Fundamental of Transport Processes

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Lecture No. # 14

Unidirectional Transport Cartesian Co-ordinates - VII (Momentum Source in the flow)

Welcome to our continuing discussion on unidirectional flows, unsteady flows where we were looking at different solution methods for flows that depend upon position and time. Variation in position is only along one direction what I have called so far the z direction and there could be a variation in time as well. We looked at two solution procedures; one for transporting to an infinite fluid where we looked at the similarity solution procedure where we could reduce from two variables time and z to just one similarity solution based on upon dimensional analysis alone. And that reduce the equation from a partial differential equation to an ordinary differential equation and that we were able to solve in order to find the profile of the concentration temperature or momentum.

When the diffusion is in a finite channel, the thickness of the layer of fluid also is important and one could use scaling on that bases of that thickness as well. In that case I showed you how to use separation of variables where we write the conservation the dependent field temperature concentration or momentum as the product of two functions; one only a function of space the other only a function of time. Separate out the two and get individual equations for each of these two components. Each of these can now be solved in order to get a solution for the entire concentration temperature or velocity fields.

And this solution procedure we had to have homogenous boundary conditions in one coordinate in this case the special coordinate. The temperature concentration and velocity were both 0 on both bounding surfaces, but, there were and some initial forcing at initial time and I showed you how to get all the constants in those equations. So, that was the separation of variables technique and last lecture we discussed how to analyze oscillatory flows. In this particular case we had a channel in which the top plate was stationary the bottom plate was oscillating with well defined frequency. And there is an imposed time period in this case in contrast to the previous problems where we just impose an additional condition and then let the system evolve in time all the way to t going to infinity.

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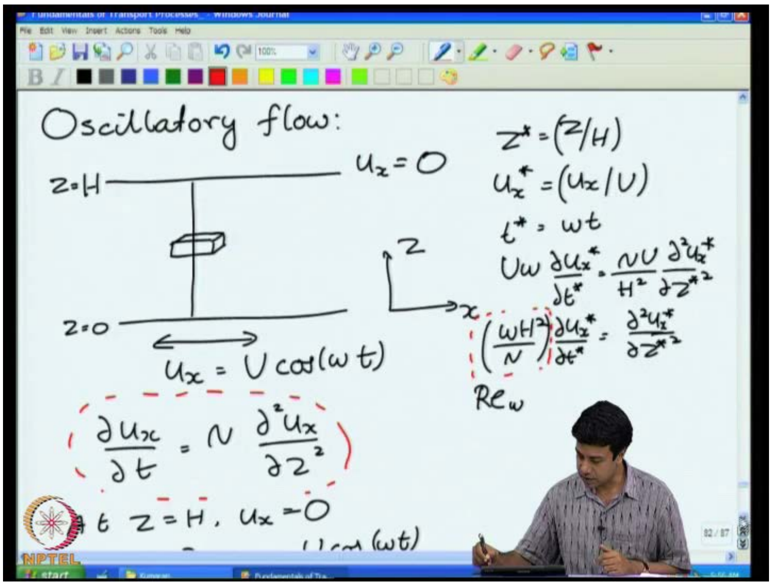

So, in this particular case we do have an imposed frequency. The momentum conservation equation was identical to what we had for all other cases. There is no source or sink. So, we had this momentum conservation equation where nu was the kinematic viscosity. The boundary conditions where that velocity 0 at the top plate, velocity is sinusoidal at the bottom plate with a very well defined frequency. We have defined scaled variables z star is equal to z by H. H is the natural length scale because it is the width of the and u x star is equal to u x by capital u where u is the velocity with which the bottom plate is being oscillated.

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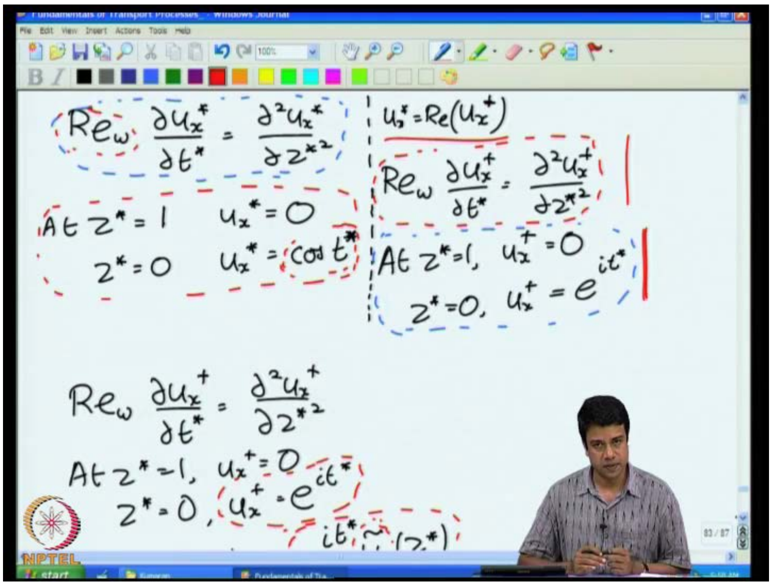

We also have an intrinsic time scale in the problem. So, we can define a non-dimensional time as t star is equal to omega times t and when we put all of these into the differential equation we ended up with a dimensionless number R e omega which was defined as omega H square by nu where H is the thickness and nu is the kinematic viscosity. And the boundary conditions are at z star is equal to one u x is equal to 0 and at 0 u x is just equal to cos t. So, this cos function varies between minus one and plus one and it is sinusoidal in time and we looked at a simple way to solve it. This cos function at the boundary is inconvenient in general to deal with. So, we defined a complex velocity field such that the actual velocity which you would like to obtain is the real part of this complex velocity field.

In terms of the complex velocity field, the equation the governing equation is exactly the same because the Renold's number is dimensionless number, it is real. At the boundaries we have the complex velocity is 0 at z is equal to 1 but there is no motion there and z is equal to 0 the complex velocity is e power i t. The real part of e power i t is the same cos t with which you are forcing the real velocity field. So, therefore, if I take the real part of the equation I get the equation for the real velocity field. If I take the real part of the boundary conditions I get the boundary conditions with the actual velocity field.

