Fundamentals of Transport Processes

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Lecture No. # 20
Unidirectional Transport Cylindrical Co-ordinates -V Oscillatory Flow in a Pipe

Welcome to lecture number 20 in our course introduction to transport processes, where we were in the mid test of discussion of the technologically important problem of the flow in a pipe. And what I had promised you was to get a relationship between the friction factor, and the Reynolds number for this flow in a pipe. If you recall we had discussed this when we did dimensional analysis, friction factor is a scale momentum, scale by the inertial scales. And for a pipe at low Reynolds number we have the relationship f s equal to sixteen by r e, and the third Reynolds numbers; there is of course a transition to a turbulent flow.

So, we were considering this problem of the flow in a pipe, we took a cylindrical differential volume momentum balance rate of change of momentum is equal to the sum of the applied forces. And as I said there is a pressure gradient down the length of this pipe.

So, in addition to the shear stress acting on the surface of the pipe, one also has the pressure forces. As you go down stream the pressure progressively decreases that is what causes the flow, and this decrease in pressure basically a results in the fluid flow. Therefore, in addition to shear stresses due to the velocity gradients acting on the cylindrical surfaces both the outer cylindrical surface as well as the inner cylindrical surface, you also have a difference in pressure between the two axial locations. And that difference in pressure results in a net pressure force acting on this differential volume.

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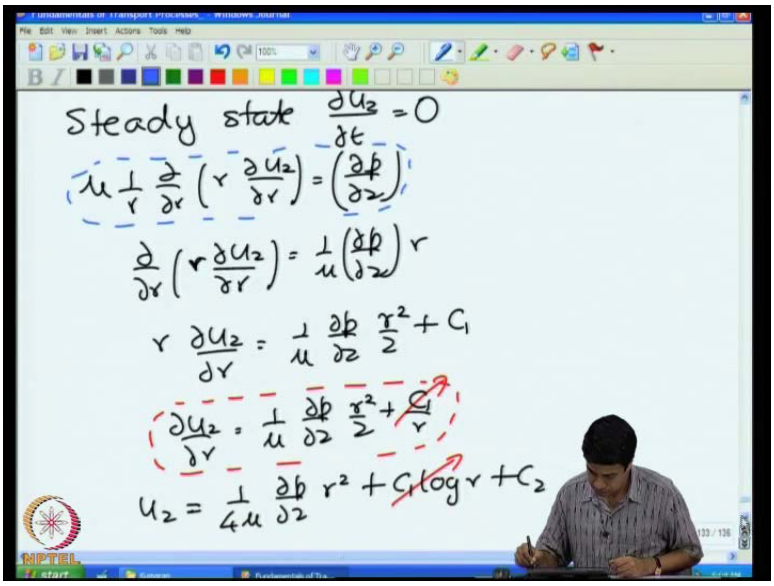

So, this also has to be included into the momentum balance equation. So, we had written the momentum balance equation and then got the differential form of this momentum balance equation. Various different ways of writing it, one in terms of the shear stress, and then if you use Newton's law of a viscosity for the shear stress you get in terms of the velocity itself, rho times d u z by d t is equal to mu times one over r d by dr of r d u z by dr minus the pressure gradient. So, if the pressure gradient is negative; that means, the flow will be along the plus z direction.

So far a flow to take place in the plus z direction you require the pressure to decrease as z increases our dp by dz has to be negative. And then we had use this to solve for the steady problem of the flow in a pipe, in that case the temperature the time dependence of the velocity is equal to 0 and I have this equation which basically relates the rate of change of velocity with r to the pressure gradient. Note this is a fully developed flow. Therefore, there is no dependence of u z on z itself. At as I, at a given radial location if we change z the velocity does not change. So, at every z location the velocity is identically the same. In other words the velocity profile is parabolic with the same velocity at each location if that is required because I have to have equal amount, the flow that is coming in is in compressible. So, volume has to be conserved.

So, u z is only a function of r, it is not a function of z for a fully developed flow similar to the cylinder case. I had the cylinder heat transfer in a cylinder infinite extent in that case temperature was only a function of r. In this case u z is only a function of r.

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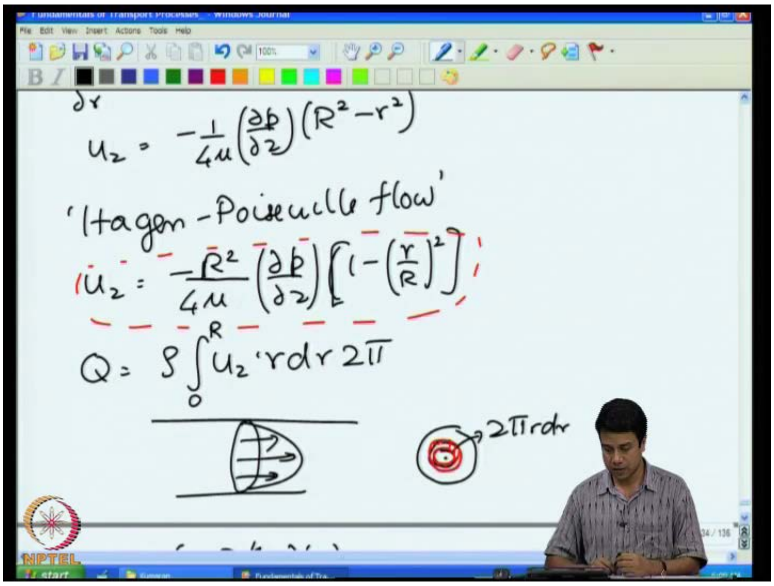

So, I had solved this equation subject to boundary conditions to get a parabolic velocity profile; u z is equal to minus 1 over four mu t p by dz into R square minus r square. Now, we had calculated the total volumetric flow rate of fluid. The volumetric flow rate is equal to the velocity times the cross sectional area. The volumetric flow rate is equal to the velocity u z times the cross sectional area along the cross section of the cylinder. Of course, the velocity depends upon r.

