Fundamentals of Transport Processes
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Lecture No. # 17
Unidirectional Transport Cylindrical Co-ordinates - II Similarity Solutions

We are at lecture number seventeen in our course on fundamentals of transport processes. And, we have just started looking at transport in curvilinear coordinates. So far, everything that we did was in a cartesian coordinate system.

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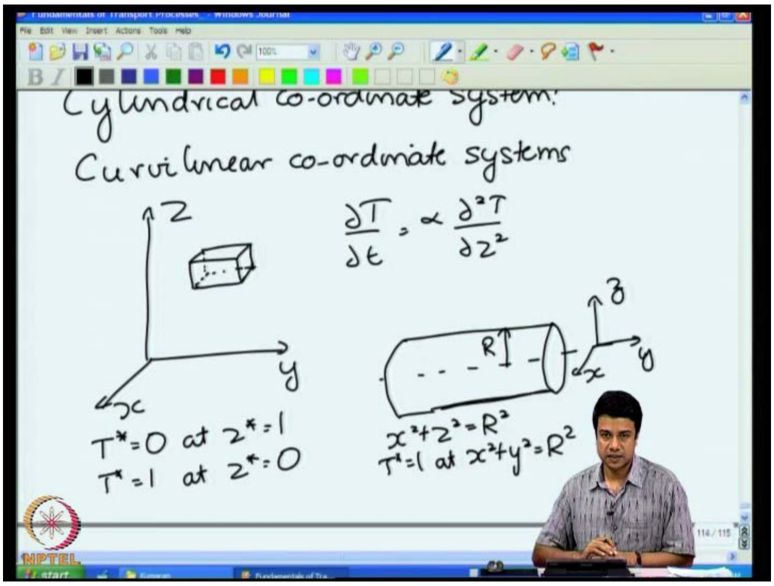

Last lecture, we started looking at a cylindrical coordinate system. This cylindrical coordinate system is an example of what are called curvilinear coordinate systems. And, these are different from cartesian coordinate systems, because the lines of constant coordinates in these coordinate systems are no longer straight lines. In a cartesian coordinate system, the three axis: x, y and z – they are all straight lines. Therefore, the planes of constant x, y and z are also straight lines. Therefore, if I construct a volume in a cartesian coordinate system – a differential volume, then all the surfaces of that volume are all planes. The surfaces of these volumes are all planes. The surface perpendicular to the x direction is in the yz-plane; surface perpendicular to y is in the xz-plane and the surface perpendicular to z is in the xy-plane. So, the lines are straight lines and the planes are flat. This is a cartesian coordinate system. This is particularly easy to work with, because we know that the planes are perpendicular to lines of constant x, y and z. And, the lines are straight. Therefore, the planes are flat.

And, when we did transport in this (Refer Slide Time: 02:35) coordinate system, we got equations of the kind dT by dt is equal to alpha times d square T by dz square for unsteady transport in only one dimension. Even though this coordinate system is the simplest to work with, it may not be a appropriate for all geometries. That is the reason that we have curvilinear coordinate systems. For example, if I wanted to analyse the flow in a pipe, which has a central axis like (Refer Slide Time: 03:24) this, this pipe is a cylinder. Therefore, if I took the coordinate system here – x, y and z, the surface of this pipe would have to be described by an equation for the surface containing the x, y and z coordinates. So, in this particular case, the surface of this pipe will be described by the equation x square plus z square is equal to R square; where, r is the distance from the axis. So, that will be the equation for the pipe.

Whereas, in this (Refer Slide Time: 04:05) case, I had fixed boundary conditions. For example, T star is equal to 0 at z star is equal to 1 and T star is equal to 1 at z star is equal to 0 in our separation of variables class. In this case, I will have to fix a boundary condition at x square plus T star is equal to 1 at x square plus y square is equal to r square. And, that makes the situation far more complicated. That is the reason why we go to a curvilinear coordinate system – a coordinate system which has the same symmetry as the configuration being analysed. In this particular case, we are analysing a pipe. So, I could choose a coordinate system in which one of the coordinates is the distance from this axis. So, that was the basis for choosing curvilinear coordinate systems.

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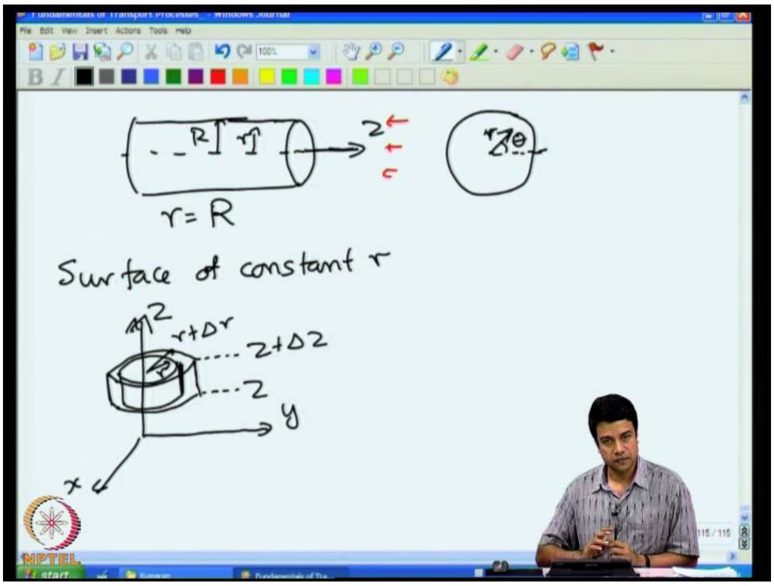

For example, if I were to look at this particular pipe, I could choose... This will be the z coordinate and one coordinate is r – the distance from the centre from the axis of the pipe – the perpendicular distance. Therefore, on the surface of this pipe, the r coordinate is just equal to capital R. Let me just make that little more clear. This is the radius r and the distance from the centre is R. So, R is the distance from the centre; z is the distance along the axis from some origin. In this case, the boundary condition gets simplified in the r quadrant.

In three-dimensional space, I need one more coordinate. In the cylindrical coordinate system, that is, the coordinate that goes around the axis. It is an angle for the coordinate that goes around the axis. So, if I look at this from the (Refer Slide Time: 06:28) side, if I look at this pipe, take a side view of this pipe, what I will see is a circle like this; the z-coordinate is coming out of the board towards you; r is the distance from the centre; and, theta is the angle that distance makes with respect to some axis. So, by fixing r – the distance from the centre, theta – the angle that this distance makes with respect to some axis, and z – the distance along the pipe have uniquely fixed the position. And then, you can express the temperature concentration, velocity fields in terms of this position.

I went to a curvilinear coordinate system, because the boundaries are more easily described in this curvilinear coordinate system. However, there is a price to pay; and, that is, that the equations get more complicated. In this particular case, since the coordinate system is a curvilinear coordinate system, the coordinates, the surfaces of constant coordinate are no longer planes. In this particular case, the surface of constant r is a cylindrical coordinate; it is no longer a planar coordinate, it is a cylindrical surface. So, the surface of constant r is a surface of (Refer Slide Time: 08:16) constant distance from the axis; that is the surface of a cylinder.

