Fundamentals of Transport Processes
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Lecture No. # 15
Unidirectional Transport Cartesian Co-ordinates -VIII (Heat and Mass Sources)

Welcome to this the fundamentals of transport processes. This is lecture number fifteen, and we are well on our way to solving problems in unidirectional flows. We had looked in series at series of problems of increasing complexity. The first few were just flow transport between 2 flat plates, where the temperature the concentration of the velocity was at 2 different values between the 2 plates, and so there was a transfer of momentum energy or concentration from the region of higher velocity temperature or concentration to a lower value.

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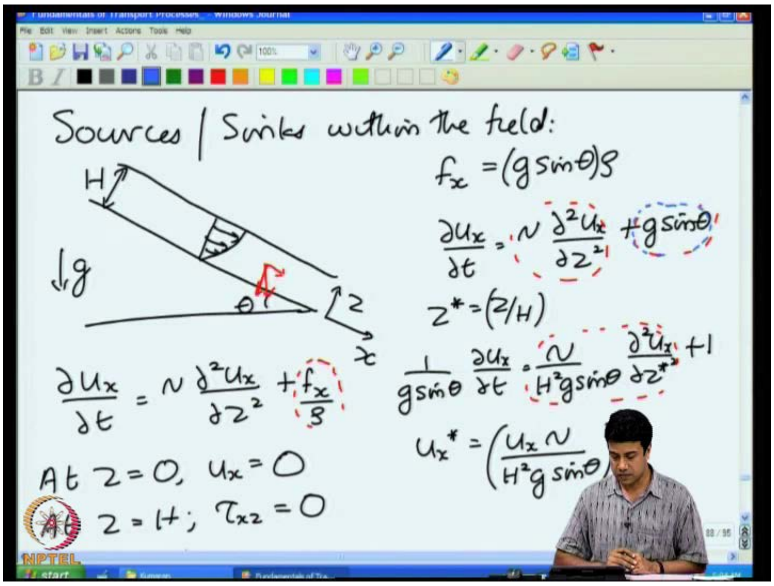

And then we were looking at some problems where there is a source or sink within the flow. In this case there is momentum generated due to forces acting on the fluid itself. Alter natively, there could be a source of mass due a to reaction taking place within the fluid or a source of heat due to various reasons, one could be because of reactions exothermic or endothermic resulting in an increase or decrease in temperature. There could also be phase transformations which result in release of latent heat, and I had also briefly told you that there could be viscous heating due to the shear stress and the fluid itself.

So, in this series of problems where there is a source or sink of mass momentum or energy within the flow. The first situation we had considered was a body force. A gravitational force acting on a fluid that is flowing down an inclined plane. So, the problem we considered was the flow down an inclined plane and there is a gravitational force acting on every element of fluid within the flow. That gravitational force has a component along the flow direction. This component is of course, if I resolve it into components along in perpendicular, I will get a component both along and perpendicular to the flow direction. The component of the gravitational force along the flow direction is what results in the fluid flow. And that exerts a force within the fluid itself. And therefore, we have an equation with inhomogeneous term here this is an inhomogeneous term. which is generating the fluid flow within the due to the force exerted on each volume element of fluid within the flow.

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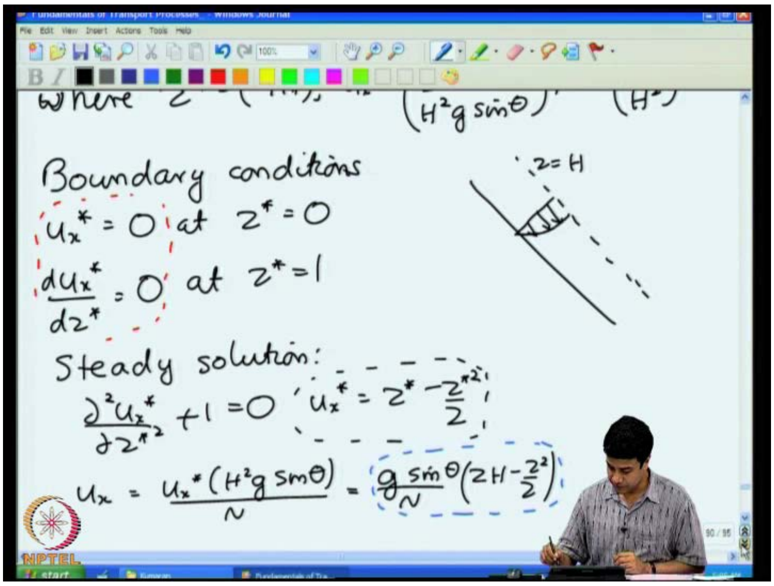

So, this is like a source of momentum and from that we had calculated what was the velocity field due to that at Steady state we get the solution quite easily. The important point is we have to impose a 0 shear stress condition at the surface if the surface is between liquid and air then the viscosity of air is much smaller than the viscosity of liquid. Due to that the air cannot exert a shear stress on the liquid and due to that we have to ensure that the shear stress at the surface is equal to 0 and on that basis we had got this velocity profile for the flow.

Now, another case where there is a force exerted on every volume element of fluid is the pressure driven flow in a channel. Pressure driven flows in channels in pipes are widely encounter. For example, if you want to pump fluid to an over head tank. There is a pressure gradient there is generated which results in the pumping of fluid to that tank. Of course, that pressure difference has to account for the potential energy of lifting the water all the way to the tank, but also for the friction losses due to the flow in the pipe. So, because of that you will have a pressure gradient which exerts a force on the fluid and we will look it how to analyze that situation in two respects one is the flow itself the flow profile itself and the second is the viscous heat generated due to the velocity profile.