So, the velocity is not a constant across the entire cross section. So, as I said we get a parabolic velocity profile u z is equal to minus 1 over four mu dp by dz times r square minus r square, it is called a Hagen-Poiseuclle flow for the flow in a profile in a pipe. This flow profile in a pipe is parabolic depends it goes as 1 minus r by r the whole square. The net volumetric flow rate is equal to the velocity times the cross sectional area. The volume coming per unit area, volume transported per unit time, the flow rate, the volumetric flow rate.

Of course the velocity is a function of r it is varying as the radius changes. So, therefore, I have to an integral over each strip of the area of the velocity within that strip times the area of that strip integrated from 0 to the wall of the pipe. Each strip of the area in this case is a strip between R and r plus dr, the velocity within that strip is u z of r which is basically given here, the u z of r is the velocity within that strip cross sectional areas two pi r times dr. So, I have to integrate two pi r dr times u z. From the center of the pipe to the wall r is equal to R. So, you put in this expression for the velocity field into this integral and then actually calculate it. So, I leave that as an exercise for you. The final result that you get is minus pi r per four by it mu times dp by dx o k you can calculate that quiet easily.

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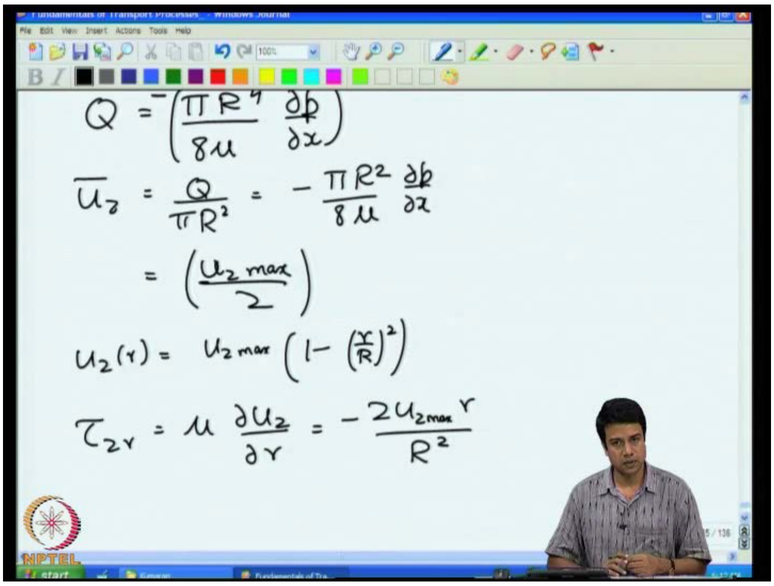

So, that is the volumetric flow rate, the volume coming out per unit time. The average velocity is the volumetric flow rate divided by the cross sectional area. So, the average velocity is a volumetric flow rate divided by the cross sectional area which is pi r square by eight mu times dp by dx. Go back to our expression for the velocity itself, this velocity is a maximum when r is equal to zero, so maximum right at the center of the pipe. So, at the center of the pipe the velocity is equal to minus r square by four mu times dp by dx. So, right at the center of the pipe is equal to 1 r square by four mu dp by dz the average velocity is r square by eight mu; that means, that u z bar is equal to u z max by two and I can also write this function u z of r as equal to u z max into 1 minus r by r whole square. So, the maximum velocity times 1 minus r by r the whole square is the velocity profile it is a parabolic velocity profile.

Next let us calculate the shear stress at the wall of the pipe. The shear stress at the wall tau z r at the wall is equal to mu times d u z by dr. This is equal to minus two u z max r by r square. This is the wall, this is the shear stress at any location.

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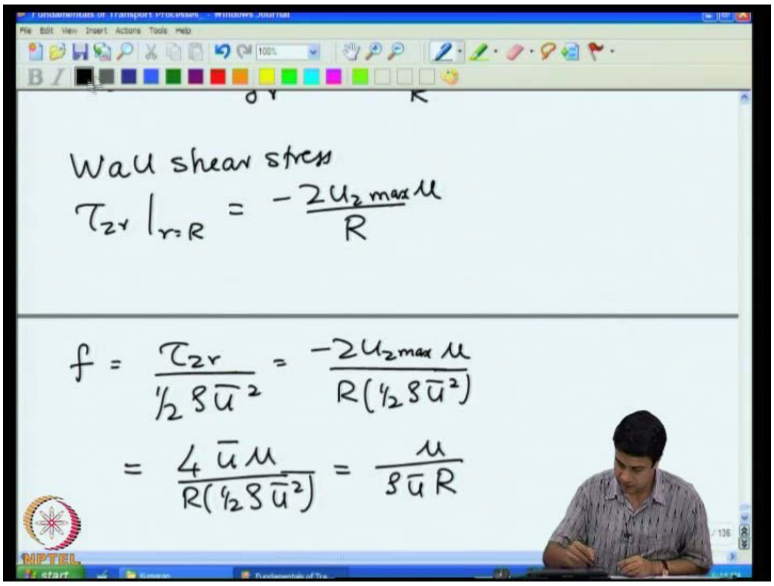

Wall shear stress tau z r at r is equal to R at the wall of the pipe is equal to minus two u z max by R, and as we had discussed in the beginning when we did dimensionless numbers the friction factor f is defined as the wall shear stress by half rho u bar square.

So, this is equal to minus two u z max by R into half rho u bar square. Note that the friction factor is defined with respect to the average velocity not the maximum velocity. So, I have to represent the maximum velocity here in terms of the average velocity. So, if I represents the maximum velocity in terms the average velocity I get four u bar I am sorry there is a mu here by r into half rho u bar square. So, this I can write it as eight mu by rho u bar r rho u bar r by mu, is a Reynolds number; however, the Reynolds number as traditionally defined is defined in terms of the pipe diameter and not the radius. So, if I little bit define it in terms of the pipe diameter D, D is equal to two times r. So, r is equal to d by two.