When I write the balance equation, I write shell balance for some differential volume; that differential volume in the case of a cartesian coordinate system was bounded. In this cartesian coordinate system, the differential volume that I write the differential equation for was bounded by surfaces of constant coordinate. The top and bottom surfaces are constant z; the left and right surfaces are constant y; and, the front and back surfaces are at constant values of x. So, when we write shell balances in a particular coordinate system, we choose a differential volume in which the bounding surfaces have constant values of one coordinate. In this particular case (Refer Slide Time: 09:18), there are six surfaces, because we have a three-dimensional space; the front and back has surfaces of constant x; the right and left constant y; and, the bottom and top constant z.

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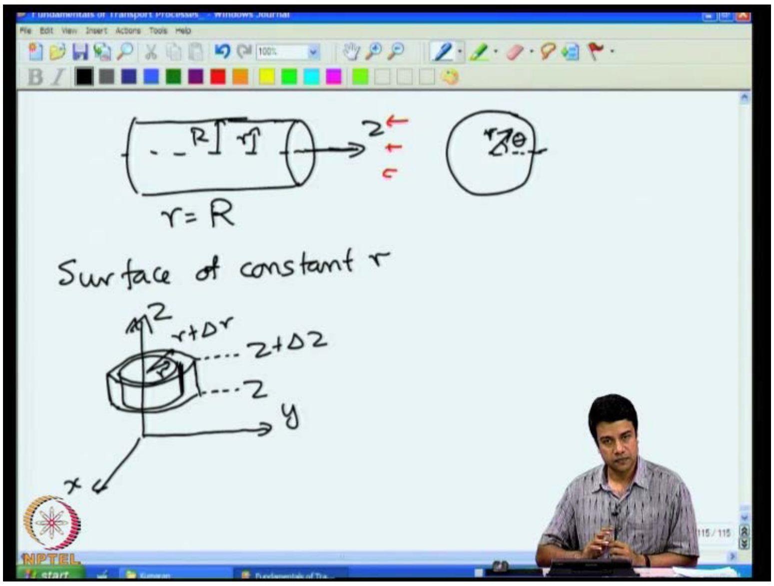

In a similar manner, when we choose a surface in a curvilinear coordinate system, when we choose a differential volume in a curvilinear coordinate system, that differential volume also has to have surfaces of constant coordinate. We saw how to do that in the last lecture. Let us say we have a cylindrical coordinate system. Now, I am aligning the axis along the vertical z axis (Refer Slide Time: 10:06). The cartesian coordinate system was this. So, I am aligning the surface at constant axis. Along the xy-plane, the distance of a point from the z-coordinate is given by r. So, if I take the projection of this on to the xy-plane, this distance from the z-coordinate is r. And, conventionally, you define theta as the angle that the r vector makes with respect to the x-axis. So, that is how it is conventionally defined.

Now, if the problem is axis symmetric – what axis symmetric means that as you go around the axis, there is no change in the temperature concentration or momentum fields. So, if it is axis symmetric, there is no dependence on theta; in which case, the fields will depend only upon r and z. The flow in a pipe for example, is axis symmetric at a given distance from the axis. If I go around the axis, there is no change in the velocity. So, it is an axis symmetric field. So, in that case, I choose the coordinates to be at constant values of r and constant values of z. The differential volume is bounded by surfaces at constant values of r and constant values of z. So, how do we write that down?

Surface at constant value of r is a cylinder around (Refer Slide Time: 12:19). So, that is one surface at constant value of r; a cylinder around the axis. And, the volume has to be bound by two surfaces in each coordinate. So, I need to define a second surface. This first surface is at radius r and the second surface is at radius r plus delta r. Therefore, the bounding surfaces are two cylindrical surfaces of constant r at r and r plus delta r. And, the region in between these two is the volume that we will write the balance equation for.

Similarly, you will have a surface in the z-coordinate at z and z plus delta z. Planes perpendicular to z are flat. So, in this case, since the z coordinate is aligned, planes perpendicular to z are flat. So, we will choose the differential volume to be bound by surfaces of constant coordinate. In this case, two surfaces of constant r at r and r plus delta r; two surfaces of constant z at z plus delta z and z. If there were variation in the theta coordinate as well we would have two surfaces at theta and theta plus delta theta. We will see that a little later when we derive differential equations in all three dimensions. For now, we will consider only axis symmetric flows. So, this is going to be the differential volume that we consider.

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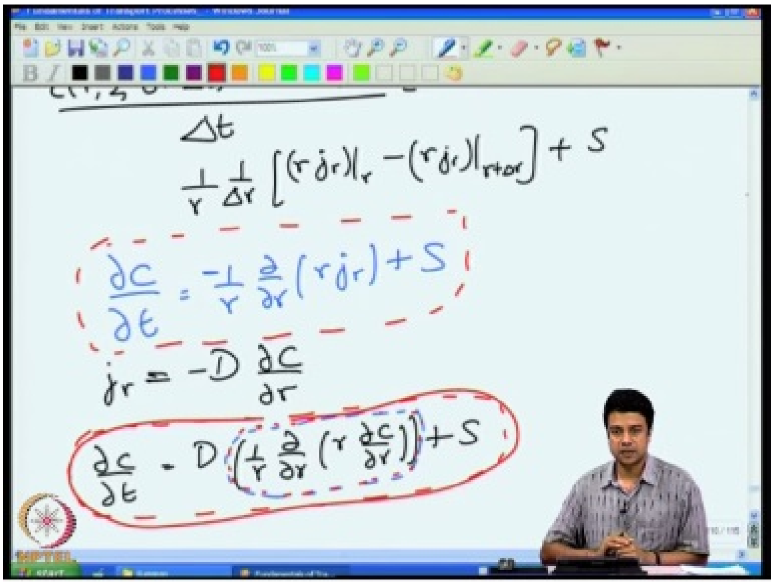

And, you write a balance equation for this differential volume. And, what you get is an equation of the type d c by d T is equal to minus 1 over r d by d r r times the flux j r plus a source term. And, if you write the flux as minus D times minus d c by d r, then you get this d c by d T is equal to d times 1 over r d by d r of r d c d r. And, as you can see, this term is more complicated than the simple second derivative that we had in the case of a cartesian coordinate system; just the price that you pay for going from a cartesian to a cylindrical coordinate system. This more complicated form of this term is because the surface area is changing as the r-coordinate changes. The cylindrical surface area is equal to 2 pi r times delta z; that surface area is changing as r changes. So, it is going to be a contribution to accumulation within that volume or transport across a surface due to the change in the flux as well as the change in the surface area. And, this more complicated form – 1 over r d by d r of r d c by d r takes into account the fact that in a cylindrical coordinate system, the surface area is changing as the position changes. So, that is the reason for a more complicated form in a cylindrical coordinate system.

