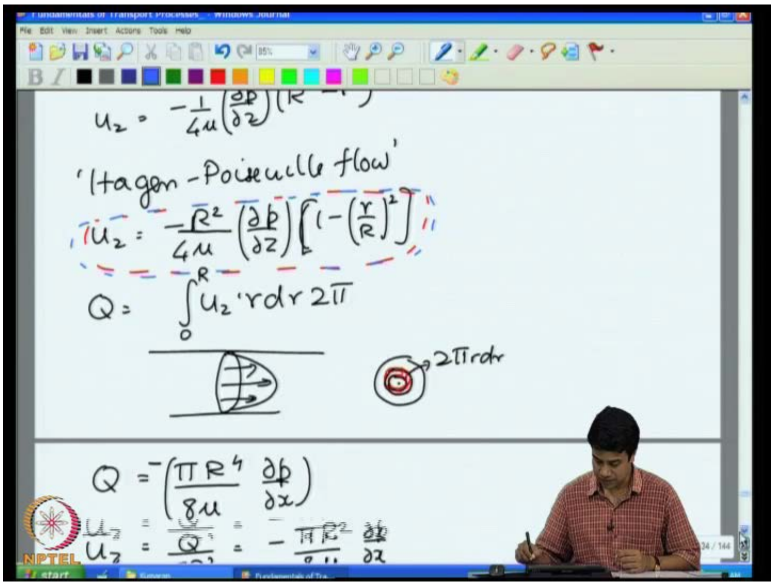


And, we had solved this to obtain the Hagen Poiseuille law for the flow in a pipe as the function of the pressure gradient. And of course, this gives you the variation velocity along the radial direction, the total volumetric flow rate through the pipe, if its volumetric flow rate I should not be having a density there.

So, the total volumetric flow rate through the pipe is equal to the velocity times the cross sectional area, but however, since the velocity is changing as a function of radius, I need to take a small section of cross section, find out the velocity on that, multiplied by the that by the area to 2 pi r times delta r and then, integrated over the entire cross section.

And that gives me flow rate, which causes pi r power 4 by 8 mu times d p by d x. The mean velocity is the flow rate divided by the cross sectional area, which turns out to be half the maximum velocity at the center of the pipe.

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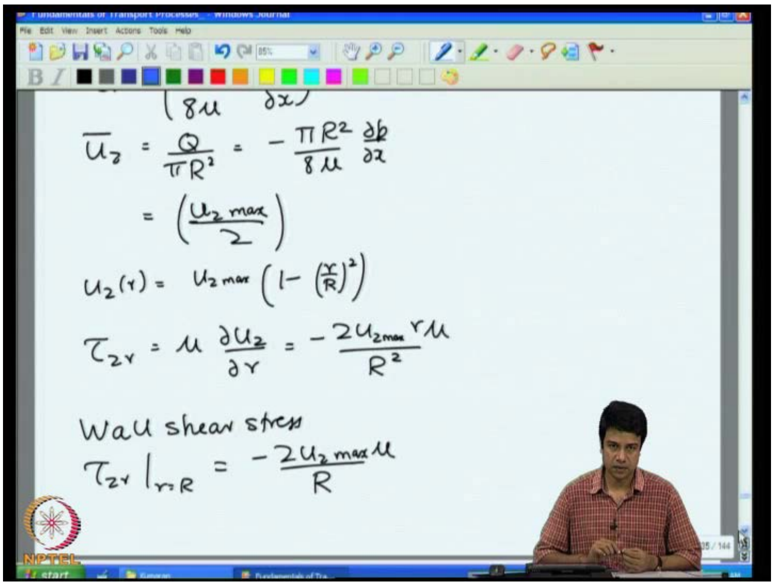


From that, we got shears stress, the shears stress at any point in the fluid and the shears stress at the wall by setting r is equal to capital R and from this, we got the shears stress as the function of the maximum velocity, we know how the maximum velocity is related to the pressure gradient. So, you get the shears stress as the function of pressure gradient.

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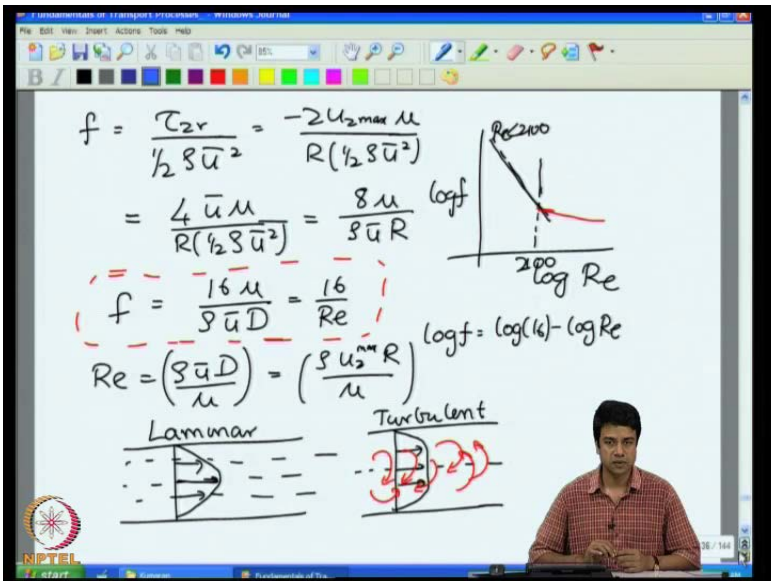


And from that, we got familiar friction factor verses Reynolds number relationship, f is equal to 16 by Re the for the laminar flow in a pipe and as I told you in the last class, the laminar flow is valid, when the Reynolds number is less than about 21 100, when the

Reynolds number goes beyond 21 100, there is a spontaneous transitions from the laminar flow to a more complicated flow profile called a turbulent flow.

Even when the Reynolds number is more than 21 100, the laminar velocity profile is still solution of the equations; however, that solution becomes unstable and any small disturbance will make the solution spontaneously go to some other solution. So, there is a transition from one solution, there is became unstable to another solution, that is transient for stable. And this turbulent velocity profile, as I told you consists of large fluctuations in the velocity, both in the stream wise and the cross stream direction.

