Fundamentals of Transport Processes

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Module No. # 03

Lecture No. # 10

Unidirectional Transport Cartesian Co-ordinates-III (Similarity Solutions)

Welcome to the tenth lecture in the series on fundamentals of transport processes and we are now starting to look at how to frame differential equations from mass momentum analogy balances and different ways of solving them. We looked in the previous lecture at unsteady diffusion from a surface the solution that we get. The equation that we get, is a partial differential equation. It is a partial differential equation in both time, contains derivatives with respect to both time and space.

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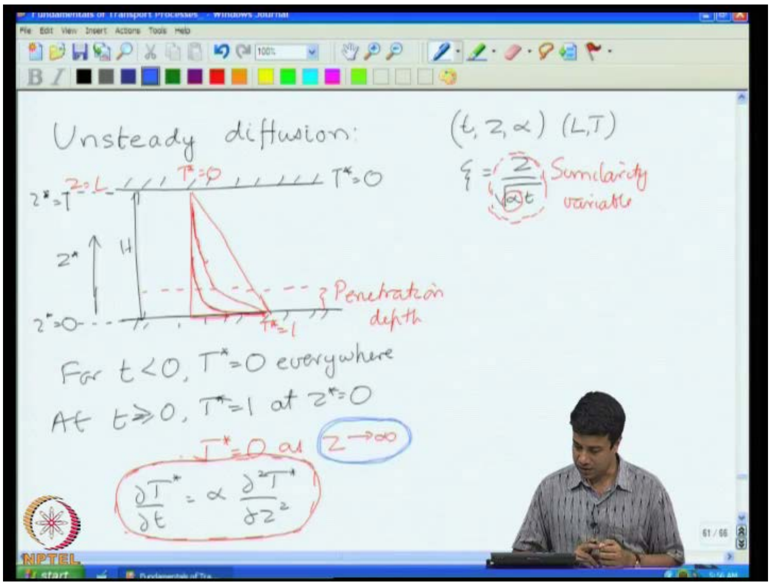

We saw how to get this equation from a shell balance on a differential volume. Again differential volume the thickness of that volume has to be small compared to the macroscopic scale H. In this case large compared to molecular scales. At this mesoscale we can define a continuum field for the concentration temperature and momentum and then write equations for that. Writing an equation is only half the task. The second half is to actually solve it in the case of partial differential equations, there are no systematic ways of solving them. We need physical insight. And this example illustrates the first such example where physical insight is used to simplify and then solve the problem.

In this particular case, we consider that the penetration depth was small compared to the total height of the flow and because it is small, the effectively that other boundary that boundary at Z is equal to L is so far away compared to the penetration depth that you can apply the boundary condition in the limit as Z goes to infinity. And we use that insight to define a similarity variable size equal to Z by square root of alpha times T and once we had that we could express the entire equation.

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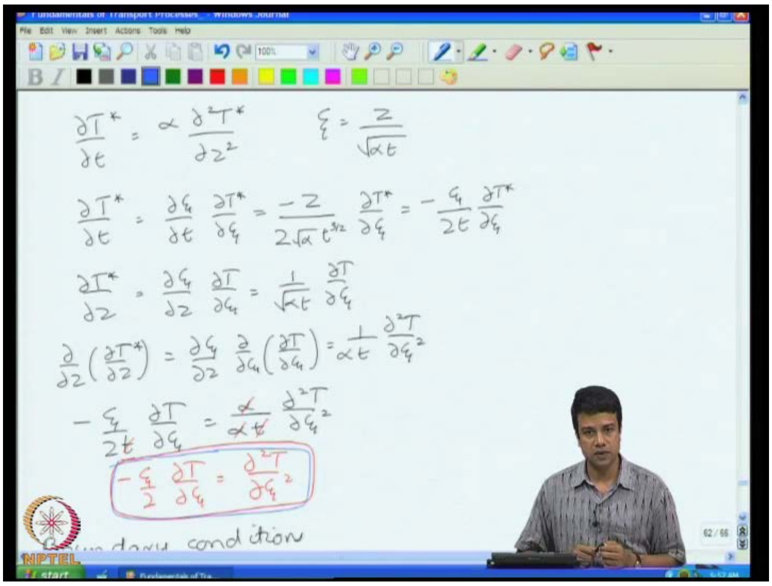

In terms of the similarity variable psi alone and since this reduces the equation from a partial differential equation in time and space to just an ordinary differential equation. Only in the similarity variable psi there are easy ways to solve ordinary differential equation especially if they are linear and we found out what the solution was in this particular case.

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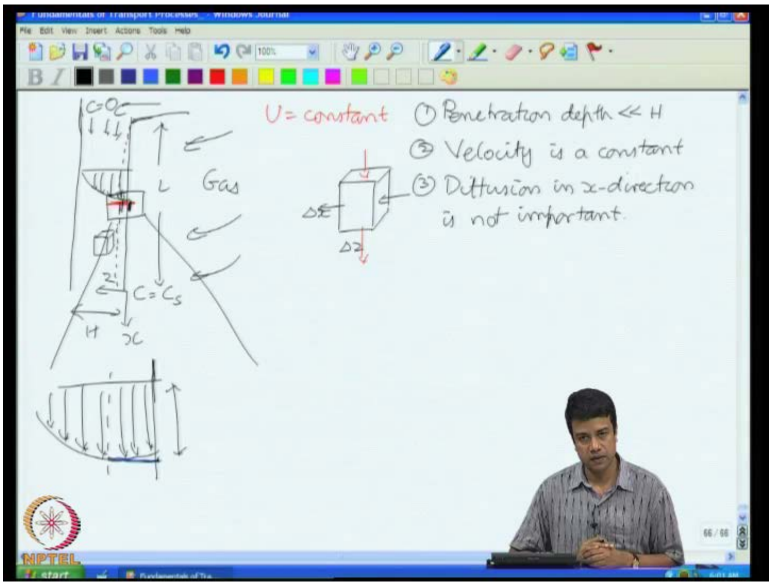

And I told you that even though we use similarity variables to solve this problem; I am sorry we use dimensional analysis to solve this problem. Similarity solutions can be used even in cases where dimensional analysis does not reduce it to just one variable and this was the example that we took a liquid gas contactor where there is a thin film of liquid flowing down the wall of the contactor and it comes into contact with gas which contains a soluble species and we would like to remove the soluble species from the gas into the liquid by dissolution at the interface in transport into the liquid film. So, that was the problem.

The concentration of the solute is 0 at the entrance. At the surface the liquid gas interface itself the concentration is C s where C is equal to C s is the solubility of the of the of the solute within the gas film and we started off making a couple of assumptions. The first was that the penetration depth of the concentration field into the film is small compared to the total height of the film H itself. This is reminiscent of the assumption that we had made in the previous lecture that the penetration depth of the temperature concentration or velocity fields was small compared to the total height H. Except in that case the penetration depth was a function of time. In this case the system is at steady state and you would expect the penetration depth to be a function of the x coordinate as more and as as as the film travels downstream; more and more gas gets absorbed into the, gets dissolved into the film and therefore, the penetration depth is likely to increase.

So that was the first assumption. Penetration depth is smaller compared to the film thickness. A second assumption was that the velocity is approximately a constant over distances comparable to the penetration depth. So, rather than having a velocity which is a function of position in the real film of course, the velocity is going to be a function of position and that is something we are going to calculate a little later.