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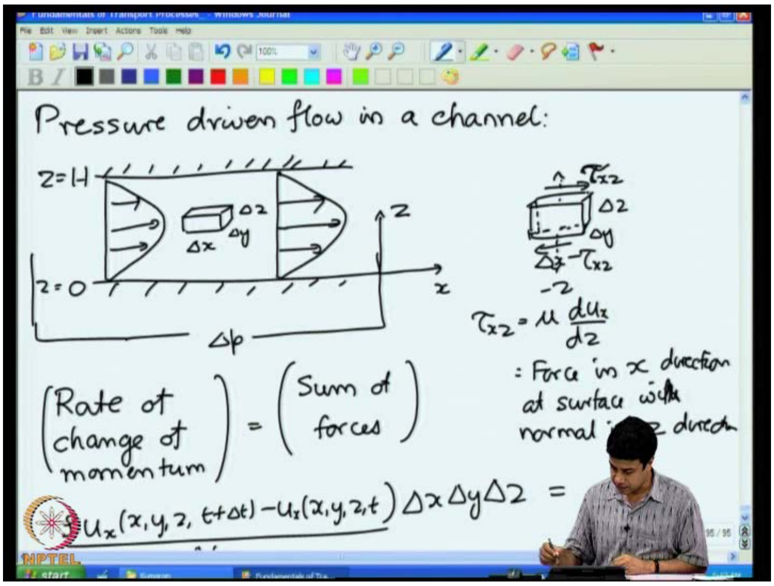

So, this is a Pressure driven flow in a channel this channel is in between 2 flat plates. So, let us draw the configuration here, you have 2 plates one is at z is equal to 0 the other is at Z is equal to H. and these are solid plates we will assume that they are of infinite extent into the boat. This is the x coordinate this is the z coordinate. And then we have to do find out what is the velocity profile within this channel. Now; obviously, this flow is being generated because there is a difference in the pressure between the inlet and the outlet there is a difference in the pressure between the inlet and the outlet and that is what is generating a velocity profile.

So, the pressure at each and every cross section in the x direction is not a constant the pressure is steadily decreasing as you go downstream. So, how do we do the momentum balance we use a shell balance once again of thickness delta x delta y delta z. And the balance equation is the same. The rate of change of momentum is equal to the sum of forces previously, I had separated out forces into 2 types one was body forces and the other was surface forces. In this particular case for simplicity, we will assume that there are no gravitational forces exerted on the fluid. So, we will neglect body forces for the time being and consider only surface forces the rate of change of momentum within this differential volume is given by rho u x at x y z t plus delta t minus u x at x y z t this is the momentum per unit volume. And therefore, I have to multiply this by the volume. So, the rate if change of momentum changes in momentum per unit time multiply it by the volume this is equal to the sum of forces.

Now, what are the forces that are active on the fluid. Let us go back to the configuration first thing we are in a steady state configuration. So, there is no time variation. And. secondly, the flow is fully developed; that means, that the velocity profile is the same at each and every cross section within the flow. The velocity profile does not change therefore, u x is independent of x it depends only upon z. Since we have assumed that the plate is infinite in the perpendicular direction in the y direction there is no variation in y either. This is different from the flow in a pipe, which was we will see a little later. In a pipe its cylindrical and therefore, there is a variation from the center of the pipe towards the wall in this case the variation is only in the x direction and not in the z direction.

So, if I take this differential volume here if I take this differential volume delta x delta y delta z. The shear stress is acting in the x direction. So, tau x z which is equal to mu times d u x by dz is equal to force in x direction at surface with normal in z direction is the force per area acting at a surface was unit normal is in the z direction at the upper surface the unit normal is in the plus z direction. Therefore, the force exerting here is tau x z i had explain to you earlier that when you reverse the direction of the unit normal the direction of the force also reverses. So, at the bottom surface unit normal is in the minus z direction therefore, the force is minus tau x z. So, I have the shear stresses acting on the top and the bottom surfaces.

So, the force exerted due to the shear stresses is going to be equal to tau x z at the top surface into the area. The area of the top surface is delta x times delta y. Delta x delta y at the bottom surface the shear stress is minus tau x z if I will get minus tau x z at z delta x delta y. So, that is the force acting on the top and the bottom surfaces. However, I told you that there is a pressure gradient along the pipe as well. The pressure at each location is not the same pressure is higher on the left hand side its lower on the right hand side and that is what it is driving the flow through the tube.

So, in addition there are also pressure forces acting on these differential volumes. There are pressure forces acting on every surface on all six surfaces of this differential volume you will have pressure forces. However, for the top and bottom there is no difference in pressure because I have not imposed a pressure gradient there. For the front and back there is no difference in pressure because there is no pressure gradient. However, along the pipe along the x direction there is a pressure gradient therefore, the pressure at x is different from the pressure at plus delta x.

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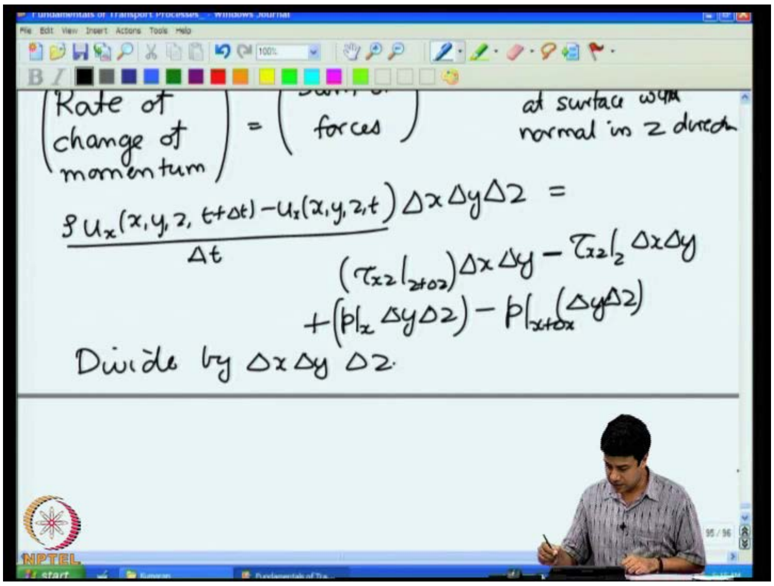

So, therefore, for this differential volume, I also have pressure forces acting there is a pressure acting at x and the surface at x there is a pressure acting. That is acting in the plus x direction. Pressure always acts invert to the differential volume that I am considering it is a compressive force that acts invert. So, the pressure acting invert at x which is in the plus x direction there is a pressure acting invert at x plus delta x which is acting in the minus x direction. So, these 2 pressure forces also enter into the momentum balance equation. So, the pressure force at x is in the plus x direction p at x. times the surface area which is delta y delta z minus p at x plus delta x delta y into delta z. So, in addition to the shear stresses whenever I have flow in a pipe or a channel there are also pressure forces acting and these pressure forces are what are driving the flow, in this case.

