Fundamentals of Transport Processes

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Module No. # 03

Lecture No. # 12

Unidirectional Transport Cartesian Co-ordinates – V

(Separation of Variables)

Welcome to lecture number 12, fundamentals of transport processes, where we were looking at unidirectional transport using the method of separation of variables. This is an important solution procedure. So, we will go through it in some detail to illustrate some of the intricacies involved in nationally implementing it properly. Because once we implement it properly, then it is a very powerful tool to be used for solving not just transient problems, but also problems involving variations in different coordinate directions.

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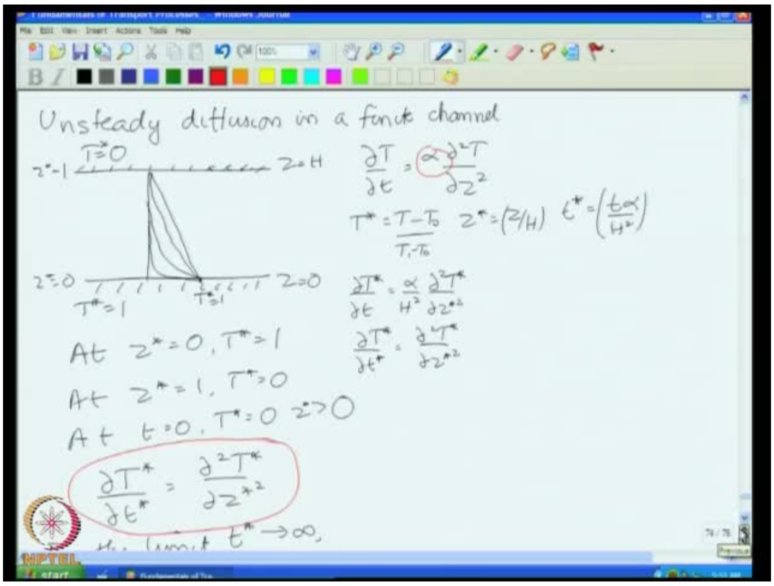

So, the problem that we were considering was as follows: This was unsteady diffusion in a finite channel; I have 2 plates, 0 and H, the temperature at the bottom is one, the temperature of the top is 0 in scaled variables.

In the final steady state, one would expect that the temperature in the limit T going to infinity, reaches a linear temperature profile, where the temperature varies linearly with position. However, the transient temperature profile is not linear, it evolves from initially 0 temperature throughout the channel to a final linear temperature profile. Because had it applied the initial condition that both the bottom and the top plate temperatures were 0 initially, at T equal to 0, the entire fluid was at T star equal to 0, at time T equal to 0. And at T equal to 0, I instantaneously switched the temperature at the bottom from 0 to 1 and then I look at how the temperature field evolves as a function of time. So that was the problem to solve for the unsteady evolution of temperature within this channel.

The differential equation that we consider is the usual one for the unsteady case, with no sources and no sinks. It is just partial of temperature with respect to time is equal to alpha times d squared T by dZ square, and T star is equal to 1 at Z is equal to 0 and 0 at Z is equal to H.

We had defined the scaled Z coordinate as Z star is equal to Z by capital H and we put into that equation and managed to get out a scaled time T star is equal to t alpha by H square. This is a dimensional necessity, because the only parameters in the problem are alpha, the thermal diffusivity, and H, which is the distance between the two plates. Therefore, I can get only one time scale out of this problem, which is H square by alpha. So, I can define only one dimensionless time as t alpha by H square.

So that was the rational for choosing this dimensionless time. Physically this can be interpreted as follows: alpha by H square, if the time is H square by alpha, which is the time it would take for thermal diffusion of energy over a distance proportional to edge. So, this H square by alpha is time scale over which would expect diffusion of heat over a distance comparable to edge and therefore, equilibration of the temperature field over a distance H.

So, when T is large compared to alpha by H square, the diffusion has already taken place over the entire extent and the system has reached a final steady state; when T is small compared to alpha by H square, the diffusion does not progress very fast. So, because of that, it is confined to a thin boundary layer near the surface; a penetration layer of depth is equal to square root of alpha T.

So, T star gives me some indication of the time over which diffusion would takes place. If T star is large compared to 1 or T is large compared to H square by alpha, I would be attaining the steady state temperature, that is, if T star is small compared to 1 or T is small compared to H square by alpha, the temperature should be in the very initial stages of development, where the penetration depth of temperature into the medium is small. So, there is physical interpretation of this time scale T star, when expressed in terms of the scaled time and the scaled coordinate, the final equation that I get has no dimensional parameters in it. So, it is a completely dimensionless equation, when expressed in this form.

As I said, in the limit as time going to infinity, I should reach the final steady temperature. That final steady temperature is equal to 1 minus H star and that satisfies the equation d square T by dZ star square is equal to 0, with temperature is equal to 1 at the bottom and 0 on top.

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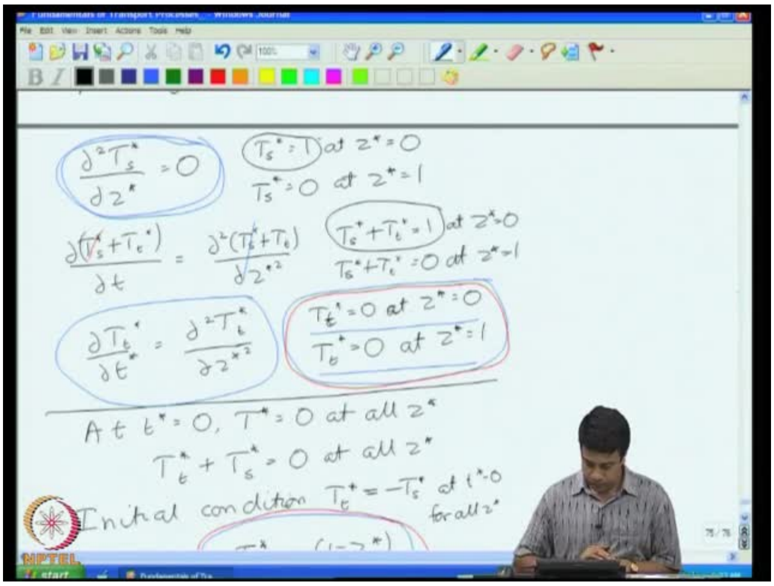

So, in the limit as t is going to infinity, you should reach this final steady state temperature. So, before progressing we made decomposition in the temperature. We wrote it as two parts: one is steady part and the other is the transient part. The idea is that if the system reaches the final steady state, the transient part has to go to 0 and T should just be equal to T s.

So, therefore, the transient part of the temperature is the correction to the temperature profile, for T being of order 1 or small. In the limit of T being large, this will go to 0. So, having separated the temperature into a steady and the transient part, we have to separate the equations also into individual equations for the steady and the transient part.

The steady part, d square T s by dZ square is equal to 0, is the equation for the steady temperature profile, with boundary condition T s is equal to 1 at Z is equal to 0 and T s is equal to 0 at Z is equal to 1; hot at the bottom and cold at the top.

For the total temperature field, that is T star is equal to T s plus the transient part, I can write that same differential equation once again, partial of T s plus T t, the partial time, is equal to the second derivative with respect to Z of the transient plus the steady part. The steady part is independent of time and its time derivative is 0. The steady part also satisfies d square T s by dZ square is equal to 0. So, my differential equation for the transient part reduces to just a simple form; the exact same form that I had for the original temperature profile, dT transient by dT star is equal to 0 and is equal to d square T transient by dZ star square.

