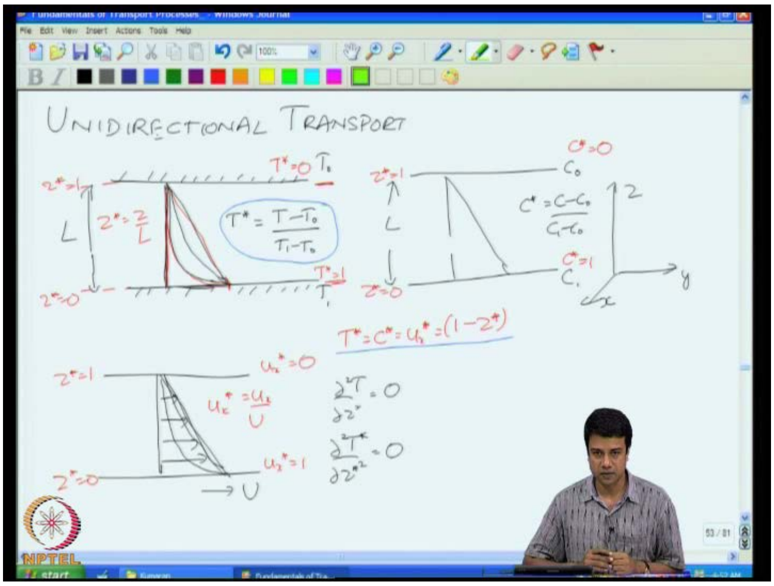
Fundamentals of Transport Processes
Prof. Kumaran
Department of Chemical Engineering
Indian Institute of Science, Bangalore

Module No. # 03
Lecture No. # 13
Unidirectional Transport Cartesian Co-ordinates-VI (Oscillatory Flows)

Welcome to this the thirteenth lecture in the series on the fundamentals of transport processes. I will just briefly go through where we are now and how we are going to proceed. So, far we have been discussing unidirectional transport of mass heat and momentum. Unidirectional transport implies that there is transport only in one particular direction. It could be unsteady, it could be a function of time, but, spatially there is transport only in one particular direction.

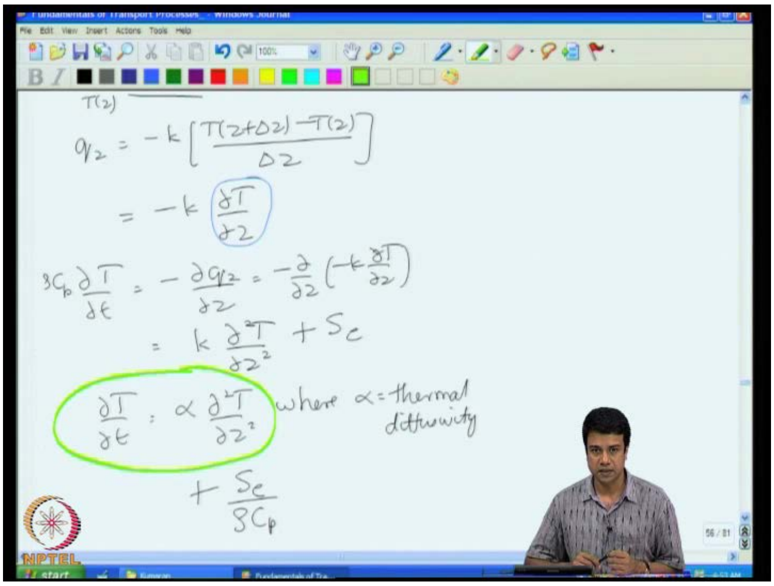
(Refer Slide Time: 00:51)



So, for example, the transport between two flat plates in the case of heat transfer problem there are two plates kept at two different temperatures T 1 and T 2 and there is transport of heat between these two. The case of concentration two plates at two different concentrations of solute c 1 and c 2 and then there is transport between these two due to the difference in concentration. The fluxes in the case of heat transfer are given by

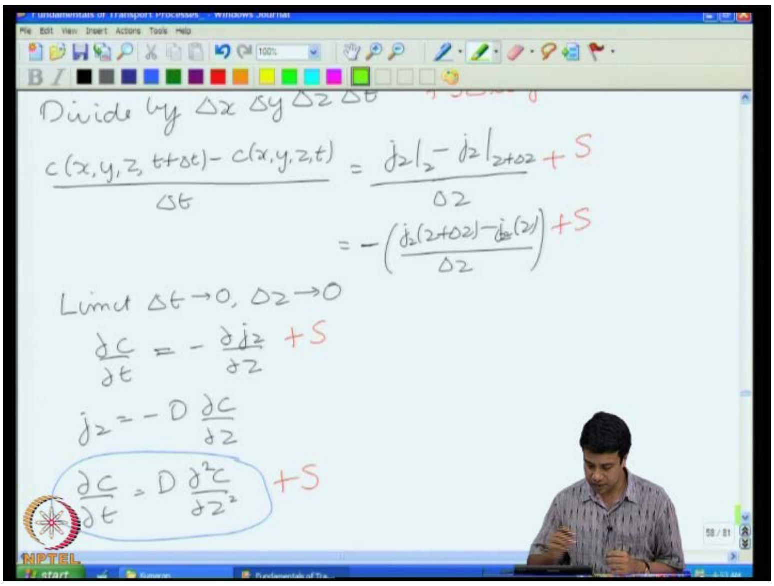
Fourier's law. In the case of mass transfer is given by Fick's law. Momentum transfer, two plates one moving the other stationary. In this case the shear stress is related to the gradient in the velocity by Newton's law of viscosity and of course, there could be sources or sinks of heat mass and momentum within the domain. A source of heat if there is a reaction exothermic or endothermic, if there is a physical transformation process which either takes in or gives out heat; in the case of mass there could be a source due to reactions. If the constituent whose concentration is being described here is a product then there is a source of mass. If it is a reactant there is a sink of mass. We will see in the case of momentum transfer, a source refers to a body force force acting per unit volume within the fluid.

(Refer Slide Time: 02:44)



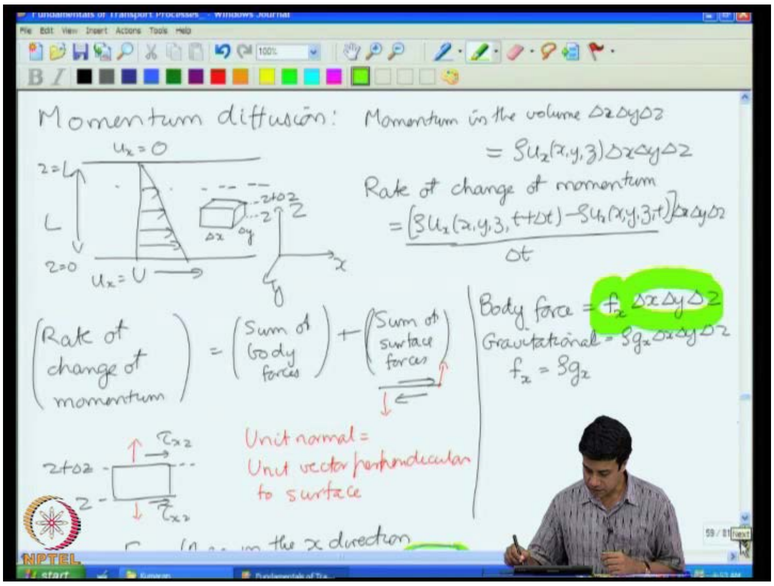
And an all these three cases we get remarkably similar equations for heat mass and momentum. So, for example, in the case of heat transfer we get an equation of this form; the time variations of temperature is equal to the diffusivity times the second derivative with respect to the z coordinate. The z coordinate is the coordinate in which there is variation of temperature plus any source of heat that is there.

(Refer Slide Time: 03:10)



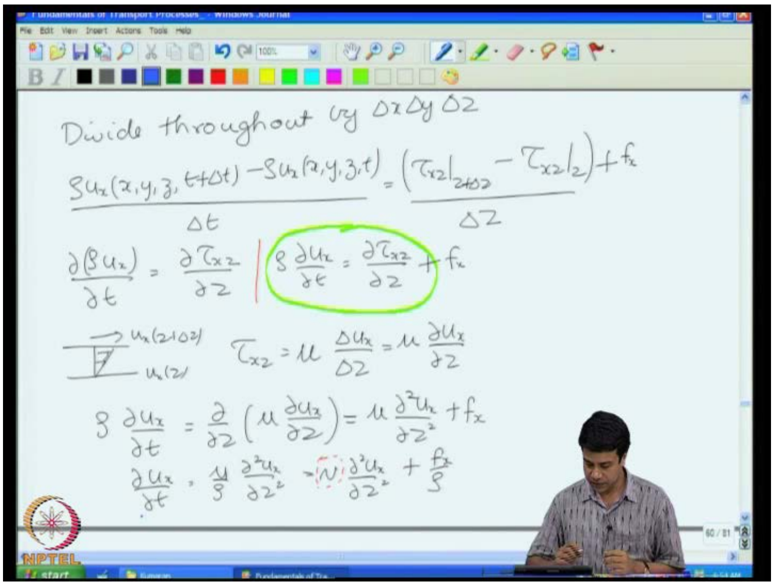
In the case of mass transfer we get an equation that looks exactly the same except that the concentration is substituted for the temperature and the mass diffusion coefficient is substituted for the thermal diffusion coefficient.