There are eddy's correlated parcels of moving fluids of various length scale within the flow all the way from the large scale to a small scale is called the (( )) scale and because, there are the eddy's these also transfer momentum across the flow. In addition to the molecular diffusion mechanism; which transfers momentum across the flow, there is also the eddy diffusion mechanism due to passels of fluid moving in a co-related fashion; and that results in much higher rate of transfer, then what would you expect for a laminar flow, because the co-related motion of the eddy's transfers momentum across the flow for more efficiently than the molecular diffusion mechanism in a laminar flow.

And that results in a much higher friction factor or or or a drag force, the wall shear stress in a turbulent flow is much higher than what you would expect for the laminar flow, because of this efficient momentum transport mechanism and also because of the efficient momentum transport mechanism, the velocity profile is for flatter than the parabolic profile in laminar flow, it looks very much like a plug flow at the centre to the short transition to zero velocity in near the walls.

So, then we started looking at the problem of an unsteady flow in a pipe, an oscillatory flow. So, as I said for example, for the the pumping of the blood by the heart is oscillatory in nature; it is not in exact sine wave, but it still in periodic in time. So, in order to model these kinds of flows for example, take an oscillatory flow, where the pressure gradient or the pressure difference across the two ends is an oscillatory function of time.

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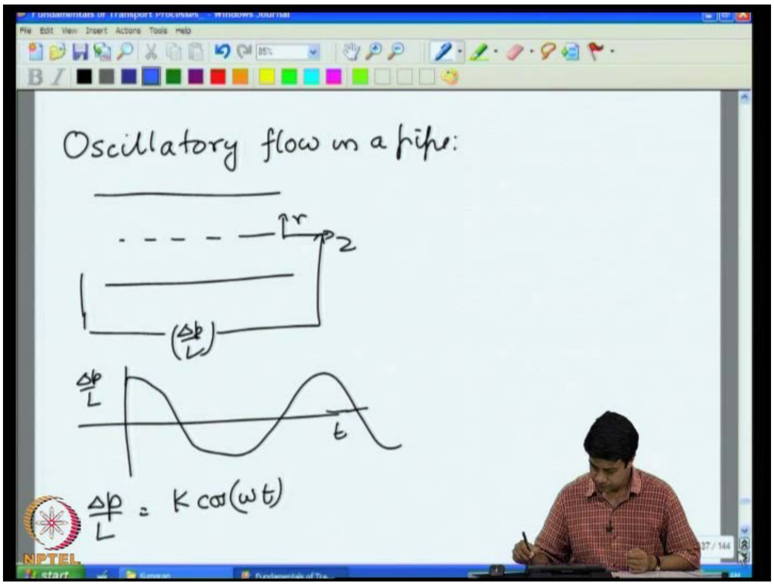


In this particular case, we took that oscillatory function to be cos of omega times t, where omega is the frequency of oscillation; however, this procedure can be used for any type of time periodic flows, because any periodic function can be expressed as the sum of sine wave of or cosine wave of that frequency plus it's higher harmonics.

So, it could separate out the wave form into fundamental mode and the harmonics, solve for the velocity field individually for each of these and then, add them of altogether to get there is the response for the entire periodic function that I have. So, in that sense, this procedure can be used in for more complicated modulations of the the the the pressure gradients across the tube.

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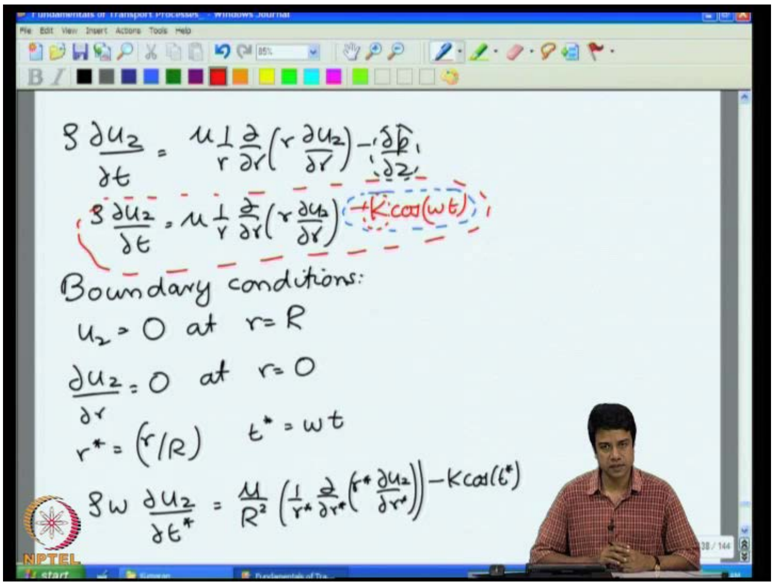


So, we have a differential equation for the velocity field, which contained an in homogenous term, this equation for the velocity field contain in a homogeneous term that was k cos omega t. The boundary conditions for the the flow through the pipe, no slip conditions at the wall, the velocity has to be equal to 0 at the wall, at r is equal to capital R; at r is equal to 0 we have the symmetric condition, that we had discussed earlier.

Because, the the the velocity gradient cannot be discontinuous at the centre, the the derivative of the velocity with respect to radius has to be equal to 0, only then the value the derivative be the same, when you approach it from different directions. And reduce the scaling t star square is equal to omega t and r square is equal to r by capital R and then, we had scale the equation.

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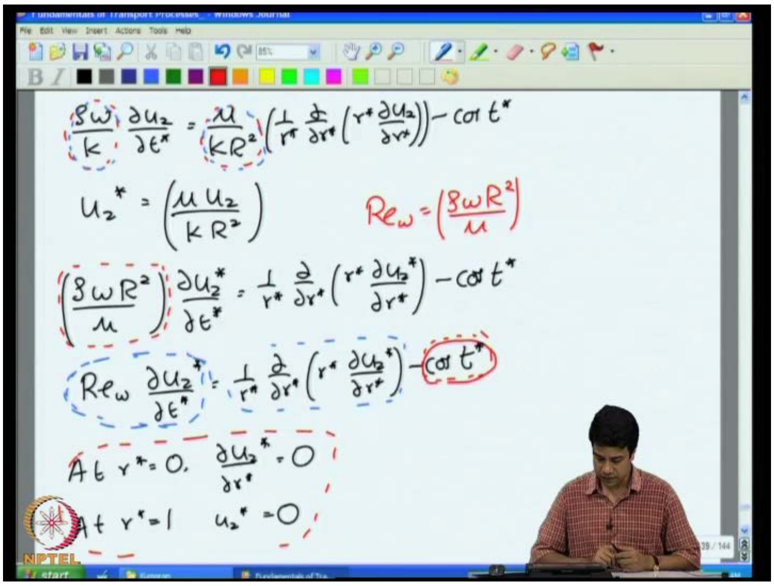


As I said, there are two ways to scale it; one is, with the inertial scale rho omega by k and other way is, the viscous scale mu by k R square you could choose either of these, either of these would give you a mathematically accurate result. However, if you want to use physical insight to solve the problem, you should scale it by the viscous scale, when the Reynolds number is small, so that viscous effects are dominant, because it expect viscous term to be large compared to inertial term in that case, whereas you should scale it by the inertial scale at high Reynolds number.