So, if I can solve the equation for this complex velocity field and take its real part, I will get the actual velocity field. That was the first simplification. The second one was that the equation is linear in u x plus. So, the equation that I have is a linear function of u x plus and it is being driven by a sinusoidal velocity profile at the boundaries u x plus is equal to e power i t. So, the equation is linear, the driving is sinusoidal, the response also has to be sinusoidal with exactly the same frequency.

So, that was the second simplification that I used. That straight away permitted me to write u x plus as e power i t star times something that is only a function of z. So, that was a significant simplification. u x plus I separated out into two parts; one which is dependent on time and the other which is dependent only on space. Implication of that is that at every point within the flow the velocity is oscillatory with exactly the same frequency as the frequency of the bottom surface. It can have a phase that is different from the phase of the bottom surface, but, it is oscillatory at exactly the same frequency. There can be a phase shift. The phase shift can be different at different locations, but, the frequency is exactly the same.

And since I know the dependence on time exactly, I can do the time derivator and I finally, get an equation which is an equation dependent only upon the z coordinate. It is a second order differential equation can be solved quite easily as we saw in the last class.

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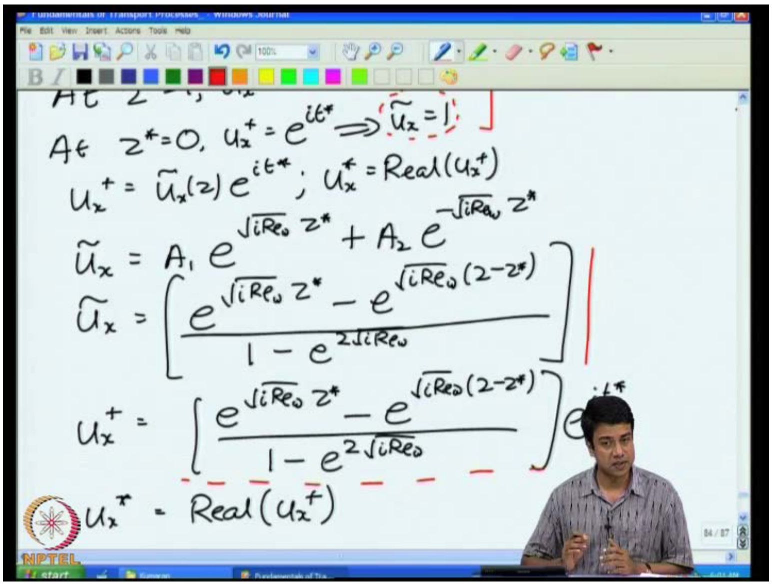

The boundary conditions since the boundary condition for the complex velocity is e power i t at the bottom surface for this u x tilde, the boundary condition has to be u x tilde is equal to 1 because this this time c power omega t is the complex velocity field.

So, now both the differential equation and the boundary condition are independent of time when expressed in terms of this variable u x tilde. They depend only upon space. So, once again I have reduced the equation from a partial differential equation in time in z to just an ordinary differential equation in z alone knowing that the variation in time is going to be sinusoidal with the given frequency at all instance of time and at all locations.

So, we solve this quite easily and subject to the boundary conditions to get a solution and we took the real part of that solution to find out what is the actual velocity field. This of course, has to be solved numerically as function of time. However, you can get some additional physical insight into this by just looking at the limits. One the limit where R e omega is small compared to one and the other is the limit where R e omega is large compared to one because that is the only parameter in the problem.

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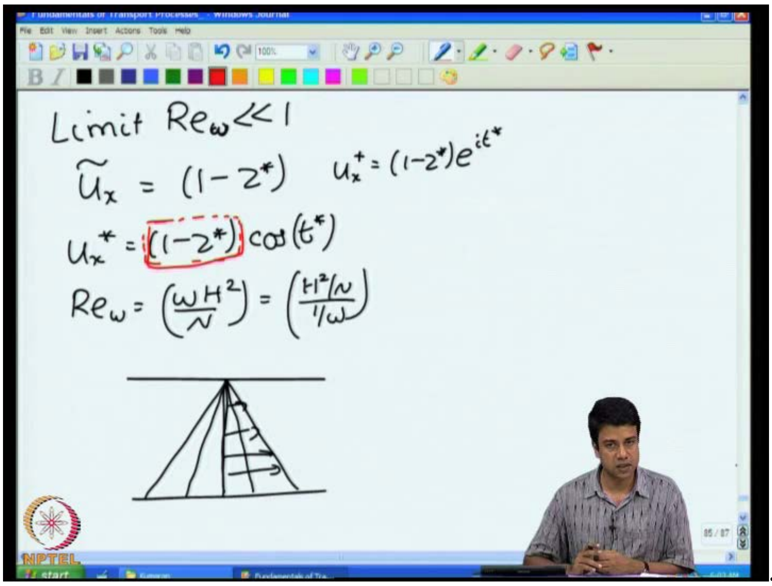

And we look looked at the physical meaning R e omega small compared to one. In that case you just kept the solution as the steady solution, one minus z times cos t.

If the steady velocity your u x is equal to 1 on the bottom surface the steady solution would have been 1 minus z. However, the solution is varying in time and. So, I am just getting 1 minus z times cos t. So, basically the solution in the limit R e omega small compared to 1; it is the same as the steady solution except that the velocity the bottom surface is the instantaneous velocity, a function of time at that particular instant whatever the velocity is. That is the maximum velocity at the bottom and you get a linear velocity profile. And we looked at physically why that is so. One can interpret the Reynolds's number R e omega, omega H square by nu as the ratio of H square by nu divided by 1 over omega. H square by nu has dimensions of time it is the time it takes approximately for momentum to diffuse over a length H. So, it is the time it takes for a momentum disturbance generated at the bottom to diffuse all the way to the top of the channel.

