Fundamentals of Transport Processes

Prof. Kumaran
Department of Chemical Engineering
Indian Institute of Science, Bangalore

Module No. # 04
Lecture No. # 16
Unidirectional Transport Cylindrical Co-ordinates-I Conservation Equations

Welcome to lecture number sixteen – the fundamentals of transport processes. So far, we have been discussing transport in one dimension in Cartesian coordinates, where we solved problems using various methods. We started off with just steady diffusion in one dimension; in which case the velocity profile or the temperature profile or the concentration profile is just a linear function of position. That is because you have transport only in one dimension with no generation or consumption of heat, mass of momentum within the flow. That was at steady state, where there is no dependency on time.

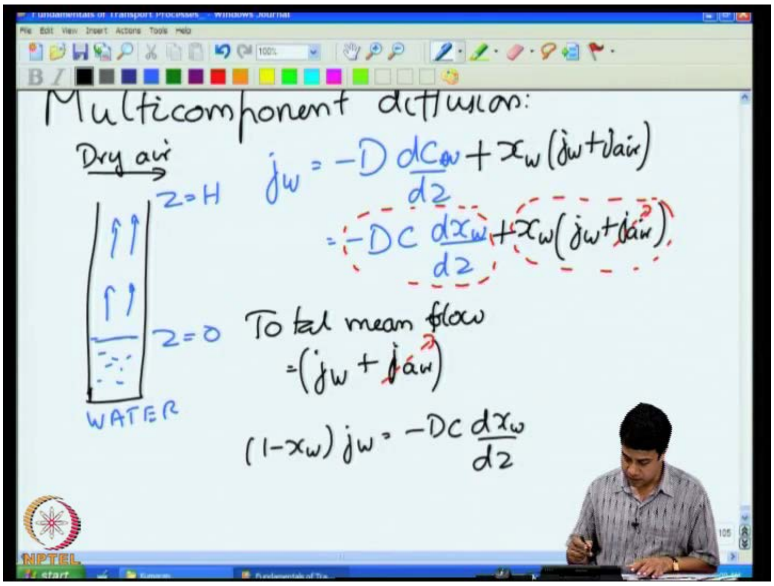
One could also consider unsteady situations. In this case, we encountered partial differential equations – equations which depend both upon position as well as time. And, we looked at a couple of ways of solving those equations. The first one was by similarity transform. Similarity transform is particularly appropriate for transport it to infinite media, because in that case, there are no length or time scales in the problem. And therefore, one can use a similarity transform in order to reduce the equation from a partial differential equation in position and time to an ordinary differential equation.

The second method that we looked at was separation of variables. And, I showed you how one can write a function of position in time into the product of two functions: one of which is only dependent on position; the other is only dependent on time. The variables are separated and you get both of two ordinary differential equations: one for the function of position; the other for the function of time. These have multiple solutions, discrete solutions with discrete eigenvalues and bases functions. And, I showed you how to put them all together and use orthogonality relations to find out the solutions. We looked at oscillatory flows, where we can use complex variables to simplify the equations and then solve them. And finally, we were looking at situations, where there are sources or sinks within the floor. And, in that case, one has size of body forces, a source of heat as in the case of viscous heating or a source of momentum due to body forces or a source of mass due to reactions.

Now, the next step is to go to balances and cylindrical coordinates. Very often one encounters geometries, which are cylindrical, for example, a pipe, a tubular reactor, a stirred-tank vessel. These are all cylindrical geometries. One could analyze them in a cartesian coordinate system. However, in a cartesian coordinate system, it is very difficult to right down the equation for the surface itself. So, for example, a circle in 2 dimensions will have an equation of the form x minus x c whole square plus y minus y c the whole square is equal to r square; where, x c and y c are the centers of the circle; and, r is the radius. So, the surface has a complicated definition – the surface of a cylinder in cartesian coordinate has a complicated definition; very different from the definition that we had for plane surfaces.

In plane surfaces in all cases, our definition was just at z is equal to 0 and z is equal to h. So, if we use a cylindrical coordinate system, the definition of surfaces gets considerably simplified in a cylindrical coordinate system. However, the equations are little more complicated. They have to take into account the fact that we are actually working with cylindrical surfaces and not flat surfaces. So, the next step is to look at how the equations are modified when we have cylindrical surfaces. But, before that, I would just like to briefly give a warning that much of what we did has to be modified in the case of mass diffusion due to the center of mass velocity of the entire system.

(Refer Slide Time: 04:33)



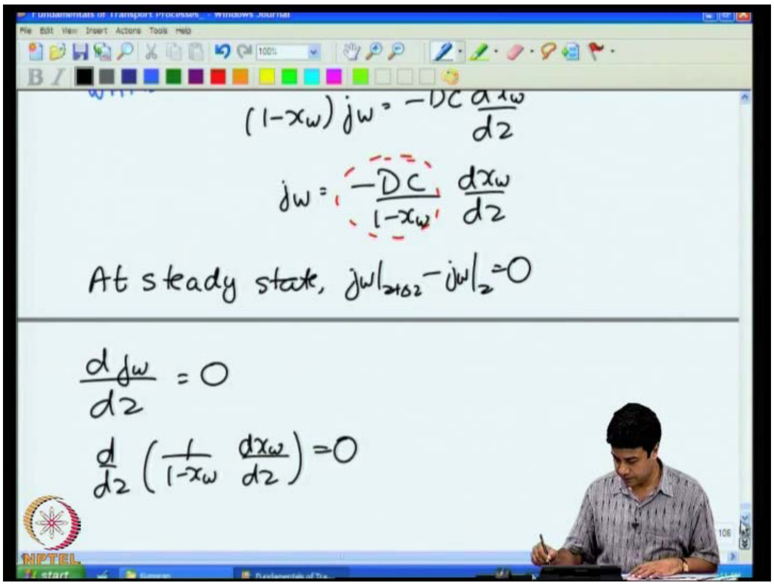
So, a simple example to illustrate this multicomponent diffusion – let us say that we have a glass rod, which is filled with water at the bottom. Then, there is dry air going across the top. Because of the dryness of the air, there is going to be a flux of water coming from the top water to the top. So, let us take this is at z is equal to 0; this is at z is equal to H. Now, this flux of water as it comes up, it is going to get entrained by the air and go away. How is this flux related to the gradient in the concentration of water or the humidity of water?