(Refer Slide Time: 03:26)



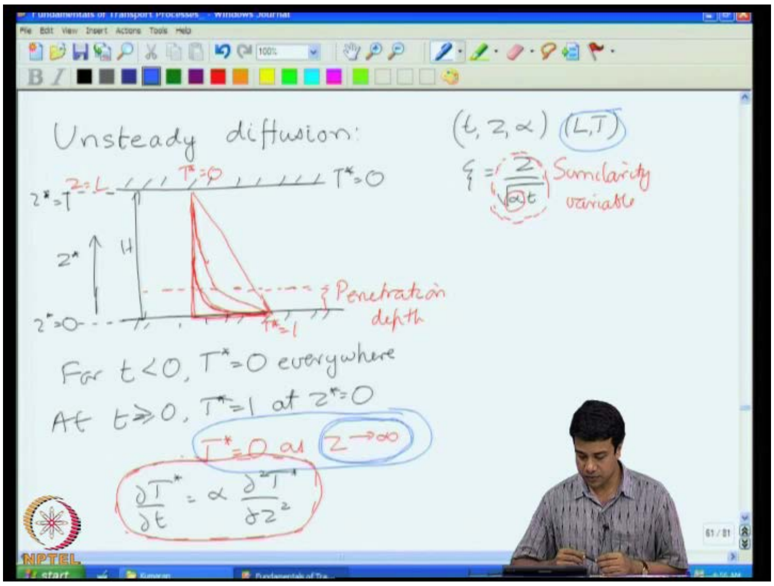
In the case of momentum transport; the route that we followed to get the momentum transport equation was a little different from heat and mass transport. In the case of momentum transport the fundamental equation that we used was that the rate of change of momentum is equal to the sum of the applied forces. Forces could be of two types; one is surface forces due to deformation and the other is body forces. However, despite the small difference in the way that we derived these equations the final equation that you get looks remarkably the same.

(Refer Slide Time: 04:06)



The final equation looks of this form d u x by d T is equal to new times d square u x by d z square plus anybody force divided by density. Note that we have written down the momentum conservation equation only for the x component of velocity. Therefore, in this case we are considering only the x component of the momentum. How do we treat momentum as a vector? That we will deal with later on. For now we only deal with one particular component of the momentum; flows only in the x direction there is a variation of the velocity only in the z direction and the momentum conservation equation is identical to the mass and heat conservation equations except that the velocity u x is substituted for c and T. The kinematic viscosity nu or the momentum diffusivity is substituted for them mass in the thermal diffusivity and then we have a source term which is basically the body force divided by the density.

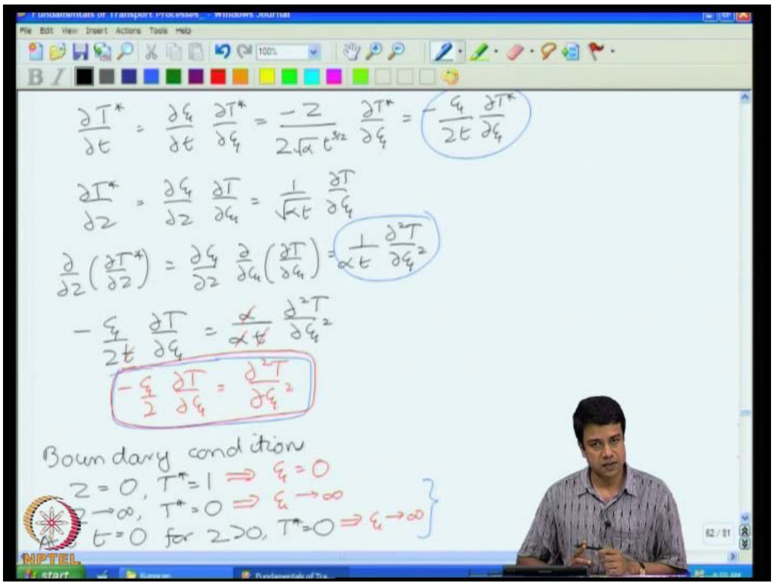
(Refer Slide Time: 05:06).



Now, we use this. So, far to solve two different types of problems; the first was an unsteady diffusion problem when written for heat transfer the situation was as follows. I have two plates; the temperature of both of these plates is equal to 0. Even if it is not 0 as I told you I can always define a scaled temperature which is the temperature minus the temperature of the plates which can be defined to be 0 throughout the domain. At the initial time, at time T is equal to 0; I increase the temperature of the bottom plate instantaneously by heating it to another temperature which is T star is equal to one, the scaled temperature is equal to one. We discussed how one can define the scaled temperature in such a way that it varies between 0 and 1. So, this was the situation. Temperature everywhere is 0. At time T is equal to 0 the temperature of the bottom plate is switched to one and then we want to see how the temperature evolves in time. The fundamental equation was the thermal diffusion equation and in this equation we neglected the source and sink terms because there are no sources and sinks in this configuration and secondly, we would expect that initially only the bottom surface is at temperature one at time T is equal to 0. Everywhere else the temperature is equal to 0.

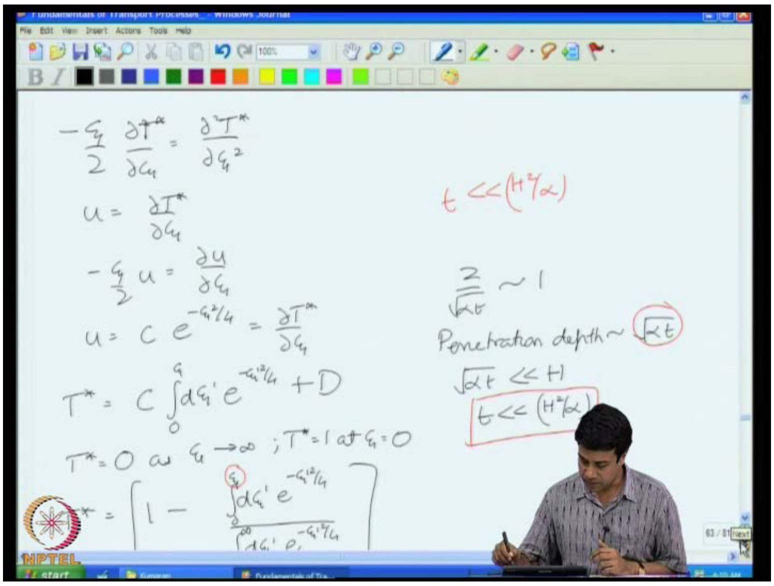
As time progresses the effect of heating from the bottom plate penetrates through the fluid until in the longtime limit you recover the linear temperature profile between the two plates. The first problem we considered was one where this penetration depth for the temperature is small compared to the height h. So, that the boundary condition T star is equal to 0. At z is equal to 1 z is equal to H T star is equal to 0, can effectively be written as T star is equal to 0 as z goes to infinity because the top surface is far off that effectively the temperature distribution is not affected by the location of that top surface. So, if we said the boundary condition T star is equal to 0 as z goes to infinity then, we tried to obtain a scaled coordinate. If the penetration depth was comparable to the height H we would simply have scaled the z coordinate by H itself. However, the penetration depth is small. So, H cannot be a factor in determining the temperature profile and it cannot be used for scaling the z coordinate. So, we had three dimensional variables; time, z and the momentum the thermal diffusibility alpha. These were the only three that were left over and they contained two dimensions length and time. On that basis you can define only one dimensionless group and that group we had defined as z by root alpha T. That is a similarity variable psi and just dimensional analysis was telling us that that temperature should depend only upon this parameter. It should not depend independently on z alpha and T. But, it should depend on those only through this particular combination. That is what dimensional analysis told us.

(Refer Slide Time: 09:01)



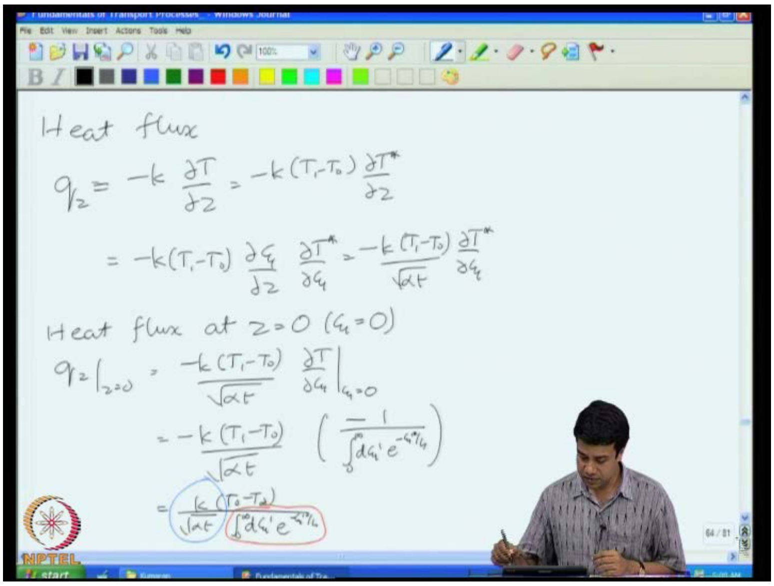
So, we had converted the equation from independent variable z and T to the independent similarity variable psi. And sure enough we ended up with an equation which was an equation only in terms of psi itself. The initial equation was second order in space, first order time. So, you need two boundary conditions, one initial condition. When we re-expressed in terms of psi we found that one of those boundary conditions at z going to infinity was identical to the initial condition at T is equal to 0.