However, there is a difference in the boundary conditions. For the steady part, I have T s is equal to 1 at Z is equal to 0, T s is equal to 0 at Z is equal to 1. For the total temperature profile, I have T s plus T t is equal to 1 at Z is equal to 0, T s plus T t is equal to 0 at Z is equal to 1; subtract the two and I will get T transient is equal to 0 at both Z is equal to 0 and at Z is equal to 1. So, as I said, these are homogeneous boundary conditions in the Z coordinate. The transient part of temperature is 0, both at Z is equal to 0 as well at Z is equal to 1. So, it satisfies homogeneous boundary conditions in the Z coordinate.

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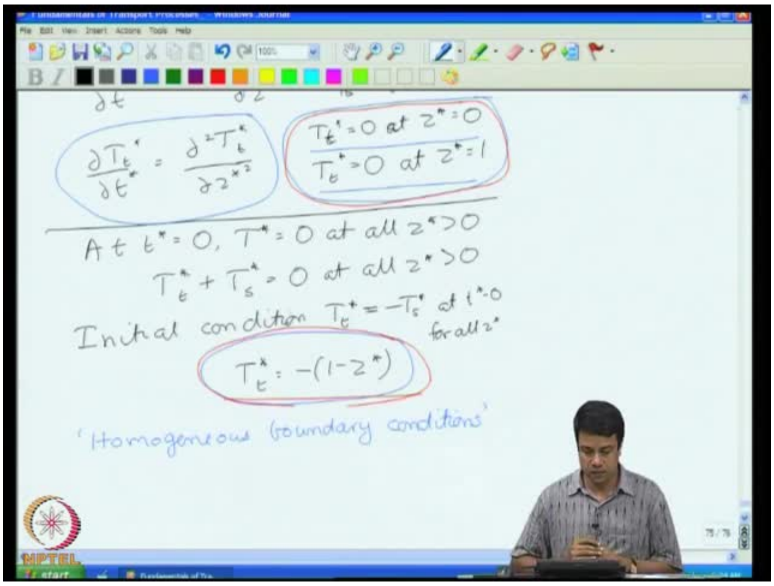

However, even though the boundary conditions are homogeneous in the Z coordinate, they are not homogeneous in time. At time t is equal to 0, the initial condition for the temperature field was that T star is equal to 0 at all Z greater than 0, that means the transient plus the steady part has to be equal to 0 at all Z greater than 0. That means for the transient part alone, the boundary condition is T transient is equal to minus the steady state at Z is equal to 0 or T transient is equal to minus 1 minus Z star, and it explained the physical significance of this in the previous class.

These homogeneous boundary conditions in the two spatial coordinates, means that there is no temperature forcing for the transient part alone at the two boundaries. The steady part thus have T star is equal to 1 at Z equal to 0 and T star is equal to 0 at Z is equal to 1, but for the transient part the boundary conditions are both T star is equal to 0 at both Z is equal to 0 and Z is equal to 1.

Whenever boundary conditions are of the form either the temperature or its derivative is equal to 0, these are referred to as homogeneous boundary conditions, because there is no force sink; over there is no finite temperature that is imposed at those boundaries.

However, the initial condition is not homogeneous. We have a non-homogeneous initial condition, which is actually the reason why T transient is not equal to 0. In the limit as T going to infinity, it will of course, decrease to 0 in the long time limit, but at intermediate times is not equal to 0, because the transient temperature field has been forced at the initial time; at T star is equal to 0.

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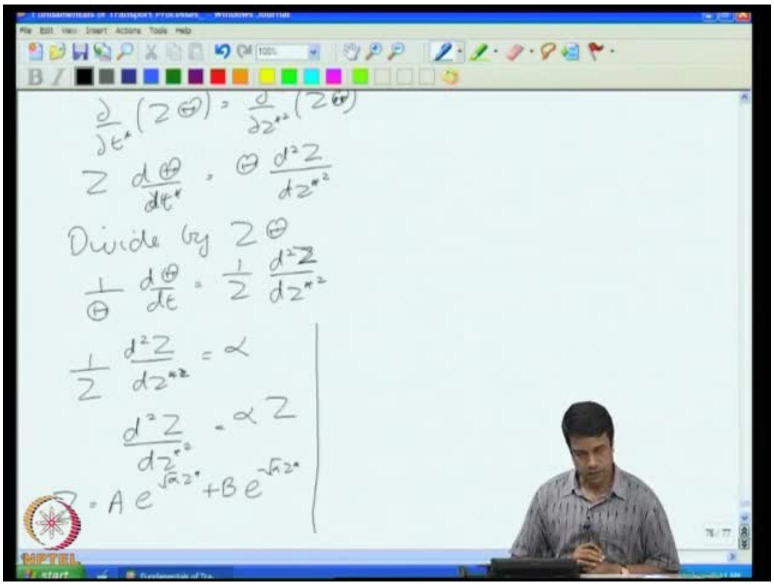

So, the solution of these equations, which we had briefly started it in the previous lecture is a method called separation of variables. So, we write this temperature which is a function of Z and t, as the product of two terms, one is only the function of Z and the other is only a function of t. So, the function of both Z and t is separated into two parts, one of which is only a function of Z and the other is only a function of, and this is put into the equation. So, I get d by dt of Z theta is equal d square by dZ star square of Z theta. Note that Z is only a function of Z star and theta is only function of t star. So, this I can write it as Z times d theta by dt is equal to theta times d square Z by dZ star square.

Note that I have used the total derivative d here, the reason is because theta is only a function of t. Therefore, the partial derivative also equal to total derivative, because it is only a function of one variable. Similarly, Z is a function only of Z star and therefore, it is a total derivative. I divide throughout by Z times theta to get 1 over theta d theta by dt is equal 1 over Z d square Z by d Z star square.

Now, in this equation, the left hand side is only a function of time, and the right hand side is only a function of Z, that means that both of these are equal to constants and the reason is as follows: Let as assume that it is not equal to a constant. Let us assume that the left hand is only a function of t and the right hand side is a function of Z and both are equal.

In that case, I can keep Z a constant and change t. If I do that, the left hand side changes, the right hand side does not, because I kept Z a constant and changed t. So, if the left hand side changes and the right hand side does not, then the equality is no long valid. Conversely, I can keep t, a constant and change Z in that case, the right hand side changes and the left hand side does not and therefore, I will end up with a situation where the equality is no longer satisfied.

The only way that the equality can be satisfied for all values of Z and t is, if both of these are equal to constants. So, therefore, I require, just from the condition, that the left hand is only a function of t and the right hand side is only a function of Z. From that I can infer, from an equation of this kind both of these, left hand and the right hand side, has to be a constant.

So, what is that constant? Now, let us look at how to solve for this constant. So, I have an equation 1 over Z d square Z by dZ star square is equal to some constant alpha. Is that constant positive or negative? Let us try to solve this and see. So, therefore, the equation d square Z by dZ star square is equal to alpha times Z. For the moment we will assume that this constant is positive. The solution of this will be of the form Z is equal A e to the power root alpha Z star plus B e to the power minus root alpha Z star.