Simplistically, one would write the flux of water as (Refer Slide Time: 05:43) minus D times dc of water by dz is equal to minus D times the total concentration dx – the mole fraction of water multiplied by dz. However, that is not all. There is also a motion due to the center of mass. There is a mean flow. And, the mean flow total is equal to j of water plus j of air. So, there is a mean velocity because both water and air together are going upwards. So, this mean flow is also entertaining some water with it. There is some motion with center of mass. And, because of that, we are going to have an additional term, which is equal to the fraction of water times j water plus j air. So, this equal to the fraction of water times j water plus j air. So, this now is going to be the equation.

This term is the diffusion (Refer Slide Time: 07:09) relative to the center of mass; and, this term is the entrainment of water due to the fact that water and air are moving. Strictly speaking, in this problem, water and air are moving in opposite directions. As the water gets evaporated, the surface of the water moves downwards and because of that air has to come downwards in order to displace the water that has been evaporated. You can imagine that if it was right at the top, as the surface comes downwards, air has to come in to fill the gap, because the water is going out. So, strictly speaking, there is actually a flux of air, which is actually opposite in direction to the flux of water.

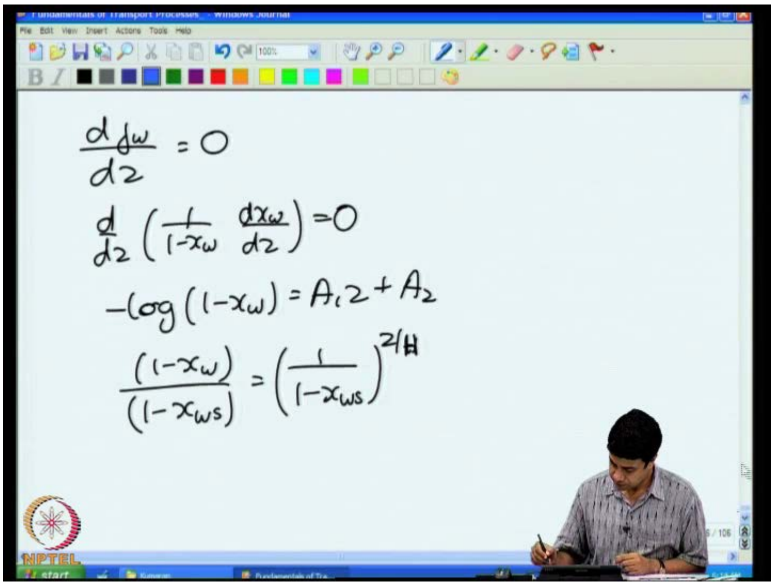
However, this flux is actually small. The reason is as follows: when water evaporates, the volume of water vapour as I told you is about a 1000 times larger than the volume of the water; liquids have volumes that are about 1000 times less than the volume of gases; the density of liquid is about 1000 times more than the density of gases. So, therefore, the volume of water displaced is actually small compared to the volume of the vapour that is coming out. And, the volume of air going downwards is equal to the volume of the water that is displaced. And, because of that, the volume of air coming in can actually be considered to be small compared to the volume of water going downwards. However, there is still a mean flux upwards. There is still a flux of water that is going upwards. And, that center of mass motion of the water has also to be accounted for when we calculate the flux. So, in this equation, if I neglect the flux due to air, then I get an equation of the form (Refer Slide Time: 09:16) 1 minus x w times j w is equal to minus D c dx w by dz.

(Refer Slide Time: 09:29)



And therefore, the flux j w is equal to minus D c by 1 minus x w dx w by dz; where, x w is the mole fraction of the water. So, note that this now has a different form. It is not just the gradient; it has 1 minus x w in the denominator. If the mole fraction is small, then this will of course be small; whereas, if the mole fraction is significant, this has to be included in the balance equation. The fact that there is a mean motion has to be included in the balance equation as well. And, if I write a balance between two locations: z plus delta z and z, the balance will just tell me that at steady state, j w at z plus delta z minus j w at z will be equal to 0 at steady state. The flux going from one surface to the other has to be equal to 0; otherwise, there will be an accumulation within that volume if the fluxes on both surfaces are not balanced. Written in differential form, this is dj w by dz is equal to 0 or d by dz of 1 by 1 minus x w dx w by dz is equal to 0.

(Refer Slide Time: 11:19)

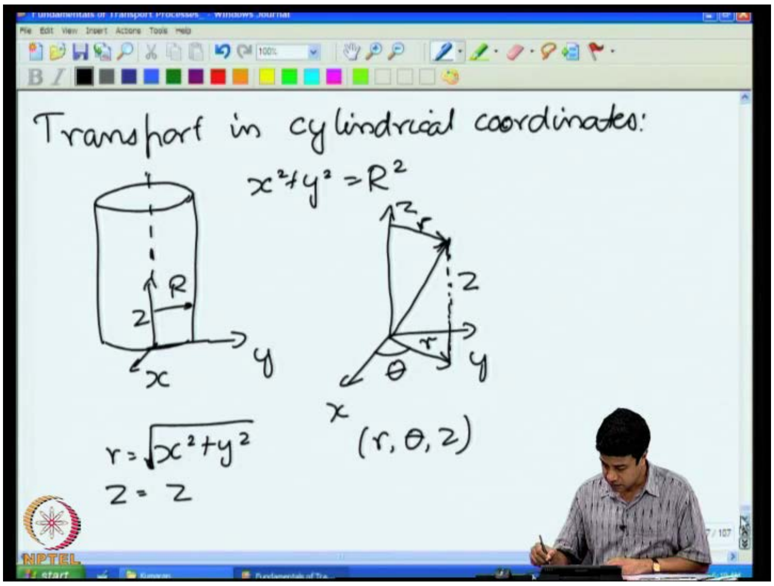


One can solve this equation quite easily to get minus log of 1 minus x w is equal to A 1 z plus A 2; and then, one has the boundary conditions. What are the boundary conditions in this case? At the surface (Refer Slide Time: 11:41) itself, the concentration is equal to the saturation concentration of water, so that x w is equal to x ws; the saturation concentration of water vapour that is in equilibrium with the air at the temperature. As I said z is equal to H, I have that c w is equal to 0 or x w is equal to 0, because it is dry air. It is the dry air that is taking the moisture along with it. Therefore, the mole fraction or the mass fraction of water in the air is equal to 0 at the top surface. So, I can solve this equation subject to these constraints.