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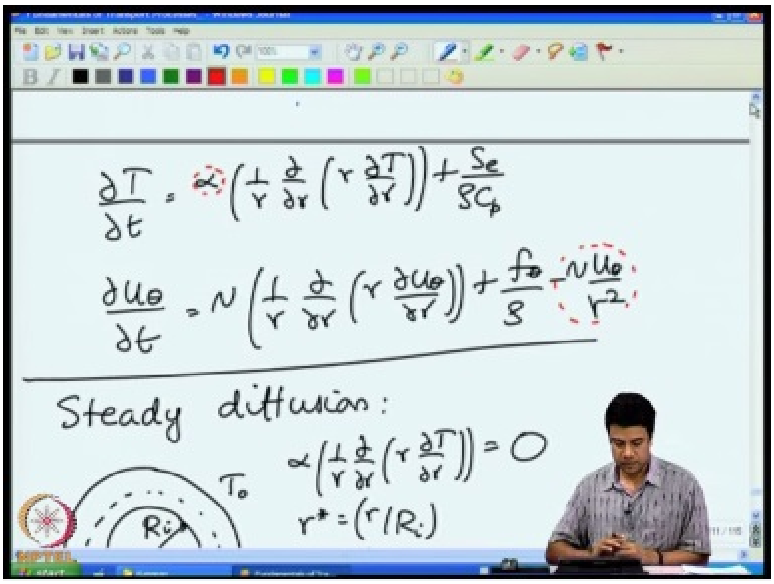

I had also told you that you can also write a similar equation for the temperature field and the velocity field. The velocity field is little more complicated than this. I would not go into the details right now, but you should also have another term that goes as nu u theta by r square. The reason for this additional term we will see a little later; we would not go into the details right now. It is a little more complicated to derive. And, in order to derive it, you need to do a little more complicated vector analysis. So, the differential form for the equation is slightly more complicated in the case of a cylindrical coordinate system. A consequence of this – the heat conduction across the wall of a cylinder is not the same as the heat conduction at a flat surface. We saw that by looking at steady diffusion in the last lecture.

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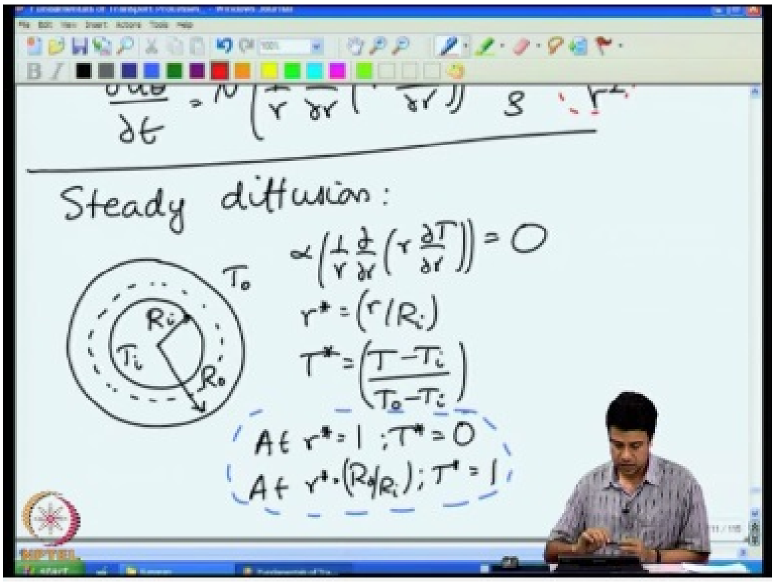

In the case of steady diffusion, the equation of motion just reduces to alpha times 1 over r d by d r of r d T by d r is equal to 0. We had looked at a problem, where the temperature was T i on the inner surface and T o on the outer surface; the radii over R i and R o on the inner and the outer surfaces. And, we wanted to find out what is the temperature profile throughout the annular region between the inner and the outer surfaces. And also, to calculate what is the heat flux due to this temperature difference. As I reminded you in the last lecture, this is of consequence especially in heat transfer problems.

Most heat transfer problems involve some kind of transport across the tube of a heat exchanger. And, in that case, it is essential to know given a temperature drop across the wall, what is going to be the heat flux across. So, we have done the scaling. The scale trade is r by R i and T star is equal to T minus T i by T o minus T i. I cannot just scale r by the difference R o minus R i, because r star going to 0 gives me a surface of 0 area. So, one has to be careful there. Necessarily, the two axis have to be at r is equal to 1 and r is equal to something else. In this (Refer Slide Time: 18:47) case, I have scaled it by the inner radius, and therefore, the other surface is at r star is equal to R o by R i.

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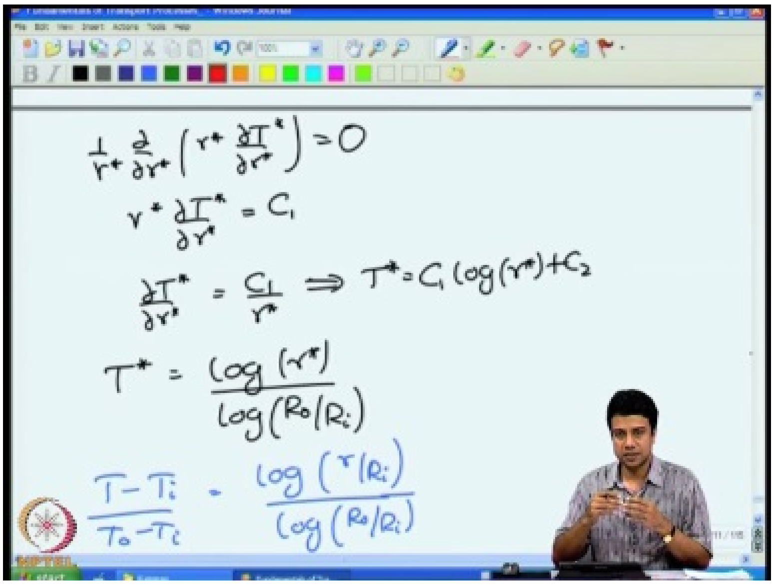

We solved this equation and we got a logarithmic temperature profile. It is not a linear temperature profile like in the case of flow past of transfer in cartesian coordinate systems. Because the surface area is changing with position, the heat that is being transported... In this particular case, there is no sources, sink of heat; the system is at steady state. So, the heat that is being transported has to be a constant; that means that the heat flux has to go as 1 over the area. The area is changing. So, the flux will change. And therefore, the temperature will change.

In a cartesian coordinate system, at every z, the cross-sectional area was the same. And therefore, if the heat transported is unchanged, the heat flux is also unchanged. Here even when the heat transported is unchanged because the area is changing, the flux will change. So, I got the temperature field quite simply as (Refer Slide Time: 20:09) log r by R i by log of R o minus R i. And, from that, we got the expression for the heat flux.

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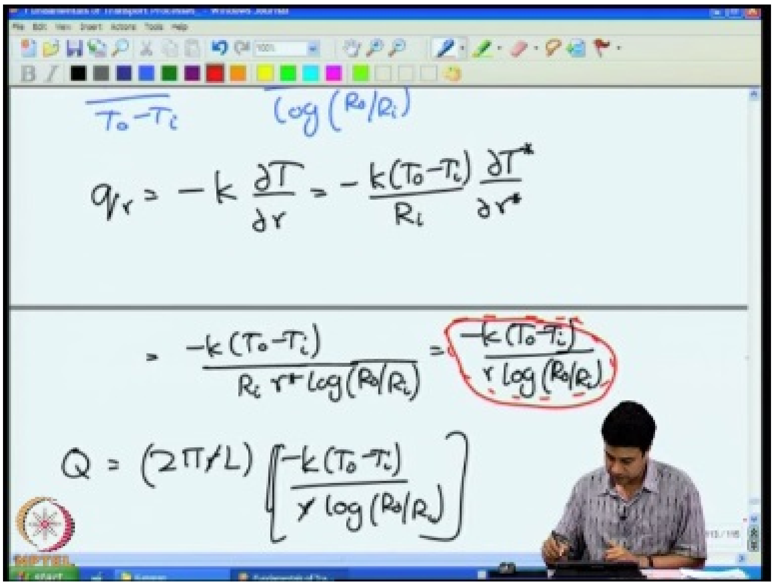

And, the heat flux had an expression of the form k into T o minus T i by R i r star times log of R o by R i.

