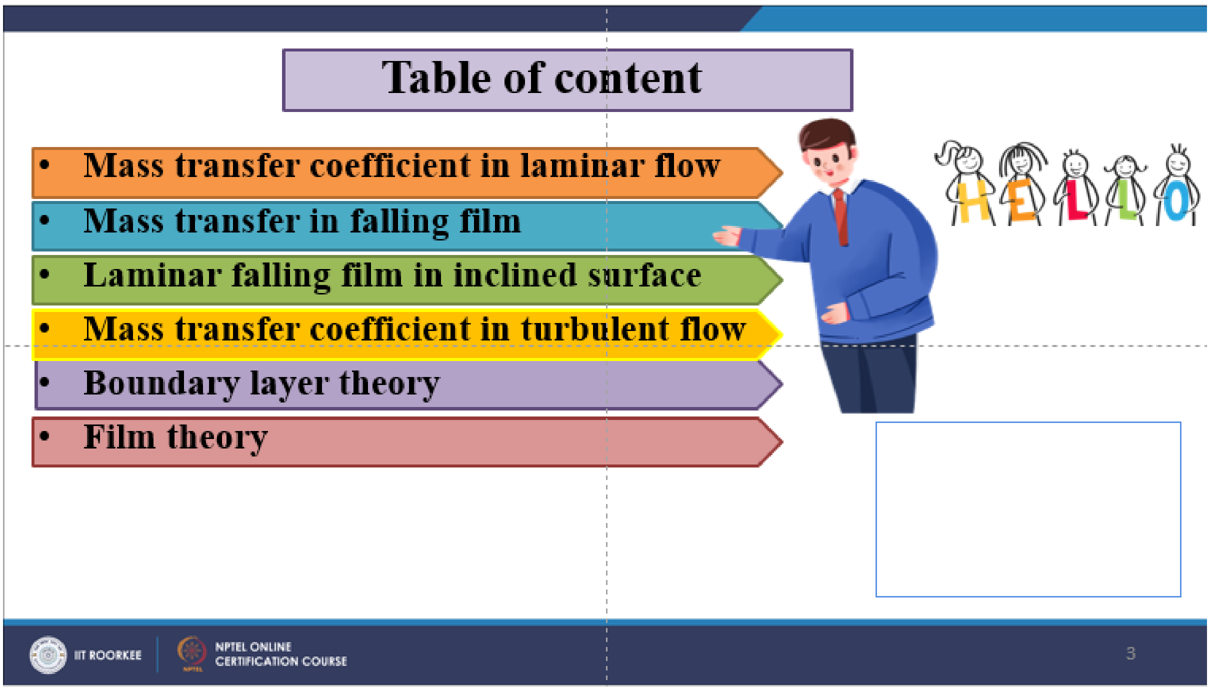
Polymer Process Engineering

Prof. Shishir Sinha
Department of Chemical Engineering
Indian Institute of Technology-Roorkee

Lecture – 24

Mass transfer phenomenon in polymers: Laminar flow and boundary layer conditions

Hello friends, welcome to the next episode of Mass Transfer Operations. In this particular segment, we are going to discuss the laminar flow and boundary conditions in mass transfer operations. So, let us have a brief outlook on what we discussed in the previous lecture, we discussed the mass transfer coefficient in the previous lecture, then different types of mass transfer coefficient, and then we discussed the equimolar counter diffusion in two components. We established the relationship between the mass transfer coefficient and we discussed the mass transfer coefficient and film thickness. Apart from this, we discussed the dimensionless numbers in mass transfer operations. In this particular segment, we are going to discuss the mass transfer coefficient in laminar flow.



We are going to discuss the mass transfer in falling film. Apart from this, we will discuss the laminar falling film in the inclined surface. We will also discuss the mass transfer coefficient in turbulent flow. Apart from this, we will discuss boundary clear theory and film theory, all these things we are going to discuss in this particular section.

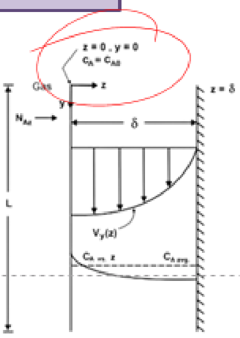
Mass transfer coefficient in laminar flow

- **In principle** we do not need to study **the mass transfer coefficient in laminar flow conditions.**
- **A uniform method** of dealing both **laminar and turbulent flow** is nevertheless desirable.
- We shall **choose one relatively simple situation** to illustrate **the general technique** and to provide some basis for considering **turbulent flow.**

Now, let us talk about the mass transfer coefficient in laminar flow. In principle, rather we did not, we do not need to study the mass transfer coefficient in laminar flow conditions. A uniform method dealing this both laminar flow and turbulent flow is nevertheless desirable. So, we shall choose one relatively simple situation to illustrate the general technique and to provide some basis for considering turbulent flow. Now, this is the particular figure.