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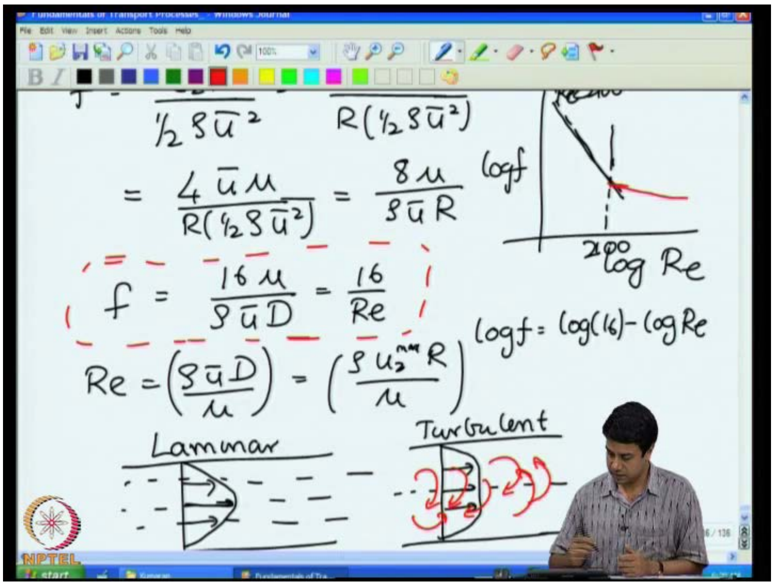

So, I finally, get 16 mu by rho u bar times D, where D is the pipe diameter this is 16 by R e, where R e is the Reynolds number rho u bar t by mu based upon the average velocity and the diameter of the pipe, this also is equivalent to rho times u z max r by mu because r is equal to d by two and u z max is equal to two times u bar.

So, the Reynolds number based upon the diameter and the average velocity is the same as the Reynolds number based upon the maximum velocity and the radius. So, this is the Reynolds number where is a friction factor correlation for the flow in a pipe. When you plot it on a log scale log r e versus log f, you get a straight line whose slope is equal to minus 1 because log f is equal to log of sixteen minus log r e, because you get a slope of minus 1.

However, this parabolic velocity profile that we calculated is valid only up to Reynolds number of about 2100 only for low Reynolds numbers with the Reynolds number r e less than 2100 is this flow through a pipe is this friction factor valid. At a Reynolds number of 2100 there is a transition from a laminar flow to a more complicated flow profile called a turbulent flow.

So, in a laminar flow, you have a smooth velocity profile, you have straight stream lines, the fluid parts are all straight and you have a smooth parabolic velocity profile. This profile is of course, a solution of the equations for Reynolds number less than 2100 it is the solution of the equations for Reynolds number is higher than 2100 it is still a solution of the equations.

However, approximately at a Reynolds number around 2100 this solution becomes unstable and the system spontaneously goes to another solution and this solution is called this is a Laminar. And this other solution is called a Turbulent solution. In a laminar profile the transmission of stress is due to molecular diffusion, due to the kinematic viscosity of the fluid therefore, the shear stress is given by the viscosity times the velocity gradient.

So, the transport of momentum across streamlines is by the molecular diffusion mechanism that we had discussed extensively. In turbulent flow actually, the velocity profile is no longer parabolic it is very flat , almost plug like at the center in a turbulent profile the transport of momentum actually occurs due to the formation of a highly keotic eddies which are highly fluctuating highly dissipative they are basically parcels of fluid undergoing correlated motion was in the flow. And these eddies transport momentum in a far more efficient manner than the molecular diffusion, because you actually have diffusion due to that actual transport of fluid across fluid velocity fluctuations themselves in the case of laminar fluids due to molecular fluctuations in the case of turbulent flow it is due to microscopic fluid velocity fluctuations. In these eddies which develops spontaneously, because the efficiency of this transport process is far higher, the stress transmitted is also much larger than in a laminar flow, and due to that the friction factor is also much higher than what you would expect for a laminar.

So, it follows this sixteen by r e law up to the point at which the laminar flow becomes unstable. Beyond that point you have a turbulent flow which is highly mixing, highly keotic with large fluctuations. The fluid velocity not just the molecular velocity, but also the fluid velocity and these large fluctuations transport momentum far more efficiently across the flow and this results in a much higher friction factor. So, afterward 2100 you can use the friction factor sixteen by r e for Reynolds numbers higher than that the friction factor depends on other things also such as the roughness of the walls of the pipe and so on.

Velocity profile is no longer parabolic near the walls there is actually a logarithmic law and towards the center it is plug like and this results in a highly dissipative system with the friction factor is much higher than what you would expect. So, I have derived for you the friction factor for the laminar flow in the pipe the sixteen by r e law turbulent flow is more complicated and there is no simple of the friction factor. So, this is a summary of the steady flow in a pipe.

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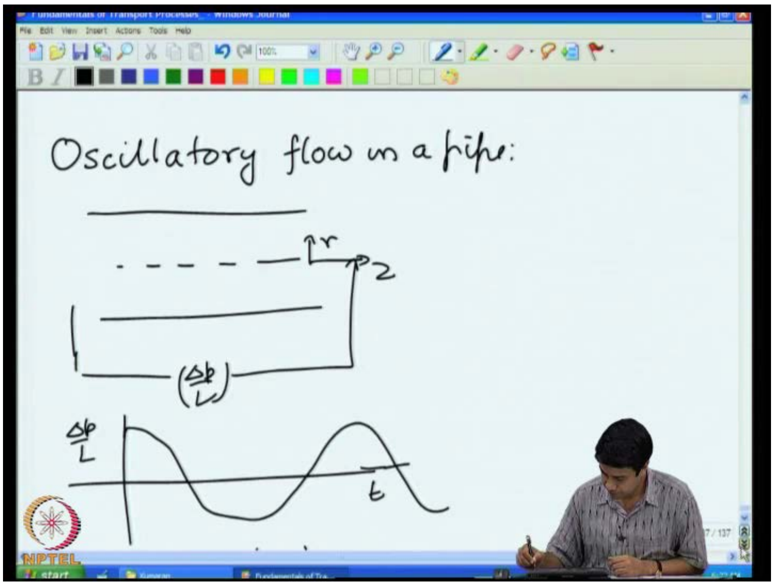