(Refer Slide Time: 09:32)



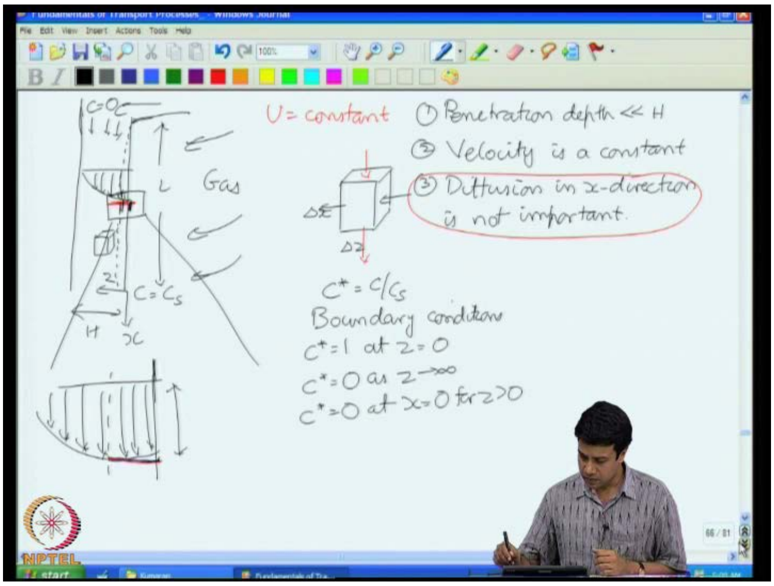
And on that basis we were able to obtain a solution for the temperature failed in terms of the variable psi alone and this this is a function that decreases as we go up. It is it is an error function and this provides you universal temperature profile as a function of z by square root of alpha T rather than in terms of z and T separately. So, this was similarity reduction that we used and this is valid when the penetration depth is small compared to the height H, penetration depth we found out was square root of alpha T. Therefore, this is valid only at vary early times when T is small compared to H square by alpha. So, in that case you can use the similarity solution.

(Refer Slide Time: 10:18)



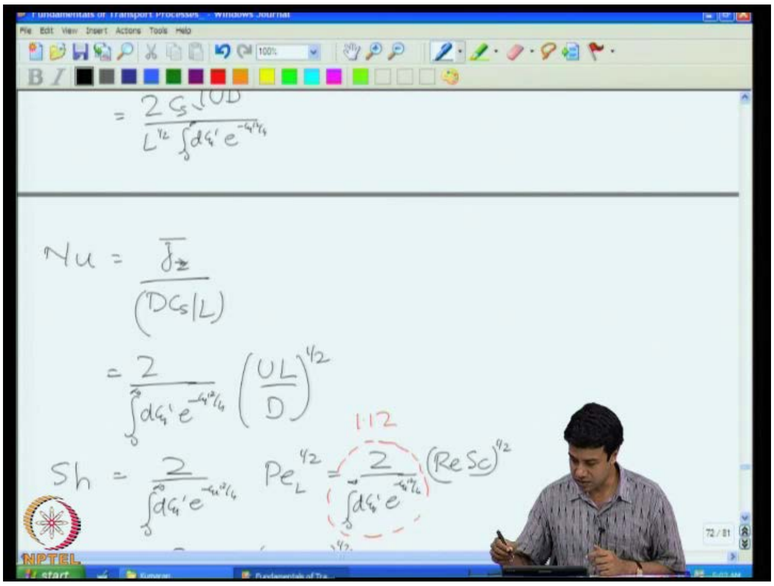
And the similarity solution basically reduces the partial differential equation to an ordinary differential equation on the basis of dimensional analysis. You can calculate the flux and the flux decreases at T power minus half as time progresses.

(Refer Slide Time: 10:36)



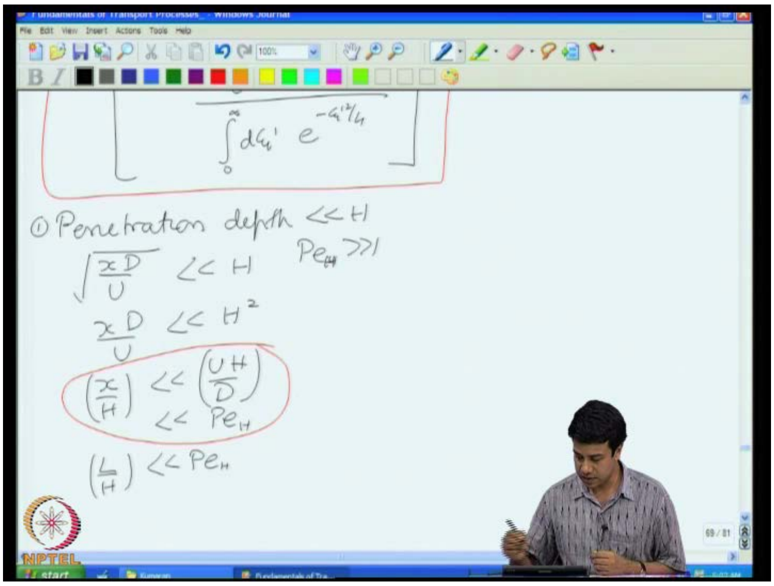
We look at another example where we can use the similarity solution even though it is not a time dependent flow, even though it is a steady flow. That was the absorption into a falling film. Even though this is a steady flow when I write out the equation after I do the shell balance, the equation has exactly the same form as you can see in the red here at the bottom left. The equation has exactly the same form as the unsteady balance equation and so, I could just define a similarity variable where T was replaced by x by u where u was the velocity itself. And from this we got our, the first of all Nusselt number correlations for the diffusion into a falling film.

(Refer Slide Time: 11:39)



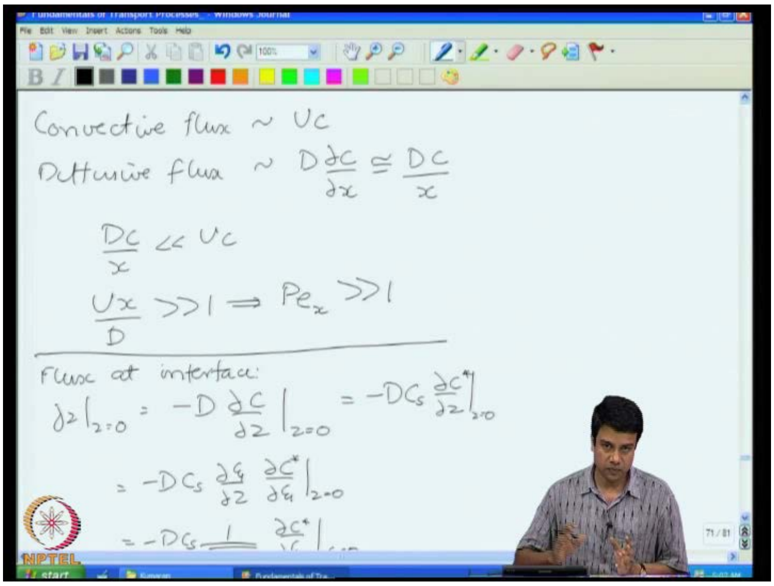
We calculated the flux and from that we manage to get correlation which was for the Nusselt number or sherwood number in terms of the peclet number. So, this correlation was obtained on the basis of dimensional analysis. It is valid only in restricted cases. There were conditions on this we made assumptions.

(Refer Slide Time: 11:50)



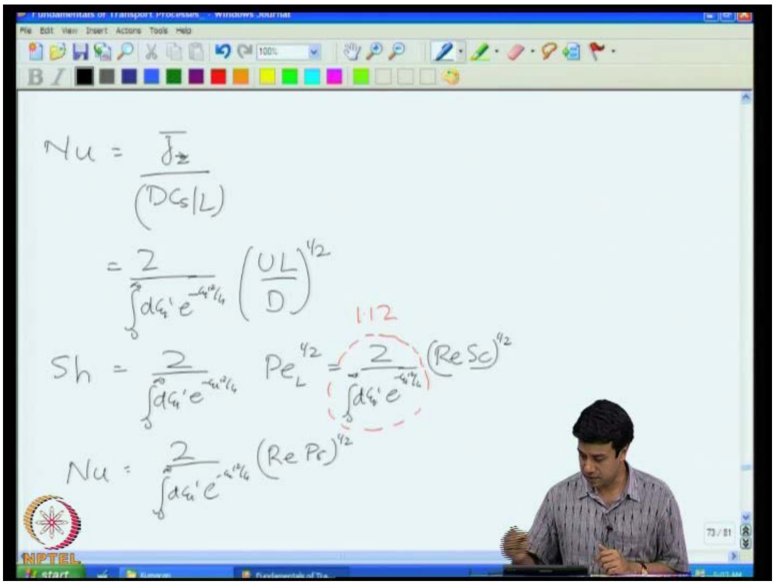
When we derive these equations the first assumption was that the penetration depth is small compared to the height H and that is when one by H the length of the film divided by the thickness is small compared to a peclet number based upon H.