Mass transfer coefficient in laminar flow

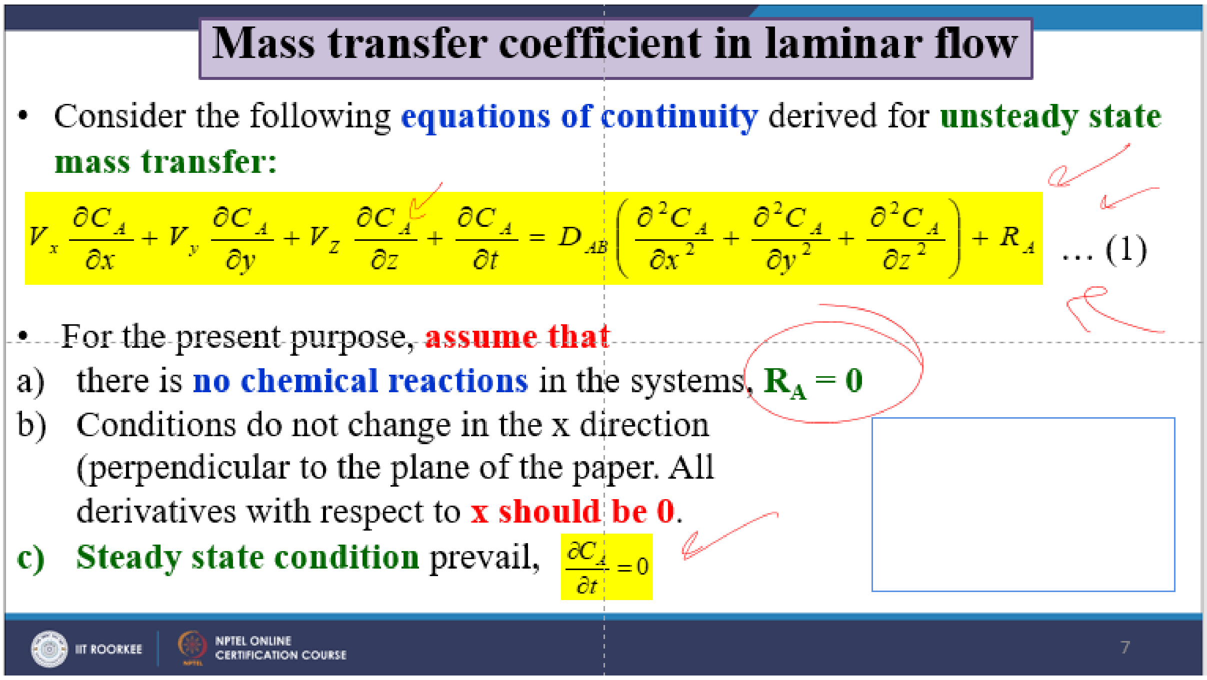
- Figure shows **a liquid falling in a thin film** in a laminar flow **down a vertical flat surface** while being exposed to a gas A, which dissolves in the liquid.
- The liquid contains a **uniform concentration C_{A0}** of A at the top.
- At the liquid surface **the concentration of the dissolved gas is C_{Ai}**. in equilibrium with the pressure of A in the gas phase, since $C_{Ai} > C_{A0}$ gas dissolves in the liquid.



Treybal R.E., "Mass Transfer Operation", 3rd Ed., 1980, McGraw Hill.

This particular figure shows a liquid falling in a thin film in a laminar flow and down a vertical flat surface while being exposed to gas A, in that case, which dissolved in that particular liquid. So, this is a very common phenomenon when a gas is usually dissolved in a particular liquid like in a scrubbing operation. So, this liquid contains a uniform concentration of C A, not of A at the top. At the liquid surface, the concentration of the dissolved gas is C A1.

In the equilibrium with the pressure A in the gas phase, now since C A1 is greater than C A0, gas dissolves in the liquid. The problem is to obtain the mass transfer coefficient kl with which the amount of gas dissolved after the liquid falls a certain distance l, this can be computed. The problem is solved by the simultaneous solution of the equation of continuity for component A with the equation which usually describes the liquid motion in the Navier-Stokes equation. The simultaneous solution of a formidable set of partial differential equations becomes possible only when several simplifying assumptions are made. Now, let us consider this particular equation which is derived for unsteady state mass transfer.



