Fundamentals of Transport Processes

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Module No. # 01

Lecture No. # 02

Dimensional Analysis

Welcome to this, the second lecture, in the course on Fundamentals of Transport Processes. Let us briefly review what we did in the last class and then come back to what we will do in this class.

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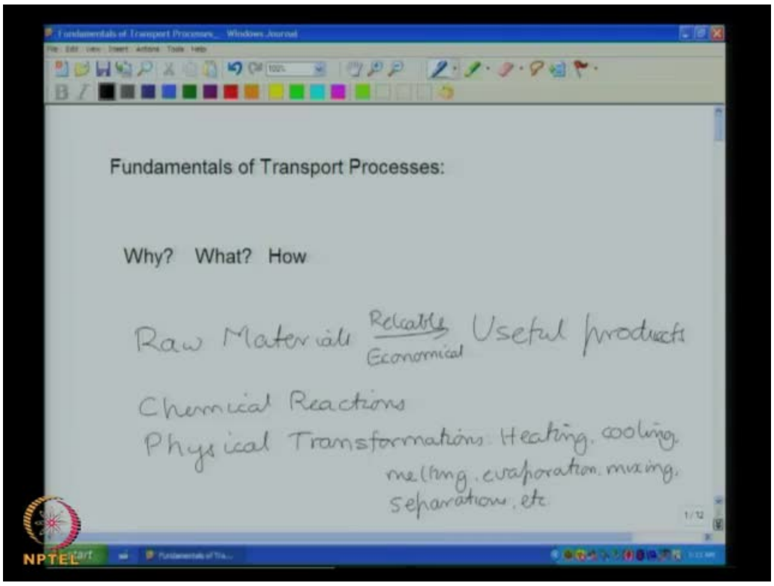

Last class, we had seen the Fundamentals of Transport Processes. I had tried to give you a motivation for why we want to do this and what is it that we are going to do.

Basically, all engineering processes deal with the conversion of raw materials to useful products. This takes place by means of transformations. These transformations could be chemical such as chemical reactions. They could be physical transformations involving heating, cooling, melting, freezing, mixing, separating, drying, various other kinds of physical operations. In all of these physical operations or chemical reactions, they involve the transport of materials, raw materials, converted into products, transfer of the products, Reynold transfer of heat.

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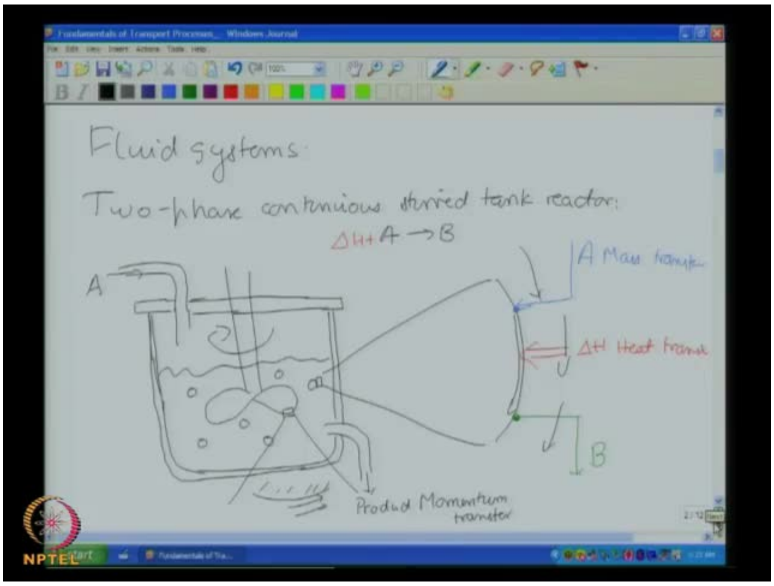

We looked at some concrete examples in the last class. For example, we should take a reactor, a two-phase system in which you have the conversion of the raw material A to the product B. A comes in at the entrance of the reactor, it gets converted to B, and then it goes out of the exit state.

Now, in this case, we looked at the specific example of solid catalyst reaction, a reaction catalyzed by a solid pellet. It is not just sufficient to ensure that there is sufficient amount of raw materials coming in and products going out, because the reaction is taking place at the surface of the catalyst. So, if you look at the surface of the catalyst, we have to make sure that there is sufficient amount of material continuously of reactant coming to the surface, reacting and product leaving the surface. It is important that reactant comes in continuously, so that the reaction can takes place. It is also important that the product leaves the surface, so that if the product does not leave, there will be no place for further reactant to come in.

In addition, the reaction can be either exothermic or endothermic. So, you have to supply requisite amount of heat or remove requisite of amount of heat from the surface; otherwise, if it is an exothermic reaction, we do not remove heat. There is going to be hot spot that are formed and then it could lead to run away reactions. If it is endothermic, if heat is absorbed, then the catalyst surface will cool down and the reaction would not proceed at the desired rate.

In addition, I had told you in the last class that there is also the problem of momentum transfer. You are stirring this reaction vessel with an impeller and supplying power to the impeller to compensate for the frictional laws of energy due to the fluid friction. The transmission of momentum due to fluid friction is a subject of momentum transfer.

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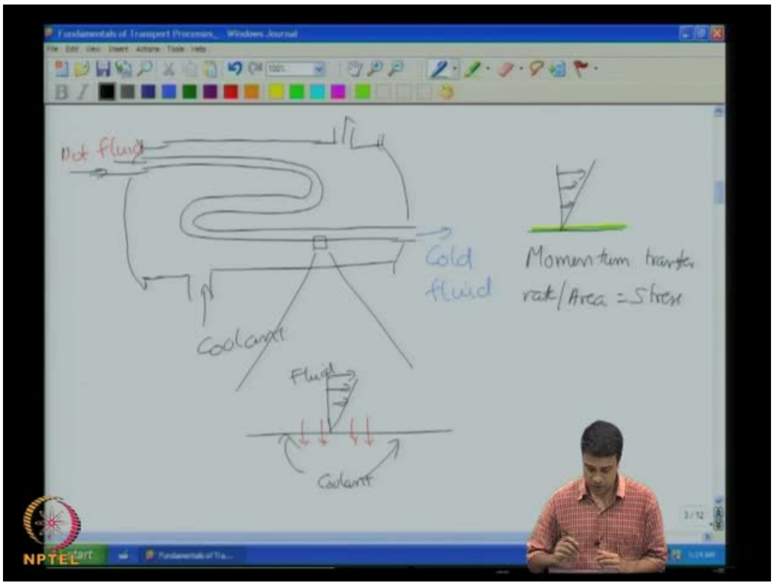

We looked at two other examples: one of heat transfer through a heat exchanger, where one has to have heat transport across the surface of the heat exchanger.

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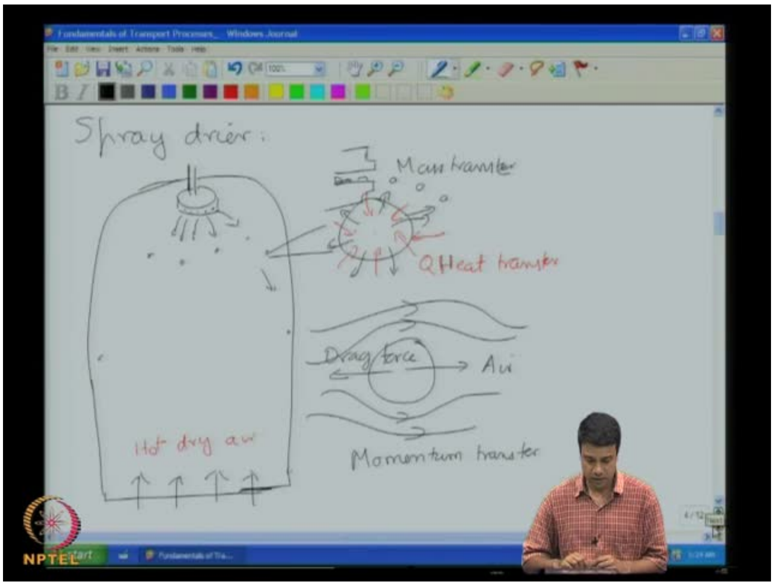

The second example was of a dryer, a spray dryer in which you are drying droplets of some material. This is used often for food products. One has to ensure that the temperature difference is not too high; otherwise, the material gets spoiled. So, in this case, for the drying to take place effectively, firstly, one has to provide sufficient latent heat for the moisture to evaporate, and then sufficient time for the evaporated moisture to come out of the surface before the material hits the wall of the dryer.

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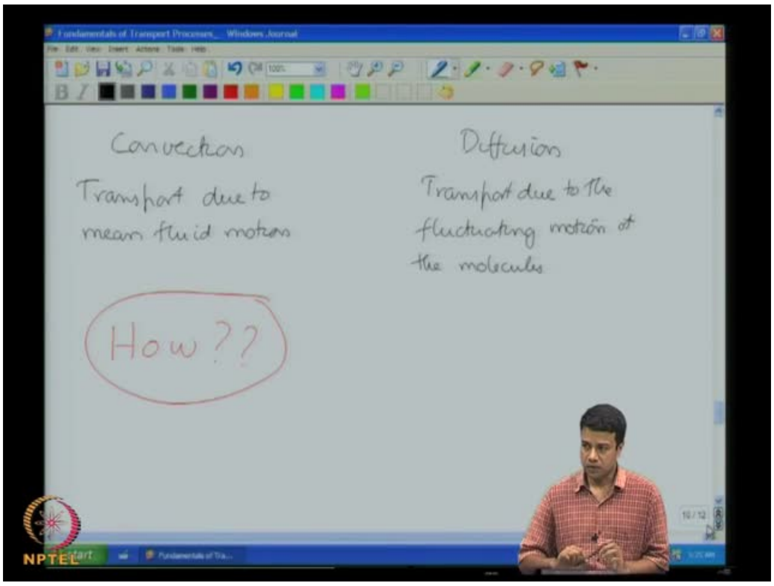

In the last class, I had discussed in detail what we are doing and why we are doing it. I briefly discussed that there is some… It is all determined by a balance between convection and diffusion. I said we would start on the topic of how we are going to do this analysis in the present lecture. However, before we do that, I would like to briefly review how the analysis is done in earlier courses such as unit operations. In unit operations, the basis is dimensional analysis.