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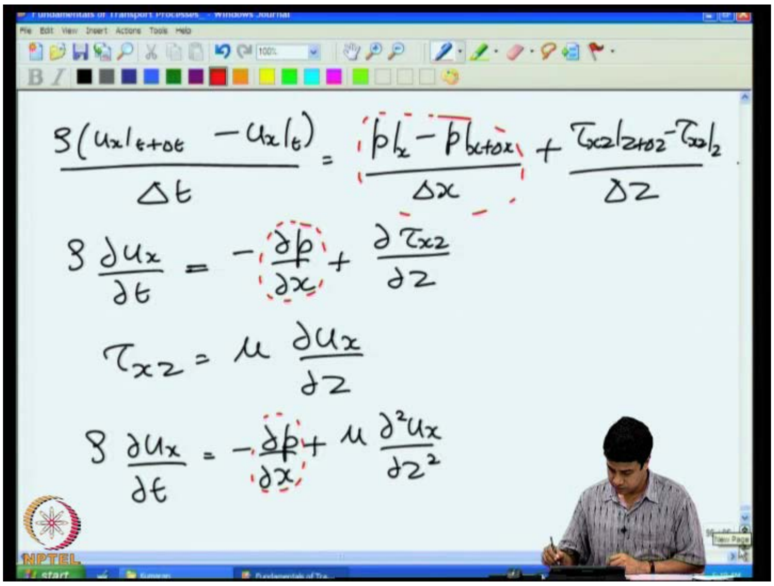

So, to get the balance equations I once again Divide by delta x delta y delta z. And then I will get the differential equation rho times u x at t plus delta t minus u x at t by delta t is equal to p at x minus p at x plus delta x divided by delta x plus the shear stress tau x at z plus delta z minus tau x z at z by delta z. So, that is now my momentum balance equation and taking the limit delta x delta y delta z and delta t going to 0. The balance equation becomes rho times d u x by dt is equal to. The second term here is p at x plus delta x minus p at x. So, this is equal to minus dp by dx and the last term is tau x z at z plus delta z minus tau x z x z which is partial tau x z by partial z.

So, that is my momentum balance equation. Rho times d u x by dt is equal to minus dp by dx this is the additional term that comes in due to the pressure gradient. This is the additional term that into the pressure gradient previously, we had a body forced term in this case there is a pressure gradient because the applied pressure on the 2 ends of the tube are different.

So, and then I can use Newton's law for viscosity tau x z is equal to mu times d u x by dz. To get rho d u x by dt is equal to minus dp by dx plus mu d square u x by dz square.

So, this is the balance equation and it contains this pressure gradient which is an imposed pressure gradient. It is an imposed pressure gradient which is a constant along the length of the tube and this is what is driving the flow. Note that the effect of the pressure gradient is remarkably similar to the effect of a body force. If we go back to the equation for the flow down of plane. I have a similar equation d u x by dt is equal to the convict viscosity times d square u x by dz square plus an additional term which is g times sin theta in the case of a pressure driven flow I get a term that is exactly similar to this. Except that it this is a pressure gradient and not a body force.

So, the effect of a pressure difference across the tube is similar to the effect of a body force acting on each element of the fluid. I could also write this in terms of the kinematic viscosity d u x by dt is equal to minus one over rho dp by dx plus the kinematic viscosity d square u x by dz square.

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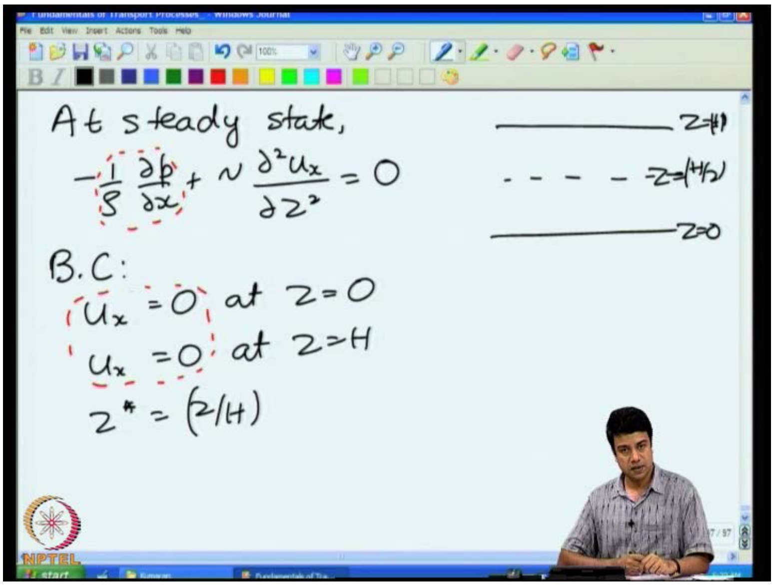

And that steady state At steady state d u x by dt is equal to 0 and therefore, my equation just becomes minus 1 over rho dp by dx plus mu d square u x by dz square is equal to 0. So, this is the momentum balance equation. And I had 2 boundaries for the channel this is at z is equal to 0 z is equal to H and the center line of the channel was at z is equal to H by 2. So, my Boundary Conditions require that the velocity is equal to the wall velocity at the 2 surfaces. At the 2 surfaces at u x is equal to 0 at z is equal to 0 u x is equal to 0 at z is equal to H. So, those are the boundary conditions that I have to solve this equation subject to.