We started of scaling the viscous scales and procedure it to see what happens and outcomes Reynolds number, Re omega is equal to rho omega R square by mu, which is the ratio of the inertial term, the unsteady term and the viscous term; the the term due to viscous, when diffusion of momentum. And we got an equation for the velocity field in terms of Re omega and there is a homogeneous term, cos of t and the boundary conditions turn out to be homogeneous once again.

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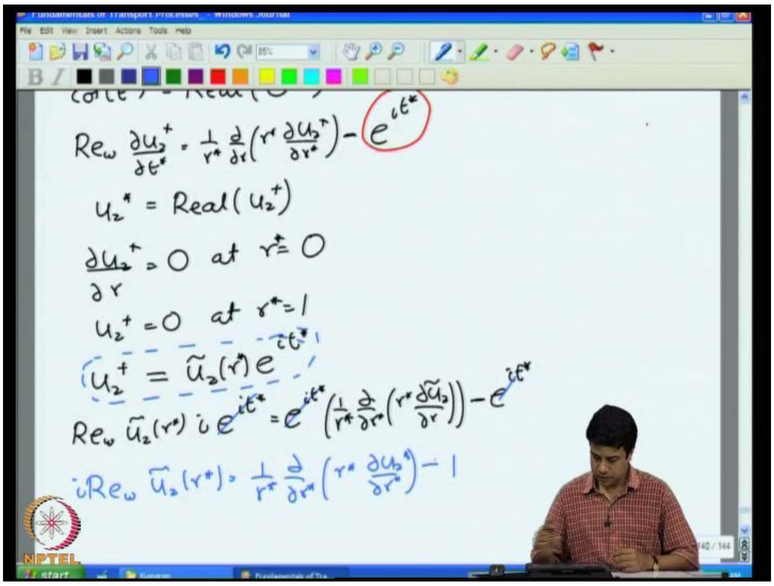


And, we solve this equation subject to boundary conditions, there is a mistake here subject to boundary conditions, the solution comes about quite easily, if I assume that I work with complex velocity, which is u z times e power i t. Basically, the system is being forced by an oscillatory pressure gradient, which is proportional to e power i t; that means, that you would expect the response also to have modulation with that same frequency. It may not have the same phase, but it has to have the same frequency, because my equation is linear. So, we put in trial function of the form u z plus is equal to u z tilde the e power i t and from that, we got from that we finally managed get the solution.

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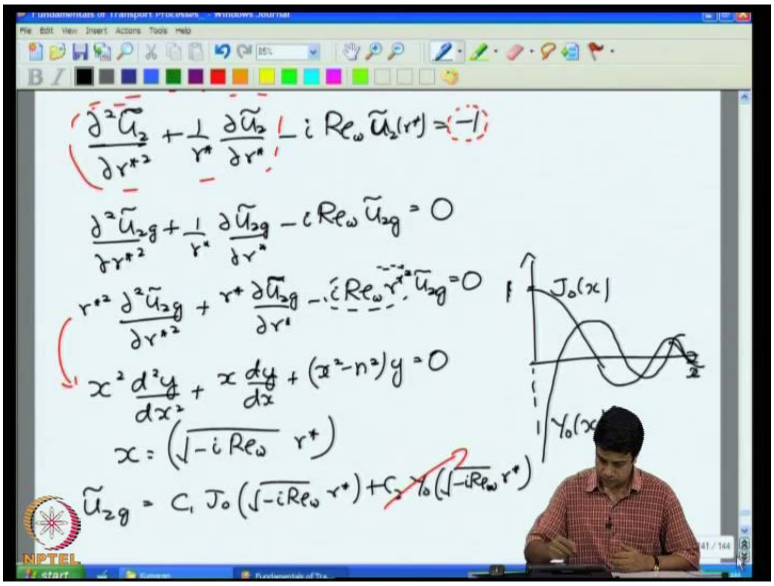


By using a separation into a particular solution and homogeneous solution; the homogeneous solution was in the form of Bessel functions, J naught and Y naught; straight away we could set the co-efficient of the term proportional to Y naught equal to 0, because we know that, the Bessel function Y naught goes to minus infinity at r star equal to 0. We had discussed the forms of the both J naught and Y naught as a function of x and J naught of x has an oscillatory form its starts at 1 and it has an oscillatory form, this is J naught of x, whereas Y naught of x starts at minus infinity.

Since, the Y naught of x starts at minus infinity, if the constant C 2 where nonzero then, the velocity the general solution goes to an infinity at at 0. So, since we cannot have that therefore, the constant C 2 has to be equal to 0 and this has to be plus 1.