Now, what are the boundary conditions for Z? I have got the two constant, I have solved a second order differential equation and now, those two constants have to be determined by boundary conditions for Z. What are the boundary conditions for Z? I require that the temperature has homogeneous boundary conditions, both at Z is equal to 0 and at Z is equal to 1, the temperature is 0. It is 0 at all times and therefore, I require that Z has to be equal to 0, both at Z star is equal to 0 and Z star is equal to 1.

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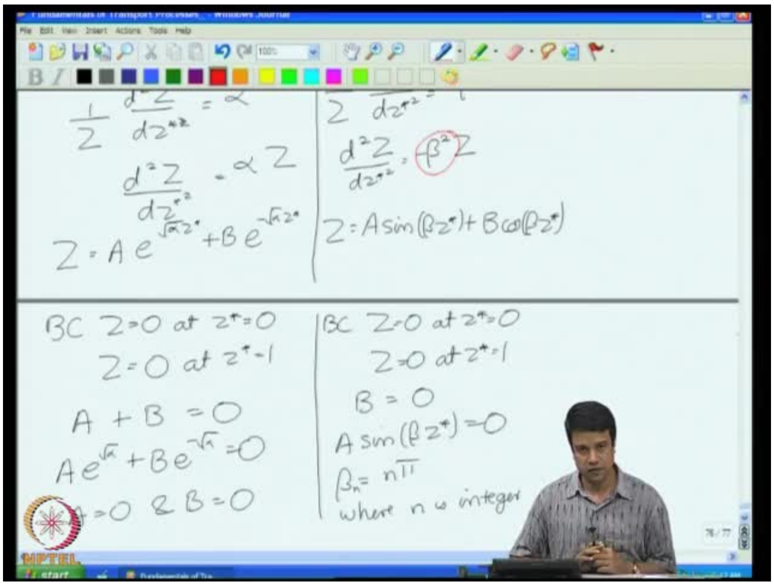

So, therefore, in this case, I require the boundary conditions Z is equal to 0 at Z star is equal to 0, and Z is equal to 0 at Z star is equal to 1. Insert those boundary conditions here and I will get A plus B is equal to 0 and A times e to the power root alpha plus B e to the power minus root alpha is equal to 0. I just put Z star is equal to 0 in the first equation, and Z star is equal to 1 in the second equation. From these equations, you can easily see that only solution that I will get is that A is equal to 0 and B is equal to 0, which means that capital Z is equal to 0. Therefore, in this case, I do not get any non-trivial solutions for Z. The solutions end up just being the trivial solution that Z is equal to 0 everywhere and therefore, T transient will be equal to 0.

Now, that was because I happened to choose a value of alpha, which was positive. What happens if I choose the value of alpha that is negative? So, let us look at that. This was one particular choice of alpha, let us look at some other negative choice of alpha. So, I will take the equation of the form A is equal to minus some constant square so that this beta is negative so that the constant that I have here is a negative value. Beta square is always positive and therefore, the right hand side is negative. The equation reduces to d square Z by dZ star square is equal to minus beta square Z. The solution for this is Z a combination of a sin and a cos function.

The boundary conditions are Z is equal to 0 at Z star is equal to 0, and Z is equal to 0 at Z star is equal to 1. It is sin 0 anyway. Therefore, I require that B is equal to 0 from the first condition, because at Z star is equal to 0 sin of 0 is 0; cos 0 is 1 therefore, B has to be equal to 0. At Z star is equal to 1 since B is already equal to 0, I require that A sin of beta Z star has got to be equal to 0. One option is of course, that A is equal to 0 in which case, I do not get any non-trivial solutions once again, I get just the trivial solution.

But wait, there is another possibility and that is that sin of beta Z star is equal to 0. If sin of beta Z star is equal to 0, then the boundary condition is satisfied. But for that, I require to take beta to be specific values. If I take beta is equal to n times pi, where n is an integer, I will call this as beta n, where n is any integer value. So, n is equal to 0, sin of 0 is 0, n equals pi, sin pi is equal to 0 and Z star is 1pi, 2 pi, 3 pi, etcetera.

Any value of the integer will satisfy this equation. So, I do get non-trivial values for the solution to this equation, but I have to choose the value of beta to be specific values. So, therefore, this first guess that we had of a positive value did not give any non-trivial solutions. So, I chose positive value and yet, did not get any non-trivial solutions. However, when I choose a negative value I managed to get non trivial solution, but only for the case, where this coefficient beta has a specific value.

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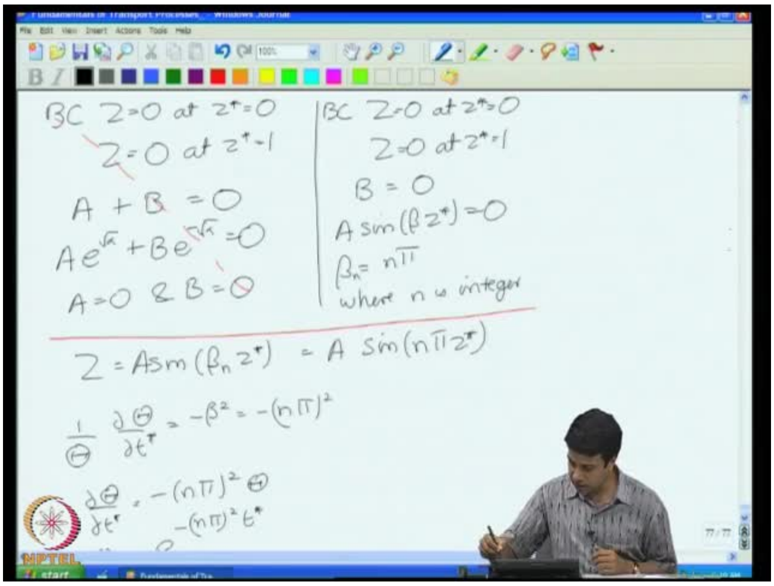

So, therefore, the solution for the case, where beta has a specific value is, Z is equal to A sin beta n Z star is equal to A sin of n pi Z star. What about the other part of the equation which was the variation in time? So, this was equal to minus beta square and this is also equal to minus beta square. Therefore, I can solve for the second part of the equation, 1 by theta d theta by dt is equal to minus beta square, which is minus n pi the whole squared or d theta by dt star is equal to minus n pi the whole squared theta or theta is equal to e to the power minus n pi the whole squared times t star.

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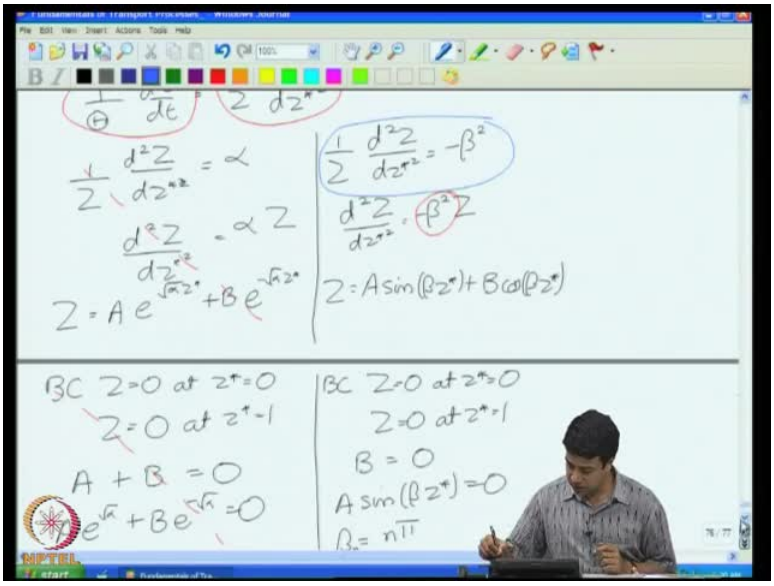

Now, we had chosen the value of beta, we had chosen the constant to be negative, because we got a non-trivial solution for this constant beta here. In this particular equation, we chose it to be negative so that we got a non-trivial value of the beta for the separation of variables problem for this equation. If we chose it to be positive, we would get the solutions of the equation to just be 0.