Finally, the solution that I will get is that (Refer Slide Time: 12:35) 1 minus x w by 1 minus x ws is equal to 1 by 1 minus x ws to the power z by H . So, this is a more complicated dependence. And, that more complicated dependence comes about, because in multicomponent diffusion problems, one has to take into account the motion of the center of mass while defining the flux itself. The motion of the center of mass has to be taken into account while defining the flux; and, that accounts for this additional term in the conservation equation. Therefore, when the volume fraction, the mass fraction, the mole fraction of the two components are roughly the same; one has to take into account the motion of the center of the mass.

Everything that we did for the concentration diffusion assumes that the fraction of the diffusing component is actually small. In that case, one can linearize; x w is small in this (Refer Slide Time: 13:51) case. One can neglect that and just worry about gradient and concentration alone, and only take into account this part of equation. So, when you have multicomponent diffusion, one has to be careful about how to define fluxes. So, that is one thing to be kept in mind while dealing with mass transfer problems. We will not discuss this in further detail; I will only assume that the concentration diffusion equation is simply defined as I have done it earlier; the flux is equal to minus D dc by dz; and, that is simplifies the problem.

(Refer Slide Time: 14:33)



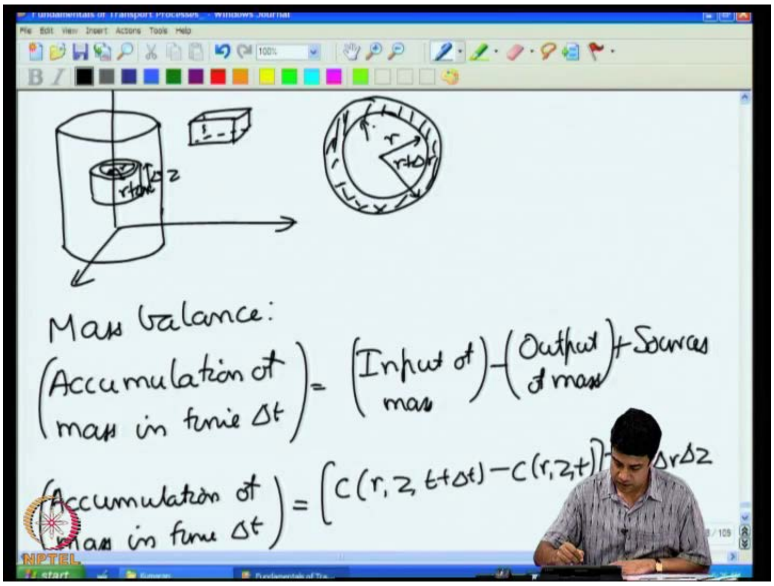
The next topic – transport in cylindrical coordinates – let me draw for you a cylinder here. You can consider this as a section of a pipe or reactor or whatever. The coordinate system that we have been using so far – x, y, z coordinate. Within this coordinate system, the definition of the surface of the cylinder… Note that I placed the coordinate system at the center point of the cylinder, the access of symmetry. Within this coordinate system, the surface of the cylinder, if the radius is r, surface of the cylinder is given by x square plus y square is equal to R square. This is inconvenient to apply boundary conditions at a surface like this. Therefore, this cylinder has an axis of symmetry. In this particular case, it is the z-axis. As you go around the z-axis, nothing changes in the cylinder. And therefore, it would be more convenient to define a cylindrical coordinate system and that is defined as follows. So, this was the original cartesian coordinate system.

In a cylindrical coordinate system, any point is defined by three coordinates. If I take this (Refer Slide Time: 16:52) as the position vector of this point, I can take the projection of this position vector on to the xy plane. So, this distance is r. The distance of the projection of this vector on to the xy plane is r; the height is z. And, this angle – the angle of the projection from the x-axis is equal to theta. So, these are three coordinates in the cylindrical coordinate system. The distance – whether I take the distance of the projection or the distance of the point itself from the z axis, both of these are equal to r. So, any point in space in a cylindrical coordinate system is defined by three coordinates. One is the z coordinate – the z is the height from the xy plane. So, this z coordinate is the height of the point from the xy plane. The r coordinate is the perpendicular distance from the z-axis. From the z axis, if I draw a perpendicular to cut this point, then that distance is equal to r. And, theta is the distance of the projection of this position vector from the x direction. It could be taken from any direction. But, in general, as a convention, it is defined from the x direction. Therefore, in a cylindrical coordinate system, the coordinates are r, theta and z. And, these coordinates can be related to the equivalent coordinates in a cartesian coordinate system.

Clearly, the distance from the z-axis is going to be equal to (Refer Slide Time: 18:46) the square root of x square plus y square. The square root of x square and y square is going to be the distance from the z-axis. The cylindrical coordinate system, z is just equal to z. And, how is theta related? Tan theta as you can clearly see is equal to y by x. So, this is the relation between the coordinates in the cylindrical coordinate system and in the cartesian coordinate system.

What is the advantage of using this cylindrical coordinate system? The big advantage is that if the z-axis is along the axis of the cylinder, then this (Refer Slide Time: 19:32) surface – the entire surface is one along which r is equal to a constant; r is equal to R along this entire surface, because it is a cylinder. And, all positions on the surface of that cylinder are equidistance from the axis of the cylinder. And therefore, rather than having this complicated boundary condition, I get a simple boundary condition of the form r is equal to capital R. So, it becomes easier to define the boundaries if the system has cylindrical symmetry by using a cylindrical coordinate system.

(Refer Slide Time: 20:34)



Now, how do I do balances in the cylindrical coordinate system? Clearly, the shell balance that I did in the cartesian coordinate system is going to be different here. In the case of the cartesian coordinate system, the three coordinate planes were perpendicular to each other. So, I managed to take a cubic volume for doing the balances. Clearly, when I have a cylindrical geometry, I cannot use that cubic volume. A cubic volume cannot be used when I have a cylindrical geometry, because it is not parallel to the coordinates in the cylindrical geometry. I have to use a volume that is parallel to the coordinates in the cylindrical geometry in order to do the energy balance or the mass balance.