Let us take a slightly more complicated situation and that is Oscillatory flow. Oscillatory flow in a pipe due to an oscillation in the pressure gradient, so the configuration is as follows and applying a pressure difference across the pipe delta p by l. So, if you look at this pressure gradient. So, delta p by l as a function of time this has an oscillatory behavior with a frequency omega. So, the pressure gradient is of the form k cos omega t. So, the pressure gradient has the form k cos omega t. So, that means, that the (refer number: 19:00) pressure is from left to right for half the cycle and right to left for the other half of the cycle. Average pressure difference across is equal to 0. The flow is still fully developed in the sense that if I have a coordinate system z and r the velocity u z does not depend upon z. So, in that sense the flow is still a fully developed flow, but it is not steady it is varying with time, the because the pressure difference is a function of time the flow is also a function of time. This is of course, encountered in a biological systems in the human body you know that the heart pumps in oscillatory manner and therefore, the flow through blood vessels is actually oscillatory in nature. There is a steady component also, but as I told you earlier by linear superposition we can add up the flows due to the steady and the oscillatory parts.

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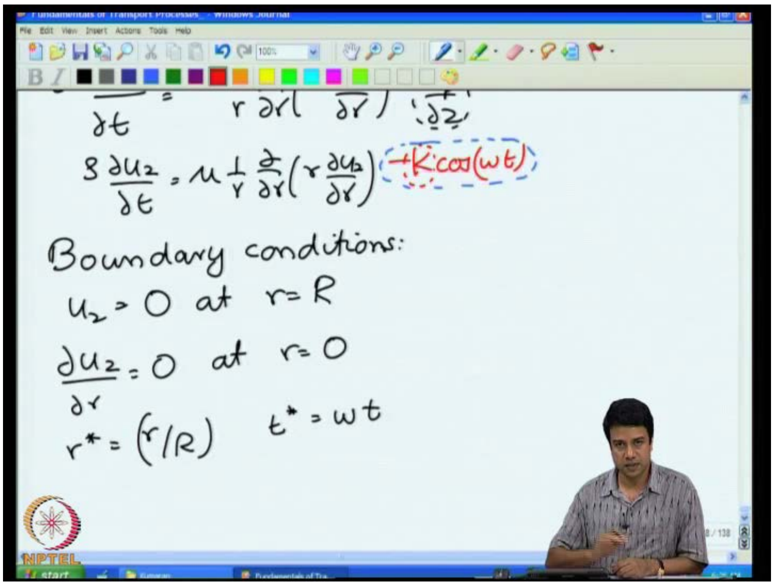

So, the differential equation that I had rho times du z by d t is equal to mu into 1 by r d by dr of r du z by dr minus dp by dz, this is now oscillatory in time, so this minus K cos omega t, so this linear differential equation for u z which is being driven by this inhomogeneous term. I have to find the solutions for these equations. This is oscillatory in time. Boundary conditions u z is of course, equal to 0 at r is equal to R, u z is equal 0 at r is equal to R. At the wall of the pipe itself the velocity has to decrease to 0 and then I have my symmetry boundary condition du z by dr is equal to 0 at r is equal to 0. Its preferable to work in terms of scaled coordinates as always, the natural scaling for the radial direction is r star is equal to r by R. So, that gives me the special scaling.

What about the scaling for time. In this particular case I am having the flow being driven by a well defined sinusoidal wave form with a well defined frequency. Since I have a well defined frequency, I could very well scale t star is equal to omega times t, the time period is two pi by omega the frequency is omega. So, I could define a scaled t star is equal to omega times t. What about the scaling for the velocity, the scaling for the velocity will come somehow from the pressure gradient that I am applying over here is somehow from this pressure gradient if that is large the velocity will be large if that is small the velocity will be small. So, scaling for the velocity has to come out of the scaling for this pressure gradient. So, we will see how that comes above.

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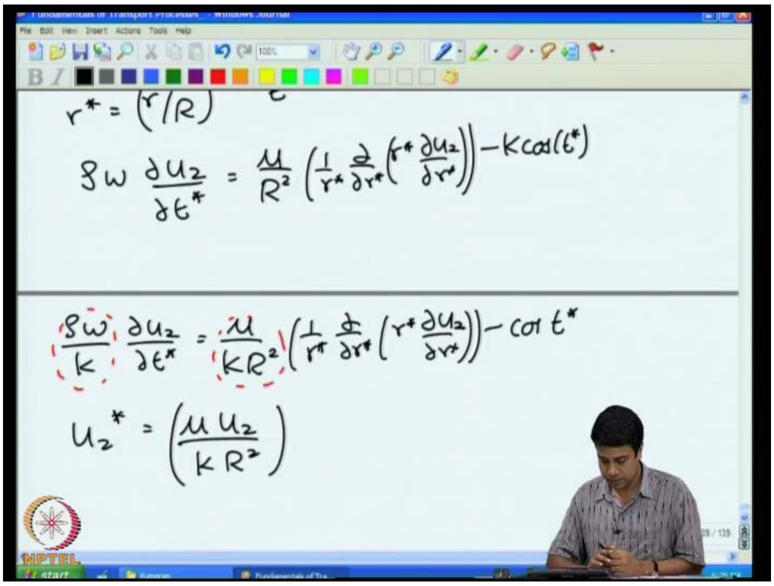

If I just write down the scaling for the time and special coordinates I will get an equation of the form rho omega du z by d t star is equal to mu by r square minus K cos t star. So, that is the equation that I get in terms of the scaled coordinates.