What function of position the velocity is? What is the dependence of velocity on position? For the present, we will assume that within the thickness comparable to the penetration depth itself the velocity is approximately a constant. A velocity is approximately a constant and then we will use a shell balance in order to find out differential equation for the concentration field. That differential equation is going to have variations only in the x and Z directions.

The third assumption that we had made so let us just list out that so first is, penetration depth small compared to H velocity is a constant and the third assumption that we will make is that diffusion in x direction is not important. This once again comes back to our earlier discussion about the relative magnitudes of convection and diffusion in the Z direction across the flow. There is no fluid velocity and therefore, transport has to take place due to diffusion alone along the flow in the x direction. There is convection because the fluid is moving and there is diffusion and the assumption I am making here is the diffusion in the x direction is not important compared to convection and that will be true only if the speckle number defined in some way. Ratio of convection diffusion is large. We will see after we solve the problem what that speckle number should be.

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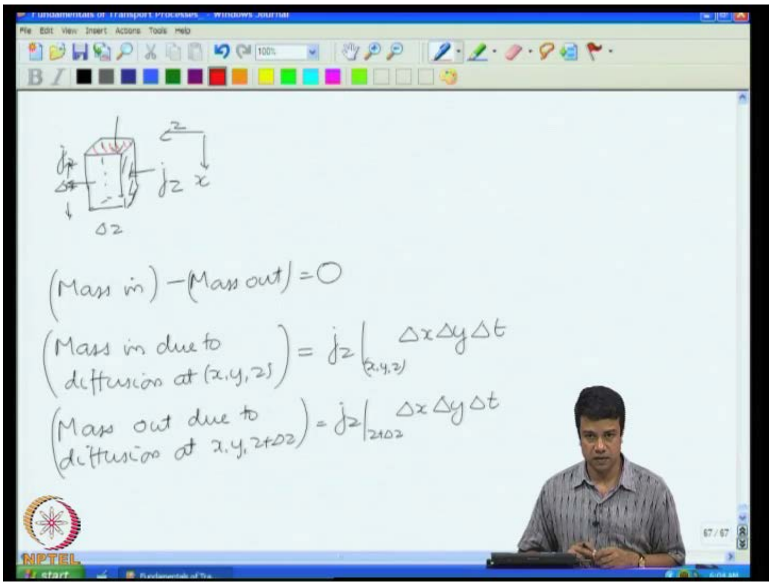

So, this is the differential volume that we consider. So, this width is delta Z and delta x is the stream wise distance. So, we have a coordinate system where this axis is Z and this axis is x. So, basically since this is a steady state problem the differential equations that I am going to get is going to basically say that mass in minus mass out has to be equal to 0 at steady state because there is no rate of change of mass concentration had to given position is independent of time.

So, the mass within a differential volume is not changing in time. There is no rate of change of mass and therefore, mass in minus mass out has to be equal to 0 and mass is coming in both due to diffusion as well as due to convection. The mass in due to diffusion at the location x y Z within a time delta T is going to be equal to the flux at x y z times. The area the area perpendicular to the Z direction is delta x delta y times delta T.

So that is the mass in on the left phase at the location Z due to diffusion mass out due to diffusion at x y z plus delta z. This is the mass that is going out at the location Z plus delta Z this is equal to j Z at Z plus delta Z delta x delta y delta T. So there is a mass going out due to diffusion at the right and left phases. How about the top and bottom phases? At the top phase we are neglecting diffusion in the x direction. Therefore, mass is coming in only due to convection mass is coming in only due to convection.

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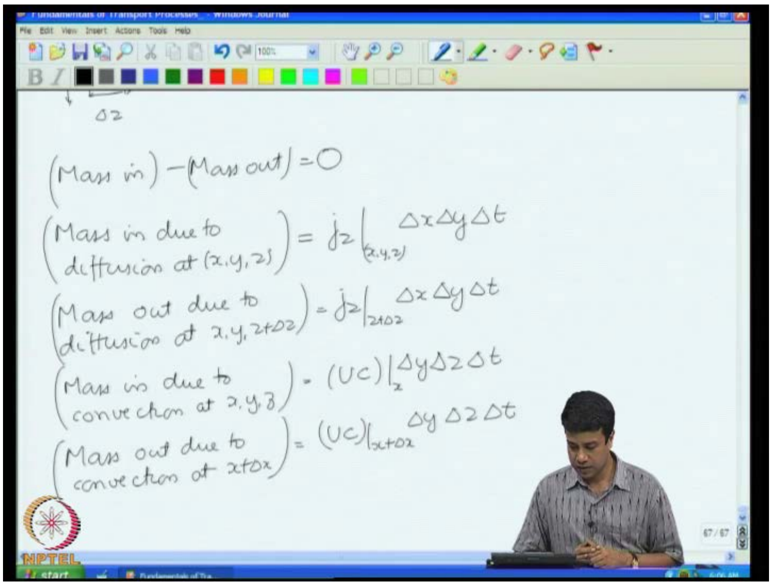

Therefore, I will have mass in due to convection at x y z is equal to the convective transport is equal to the concentration times, the velocity times the area. So, this is going to be equal to the velocity times the concentration at x times the area area perpendicular to the x direction is going to be equal to delta y delta z times delta T.

So, there is mass coming in at the top surface at x. There is mass going out at the bottom surface at x plus delta x and what is that mass out due to convection at x plus delta x will be equal to U times C at x plus delta x delta y delta Z delta T. So, that is the mass that is going out at x plus delta x when mass conservation equation states that mass in minus mass out is equal to 0.

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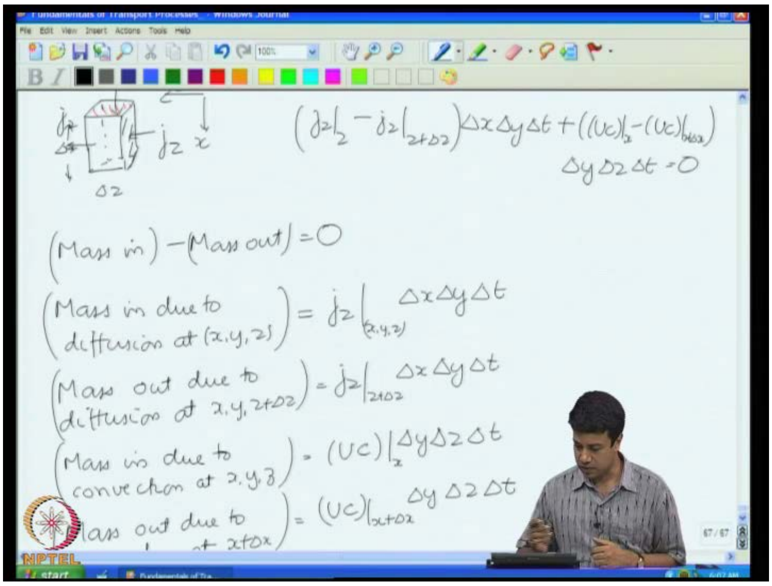

So putting all this together, I will get j Z at Z minus j Z at Z plus delta Z into delta x delta y delta T plus U times C at x minus U C at x plus delta x into delta y delta Z delta T. So, equal to 0 and I divide throughout by the delta x delta y delta Z delta T. So, if I divide throughout by delta x delta y delta Z delta T I will get j Z at Z minus j Z at Z plus delta Z divide it by delta Z plus U times C at x minus U C at x plus delta x by delta x is equal to 0 and now I take the limit of delta x and delta Z going to 0. These differences now become partial derivatives.