For definite (( )) let us take the mass balance. I will assume for the present; we will do away with that assumption later. But, for the present, I will assume that there is no variation in the theta direction. In other words, the temperature, velocity, concentration fields do not vary as I go around the z-axis, so long as the distance from z-axis remains the same. So long as r remains the same and the z-coordinate remains the same, nothing changes as I go around the z-axis. So, that is an assumption that I will make for now. We will see a little later how the results change if we relax that assumption.

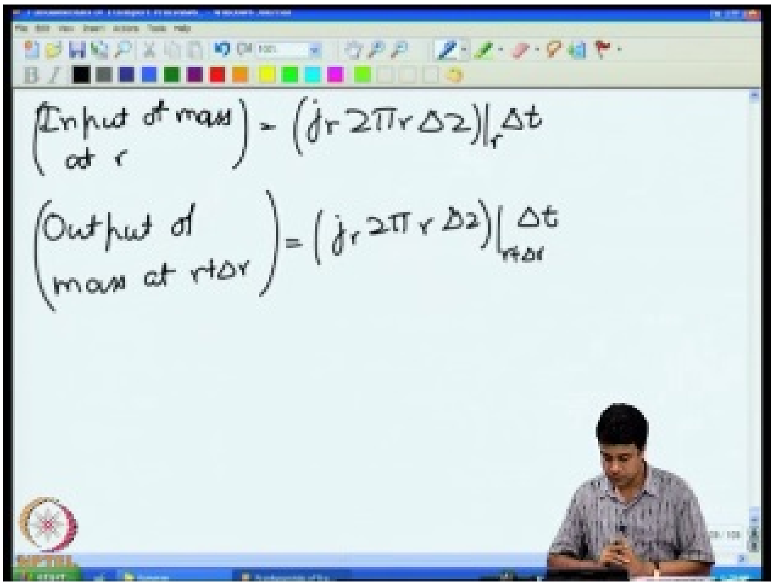
Since nothing is changing as I go around the z-axis, at a given distance r, I could use a cylindrical shell. This is a cylindrical shell of height delta z. And, it is between radius r and r plus delta r. Let me draw it out in detail for you. If I look at this from the top of the cylindrical shell, it will look like two circles (Refer Slide Time: 23:20). So, this is a shell contained between the radius r and the radius r plus delta r. It is a cylindrical shell between the radius r and the radius r plus delta r of height H. So, this is the differential volume that I will consider for doing the balance.

What is the balance equation (Refer Slide Time: 23:52)? The rate of accumulation of mass – the accumulation of mass within the cylindrical shell in time delta t is going to be equal to input of mass minus output plus any sources. So, that is the balance equation for the mass conservation. The volume that we considered for this is this volume (Refer Slide Time: 24:40) – the volume between the inner and the outer cylinder – between r and r plus delta r.

What is the accumulation of mass within this (Refer Slide Time: 24:55) shell in time delta t? This is going to be equal to the concentration at r, z, t plus delta t minus the concentration at r, z, t times the volume. The volume in this case is equal to 2 pi r delta r. The area of this (Refer Slide Time: 25:50) cross section is going to be equal to the circumference times the thickness. The thickness is delta r; the circumference is 2 pi r. Therefore, this area is 2 pi r delta r times the height, which is delta z. So, this is the accumulation within the time delta t.

Now, how about the input and the output of mass? In this cylindrical volume, there is an input of mass (Refer Slide Time: 26:20) at the surface at r due to the flux j r at r. There is also an output due to flux j r at r plus delta r. So, there is an input at r and an output at r plus delta r. And, both of these have to be incorporated in the mass balance equation.

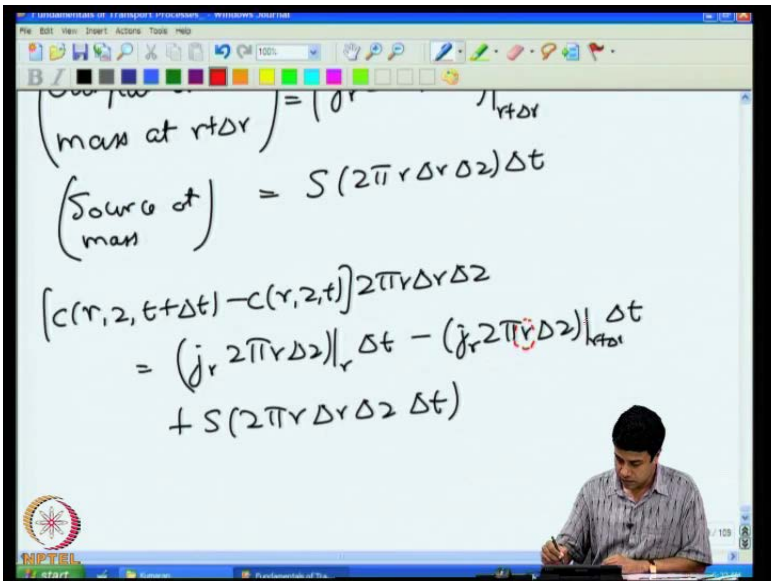
(Refer Slide Time: 26:50)



What is the input of mass at r? It is equal to j r times the surface area. The surface area for the input of mass is this (Refer Slide Time: 27:09) cylindrical surface area. It is the cylindrical surface area; it is the surface area for the input of mass. Therefore, this cylindrical surface area is 2 pi r times delta z. This at location r. This has to be multiplied by (Refer Slide Time: 27:38) delta t, because the flux is mass per unit area per unit time. So, for the input of mass, I have to take the flux multiplied by the curved surface area multiplied by time.