One over omega is the time period of the oscillation. So, when the Reynolds's number is small compared to 1, the time required for momentum diffusion is small compared to the period of the oscillation. So, whatever the velocity at the bottom plate the momentum diffuses instantaneously because a velocity is varying much slower than the time than the rate of diffusion of moment and because of that at every point in the oscillatory cycle the momentum field equilibrates to the steady value it would have had for that particular bottom velocity. So, in the case of R e omega small compared to 1, we recover the linear velocity profile except that the velocity at the bottom surface of that linear velocity profile is the instantaneous velocity of the bottom plate. That that is restricted to the case where the time required for momentum diffusion is small compared to the time required for the period of oscillation.

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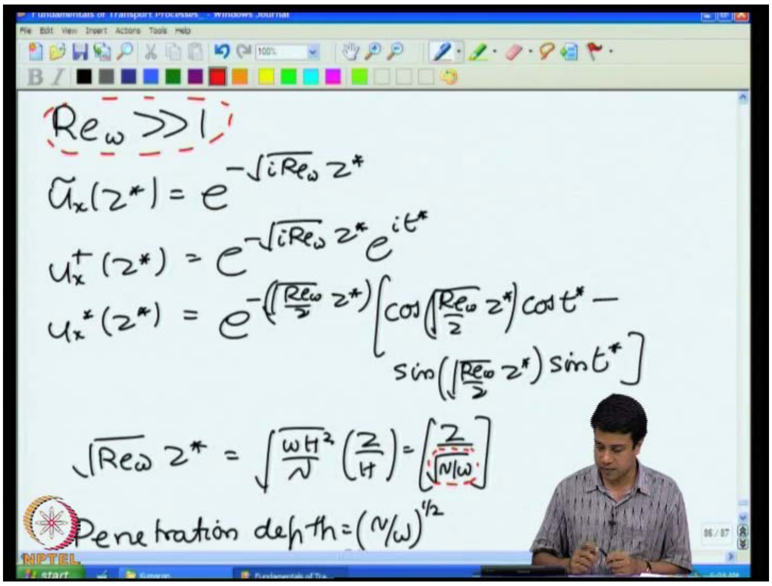

The opposite case when R e omega's large compared to 1; we saw that there is an exponential decay in the velocity field into the fluid. That decay takes place over a length scale which is approximately square root of nu by omega. Nu is the kinematic viscosity has dimensions of length square per time, omega is the frequency. So, square root of nu by omega is the time it takes, the square root of nu by omega is the distance to which the momentum diffuses within a time period of two pi by omega or within the time required for the oscillation.

So, this is the distance over which the diffusion takes place over the time period two pi by omega and the Reynolds number can also be interpreted as H by nu by omega power half the whole square which is the square of the ratio of H and the distance to which momentum diffuses. So, if Reynolds number is large; that means, H is large compared to the distance over which the momentum diffuses. It looks like momentum diffusion into an infinite fluid and because the distance H is large, the momentum diffuses only to a finite distance before the plate stops its forward motion and comes backward. So, over a time period comparable to two pi by omega the distance is the momentum diffuses is given by square root of nu by omega and that is the distance the penetration depth or the boundary layer thickness over which this velocity is finite.

And we got a solution also for that For that velocity field in the limit of R e omega large compared to one and this solution is not in phase. It has both sine and cosine functions in time. So, this solution is not exactly in phase with the driving frequency even though the frequency of oscillation of the velocity at every point is exactly the same.

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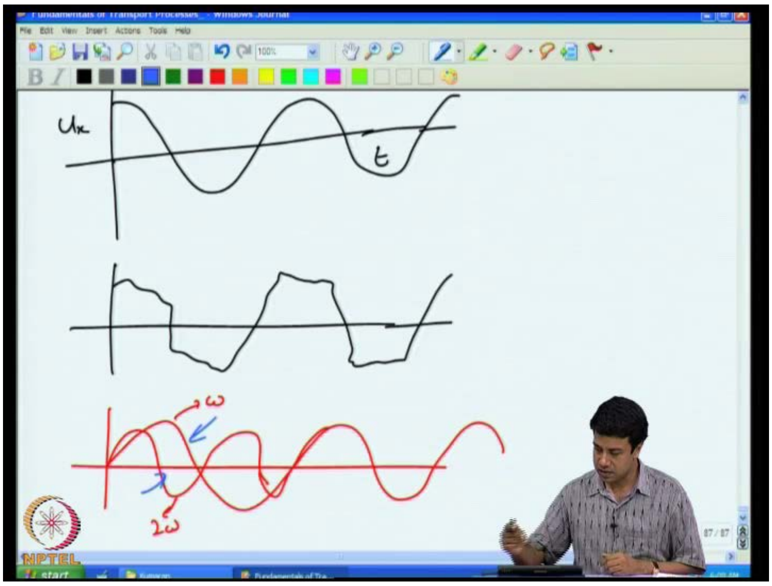

So, one last comment about the discussion on oscillatory flows; so, far we have solved only for u x at the bottom being oscillatory in time. Ah what we actually solved for was a cos function solution what we actually solved for was a cos function, sine function will be exactly displaced by pi by 2. But, what we solved for was a cos function. One could have an a forcing which is not exactly a sine function or a cos function. For example, one could have a forcing that took something like that, but so long as it is periodic in time with a frequency omega. So, long as it is periodic in time, the frequency omega this thing can always be written as the sum of cos and sine functions which are the lowest harmonic and its submultiples. There is the period, is the frequency is equal of this is omega this is two times omega three times omega and so on.

So, any function can be written as can be decomposed into its fundamental mode the frequency of the largest oscillation omega plus its entire harmonics and so, I can solve individually for each of these. I can solve individually for this function, this function and all other harmonics. Solve the equation get the solution here for each of these harmonics get the solution for each of these harmonics and then add them all up and I will get the solution for this more complicated form of the wave ok. That is because in Fourier transforms we know that any function that is periodic with the frequency omega can also be can always be decomposed into a fundamental mode plus all of its higher harmonics.