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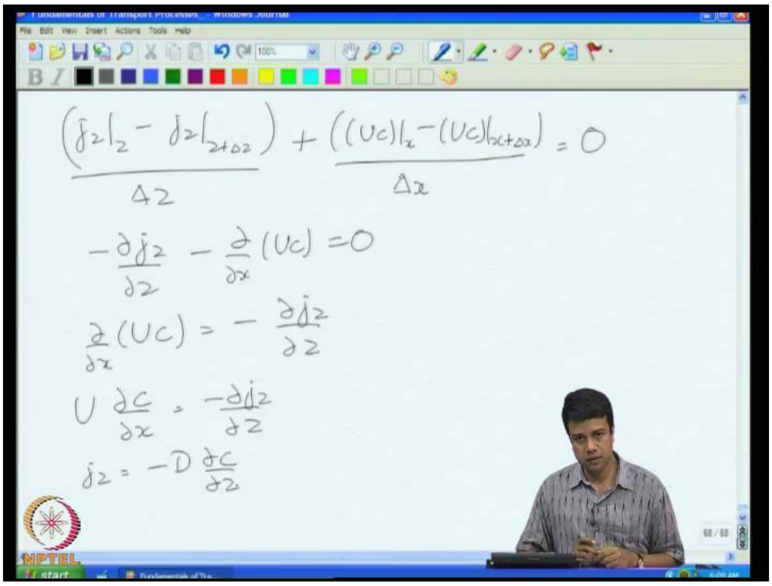

So I will get minus D j Z at D Z minus D by D x of U C equal to 0. I can alternatively write this as D by D x of U times C is equal to minus partial j Z by partial Z velocity s was at constant. In this case the velocity was a constant therefore, equivalent to writing U D C by D x is equal to minus D j Z by D Z and of course, I know that j Z is equal to minus D partial C by partial Z.

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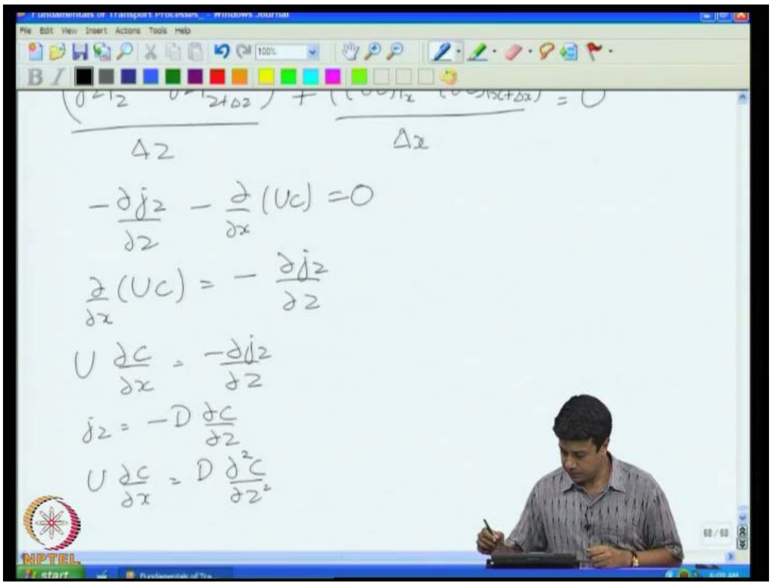

So, putting all this together I finally, arrive at the differential equation that governs the concentration field here. U times D C by D x is equal to D partial square C by partial Z square. Now, I had said that the concentration of the surface was equal to C s the concentration far away was equal to 0.

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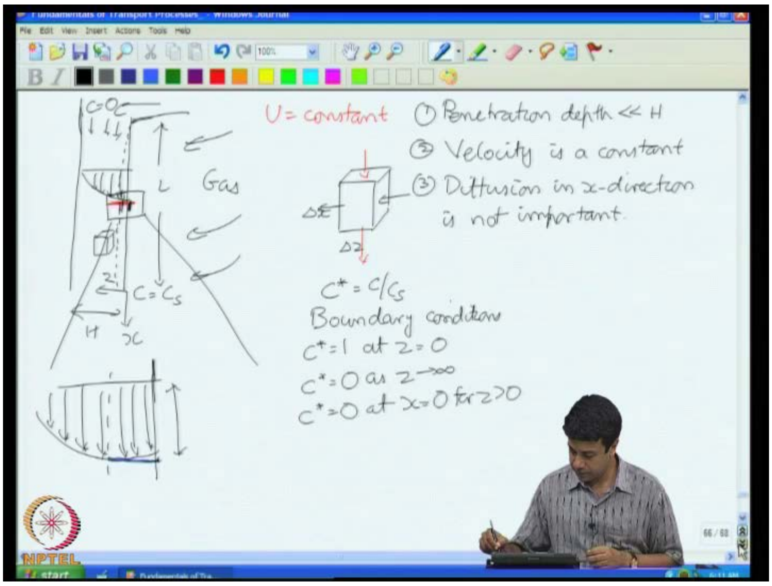

So, I could define a scale concentration field C star is equal to C by C s where C star is this scaled concentration field which varies from 1 at the interface to 0 very far away. How about this scaling for the penetration depth? I do not yet have a scaling for the penetration depth it has. I know it has to be some function of the Z coordinate. What I do now is that the boundary conditions boundary conditions in this case are that C star is equal to 1 at Z is equal to 0 at the interface itself C star is equal to 1. So, scale concentration field is 1 at Z is equal to 0 and C star is equal to 0 as Z goes to infinity.

Since the height penetration depth is small compared to height, I should strictly be enforcing a 0 flux condition at the left surface. If it is impenetrable, if the left surface were impenetrable, I would be imposing a 0 flux condition there. But, however since the left surface the distance H is large compared to the penetration depth, I can effectively enforce a boundary condition that C star has to go 0 as Z goes to infinity. In addition I also have a condition for x that is that the concentration is 0 at x is equal to 0 because the film is just entering the the contactor at that point. And therefore, the concentration has got to be equal to 0 at x is equal to 0 for Z greater than 0. That is the third boundary condition that I will use. The concentration has to be 0 at the point at which the fluid is entering into the contacting chamber.