(Refer Slide Time: 12:04)



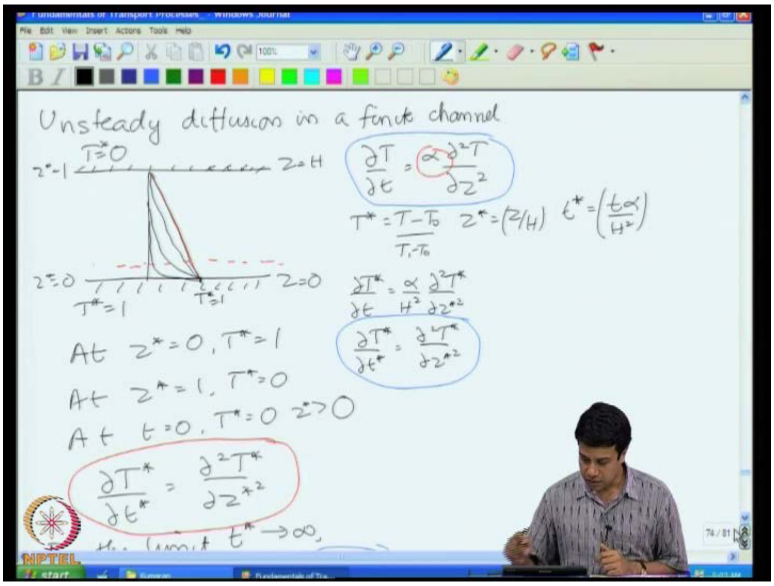
Second assumption we made was that the velocity is nearly a constant and that gave the same identical condition that one by H has to be small compared to the peclet number and the third one was that we neglected diffusion in the stream wise direction and that is when the peclet number is large compared to one based upon downstream distance x. So, that is a summary of how we derived correlation for the average flux for mass transfer into a falling film of liquid.

(Refer Slide Time: 12:37)



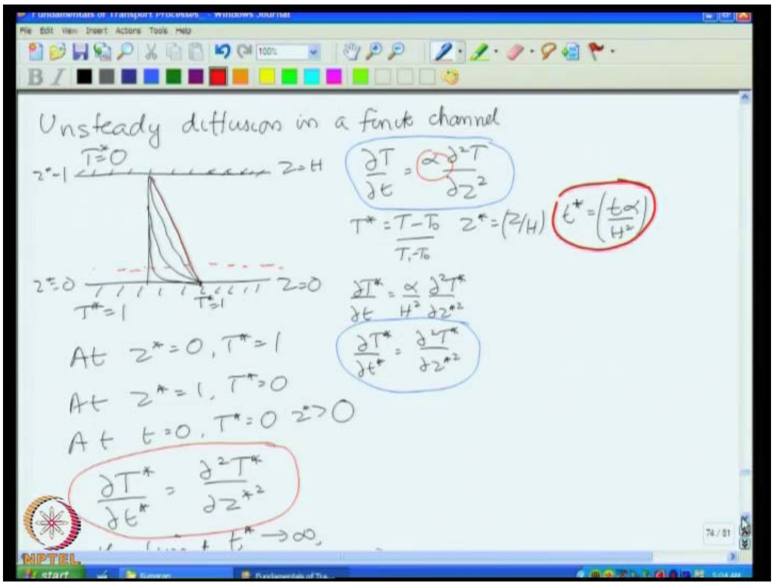
And this was once again based on similarity solution. But, not on dimensional analysis we just used the fact that the equations for the unsteady flow and for this situation are exactly the same, boundary conditions are the same. So, the solution is the same. That works for any linear problem if the if the problem is linear in the concentration field and you have well specified boundary conditions there exists solution and that solution is unique.

(Refer Slide Time: 13:07)



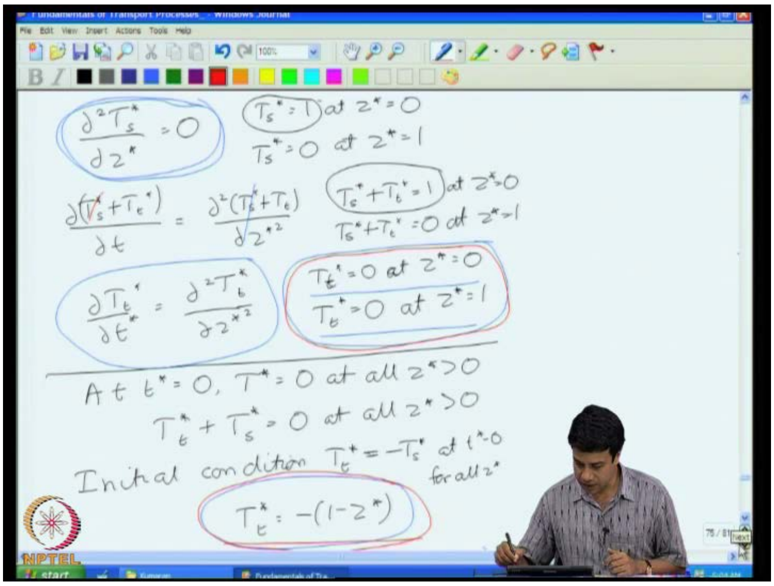
Now, what happens if the penetration depth is not small compared to the thickness? So, in that case there is an additional scale in the problem which is the height h. So, we can define a non-dimensional distance as z divided by H. Now, we get a scaled equation of the form d square T star by d time square is equal to d square T star by d z star square where we have scaled time by the time required for the diffusion over a distance of the order of H, scaled time by the time required for the diffusion over a distance of the order of H.

(Refer Slide Time: 13:57)



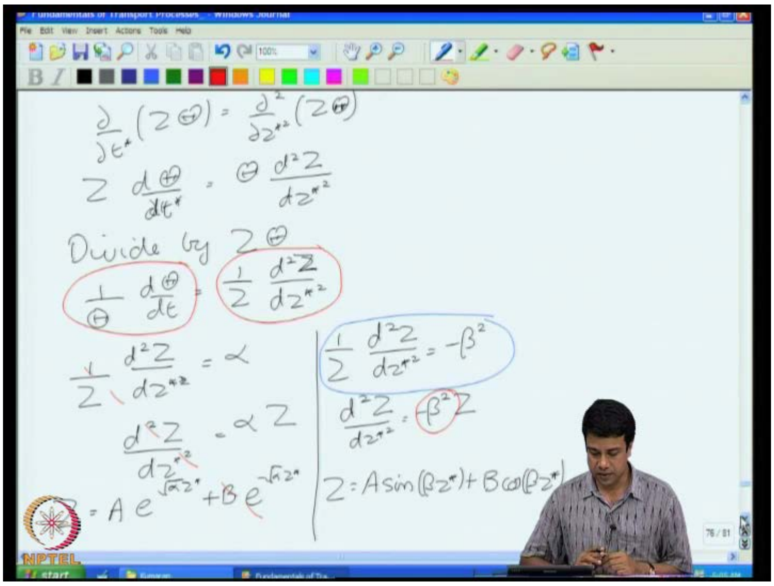
And at steady state, you get a linear profile 1 minus z which is shown here. So, there is a steady linear profile. However, at initial times when you just switched on the temperature at the bottom, the temperature is different from this steady solution. So, we separated out the temperature into a steady plus a transient part. In the limit of time make to 0 you recover the steady solution, but, at initial time when you just switched on the heating at the bottom, the temperature is different. The difference between the actual temperature at that time and the steady solution as the transient part of the temperature.