However, that is consistent with the solution that I am getting for T. Because I am getting something that is exponentially decreasing in time and the transient part will go to 0 only if it decreases exponentially in time. If I chose the constant here to be positive, I would have got solutions that increase exponentially in time and clearly that is not going to reach a steady state.

So, I require solutions that decrease exponentially in time and therefore, I require this constant to be negative. So, therefore, both of these parts of the solution of the separation of variables are consistent with each other.

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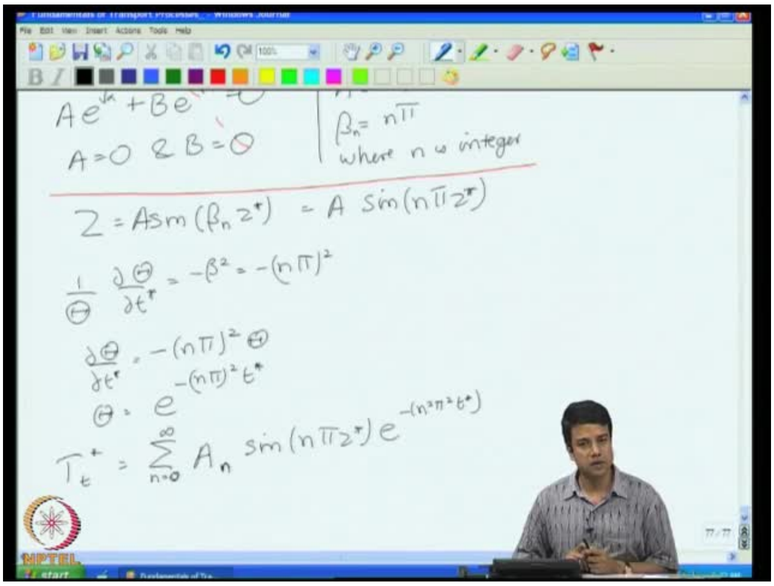

So, let us put all of that together and finally we get T t is equal to A sin n pi Z e to the power minus n square pi square t. So, this is one solution, it satisfies any differential equation. So, this satisfies the differential equation and the boundary condition, if n is equal to 0, 1, 2, 3, etcetera. So, no matter what the value of n, this form of the solution satisfies the boundary condition as well as the differential equation. We still have not imposed the initial condition; we will come back to how to impose that little later. But this is one particular solution of the differential equation, which satisfies both boundary conditions. This is true for n is equal to 0, n is equal 1, etcetera. So, the most general solution is the sum of all of these solutions. So, the most general solution that I can get is summation n is equal to 0 to infinity of A n times this. So, since it satisfies it for n is equal to 1, satisfies it for n is equal 2, n is equal to 3, etcetera, the most general solution is one which is the summation of all these solutions. Now, we have to determine what is the coefficients A n we have to find out what these coefficients are in order to find out what the final solutions.

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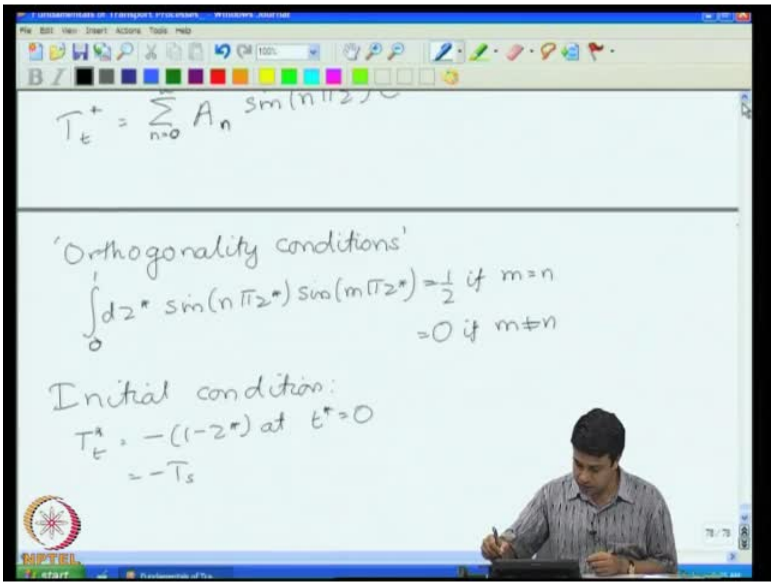

These coefficients can be determined using something called orthogonality conditions. And these conditions are defined on the domain between 0 and 1; Z star is equal to 0 Z star is equal to 1 was the domain of interest and these orthogonality conditions are defined on that domain.

These are easy to do by simple integration. If I take integral 0 to 1 dZ star sin of n pi Z star sin of n pi Z star. So, if I just take two functions, sin of n pi Z star and sin of m pi Z star and integrate from 0 to 1, this is equal to 1 by 2 if m is equal to n and it is equal to 0 if m is not equal to n.

So, in a sense, these functions defined on this domain 0 to 1 are all orthogonal to each other. If you take one function multiplied by itself and then integrate it from 0 to 1, you get a half. You should take one function multiplied by some other function and integrate from 0 to 1, you will get 0. So, every function, when integrated with any of the other functions, all the way from n is equal to 0 to infinity, it ends up being 0. How does this help us?

The initial condition we wanted was T star transient is equal to minus of 1 minus Z star at t is equal to 0. So, this was equal to minus the steady state temperature, because T transient plus T steady was equal to 0 at time T s equal to 0. Therefore, T transient was minus 1 minus Z star at t is equal to 0.