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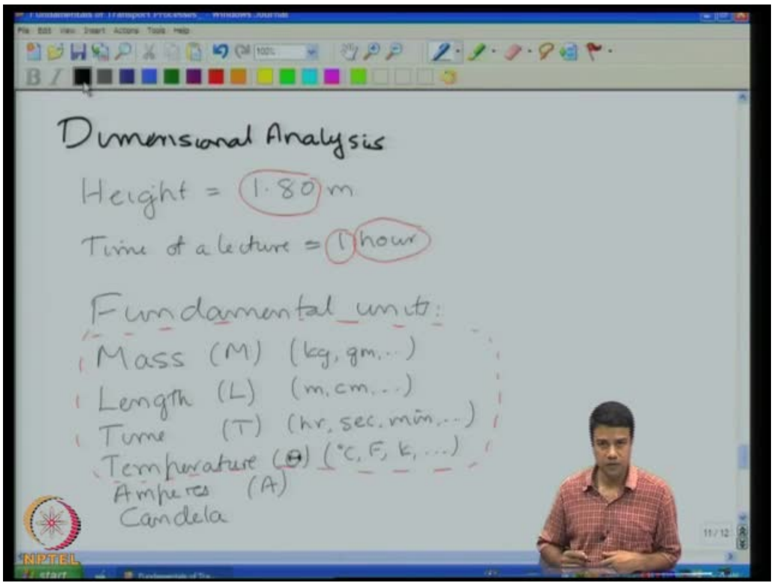

First, I briefly review dimensional analysis, how one uses dimensional analysis to advantage in determining the relationships between the fluxes and the concentrations at various locations, the heat transfer rate and the difference in temperature and so on. As I said, in dimensional analysis, one can only use dimensional analysis to get relations between average quantities. For example, in the reactor example, one can use dimensional analysis to get the flux, the transfer rate as a function of the difference in the concentration between the catalyst surface and the bulk of the fluid. So, this is a relationship between the average concentration difference and the flux.

Similarly, in the case of heat transfer, one can use dimensional analysis to get the relation between the heat flux and the difference in temperature between the surface and the bulk. So, these are average quantities averaged over the entire system. In the heat exchanger example, one can use dimensional analysis to get a relationship between the average temperature difference and the total flux across the surface.

In this course, in the subsequent lectures, we are going look at how to get those relationships at every point within the floor. However, first I would like to go through dimensional analysis to show you how you get these quantities in dimensional analysis here and how does one get empirical correlations. In the next lecture, I will try to give you a little more deeper understanding of what those dimensionless numbers mean. One way to look at it is to say that one has to just have the net dimension of a dimensionless number equal to 0 and that just defines the dimensionless number. However, there are multiple ways to define dimensionless numbers for a given set of quantities. Why are correlations written in one particular form and not another particular form? That is because each of these dimensionless numbers has a particular physical interpretation to it. That physical interpretation will be a subject of the next lecture. This lecture, we will look just that dimensional analysis.

Let us start at the basics. Every quantity has a unit, which tells you what is the measure and a number. For example, the height of some person is 1.80 meters. There is a unit that is meters. It tells you the unit of length. Then, there is a number in front, which tells you how many units correspond to the height of this person. The time of a lecture is 1 hour. This is the number (Refer Slide Time: 08:04) and this is the unit, the unit of time. There are various units of time, for example, hour, minute, second, but all of these are multiples or sub multiples of each other.

When one equates quantities, one has to equate quantities with the same unit; one cannot say that 1.8 meters is equal to 1 hour, because they are two different quantities. However, one can write a relationship for example, between the flux and the concentration gradient, because that is an equation in which both sides of the same unit. So, it is first important to list out the fundamental units, and then look at how these quantities can be expressed in terms of those fundamental units.

The fundamental units: mass - I will use the symbol M in this lecture; the units are kilograms, grams, etcetera. Length: this is symbol L; meters, centimeters, etcetera. Time: I will use capital T for time; the unit of time - hours, seconds, minutes, etcetera. So, these are the fundamentals units used for most mechanical operations. There are three others: one is temperature; I will use the symbol here - a capital theta; capital theta is a symbol for temperature. There are different units of temperature: degree, centigrade Fahrenheit, Kelvin, etcetera. There are two others, which we shall not use in this course. However, I will write them for completeness. That is, amperes, unit of current; it goes by the symbol A; candela, which is the unit of light intensity.

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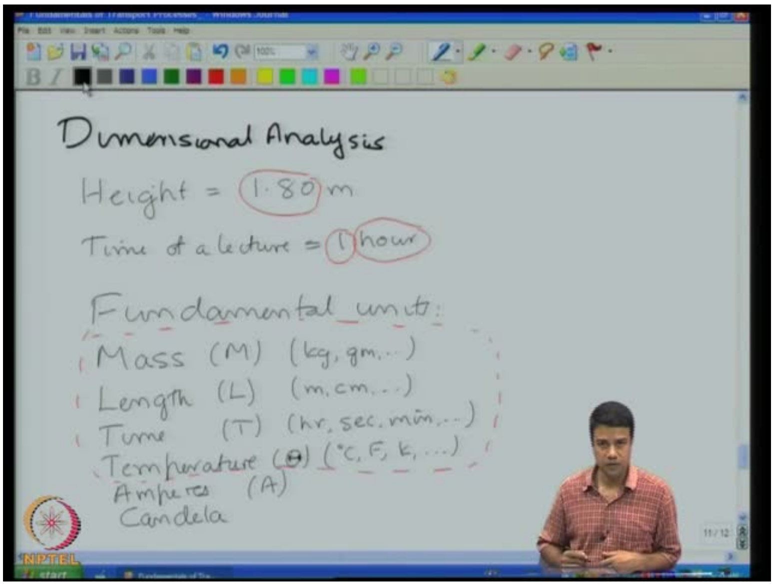

In this course, we will be using these fundamental units. For most mechanical momentum transfer processes, we will just use mass, length and time. Temperature becomes important when we have to deal with heat transfer. Now, how do we get derived units from these fundamental units?

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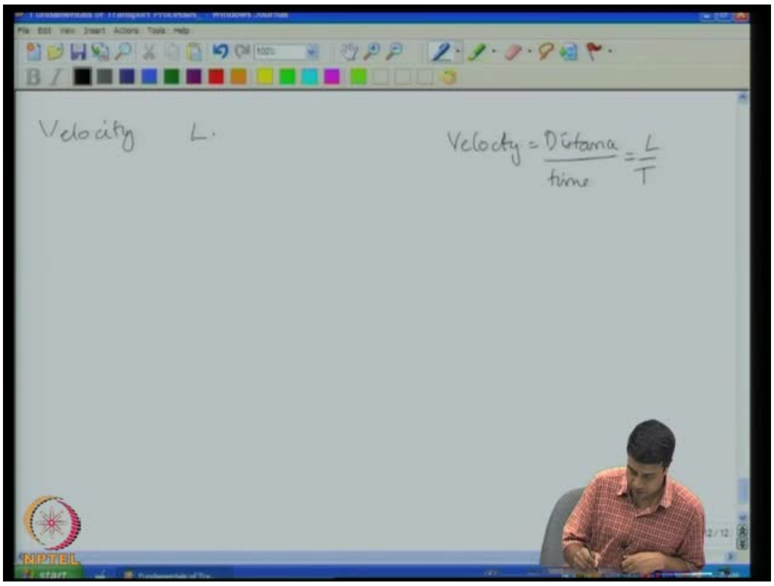

For example, velocity; velocity is distance moved per unit time. So, velocity is equal to distance by time. So, this has units of distance divided by units of time, which is L by T. So, the velocity has dimensions of LT inverse.

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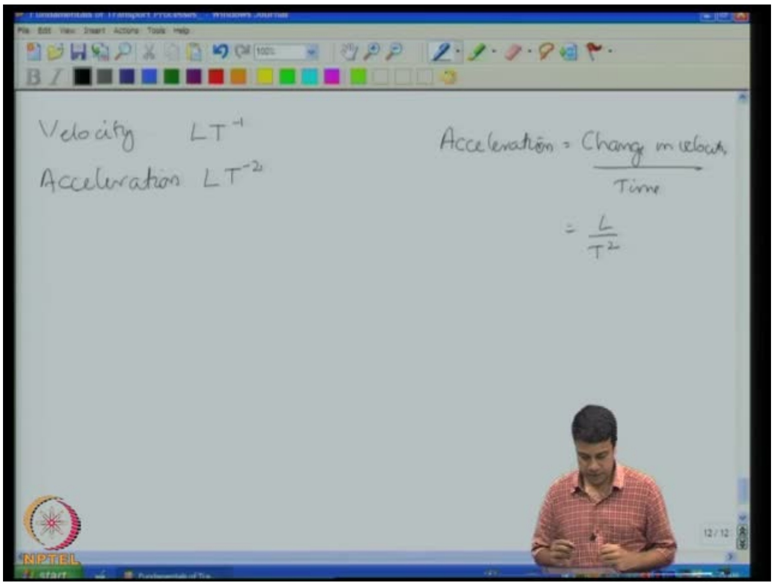

Acceleration; acceleration is the rate of change of velocity. So, acceleration is equal to change in velocity by time. So, it has the dimensions of velocity divided by time; that is, length by time square. So, this has dimensions of length T to the power minus 2.