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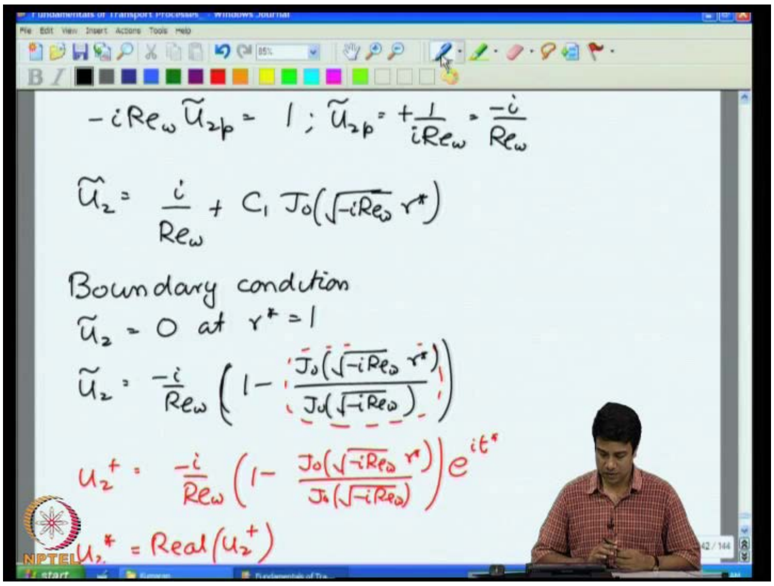


So, we got the particular solution as just a constant that is the simplest particular solution that will satisfy this equation. So, we got the particular solution as just a constant, the general solution as a Bessel function and from that we can constructed the total solution. And u z star is of course, the real part of complex velocity and from that, I can get the velocity variation as a function of time. So, this is the mathematical solution does not quite give us very much physical insight unless, we actually plotted it out and see how it looks.

In order to get more physical insight, one can look at the limits of low and high Reynolds number; in the limit of low Reynolds number, you can do one of two things; the first thing you could do is actually just take this solution, just take this solution and expand it in series in the Reynolds number, so that is one way to do it. The other way to do it is to take starting governing equation and then, expand in it series in Re omega. So, at the limit of Re small compared to 1, if I completely neglect the inertial terms what I get is this equation that looks something like this, this is 1 by r and d by d r of r d u z by d r minus cos t is equal to 0.

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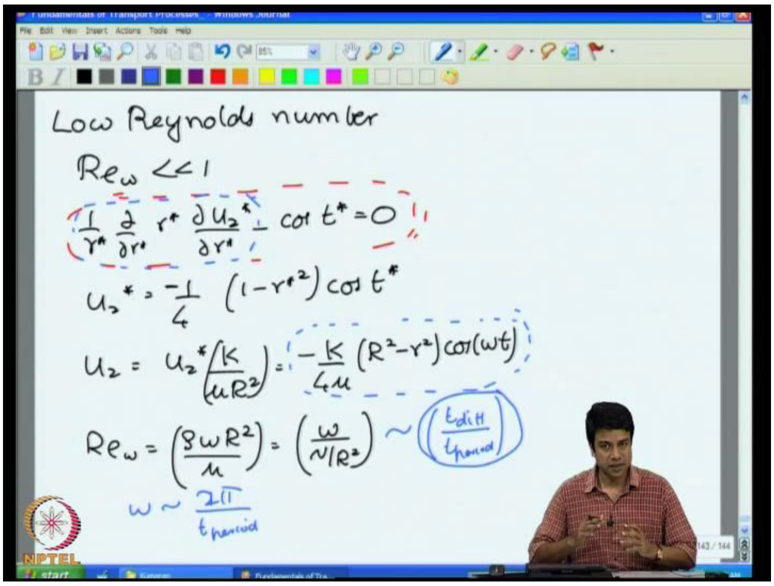


Now, this first term does not have any time derivatives in it, so because of that I can straight away integrate this in time to get the velocity as minus 1 by 4 into 1 minus r square into cos t and from that, we got the velocity about the scale velocity, dimensional velocity in terms of the pressure gradient and this dimensional velocity is identical to what you would have for the steady flow, if the pressure gradient was just given by k cos omega t, this is the same Hagen Poiseuille law for the flow in a pipe except that instead of having the steady pressure; the pressure in this case given by K cos omega t and I explained the reason for this in the previous lecture.

The Reynolds number, rho omega R square by mu I can write this as omega divided by nu by R square. So, this is equivalent to the time, the time it takes for diffusion, because divided by the time period of the oscillation, because omega is goes as 2 pi by the time period of the oscillation, omega 2 pi by the time period of oscillation. The time it takes for diffusion across a length proportional to R is equal to R square by mu, where R is the pipe radius and mu is the kinematic viscosity, kinematic viscosity has dimensions of length square per time therefore, the time taken goes as R square by mu.

So, therefore, time for the time for diffusion is R square by mu and I can write the Reynolds number as the ratio of the two time scales. Reynolds number is small implies that, the time taken is small compared to the period of oscillation by the time the the pressure changes it’s value over a time comparable to the period of oscillation, the diffusion takes place very fast compared to that and therefore, the velocity field looks like instantaneous velocity field you would have had, if the pressure gradient where given by that instantaneous value of the pressure gradient, because the the response of the fluid is much faster than the the rate at which the pressure is oscillated; the time for diffusion is as much smaller than the period of oscillation, so in that case you get something that is close to the steady well.

So, this is just the steady solution what happens, if the Reynolds number is not 0, but still a small number what happens the Reynolds number is not exactly 0, but still very small.

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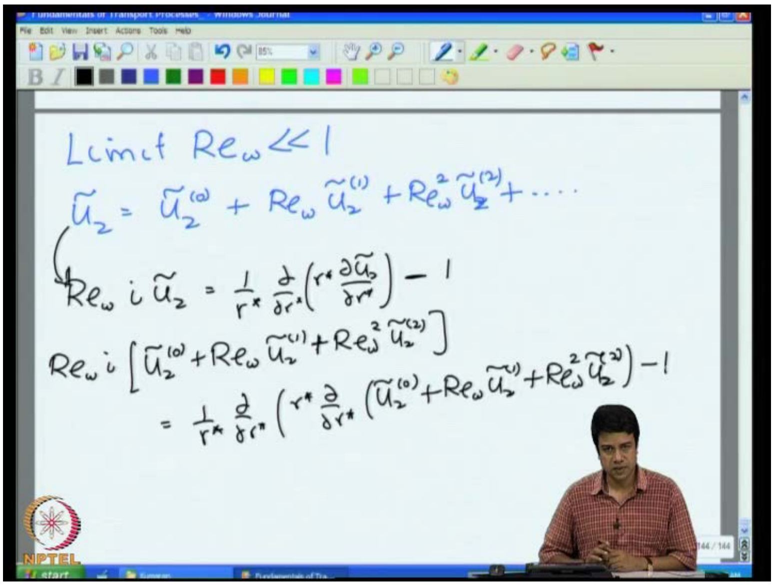


So, we will come back to this little later, what happens in the limit Re omega small compared to 1, I can consider the Reynolds number as a small parameter in that case, I can expand my velocity; I can expand u z tilde is equal to plus plus etcetera. So, I am using an expansion for the velocity field in this small parameters, the original equation that I had was i u z tilde is equal to minus 1, so that was the original equation that I had.