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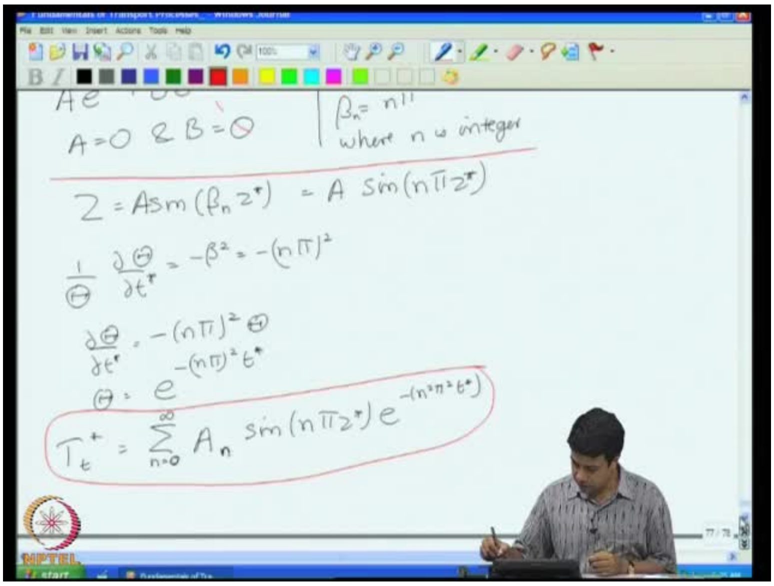

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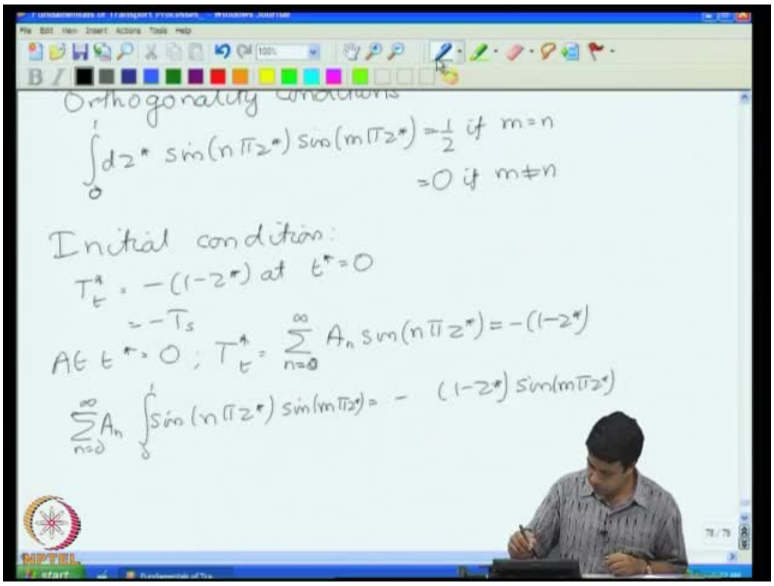

Now, we have the solution for the temperature field. Therefore, at t star is equal to 0, T star transient is got to be equal to summation n is equal 0 to infinity of A n sin of n pi Z star. And this, from the initial condition, is got to be equal to minus of 1 minus Z star. So, I have to determine all of these coefficients A n, all the way from minus equal to 0 to infinity from this particular equation.

So, in general, this equation contains an infinite set of coefficients, but this is where the orthogonality relations can be crucial in determining each of these coefficients and that is done as follows.

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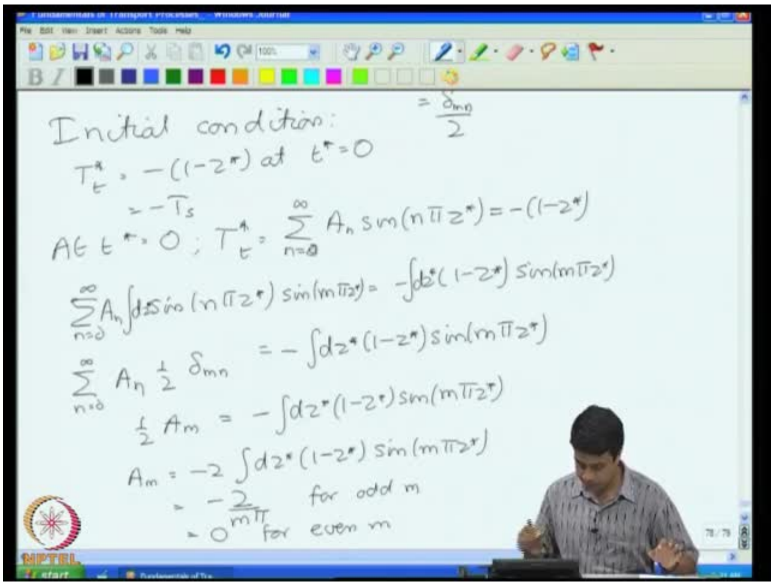

So, this equation that I have, I take both the left and the right sides multiplied by sin of m pi Z and then I integrate this from 0 to 1. So, I multiplied by sin n pi Z on both sides of the equation and I integrate from 0 to 1. What do I get on the left hand side? Summation n is equal to 0 to infinity times A n times half times something, which is 1 if n is equal to m and it is 0 if n is not equal to m. This thing, I will call it as the delta function; this delta mn is 1 if n is equal to m and it is 0 if n is not equal to m.

So, I just get half A n times delta mn sin Now, on the left hand side, I have the summation delta mn times A n from n is equal to 0 to infinity. So, I have n going to 1, 2, 3, 4, 5, 6 all the way to infinity, but delta mn is nonzero only when n is equal to m. For example, if m is equal to 23, delta mn is not nonzero when m is equal to 23. Therefore, on the left hand side, I will get only A23. If m is equal to 2, then the only term on the left hand side I will get is A2 divided by 2.

Since it is nonzero only when n is equal to m, this can effectively be written as half A m times this. So, this now gives us each and every one of the constants in that expansion. So, A m is equal to minus 2 integral dZ star 1 minus Z star sin of and you can evaluate this constant and this turn out to be equal to minus 2 by m pi for odd; m is equal to 0 for even.

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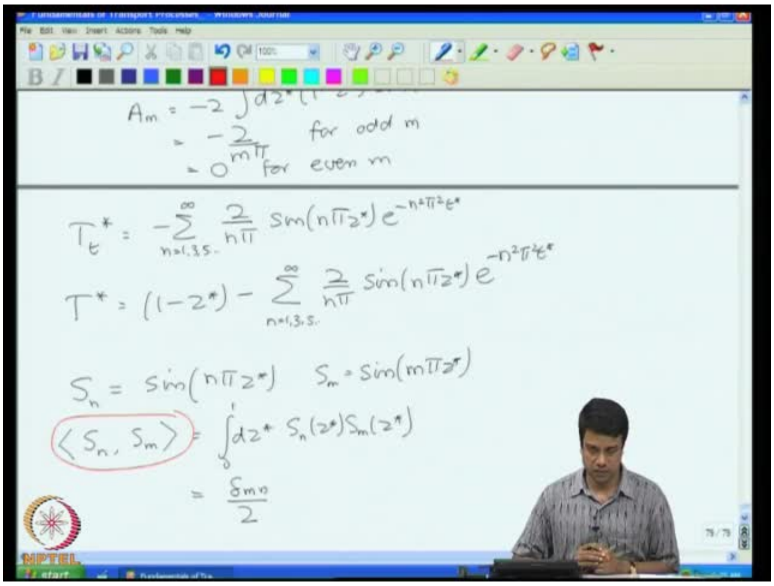

So, putting all these together, with this coefficient for m, I will get finally the transient temperature field, T star is equal to minus summation n is equal to 1, 3, 5, etcetera to infinity of 2 by n pi sin of n pi Z star e to the power minus n square pi square times t star.

So that is the final solution for the temperature field. This is only the transient part and you can see that it is equal to 0 at Z is equal to 0, it is equal to 0 at Z equal to 1, it is equal to 0, when the limit as t goes to infinity. Everything that was required with the transient part of the temperature field and the total temperature is equal to this plus the steady state that is 1 minus Z star minus 2 by n pi sin of n pi Z. So that is the final solution for the temperature field.