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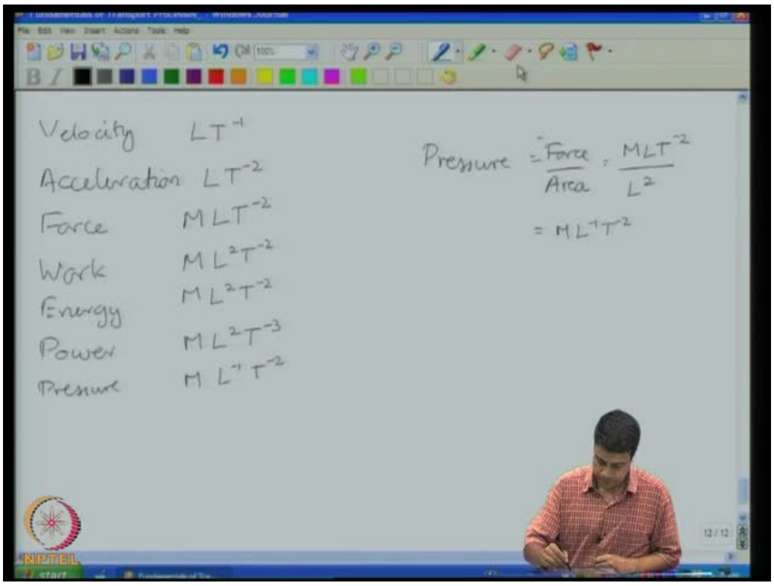

Next is force. From Newton's laws of motion, we know that force is equal to mass times acceleration; MLT to the power minus 2. Work has the same units. Work has the units of force times the distance. So, work is equal to the force multiplied by the displacement. So, it has units of force times distance; that is, M L square T to the power minus 2; that is, force times distance. Energy has the same dimensions as work; M L square T to the power minus 2. Power is the rate of input of work; the work input per unit time. So, it is going to have the dimensions of energy per unit time.

Now, let us look at some dimensions of some quantities that will be of interest to us in the present course. The pressure acting on a surface is the force per unit area. Pressure is equal to force by area, which is equal to M L T to the power minus 2 divided by the area. Area has dimensions of the square of length; by L square. So, this equal to M L inverse T to the power minus 2.

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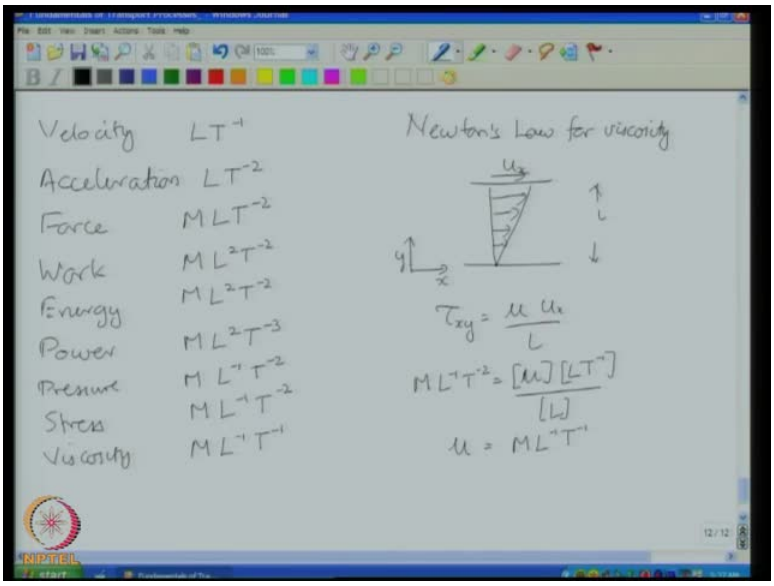

Now, the stress has exactly the same dimensions as the pressure. The stress is also a work per unit area except that this stress acts tangential to a surface; whereas, the pressure acts normal to the surface. So, the stress has exactly the same dimension. How about viscosity? The dimension for viscosity has to be obtained from some relation that contains the viscosity in it. That relation we know is the Newton's law of viscosity. If I have a fluid of length L between two plates, the top plate is moving with velocity u x, the bottom plate is stationary. At steady state, I will have a linear velocity profile between these two plates. Newton's law of viscosity tells us that the shear stress… Let us put the coordinate system here - x and y. Newton's law of viscosity tells us that the shear stress tau x y is equal to mu u x by L. The shear stress is equal to the viscosity times the difference in velocity between two locations divided by the distance between those two.

Let us try to write this in terms of dimensions to get the dimension of viscosity. The dimension of stress is M L inverse T to the power minus 2 is equal to the dimension of viscosity times the dimensions of velocity divided by the dimension of length. So, from this, one can easily see that viscosity has dimensions of M L inverse T inverse. So, viscosity is M L inverse T inverse. So, from the expression for the stress, we know the dimensions of the stress itself; we know the dimensions of velocity and distance. From that, we have got the dimensions of viscosity.

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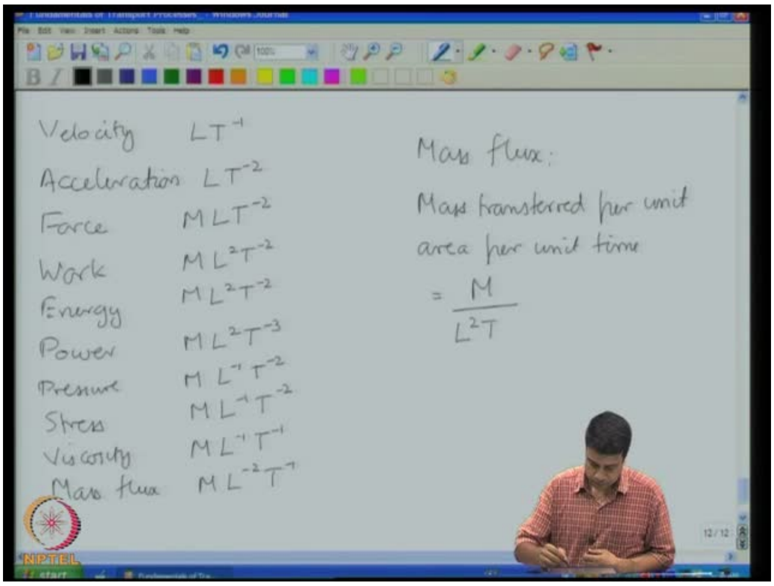

In addition to stress, in the last class, I had focused on two other important things: that was the heat flux and the mass flux. Let us look at the dimensions of those two. The mass flux is mass transferred per unit area per unit time. So, it is going to have dimensions of mass divided by unit area, which is the square of length divided by time. So, the mass flux is equal to mass L to the power minus 2 T inverse.

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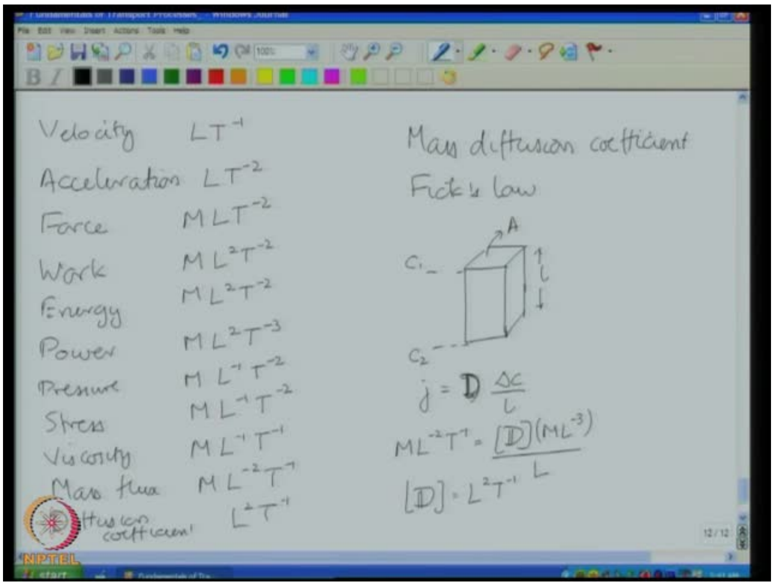

How about the mass diffusion coefficient? The diffusion coefficient for mass will be determined from Fick's law for diffusion. What does Fick's law state? If I have some fluid material, whose length is L and it has a cross sectional area A; if I put a difference in concentration of some solute across this material - this concentration is C1 and this is C2. There is going to be a flux of mass across this material, because there is a difference in concentration mass travels from a region of higher concentration to a region of lower concentration.

This mass flux is related to the diffusion coefficient by the diffusivity - I will use the script D for the diffusion coefficient – times the difference in concentration divided by the length. Therefore, the mass flux has dimensions of M L to the power minus 2 T inverse. This is equal to the dimension of the diffusion coefficient times delta C - delta C is a concentration. Concentration is mass per unit volume divided by length. By doing cross multiplication, it is easy to see that the dimension of the diffusion coefficient is equal to L square T inverse. So, I have diffusion coefficient, L square T inverse. So, these are quantities that are related to transport of mass.