So, within this equation I substitute this expansion in the limit Re small compared to 1, this expansion for u z as substitute into this equation, so what do I get is minus 1, this is minus 1, so this is the expansion of u z in a series Re omega. Now, I can collects terms that are multiplied by Re omega, Re omega square, as well as terms that are independent of Re omega, because an expanding the series in the small parameter Re omega in the limit as Re omega goes to 0.

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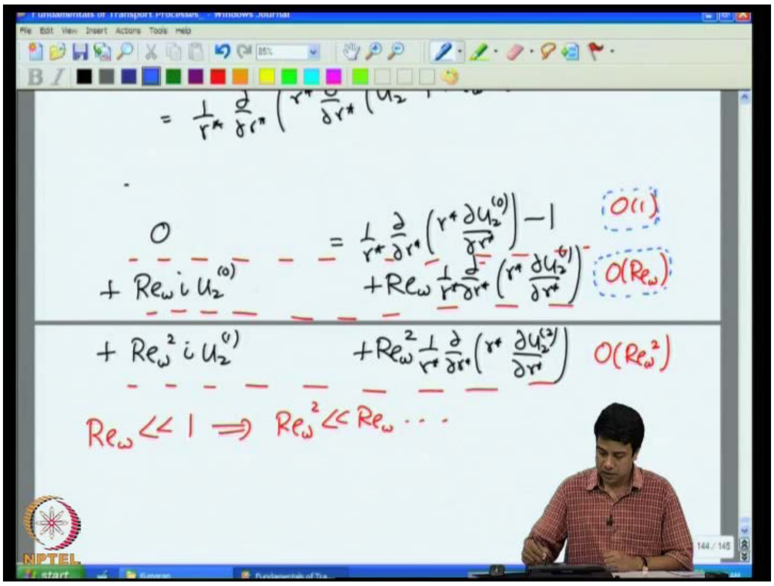


So, if Re omega where identically equal to 0 then, I could neglect all the terms proportional to Re omega and I would get on on the left hand side, I will just get 0 because, in the left hand side I have Re omega is multiplying by everything. So, on the left hand side I will just get is 0, on the right hand side I will get is equal to minus 1. So, these are term are independent of Re omega in the limit as Re omega is goes to 0.

However, I do have the terms are proportional to Re omega in the limit is Re omega goes to 0. In particular on the left hand side, I have Re omega i times u z naught, on the right hand side I have plus Re omega 1 by r d by d r of r du z 1 by d r that is on the right hand side and then, I can collect the terms that are proportional to Re omega square u z 1 is equal to and on the right hand side I have plus Re omega square 1 by r d by d r. So, this is the expansion of left and right hand side in a series in Re omega. So, this expansion in series in Re omega now I am taking the limit as Re omega goes to 0. So, when I take the limit Re omega is goes to 0, if Re omega was identically equal to 0 then, there will be only the first term the underline term that is entering in to the balance; but of course, Re omega small but not 0 in that case, if Re omega small very much less than 1 implies that, Re omega square is small compared to Re omega, etcetera Re omega cube is small compared to Re omega square and so on.

If this balance is to hold for all values of Re omega in the limit as Re omega goes to 0; that means, these individual co-efficient have all got to be equal to 0, if this balance is to hold for all values of Re omega in the limit as Re omega goes to 0, then the individuals co-efficient of the equation of 1, Re omega, Re omega square, etcetera they all got to be 0. So, this equation is termed as the order one equation, this is termed as order Re omega equation, keep this is order Re omega square and so on, you can keep expanding to higher and higher orders in this manner and get the higher and higher correction to this equation.

So, the important point is that, if the entire equation the entire expansion is to be valid in the limit Re omega goes to 0; that means, the order one equation has to be 0, the order Re omega has to be 0, the order Re omega square equation has to be 0 and so on. This order one means that, the terms in this equation remain finite as Re omega goes to be 0; order one the terms in the equation remain finite as I take the limit Re omega going to be 0. The order Re omega means that, the terms of the equation decrease to 0 proportional to Re omega in the limit Re omega going to be 0 that is the meaning of order Re omega, this capital O stands for order.

Similarly, the order Re omega square term this implies that, the terms in the equation decrease proportional to Re omega square in the limit as Re omega goes to 0.

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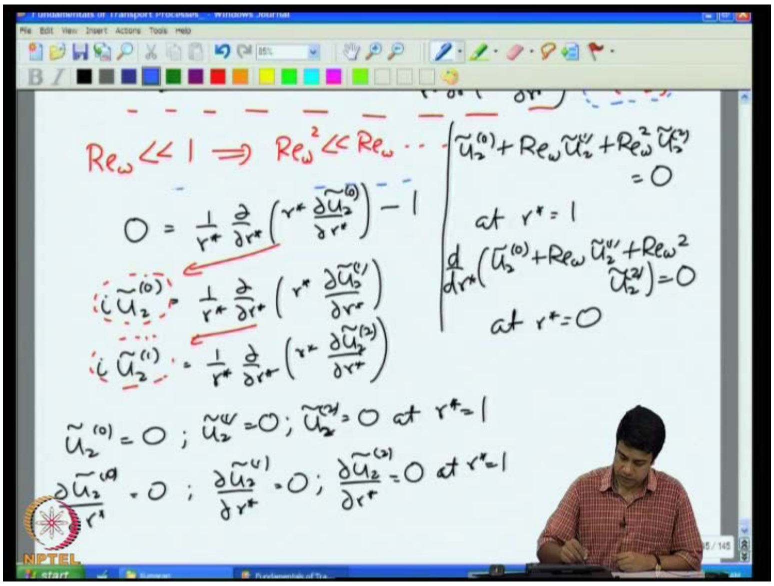


So, now each of these equation individually has to be equal to 0; it was that implies is that 0 equal to 1 by r d by d r of r d u z by d r minus 1. So, that is the order one equation. The first correction is i u z 1 I am sorry i u z naught is equal to 1 by r d by d r of r d u z 1 by d r tilde here and then, the second equation is I u z 2 is 1 by r d by d r of r d u z 1 by d r and so on, you will get series whole series is of equation and you can cut off that series at any at any desired value to get a solution of sufficient accuracy.