(Refer Slide Time: 14:38)



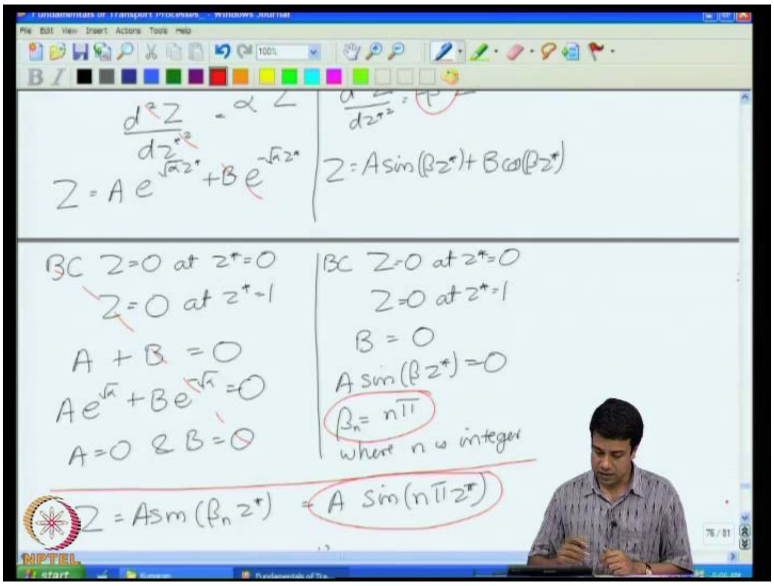
And we wrote an equation for the transient part of the temperature. It looks identical to the equation for the total temperature. And we wrote the boundary conditions for the transient part of the temperature and this is important these are homogeneous boundary conditions. That means, that transient part is 0 at both boundaries. However, at time T is equal to 0 the transient part is different from 0 at time T is equal to 0 the temperature is 0 everywhere the steady temperature is still 1 minus z. So, the transient part which is the difference between the temperature and the steady temperature is non 0. So, therefore, the flow is being forced by this non 0 value of the transient part of the temperature at time T is equal to 0. There is no forcing at the boundaries because T star is equal to 0 at both boundaries.

(Refer Slide Time: 15:36)



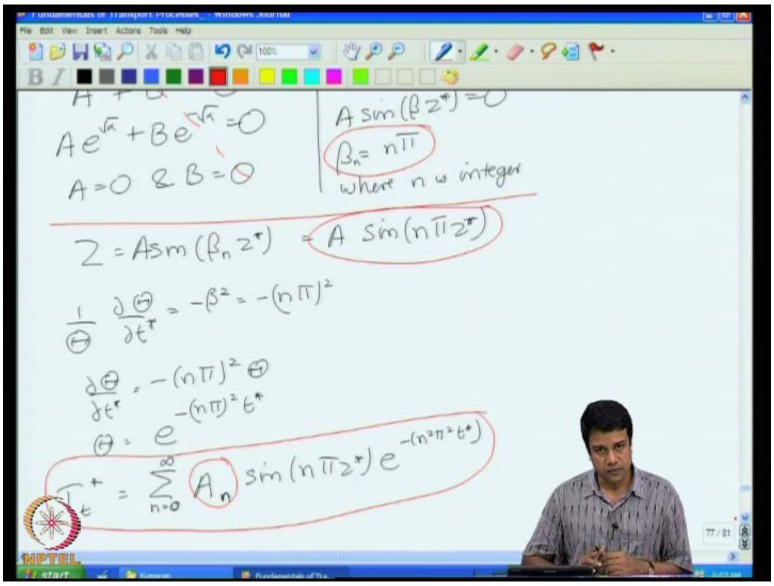
We went through the separation of variables procedure to calculate the time dependence. You write the function as a function of T times function of z and insert this into the equation, divide throughout by z times T; you end up with an equation in which the left hand side is only a function of time, right hand side is only a function of distance z. Therefore, both of these have to be constants. Are they positive or negative constants? From the z equation we saw that if it is a positive constant, we get a solution z is equal to 0 because you get exponentially increasing and decreasing functions which cannot be 0 on both boundaries unless both constants are 0. It is a negative constant you get sine and cosines and in this case we get a nontrivial solution.

(Refer Slide Time: 16:31)



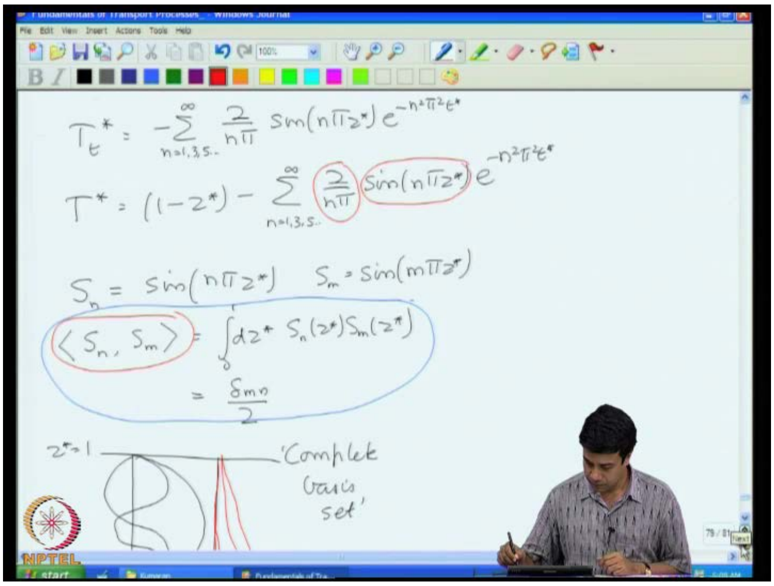
If my solution which is of form sine and pi z pi z star then this is 0 both at z is equal to 0 as well as z is equal to 1 provided n is an integer. So, the boundary conditions place a restriction on this constant beta. The boundary conditions place a restriction on the constant beta not just on the constants you get from the solving the differential equations a and b. So, these are the Eigen values for this problem.

(Refer Slide Time: 17:04)



You put the constant into that equation for the part theta that depends upon time and then solve it and you get an exponential decrease in time and you put these two together and you get a series solution with the coefficient a n the Eigen function or the basis function in the special coordinate sine of n pi z and then this part that depends upon time which basically tells you the decay rate of this component of the solution proportional to the basis function sine and pi z. And then we looked at how to obtain the coefficients a n using orthogonality conditions.

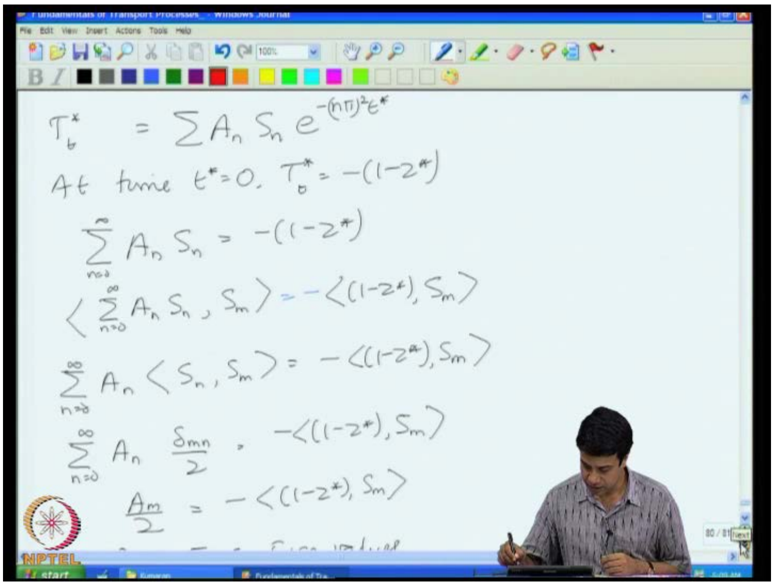
(Refer Slide Time: 17:46)



So, if I write S n is equal to sine n pi z and I define the product, in a product S n times s m as integral 0 to 1 d z of S n of z times s m of z; this 0 if n is not equal to m is equal to half when n is equal to m and it is given by this symbol, this delta function symbol.

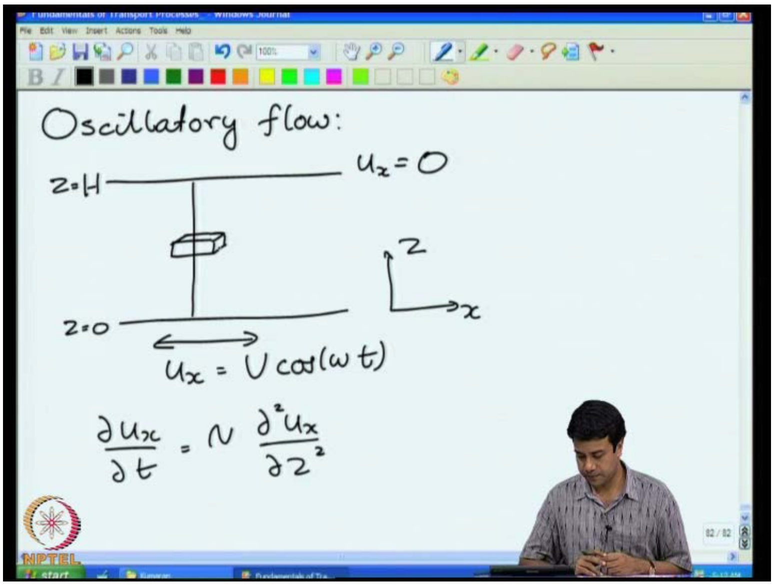
So, basically the solution is being separated into a series of bases functions with each with coefficients in front and those coefficients are similar to for example, the representation of a vector in three dimensional space is equal to the unit vector times a component. The unit vector is the basis vector the component gives you the quantity the coefficient in front. In a similar manner the solution is being expressed as a component this coefficient a n times the basis function S n and that S n is being determined from orthogonality relations for the bases functions.