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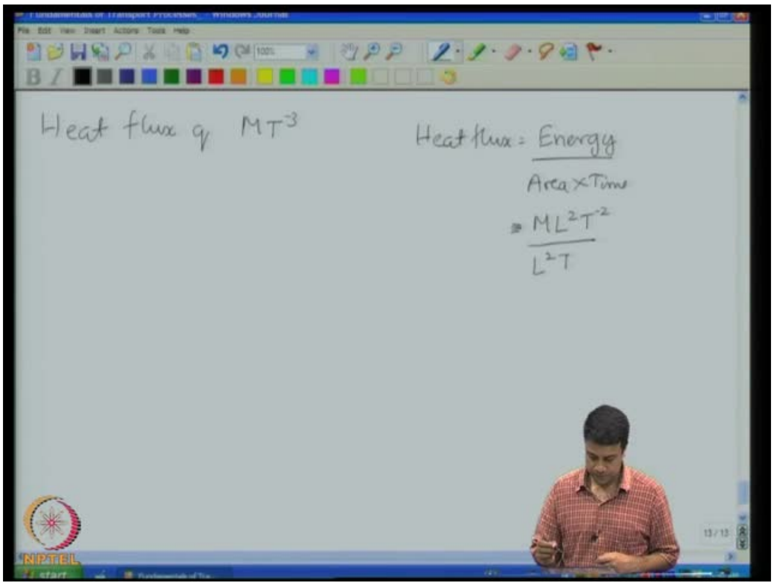

How about transport of heat? The fundamental quantity here is the heat flux. Heat flux is the heat energy that is transported, is equal to energy transported per unit area per unit time; the rate of transport of energy per unit area per unit time. Energy has dimensions of mass L square T to the power minus 2 divided by area, which has dimensions of L square and time, which has dimensions of T. So, you can easily see that the heat flux q has dimensions of M T to the power minus 3.

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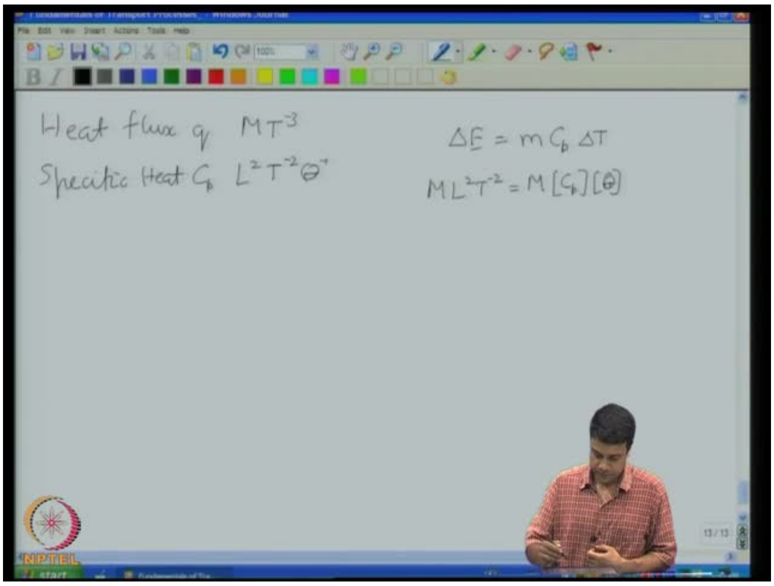

Another quantity of importance in heat transfer of applications is what is called the specific heat. It is written as C p for processes at constant pressure and C v for processes at constant volume. How do we get the dimensions of this C p? You can get this from the relationship that the change in thermal energy due to a change in temperature is given by m C p delta T. The change in energy due to a change in temperatures is given by m C p delta T at constant pressure.

The change in energy has units of energy. So, it has units of energy. It is M L square T to the power minus 2. It is equal to the dimension of M, which is mass, the dimensions of specific heat times the dimension of temperature, which is theta. From this it is easy to see that the dimension of specific heat is L square T to the power minus 2 theta inverse. So, that is the specific heat.

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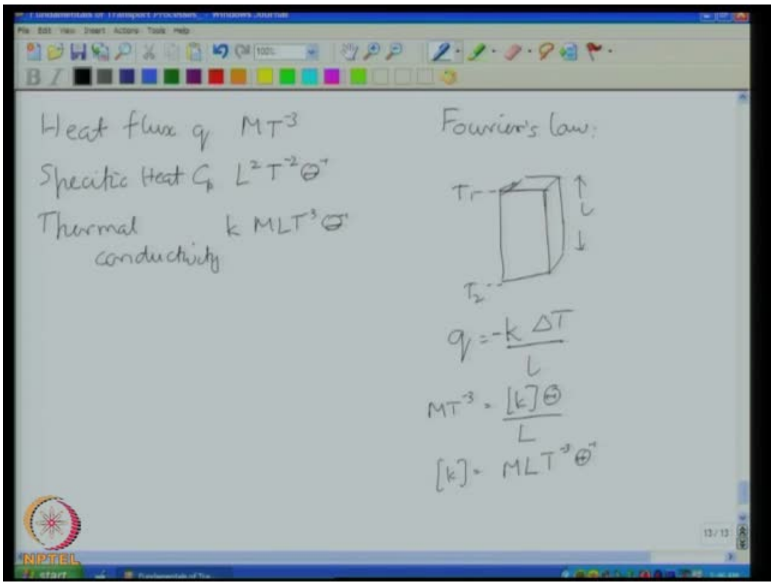

The other quantity of interest in heat transfer is the thermal conductivity. It is denoted by the symbol k. The thermal conductivity can be obtained from Fourier's law for heat conduction. Once again if we have some slab of material of length L across which I apply a temperature difference; T1 here and T 2 here, there is going to be a heat flux through the material. The total heat transfer will be proportional to the surface area of the material. However, the flux, which is a heat transfer per unit area, will be directly proportional to the difference in temperature and inversely proportional to the distance; to the length L. So, I will have the q is equal to the thermal conductivity times the difference in temperature divided by L. So, that is Fourier's law of heat conduction. Heat is transferred from a region of higher temperature to a region of lower temperature. So, it goes from higher temperature to lower temperature. Therefore, the direction of heat transfer is actually opposite to the direction of the temperature difference.

Writing down the units of this; M T to the power minus 3 is equal to the dimension of the thermal conductivity times the dimension of temperature theta divided by the dimension of length, which is L. So, putting all of these together, we find that the dimension of thermal conductivity is equal to M L T to the power minus 3 theta inverse. So, this is M L T to the power minus 3 theta inverse. Now, one could look at other simple dimensions, for example, gravity. Acceleration due to gravity has the same dimension as acceleration itself.

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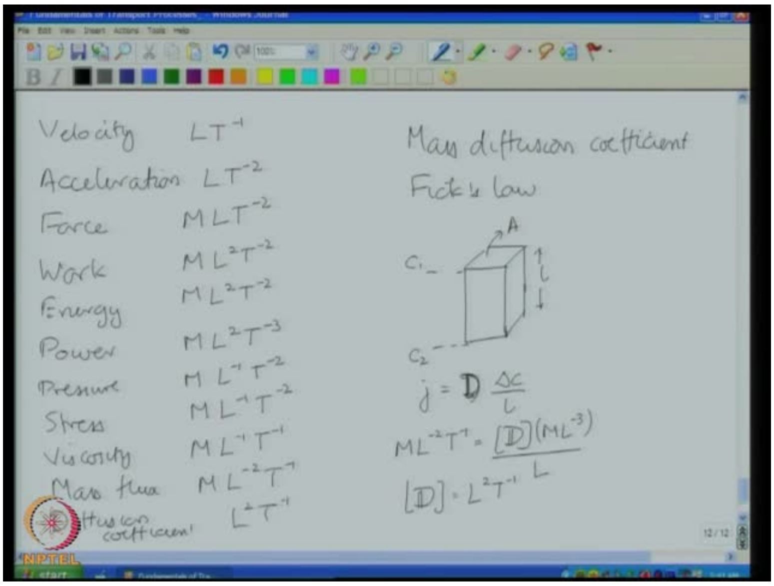

We have listed out the dimensions of various quantities, both for mass transfer and for heat transfer and for momentum transfer.

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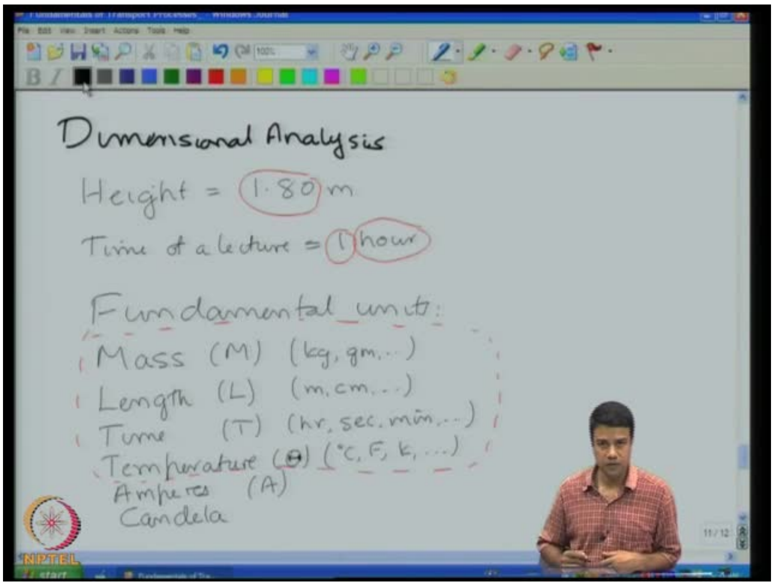

In the case of momentum transfer, the important quantities are mass, length, time as well as this (Refer Slide Time: 25:47) pressure, stress, viscosity. Mass flux and mass diffusion coefficient are two important quantities for mass transfer, as well as concentration difference. For heat transfer, I have heat flux, (Refer Slide Time: 25:58) the specific heat, and the thermal conductivity. Now, how do we use this to simplify our problems?