It is an infinite series, but, still the decomposition is always possible and because the decomposition is possible this procedure for an oscillatory flow can be applied to any wave form not just a sinusoidal wave form, but, to any wave form. So, in that sense it is more general. All I require is that there should be bottom plate velocity should be periodic in time with the frequency omega. Once that condition is satisfied does not matter what the wave form is I can always apply the solution procedure.

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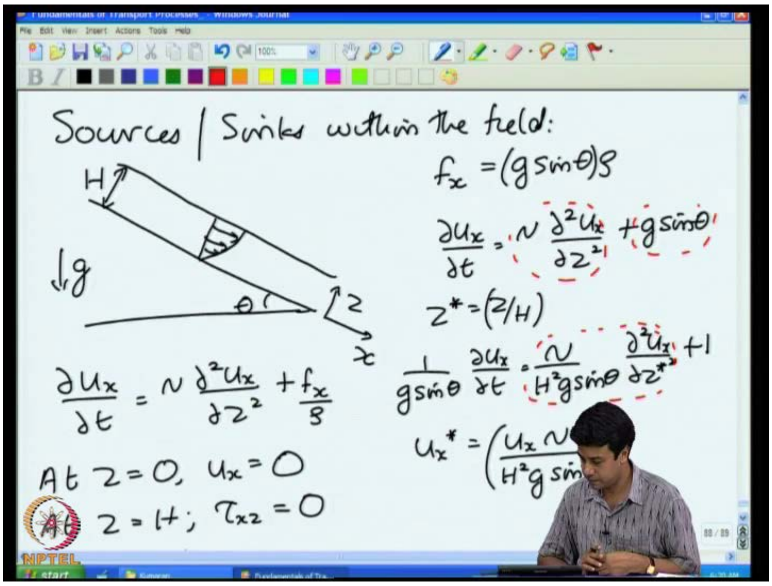

So, that completes our discussion on oscillatory flows. We will come back and see later on oscillatory pressure driven flows, but, before we go there I would like to first spend some time on what happens when there are sources or sinks within the field. So, far we always took two plates; one had one temperature the other had another temperature and there was no generation or destruction of mass momentum energy within the domain.

What about sources where there, what about cases where there are sources or maybe there are forces acting on the fluid within the domain? In that case how do us solve these problems? So, we will just look at a few brief examples of this. In the case of momentum transfer the simplest case that you can think of is the flow of a fluid layer down an inclined plane. So, I have an inclined plane, inclined at some angle theta to the horizontal and the acceleration due to gravity is acting downwards and I have a layer of fluid flowing down the plane. We will get a velocity profile that looks something like this.

So, I, as usual I take x as the flow direction and z as the direction in the perpendicular to the flow. Fluid velocity is in the x direction and the velocity is varying only in the z direction. There is no change in velocity as you go along the x coordinate. So, if I were to write from momentum balance equation for this I would have to write it of the form; d u x by d t is equal to nu times d square u x by d z square. However, there is also a force acting on this due to the gravitational field. There is a force acting on this along the x direction due to the gravitational field because it is inclined at a fixed angle to the horizontal.

Once again I will take the layer thickness as H and I would like to find out what is the velocity profile as a function of H. What are the boundary conditions for this case at the bottom surface there is a solid wall which is not moving? Therefore, I require that at z is equal to 0; u x is equal to 0. What about the top surface? At the top surface, the fluid layer is in contact with a gas or with vacuum. Even if it is in contact with a gas, the viscosity of the gas is much smaller than the viscosity of the liquid. Liquids usually have viscosities of two to three orders of magnitude higher than gases. Therefore, the viscosity of the gas is much smaller than the viscosity of the liquid.

The shear stress applied by the gas is much is negligible compared to any stress that would be exerted by the liquid. Therefore, at the free surface itself since the viscosity of the gas is small. The sheer stress exerted by the gas is small at the interface you require continuity of stress between the liquid and the gas. That means, that at z is equal to H you require that the sheer stress in the liquid go to 0 because the viscosity of the gas is small compared to the viscosity of the liquid. That means, if the shear stress that is exerted in the gas is small compared to the internal stresses within the liquid. So, as you approach the interface from the bottom, you require that the internal stresses in the liquid should go to zero. So, that they match with the stress in the gas.

Tau x z is equal to 0 implies that mu times d u x by d z is equal to 0 or the boundary condition is d u x by d z is equal to 0. So, the derivative of the velocity with respect to z has to be 0 at the top surface. We should put in here force per unit volume. So, what is the force acting in the horizontal direction? This is as I said a force per unit volume f x is a force per unit volume acting along the x direction. The component of the gravitational acceleration is g times sine theta along the x direction. The component of the gravitational acceleration along the x direction is equal to g times sine theta because theta is the angle of the inclined plane with respect to the horizontal. Therefore, the force per unit volume acting on every volume of fluid has to be g times sine theta times the density.

So, this is the force acting along the x direction on every differential volume of the fluid. So, therefore, if I put this into the governing equation I get d u x by d t is equal to nu times d square u x by d z square plus g sine theta. So, that is my equation for the velocity field. How do I non-dimensionalize my variables? Once again H is the natural length scale in the z direction. So, I can define z star is equal to z by H. What about u x? In the previous examples there was a well defined velocity of the boundary. I had the boundary condition u x is equal to capital u at z is equal to 0 and u x is equal to 0 and z is equal to 1. In this case the boundary conditions are both homogenous. There is no velocity at the boundary u x is equal to 0 on one surface and the derivate is 0 on the other surface. So, there is no velocity scale emerging from the boundaries.

However there is a velocity scale that emerges from the fact that there's a body force acting on the fluid. So, therefore, what I can do is to divide throughout by g sine theta and use that to scale the velocity. So, if I divide throughout by g sine theta, I will get one by g sine theta partial u x by partial t is equal to mu by H square g sine theta partial square u x by partial z star square plus one. Now, this thing gives us a clue about how we should scale the velocity because if I define u x star is equal to u x times nu by H square g sine theta.