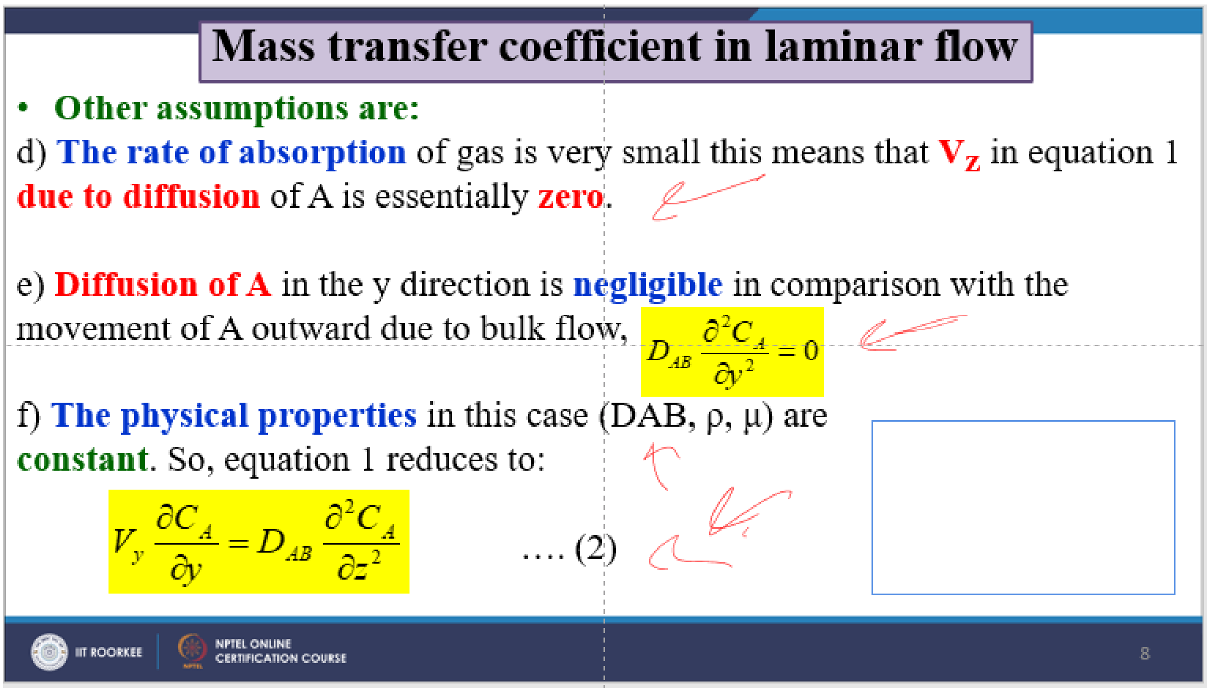
Continuity equation

$$V_x \frac{\partial C_A}{\partial x} + V_y \frac{\partial C_A}{\partial y} + V_z \frac{\partial C_A}{\partial z} + \frac{\partial C_A}{\partial t} = D_{AB}\left(\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} + \frac{\partial^2 C_A}{\partial z^2}\right) + R_A$$

At steady state condition;

$$\frac{\partial C_A}{\partial t} = 0$$

Here, vx del C over del x plus vy del C A over del y plus vz del C over C A over del z plus del C A over del t is equal to D AB, del 2 C A over del x2 plus del 2 C A over del y2 plus del 2 C A over del z2 plus rA. This is the equation number 1. For the present purpose, assume that there is no chemical reaction within the system. So, we can take rA is equal to 0 and condition do not change in the x direction perpendicular to the plane of the paper. All derivatives with respect to x should be 0.



$$D_{AB}\frac{\partial^2 C_A}{\partial y^2}=0$$

$$V_y\frac{\partial C_A}{\partial y}=D_{AB}\frac{\partial^2 C_A}{\partial z^2}$$

Now, if we take the steady state condition previous, in that case, del C A over del t is equal to 0. Now, we can take the other assumptions too. One is that the rate of absorption of gas is very small. This means that vz in equation 1, this is equation 1, due to the diffusion of A is essentially 0. A diffusion of A in the y direction is negligible in comparison with the movement of A outward due to bulk flow and that is D AB del 2 C A over del y2 is equal to 0 and the physical properties in this case, that is D AB rho mu are constant. So, equation 1 which can this equation can be reduced to v at y del C A over del y is equal to D AB del 2 C A over del z2 and we can say that this is equation number 2. Now, if we talk about the mass transfer in falling film, now, this is states that A added to the liquid running down at any location z over an increment of in phi.

$$\mu\frac{\partial^2 V_y}{\partial z^2}+\rho.g=0$$

$$V_y=\frac{\rho.g.\delta^2}{2.\mu}\left[1-\left(\frac{z}{\delta}\right)^2\right]$$