You can see that higher order terms in that expansion are decaying exponentially as e to the power minus n square. So, at any finite time these higher terms will become smaller because as n increases, this exponential is going as e to the power minus n square. So, because of that, the higher order terms will decay even faster. So, if I want a good numerical approximation, I can cut off the series at some particular finite value of n and I will get a good numerical approximation for this temperature field.

So, we managed to get a mathematical solution for this temperature field. How about a physical field for what it all means? What do these orthogonality relations mean? So, let us assume that I define two functions, S n is equal to sin of n pi Z and S m is equal to sin of m pi Z. I can define what is called an inner product of these two functions. I will use angular brackets for here, S n comma S m and is equal to integral 0 to 1 dZ S n of Z S m of Z and I did the calculation to show you that this is equal to delta mn by 2.

So, these two functions are orthogonal to each other. In a similar sense that for example, in 3-dimensional space, unit vectors are orthogonal to each other. For example, if you had a 3-dimensional coordinate system with unit vectors e x, e y and e z they are all orthogonal to each other. That means if I take the dot product of two vectors, e x dot e y, I will get 0; if I take the dot product of the same vector itself, e x dot e x, I will get 1.

So, the inner product here, which I have defined for you, is analogous to the dot product of two vectors. Therefore, the inner product is nonzero only if you take the inner product of one function with itself. If we take the inner product of one function with some other function, it turns out to be 0.

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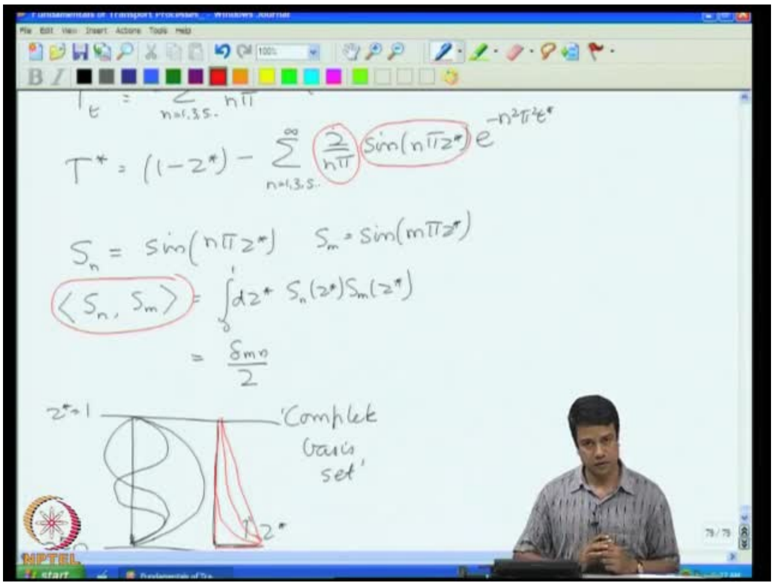

So, in that sense, each of these functions is orthogonal to every other function. What are these functions? Within this domain, I have Z star in this direction. For n is equal to 0, sin pi times Z star is 0, both at Z is equal to 0 as well as at 1. So, it goes to 1 at a half.

So, it has a behavior that goes something like this. Then I have sin 2 pi, which will go something like this, 3 pi and 4 pi, which will be a still higher harmonic and so on. So, basically, I have a series of function each of which is orthogonal to every other function within this domain and this forms what is called a complete basis set functions. So, any function in this domain, for example, the functions that we had for the temperature profile, which was initially was something like this and then it decreased in time until it went to the final linear profile. Each of these functions, at all intermediate times, can be expressed as the sum of these basis functions multiplied by suitable constants. So, any function, just as in a 3- dimensional vector space, can be expressed in the sum of the components times the unit vector.

So, one should think of these things as the unit vectors or the basic functions in this function space and these as the components of that basis function. So, basically you are decomposing the temperature profile into a set of bases functions. We know there is this conflict that any function can be expressed as sum of all of these functions. Each of these functions is orthogonal to every other function. And the only difference is that in vectors, you have only three directions. In this case, there is an infinite set of basis functions. As far as the analysis is concerned, that makes no difference.

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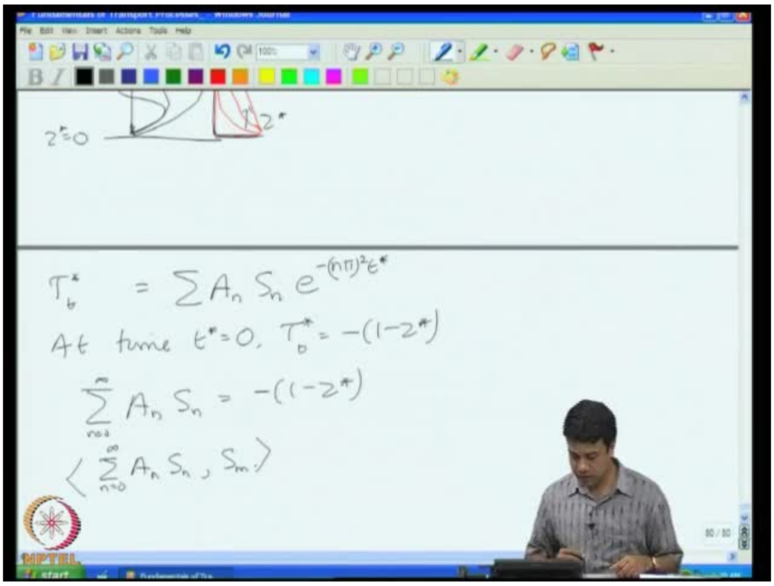

So, in my solution procedure, what I have done is to express T star of t as some summation of A n times S n times e power minus n pi the whole square times t star, where S n is a function of Z is equal to sin of n pi Z. I know that at time t star is equal to 0, this capital T, transient is equal to minus of 1 minus Z star. Therefore, the summation A n S n has to be equal to 1 minus Z.

Now, what I have done is I have taken the inner product of both sides with respect to S n. So, I have taken the inner product of summation n is equal to 0 to infinity of A n S n comma S n ,where the inner is defined as follows.

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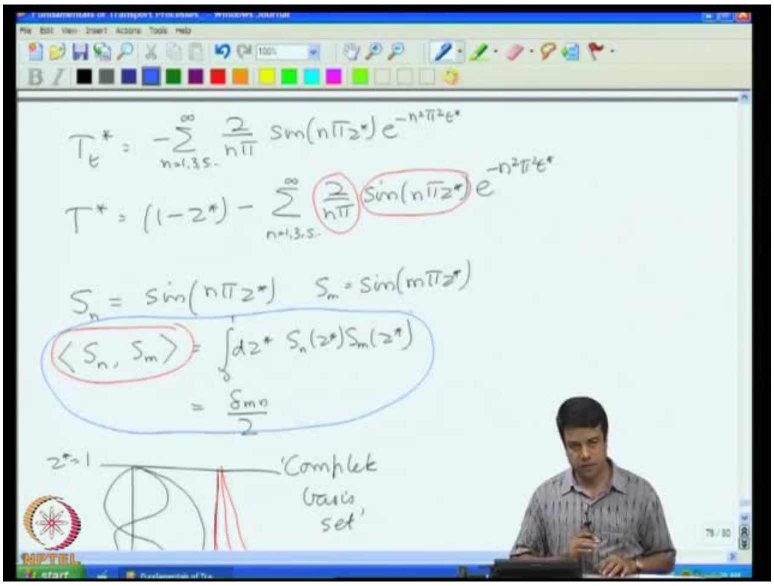