If I define it this way then that entire term becomes dimensionless. One is dimensionless anyway. So, this gives us a way to scale the velocity and you can easily see that it is dimensionally consistent. On the top we have velocity which is length per unit time, kinematic viscosity is length square per unit time. So, you get length cubed by time square. On the bottom H square length square and g is length per unit time square. So, length cube by time square. So, it is dimensionally consistent.

So, therefore, in this case the velocity scale is coming from the forcing within the fluid not from the boundaries. So, if I put this in then I will get an equation of the form partial u x star by partial t star is equal to partial square u x star by partial z star square plus 1 where z star is equal to z by H and u x star is equal to u x nu by H square g sine theta and you will find that t star is the same that I had earlier t star will be equal to t nu by H square. So, the time scale is once again scaled by the time. It takes for momentum to diffuse across the entire channel the distance is scaled by H itself and u x star has a scaling which is given by a balance between the body forces. So, the scaling for u x star that we got was by taking a balance between the body forces which are here and the viscous stresses over a layer of thickness each.

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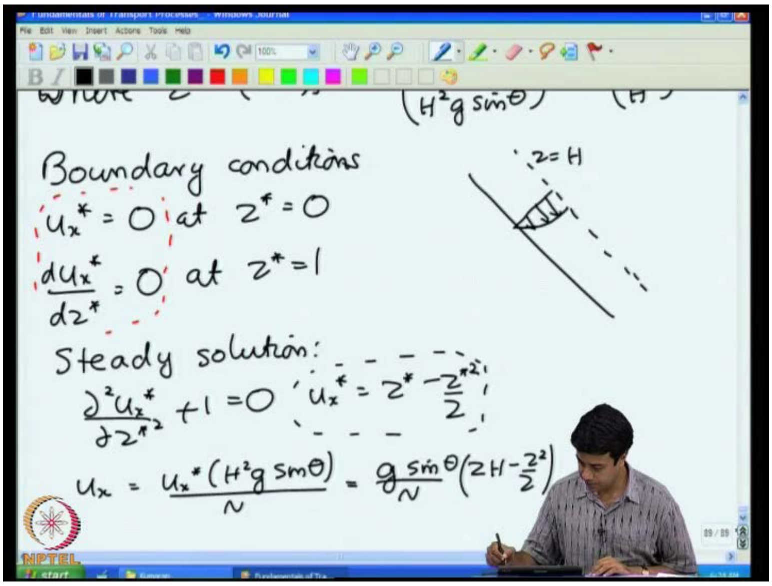

So, this has given us the scaling for u x star. So, what are the boundary conditions? u x star i equal to 0 at z star is equal to 0 at z star is equal to 0 and d u x by d z is equal to 0 at say this star is equal to 1. So, these are the boundary conditions for the flow and let us solve this differential equation subject to boundary conditions. Note that the boundary conditions are homogeneous. The boundary conditions do not contain any inhomogeneous term in them. So, far whenever we have solved diffusion in the channel, the boundary conditions have contained an inhomogeneous term. There was a velocity u at the boundary there was a temperature t star is equal to 1 at the boundary or constasion c star is equal to 1 at the boundary. In this case the boundary conditions do not contain any homogenous terms. The driving of it comes from the source within the equation itself. In case the driving comes from the existence of a source within the equation itself.

So, first thing steady solution; in the absence of a time dependence the equation becomes d square u x by d z square plus 1 is equal to 0 and the solution is quite easy to get. The solution for this is u x is equal to z minus z square by 2. It is easy to verify that this solution z minus H square by 2 satisfies both of these boundary conditions at z is equal to 0 u x is equal 0 and at z is equal to 1 the derivative is equal to 0. So, this satisfies both boundary conditions.

So, this is the final solution for the velocity profile at steady state. I express this back in terms of the dimensional velocity u x is equal to u x star into H square g sin theta by mu and this will be equal to g sin theta by mu into z H minus z square by 2. So, this is a parabolic velocity profile. So, velocity profile for the flow down in an inclined plane this quadratic in the velocity z it starts out with 0 then you get a parabolic velocity profile and at z is equal H the derivative of the velocity with respect to z is equal 0. The sheer stress is 0 and therefore, it satisfies the stress balance condition between the fluid and the gas that is in contact. So, this is the velocity profile for the flow down inclined plane.

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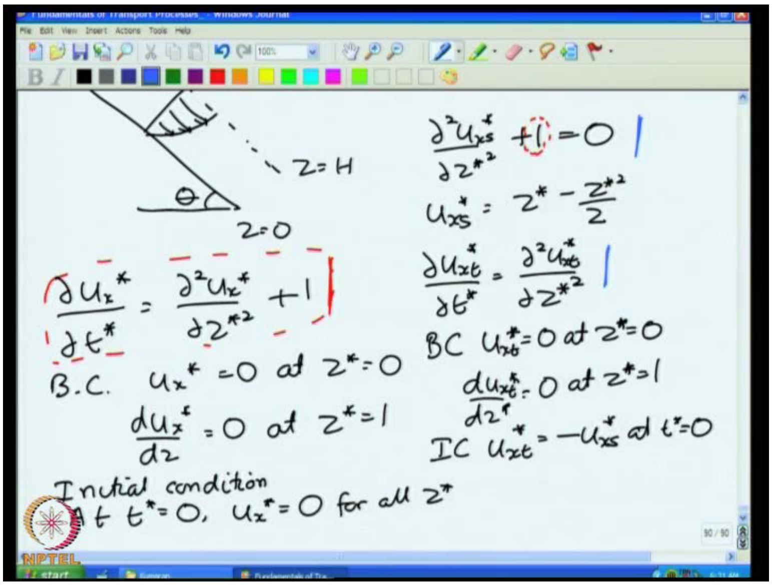

Now, this is the steady solution. Now what about the n steady solution? So, we can consider for example, get the flow down inclined plane. This is the final steady solution and the solution satisfies the boundary conditions at z is equal to 0 and z is equal to H. So, the equation is d square sorry d u x by d t is equal to d square u x by d z square plus 1 in non dimensional terms. The boundary conditions are u x is equal to 0 at z equal to 0 and the stress is 0 at z is equal to 1.