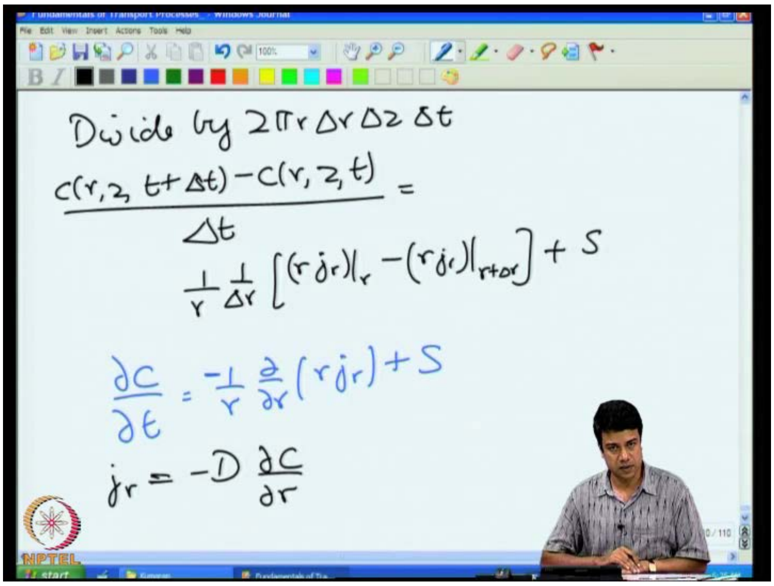
The output of mass is (Refer Slide Time: 28:04) equal to j r 2 pi r delta z at the location r plus delta r times delta t. Let me point out one thing here itself. The input of mass is through this (Refer Slide Time: 28:31) surface area. The output of mass is through this surface area. The two surface areas are not the same, because the two radii are not the same. In this case of the flow in cartesian coordinates, whether I take the top surface or the bottom surface, the areas are the same, because they are in cartesian coordinates. In cylindrical coordinates, the surface areas of the two cylinder surfaces are not the same. That is going to be important when we do the mass conservation condition. Therefore, the equation becomes… I have an accumulation within the volume and I have the input and the output.

(Refer Slide Time: 29:28)



There is an additional source and that is of the form S times the volume. S was defined as amount of mass created per unit volume per unit time. Therefore, the source of mass is going to be S times the volume, which is 2 pi r delta r delta z times delta t. Therefore, this accumulation of mass and time, delta t has to be balanced by the input, output and the source. So, I will have c of r, z, t plus delta t minus c of r, z, t into the volume, which is 2 pi r delta r delta z is equal to j r into 2 pi r delta z at r times delta t minus j r into 2 pi r delta z at r plus delta r into delta t plus S into 2 pi r delta r delta z delta t. Note that I have kept the radius within r plus delta r and r, because this is at r plus delta r. This one is at r; and, the two are different.

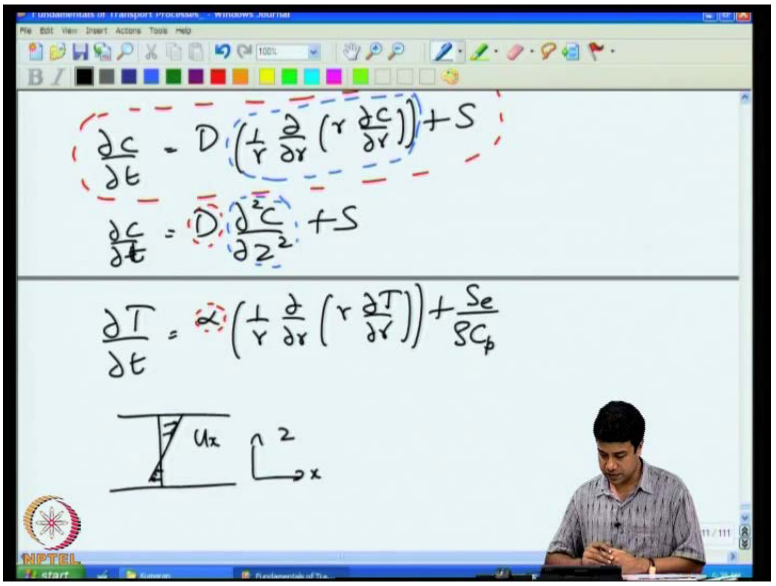
(Refer Slide Time: 31:57)



We divide throughout by 2 pi r delta r delta z delta t. And, this becomes c at r, z, t plus delta t minus c at r, z, t by delta t. This is equal to 1 by r 1 by delta r of r times j r at r minus r times j r at r plus delta r plus S. Once again, in this equation, 2 pi is a constant (Refer Slide Time: 33:16). So, it can be taken out. Delta z is also independent of r; delta z is just the height. It does not matter whether the surface is at r or at r plus delta r; the height is the same; it is independent of r. Therefore, delta z and 2 pi can both be taken out. However, this r depends upon r. So, it has to be taken into account when we do the differentiation; that is the basic issue.

Now, if I take the limit delta t and delta r going to 0, this equation (Refer Slide Time: 33:57) becomes dc by dt is equal to 1 by r d by d r of r j r plus S. Note that this is slightly more complicated than the form that we had earlier. It is a negative sign, because I am taking the value at r minus the value at r plus delta r. This is more complicated, because as I said earlier, the surface area (Refer Slide Time: 34:35) is changing as a function of position. And, because of that, even if the flux does not change, the mass that is transferred will change, because the surface area is changing. The total mass transfer is equal to the flux times the area. So, it is one component of the mass due to the change in the flux. There is another component of the mass due to the change in the surface area and this is inescapable in the case of cylindrical coordinates. You will end up with the more complicated form for the mass conservation equation simply because the surface area is changing as the radius is changing. And then, I can use the constitutive relation for the flux; j r is equal to minus D times partial c by partial r to finally, get an equation for the concentration field.

(Refer Slide Time: 35:35)

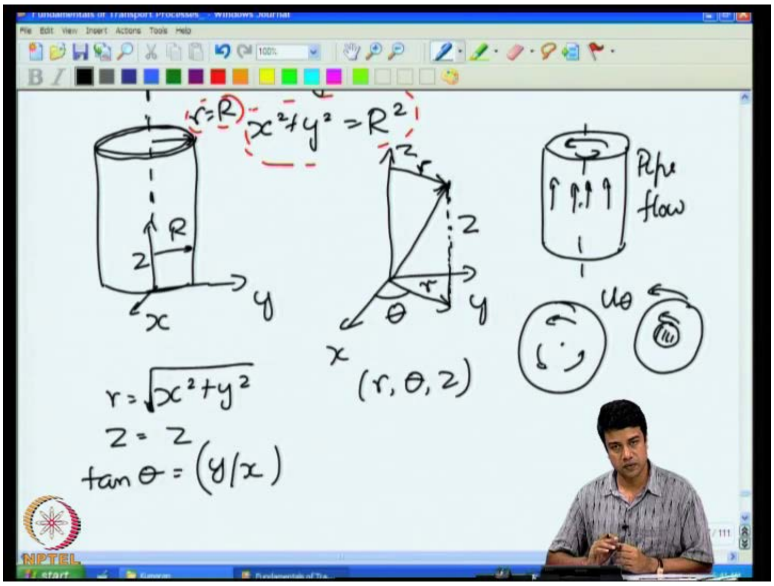


Partial c by partial t is equal to D into 1 by r d by d r of r d c by d r plus S. So, this is the mass conservation equation for unidirectional transport in a cylindrical coordinate system. Contrast this with the equation that I had for the cartesian coordinate system, which was of the form dc by dt is equal to D times partial square c by partial z square plus S. So, the difference is that this operator, which was just a second derivative in the cartesian coordinate system, now, has more complicated form in a cylindrical coordinate system. So, that is the big difference between cartesian and cylindrical coordinates.