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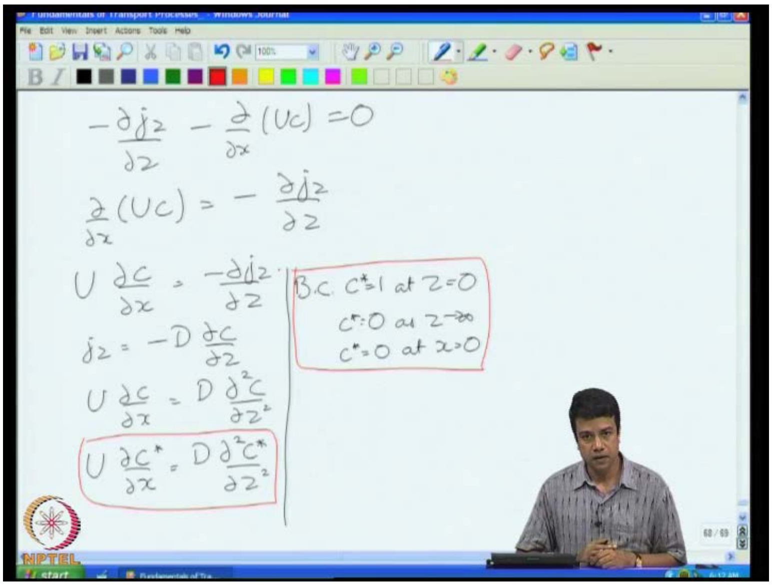

So, these are three boundary conditions. So, this is my differential equation when written terms of scaled coordinates becomes D C star by D x is equal to D partial square C star by partial Z square and I have boundary conditions and that is that C star is equal to 1 at Z is equal to 0 C star is equal to 0 as Z goes to infinity and C star is equal to 0 at x is equal to 0.

Look at this differential equation and these boundary conditions. So, you have to solve this differential equation subject to these boundary conditions. It looks exactly the same as the differential equation that I had for the unsteady flow case for the unsteady flow case. This was my differential equation D T by D T star by D T is equal to alpha times del square C and the boundary conditions that I used were that for T less than 0 T star is equal to 0 everywhere T star is equal to 1 at Z is equal to 0 and T star is equal to 0 as Z goes to infinity.

So, this equation with these boundary conditions is identical to this equation with these boundary conditions except that instead of T I have substituted x by U. Instead of T I substituted x by U except for that these two are the same and that means that the solutions also have to be the same.

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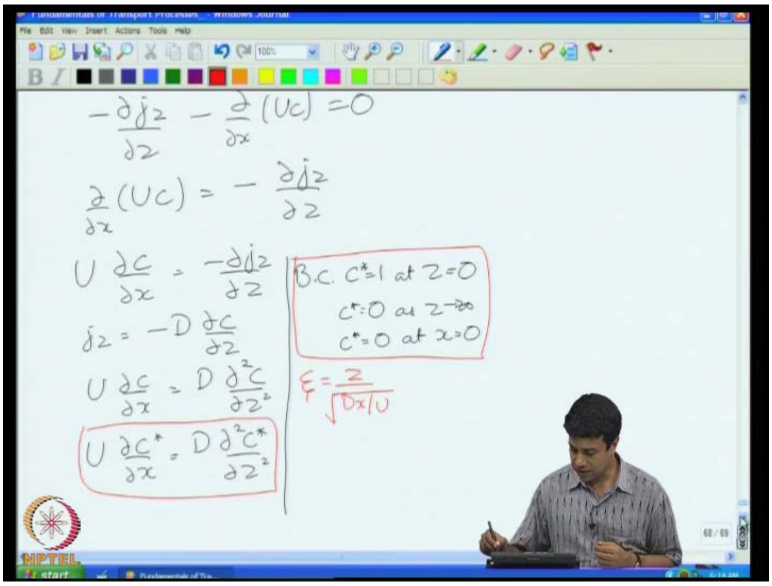

So I could for example, write a similarity variable psi as Z by root of alpha x by U I should write in terms of D square root of D x by U. If I write the similarity solution, this way only thing is from my previous similarity solution instead of writing the time T Z by root of D T I have written and said instead Z by root of D x by U. So, x by U takes the position of T in the similarity transform and so with that I will get quite easily the solution of this equation.

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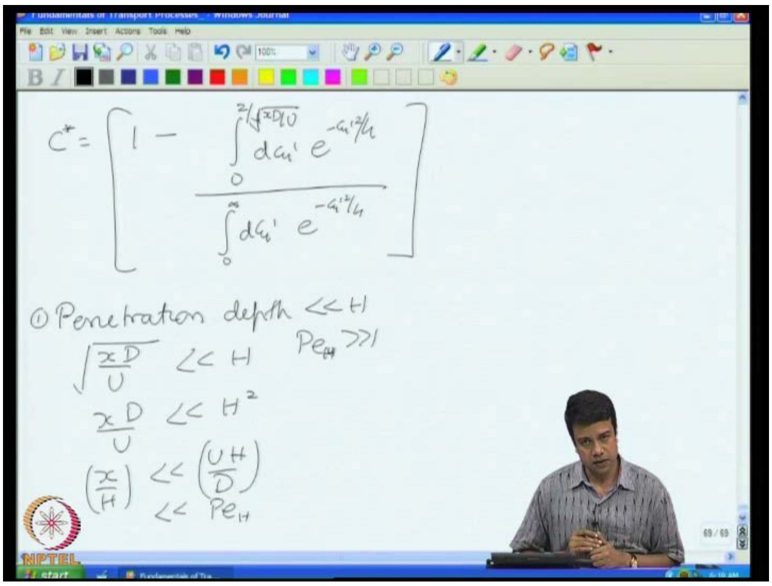

And that solution will be of the form C is equal to C star is equal to 1 minus integral 0 to square root of integral 0 to Z by root of x D by U D psi prime e power minus psi prime square by four by integral 0 to infinity D psi prime e power minus psi prime square by four. That is my final solution for the concentration field instead of Z by square root of D T. I have just substituted Z by square root of D x by U.

In the previous unsteady problem, the solution Z by square root of D T was a dimensional necessity. That was the only dimensionless group that I could get. The similarity variable was the only dimensionless group that I could get. In this case it is not a dimensional necessity I could define other dimensionless groups Z by x is one of them. So, it is not a dimensional necessity. However, we use the fact that the equations are of identical form. In order to write the solution quite simply so we solve this problem making various assumptions. Now, is the time to check all those assumptions and in order to prescribe the parameter regimes where this solution will be valid.

Assumption number one; penetration depth is small compared to H in the previous case. The penetration depth was square root of D times T in this case. The penetration depth is square root of x T by U. This is the penetration depth in this case. This has to be small compared to H that means that x D by U has to be small compared to H square or x by H has to be small compared to U H by T.

So, this is the first assumption U H by D is also effectively a speckle number based upon the length scale H based upon the thickness H. So, the first assumption, the penetration depth is small compare to H when the ratio of x by H is small compared to the speckle number. Therefore, the solution will be valid for distances large compared to the height H, the thickness H only if this speckle number is large. In other words U was large enough such that U H by D is large.

So, an implicit requirement here is the P H has to be large compared to one for this solution to be valid if P H is large compared to 1. The distance, the downstream distance x up to which the solution would be valid would be x by H is small compared to speckle number. The total length of the contactor was L in the beginning. I had assumed that the total length of the contactor was L.