Mass transfer in falling film

- This states that A added to the **liquid running down at any location z** over an increment in y, got there by **diffusion in the z direction**.
- The equation of motion under this condition will be again reduces to

$$\mu \frac{\partial^2 V_y}{\partial z^2} + \rho . g = 0 \quad ..... (3)$$

- The solution to equation 3 with the conditions $\mathbf{V_y = 0}$ **at** $\mathbf{z = \delta}$ and that $\mathbf{dV_z/dz = 0}$ **at** $\mathbf{z = 0}$, is well known

$$V_y = \frac{\rho . g . \delta^2}{2 . \mu}\left[1 - \left(\frac{z}{\delta}\right)^2\right] \quad ..... (4)$$

$$V_{y,max} = \frac{\rho . g . \delta^2}{2 . \mu}$$

$$V_{y,avg} = \frac{1}{A}\int V_y dA = \frac{1}{W\delta}\int_0^W \int_0^\delta V_y dxdz = \frac{W}{W\delta}\int_0^\delta V_y dz = \frac{1}{\delta}\int_0^\delta \frac{\rho . g . \delta^2}{2 . \mu}\left[1 - \left(\frac{z}{\delta}\right)^2\right] dz$$

$$V_{y,avg} = \frac{\rho . g . \delta^2}{3 . \mu}$$

So, we have to be the diffuser, we have to be called as the diffusion in the z direction and the equation of motion under this condition can be reduced to mu del 2 vy over del z2 plus rho g is equal to 0 and that is the equation number 3. Now, the solution of equation 3 with the condition vy is equal to 0 at z is equal to delta and that dv over dz is equal to 0 at z is equal to 0 is well known and this can be represented like this vy is equal to rho g del 2 over 2 mu into 1 minus z over delta to the power 2 and that can be represented as equation number 4. So, the maximum velocity that occurs at z is equal to 0 in this particular equation vy max is equal to rho g del 2 over 2 mu and that is equation number 5.

Mass transfer in falling film

- **The maximum velocity** which occurs at z = 0 in equation 4

$$V_{y,\max} = \frac{\rho . g . \delta^2}{2 . \mu} \quad \ldots (5)$$

- **The bulk average velocity** can be obtained as follows:

$$V_{y,avg} = \frac{1}{A}\iint V_y dA = \frac{1}{W\delta}\int_0^W\int_0^\delta V_y dx dz = \frac{W}{W\delta}\int_0^\delta V_y dz = \frac{1}{\delta}\int_0^\delta \frac{\rho . g . \delta^2}{2 . \mu}\left[1-\left(\frac{z}{\delta}\right)^2\right]dz$$

$$V_{y,avg} = \frac{\rho . g . \delta^2}{3 . \mu} \quad \ldots (6)$$

So, the bulk average velocity can be obtained like vy average this can be represented as a mathematical formula like this and vy average can be represented if we adopt all kind of integration and another thing can be put as rho g del 2 over 3 mu and this can be written as equation number 6.

$$\delta = \left(\frac{3 . V_{y,avg}\mu}{\rho . g}\right)^{\frac{1}{2}}$$

$$\frac{3}{2}V_{y,avg}\left[1-\left(\frac{z}{\delta}\right)^2\right]\frac{\partial C_A}{\partial y} = D_{AB}\frac{\partial^2 C_A}{\partial z^2}$$

Mass transfer in falling film

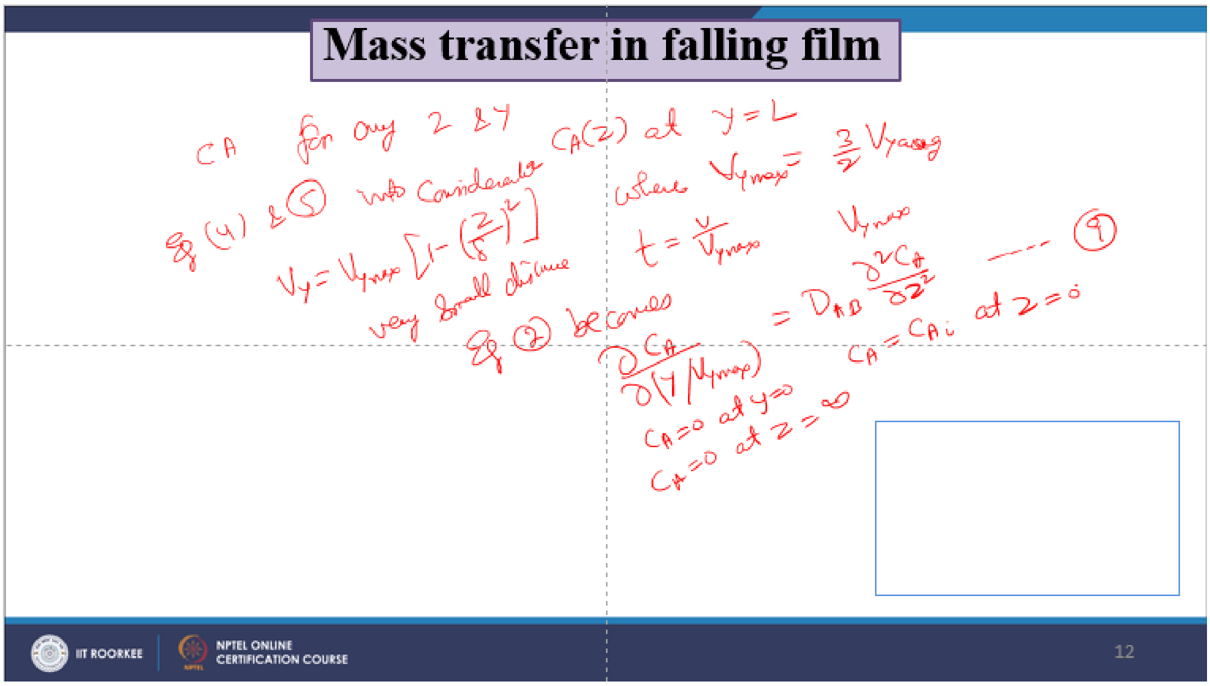
- **The film thickness** is then $\delta = \left(\frac{3 V_{y,avg}\mu}{\rho . g}\right)^{\frac{1}{2}}$ ….. (7)
- Substituting equation 4 into equation 2 and then by using equation 6, we get:

$$\frac{3}{2}V_{y,avg}\left[1-\left(\frac{z}{\delta}\right)^2\right]\frac{\partial C_A}{\partial y} = D_{AB}\frac{\partial^2 C_A}{\partial z^2} \quad \ldots (8)$$

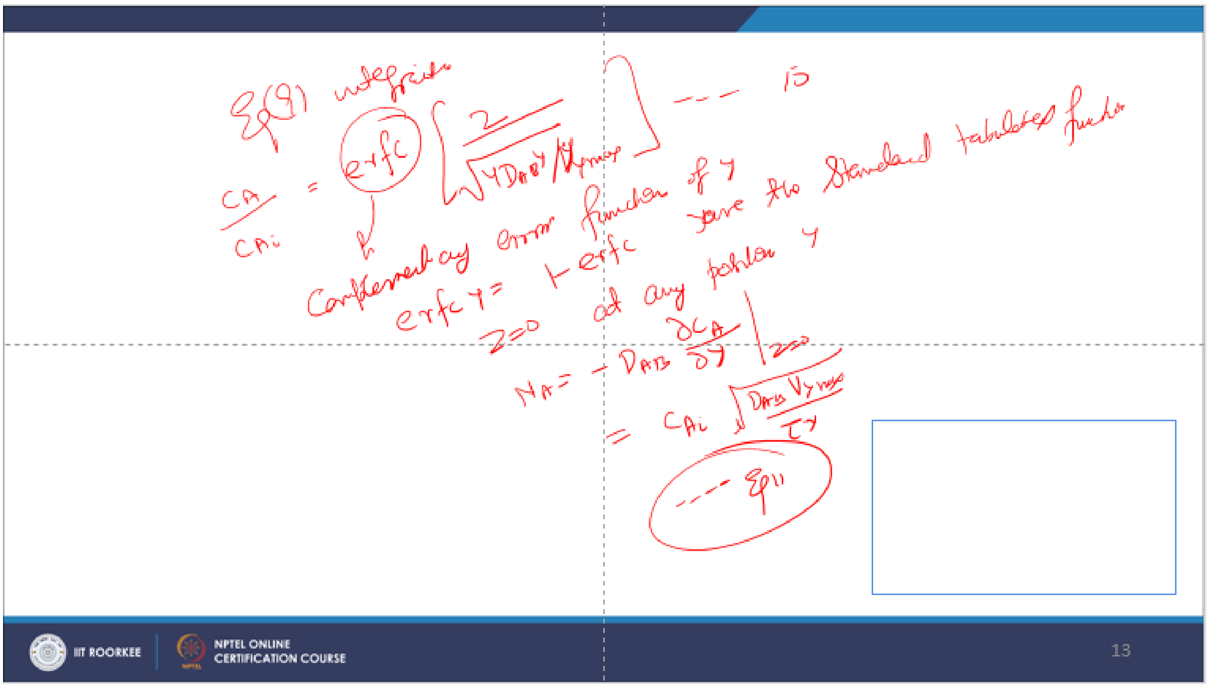
- Which is to be solved under **the following conditions**

1. At **z = 0, $C_A = C_{A,i}$** at all the values of y.
2. At **z = δ,** $\partial C_A / \partial z = 0$ at all the values of y, since no diffusion takes place into the solid wall.
3. At **y = 0, $C_A = C_{A,0}$** at all values of z