Now, the way to solve this equation is clear I can solve the first equation for u z naught put that u z naught in to the homogeneous term here and solve for u z 1 put that u z 1 into the homogeneous term over here. So, this u z 1 will go to be homogeneous this solution is going to here and this solution goes to be here, put u z 1 into the homogenous solution there and get u z 2 and so on. (( )) this is u z 1, u z 2 and so on. Boundary conditions the boundary conditions that we had used where u z is equal to 0 at r star equal to 1 and d u z star by d r is equal to 0 at r star is equal to 0; these boundary conditions also have to be expanded in a series, these boundary conditions also have to be expanded in a series.

So, I have to have u z naught plus Re omega u z 1 plus Re omega square u z 2 is equal to 0, at r star equal to 1 and d by d r of u z naught plus is equal to 0 at r star equal to 0. So, those are the boundary conditions in insert the expansion for the velocity into the boundary conditions and once again set the co-efficient of order one order, Re omega and order Re omega square, individually to 0.

So, insert the expansion into the boundary conditions and set the co-efficient of one, Re omega, Re omega square individually to 0 in the expansion. Therefore, you will get u z naught is equal to 0, u z 1 equal to 0 and u z 2 is equal to 0 at r star is equal to 1. So there is at the wall of the pipe each individual component of the velocity, the order one velocity, the order Re velocity, order Re square velocity they are all individually equal to 0. And the center you have d u z naught by d r is equal to 0, d u z 1 by d r is equal to 0, d u z 2 by d r is equal to 0 at r star is equal to 1. So, these are the boundary conditions that can be used to solving each of these individuals conditions.

Note that, the equation for u z naught is identical to the equation that I had at for Re omega is equal to 0, if you recall that when I did my approximation for low Reynolds number I had an equation of this kind 1 by r d by d r d u z by d r minus cos t is equal to 0, when expressed in the terms of u tilde that is u z equal to real part of u tilde times e power i t. So, when expressed in terms of u tilde the equation is actually identical to the leading order equation. So, this equation order, this order one equation is identical to the equation that I had exactly in the limit of 0 Reynolds number, this is 0 Reynolds number equation and therefore, I can straight away write down the solution.

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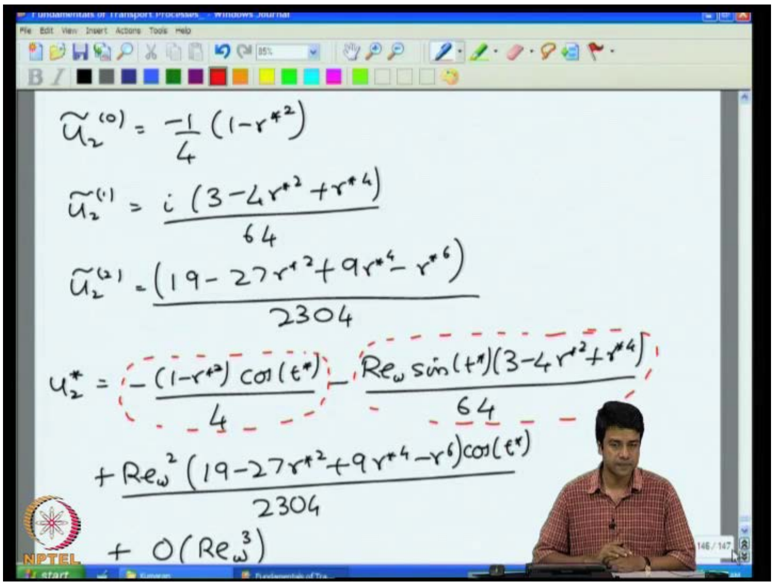


The solution is u z tilde naught is equal to minus 1 by 4 1 minus r star square. So, that is the leading order solution for u z naught and that gives me the steady velocity profile. For u z 1 this is the equation, this is the equation; it contains no homogeneous term; however, it does contain u z naught on the left hand side, it does contains the term u z naught on the left hand side. So, therefore, I can solve this subject to the condition that is u z is equal to u z naught on the left hand side. In order to get the solution for u z 1 and you will find that, u z 1 is equal to i into 3 minus 4 r square plus r power 4 by 64 you can easily verify that this solution actually satisfies both boundary conditions, at r is equal to 1 this is equal to 0, at r is equal to 0 it's derivatives is equal to 0.

So, this solution obtained from the homogeneous equation for u z 1 it satisfies both boundary conditions, this can be inserted in to the equation that I have u z 2 this can be inserted into the equation that I have for u z 2 and this can once again we can solve to get a solution for u z 2. And if you actually solve that equation you will find that, u z 2 is equal to 19 minus 27 r square plus 9 r power 4 plus r power 6 is divided by this be minus 2304 and this is the solution for u z 2, u z 2 can put as an homogeneous term in the equation for u z 3 and once again you will get the solution. So, we can get the solution to whatever order in Re omega that you want and one can put all of this together to get the final velocity profile.

So, my final equation of the velocity based upon this expansion will be u z star, which is the scale velocity is equal to minus 1 minus r square cos t by 4 minus Re omega sin of t into 3 minus 4 r square plus r power 4 divided by 64 plus Re omega square into 19 minus 27 r square plus 9 r power 4 minus r power 6 into cos of t by 2304 plus order of Re omega cubed. So, this is an approximation solution we evaluated it as a series, there are still terms in the series we are not evaluated for those terms are order Re omega cubed or smaller.

So, for example, if the Reynolds number is the 0.01 terms we have not evaluated are 10 powers minus 6 approximately if it Re omega is about 0.1 terms we are not evaluated r in the order of one and 1000 this was the leading order steady solution that we got. The pipe flow parabolic flow in a pipe for the case, where the Reynolds number is identically equal to 0, that has the phase that is exactly the same phase has the pressure itself; pressure we had imposed was cos t, the velocity goes as minus cos t, because when the pressure gradient is positive, the velocity goes in the negative direction.