Now, we want to solve for the unsteady case. So, the idea is as follows: initially the film is horizontal. So, that the velocity is identically equal to 0. At time is equal to 0 you tilt the film to this angle you tilt to an angle theta and the fluid starts to flow. The flow is still fully developed in the sense that there is no variation in the x direction. However, the flow does increase with time to reach the final steady sate. So, because of that you require to find out what is the profile at intermediate times as it is evolving to the final steady state. Similar to the problem that we solved for the instantaneous start up of a plate at using separation of variables.

So, this is also an initial condition which is that at t is equal to 0 u x star is equal 0 for all z star. At time t is equal to 0 the velocity is 0 everywhere. So, we have to solve this equation subject to the initial length boundary conditions. The solution procedure is similar to the separation of variables solution procedure that we had earlier. So, we have to follow the same steps in order to get solution for this as well. There is a complication when you have body forces and it is in order to point that out I am going through this solution procedure.

So, as in the previous case I will separate u x into a steady part plus a u transient part. So, I separate the velocity profile into a steady part plus a transient part. The steady part I already know what the solution is. The steady part satisfies the differential equation d square u x by d z square plus one is equal to 0. The steady part satisfies this equation and I got the steady solution is equal to z minus z square by 2. So, we already got the steady solution.

Now, the total differential equation satisfies this equation satisfies this equation and the steady part satisfies this equation. Subtracting the two, I can find out what is the equation for the transient part. So, from the equation for the total velocity profile, I subtract out the equation for the steady velocity profile and note that d u x s by d t is equal to 0 because it is steady. It does not depend upon time and then I will get an equation for the transient part alone, d u x t by d t is equal to d square u x t by d z square. So, that is the equation for the transient part alone. Note that the equation for the transient part does not contain an in homogenous term similar to this form. The equation to the transient part does not contain this in homogeneous term.

Once I have the equation I also should impose the boundary conditions. For the total velocity field I have u x is equal to 0 at z is equal to 0, derivate is 0 at z is equal to 1. For the steady part I have exactly the same boundary conditions. u x is equal to 0 at z is equal to 0 and u x is the derivative is 0 at z is equal to 1. That means, that the equation for the transient part will also have boundary conditions and d u x by d z. So, it has the same boundary conditions as the steady part and the total velocity profile. However, the initial condition is different. Initial condition for the total velocity field is u x is equal to 0 for all z at t is equal to 0.

The steady part of course, is independent of time. It is the same at all times. Therefore, the initial condition for the transient part has to be u x transient is equal to minus u x steady at t equal to 0. Only if u x transient is equal to minus u x steady, the sum of the two will be equal to 0 which means that the total velocity u x will be equal to zero. So, one has to careful while defining the initial length of boundary conditions. In the equation itself when I subtract out the equation for the steady part from the total equation since both of them contain the in homogeneous term, the equation for the transient part does not contain any in homogeneous term. So, this equation for the transient part does not contain any in homogeneous term. The boundary conditions were homogeneous for both cases. But, we have to be careful about the boundary condition for the transient I am I am sorry you have to be careful about the initial condition for the transient part because it will contain a contribution due to the initial condition for due at time it is equal to 0.

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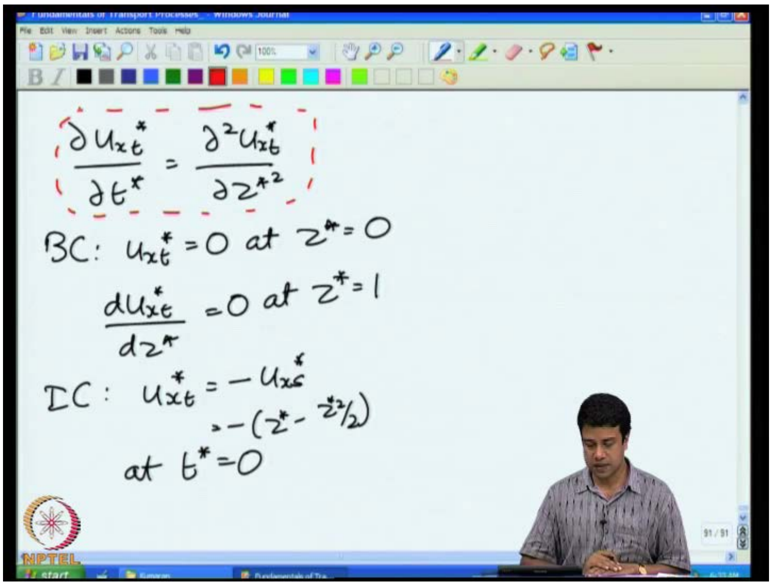

So, to summarize the equation for the transient part is d u x transient by d t is equal to d square u x transient by d z square. The boundary conditions are u x transient is equal to 0 at z is equal to 0 and u x transient, the derivative 0 at z star is equal to 1 and the initial condition was u x transient is equal to minus u x steady is equal to minus of z minus z square by 2 at t equal to 0. So, we have to solve this equation with these initial and boundary conditions. The method is exactly the same, separation of variables. Note that now I have an equation which is entirely homogenous. That is essential for the separation of variables procedure as I will show you.

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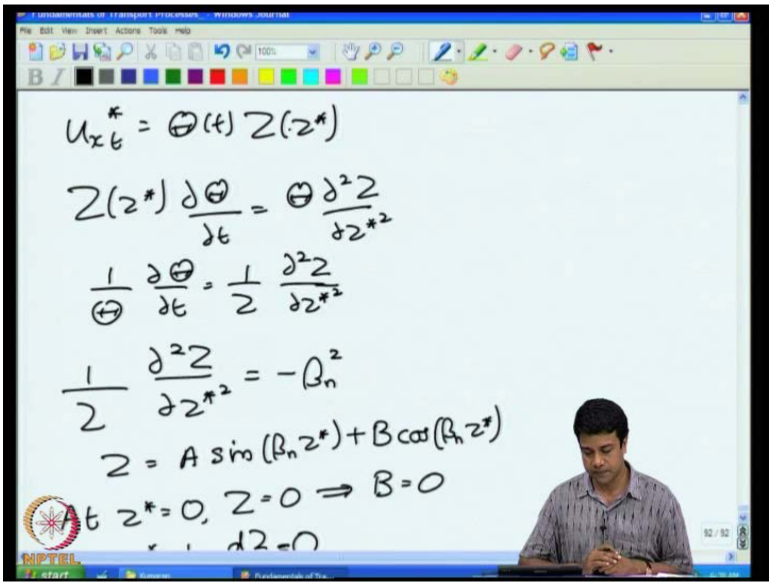

If I have, if I substitute u x transient is equal to theta of t times z of z star into the differential equation; then I get partial theta is equal to theta times t square z by d z star square and if I throughout by theta times z, I get one by theta partial theta by partial t is equal to one by z partial square z by partial z star square.