Note that once again this is an inhomogeneous equation. It contains an inhomogeneous term here. So, there is no forcing at the walls. The velocity at both the boundaries are identically equal to 0. So, both the boundary conditions are both homogeneous; however, there is a steady forcing within the equation itself due to the pressure gradient that is applied across the length of the tube. Scaling obviously, z star is equal to z by H. How about the velocity u x once again the velocity u x has to come from the conservation equation itself.

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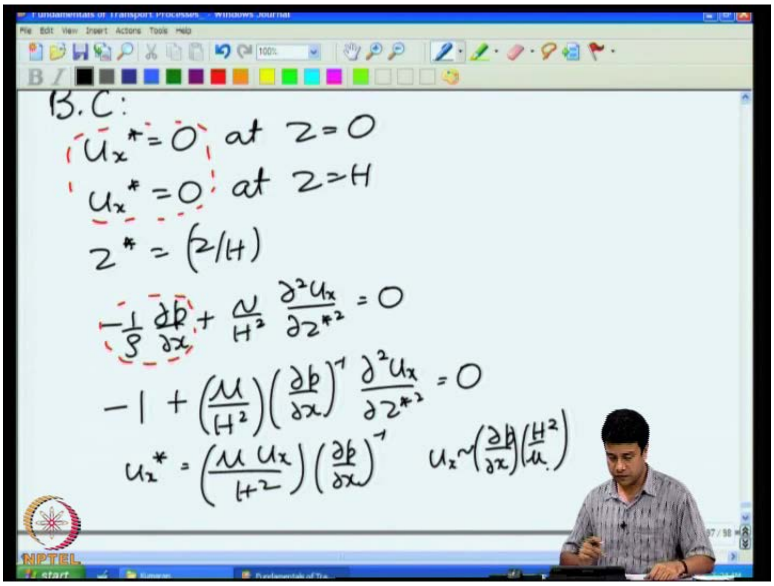

Because if I write down minus 1 by rho dp by dx plus mu by H square d square u x dz star square is equal to 0 and then I can divide throughout by this term here and set it equal to 1.

So, therefore, I will get minus 1 plus minus 1 plus u I have write this there is total viscosity mu by H square into dp by dx whole inverse d square u x by dz star square is equal to 0. Therefore, I can define a non dimensional velocity u x star is equal to mu u x by H square into dp by dx whole inverse. That is my definition of a non dimensional velocity in terms of the pressure gradient as I said there is a constant pressure gradient along the entire length of the tube

So, therefore, the velocity has to be scaled by that pressure gradient to get a non dimensional velocity. So, this gives us the scaling for the velocity; that means that the velocity will go as dp by dx times H square by mu. So, if I apply a pressure gradient the velocity along the tube is going to go as the pressure gradient times H square divided by mu.

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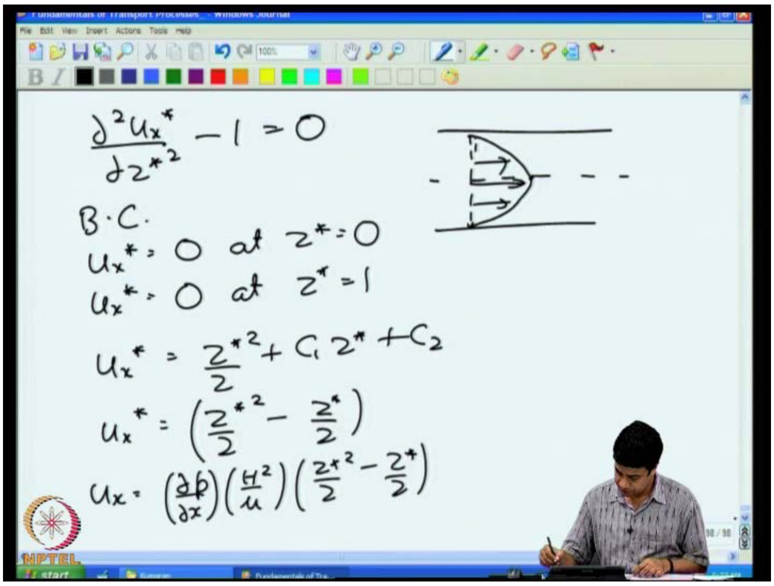

So, this gives me a scale for the velocity. And once I put that in the equation just becomes d square u x by dz square minus 1 is equal to 0. With Boundary Conditions u x is equal to 0 at z is equal to 0 and u x is equal to 0 at z is equal to 1. This equation is easily solved. The second order differential equation which solution is quiet easy to get the solution is just of the form u x is equal to z square by 2 plus C 1 z plus C 2. where the constant C1 and C 2 are to be determined from the boundary conditions say z is equal to 0 and z is equal to 1. and this C 1 and c 2 can be determined quiet easily and finally, I will get an expression for u x which will be equal to z square by 2 minus z by 2.

So, that is the final equation for the velocity profile. It is a parabolic velocity profile within that within the channel. The velocity is equal to 0 at both z is equal to 0 and at z is equal to one. So, the velocity is equal to 0 at both z is equal to 0, and z is equal to 1 at the center of the channel at z is equal to half the derivative of the velocity is equal to 0. So, I get a parabolic profile that looks something like this in the channel. And this parabolic profile is characteristic of all pressure driven force we will see a little later then when we do the flow in a pipe that you get a parabolic flow in that case as well.

So, now I can express this back in terms of a dimensional velocity. To get a dimensional velocity all I need to do is multiplied by dp by dx times H square by mu. So, my dimensional velocity will just be equal to dp by dx into H square by mu into z star square by 2 minus z star by 2 and this is equal to minus 1 by 2 mu dp by dx into z into z minus H. So, this is the parabolic velocity profile for the flow in a channel. It is called the plane poiseuille flow the parabolic profile for the flow in a channel.