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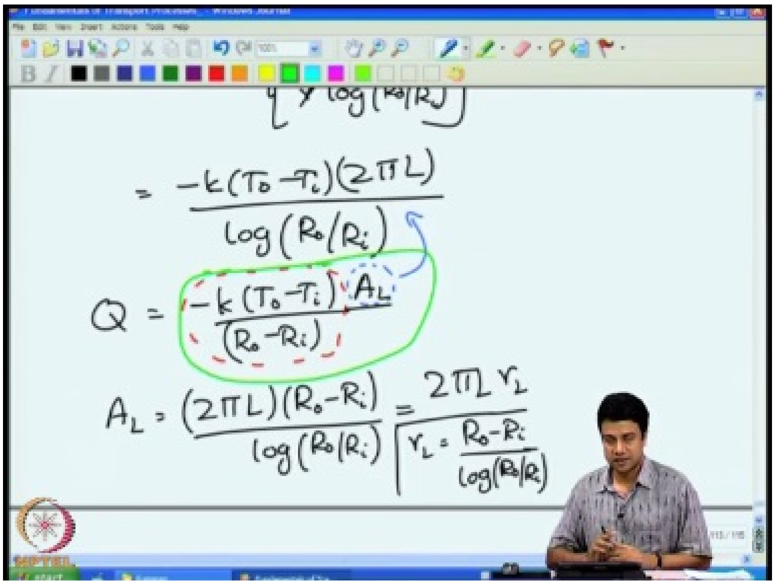

Now, conventionally, what is done in unit operations is to express the total heat transported as Q is equal to minus k into T naught minus T i by R o minus R i into an area A L. In a cartesian coordinate system, this would be exactly the same area everywhere, because the planes have constant area. In a cylindrical coordinate system, the area changes. This is obtained just by analogy with the heat transfer in one dimension. You would expect that the heat flux is going to be equal to k into the temperature difference by the distance. In this particular case, the distance is the distance between the inner and the outer surfaces. So, the heat flux that I get will be equal to minus k into a temperature difference divided by a distance.

The total heat is going to be equal to that heat flux times an area. This area (Refer Slide Time: 21:51) A L can be determined by comparing it with this equation. Just by comparing the two, you can see that A L has to be equal to 2 pi L into R o minus R i by log of R o by R i. That is the area. That is the effective area that is required to get the correct heat flux in this expression, where I have written it as the temperature difference times a conductivity divided by the distance between the inner and the outer walls. So, this is equal to 2 pi L times r L; where, r L is the logarithmic mean temperature; where, r L is defined as R o minus R i by log of R o by R i, the logarithmic mean radius. So, for heat conduction through a cylindrical annulus, we have to use the logarithmic mean radius to calculate the effective area if you use an equation of this (Refer Slide Time: 23:33) kind in order to calculate the heat flux. And, that is because the surface area is changing as the radius changes. And, one has to keep that in mind whenever 1 works with curvilinear coordinate system.

Now, this exact same expression between the heat flux and the temperature difference will also work for the relationship between the mass flux and the concentration difference. The same expression for the concentration profile here, the scaled concentration (Refer Slide Time: 24:16) field will also work when we write a relation between the concentration and radius for a mass diffusion problem. And, instead of the heat flux, we will just get an expression for the mass flux (Refer Slide Time: 24:32). All we need to do is substitute. Instead of the thermal conductivity, we substitute the mass diffusivity; instead of temperatures, we substitute concentrations. That is the only difference. A similar expression will not work for the velocity fields.

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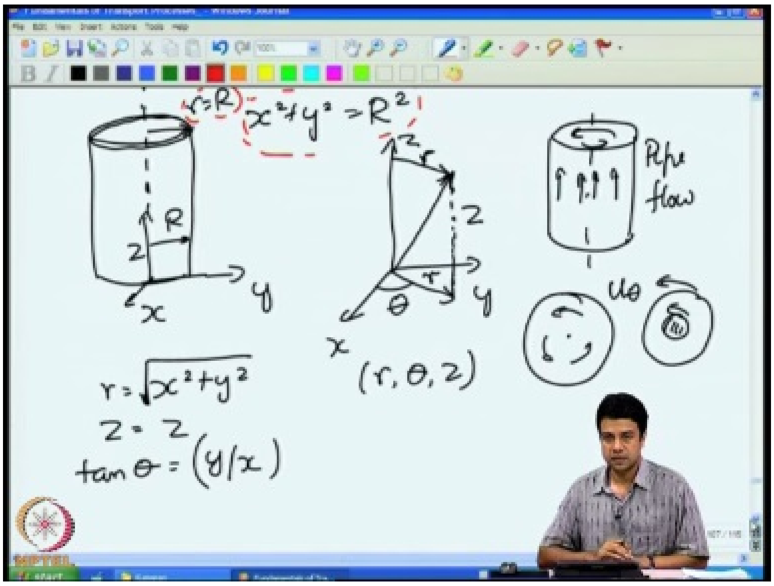

As I said in the last lecture, one can consider the velocity perpendicular to the surface to be in two directions. As I said, we can consider the velocity perpendicular to the surface to be in two directions. One is along the axis for a pipe flow. That is, in this particular case, the velocity is along the z direction for a pipe flow. In the case of rotating cylinders, the velocity is along the theta direction. So, one can write balance equations for both of these. We will come back and see why the equation for the pipe flow is actually identical to the equations that we get for the mass and heat diffusion problems. The equation for the pipe flow is exactly the same. Equation for u theta is a little bit different. And, we will come back later and see why when we deal with velocity (( )).

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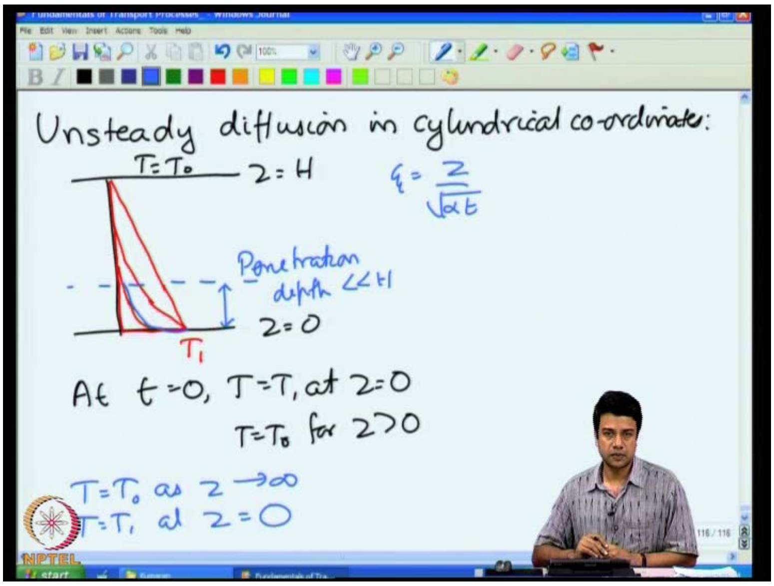

In the last lecture, we solved the simplest problem and that is for the steady diffusion in cylindrical coordinates. Now, we will go a step further and look at unsteady diffusion. When we looked at the unsteady problem in cartesian coordinates with the flat surfaces, the first thing we looked at was unsteady diffusion in one dimension using a similarity solution. In that particular case, we had a surface at z is equal to 0 bounding effectively an infinite fluid. We considered a situation where the penetration depth of the concentration or temperature disturbance at the bottom surface is small compared to the thickness of the channel. In that case, the condition at the other surface was not really affecting the temperature field near this bottom heated surface. So, we had a situation like this z is equal to 0, z is equal to H.