Note that on the last terms on the right hand side cos of t star is dimensionless. Therefore, if I divide the equation throughout by K, then the last term on the right becomes dimensionless by dimensional consistency then every term has to be dimensionless. So, I divide throughout by K and I will get rho omega by K d u z by d star is equal to mu by K R square minus cos of t star divide it throughout by K.

Now, the last term on the right hand side is dimensionless; that means, that all terms are dimensionless just by dimensional consistency. So, I cloud define a scaled velocity by either dividing it by this term or dividing it by this term I could use either one of those for defining a scaled velocity. If divide a scaled velocity as mu u z by K R square; that means, I am scaling the velocity by the viscous scales, where as we scale it by rho omega by k then I scale it by the inertial scales.

Typically, if the Reynolds number is small you would expect viscous of x to be dominant therefore, you would scale it by the viscous scales whereas if the Reynolds number is large you would expect inertial effects to be dominant in which case you have to scale it by the inertial scales. For the present we will use the viscous scaling for the present case.

So, let me define u z star is equal to mu u z by K R square. So, that is my definition for u z star.

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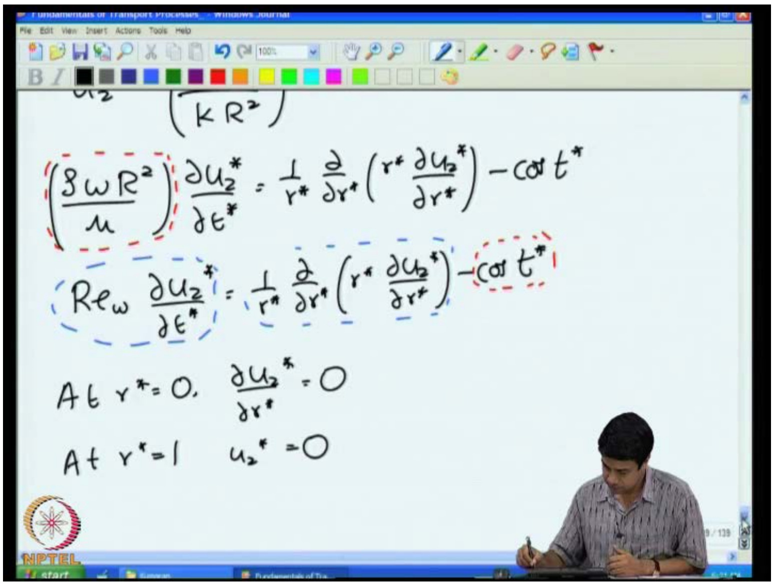

So, if I insert this into the equation ,I am getting rho omega r square by mu du z star by dt star is equal to 1 by r d by dr of r d u z star minus cos t star. And this is basically a Reynolds number based upon the frequency of oscillations and the tube radius. So, this I can define it as some number R e omega minus cos of t star.

So, note that this is the inhomogeneous term this is the forcing in time this is the term that is forcing the velocity field. This is balanced by 2 terms, this one is a viscous term and this one is an inertial term. So, at higher Reynolds number is expects the inertial term on the left to be dominant. The limit of lower Reynolds number the viscous term the first term on the right will be the dominant term. And then there are Boundary conditions that at r star is equal to 0 u z star by dr star is equal to 0 and at r star is equal to 1 r is equal to capital r means that r star is equal to 1 u z star is equal to zero. So, I have to solve these subject to these Boundary conditions. Note that the Boundary conditions are both homogenous there is; however, a forcing within the equation that is forcing the flow.

So, let us see how we can solve it. This is a flow with an oscillatory pressure gradient the equation the conservation equation itself is linear in u z. It is being forced by an inhomogeneous oscillatory term. So, a linear system being forced with a certain frequency in this case the scaled frequency is 1, because when I scale the frequency, when I define omega t is equal to t star, the scaled frequency in this case is equal to 1. Therefore, the response also has to have the same frequency as the forcing. We did this earlier when we looked at the oscillatory flow past a flat plate when you had a bottom plate which was oscillating back and forth. It is oscillating with a certain frequency; therefore, the velocity field everywhere in the flow also had to have that exact same frequency. So, rather than solve this equation what we can do is to solve an equation

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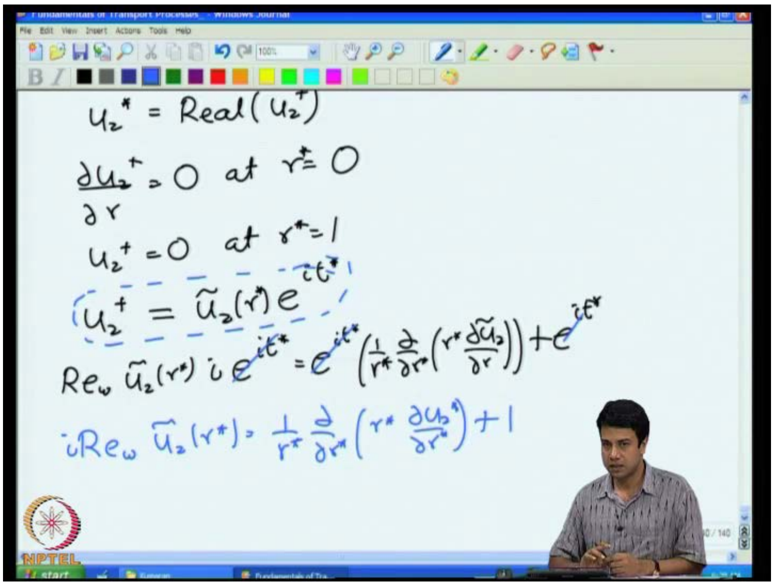

You know that cos t star is equal to the real part of e power i t star cos t star is a real part of e power i t star. Therefore, I can solve an equation for a complex velocity field defined by this equation R e omega du z plus by d t star is equal to 1 by r d by dr of r star d u z plus dr star e power i t star. Note that cos t star here this cos t star is the real part of this inhomogeneous driving term. Therefore, if I solve the equation for this complex velocity then the velocity u z that I get will just be equal to the real part of this complex velocity u z plus. Since the forcing term in my equation for u z star is the real part of the forcing term in my equation for u z plus; that means, that the solution u z star will also be the real part of u z plus. This is always possible when we are working with complex variables.