However, there are corrections to this due to inertia; the first correction due this inertia is this one, is proportional to the Reynolds number in the limit of small Reynolds number and this thing has phase shift of pi by 2, it goes as sin t. So, the the inertia causes the phase shift between the pressure gradient and the velocity and then, there is a second correction which once again the goes as the cos t is proportional to the Re omega square and using this I can evaluate the all the higher order terms in the series.

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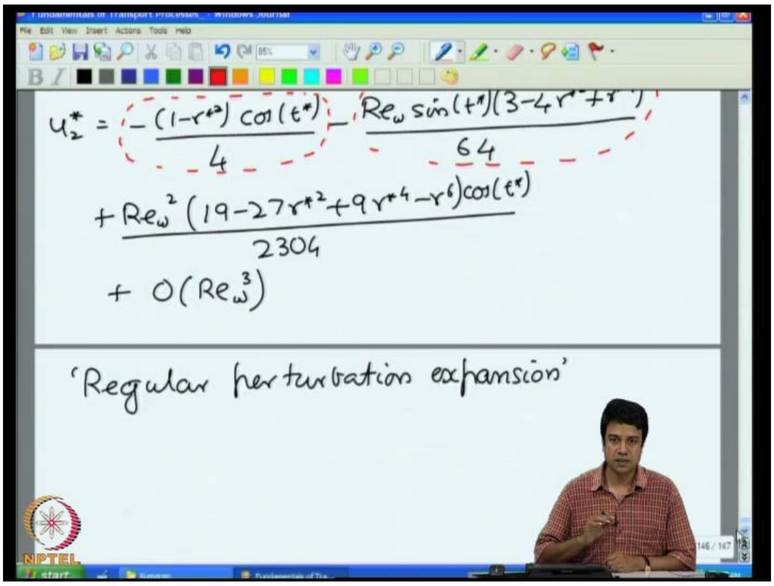


So, this procedure of expansion is what is called as regular perturbation expansion; in order to get an approximate solution, whenever you have a small parameter in this problem, in this particular case we had a small parameter that was Re omega and so we expanded out the velocity u z as a series in Re omega and we inserted that expansion in to the governing equation, as well as the boundary conditions. In order to get the both the governing equation and boundary conditions of as a series in Re omega in the limit as Re omega goes to 0, the term proportional to Re omega will be small compared to the term proportional to the order of one, because the order one term remains finite even as Re omega goes to 0.

So, order term there is proportion to Re omega will be small compared to the leading order term, which is order one. The term Re omega square will be small compare to the term is proportional to Re omega therefore, each of those individual co-efficient can individually be set equal to 0. I said the order one equation is equal to 0, the order Re omega is equal to 0 and the order Re omega square is equal to 0 and so on and you do the same thing with the boundary conditions and once you do that, you can solve each of these equations individually. In this particular case, the order one equation give the parabolic velocity profile for the flow in a tube, which is exactly opposite to the pressure gradient; it has the same phase, but it opposite insight to the pressure gradient.

However, there are correction to this due to inertia and we can calculate systematically what is the correction at order Re omega, the correction at order Re omega square and so on. We can calculate it each individual correction in it, because we have a series of the equation in which as you can see the leading order equation contains only u z naught and in an homogeneous term, the first correction contains a term derivative of u z 1 and in homogeneous term which is proportional to u z naught that in homogeneous term as already been evaluated in the order one equation.

So, I can put insert that in homogeneous term into the order Re equation get the solution, insert that solution to the order Re square equation and get the solution of that and continue that series, continue that series up to the extent required to get a solution of the accuracy that I need. And this is illustrated to you the way that I would do that, it just a matter of simply solving these equations in order to get the solution. As expected we get the steady solution with the pressure given by cos t as the leading order solution and then, there are corrections to that, there is a Re omega correction which is the first effect of inertia on the leading order solution and then, thus the Re omega square correction and so on.

And in this particular solution we have neglected the terms that are proportional to as Re omega cube and higher order terms And as I told you this illustrate the procedure of the regular perturbation expansion we we had got a solution for the complete equations earlier in terms of Bessel functions, but as I said that does not give you very good physical insight into the problem, in this particular case we chosen one particular limit of this equation in the limit, where the Reynolds number is small and minus to get solution in terms of an expansion.

This expansion procedure will be useful even when we cannot get analytical solutions to the equations, this particular case we manage to get an analytical solution as a Bessel function, but there are problems where there are multiple equations can be solved in that case, you might not able to get an analytical solution to the equation, perturbation expansion procedure would still work in that case to give you an approximate solution in the particular limiting case that you are interested.

So, that is the the power and usefulness of these regular perturbation expansion. So, far we look at the limit where Re omega is the small compared to 1, what about the limit where Re omega is large compared to 1 in that case, you would expect the inertial terms to be large compared to viscous terms and as I said, one has to go back and scale the velocity by the inertial terms, so that the velocity scale by the inertial terms is an order one number. So, let us just look at that scaling briefly, before we look at the procedure for solving that equation, so my original equation in terms by u z was the rho times d u z by d t is equal to the mu 1 over r d by d r of r d u z by d r minus k cos omega t.

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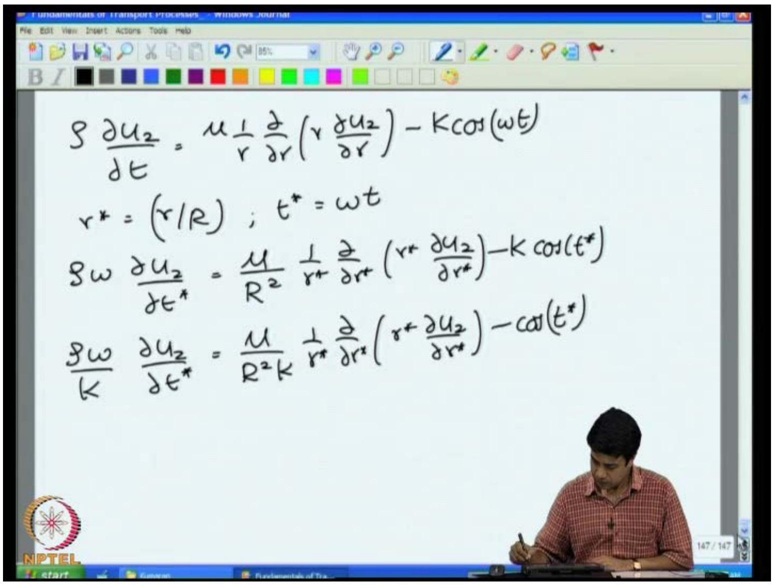


Now, I define as usual r star is equal to r by R, where capital R is the radius of the pipe and t star is equal to omega t. Once you do that you get rho omega partial u z by partially t star is equal to the mu by R square 1 by r d by d r of r d u z by d r minus k cos of t star as before I divide throughout by k to get dimensional less equation. So, I will get rho omega by k d u z by d t star is equal to mu by R square k 1 by r minus cos t. And I am interested in the limit of high Reynolds number I have to scale velocity by the inertial scales, if I am interested in the limit of high Reynolds number I have to scale velocity by the inertial scales.