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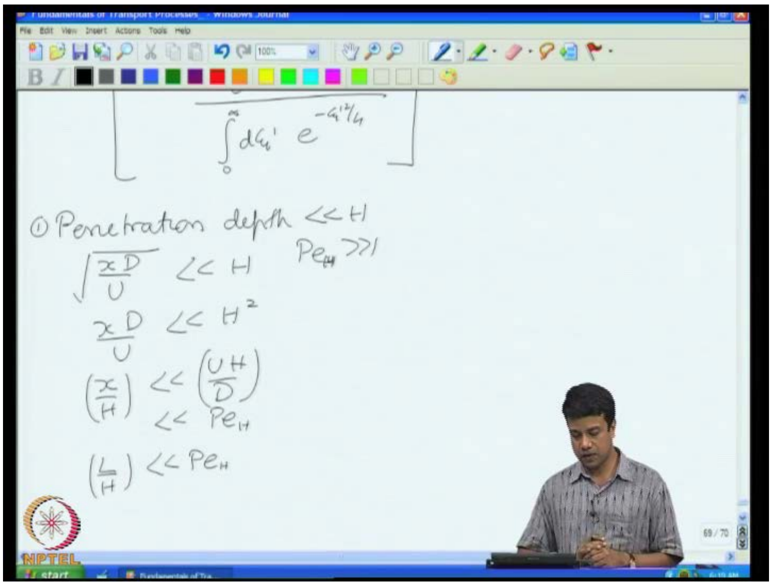

So therefore, the similarity solution is valid throughout the total length provided L by H is smaller than e H. So, there is a requirement for the solution to be valid throughout the entire depth. The second assumption that we had the velocity is at constant velocity is a constant over distances comparable to the penetration depth. So, what is the variation velocity very close to the surface? We will solve this problem a little later but, for the present we will use results that we will be deriving later in order to get limits on this because the gas is in contact with the liquid the shear stress at the surface of the liquid is actually very small because the gas viscosity is much smaller than the liquid viscosity. And because of that, as you approach the liquid interface the shear stress in the liquid has to go to 0 because otherwise you will have a large shear stress acting on gas with very low viscosity usually generate large velocities. Therefore, the shear stress in the liquid goes to 0. As you go very close to the surface shear stress goes to 0 means that the velocity gradient goes to 0 the velocity gradient goes to 0.

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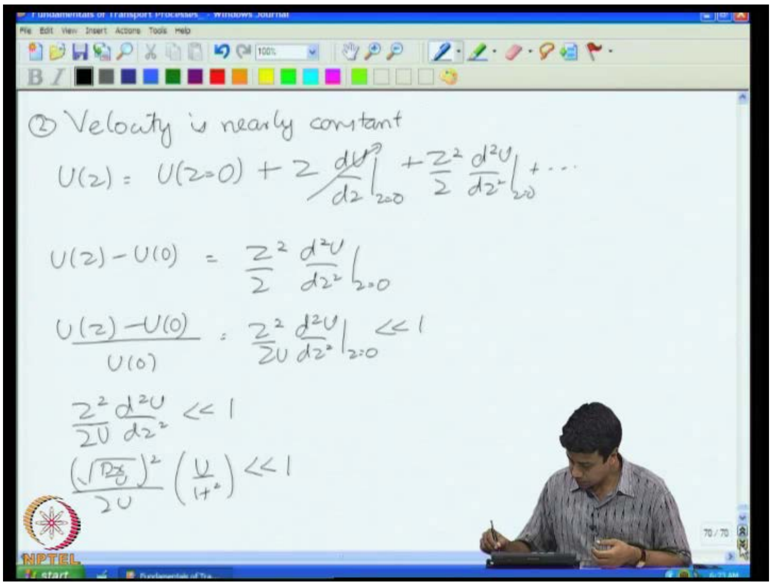

So assumption two; velocity is nearly constant over a distance Z. The velocity U at Z is equal to the velocity at Z is equal to 0 plus Z times D U by D Z at Z is equal to 0 plus Z square by 2 D square U by D Z square and Z is equal to 0 plus etc etc. Very close to the surface. I said that the shear stress at the interface is 0 which means that this term is equal to 0. So, if you are very close to the surface then U of Z minus U of 0 will be equal to Z square by 2 D square D U by D Z square and Z is equal to 0.

So, this is the velocity variation over a distance Z. You can consider the velocity to be approximately a constant. If this velocity variation over the distance Z is small compared to the velocity itself is the variation is small compare to the actual velocity then you can consider that the velocity is approximately a constant in this region. What is the distance over which we require the velocity should be a constant. That is of course, the penetration depth. Therefore, U of Z minus U of 0 divided by U of 0 will be equal to Z square by 2 D square D U by D Z square at Z equal to 0. We require the velocity to be a constant over distances of the order of penetration depth. That means that this has to be small compared to 1 U of Z minus U of 0 by U of 0 has to be small compared to 1 if Z is comparable to the penetration depth.

So, Z square by 2 D square U by D Z square has to be small compared to this to be 2 U has to be small compare to 1 Z is proportional to the penetration depth. So, Z Z is proportional to square root of D x by U. This is the penetration depth the whole square by 2 U and D square U by D Z square. We will see later on when we solve the problem. This is approximately U by H square has to be small compared to 1.

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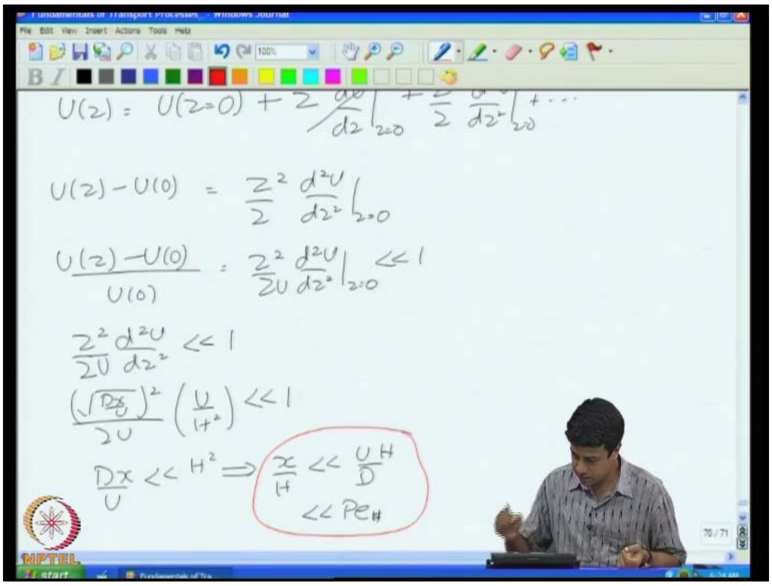

So, putting all this together we get D x by U has to be small compared to H square or x by H has to be small compared to U H by D speckle number based upon the height H. Exactly the same condition that we got over here. So, the condition for the penetration depth to be small compared to H is also the same as the condition for the velocity to be approximately a constant in this particular problem.

So so long as L by H is small compared to the speckle number, the speckle number is sufficiently large that L by H is small compared to the speckle number based upon the total length of the film. We will find that we can approximately assume that the penetration depth is small and that the velocity is approximately a constant near the surface.