One could do this quite easily for the energy balance equation; you will get exactly the same result except that you substitute T for c and the thermal diffusivity for the mass diffusivity. So, the equation for that will be of the form dc by dt is equal to alpha of 1 by r d by dr of r dT by dr plus a source term (Refer Slide Time: 37:07). So, this is for thermal diffusivity for the temperature field. It has exactly the same form, except that the mass diffusivity is substituted for the thermal diffusivity.

How about momentum transfer? When we did cartesian coordinates, invariably, we took a configuration like (Refer Slide Time: 37:35) this – x, z; the velocity was in the x direction. So, velocity was u x and that was the function of the z-coordinate. So, the velocity is tangential to the surfaces. So, the variation in velocity is perpendicular to the surfaces; whereas, the velocity itself is parallel to the surfaces. In our cylindrical coordinate system, there are two possible ways that we could take the (Refer Slide Time: 38:09) velocity, which is parallel to the surface. One is for the velocity to be in the tangential direction along the theta axis.

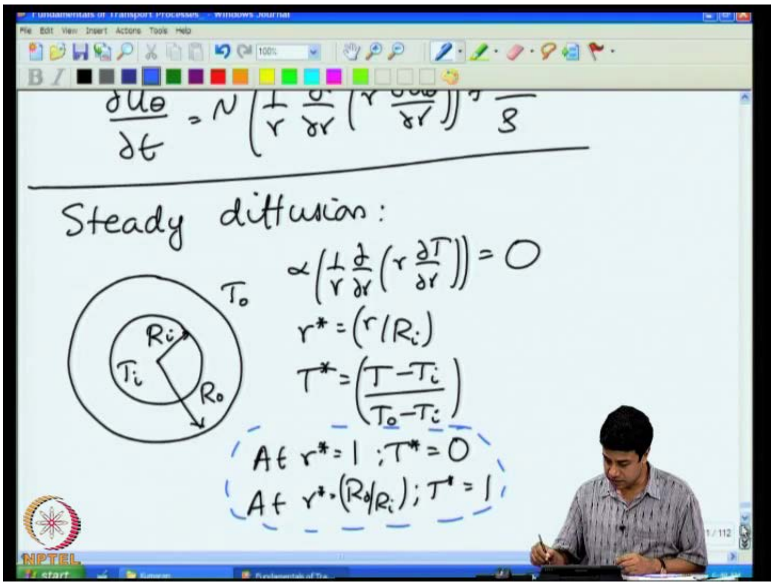
(Refer Slide Time: 38:24)



In other words, if I had a cylinder in which there was a swirling flow, then if I look from the top, the velocity is in the theta direction. So, I will have a velocity u theta, which is nonzero. This is relevant for rotating flows within cylindrical geometries. I could also have a velocity, which is parallel to the surfaces, but which is in the z direction. So, there are two directions that are parallel to the surfaces. Since the surface is at constant r, there are two directions parallel to the surfaces: one along the theta direction; the other along the z direction. This is relevant for circulating flows in cylinders. This is relevant for example, for a pipe flow. You have a flow along the axis. So, you could have flow either along the axis or around the axis. And, one could write different momentum balance for each of these.

The pipe flow along the axis is usually encountered in combination with a pressure gradient along the pipe, because the requirement in this (Refer Slide Time: 39:47) case is that the flow at the wall itself is equal to 0. Therefore, this is usually encountered in combination with a pressure gradient along the pipe axis. The velocity at the wall itself has to be 0. If the pipe is bounded by a rigid wall, then the velocity of the fluid at the wall itself has to be equal to 0. The velocity will have a parabolic profile as we will see a little later. Swirling flows are usually encountered when you have some circulation either at the wall or one could also have an annular region; one would have an annular region between two plates in which one of them moves with one velocity, the other is moving with another velocity. And, one could have a velocity of the fluid in between. So, this is another type of profile that is seen whenever you have rotating cylinders. And, one can write momentum balance equations for both of these.

(Refer Slide Time: 41:03)



The momentum balance equation for the swirling flow is actually quite easy; it is just analogous to this. Partial u theta by partial T is equal to the kinematic viscosity times 1 over r d by dr of r d u theta by dr plus any force (( )) direction divided by the density itself. So, this is the solution for a swirling flow. The solution for flow along the axis we will see when we deal with pressure gradients a little later. We will explicitly look at the flow down along a pipe and (( )) momentum balance equation for that flow. So, that we will see a little later.

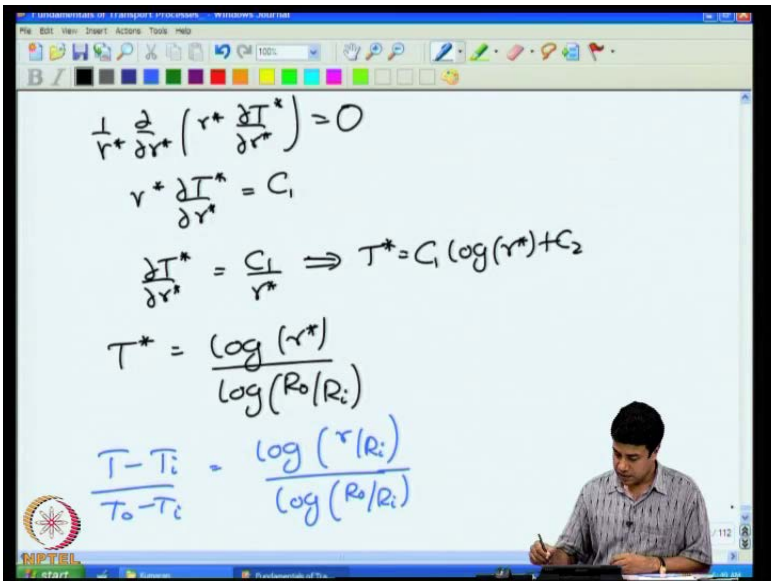
In cylindrical coordinates, these are the (Refer Slide Time: 41:44) mass momentum and energy balance equations. And, they are similar to what you get in cartesian coordinates, except that the viscous derivative of operator of the diffusion term is more complicated. It contains an additional contribution due to the fact that the surface area is increasing as the radius increases. And because of that, we have a slightly more complicated term in the diffusion term in the equation.