And, in the very initial stages, we considered our... Initially, at time t is equal to 0, here T was equal to T naught; and, for at t is equal to 0, we consider that T is equal to T 1 at z is equal to 0; and, T is equal to T naught everywhere else. So, initially, the temperature everywhere was just 0. At time t is equal to 0, a temperature disturbance T 1 was imposed at the bottom surface. And, this propagated through the fluid due to diffusion. So, in the very initial stages, the temperature profile looks something like this (Refer Slide Time: 28:17). And, as time went on, it looks something like this. When the final time, we get a linear profile. In the final steady state, the profile turns out to be linear.

For the similarity solution, we concentrated on the very early stages, where the temperature disturbance had not propagated very far into the channel. So, if this penetration (Refer Slide Time: 28:43) depth, a small compared to the height of the channel, I can effectively impose a boundary condition of the kind T is equal to T naught as z goes to infinity and T is equal to T 1 at z is equal to 0. So, effectively, we assumed that the boundary condition at z is equal to H was equivalent to putting a boundary condition at z is equal to infinity, because the penetration depth is much smaller than the height of the channel. And, in that case, we had no length time scales in the problem. And therefore, we were able to define a similarity variable psi is equal to z by square root of alpha t. And, we wrote down the conduction equation in terms of this parameter psi.

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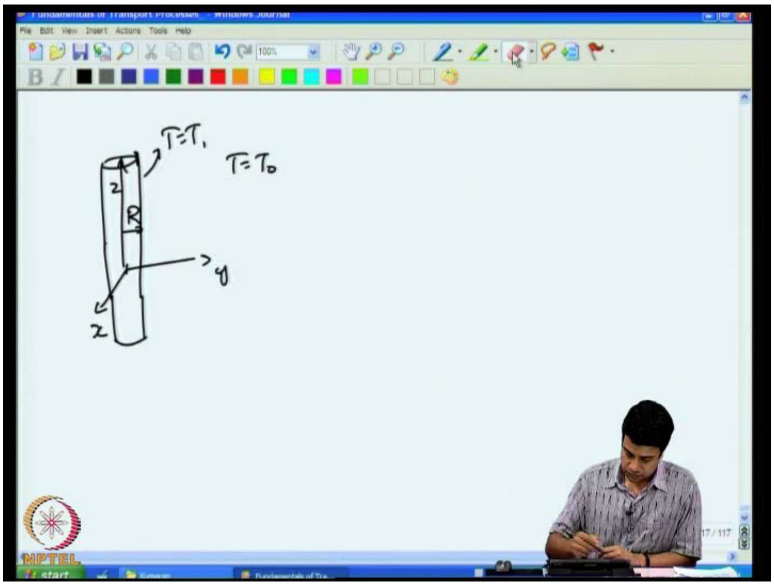

Now, it is difficult to have a cylindrical analog for this particular problem. The reason is as follows. Let us say I had a cylinder in an infinite fluid. I mean the immediate analog that one would think of is to look at a cylinder in an infinite fluid. So, if I had a cylinder in an infinite fluid and the fluid is at T is equal to T naught far away; and, it is equal to T is equal to T 1 at the surface. Just because the fluid is infinite, I cannot just go and apply similarity solution. The reason is because in this problem, there is still a length scale present and that is the radius of the cylinder. Because there is a length scale present, which is the radius of the cylinder, this radius r also enters into the problem. And therefore, you cannot just use a similarity variable, which assumes that there are no length scales in the problem.

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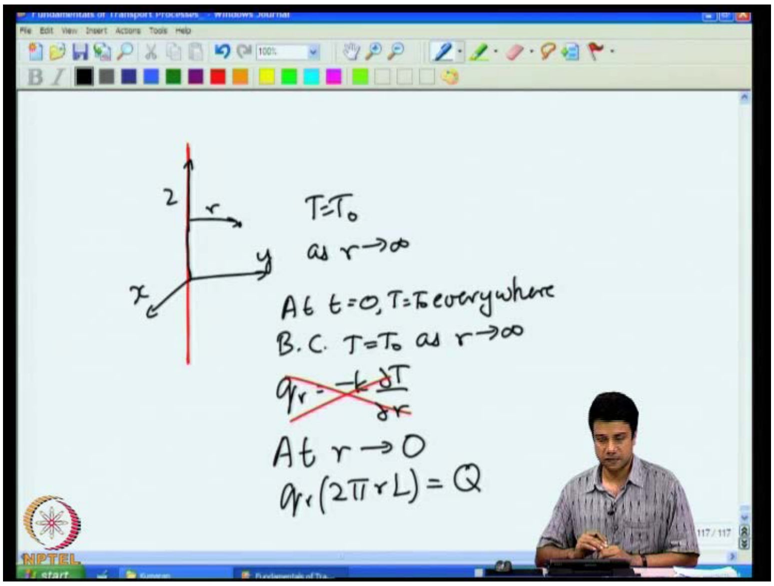

Finite cylinder in an infinite fluid cannot be analysed using a similarity solution. The analog of diffusion into an infinite fluid in cylindrical coordinates is actually diffusion from a line source – a wire of infinitesimal thickness, which is heated in an infinite fluid. I have temperature T is equal to T naught. As r goes to infinity far away from the wire T is equal to T naught, at the surface of the wire, there is some heating going on. And then, we can find out what is the temperature profile due to this. So, I had used the cylindrical coordinate system, where this was the z-axis and the x and y are in the plane; and, the distance r is the distance from the z-axis to some position on the surface.

The analog of this problem is to save that initially at time t is equal to 0 (Refer Slide Time: 32:37), T is equal T naught everywhere. So, this is the initial condition for the unsteady problem. The boundary condition T is equal to T naught as r goes to infinity, the cylinder, the wire in this case is located at r is equal to 0. I said it is an infinitesimally a thin wire. And, I required the wire to be infinitesimally thin because if it is a finite wire, I will have a length scale in the problem. And then, a similarity solution cannot be used. So, I have to consider a wire that is of infinitesimal thickness.

But, how about the temperature at the surface of the wire? Turns out I cannot prescribe a temperature at the surface of the wire. The reason is as follows. At the surface of the wire itself, the heat flux coming out is the thermal conductivity times the derivative of the temperature. q r (Refer Slide Time: 33:50) is equal to minus k d T by d r. So, that is the heat flux that is coming out. So, if I had some temperature profile with the gradient at the wire itself, this would give me the heat flux that is coming out. However, the total heat coming out is the heat flux times the surface area. For the cylinder, the wire – since it is of infinitesimal thickness, the radius is 0 at the surface of the wire. And therefore, the surface area is also 0.