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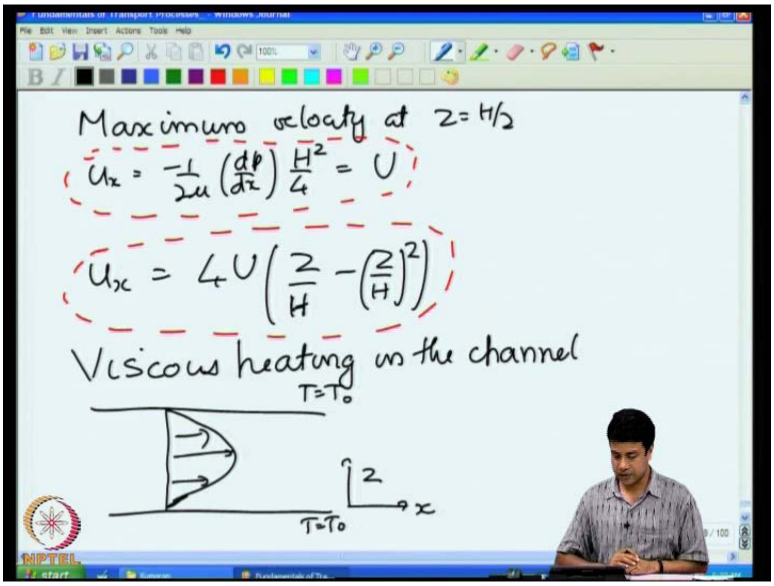

Now, this can also be written in terms of mean velocity. if I take for example, the maximum velocity of the center of the channel. So, at the center of the channel z is equal to H by 2. So, u x will be equal to minus 1 over 2 mu into dp by dx into H square by four which is equal to the velocity u at the center of the channel. So, this is the maximum velocity at the center of the channel the Maximum Velocity u is equal to is given in terms of pressure gradient by this expression.

So, I could also express the velocity u x in terms of the velocity at the center of the channel the maximum velocity. So, u x in terms of the maximum velocity is equal to four U into z by H minus z by H the whole square where U is equal to this expression the maximum velocity at the center of the channel. So, that is the velocity profile under a pressure gradient, for the flow in a channel. And as I said the velocity is remarkably similar to the velocity under they under the application of the gravitational force. If you go back to the previous example, the velocity in this case was also parabolic the velocity in this case was also parabolic. Accept that it had a derivative equal to 0 at the top surface It had derivative equal to 0 at the top surface. That is because we had imposed a 0 shear stress condition at the top surface. Where as for our flow in a channel, the shear the velocity was equal to 0 on both sides the solution was parabolic except that it came back to 0 at both walls that is the only difference. If, instead of having a free surface at the top if I had had a flat solid wall at the top I would have got the exact same velocity profile except that you parabolic and come back to 0 because I require to enforce the 0 velocity condition at the top surface if the flow is in between 2 flat surfaces.

So, instead of the gravitational force here between 2 flat surfaces tending to flow the fluid down the slope I instead have a pressure difference which is trying to flow fluid down the slope. So, that is the only difference. So, pressure differences and body forces act in a very similar manner. Now this problem of the flow in a channel we have used it to determine what is the velocity profile in this flow another situation we could consider is the effect of viscous heating due to viscosity within the flow. So, in that case because of the flow there is a source of energy and that tends to heat up the channel. And the objective is to find what is the temperature due to this viscous heating within the channel?.

So, let us look at that problem which is now a problem of a source of heat within the channel. So, this is the problem of Viscous heating in the channel. So, I am given a velocity profile. This is x this is z i know that the velocity profile is given by this profile and I want to know what the temperature. So, we are maintaining for example, in a heat exchanger problem we maintain the temperature T is equal to T naught on both walls and we would like to know what is the temperature within the fluid due to the viscous heating?

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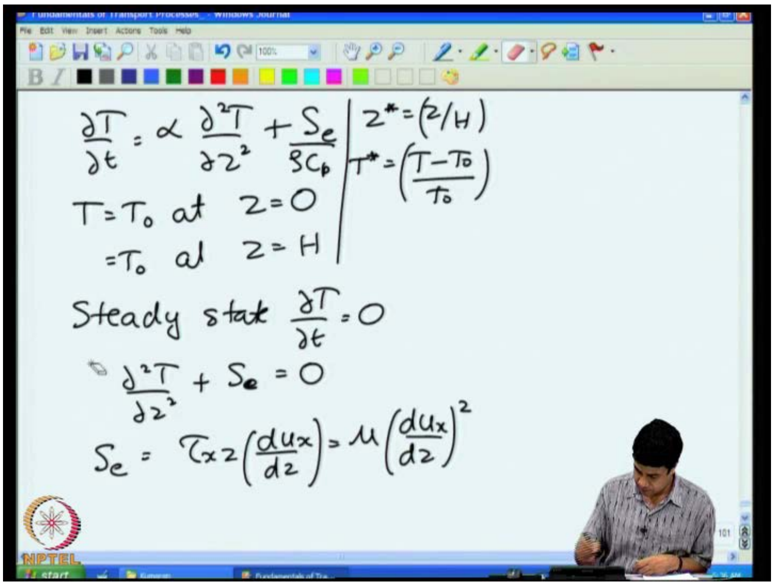

So, that is the problem. We will not derive in detail here, but the source of energy due to viscous heating is given by the source per unit volume is equal to the shear stress times the velocity gradient. The source of heating per unit volume energy generated per unit volume per unit time is the product of the shear stress and the strain rate. And we will assume that this formula for the shear stress and strain rate is given to us. So, you do not try to obtain it.

So, with this expression for the rate of viscous heating the heat transfer equation is of the form d T by dt is equal to the thermal diffusivity d square T by dz square plus S e. With boundary conditions T is equal to T naught at z is equal to 0 and is equal to T naught at z is equal to H. So, those are the boundary conditions. So, using these I have to solve this equation scaling as usual we can define the z coordinate as z star is equal to z by H what is the scaling for the temperature?