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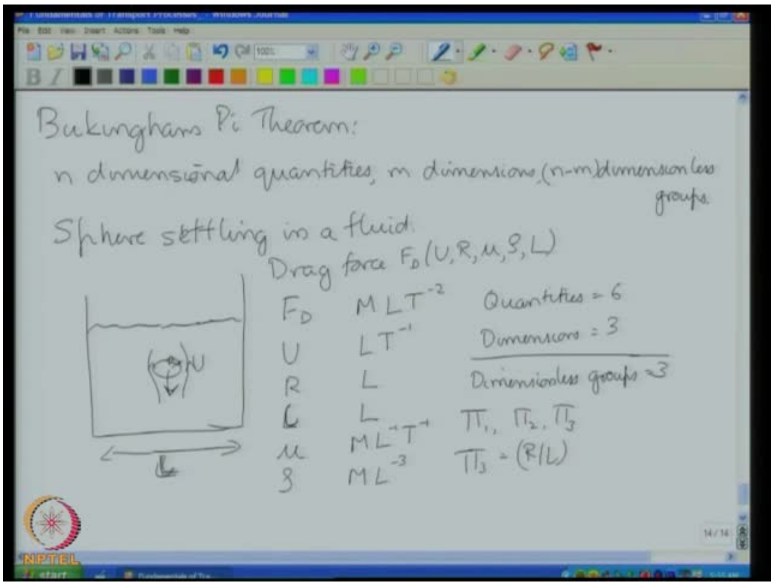

The fundamental theorem here is what is called the Bukingham pi theorem. Now, what that says is that if I have problem in which there are n dimensional quantities like force, velocity, viscosity, density, etcetera, these quantities involve the total of m dimensions. That means that one can write this as a relationship between n minus m dimensionless groups. So, these n dimensional quantities in this problem if they contain n dimensions, can be organized in to n minus m dimensionless groups.

Two important points here: One is to decide what are the quantities of dimensional quantities that are of relevance in the problem - not always an easy task. I will show you an example here. Then, to identify what are the dimensions in that problem. Then, you can find out how many dimensionless groups are there - n minus m independent dimensionless groups. So, in general, you can multiply two dimensionless groups to get a third dimensionless group, but that is not counted. There must be n minus m fundamental independent dimensionless groups.

Let us consider a concrete example, a sphere settling in a fluid. So, I have some tank of fluid and a particle that is settling under gravity in this tank of fluid. So, there is a gravitational force acting on this particle and my task is to find out what is the velocity U with it is settling. That is, of course, done by a force balance. The forces here - there is one gravitational force acting on the particle downwards; there is a buoyancy force acting upwards.

In addition, there is a drag force acting on the particle. The drag force is due to fluid friction. As the particle moves downwards, the fluid has to move around the particle relative to it. That fluid motion around the surface of the particle causes fluid friction, which acts as a resistive force to the motion of this particle. So, our task is to find out what is this drag force, FD. If I know what the drag force is, what the force due to gravity is, what the force due to buoyancy is, then from the relation, mass and acceleration is equal to some of forces, I can find out the rate at which the sphere settles.

Now, the task is to find out what is the drag force. The drag force in general can depend upon various things. First of all, it has to depend upon the velocity with which the sphere is settling. Obviously, if it goes faster, there we will be more force. If it goes slower, there is going to be less force. Secondly, it has to depend upon the radius of the sphere. A larger particle settling in a fluid is going to feel a larger force than a smaller particle settling in a fluid.

Fluid friction is called caused by fluid viscosity. Therefore, it has to depend upon the coefficient of fluid viscosity. In general, the fluid flow around the particle, the friction is determined by viscosity; whereas, the inertia of the flow around the particle is determined by the density of the fluid. The density of the fluid is also important. In addition, you can have other length scales. For example, it could also depend upon the dimensions of the tank. If the tank is very small so that the sphere just about fix into the tank, it is going to experience a large force; whereas, if the tank is large, it is going to experience a small force. It is also going to depend upon some dimensionless length, which gives you a measure of the size of the tank.

I told you that density was the density of the fluid. Can it depend upon density of the particle itself? Can it depend upon acceleration due to gravity? These are things where one must be careful. Gravity only affects the gravitational force acting on the particle. It does not affect the fluid friction force on the surface of the particle. The drag force is due to fluid friction on the surface. So, gravity has no direct effect on the fluid friction. Therefore, gravity cannot appear in this relationship.

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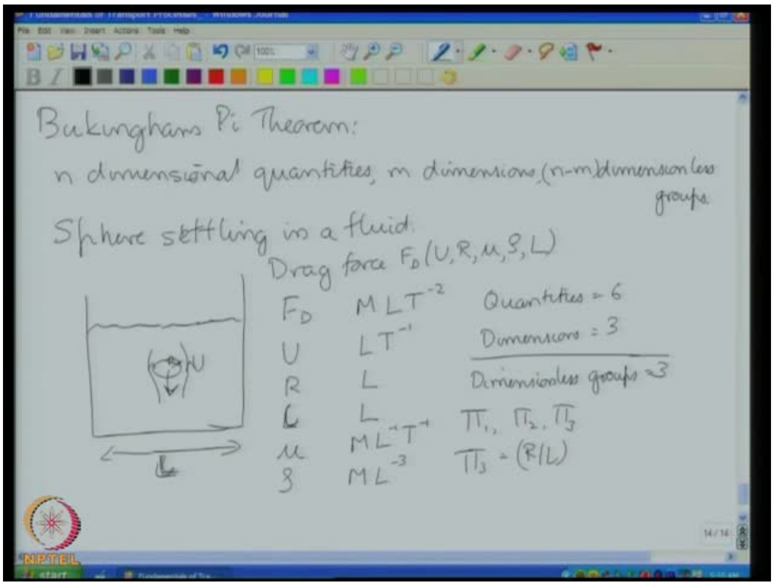

Similarly, the density of the particle is not important as far as the frictional force is concerned. The frictional force depends only upon how the fluid is flowing around the particle. How it flows around the particle depends upon the velocity around the particle and the balance of forces within the fluid. The solid density does not appear anywhere in this balance of forces within the fluid. Therefore, the solid density cannot be a relevant parameter here. So, in that sense, one has to be careful in assembling dimensionless groups. What exactly is the quantity that you are assembling the dimensionless group for? If it is for the drag force, acceleration due to gravity cannot be important. The acceleration acting on the particle should be the same whatever condition it is in. Only the downward force, the gravitational force acting on the particle, which appears later in the force balance, will be affected by the acceleration due to gravity.

Now that we have determined what are the dimensional quantities, we list out the dimensionless groups. FD is a force - mass times acceleration. U is a velocity, length per unit time. R is a radius, which is length. L is also some dimension of this tank; I will call it is small l in order to be unambiguous. The viscosity, coefficient of viscosity we just determined - M L inverse T inverse. The density is mass of fluid per unit volume; mass per unit volume; that is, M L to the power minus 3. So, these are the dimensional quantities.

How many dimensions do these quantities contain? There are only three dimensions; that is, mass, length and time. So, I have six quantities, three dimensions. That means I can have three dimensionless groups. Number of quantities is equal to 6; number of dimensions is equal to mass, length and time; there is no temperature here. So, this equal to 3 (Refer Slide Time: 34:00), which means that dimensionless groups is equal to 3. These are three dimensionless groups and we will call them as conventional as pi 1, pi 2, pi 3. One of these dimensionless groups is quite easy to see. There are two quantities, which have dimensions of length. So, one can form a dimensionless group, which is the ratio of these two quantities with dimensions of length. So, I will call pi 3 is equal to R by L - the ratio of the two lengths.

Now, once pi 3 has been defined with respect to R and L, it is sufficient for me to include just one length scale in all subsequent calculations. I do not have to consider R and L separately because I already have a dimensionless group, which contains the ratio of these two (Refer Slide Time: 34:58). So, it is sufficient to work with just one of these. What are the other dimensionless groups?

Now, what we want to find out is the force. So, obviously, the force, the dependent variable, which depends upon all of these other independent variables has to appear in one dimensionless group. So, one should be able to form a dimensionless group, which contains the force and three of the other quantities, because the force contains three dimensions. Therefore, I need another three of the other quantities in order to form a dimensionless group. There is some subjectivity in how you choose these dimensional quantities for assembling a dimensionless group. For the present, I will choose the dimensional quantities as force, the velocity, the radius and viscosity, and assemble a dimensionless group from these four.