Now, let us solve these (Refer Slide Time: 45:26) equations for some simple cases. The simplest case to consider of course, is steady diffusion. In cylindrical coordinates, the simplest case is to consider is steady diffusion along the wall of a pipe. Let us say that the pipe has an inner radius. It has an inner radius, which is R i; outer radius is R o. We can consider this as for example, the wall of a shell and tube heat exchanger. There is hot fluid flowing inside; there is cold fluid flowing outside. So, the temperature on the inner surface is T i; the temperature on the outer surface is T 0 – T out. And, our task is to find out what is the heat flux as a function of the difference in temperatures and the radius of the two sides. So, you have to solve the mass conservation equation in order to get this difference and the heat flux.

The conservation equation at steady state – if I just neglect variations in time and consider steady state alone, the conservation equation is of the form alpha times 1 by r d by d r of r d T by d r is equal to 0 provided there are no sources or sinks within the flow. And now, I can nondimensionalize the variables. I can choose either R i or R o to nondimensionalize the variables, because ultimately what will matter is only the ratio of the two radii. So, I can choose either of them to nondimensionalize the variables. So, I will choose r star is equal to r by R i and T star is equal to T minus T i by T o minus T i.

I should note that you cannot just set one boundary at r is equal to 0 at r star is equal to 0. So, in this case, the boundary conditions become (Refer Slide Time: 45:06) at r star is equal to 1, T star is equal to 0. At r star is equal to R o by R i; T star is equal to 1. So, at the outer surface, T is equal to T 0. So, T star becomes 1. At the inner surface, T is equal to T i. So, T star becomes 0. The surfaces are at r star is equal to 1 and r star is equal to the ratio of the two radii. Previously, we always took one surface at z star is equal to 0; and other is at z star is equal to H. Even if the value of z itself was some other value, I could just subtract out a constant length and reduce the surface to 1 at z star is equal to 0. In this case, I cannot do that. At r is equal to 0, the surface area becomes 0, because 2 pi r T r is the surface area of the surface. So, I have to have a surface between two finite values of r whenever I do the calculations.

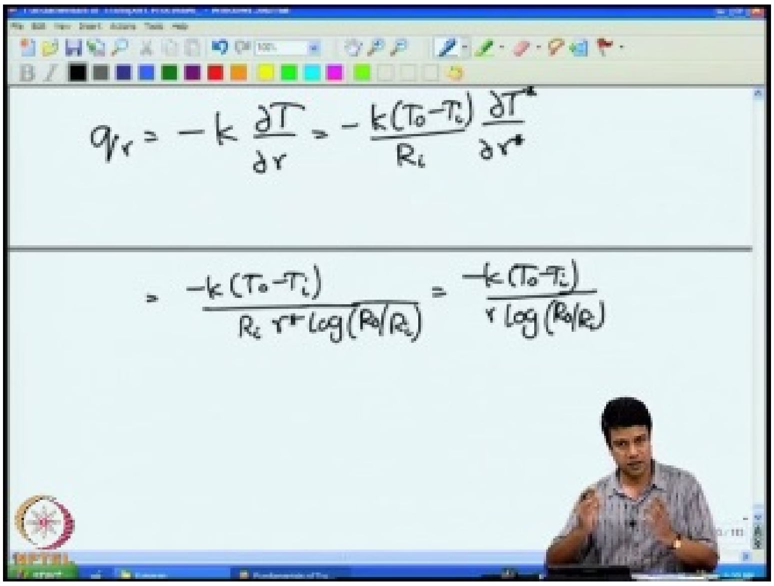
(Refer Slide Time: 46:21)



The differential equation expression in terms of this is 1 over r star d by d r star of r d T by d r is equal to 0 . So, if I integrate once, I will get r d T by d r is equal to C 1, some constant of integration or d T by d r is equal to C 1 by r, which implies that T is equal to C 1 log r plus C 2. Note that this is a diffusion dominated system; we do not have a linear profile in this case. The reason is because the surface area is changing as a function of radius. And therefore, if you have diffusion dominated transport in a cartesian coordinate system, we just got a linear variation. In this case, we do not get a linear variation. It is a logarithmic variation. And, C 1 and C 2 can be solved subject to boundary conditions to get T star is equal to log of r by log of R o by R i.

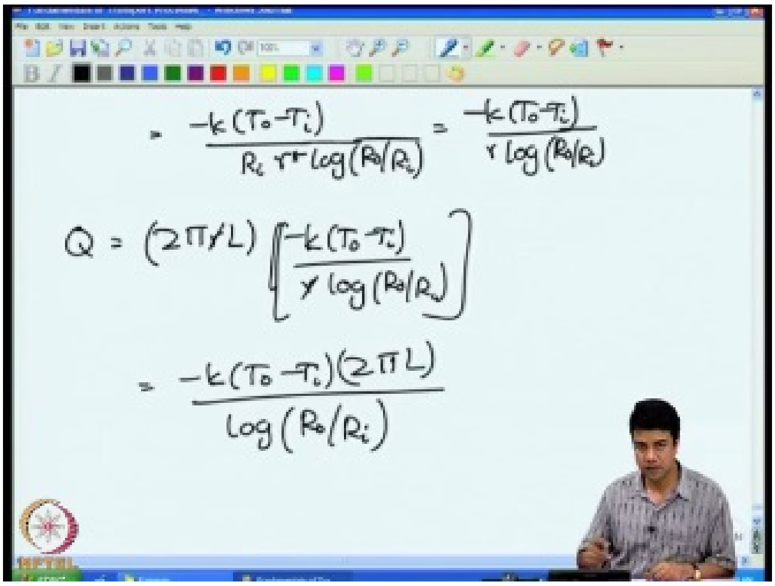
You can easily see that when r star is equal to 1, T star is equal to 0. When r star is equal to R o by R I, then T star is equal to 1 (Refer Slide Time: 47:52) as required by these boundary conditions. This solution (Refer Slide Time: 48:02) satisfies these two boundary conditions. So, I have chosen C 1 and C 2 in such a way that these two boundary conditions are satisfied. So, this is the temperature profile. If I express it back in terms of dimensional variables, this is T minus T i by T o minus T i is equal to log of r by R i by log of R o by R i. So, that is the dimensional solution.