Having a finite temperature would basically tell me that there is no net heat coming out of this surface. So, this Q (Refer Slide Time: 34:40) will be equal to 2 pi r L into q r, which should go to 0 if the temperature were finite, because the radius r is going to 0 in this cylindrical coordinates system. So, basically, I cannot prescribe a temperature condition. What I have to say is that what is the total heat coming out of the surface. The total heat coming out of the surface... At r going to (Refer Slide Time: 35:35) 0, q r into 2 pi r L is equal to the total heat coming out per unit time from the surface. That is because the wire is of infinitesimal thickness. This problem actually has a lot of physical applications especially in heating applications. Very often, in order to convert electrical energy into heat, electrical resistance wires are used. And, these wires are very thin. I said that the wire has to be of infinitesimal thickness, because only then, can I apply a similarity solution, only if the thickness of the wire is small compared to the size of the system. So long as the thickness of the wire is small compared to the size of the system, the macro scale, I can apply this analysis.

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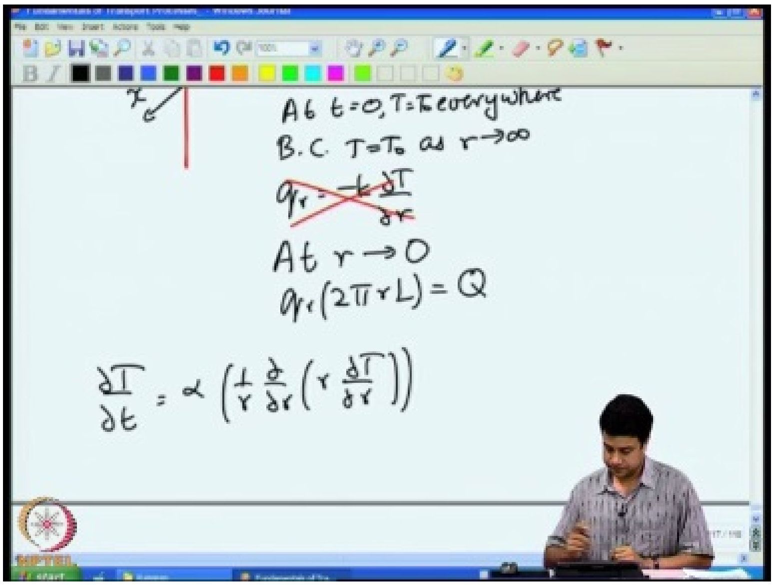

Consider the wire to be of infinitesimally thin wire at r is equal to 0. In that case, I cannot specify the heat flux itself or the temperature itself at the surface; I had to specify the total heat coming out of the surface. We will see that when we solve this problem. So, for this particular problem, the energy conservation equation is given by d T by d T is equal to alpha times 1 by r d by d r of r d T by d r. We just derived the conservation equation. With boundary conditions... T is equal to T naught everywhere at time T is equal to 0 – that is the initial condition. Boundary condition is T is equal to T naught as r goes to infinity. And, from the wire itself, I know what is the heat flux; that is, the heat energy coming out per unit time.

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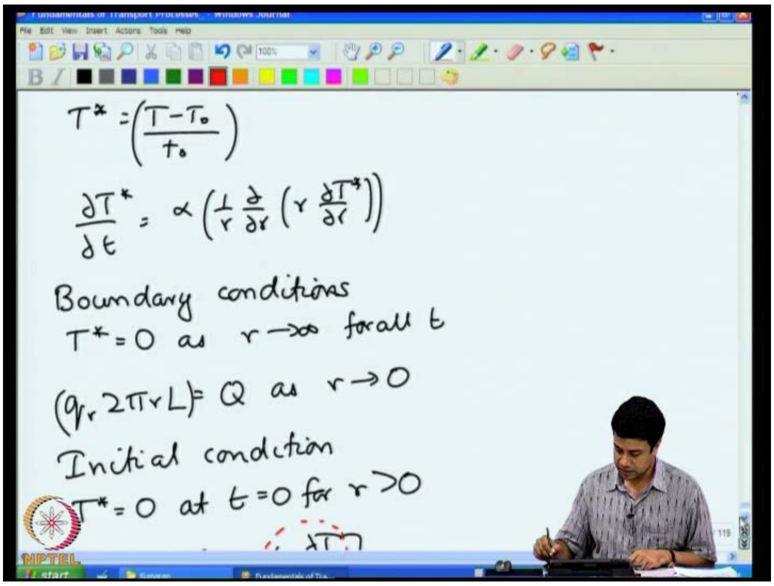

Scaling – I can define T star is equal to T minus T naught by t naught, which means that T star is equal to 0 as you go far away. So, the differential equation in terms of this scale coordinate becomes d T star by d t is equal to alpha 1 by r d by d r of r d T star by d r with boundary conditions T star is equal to 0 as r goes to infinity for all time. So, that is one boundary condition. The other boundary condition was that q r times 2 pi r L – the heat flux times the area has to be equal to Q as r goes to 0. And, the initial condition T star is equal to 0 at t is equal to 0 for r greater than 0. That is, everywhere in the fluid, except at the wire itself, the temperature is 0, because we switched on the heating at time t is equal to 0; that means, the temperature is equal to 0 everywhere, except at the surface of the wire. So, these are the initial and boundary conditions.

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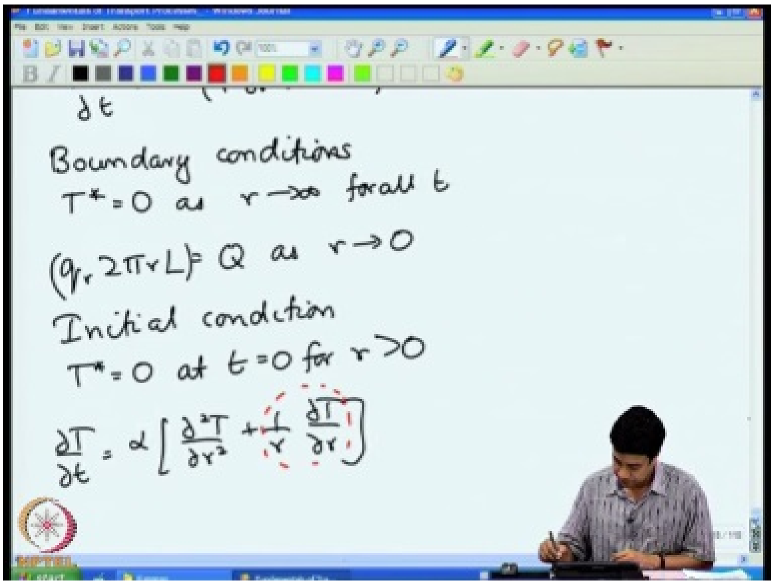

I can expand out this equation – the energy conservation equation as d T by d t is equal to alpha into d square T by d r square plus 1 by r into d T by d r. Note that when we did the solution in a cartesian coordinate system, this term was not present. This term is present primarily, because we are dealing with a curvilinear coordinate system; surface area is changing as r changes. So, the argument for similarity transform – there are no length or time scales in the problem. Therefore, I can get only one dimensionless group.