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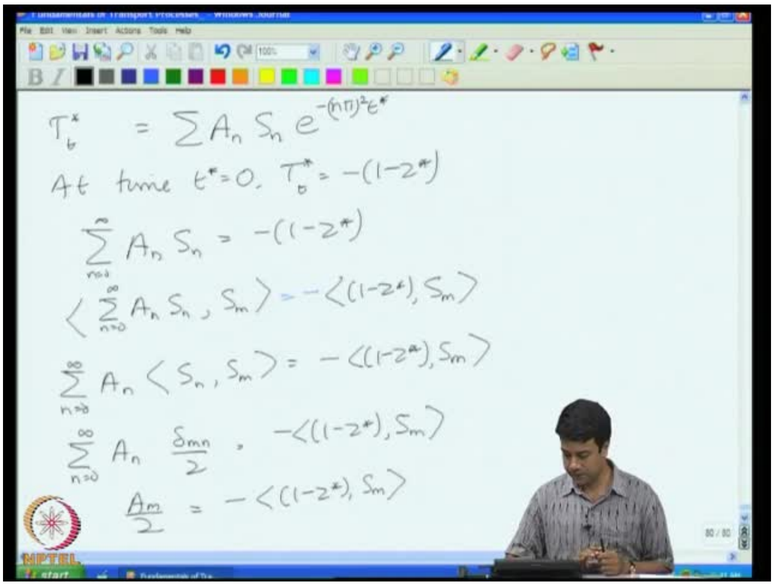

So, I take the inner product of both the right and the left hand sides, with respect to the basis functions. It is like saying if I have an equation, which says that one vector is equal to another; I can take both the inner those sides with respect to the unit vectors in a similar manner and taking the inner product of both these sides with respect to the basis functions comma S n.

Since the A ns are all constants, this is equivalent to writing summation n is equal to 0 to infinity of A n times S n, S m is equal to minus this inner product. S n and S m are orthogonal to each other. Therefore, n is equal to 0 to infinity A n by delta mn by 2, and delta mn is nonzero only when m is equal to n. Therefore, this will give me A m by 2 minus and that was how I determined all of those infinite set of constants in the equation.

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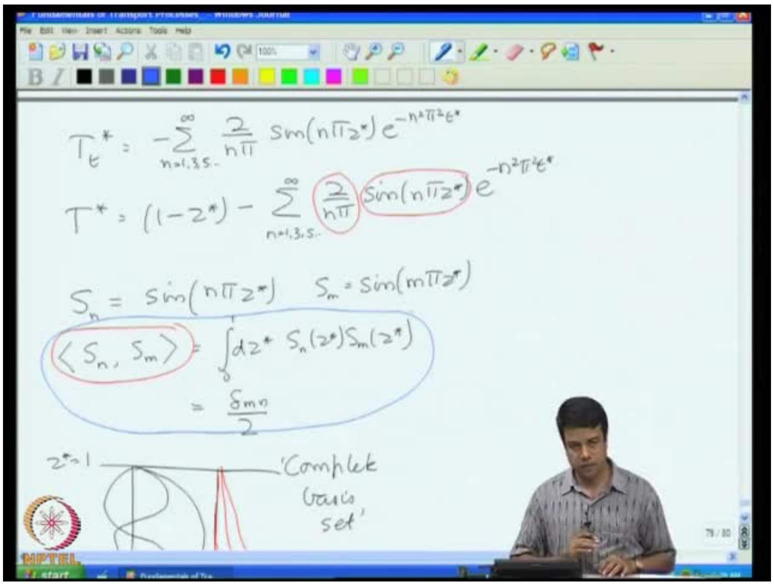

So, this process of expanding out the equation in terms of these basis functions here. In expanding out the entire domain from 0 to 1, in terms of this basis function, is equivalent to writing a vector in terms of the three basis vectors. In this particular case, you have infinite sets of basis functions, but any function can be expanded in that infinite set. The set is infinite, but there is no ambiguity in determining the components and the reason is because these functions are all orthogonal to each other. If they were not orthogonal to each other, when I took this inner product, I would get an a series of linear equations, which involved all of these coefficients.

However, since they are orthogonal to each other, these coefficients are nonzero only when n is equal to m and therefore, without any ambiguity, I can determine what each of these functions are. The values of beta n, that I had is equal to n pi and these are called the Eigenvalues of this problem. And this S n, which was sin n pi Z by L, are called the Eigenvectors or the basis functions for this problem, and we have defined the inner product of this in such a way that these basis functions are all orthogonal to each other, and in that way we have manage to determine the entire solution to the equation.

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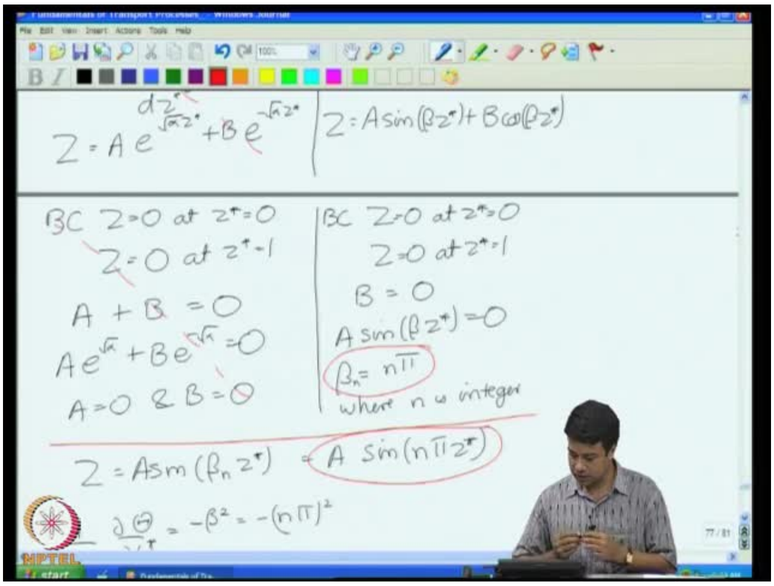

Important points to note: At the beginning, I said that I require homogenous boundary conditions in the Z direction. The reason is because it is only with these homogenous boundary conditions that I manage to get a discrete set of values of beta which satisfy this equation. It was only because I had homogeneous boundary conditions on both boundaries, that beta was equal to n pi. So, it is important in all these cases to ensure that you have homogeneous boundary conditions in all directions expect one. You have to reduce the problem to one, where there is homogeneous boundary conditions in all directions expect one and that one in this case, the initial time t is equal to 0 that is where the forcing is taking place. It is only if you have homogeneous boundary conditions that you will end up with a discrete set of Eigenvalues and a discrete set of Eigen functions.