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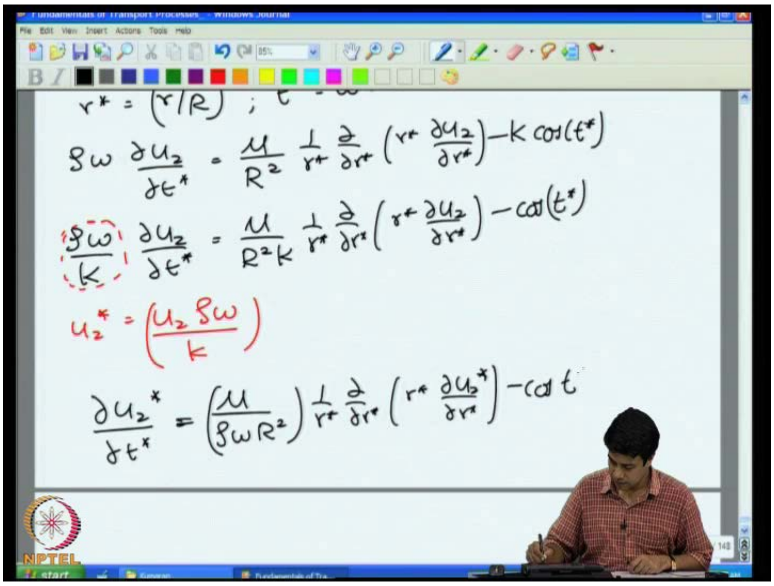


So, I should define u z star is equal to u z rho omega by k I should be defining u z star in this manner and if I define u z star in this manner my equation will become d u z star by d t star is equal to mu by rho omega R square 1 by r d by d r of r d u z star by d r minus cos t and this of course, this is 1 over the Reynolds number, this is the inverse of the Reynolds number 1 divided by the Reynolds number.

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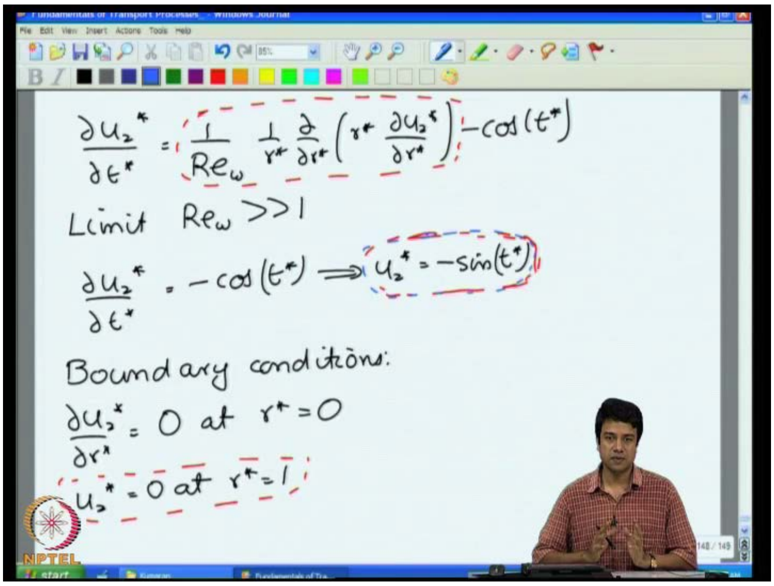


So, therefore, the equation can be written as d u z star by d t star is equal to 1 over Re omega minus cos of t. So, as I said I am considering the limit of high Reynolds number I am considering the limit of high Reynolds number; that means that, 1 by Re is small therefore, (( )) if I where to try to solve this problem simplistically I would say, why not we just neglect this entire terms here we just neglect the term and solve the rest of this equation, because the reynolds number is large, so 1 over R is small. So, we just neglect that term go head and solve the rest of the equation. So, what happens if you do that, you get d u z by d t is equal to minus cos t this can be integrated quite easily d u z by d t is equal to minus cos t implies that, u z is equal to minus sin t.

So, this is the solution in the limit of of high Reynolds number. Now, we have to satisfy the boundary conditions, boundary conditions d u z by d r is equal to 0 at r is equal to 0 clearly that boundary conditions is satisfied, if I take derivative of u z that I have since u z infinite of r, it is derivative that is identically equal to 0. How about the boundary condition at wall of the pipe? How about the boundary condition at the wall of the pipe, which as that u z equal to 0 at r is equal to 1.

Note that, boundary condition has to be satisfied at all instance in time; boundary conditions has to be satisfied at every value of time, the velocity is 0 for each and every time value can be satisfied that boundary condition with this solution clearly not, since the solution was independent of r, the velocity at the boundary does not go to 0 at r is equal to 1 and there is no way for as to satisfied this boundary conditions by using this solution. So, this solution for the equation cannot satisfy this boundary condition.

So, clearly in the limit of very high Reynolds number we have solution that does not not satisfies the boundary conditions, if we just simplistically go ahead and neglect the viscous term in the equation, because there is a coefficient 1 over Re omega in front of the viscous term, why is that, why we cannot satisfies the boundary conditions and what should we do to ensure that the boundary conditions is satisfied, because in the real physical system in the real pipe, the velocity is actually 0 at the wall at all times whereas, the mathematical solution that we have so far, they seems to be no way to satisfy that.

So, we look at the reasons for that we will continue this in the next lecture, think about it what is that we did while we are trying to solve the problem, which made it impossible for us to satisfy that boundary conditions. We will come back and look at this in the next lecture, we will see you then.