So, for simplicity I will define the scaling here for the temperature as T star is equal to T minus T naught divided by T naught the fractional rise in temperature due to the viscous heating we will work with this and then we will see that we will get a dimensional less number which gives you the rate of viscous heating as compared to the temperature rise. So, inserting this into this equation and assuming steady state. So, at Steady state d T by dt is equal to 0. So, this is a steady configuration and there is viscous heating. So, my differential equation becomes alpha d square T by dz square plus S e is equal to 0. Now what is the source I told you that S e was equal to tau x y times be careful here tau x z times d u x by dz, but from Newton's law of viscosity tau x z is equal to mu times d u x by dz fine. So, I can write this as mu times d u x by dz the whole square. So, that is my expression for the viscous heating per unit volume per unit time in the energy balance equation. This becomes k is the thermal conductivity.

Now, we know that u x is equal to four U into z by H minus z by H the whole square which means that d u x by dz is equal to sixteen U square by H square sixteen u by H 1 minus 2 z by H.

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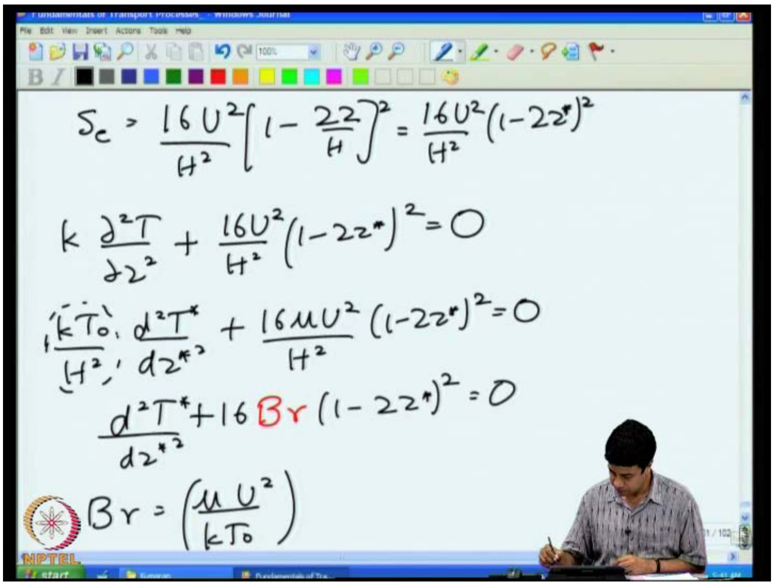

Therefore, s e will be equal to (( )) 16 U square by H square into 1 minus 2 z by H the whole square is equal to 16 U square by H square into 1 minus 2 z star square.

So, that is the final expression for the rate of production of energy. And if I put that into by configuration equation I have k times d square T by dz square plus 16 U square by H square into 1 minus 2 z star the whole square is equal to 0. So, this is my energy configuration equation in the presence of viscous heating within the flow. Scalings I defined z star is equal to z by H and T star is equal to T minus T naught by T naught.

So, that I have T star is equal to 0 at both boundaries with that scaling you can write this as k T naught by H square d square T star by dz star square plus 16 U square by H square 1 minus 2 16 mu is equal to 0 and I can divide throughout by this first factor in order to get an equation of the form d square T by dz square plus a dimensionless number times 1 minus 2 z star square is equal to 0. This number is usually defined with a pre factor there I will just leave that definition of the pre factor there. I will leave that definition of pre factor there this must (( )). So, I will define it with pre factor well this thing this dimensionless number is given by mu U square by k T naught. sometimes called the brinkman number. I will just live it as a dimensionless number here.

So, this basically gives me the ratio of viscous heating to the temperature rise due to a temperature difference proportional to T naught. The mu u square is the rate of energy generation per unit volume due to viscous heating mu U square by H square and k T naught by H square is a rate of change of energy within a volume if there were temperature gradient of magnitude T naught over a distance H. So, that is physical significance of this dimensionless number. This equation can be easily solved to get the temperature profile.

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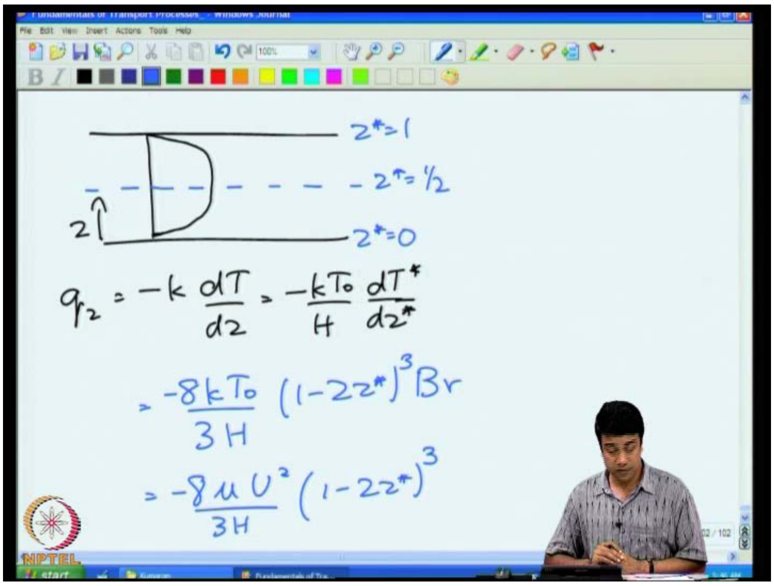

The final solution for this equation is T star will be equal to this Brinkman number into eight z 1 minus z star 1 minus 2 z star plus 2 z star square by 3 that is the final solution for the temperature increase. And if you actually plot out this temperature profile this temperature across the channel it has to come to 0 at the 2 walls. At the center of the channel itself it is very flat the temperature actually looks something like this. So, this is the temperature as a function of the height function of z this shows you the temperature profile. It is much flatter than the parabolic profile for the velocity itself. The reason is because at the center of the channel the shear stress goes to 0. The slope of the velocity is equal to 0 at the center of the channel therefore, d u x by dz is equal to 0 at the center of the channel. Therefore, at the center of the channel there is no viscous heating and because of that you do not have any variations in temperature right at the center at the walls of course, the shear stress is nonzero. In fact, the shear stress the slope of the velocity profile is largest at the walls because of that the heating is also largest at the walls and therefore, you have the largest temperature difference at the walls.