(Refer Slide Time: 18:56)



And on that basis we are actually found out what those coefficients were and from that we got the final solution for that concentration field. So, this ended up being the final solution for the temperature field. Similar solutions arise for the concentration and momentum fields. In the case of a concentration field you the entire system is at concentration initially and at time T is equal to 0 you increase instantaneously the bottom surface concentration, momentum field. Everything is at rest initially. At time T is equal to 0 you start the motion of the bottom plate. That is separation of variables technique. The first one was a similarity solution.

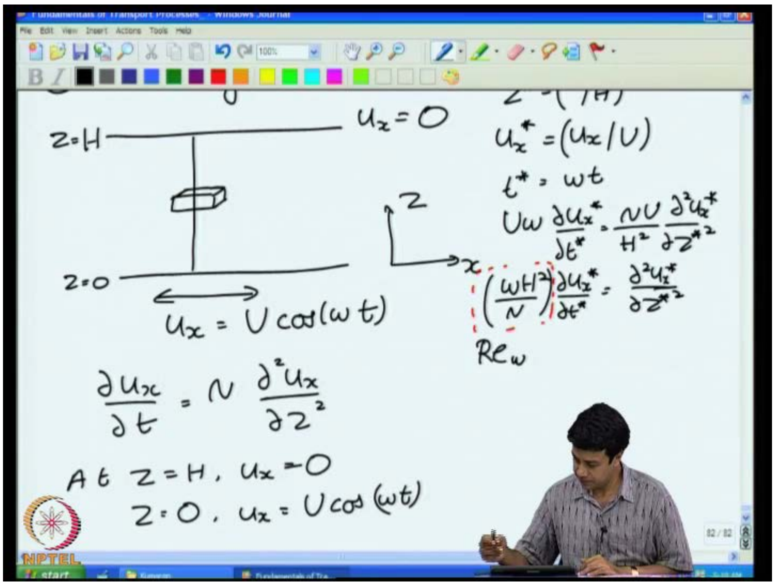
(Refer Slide Time: 19:54)



Now, we will look at one more way of solving these equations. For the specific flow which is oscillatory in time, same configuration as before z is equal to 0, z equals H, my coordinate system x z with a fluid in between. This top surface is stationery. u x is equal to 0 where as the bottom surface is oscillating. u times cos where omega is the frequency of oscillations and we would like to know what is the velocity field within the flow. So, the problem is for an oscillatory velocity field at the at some bottom at the bounding surface what is the entire velocity field within the flow. So, that is the problem.

These oscillatory flows have many applications. For example, the flow in the human body, blood in the human body is an oscillatory flow. Many machines have oscillatory motion like reciprocating machines and so on and the forcing is oscillatory in time. It could be with a fixed frequency, it could be with a way from there is not exactly sinusoidal, but so long as it is periodic in time these oscillatory flows can be treated by the method that I am going to show you. So, the governing equations again d u x by d T is equal to the kinematic viscosity times d square u x by d z square. So, that is the equation for the velocity field which we had derived by taking a balance over a small shell within the fluid earlier. If we take a balance over a small shell and then use Newton's law of the shear stress; we get an equation of this form where nu is the kinematic viscosity.

(Refer Slide Time: 22:32)

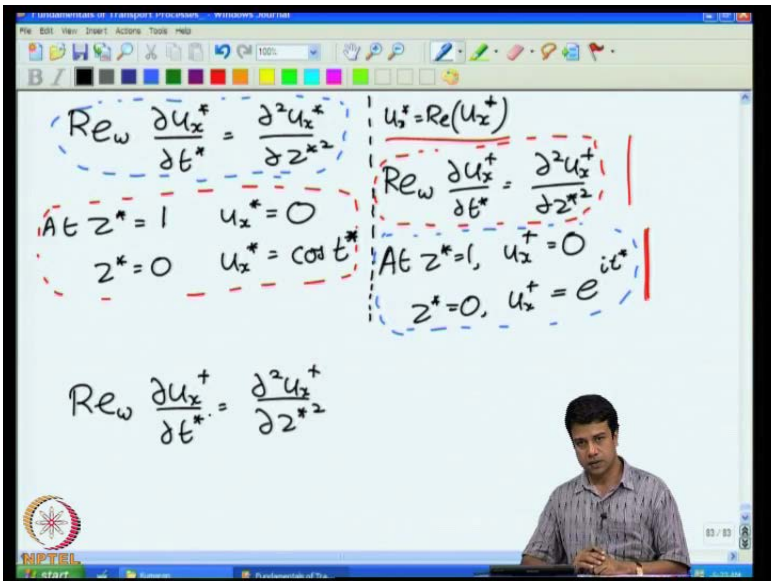


How do we, what are the boundary conditions at z is equal to H the velocity u x is equal to 0 and at z is equal to 0, u x is equal to capital u times cos omega T. How do we scale the variables in this case? The scaling for z is natural, z star is equal to z by H. That is the scaling that we have used repeatedly throughout the section because H is the distance of relevance here. The distance over which there is variation in the velocity. How about velocity u x? The obvious way to scale it is by the amplitude of the oscillations capital u because that gives you the maximum velocity that is attained during the oscillations. So, it is natural to scale u x star is equal to u x by u.

How about time when we saw of the unsteady flow problem, we scaled time by H square by nu because there was no time scale in the problem itself. Therefore, we felt it natural to scale it by the time it takes for diffusion to take place across the channel of distance H. The time it takes approximately H square by the diffusion coefficient. In this particular problem we have a well defined frequency for oscillations omega well defined time period for oscillations there is 2 pi by omega. Since there is an intrinsic time scale in the problem the time scale for the oscillation itself, it is natural to scale the times by the frequency of oscillations itself that is to define T star is equal to omega times T. So, these are the three; length, time and velocity scales in the present problem. This differs from what we have been doing earlier in the sense that there is an intrinsic time scale in the problem that is the frequency of the time period of the oscillations and therefore, it is natural to scale time by, the define the non-dimensional time as omega times T.

Put all of these together into the governing equation what I will get is u times omega d u x star by T star is equal to nu u by H square d square u x star by d z star square. Just putting in the time n, distance and velocity scales and of course, I can cancel out u on both sides and finally, I will get omega H square by nu d u x star by d T star is equal to d square u x star that is z star square. So, that is my final dimensionless non-dimensional equation which contains this non-dimensional parameter. It contains this non-dimensional parameter here. What is that parameter? It is equal to H times omega by the kinematic viscosity. Kinematic viscosity has dimensions of length square per unit time. Omega times H square also has dimensions of length square per unit time. So, this is basically ratio of the time scale for diffusion and the time scale for for for oscillation. The time scale for oscillation is 2 pi by omega, the time scale for diffusion is H square by nu, the ratio of these two the time scale for diffusion divided by time scale for oscillation is this non-dimensional number. I will refer to this in the present lecture as a Reynolds number based upon the frequency of oscillations omega. So, this is a Reynolds number based upon the oscillation frequency omega. In a sense it is gives you a ratio of inertial and viscous forces because the right hand side of the equation contains the viscous the viscous stresses, the left hand side is a rate of change of momentum an inertial term. So, this R e omega gives you the ratio of the inertial and the viscous forces

(Refer Slide Time: 27:43)



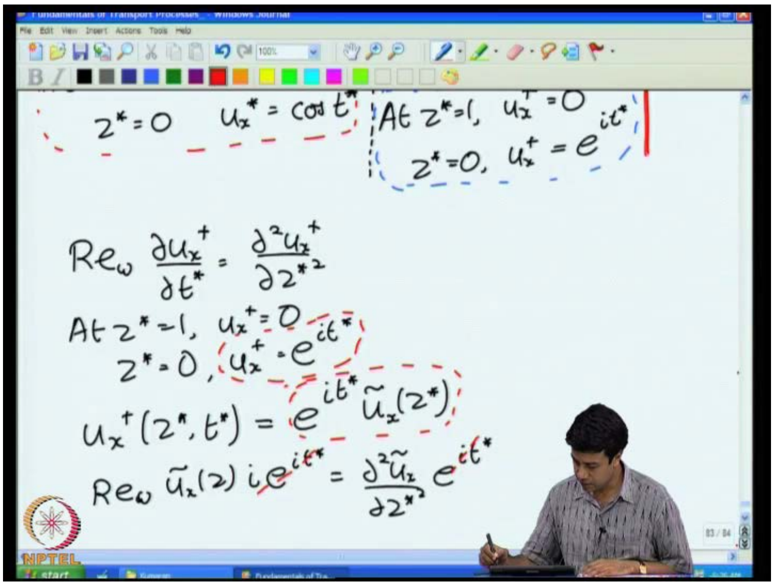
So, this is our conservation equation; R e omega times partial u x by partial T is equal to partial square u x by partial z square and the boundary conditions on this, at z star is equal to 1 top plate z is equal to H and therefore, z star is equal to 1. You have u x star is equal to 0. Bottom surface z star is equal to 0, u x is equal to u cos omega t. So, u x star is u x by u and T star is omega T. Therefore, I will get u x star is equal to cos T star. So, this is the partial differential equation that we are trying to solve for an oscillatory flow. Now, this has an initial condition that is oscillatory in time, cos T and cos function is generally inconvenient to handle. A simpler way of handling it is to actually define a complex velocity field. So, let me just say how it is defined and then justify it.