Once I have this, the left hand side is only a function of time the right hand side only a function of z and I can effect a, I can set both of those equal to constants. If on the other hand I had an in homogeneous term here, which would look something like plus one in the present case. In this case I would have got something that goes as one over theta times z and then I could not have done a separation of variables because the right hand side also contains the function of time. So, it is essential when we do separation of variables that the equation be homogeneous. It should not contain any in homogeneous terms and you reduce it to homogenous equation by subtracting out the steady part which already has the in homogeneous term in it. Because we do not have these terms, we are able to do a separation of variables solution. So, it is a essential, if you recall when we did the flow between two flat plates we had in homogeneous boundary conditions u z was equal to 0 at z equal to 1 I am sorry t was equal to 0 at z is equal to 1 1 at z is equal to 0. However, when we subtracted out the steady part for the transient part alone we had temperatures equal to 0 on both surfaces and a similar reduction and and because of that we got homogenous boundary conditions on both surfaces. A similar reduction has to be done when there are in homogeneous terms in the equation as well due to sources, sinks, body forces and so on.

Once that is done then I can set both of these equal to constants. What constant should they be? Once again one by z d square z by d z star square if it were a positive constant I would get exponentially increasing and decreasing functions and it would not satisfy boundary conditions. If it were negative constant I would get sinusoidal solutions and then there is a chance to satisfy boundary conditions if I chose the constant appropriately.

So, this I can write it as a negative constant beta n square. That means, that z is equal to a sin beta n z star plus b cos beta n z star and then I have the boundary conditions on the velocity at z is equal to 0 capital z is equal to 0 which means that b is equal 0. At z is equal to 1, I have at z is equal to 1 I have the boundary condition d u x by d t is equal to 0 I am sorry I have d u x transient by d z is equal to 0. Since all of the dependence on z is contained within the function capital z itself if I want to satisfy the boundary condition; I need to a ensure that d z by d z star is equal to 0 at z equal to 1. Because the derivative of velocity with respect to the z coordinate has to go to 0. Velocity is the product of two functions; one is the function of time, the other is a function of z. When I take the derivative of velocity with respect to z. This first theta is independent of z and therefore, I just get d z by d z star and if d u x by d z star has to be 0. That means, d f capital z with respect to d z star has to be equal to 0 and this implies that beta n has to be the slope of the sin function is equal to 0 sin beta and z star d by d z star of that is equal to 0. That means that cos beta and z star has to be equal to 0. That means, that beta n has to be equal to half 3 by 2 5 by 2 etc to infinity. Only f beta n is these values will you have the slope of the sine function being 0 at z is equal to 1. I can write this compactly as n plus 1 2 n plus 1 by 2. So, for n is equal to 0 its half for n is equal to 1 its 3 by 2 n is equal to 2 its 5 by 2 and so on.

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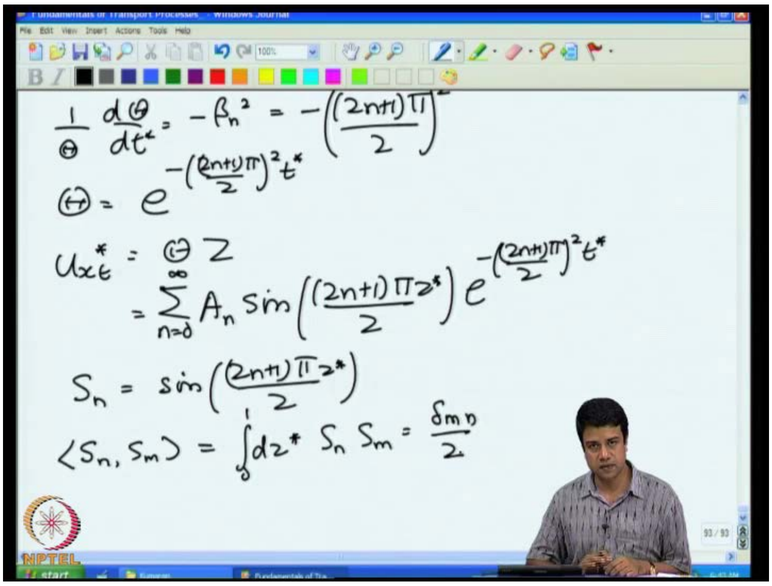

Therefore, my solution for the function z is a sin of 2 n plus 1 pi z by2. I made a mistake in the previous one; this should be pi by 2. So, that is the equation for capital z and I also have one by theta d theta by d t is equal to minus beta n square is equal to minus of 2 n plus 1 pi by 2 whole square. Solving this I will get the exponential function theta is equal to e power minus 2 n plus 1 pi by 2 the whole square times t.

So, that is the final solution. u x transient will be equal to theta times z is equal to a times sin whole square t star. So, this is the solution. This satisfies the equation for any value of z. So, the most general solution is a summation n is equal to 0 to infinity of A n times this; a linear combination of all of the solutions. How do we determine the coefficients A n? Using the orthogonality relations once again.

So, for this particular problem if you recall for the flow in a channel the basis functions were sin of n pi z by n pi times z star whereas, here the basis functions are two n plus one pi by z star by 2. So, I define S n is equal to sin of 2 n plus 1 pi z star by 2 and the orthogonality function the inner product S n comma s m is equal to integral 0 to 1 d z star S n times s m which shows once again equal to delta m n by 2. You can work it out for this function as well. It gives you the same identical result as that of sin of n pi z.