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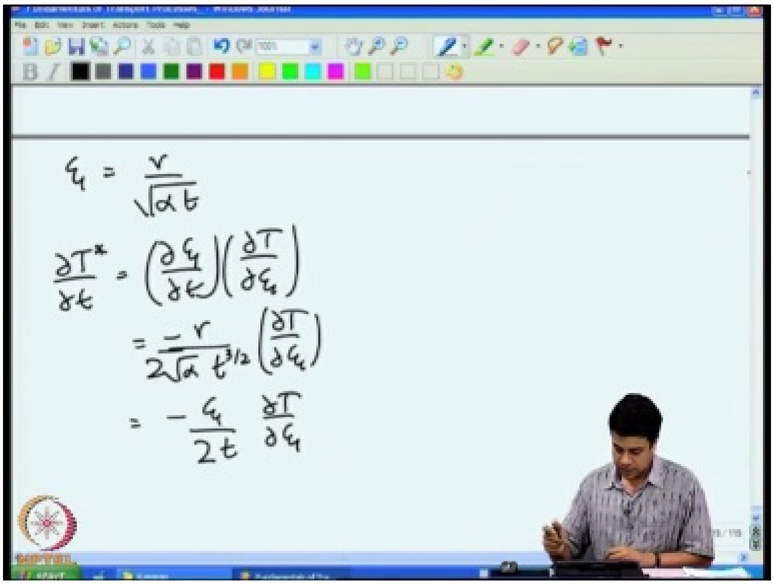

There are only three dimensional variables left: time, r and alpha. Out of these, I can get only one dimensionless group. And, that is the similarity variable – psi is equal to r by square root of alpha times t. That is the dimensionless group. I need to express this equation in terms of that dimensionless group. Let me just put the non-dimensionalization here. Therefore, partial T by partial t is equal to d psi by d t into partial T by partial psi, which is minus r by square root of alpha t power 3 by 2 into d T by d psi. Taking derivatives; and, that should be the factor of 2 here; is equal to minus psi by 2 t. So, this is the time derivative.

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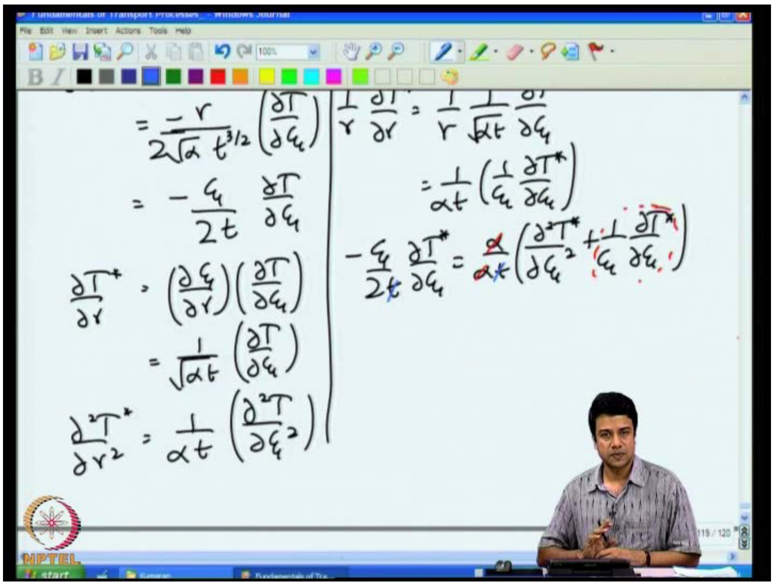

Then, d T by d r will be equal to d psi by d r times d T by d psi. It will be equal to 1 over square root of alpha t d T by d psi. And, I can take the second derivative d square T by d r square is equal to 1 by alpha t d square T by d psi square. In addition, I have also a term that is proportional to 1 over r d T by d r, which will be equal to 1 over r into 1 by root of alpha t d T by d psi. Since r is equal to square root of alpha t times psi, I can write this as 1 by alpha t times 1 by psi d T by d psi. Put all of these together into the differential equation. So, I will have minus psi by 2 T into d T by d psi; this is the equivalent of the time derivative here – this is (Refer Slide Time: 43L51) the time derivative on the left-hand side. This thing is equal to (Refer Slide Time: 44:06) alpha by alpha t times d square T by T psi square plus 1 over psi into d T by d psi.

And, note this (Refer Slide Time: 44:22) additional term, which was not present when we did the equation in the cartesian coordinate system. And, as you can see, this alpha cancels of and T cancels of on both sides. So, I am left with an equation, which is only in terms of psi, which is the requirement if the similarity procedure has to work. If the similarity solution has to work, once I have changed the equation from r and T to the psi coordinate, I should end up with an equation, which does not depend upon individually on r, T and alpha, but depends only upon psi.

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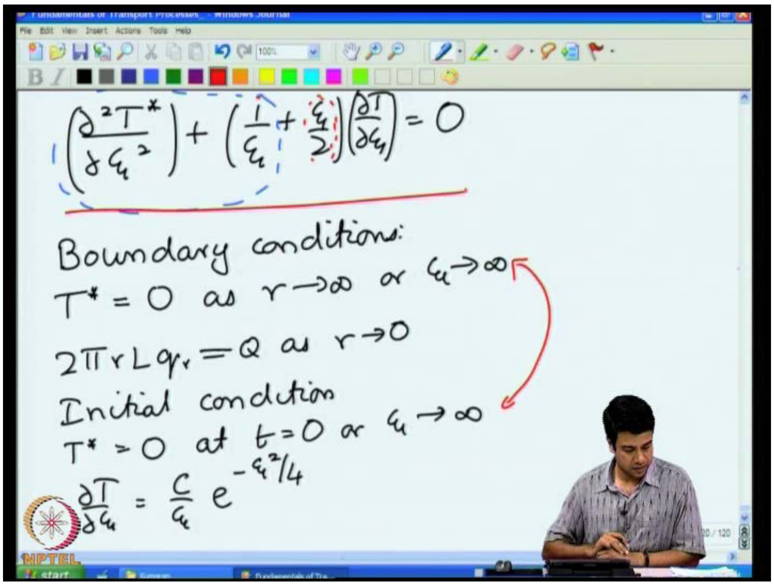

This equation – I can rewrite it as d square T by d psi square plus 1 over psi plus psi by 2 d T by d psi is equal to 0. This psi by 2 – this term here came from the time derivative; whereas, these other two terms – coming from the diffusion terms. These other two terms basically come out of the diffusion term. Boundary conditions – T star goes to 0 as r goes to infinity or psi goes to infinity, because psi is equal to r by square root of alpha t. So, T has to go to 0 as psi goes to infinity. The other boundary condition is that 2 pi r L q r is equal to Q as r goes to 0. We will see how to enforce this a little later. And, the initial condition T star is equal to 0 at t is equal to 0 at finite r. Note that psi was equal to r by square root of alpha t. So, T is equal to 0 is equivalent to psi going to infinity.

These (Refer Slide Time: 47:10) two: one boundary condition and one initial condition turn out to be the same as in the case of cartesian coordinates. So, in common with cartesian coordinates, in this case as well, one initial and one boundary condition turnout to be the same. So, we can solve this subject to these two constraints. Integrating this equation (Refer Slide Time: 47:37) once, I will get d T by d psi is equal to some constant by psi e power minus psi square by 4; that is, d T by d psi. Integrating this is equation one time.