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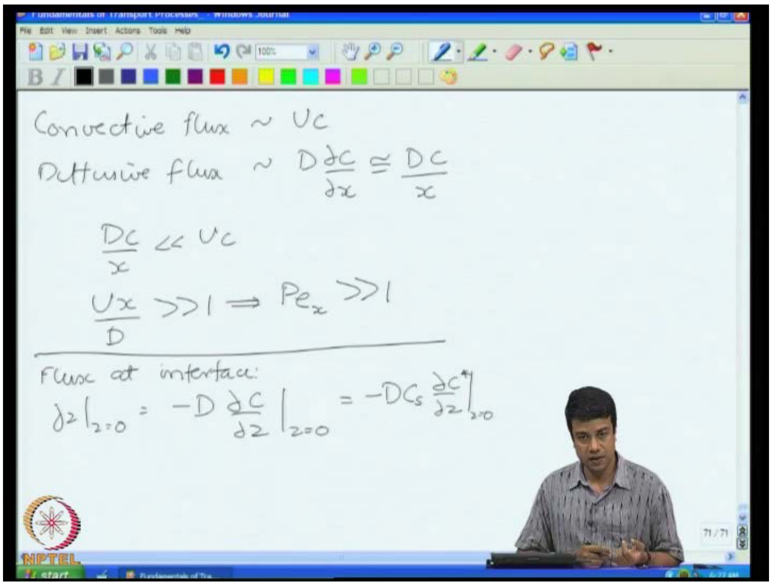

Approximation number three: Diffusion is not important in the Z direction. So, the convective flux in the Z direction is approximately equal to U times C diffuse flux goes as D times partial C by partial x which is approximately equal to D C by x. So, the diffusive flux can be neglected only when D C by x is small compared to U times C or U x by D is large compared to 1. This is a speckle number based upon the downstream distance x, not upon the thickness H. It is a speckle number based upon the downstream distance H rather than the thickness H x. So, this has to be large compared to one for the assumption that diffusion can be neglected in comparison to convection in the x direction.

So, we have looked at conditions under which the the assumptions are valid and now comes the time to actually calculate the correlation for the average flux as a function of the various other dimensionless proofs in the problem. First things first, we will first calculate the flux at a given location flux at interface j Z at Z is equal to 0 is equal to minus D times partial C by partial Z at Z is equal to 0. So, the flux at surface is equal to minus D times the concentration gradient at the surface expressed in terms of the scaled concentration field. This is minus D C s times partial C star by partial Z and Z is equal to 0. So, I am just expressing that C is equal to C s times C star I have a similarity variable. So, I have to express Z in terms of that. Similarity variable because that is where the solution came from.

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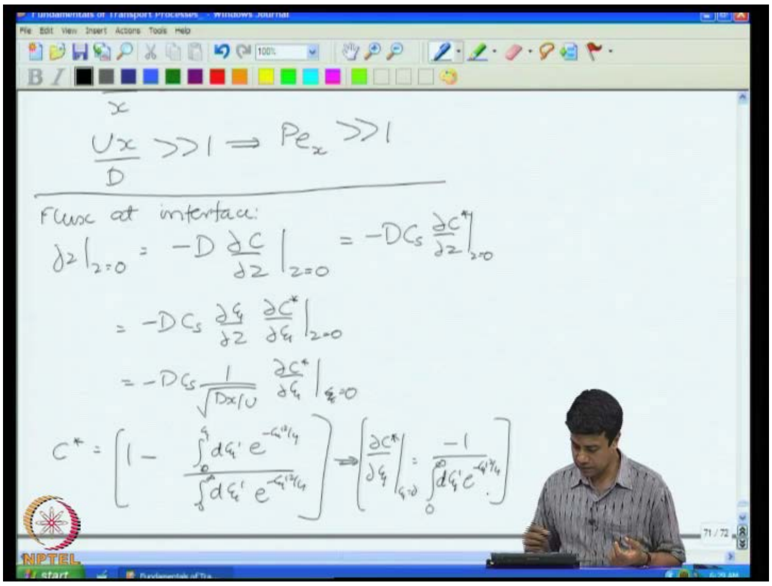

So therefore, this is equal to minus D C s into partial psi by partial Z partial C by partial psi C star by partial psi at Z is equal to 0. This is expressed in terms of the similarity variable and D C by D Z is equal to 1 by square root of D x by U instead of D T I have D x by U I have just partial C star by partial psi. Psi is equal to 0 and then I had my solution C star is equal to 1 minus integral 0 2 psi e psi prime e power minus psi prime square by four by integral 0 to infinity which implies that D C star by D psi at psi is equal to 0 is equal to minus 1 by integral 0 to infinity. Just taking the derivative right and then taking the value at psi is equal to 0.

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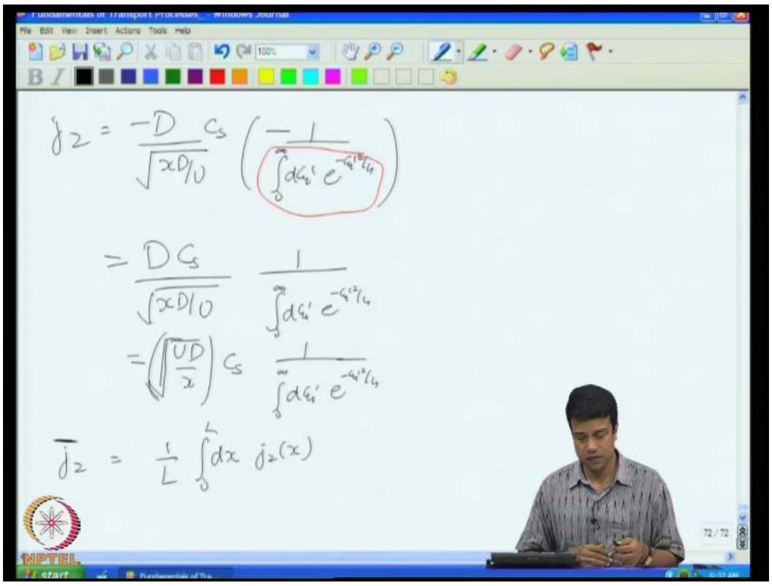

So putting all that together I get j Z is equal to minus D I should yeah by square root of x D by U into C s into minus 1 by integral 0 to infinity T psi prime T power minus psi prime square by four. Note that, this term here is a definite integral therefore, it is just a constant it has independent of any of the independent variables.

So therefore, this is equal to this is equal to D C s by square root of x D by U times 1 by integral 0 to infinity D psi prime e power minus psi prime square by four. I can also write this as square root of U D by x into C s into 1 by integral 0 to infinity. So, this is a flux. Flux is a function of position. So, function of downstream distance it decreases as 1 over square root of x. In the case of the transient problem the flux decreases 1 over square root of T. In this case we replace T by x by U and therefore, flux decreases 1 over square root of x. But, this is the local flux a correlation that you would write down, would involve the average flux over the entire length of the of the film over the entire length of the film integrated from 0 to L.

So let us write that the average flux j Z. I will use an over bar for average will be equal to 1 over L integral 0 to L D x of j Z of x. So, I sum up the flux at every location integrate over the entire length from 0 to L and divide by L in order to get an average flux. So, that is my procedure for getting the average flux.