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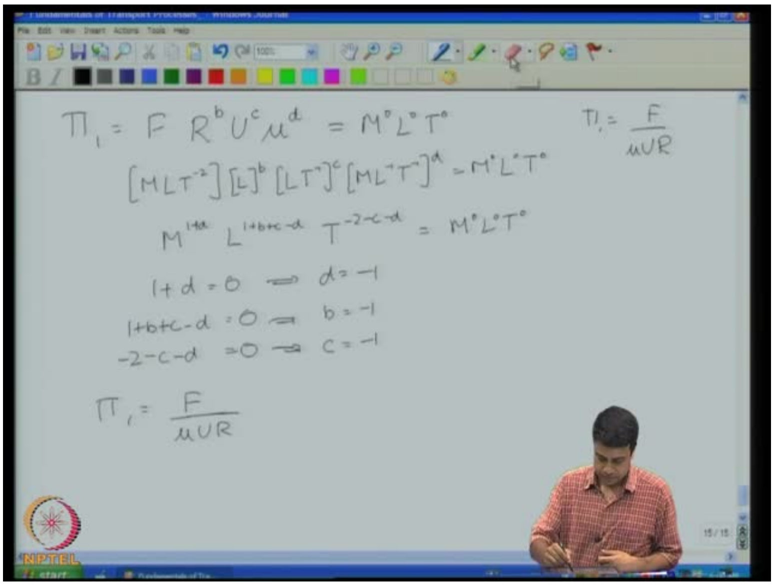

Dimensionless group, pi 1 is equal to force to the power a radius to the power b velocity to the power c and viscosity to the power d. That is the most general relationship that one can write now. However, if pi 1 is a dimensionless group, then pi 1 raise to any power is also a dimensionless group. Without loss of ambiguity, I can take one of these exponents as just one, because if I take this dimensionless group pi 1, take it to the power of 1 over a, the exponent on f will become just 1. So, because of that, I can take any one of these to be 1 and I will take the coefficient a to be 1. So, I get a dimensionless groups in all is force, distance, velocity, and viscosity. This has to be dimensionless. So, it has to have mass to the power 0, length to the power 0, and time to the power 0. So, force has dimensions of M L T to the power minus 2 - mass times acceleration - times length to the power b - velocity, which is length T inverse to the power c and then viscosity, which is ML inverse T inverse to the power d, which is equal to M to the power 0 L to the power 0 T to the power 0 - dimensionless.

Let us assemble the dimensions of each individual – mass, length and time. So, M to the power 1 minus d. Now, L has 1 plus b plus c minus d; that is, it has one dimension from the radius, one from the velocity, and minus d from the viscosity. Dimension of time is T to the power minus 2 minus c minus d, is equal to M to the power 0, L to the power 0, T to the power 0. From this, I get 1 minus d is equal to 0, 1 plus b plus c minus d is equal to 0, and minus 2 minus c minus d is equal to 0. So, I can solve these equations. This will give me… Sorry, 1 plus d here; d is equal to minus 1. This equation will give me c is equal to minus 1. This equation from that I will get b equals minus 1. So, this dimensionless group pi 1 is equal to F by mu U times R. So, that is the first dimensionless group, pi 1 is equal to F by mu U R.

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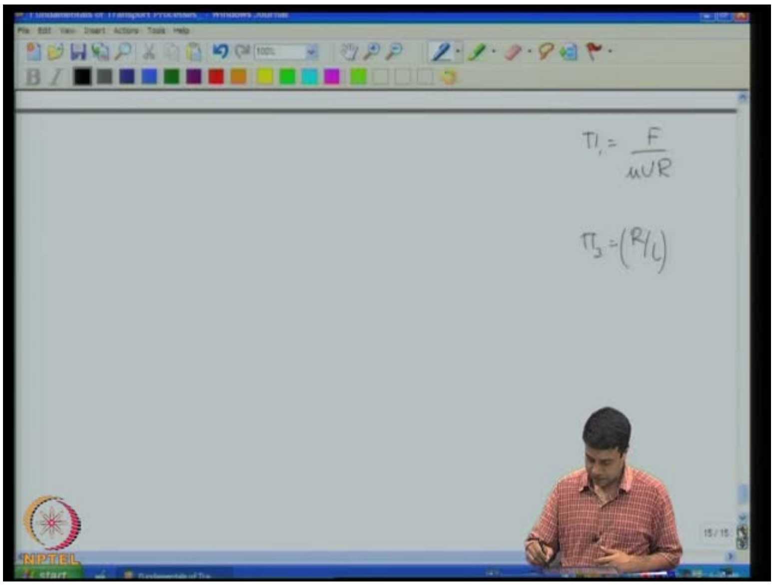

So, that is one dimensionless group. I already said that the third dimensionless group can be written as the ratio of length scales R by L. What about the second dimensionless group?

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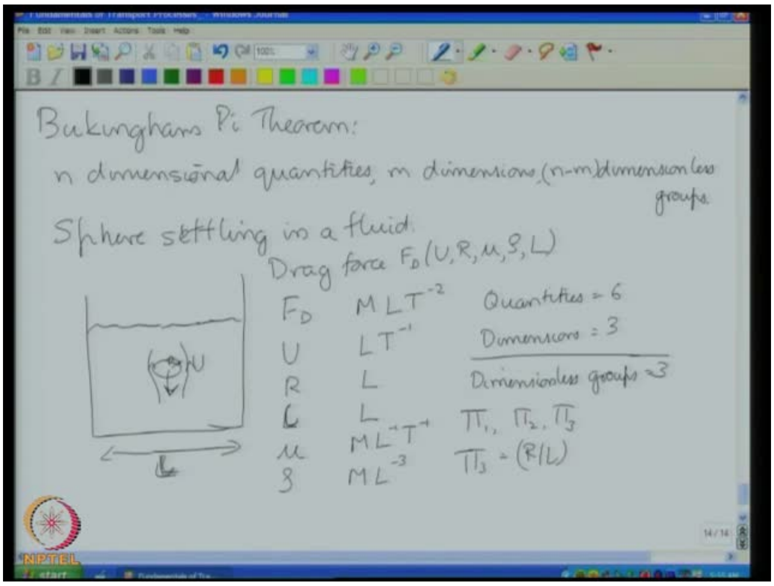

Let us go back to the previous example. The second dimensionless group that I can assemble… I have already assembled one with the force. Force is a dependent variable. So, the second one does not to have force. Therefore, it has to have the other four quantities; it has to have the velocity, the radius, the viscosity, and the density.

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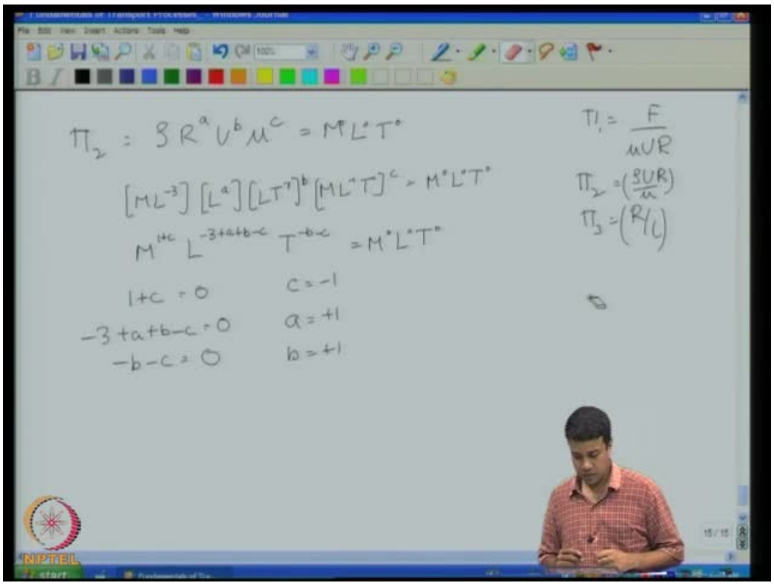

The second dimensionless group has to have the form, pi 2 is equal to… As I said, you can put the dimension of any one quantity as 1 without loss of generality. So, I will put the dimension of rho as 1 – R to the power a – U to the power b – times viscosity to the power c. This is dimensionless. Therefore, I have M L to the power minus 3; density is mass per unit volume – times the length to the power a – the velocity; L T inverse to the power b – times the viscosity; M L inverse T inverse to the power c, is equal to M to the power 0 L to the power 0 T to the power 0.

Expand this out; M to the power 1 plus c because it is there in density and the viscosity. Then, L to the power minus 3 plus a plus b minus c times time minus b minus c, is equal to M to the power 0 L to the power 0 T to the power 0. So, from this I have the equations: 1 plus c is equal to 0, minus 3 plus a plus b minus c is equal to 0, and minus b minus c is equal to 0. From this, I get c is equal to minus 1. From this, (Refer Slide Time: 42:06) I will get b equals plus 1. From this, I should get a is equal to plus 1. Therefore, the dimensionless group is pi 3 pi 2 is equal to rho UR by the viscosity. This will be familiar to most of you – rho UR by mu is the Reynolds number; ratio of inertia and viscosity.

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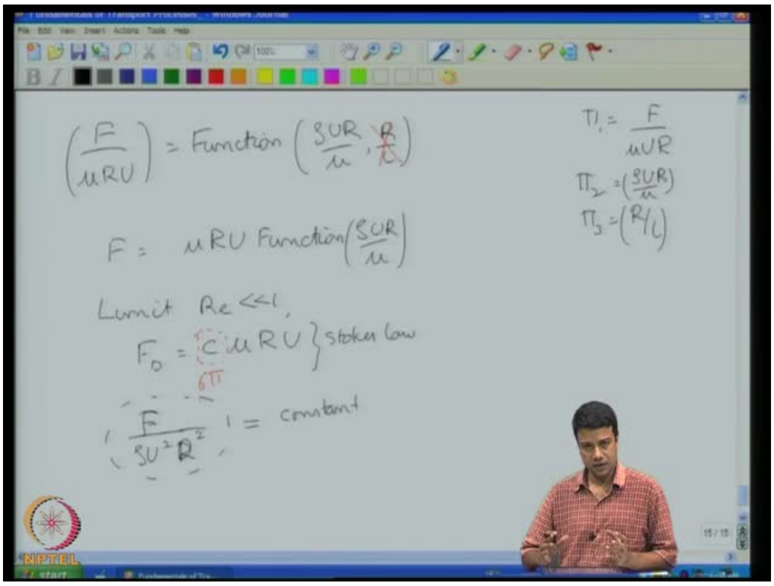

Rather than writing the force in terms all of those other dimensional variables, I could very well write a relation of the form F by mu R U, which is pi 1 is equal to some function of rho UR by mu and R by L. So, rather than having a relationship between F and all of these quantities, now, I only have relationship between one dimensionless group and two others. So, this has significantly reduced the amount of experimentation that I need to do in order to find out what those dimensionless groups are. So, rather than varying the density, velocity, radius, viscosity, I just need to vary the Reynolds number and this ratio of lengths alone.