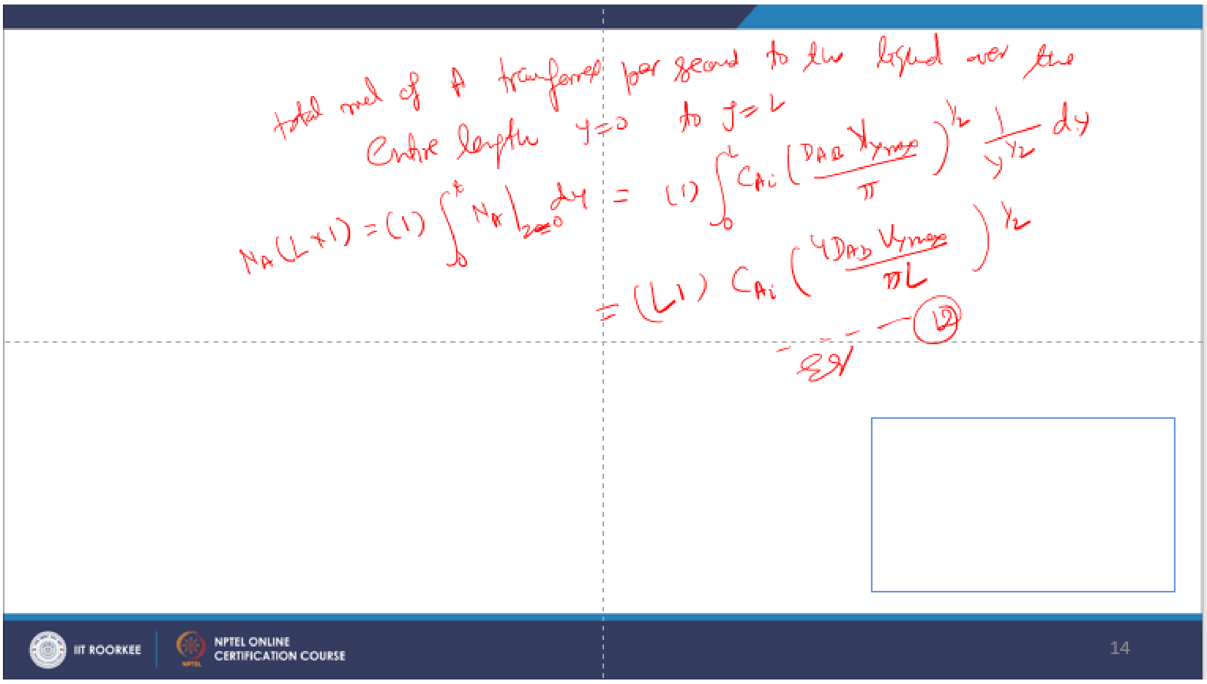
So, the film thickness can be represented as delta is equal to 3 by average mu over rho g to the power half and that is equation number 7. So, if we substitute equation 4 into equation 2, which we derived previously, and then by equation 6, we get this final equation 3 over 2 vy average into 1 minus z over the delta to the power 2 del c over del y is equal to dAb del 2 c over del z2. So, which is to be solved under the different conditions like at z is equal to 0, ca is equal to cai at all the values of y and at z is equal to delta, ca over del z is equal to 0 at all the values of y since no diffusion takes place into the solid wall and at y is equal to 0, ca is equal to ca naught at all values of z. So, the solution in the result in the general expression and infinite series gives ca for any for any z and y therefore, the providing the distribution that ca z at y is equal to solid wall L.



So, if we take equation 4 and 5 into consideration then vy is equal to vy max 1 minus z over delta where vy max is equal to 3 by 2 vy average. So, if the solute is penetrated only a very small distance, very small distance into the fluid and that is a short contact time that is t is equal to v over vy max, then the solute a that has diffused this has been carried out along with the velocity vy maximum. So, the previous equation 2 which becomes del ca over del y over v is equal to d A v del 2 ca over del z 2 this is equation number 9. So, if we use a if we use ca is equal to 0 at y is equal to 0 and ca is equal to ca i at z is equal to 0 and ca is equal to 0 at z is equal to infinite. So, if we integrate equation 9 then it can be this is equation 9 if we integrate then ca over ca i this is equal to f c over z 4 d A b y over v y this is equation number 10.



Now, where this particular is a complementary error, error function of y it can be written as E r f c y is equal to 1 minus E r f c y r the standard tabulated functions. So, the local molar flux at the surface z is equal to 0 at any position y from the top of the entrance can be given as n A is equal to minus d A b del c A over del y z is equal to 0 and this can be c A i in d A b v y over tau y this is equation number 11.



So, the total mole of A transferred total mole of A transferred per second to the liquid over the entire length y is equal to 0 to y is equal to L where the vertical surface is unit with which can be calculated like n A L into 1 1 0 to t n A z is equal to 0 to L c A i d A d y v y maximum which we are targeting to the power half into 1 over y to the power half d y which is equal to L c A 4 d A b v y pi L to the power half this can be written as equation number 12.

Problem-1

Question: The absorption of pure carbon dioxide is carried out at 1 atmospheric pressure and at 25 degree centigrade by using water film flowing down a vertical wall of 1 meter long. The water is essentially CO_2 free initially. The average velocity of the liquid is 0.2 meter per second. The solubility of CO_2 in water at 25 ^oC and at 1 atmosphere is $C_{A,i}$ = 0.0336 kmol/m^3

Calculate the film thickness and the rate of absorption of carbon dioxide ?