So, I will solve this equation for u z plus with Boundary conditions partially u z plus by partial r is equal to 0 at r is equal to 0 and u z plus is equal to 0 at r star equal one. So, I will solve this equation with these Boundary conditions which are the same as the Boundary conditions that I had for u z star these are same homogenous Boundary conditions that I had for u z star. So, I will solve the equation for u z plus with these boundary conditions and then take the real part of that and also take the real part I will get the solution for u z start in this equation. That is the strategy that we will follow for dealing with this oscillatory flow.

So, how do we solve this equation for u z plus. Its being forced with a function of the form e power i t in this particular case the frequency is one in scaled variables omega is one in scaled variables is being forced with e power i t; that means, that u z plus also has to have the form some function of r e power i t. It has to have the same frequency there may be a phase shift, but the frequency has to be the same a phase shift will basically be reflected in the complex nature of u z. The real if there is no phase shift it will be real, if there is a phase shift it will be a complex number and tan theta tan of phi where phi is the phase shift equal to the ratio of the imaginary and real parts.

So, I take this form of the equation and put it into the differential equation for u z plus and then solve it . So, this I take, so note that u z tilde that I have here is only a function of the radius star it is only a function of the radius. So, when I take the derivative with respect to time I will get r e omega times u z of r times i e power i t star when I take the derivative with respect to t I will just get i times e power i t star. And the I have a derivative with respect to r of u z plus. So, this will be of the form e power i t star into 1 by r d by dr of r d u z plus by dr I am sorry plus e power i t star. Note that within this equation all terms have e power i t star multiplying them; that means, that I can cancel out e power i t star on each of these equations.

I can divide throughout by e power i t star and then i get an equation that is not dependent on time at all. It is completely independent of time. It depends only upon r that is the reason that we were able to do this substitution of the first place. Because I knew that if I do this substitution inserted into the equation the time derivative this gives me I times e power i t. Once I have done that this e power i t on each of those terms and I can just cancel those out to get r e omega into i into u z r is equal to 1 by r d by dr of r du z by dr plus one. So, this is the differential equation I have reduced it from a partial differential equation in terms of r and t to just an ordinary differential equation in terms of r alone because I know that the flow is oscillatory in time. Therefore, since the pressure forcing is of the form e power i t star the velocity depends on time also has to be of that same form. So, I have used that here.

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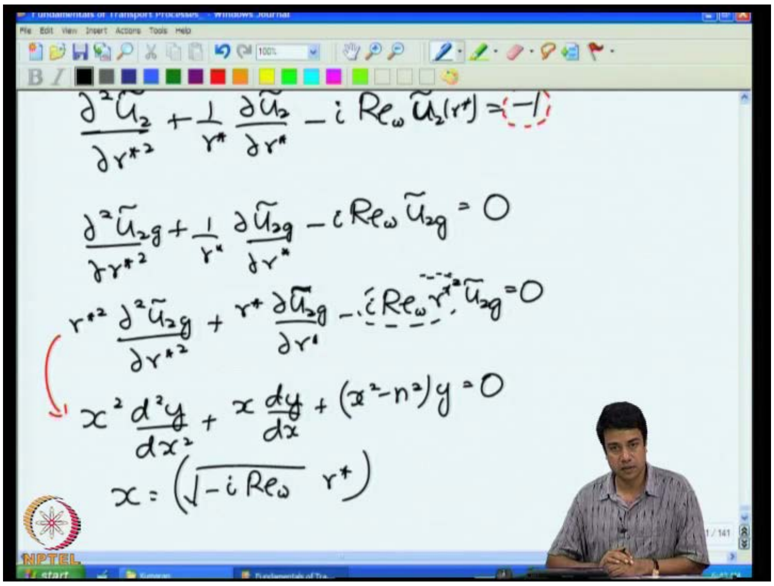

So, now, I can simplify this equation. I will get 1 by d square u z by dr star square plus 1 over r du z by dr minus i R e omega u z is equal to minus 1. So, I have taken minus this plus 1 to the left hand side and I have taken the r e dependence on the right hand side. So, this is the differential equation that I have to solve in order to get what is u z.

This is a first this is a second order linear differential equation with an inhomogeneous term. First order linear differential equation is (( )) this is a first order linear differential equation with an inhomogeneous term; that means, that I can write the solution as the sum of two parts. One is general solution and the other is a particular integral. The general solution is the one that satisfies the homogeneous equation without the inhomogeneous term on the right hand side. That means, that the general solution satisfies the differential equation partial square u z by partial r square plus 1 over r partial u z general by partial r minus i R e omega u, u z general is equal to zero. So, this the general solution satisfies the equation without homogeneous term. And how do we solve this equation we already saw what was the form of this solution. So, first thing first I multiply throughout by r square. So, I get r star square d square u z general by dr star square plus minus i R e omega r star square u z general is equal to 0.

You have seen this equation before you have seen an equation of the form x square d square y by dx square plus x dy by dx plus x square minus n square y is equal to 0 . This is the Bessel equation and the solution for these are the Bessel functions j n and y n. I can convert this equation to this form using the substitution x is equal to square root of minus i r e omega times r star I can convert this equation to this form by using this substitution x is equal minus i r e omega times r star square root of. So, at this whole thing become just equal to x square this whole thing becomes equal to x square and as I said that these first two terms are equi dimensional in r. So, they do not change if you scale r.