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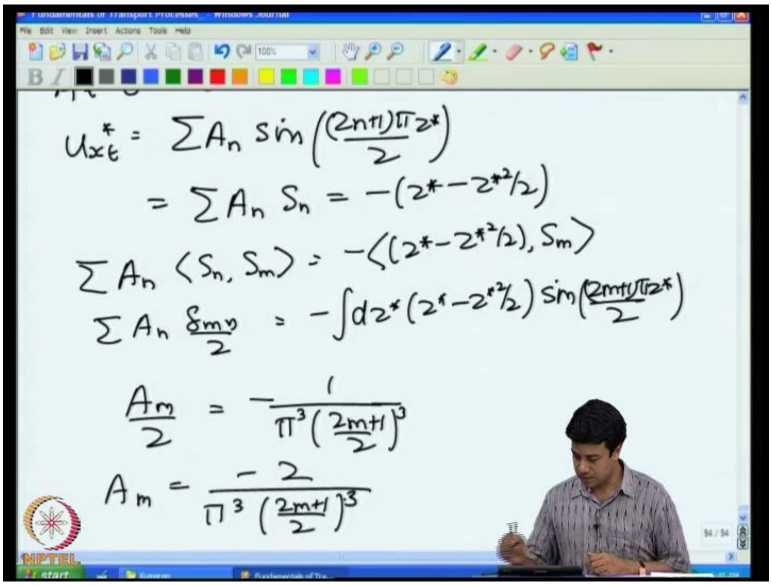

So, now I to use the orthogonality conditions at the initial time t is equal to 0. At t star is equal to 0 like u x t is equal to summation A n sin of 2 n plus 1 pi z by 2 which is equal to summation of A n times S n. That is equal to minus of the steady solution which was minus of z minus z square by 2. So, minus is the steady solution. So, how do I find out the coefficients and I take the inner product of both the left and the right hand side with respect to S n. So, summation of A n; S n , s m is equal to inner product of z star minus z square by 2 comma s m. This is equal to summation A n into delta m n by 2. So, it is minus integral d z of z star minus z star square by 2 sin of two m plus 1 pi z by 2.

So, this is A m by two and you can work out this integral quite easily to get a solution which is minus 1 by pi cubed into 2 m plus 1 by 2 whole cubed and therefore, A m is equal to minus 2 by pi cubed into 2 m plus 1 by 2 whole cubed.

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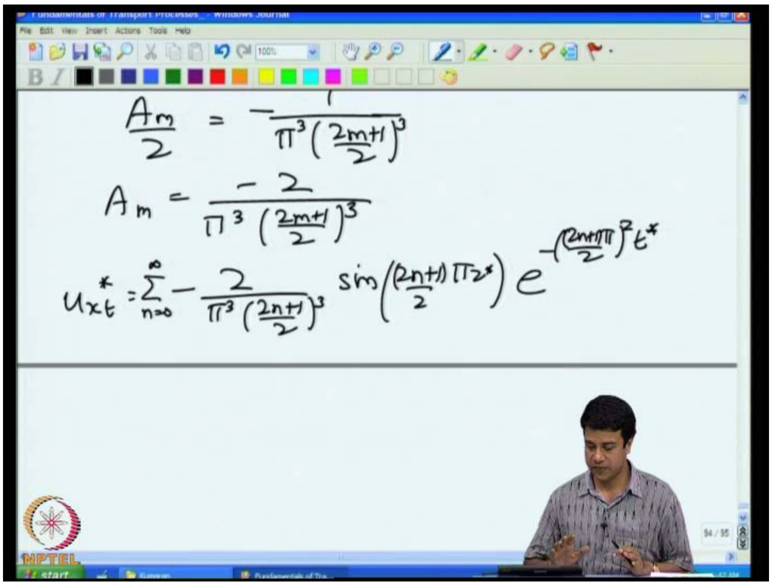

So, that is the final solution. u x transient is equal to minus x summation n is equal to 0 to infinity minus 2 by pi cubed sin into the exponential part 2 t.

So, this solution has all of the features of previous solution by separation of variables. You have a series of harmonics complete basis set and the higher order terms in the series decay exponentially in time and as n increases, the rate of decrease exponentially in time is faster. So, these are the same characteristics as the solution for the flow between flat plates except that the basis functions are not slightly different. The previous case we required the basic functions to be 0 on both boundaries. So, it has sin and pi z the present case we require the basis functions to be 0 on one boundary and the slope to be 0 on the other boundary. That is the only difference and because of that the basis functions were sine of 2 n plus 1 pi z by 2 and they have an exponential decay.

The decay rate will be approximately the same in both the cases because two n plus one by two for large n is approximately equal to n. So, this gives you a flavor of how one solves a separation of variables for this particular case. When we did the separation of variables for the flow between flat plates, we have to ensure that the velocity on both surfaces for the transient part was 0. In this case we have to ensure that there was no in homogeneous term of equation of motion. So, that is the only difference.

So, this is second example of separation of variables that I have showed you in order to solve the transient problem. So, this is an example of momentum transfer were there are sources and sinks. Next class we will look at a couple of problems I heat and mass transfer were there are sources and sinks. After that we will go into looking at other configurations such as cylindrical coordinates and so on. So, we will continue unidirectional flow in the next class with other examples of flows in with sources and sinks and then we will proceed later on to looking at cylindrical and spherical coordinate systems.

So, that completes the separation of variables part of this lecture. We will see it again when we do cylindrical coordinates, but, as far as Cartesian coordinates is concerned; this completes the various solution procedures that we have used. Similarity variables, separation of variables and oscillatory flows; three techniques that I have thought you for Cartesian coordinates we will see them again in other coordinate systems. But, before that we will solve a couple of problems whether our sources and sinks within the flow for heat and mass transfer. So, we will see you in the next lecture and we will continue unidirectional flows there.

Thank you.