Now, one can define calculate the heat flux q z is equal to minus k times d T by dz. This is true of any location q z is equal to minus k times d T by dz. I can write this in terms of the dimensionless variables T star and z star as minus k T naught by H d T star by dz star and if I put in my expression for the heat for the temperature profile here, if I put in this expression for the temperature profile, what you get is that q z is equal to eight k T naught by 3 H into 1 minus 2 z star the whole cube into the Brinkman number. alternatively, I can write it in terms of viscosity and the velocity of the fluid just from the definition of the Brinkman number mu u by k T naught as eight mu U square by 3 H into 1 minus 2 z the whole square whole cube I am sorry.

Clearly this is a maximum at z is equal to 0 and at z is equal to z star is equal to 1 at the top and the bottom surface is where z star is equal to 0 and at the top surface by z star is equal to 1. This heat flux is the maximum is going outwards; that means, that it goes downwards at z star is equal to 0 and therefore, I should go up at z star is equal to plus 1 at the center itself the flux is equal to 0 at the center z star is equal to half and therefore, the flux is equal to 0 at the center itself and that was because the velocity gradient is 0 at the center as I had explained to you little earlier.

So, the maximum heat flux due to viscous heating q z is equal to 8 mu U square by 3 H that is the maximum heat flux due to viscous heating and this heat flux actually has relevance in the context of the heat exchanger problem that we had solved in the second or the third lecture. If you recall we had solved this problem using dimensional analysis.

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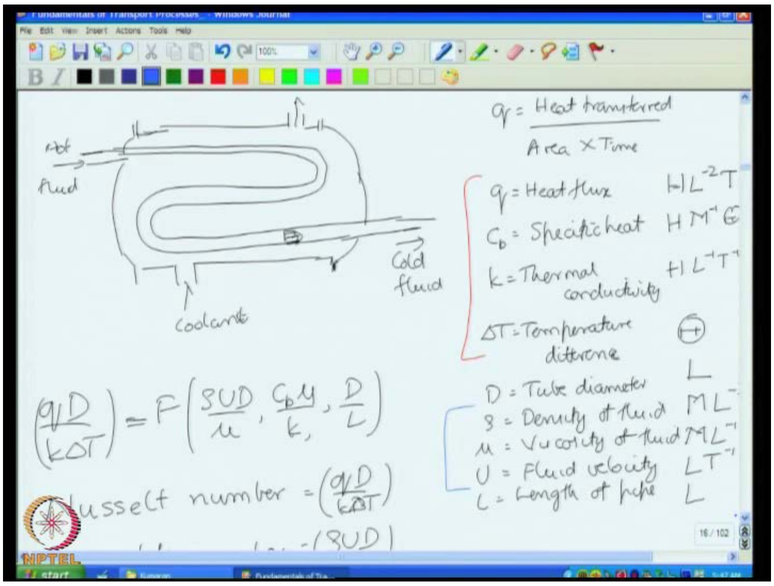

Yeah in this problem when we did that dimensional analysis we had separated out the quantities into thermal and mechanical quantities we had said that there is no inter conversion of energy from thermal to mechanical energy. Strictly speaking I had 1, 2, 3, 4, 5, 6, 7, 8, 9; 9 variables and 4 dimensions. So, I should have been able to get 5 dimensionless groups in this problem. However, I said if there is no conversion inter conversion of energy from thermal to mechanical then I can consider heat energy to be separate from mechanical energy and if heat energy is considered to be separate from mechanical energy then I have five dimensions because heat energy is a separate dimension and I have five dimensions nine dimensional groups; that means, that I have only four dimensionless groups which ultimately, we identified as Nusselt number the Reynolds number Prandtl number and the ratio of d by l. So, the assumption here is that heat and mechanical energy are separate there is no inter conversion and therefore, I can write a balance for the heat energy alone. I do not need to consider the fact that mechanical energy can be inter converted into heat energy.

In the problem that we just solved in the problem that we had just in the problem that we have just solved I told you what is the heat energy generated due to viscous heating this heat energy generated due to viscous heating represents an inter conversion of energy between mechanical energy and heat energy. Therefore, I can consider heat energy to be separate only if this flux due to viscous heating is small compared to the flux due to temperature difference across the wall across the tube wall. So, it is only under these conditions that I can consider the heat energy and the mechanical energy to be two different things and write different balances for this

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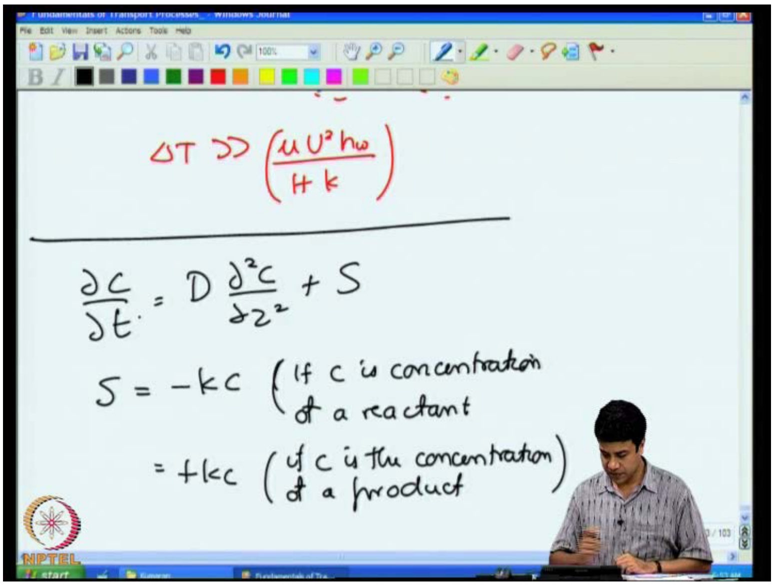

What is the flux due to the temperature difference q z is equal to k delta T by delta z where delta z is the thickness of the tube. Therefore, I require that mu U square by 3 H is small compared to k delta T by delta z. In order to be able to neglect the viscous heating in comparison to the actual energy transfer across the tube wall the energy transfer across the tube wall just depends upon the wall thickness. So, this is just equal to the thickness of the wall whereas, the viscous heating energy depends upon the thickness of the entire tube the thickness of the entire tube is usually small compared to the wall thickness and therefore, my requirement is that delta T has to be large compared to mu U square H w by H times k. So, only if this temperature difference is large compared to the temperature that is generated due to viscous heating will I be able to neglect viscous heating in comparison to the heat exchange due to the temperature difference across the tube. And it is only in that case that the assumption that the heat energy is separate from mechanical energy is a valid one.