So, I will define a complex velocity field u x plus such that the actual velocity field that I have in my problem u x star is equal to the real part of u x plus. So, I am defining the complex velocity field the actual real velocity field that I am after is the real part of this complex velocity field. The differential equation for this complex velocity field R e omega d u x plus by d T star is equal to d square u x plus by d z star square. Identical to the differential equation for the actual velocity field, the boundary conditions at z star is equal to 1 u x plus is equal to 0 and at z star is equal to 0, u x plus is equal to e power I T star where I is the square root of the minus one. So, u x plus at z is equal to 0 is equal to e power I T star and you can easily see that if I take if I take the real part of this entire equation R e omega is already a real.

So, if I take the real part of this entire equation; I do get back the equation for my actual velocity field. So, I do get back the equation for my actual velocity field and I take the real part of the equation for the complex velocity field. In addition when I take the real part of the boundary conditions; note that z is still a real number, z is a coordinate is still a real number. When you take the real part of the boundary conditions, I recover the boundary conditions for my actual velocity field. So, the only inhomogeneous term in the boundary condition is that z is equal to 0 u x plus is equal to e power i T star and I take the real part of that I recover cos of T star. Therefore, if I solve this equation with this boundary condition and then it take the real part of that, I will end up with the velocity u x. So, that is the basic idea and it is more convenient for me to deal with exponentials than with sine and cosine functions.

So, this is an easier way to get the solution than to actually do it for the real sin and cosine functions. So, how do we solve this equation?

(Refer Slide Time: 32:55)

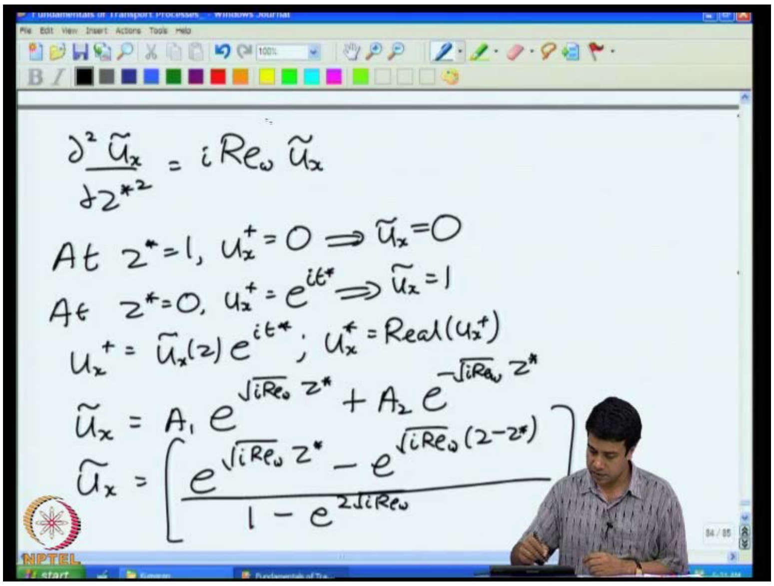


So, my equation is R e omega d u x plus by d T is equal to d square u x plus by d T and T d z square with boundary conditions at z star is equal to 1 u x plus is equal to 0 and at z star is equal to 0 u x plus is equal to e power i T star. Now, the differential equation, the partial differential equation is linear in u x plus and it is driven by an oscillatory driving at the wall. Whenever you have a driving with which is oscillatory on a linear system the response also be oscillatory with the same frequency. Because the system is linear in the velocity right and the driving is oscillatory therefore, you will end up with a solution that is also oscillatory with the same frequency.

Therefore, I know that u x plus u x plus which is a function of z and T it has to be oscillatory in time with the same frequency as the driving. It has to be oscillatory in time with the frequency as the driving. Time some function it can be any function of z, but, it has to be oscillatory in time with the same driving as the driving of the bottom plate in time. So, because it is a linear equation, it is being driven by an oscillatory velocity at the bottom surface. The final solution that I get should also be oscillatory in time. Now, this form of the solution I can now insert into the differential equation.

So, I insert this form of the solution into the differential equation and you will get R e omega u x of z into the derivative of e power i T with respect to time which is basically I times e power i T, will be equal to now, the right hand side contains it derivative only with respect to z. So, therefore, this will be equal to u x of z I am sorry d square u x by d z square and d square into e power i t. So, that is what get when I substitute this form of the equation into the, of the solution into the governing equation. And now of course, I can cancel out e power I T on both sides, I can divide throughout by e power i T to get a final equation.

(Refer Slide Time: 36:32)



Which is d square u x by d z square is equal to i R e omega times u x. Now, this equation is an equation only in the z coordinate we had got rid of the time variable by postulating that the solution has to be of this form because it is been driven by an oscillatory flow in time and once I put this form of the solution in e power i T will cancel on both sides and I get only a ordinary differential equation in the z coordinate for this variable. This u tilde; for this u tilde I get an ordinary differential equation in the z coordinate alone.

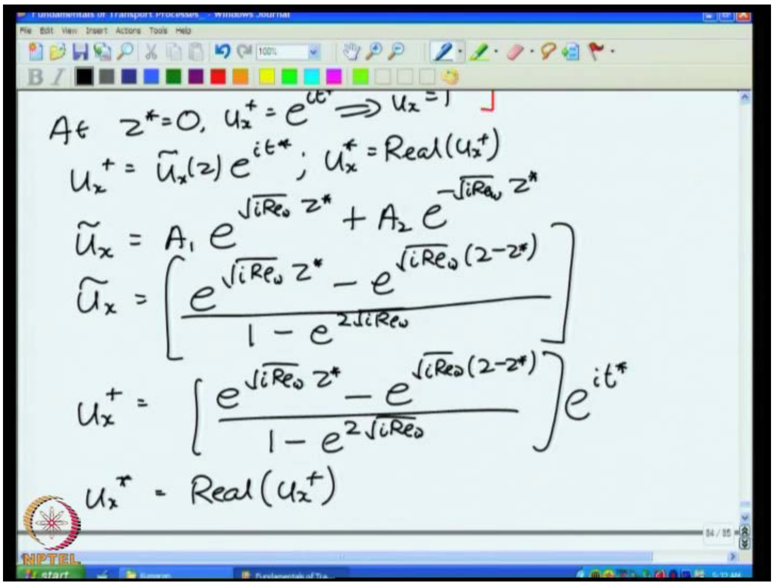
What about boundary conditions? At z star is equal to 1; u x plus is equal to 0 which implies that u x tilde is also equal to 0. At z star is equal to 0, u x plus was equal to e power i T star. Since u x plus is equal to e power i T star which is equal to e power i T star into e power i T star into u x tilde. Therefore, u x tilde will be equal to 1.

So, now I do not have any time dependence in either the equation or the boundary condition because I use the fact that is being driven by sinusoidal well profile at velocity at the bottom. The velocity everywhere will be a sinusoidal velocity for the same frequency because the linear governing equation and of course, I can solve for this. I can solve this equation to find out what is u x tilde. From that how do I get the solution? I go back and get u x plus is equal to u x tilde of z e power i T star and then u x star is equal to real part of u x plus.

So, that finally, gives me the solution for the flow within the domain. So, this equation is quite easy to solve the second or differential equation. So, the solutions are sin and cosine functions or exponential solutions. The final solution for the u x tilde will be of the form a 1 e power square root of i R e omega times z star plus a 2 e power minus square root of i R e square root of i is 1 by root 2 plus i by root 2. in Then if I impose the boundary conditions u x tilde is equal to 0 at z is equal to 1 and u x tilde is 1 at z is equal to 0 I will get the final solution for the velocity field as u x tilde is equal to e power root of i R e omega z star minus e power root of i R e omega into 2 minus z star divided by 1 minus e power 2 root of i R e omega.

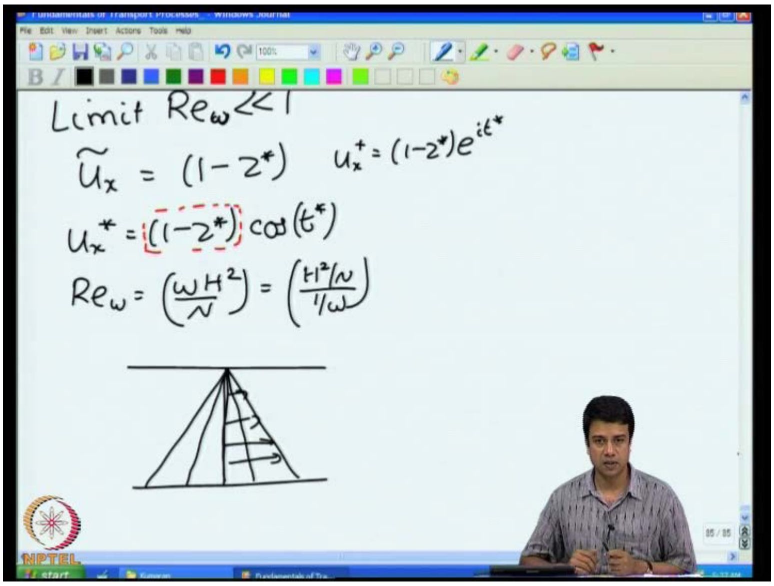
So, that is the final solution. You can verify that at z at z star is equal to 1 this will be identically equal to 0. At z star is equal to 0 this will be identically equal to 1. Therefore, it satisfies both of these boundary conditions.