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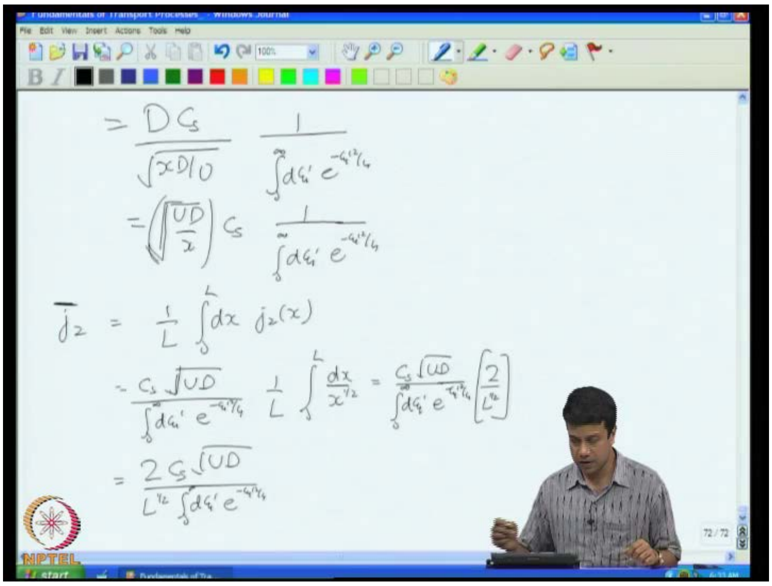

So, this is quite easy to do because the only dependence on x comes in through 1 over square root of x. So, this equal to C s square root of U D by integral 0 to infinity D psi prime e power minus psi prime square by four into 1 more L integral 0 to L D x by x power half and this integral is quite easy to do. This just becomes equal to C s square root of U D by integral 0 to infinity D psi prime e power minus psi prime square by four into 2 by L power half. So, that is my final expression for the average flux. This is equal to to 2 C s by square root of U D by L power half into integral 0 to infinity D psi prime e power minus psi prime square by four. So, that is my average flux.

Now, in order to get the nusselt number; I have to scale the average flux by the diffusion coefficient, the concentration difference and the total length in order to get my correlation for the nusselt number. Nusselt number is a scaled flux.

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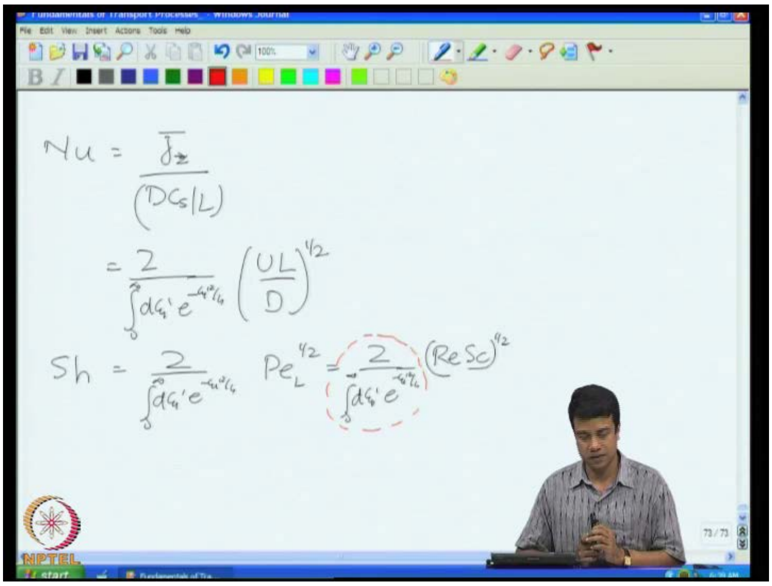

Therefore, the nusselt number; one way to define it is the average flux divided by D C s by L because D is the diffusion coefficient C is the scale for the concentration field. The average concentration difference between the surface and the fluid and L is the length scale in the proper.

So if I do this this will become equal to from this expression what I will get is two by integral 0 to infinity D psi prime e power minus psi prime square by four into U L by D power half and this is equal to two by integral 0 to infinity D psi prime e power minus psi prime square by four into the speckle number based upon L to the half. This is the speckle number based upon L to the half. I can also write this as 2 by integral 0 to infinity D psi prime e power minus psi prime square by four speckle number can be written as Reynolds number times Smith number to the half.

So this is the nusselt number for mass transfer. So, mass transfer this is also often called as the Sherwood number. The nusselt number for mass transfer is called the Sherwood number. This gives me the correlation between the Sherwood number, the Reynolds number and the schmidt number. And what is sitting in front is a constant here which we have determined explicitly by considering the entire field for the concentration, this thin film. We did not just do dimensional analysis for an average concentration difference as a function of an average flux.

We actually did solve the problem using shell balances subject to certain assumptions and got the variation of the entire concentration field within this thin film. So, this gave me the entire concentration field throughout the thin film evaluated on the basis of similarity arguments and subject to certain assumptions penetration depth is small compared to the film thickness. The velocity is approximately a constant diffusion in the x direction is neglected and we found out the conditions under which those assumptions are valid the penetration depth and the velocity approximations are always valid provided the length to the thickness ratio is small compared to the speckle number based upon the thickness of the film and we also require that the P e based upon x had to be large compared to one before the diffusion to be small in the stream wise direction in comparison to convection. From that we calculate the flux at every point on the interface. From that we calculated the average flux at the interface, scale the average flux to get a nusselt number and we found out that the nusselt number was related to the speckle number as the square root or this was the relation that I told you in case you have transfer at a free surface. You will always get a speckle number to the half power or Reynolds number time (( )) number half power.

So, that that that has been explicitly obtained here subject to the assumptions which we have defined the limits in which those assumptions are valid. So, this is the first of such correlations that we will get. We will also look at flow near surfaces where the velocity goes to 0 at the surface. In that case the power will be one third and I will tell you why the power is one third in that case. And we will also explicitly calculate the coefficients in that case.

So, in this problem of transfer to a film we use the analogy between the equations which were basically partial differential equations in x and z and the equations that we had got earlier for the steady, for the unsteady problem, unsteady diffusion into an infinite medium where it was a partial differential equation in T and Z. In the case of the unsteady problem, we use dimensional analysis. In this case, dimensional analysis alone is not sufficient. But, since the equations were exactly the same you were able to use the same solution procedure, derive at the same solution. Despite the fact that the similarity reduction is not a dimensional necessity and I just took you through how one calculates the nusselt number correlations. In this particular case we will see many more. During in the duration of this course all of similar form this illustrates for you the difference in approach between what we do here and what is done in unit operations. Here we try to solve the whole thing and from the physical understanding we try to obtain what should be the relation between the average nusselt number or the average dimensionless flux and the ratios of diffusion and convection.

So, this is a preview of the kinds of things that we will be doing much later in the course in in in different context. So, we looked at steady diffusion, we looked at unsteady diffusion, unsteady diffusion in the very limited context of diffusion into an infinite medium where the upper boundary was effectively located very far away. So, we could apply the boundary conditions in the limit as Z goes to infinity. What happens when the penetration depth is not small compared to the distance between the plates? In that case, the distance between the plates, the thickness is also a relevant parameter and that can be used to non dimensionalize the equations. So, it is not sufficient to just assume that the boundary condition can be applied as Z goes to infinity. We have to apply boundary conditions at a finite value of Z and how do we do that? So, that is going to be the next topic.