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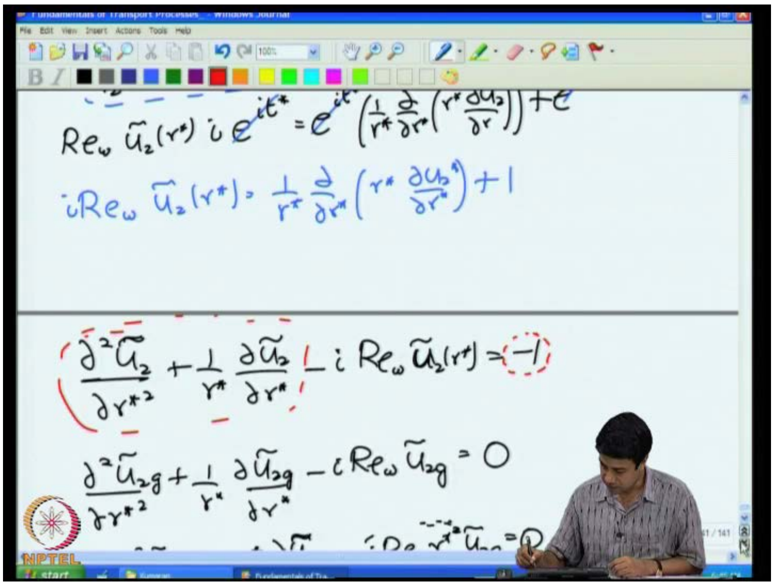

So, the solution of this is of the form u z general is equal to C 1 j naught of square root of minus i R e omega r star plus C 2 y naught of square root of . So, that is the most general form of the general solution; however, we saw in the eighteenth lecture that whenever you have an equation of this form the Bessel function y naught goes to infinity minus infinity at zero. So, if the u z general has to be finite; that means, that cos coefficient C 2 has to be equal to 0. If C 2 is not zero then the solution go to plus or minus infinity at r star is equal to zero. So, central symmetric condition itself indicates that you cannot have y naught in the solution, the solution can only have j naught in it. So, this is the general solution.

How about the particular integral? The particular integral is any one solution that satisfies the inhomogeneous equation the complete inhomogeneous equation any one solution. So, its easiest to choose the simplest possible solution that satisfies this inhomogeneous equation. The simplest possible solution in this case is just a constant. If you if I postulate that the particular solution is a constant then these first 2 terms identically become equal to 0 because when you take the derivate of a constant you get 0 and you end up only with the third term. So, constant can satisfy this equation. So, the constant has to be of the form minus i R e omega u z particular is equal to minus 1. Or u z particular is equal to minus or plus 1 by i R e omega is equal to minus i by. So, this is the constant that will satisfy the differential equation. yeah.

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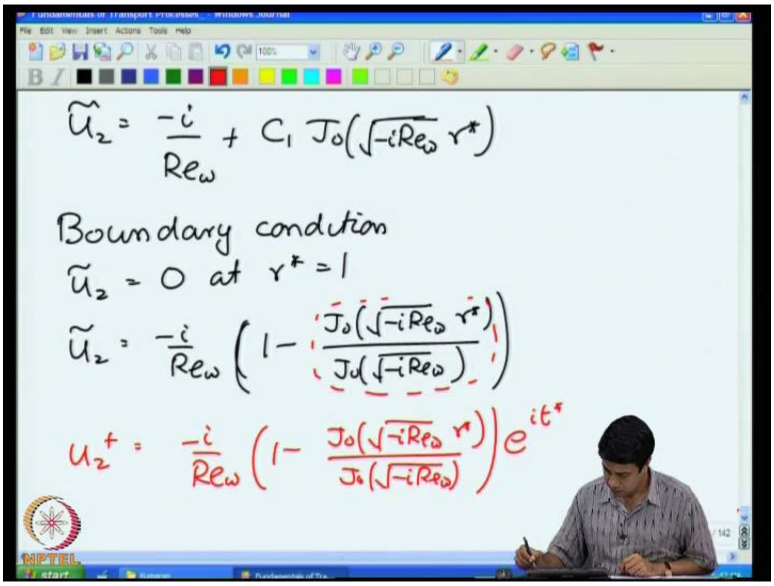

So, therefore, u z is equal to minus i by R e omega plus C 1 j naught of square root of and C 1 is of course, evaluated from the constant from the Boundary condition that we have no yet used that is the there is the Boundary condition u z tilde is equal to 0 at r star is equal to 1 and using that condition we can easily get the velocity profile as…

This gives us the final expression for the velocity profile and you can easily verify that when r star is equal to 1, in this term the numerator and the denominator are equal therefore, this becomes 1 velocity becomes zero. So, that is a final solution for the velocity profile, this is u z tilde; that means, that u z plus is equal to minus i by R e omega to 1 minus times e power i t star.

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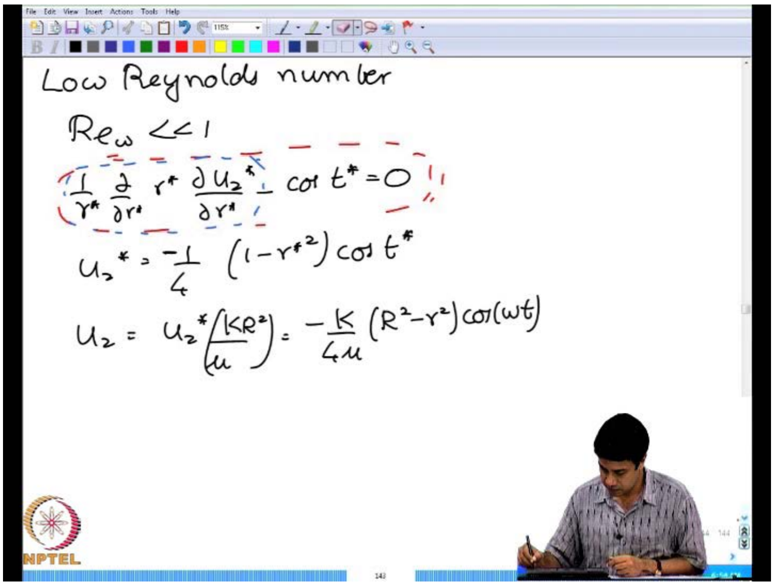

And finally, u z star is equal to the real part . So, I formally obtained a solution I obtained u z i am sorry I have obtained u z plus in terms of u z tilde it has an e power i t star on the right hand side, then I have to take the real part of this whole thing to get the actual velocity u z star as a function of time.