In this example, (Refer Slide Time: 43:41) if the tank is sufficiently large in the sense that the walls do not influence the flow near the surface and near the solid particle, then this ratio (Refer Slide Time: 43:50) will not be a function of this. So, I will only get a relationship between scaled force F is equal to some constant times mu RU times some function of the Reynolds number.

Now, here is where some physical understanding can come in to further simplify the problem. I said that the Reynolds number is the ratio of inertial and viscous stresses. Now, if the Reynolds number is small, that means that inertia is small compared to viscosity; that means that inertia no longer matters. Therefore, the force should be a function of viscosity R and U alone. So, in the limit as Reynolds number small, then I should able to get a force, which is a function of the viscosity, the radius, and the velocity alone. So, the force, drag force should have the form some constant times mu R times U. So, it is further simplified if you know that the velocity such that the Reynolds number is small. Reynolds number is small, you know that inertial effects cannot be important. Therefore, the density cannot come into the equation.

Now, this is what as far as the dimensional analysis and physical inside will get you. It will not tell you what is the actual value of this constant (Refer Slide Time: 45:23). The value of the constant can be determined in two ways. One is by experimentation. So, you just do it for different values of the viscosity, velocity, and so on and find out what the constant is. The other way is to actually do the analysis of the exact flow around a sphere, which is what we will do in this course. If you actually do that analysis, you will find that this constant is equal to 6 pi. This is called the Stokes law for the flow around a sphere.

What about when the Reynolds number becomes large? When the Reynolds number becomes large one would intuitively expect that the viscosity is not important. So, I should be able to get a dimensionless group, which includes just the force, density, velocity, and diameter. So, that dimensionless number is given by… If you can easily verify it, F by rho velocity square times the radius square. So, you would except this to be a dimensionless group in the limit… As viscosity goes to 0, the Reynolds numbers becomes large. This should be equal to some constant because the viscosity is no longer in the picture. Turns out this is not accurate and we will see why later. That has got to do with the balance with convection and diffusion that I talked about in the last class.

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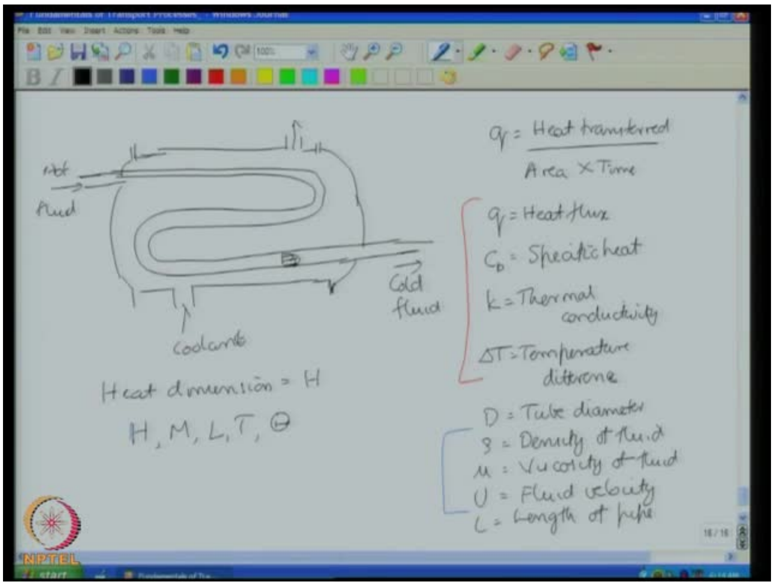

Now, let us go towards slightly more complicated problem. That is, we will try to analyze the heat transfer rate in an entire heat exchanger using dimensional analysis. There is a heat exchanger. Now, how do we do dimensional analysis for this entire system? One is to just find out what is the heat transfer rate; heat transfer per unit time from the shell side to the tube side of this heat exchanger. There is hot fluid coming in and there is cold fluid going out. This is the tube side coolant coming in and going out. So, one is the amount of heat energy that is transferred from the hot fluid to the cold fluid per unit time. So, that is the quantity that you are interested in as far as design is concerned. You want to know how much heat is transferred, so that you know what the difference in temperature between the inlet and outlet is.

However, it is more useful to talk about a transfer rate per unit area of the surface or the heat flux. The average heat flux is equal to total heat transfer per unit area per unit time. So, this is the heat flux. Now, the heat flux is dependent upon the temperature difference between the shell side and the tube side. So, there is one of the quantities that depend upon; greater the temperature difference, the more the heat flux. It will depend upon the thermal properties: the thermal conductivity, the specific heat.

Also, the heat is being swept through the heat exchanger due to convection. So, it is also going to depend upon the velocity with which the heat is going through. In addition, not only the mean velocity; it is not going to depend upon only the mean velocity, it is also going to depend upon the details of how the flow is within that q of the heat exchanger. If the flow is of a laminar type, you have nice straight stream lines; you will have a lower transfer rate. It is a highly mixed turbulent velocity profile. Then, the mixing is far more efficient and you will have a different heat transfer rate. So, it is also going to depend upon the fluid flow properties, that is, the density, the viscosity, and the velocity.

Let us assemble all of these. What are the dimensional quantities that it is going to depend upon? One is q, which is the heat flux. It is going to depend upon the specific heat, the thermal conductivity, and the temperature difference, delta T. It will of course depend upon the tube diameter. Then, it is going to depend upon the mechanical properties: the density of fluid, the viscosity of fluid, and the fluid velocity. Of course, the heat transfer rate is going to increase as the length of the pipe increases. So, the pipe length L is also going to be a factor.

Number of dimensional groups: 1 2 3 4 5 6 7 8 9; number of dimensions in these dimensional groups: there is mass, length, time, and temperature, because the temperature difference also comes in. So, there is 4. (( )) you might think that there are five dimensionless groups and that is correct. So, one should be able to get five dimensionless groups from this. However, one can do more than just this with dimensional analysis.

Here, the assumption is that the heat flux is an energy per unit area per unit time. However, if there is no conversion of heat energy into mechanical energy, then the heat energy can be considered as a dimension all by itself. There is no conversion between heat and mechanical energy. So, the heat energy has to be conserved. So, rather than considering heat and mechanical energy as equivalent things, we can consider them as different things.

Assign a different dimension, that is, heat dimension with the symbol H. So, this makes it clear that there is a separation between the thermal properties and the mechanical properties in the system. Thermal properties are different from the mechanical properties, because the thermal properties involves the transfer of heat energy; whereas, the mechanical properties only involve the mechanical energy. The two are different. So, the number of dimensional groups that I have is heat energy, mass, length, time, and temperature theta. Five dimensions, nine dimensional quantities. That means I should have four dimensionless groups. How do you determine these dimensionless groups? First, we will list out the dimensions of all of these dimensionless groups. We will take up this in the next lecture. How to calculate the dimensionless quantities from these dimensional groups and then get a relationship between the different dimensionless quantities?

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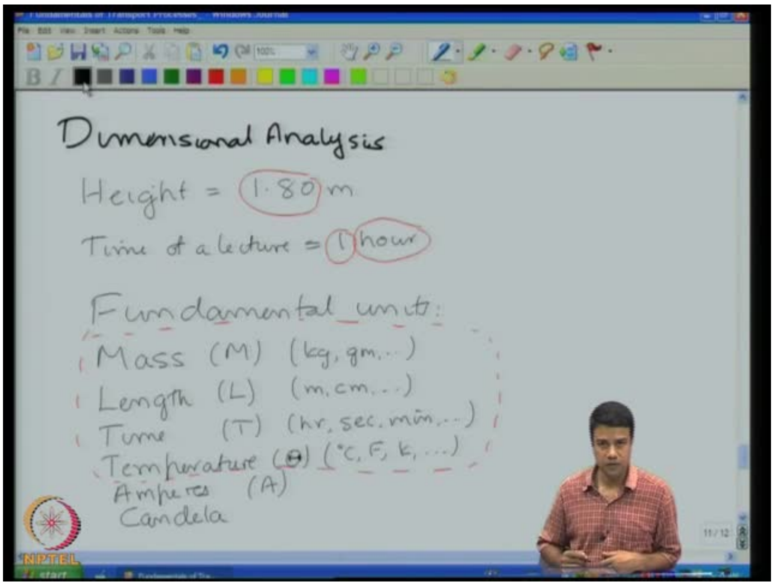

To briefly review, in this lecture, I had gone through dimensional analysis. The first thing that I said was that there are fundamental units and there are derived units. The fundamentals units are mass, length, time, temperature, amperes for current, and candela for light intensity. In this course, we will use only four of these: mass, length, time, and temperature, which I have denoted by M L T and theta.