Use the following properties, DAB: 2 x 10^{-9} m^2/s,
solution density ρ = 997 kg/m^3 and
viscosity μ = 8.95 x 10^{-4} kg/m.s at 25 ^oC.

Now, let us take up a problem the absorption of pure carbon dioxide is carried out at 1 atmosphere pressure and at 25 degree centigrade by using water film flowing down a vertical wall of 1 meter long and the water is essentially CO2 free at the outset. The average velocity of the liquid is 0.2 meters per second and the solubility of carbon dioxide in water at 25 degree Celsius and 1 atmosphere is that is c A i initially is equal to 0.336-kilo mole per meter cube. So, you need to calculate the film thickness and the rate of absorption of carbon dioxide. Now, some statistical information is given to you like that d A b is equal to 2 into 10 to the power minus 9-meter square per second and the solution density is equal to rho is equal to 997 kilograms per meter cube and the viscosity which is foremost important to solve this particular problem is given as 8.95 into 10 to the power minus 4 kilograms per meter second at 25 degree Celsius.

$V_{y,avg} = 0.2$ m/s $\quad \rho = 997$ kg/m^3 $\quad \mu = 8.95\times10^{-4}$ kg/m s $\quad g = 9.81$ m/s^2

$$\delta = \left(\frac{3V_{y,avg}\,\mu}{\rho g}\right)^{1/2} = \left[\frac{3\times0.2\times(8.95\times10^{-4})}{997\times9.81}\right]^{1/2}$$

$\delta = 2.34\times10^{-4}$ m

$C_{Ai} = 0.0336$ kmol/m^3 $\quad D_{AB} = 2\times10^{-9}$ m^2/s $\quad L = 1$ m

$$N_A = C_{Ai}\left(\frac{4D_{AB}V_{y,max}}{\pi L}\right)^{1/2}$$

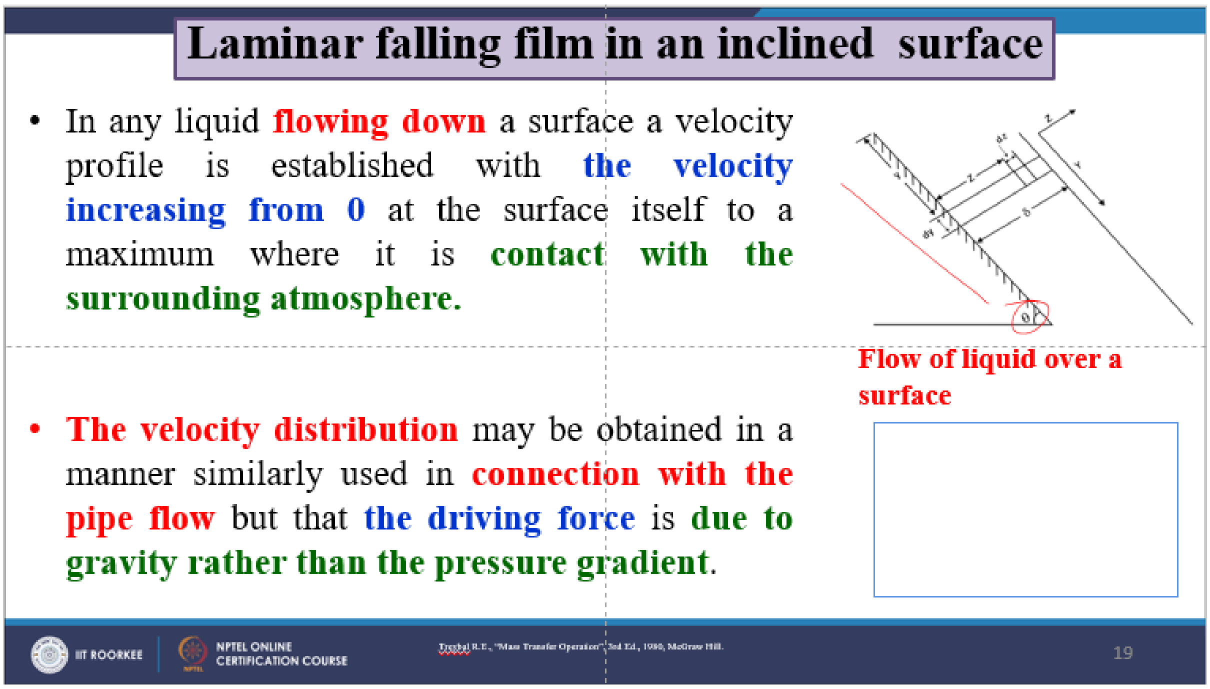
$$N_A = 0.0336\left(\frac{4\times(2\times10^{-9})\times0.2}{\pi\,1}\right)^{1/2}$$