(Refer Slide Time: 41:22)



So, from this I will get u x plus which is equal to into e power i T star and then u x star will be equal to the real part of u x plus. So, that will be my final solution. Of course, I could evaluate this real part numerically and then find out the velocity profile at very point within the flow.

(Refer Slide Time: 42:35)



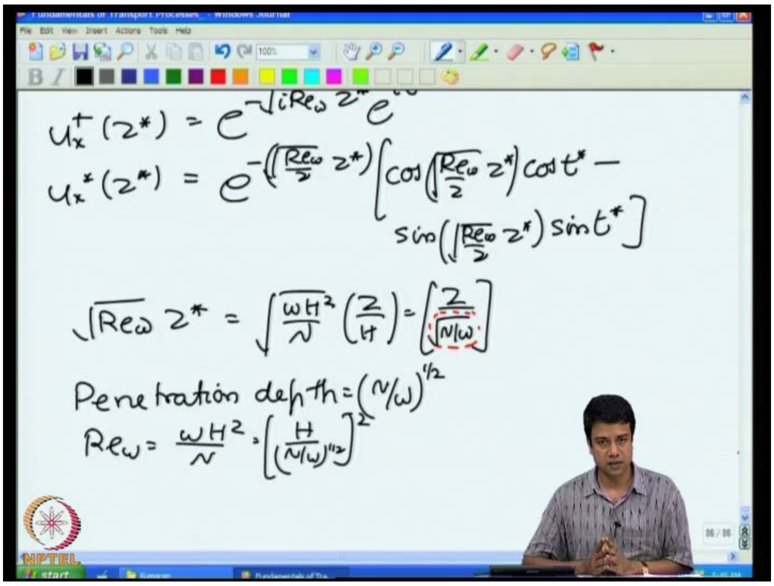
However, in order to get a physical understanding it makes sense to first look at limiting cases. The first limiting case is the limit R e omega small compared to one. In this limit you can actually take this velocity profile and do an expansion in the small parameter R e omega do an expansion in the small parameter R e omega the leading order time will be identically 0 and then you will get first correction and that first correction will be of the form u x tilde is equal to 1 minus z star. So, that will be the first correction that you get for in when you do an expansion in square root of R e in the limit of R e omega small compared to one.

This implies that u x plus is equal to 1 minus z star e power i T star and therefore, u x star is equal to 1 minus z star cos T star note that u x star is equal 1 minus to z star. This part alone is the solution for the steady velocity between two plates. If you have velocities is equal to 0 at z star is equal to 1 and 1 at z star is equal to 0; it is just a steady flow. You would get a solution of the form 1 minus z star. in this case I am getting a solution of the form 1 minus z star times cos T star. Cos T star is the instantaneous velocity of the bottom surface. So, the solution that I get is the same that I would get for a steady flow except that the velocity of the surface is the velocity at that particular time. Why is that? What does R e omega physically mean? As I discussed before R e omega is equal to omega H square by nu which is equal to H square by nu 1 over omega. H square by nu is the time taken for diffusion over a length of order H. it is a It is a time for diffusion over a distance comparable to H. 1 over omega is the period of oscillation of the bottom plate. So, when H square by nu is small compared to one over omega the time taken for momentum diffusion is small compared to the period of oscillation. That means, that the momentum diffuses almost instantaneously across the entire plate.

So, instantaneously the momentum has equilibrated over the entire surface over the entire fluid and therefore, the velocity that you get is identical to the velocity you would have got for a steady flow except that the velocity of the bottom surface is the instantaneous velocity at that instant in time. So, when momentum diffusion is fast; there is an equilibration of velocity across the channel and the velocity response profile is a steady velocity profile which responds instantaneously to the instantaneous velocity of the bottom surface. So, in that case what I would get is something that looks like this. As the bottom surface velocity if it is this way you will velocity that looks like this. As the velocity decreases, you will get a linear profile half way through the cycle you get this. Then you get negative larger and then it comes back all the way. You get a linear profile at at every instant in time a linear profile with the instantaneous velocity at the bottom surface as the driving velocity for the steady flow.

So, this is the case where R e omega is small compared to one when momentum diffusion is very rapid, when the time required for the momentum diffusion is small compared to the time period for oscillation of the bottom plate.

(Refer Slide Time: 47:03)



What happens when R e omega is large compared to one? iIn this limit I can once again solve the velocity profile. If R e omega is large compared to one, then I will have only the exponentially decreasing part of the of the solution because the coefficient of the exponentially increasing part will go to 0. Think of solution at the bottom surface into an infinite fluid. There is an exponentially increasing and decreasing part and if you are into an infinite fluid you can have only the exponentially decreasing part. You can get it by an analysis of this this equation. Taking the limit of R e omega large, but, a simpler way is to just use physical understanding and on that basis you will get u x tilde of z star will be equal to e power minus root of i R e omega times z star which means that u x plus of z star e power i T star and u x star is the real part of this one the real part of this one. I would not go into the details, but, basically you will get e power square root of by 2 into a z star into cos of.

So, you get a solution that looks something like this. First of all this solution is not in phase with the driving at the bottom surface. In the previous case where the R e omega was small, the solution was exactly in phase and the reason it was in phase was because diffusion is dominant and diffusion is if you like it a resistive element. It does not introduce a phase change whereas, this solution is not exactly in phase that is because inertia is important.

So, it has a component proportional to cos and the component proportional to sin of omega T with the same frequency of cos, but, it does have a phase shift. Secondly, this decreases as square root of R e omega times z star. It decreases exponentially when R e omega is large compared to one. So, we are considering the limit where R e omega is large compared to one, this solution decreases exponentially into the fluid. The exponential decrease is goes as square root of R e omega times z star which is equal to square root of omega H square by nu times z by H which is z by square root of nu by omega. So, in this case the penetration depth for the exponential decrease is this one. In this case the penetration depth for the exponential decrease is square root of nu by omega. Nu, recall nu has dimensions of length square per unit time, omega has dimensions of time. Therefore, this will give you a length scale. Physically what does it mean? nu by omega power half is the distance over which diffusion can take place over a time period one over omega period of the oscillation is one over omega, nu by omega power half is the distance over which the diffusion will take place over a time period which is one over omega where omega is the frequency of the oscillations. And the Reynolds's number is large; that means, that nu by omega is small compared to H.

So, the Reynolds's number is equal to omega H square by nu. This is equal to H by nu by omega over a half the whole square. So, this number is large what that means, is that H is large compared to the distance over which momentum diffusion can take place over a time period 2 pi by omega where omega is a frequency. So, we have bottom plate that is oscillating with a period omega or a frequency 2 pi by omega. Within that period omega or within that time period 2 pi by omega; the momentum from the bottom plate due it is forward motion diffuses a certain distance. By the time it has diffused further the plate has already reversed its direction and it is come the other way and therefore, you get a momentum disturbance that in the opposite direction and those two will cancel out resulting in penetration of the velocity profile only over a finite depth not throughout the distance H.

So, because of that the momentum disturbance is confined to a thin region near the surface which is oscillating. Because the frequency is large enough, the period is small enough that the period of oscillation is much smaller than the time taken for the momentum to diffuse across the entire channel. So, in that case you will get a boundary layer at the bottom where the momentum diffusion is restricted and the thickness of that is nu by omega power half. Once again, H is no longer a relevant variable because momentum does not diffuse that far when the frequency is large. It diffuses only a small distance from the bottom surface and that distance which diffuses is this, nu by omega power half.

So, taking the limits R e small compared to one and R e omega large compared to one; gives you a better physical understanding of what is happening in the two limiting cases and of course, you can always do the numerical solution of this quite easily in order to get numerical values of the solutions for the velocity profile everywhere in the channel. This is an approach that we will use regularly or repeatedly in this course. You will derive equations, do scaling, try to get limiting cases and use those limiting cases to give us a better physical understanding of what is happening within the system. Similar problems can also be posed for concentration and for mass and heat transfer. They are a little more difficult to realize in practice. In heat transfer you would have to heat the bottom plate with sinusoidal temperature. It can be done if you have a heater with correct frequencies. Mass transfer is much more difficult to realize you cannot very easily have a sinusoidal there are oscillatory reactions, but, it is not very easy to realize in practice to have a sinusoidal variation in the concentration of the bottom surface.

So, this is how we deal with oscillatory flows and so far we have looked at three different ways similarity solutions; separation of variables and oscillatory flows all for systems where there is no source or sink within the flow. Next lecture we will start looking at some systems which have sources and sinks within the flow and how do we analyze it for these cases. So, will close this discussion of oscillatory flows here and continue in the next lecture on unidirectional flows once again in the presence of sources and sinks of concentration momentum etc. So, we will see you in the next lecture.