(Refer Slide Time: 48:54)



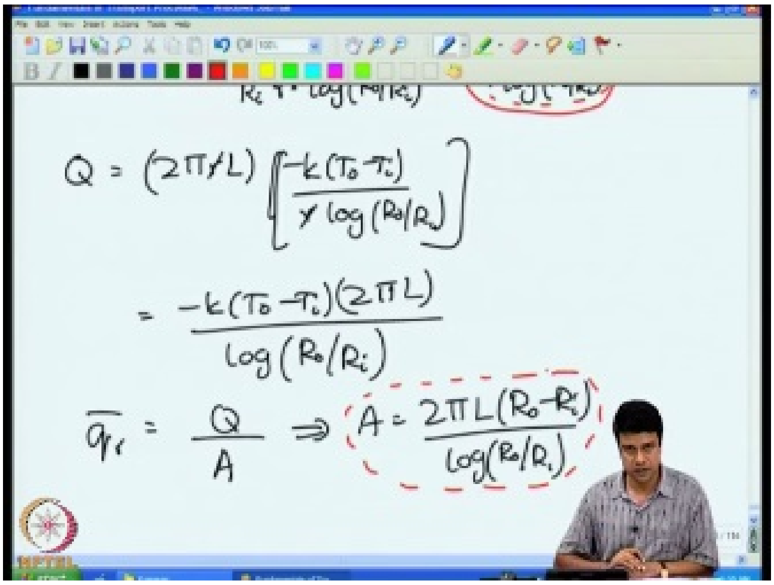
What we were ultimately after was the heat flux. So, how do we calculate the heat flux in this case? The heat flux q r is equal to minus k times d T by d r. And, using nondimensionalization, this is equal to minus k into T naught minus T i by R i into partial T star by partial r star, And, using this expression for (Refer Slide Time: 49:37) T star, I can easily get the final solution as minus k into T naught minus T i by R i r star into log of R o by R i. And, since r star is equal to r by R I, this is also equal to minus k into T naught minus T i by r log of R o by R i. So, this is the heat flux.

(Refer Slide Time: 50:25)



Total heat coming out of the cylindrical surface – the total heat is going to be equal to the surface area; the total heat coming out of any surface is going to be equal to the surface area (Refer Slide Time: 50:51) of that surface times the flux. The surface area is equal to 2 pi r times the height in the perpendicular direction of this. So, if I take a height length L of this tube of the heat exchanger, then the total heat coming out is going to be equal to 2 pi r L into minus k T naught minus T i by r log of R o by R i. And, you can see that the r cancels out; and, I just get minus k T naught minus T i times 2 pi L by log of R o by R i. Note: this total heat coming out is independent of r. The heat flux that I had was dependent upon r. The expression for the heat flux that I had was dependent upon r (Refer Slide Time: 52:07). In fact, it went as 1 over r. The heat flux went as 1 over r; the surface area is proportion to r. So, the total heat coming out of any surface is exactly the same. It is independent of the radius.

(Refer Slide Time: 52:29)



Now, one can use this to define an average flux. As I told you, the flux itself is dependent upon r. In this expression, the flux (Refer Slide Time: 52:41) depends upon r. But, the total heat coming out is not dependent upon r. So, I can define an average flux as q r average (Refer Slide Time: 53:01) is equal to Q by 2 pi L into some average. And, this I can write it as k times T naught minus T i by R naught minus R i times some function. Therefore, this average radius is effectively going to be equal to the logarithmic average of the inner and the outer cylinder radii. So, if I define the average heat flux as Q by 2 pi r into R naught minus R I, then I will get the average area coming out as (Refer

Slide Time: 54:18) Q by A. Then, it implies that the area is equal to 2 pi times L into R naught minus R i divided by the log of R naught by R i. Therefore, the actual radius of the surface at which the average flux is calculated is actually the logarithmic mean of the surface. So, this is the log law for heat transfer from a cylindrical pipe. So, logarithmic law for the heat transfer from a cylindrical pipe.

And, this logarithmic mean (Refer Slide Time: 55:03) is simply because I showed you that the heat flux goes as log of r. The reason is because t he surface area is changing. So, instead of having a dependence on r itself, the heat flux goes as log of r. And, because of that, when you calculate the average area, it is not the mean area of the surface itself for the transfer, but rather logarithmic mean of the area for transfer. So, this briefly is the fundamentals of the transport in cylindrical coordinate systems.

I derived for you the transport equation and I showed you the complication is because in a cylindrical coordinate system, the surface area changes with radius. And, because of that, the conservation equation has a more complicated form in this case (Refer Slide Time: 55:48). The conservation equation has a more complicated form. Instead of having the d square c by dz square, which I had earlier, I get a more complicated form of the conservation equation. And, in cylindrical coordinates, if I have two surfaces at two different radii with the temperature difference between them, the temperature profile in between these two is not linear, it is logarithmic. And, because of that, the flux equation for the flux, the area that you take should be the logarithmic (( )).

Next class, we will go back to looking at how we do unsteady problems. Start with the simplest case for cartesian coordinates, the similarity solution; and then, I will show you how to do separation of variables in cylindrical coordinates. Methods are exactly the same; the functions are more complicated. In the previous case, I just had a second derivative, and therefore, I got sine and cosine as the bases functions. In this case, I do not have a second derivative; the bases functions will be more complicated. Then, we will look at pipe flows and look at how to study oscillatory flows. So, will continue our discussion of cylindrical coordinates in the next class; we will see you then.