So, that depends upon as I said on the Brinkman number this number that I had if I assume that delta T is of the same magnitude as T w then the Brinkman number gives me the ratio of viscous heating and the heat generated due to temperature differences if, delta T is of the same magnitude as T w and if the brinkman number is small then I can neglect the heating due to viscous friction in comparison to the heating due to the heat flux due to a temperature difference across the tube.

So, we solved the problems where there are sources and sinks for momentum transfer. We solved 2 problems one is where there is a body force which tends to force the fluid and the other is where there is pressure difference across the ends which results in a pressure gradient along the tube. I showed you that the effect of the pressure gradient is it can is this effectively is equivalent to the pressure due to a body force acting on the fluid and then we solved a problem here of the temperature the heat generated due to the viscous flow within the channel.

In a similar manner one can write down the equations for the concentration field as well. we had seen the concentration equation a little earlier d c by dz is equal to D d square c by dz square plus any Source or Sink. In the concentration equation the source of mass or the sink of mass is due to chemical reaction and therefore, the source or sink will depend upon the reaction rate and the concentration. The source and sink are usually of the form Source is equal to either k c a minus k c if c is concentration of a reactant or will be equal to plus k times c if c is the concentration of a product.

So, this is for first order reactions. If the reaction is second order then you will go at c square if it is an nth order reaction will go c power n. The equation continues to be linear only if the reaction is first order. So, reaction is second order the equation is no longer a linear equation. So, you had to have some special ways of solving the equation if it is a higher order reaction. However, if the reaction is first order there is an easy way to solve it. The reaction at Steady state D d square c by dz square minus k c is equal to 0 and from the reaction rate and the diffusion constant you end up getting a length scale out.

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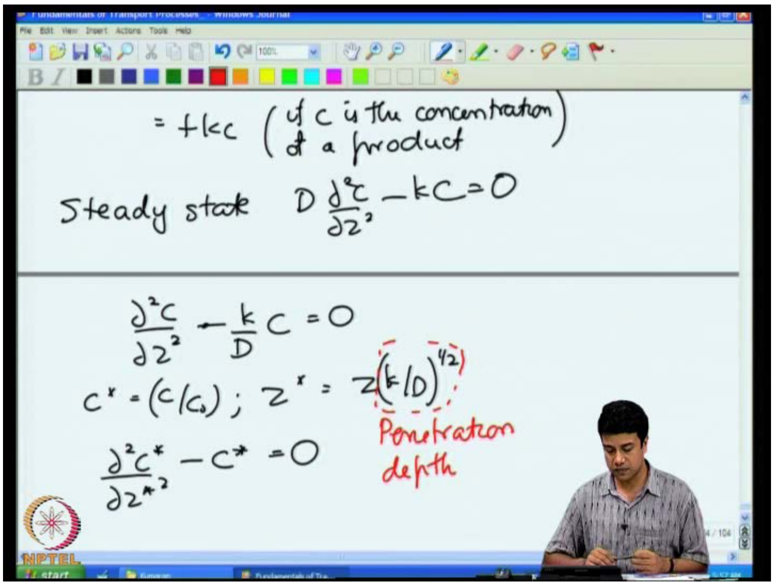

So, if I divide throughout by diffusion constant I get d square c by dz square is minus k by d c is equal to 0. if I define c star is equal to c by c naught I get a length scale out of here z star is equal to z times k by D power half this is non-dimensional because k is a reaction rate it has dimensions of time inverse and d is a diffusion coefficient it has dimensions of length square per unit time. So, this z star has is a dimensionless number. So, this dimensionless number comes out of the analysis and the equation becomes partial square c star by partial z star square minus c star is equal to 0. So, this has exponentially increasing and decreasing solutions this equation has exponential solutions.

So, this length scale is effectively a Penetration depth, the depth to which a perturbation to the concentration at a given surface will penetrate within the flow. So, if this penetration depth is small compared to the microscopic length scale then the effect of any concentration field at the bottom will be felt only to within a finite depth within the fluid. So, this thing acts as a penetration depth within the flow important point to note is that in this case where we have a reaction and a diffusion system simultaneously, the fact that you have reaction as well and that reaction rate depends upon concentration. In the previous case for viscous heating the heating rate did not depend upon temperature. So, it was independent of temperature in this case reaction rates usually depend upon concentration of reactants or products

So, if it depends upon concentration you get out penetration depth from this exercise. So, there is an additional length scale that comes into play apart from the length and the length and from the width of the channel. In the case of flow in a channel that was the only length scale in the problem in this case there is an additional length scale which is the which is related to the ratio of the reaction rate and the diffusion coefficient physically the system the concentration diffuses from the surface, but it is also getting consumed within the flow and therefore, because of that it will penetrate only to a finite depth within the fluid. So, that is the characteristic of diffusion problems in diffusion reaction problem in mass transport.

There is an additional complication in mass transport problems and that is that one has to account for center of mass velocity. The flux if the concentration of the mass being the transported is small then its transport does not result in a velocity of the center of the mass, but if it is finite there is a center of mass velocity we look briefly at that center of mass velocity in the beginning of the next lecture and then we will go on to analyzing problems in cylindrical coordinates. So, this completes sources and sinks within the flow for unidirectional transport in cartesian coordinates the next is to go to cylindrical coordinates before that we will just briefly look at mass transfer problems. So, we will see you in the next lecture.