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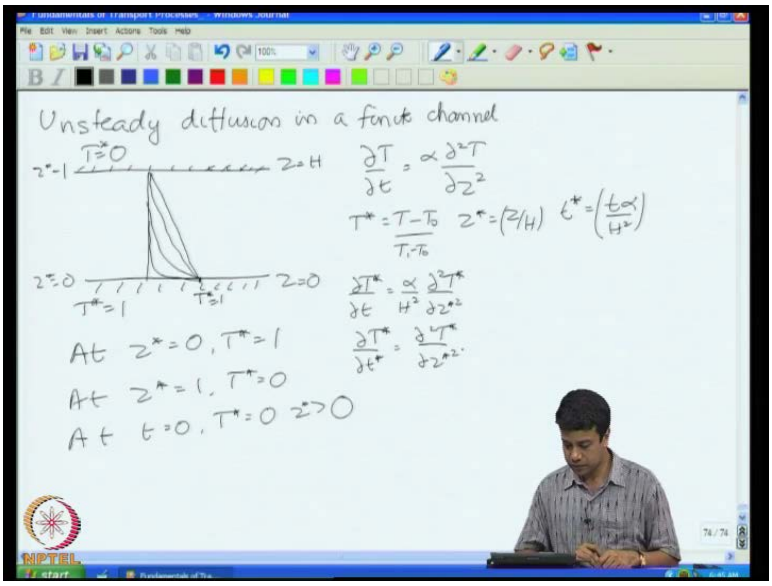

Situation is same as before Z is equal to 0 Z is equal to H problem is same as before. This is an unsteady diffusion problem. So, for example, T is equal to 0 on this surface T star the scale temperature is equal to 0 T star is equal to 1. We have already seen how to get the scale temperature from the un scale temperature and here Z star is equal to 0 and Z star is equal to 1 and initially at, so the boundary conditions are at Z star is equal to 0 T star is equal to 1 and at Z star is equal to 1 T star is equal to 0 and then the question is what is the initial condition? We will use the same initial condition as the solution of the unsteady problem. We will assume that at T is equal to 0 T star is equal to 0 everywhere for Z greater than or equal to greater than its greater than greater than 0. That is at the time T is equal to 0 and switching on a temperature T star is equal to 1. Here for T less than 0 the temperature was 0 everywhere.

So, in the final steady state the temperature has to be linear but, it will get to this linear temperature through a progression until it finally, reach finally, reaches the final steady state temperature and we want to know how it gets to this final steady state temperature. The differential equation is the same as what we have derived before partial T by partial T is equal to alpha. Partial square T by partial Z square Z, square the differential equation is same as before and now I said it is always a good idea to define dimensionless groups to non dimensionalize the variables. In the previous example, there was a deficit of dimension. So, there was only one similarity solution in the present example. One can find sufficient dimensions because it is always possible to non dimensionalize Z by H so I define T star is equal to T minus T naught by T 1 minus T naught and Z star is equal to Z by H. So that, Z is equal to H times Z star and if I rewrite my equation here, I get partial T star by partial T is equal to alpha by H square partial square T star by partial Z star square.

So, that is my equation and now I can use H square by alpha to scale temperature to get a dimensionless temperature. You can see that this left hand side has dimensions of one over time in the right hand side, the temperature is dimensionless, Z is dimensionless. Therefore, alpha by x square has dimensions of one over time alpha is length square per time, the thermal diffusivity and so alpha by H square has dimensions of one over time. Therefore, I could not dimensionalize the time by H square by alpha. So, I could define a non dimensional time T star is equal to T alpha by H square and in terms of this, my final equation becomes partial T where partial T is equal to partial square T by partial Z square.

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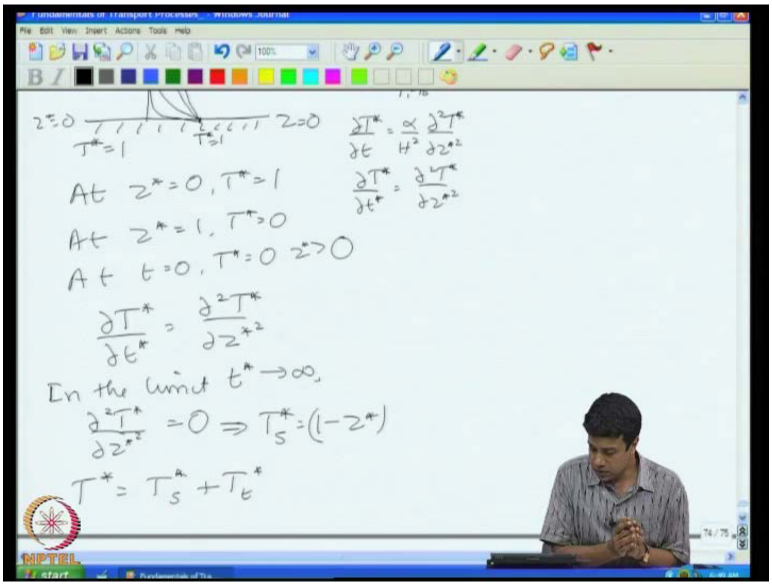

So this is my final equation for the temperature profile. This is my final equation for the temperature profile. With these boundary conditions at Z is equal to 0 T star is 1 at Z is equal to 1 T star is equal to 0 and T is equal to 0 T star is equal to 0 for all Z star greater than 0 and this is still a partial differential equation. Previous case we had reduced it to an ordinary differential equation on the basis of dimensional analysis. In the present case we cannot do that.

So, how do we solve it? The procedure that we will use for solving this is a procedure called separation of variables and I will explain it in some detail because this is a procedure that will be used repeatedly. Even we analyse more complex problems. First things first what happens at long times in the limit T star going to infinity? You would expect the solution to reach some steady state at long in the long time limit. You would expect the solution to reach some steady state and what is that steady state? That steady state is when the temperature profile has attained a linear temperature profile. Partial square T star by partial Z star square is equal to 0. This problem was solved earlier if you recall when you first read shell balances when you first read shell balances (( )). We actually solved for the temperature field in a steady state when there is no variation in time. The differential equation is D square T star by D Z star square is equal to 0 and T star was equal to 1 minus Z star. So, we solved it in the very first lecture on unidirectional flows.

So, this is the same differential equation. Therefore, this is the steady solution which implies that T star is equal to 1 minus Z star. This is the steady solution the steady solution in the long time limit. If the heat transfer had been going on for a long time finally, the the temperature would reach a linear temperature profile. However in the initial state the temperature everywhere was T 0 and because of that there was a deviation from the steady temperature.

So therefore, the temperature can be written as steady part plus a transient part. Transient means it is transient in time. It is in the in in in the limit of long times, this transient part has to go to 0 because the temperature has to be its steady value. So, the transient part gives me the departure from the final steady value because I had an initial condition which is not the same as the final steady temperature profile. That initial temperature profile was different from the final state temperature profile because of that I have some temperature which is different from the steady state value. At short times in the long time limit ultimately it will go to the steady state value.

So, there is a first first separation that I make of the temperature into a steady and a transient part. This is important. It is whenever separation of variables problems are solved. It is always necessary to separate the temperature into a steady and a transient part at the end of this the next lecture when actually derive the final temperature profiles. I will tell you precisely why it is necessary to separate into the into a steady and a transient part and once you have done that right, you have to write differential equations for the steady and a transient part. Separately one thing is here to write the differential equations for the steady and transient part separately. Secondly, you have to also write boundary conditions for the steady and transient part separately.

So, how to write the differential equations? How to write the boundary conditions and how to solve them? Will be the subject of the next lecture. The method that we will use is one called separation of variables, where you we will separate the temperature into two functions. One which depends only upon position and the other which depends only upon time, solve for each of those separately and then put them back together to get the final solution for the temperature field.

So, that will be the agenda for the next lecture. Diffusion in a finite channel solved using unsteady diffusion in a finite channel, solved using separation of variables. So, we will see you in next class where we will continue this calculation.

Thank you.