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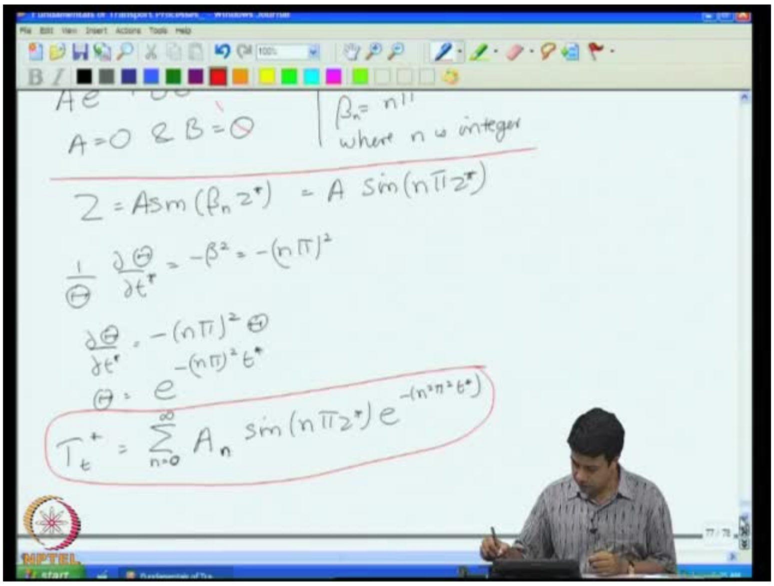

So, beta cannot take any value. It has to have value of an integer value of times pi and the Eigen function has to be of this form. For this particular equation, that Eigen function with the orthogonality relation defined in this manner gives you an orthogonal basis set and because the system is orthogonal, I can solve the equation using orthogonality condition to find out each of these coefficients uniquely. Once I have each of these coefficients uniquely, then I know the entire solution. This solution is an infinite series. So, if you wanted to determine the solution exactly, you would have to determine the infinite number of terms. However, the higher order terms in this series decay as e power minus n square; as n becomes large.

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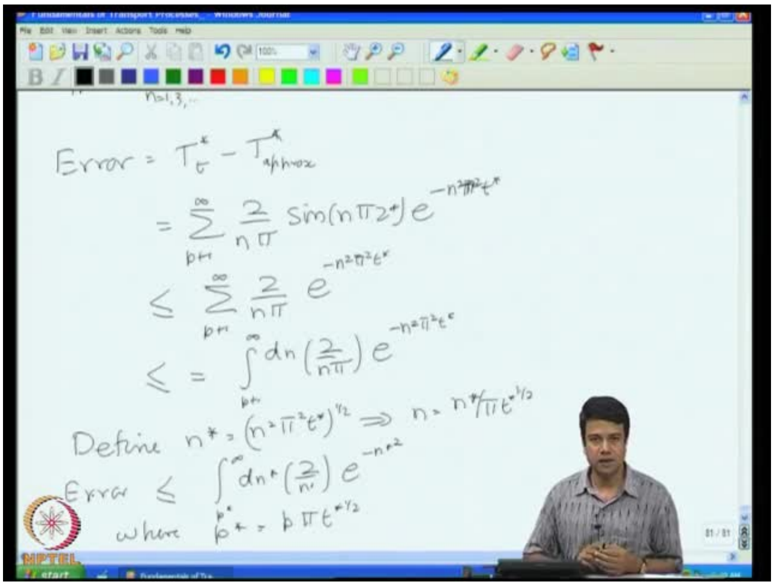

So, for example, I could get a good numerical approximation to this equation by writing it T star transient is equal to summation from n is equal 1, 3, etcetera to say up to some number p. p may be 50 or 100 and so on, of 2 by n pi sin of n pi Z star e power minus n square pi square t star. This is an approximate solution, because I have cut it off at some particular value of m; I have neglected the higher order terms in this expansion in n.

What is the error that I am likely to make because of this? The error will be equal to T star T minus the approximate solution that I have. So, this is going to be equal to the summation from p plus 1 to infinity of 2 by n pi sin of n pi Z e power minus n square times pi square t.

Now, we do not know exactly what the sin function as it is bounded, but we know that this has to be less than or equal to summation of p plus 1 to infinity 2 by n pi times e power minus n square pi square t. That is because the sin function is bounded from above; the sin function can be utmost 1. So, the error can be utmost summation of 2 by n pi times e power minus n square pi square t.

So, in the limit, as n becomes large I can actually approximate this as an integral, dn into 2 by n pi e power minus n square pi square t. And if I define another variable, n star is equal to n square pi square t to the half, which means that n is equal to n star pi by t star per half, then this error has to be less than or equal to integral from p star plus 1 to infinity of dn star into 2 by n star pi 2 by n star e power minus n star square, where p star is d is equal to p plus 1 by pi t star per half, where p star is equal to p pi t star per half.

So, for a given time, I can always choose my p in such a way that the error becomes less than some specified value. So, as time progresses, the value of p that I choose will be smaller and smaller, because if this integral is fixed to a constant value, I can truncate my series at a smaller value of p and still get the error below a prescribed value. For example, if I wanted a 1 percent error maximum, I will have to choose my p depending upon the value of t in such a way that this integral is less than that.

So, even though the series is infinite, I can use this to control the error in any expansion if I truncate it at some particular value in the series. So, we have theoretically got an infinite series, but in practice if t star is equal to 1, then this is a series, which causes 1 over n, but if t star is equal to 0, then the exponential all become 1. So, long as t star is larger than 0, the higher order terms are going to decay exponentially as n increases and therefore, I can truncate it at some point. The point that I truncated goes to lower values of n as time progresses. So, I control my error in this manner and get a good numerical approximation for the separation of variable solution.

So, you looked in some detail at the separation of variables procedure. I said that this is an important procedure and we will use it repeatedly later. I request you to go and revise this a little so that when it comes once again in later lectures, you will be familiar with this procedure and all the intricacies of this.

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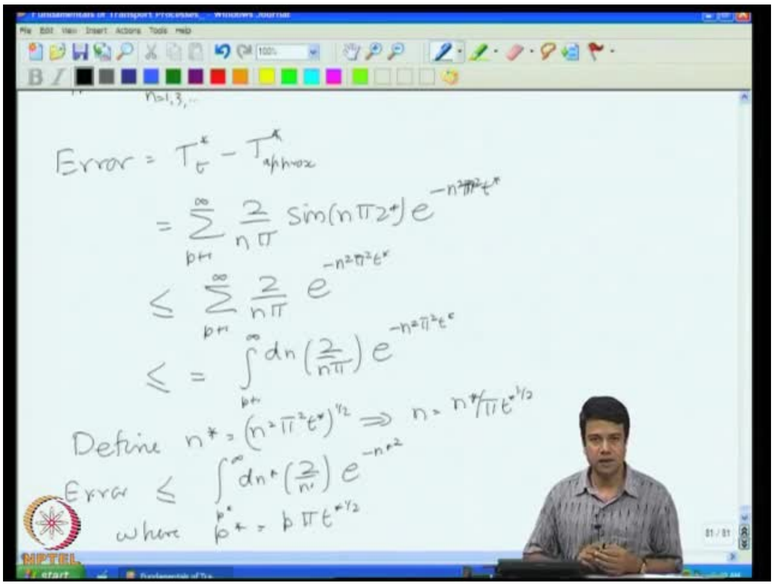

I will briefly review this once again before we go onto another method of solving the conservation equation in the next class and that is the method for oscillatory flows. If I had, for example, two plates and one was oscillated with some particular frequency, how do I find the velocity field within the entire domain? That we will continue in the next lecture, after we briefly revise the separation of variables-procedure that we conducted in this class.

So, we will see you in the next lecture and we will continue unidirectional flows in the next lecture.

Thank you.