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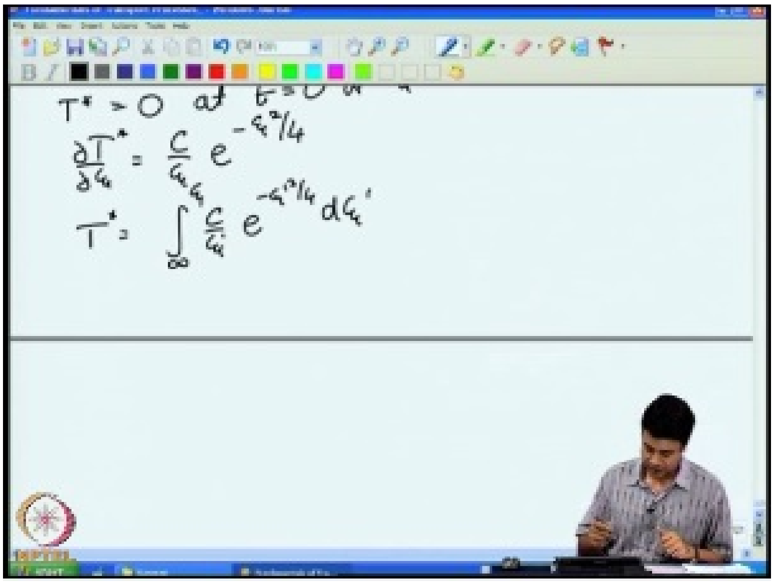

I integrate once more to get T is equal to integral of c by psi prime e power minus psi prime square by 4 d psi prime. This integral, we know that the temperature has to go to 0 as psi goes to infinity. Therefore, I can integrate from infinity to r by root alpha t. The upper limit of integration has to be the coordinate; in this case, psi. The lower limit of integration can be any value. If I had taken some value as the temperature is not 0; that lower limit of integration, I would have to add another constant here. But, since I have taken the lower limit of integration to be the value at which psi goes to 0, you can see that this integral, if psi is equal to infinity, temperature is equal to 0. So, this satisfies that condition. There is only one constant left, because I have adjusted one constant using the boundary condition. So, there is only one constant left. That constant has to be determined from the total heat output condition. So, let us look at that.

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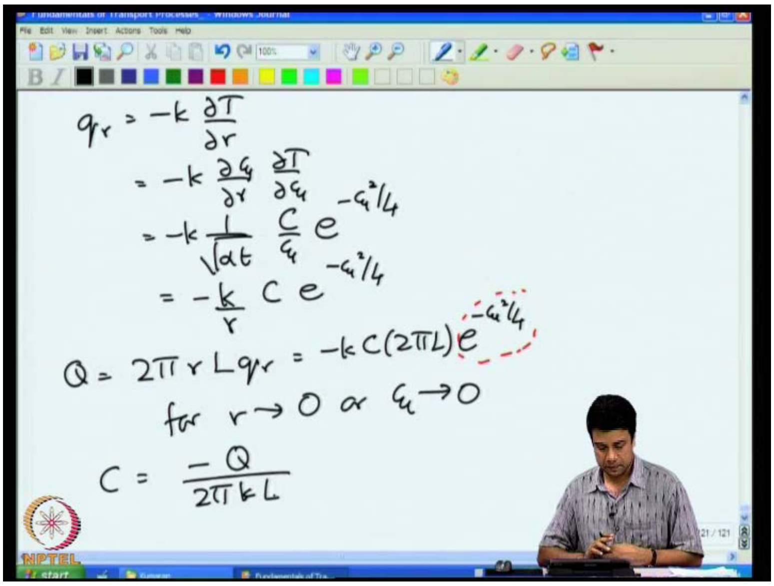

The heat flux q r is equal to minus k d T by d r; q r is equal to the heat flux at the surface. So, this is equal to minus k times d psi by d r into d T by d psi. And, I have an expression for d T by d psi over here; d T by d psi is given by (Refer Slide Time: 50:16) c by psi e power minus psi square by 4. So, this is equal to d psi by d r into... d psi by d r is just (Refer Slide Time: 50:30) 1 over square root of alpha t into d T by d psi, which is c by psi e power minus psi square by 4. Note that psi times square root of alpha t is just the radius r, because psi is equal to r by square root of alpha t. So, this becomes minus k by r into c e power minus psi square by 4.

Now, the total heat coming out (Refer Slide Time: 51:07), Q is equal to 2 pi r L q r, which is equal to minus k c into 2 pi L into e power minus psi square by 4. Now, the boundary conditions set that this Q has going to be equal to 2 pi r L q r for r going to 0 or psi going to 0. So, when psi goes to 0, this factor is (Refer Slide Time: 51:45) just equal to 1; when r goes to 0 or psi is equal to 0, this factor is just 1. So, this basically enables me to find out what the constant is. Therefore, from this, this constant of integration has got to be equal to minus Q by 2 pi k L. And therefore, the temperature field (Refer Slide Time: 52:10) has got to be equal to minus Q by 2 pi k L integral from infinity to r by root alpha t d psi prime 1 over psi prime e power minus psi prime square by 4.

Note that the heat flux actually diverges. It goes as 1 over r as r goes to 0 as we had expected. Because the surface area is going to 0, the heat flux actually goes as 1 over r if the heat coming out of this surface has to be finite (Refer Slide Time: 52:45). Even though the heat flux is diverging, the total heat coming out per unit time from this wire is a constant. And, that enabled us to find out the boundary condition.

And, you can see here (Refer Slide Time: 53:03) that in this expression for the temperature as well, d T by d psi is diverging; the temperature also diverges. Temperature has integral from infinity to psi of c by psi prime e power minus psi prime square by 4. So, in the limit as psi goes to 0, e power minus psi prime square by 4 goes to 1, but I have an integral of 1 over psi prime d psi prime. That is logarithmic. The integral of 1 over psi d psi is log of psi. So, the temperature is going to infinity logarithmically as the radius goes to 0. The heat flux goes as 1 over r. But, when I multiply the heat flux by the area in the limit of as r goes to 0, that has a finite value. And, that value essentially enables us to find out what is that temperature in this problem.

This is basically (Refer Slide Time: 54:00) the temperature field around a line source. The wire can be considered as a line source. It is of infinitesimal thickness; radius of the wire is going to 0. Even though the radius is going to 0, the heat coming out per unit time is fixed. If the heat coming out is fixed, the radius goes to 0; that means that if the radius goes to 0, the surface area for heat transfer is going to 0. Therefore, the heat flux has to go to infinity at the line source. The temperature also goes to infinity at the line source. But, we can still get the solution for the temperature field everywhere else using this similarity solution. This of course, is an approximation.

In a real system, the wire will have some thickness; it will not be of 0 thickness. Even though it has some thickness, so long as the thickness of the wire is small compared to the other length scales in the problem... so long as I can consider for example, if you had a wire in a large tank, the wire does has some thickness, but the tank size is much larger still. So, I can consider the wire to be of point source. And, we will see that this concept of a line source is a line source in three dimensions. If I just looked at it in the plane perpendicular to the wire in two dimensions in the plane perpendicular to the wire, this would be like a point source in two dimensions. And, these point sources in two dimensions as well as a point source in three dimensions – we will see that at a little later in the context of spherical coordinate system. These point sources are important later on when we look at solutions of the Laplace equation for the transport problem. So, we will encounter this once again.

So far, we have looked at similarity solution for the flow from a wire of an infinitesimal thickness. Next lecture, we will look at transport within a domain of finite thickness – heat conduction in a cylinder. We will use these methods of separation of variables to analyse that problem. So, just as we did in the case of cartesian coordinate systems, we started off with similarity solution, then separation of variables, follow the same thing here. Next class, we go to separation of variables. So, we will see you in the next lecture.

Thanks.