What does this physically mean I could of course, take the solution for the velocity profile and then use that in order to calculate numerically how the velocity profile should look, but a better understanding is obtained by looking at limits where R e omega is small compared to 1 and R e omega is large compared to 1 and R e omega small.

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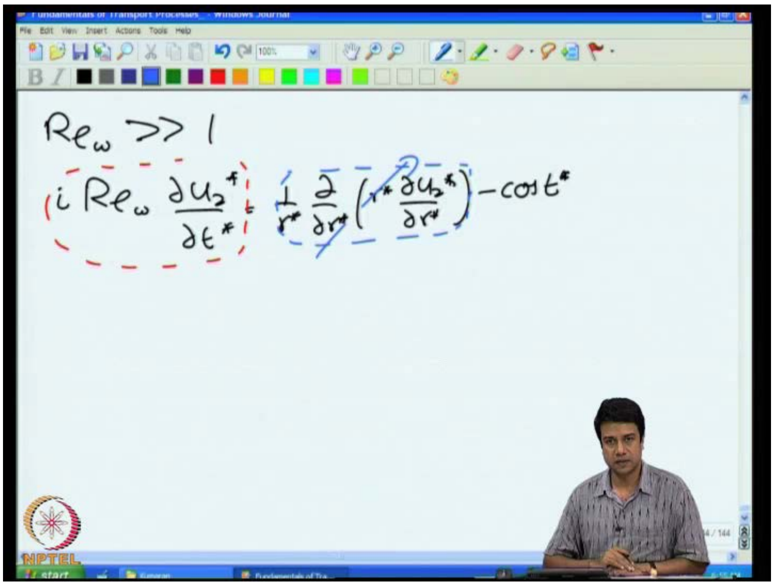

That means, in this equation I can neglect the left hand side with respect to the right hand side when R e omega small that is left hand term can be neglected in comparison to the right hand term and I will get 1 by r d by dr of r du z by dr minus cos t is equal to 0. And this you can easily solve this can be solved quite easily as if I neglect the take. So, this is the equation in the limit of low Reynolds number and I can solve this quite easily to get an equation of the form u z star is equal to 1 over four mu times 1 minus r star square times cos t star.

And I can get the dimensional velocity by using the same scaling that i had. So, u z is equal to u z star into K r square by mu is equal to minus K by four mu into r square cos of omega t, this is a parabolic velocity profiler, the exact same profile that we would get for a steady flow except that it has this k cos omega t dependence on it. So, this is a parabolic velocity profile for which the pressure gradient is the instantaneous pressure gradient at that particular instant in time. So, the limit of Low Reynolds number we get something that is the same as a parabolic velocity profile except that the mean velocity and the maximum velocity are oscillating in time with a frequency omega This is in the limit of Low Reynolds number what does Low Reynolds number mean, it means that the viscous stresses are dominant compared to the inertial stresses.

One can have another interpretation of Low Reynolds number, I can also define R e omega is equal to rho omega r square by mu is equal to omega by mu by r square. Omega is the frequency 1 over the time period omega is a frequency two pi by t, where t is the time period mu by r square r square by mu is the time it takes for momentum diffusion across a distance r.

So, if the Reynolds number is small; that means, that the time it takes for the momentum diffusion across a distance r is small compared to the time period of oscillation. The time it takes for momentum diffusion across a distance r is small compared to the time period of oscillation; that means, that the momentum diffuses throughout the tube instantaneously it diffuses over a distance r instantaneously and because of that instantaneous diffusion the velocity profile is the same velocity profile that you would have at steady state except that the pressure gradient in the velocity profile is the instantaneous value of the pressure gradient. So, the same velocity profile that you would have for instantaneous momentum diffusion except there is a pressure gradient is the instantaneous value of the pressure gradient. So, because of this the pressure and the velocity are exactly in phase the pressure gradient is exactly in phase with the velocity.

So, this is the simplified solution for Reynolds number small compared to 1 the velocity is exactly in phase with the pressure gradient and that is because the time it takes momentum to diffuse across the width of the tube is small compared to the time period of the oscillation. When the frequency is small period of oscillation is large the time it takes for momentum to diffuse across the tube is small compared to the time period of oscillation. So, that was when r e was small compared to 1.

What about the case where R e omega is large compared to 1. I have an equation of the form i r e omega d u z by d t is equal to 1 by r minus cos t star. When R e is large compared to 1, you would expect this term to be large compared to this 1. In that case can we just go ahead and neglect this term and solve the rest of the equation and get a velocity profile think about it we will continue that discussion in the next class it is a little bit complicated in this case you should think a little bit more about this frame this problem and see if you can get a solution. In the case where the Reynolds number is large is it possible to obtain a solution the same way that we did when the Reynolds number was small. This will give you some insight in to the competition between diffusion and convection in these systems. This is a first such case we have actually analyzed the balance between the convection and diffusion in this particular case it is not exactly convection it is an unsteady term, but proportional to the Reynolds number nevertheless.

So, we will continue this class the oscillatory flow in the pipe in the limit of High Reynolds number in the next lecture with a particular emphasize on the ration between inertial, and viscous terms we saw that when the Reynolds number based upon the frequency and the pipe radius was small you could neglect the inertial terms and get a solution which look like a steady flow, when the Reynolds number is large what happens we will briefly discuss this in the next lecture before you go into the next topic next topic will be on transport in another covering a coordinate system in that case it is called a spherical coordinate system appropriate for objects with spherical symmetry like spherical particles and so on. So, we will continue this finish it in the next lecture and then go on to transport and spherical coordinates. So, with this we will end this lecture we will continue this in the next class and we will see you next time.