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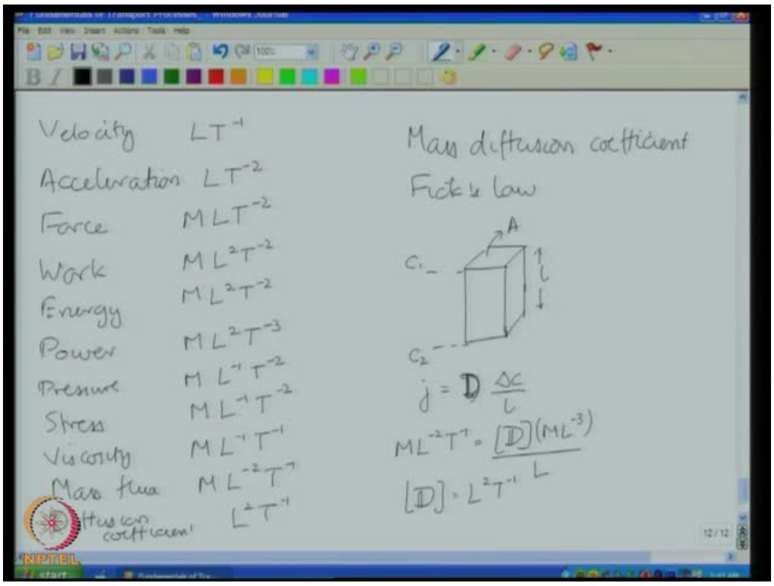

In any equation that you write, the dimensions on both sides are to be equal. So, all of these other derived quantities can be determined from relations between these derived quantities and fundamental quantities. For example, velocity is distance moved by unit time. So, velocity has dimensions of L T inverse. Acceleration: rate of change of velocity; L T to the power minus 2 and so on. Work, energy, power. Pressure is force per unit area, stress is force per unit area. They both have the same dimension.

For the present course, important dimensional groups: we saw viscosity, comes from Newton's law of viscosity, which states that the stress is equal to the coefficient of viscosity mu times the difference in velocity divided by the length across which that velocity difference is applied. The change in velocity per unit length is called strain rate. From that relation, we got the dimension of viscosity as M L inverse T inverse. Fick's law for diffusion was used to determine the equation for the diffusion coefficient from analogy of the mass flux.

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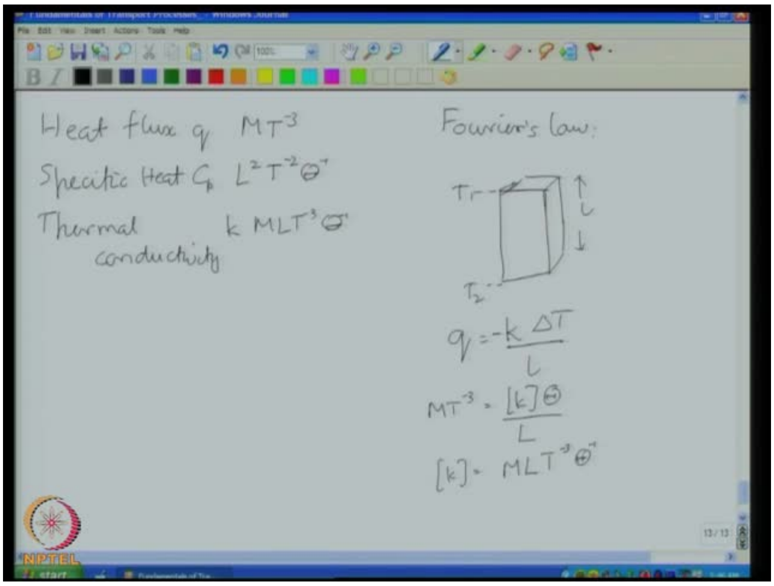

Fourier’s law of heat conduction was used to determine the dimension of the thermal conductivity by knowing the difference in temperature and the heat flux.

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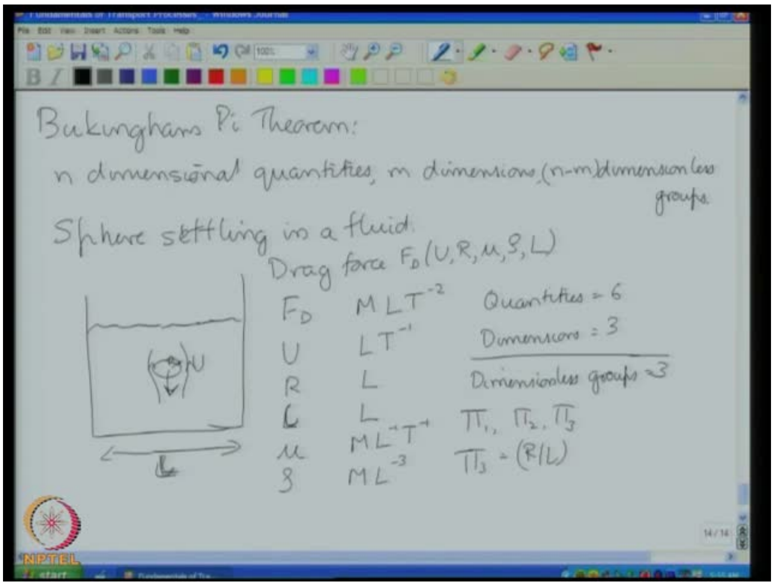

Then, we did a simple example, where we looked at the force – the drag force acting on a sphere, and tried to determine the dependencies of this drag force on various dimensional quantities using dimensional analysis. In this case, we had six quantities and three dimensions. Therefore, there are three dimensionless groups. One of these dimensionless groups was just a ratio of lengths. That was easy. We had to determine two others.

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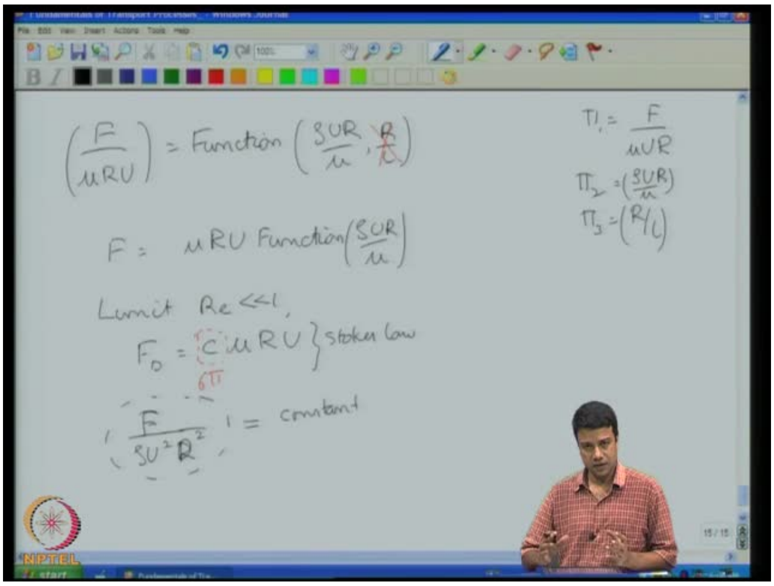

These two are others. We got as F by mu R U, which is the ratio of the force and the viscosity, velocity, and the radius of the sphere. The second one was the Reynolds number – rho UR by mu; ratio of inertia and viscosity. I told you that you can do a little more than just finding out the relationship between these different groups. If R is large, if the width of the tank is large compared to the radius of the sphere, then the fact that there are walls should not matter, because the walls of the tank do not affect the velocity field around the sphere. Then, the dependence of R by L is removed.

The force just becomes function of the Reynolds number; F by mu RU just a function of the Reynolds number. If the Reynolds number is small, there should be no dependence on inertia. So, we got a straight equation between the drag force and the viscosity, the radius and the velocity with just an unknown coefficient in it. In the other limit where the Reynolds number is large compared to 1, I said that you would expect on the basis of dimensional analysis that this is a constant. Viscosity does not matter, but that is not true. We will see later why that is not true.

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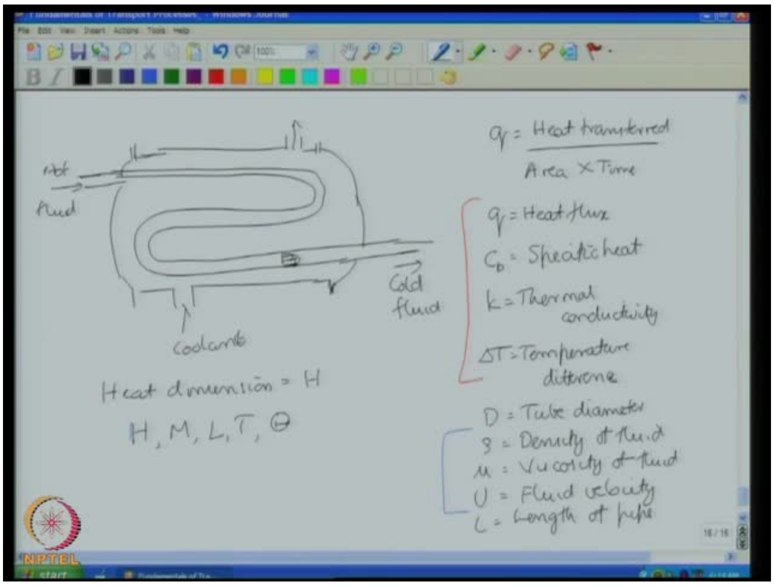

Then, we started the problem of trying to find out the similar relation for the heat transfer across a heat exchanger. We will continue this in the next class.

Have a good day. We will see you in the next lecture. We will continue the derivation of dimensionless groups for this heat transfer problem. We will later on go to the dimensionless groups for the reactor that I had talked about in the previous lecture. So, we will see in the next lecture.