$= 7.58\times10^{-7}$ kmol/m^2.s

Ans

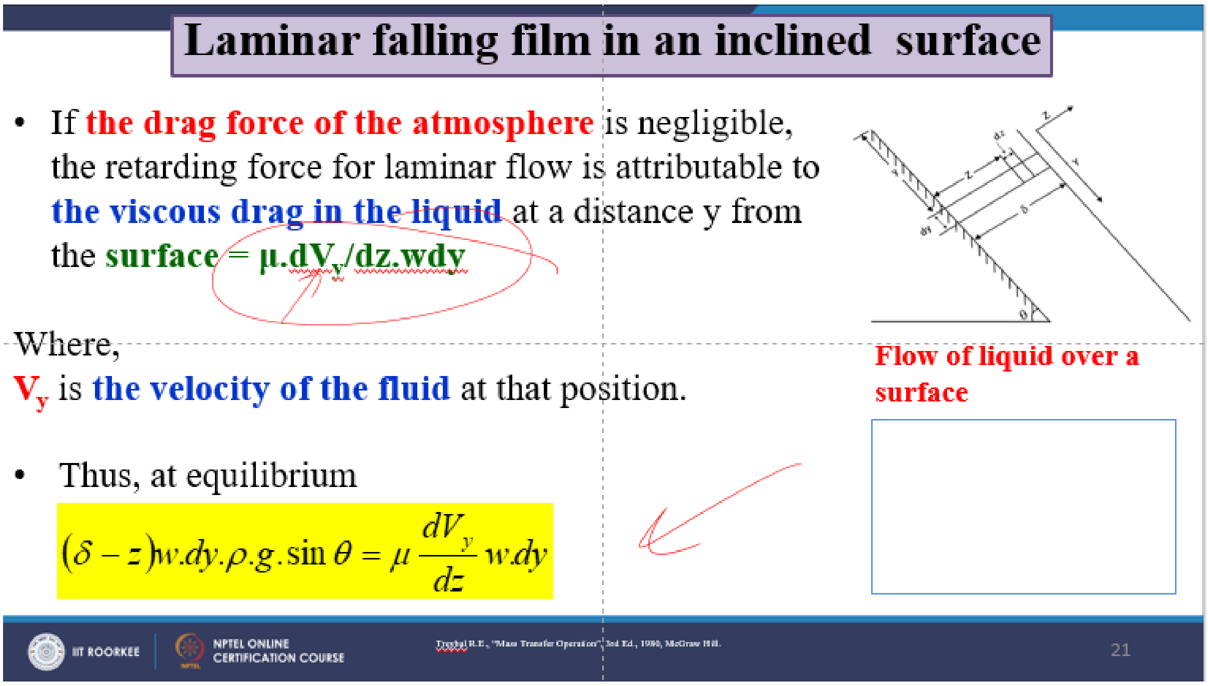
So, let us solve this particular problem. Now, it is given that v y average this is equal to 0.2 meters per second, p is equal to 997 kilograms per meter cube, mu is equal to 8.95 into 10 to the power minus 4 kilograms per meter second and g is 9.81 meters per square. So, the delta is equal to 3 v average mu this is the standard formula that we discussed earlier rho g to the power half, and this is 3 into 0.2 into 8.95 into 10 to the power minus 4 over 997 into 9.81, which is equal to one half.

So, the delta is equal to 2.34 into 10 to the power minus 4 meters. So, now, CAI is given to you that is 0.0336-kilo mole per meter cube, D AB is given to you 2 into 10 to the power minus 9 meter square per second length is given 1 meter. So, N A is equal to CAI 4 D AB v y maximum tau L to the power half. So, if we substitute all these values, then it becomes N A is equal to 0.0336 into 4 into 2 into 10 to the power minus 9 into 0.2 tau 1 to the power half and this is coming out to be 7.58 into 10 to the power minus 7 kilo mole per meter square second and this is our required answer.



Now, let us talk about the laminar falling film in an inclined surface. Sometimes this inclined surface plays a very vital role in this aspect.

Now, this is our inclined surface at angle theta. So, for any liquid flowing down a surface, a velocity profile which is usually established with the velocity increasing, let us say from 0 at the surface itself to a maximum where it is in contact with the surrounding atmosphere. So, the velocity distribution may be obtained in a manner similar what we used in connection with the pipe flow, but the driving force is attributed to the gravity rather than the pressure gradient. So, for the flow of liquid of the depth of say, delta down a plane surface with the width W, which is inclined at an angle of theta to the horizontal. So, a force balance in the y direction parallel to the surface may be written in this particular aspect.



$$(\delta - z)w.dy.\rho.g.\sin\theta = \mu\frac{dV_y}{dz}w.dy$$

Now, in an element of length dy, the gravitational force acting in the part of the liquid which is at the distance greater than Z from the surface, this can be written as delta minus Z into W dy rho g sine theta. Now, if we talk about the drag force, so if the drag force of the atmosphere is negligible, the retarding force for laminar flow is attributable to the viscous drag in the liquid at a distance y from the surface and that is equal to mu dy over dz W dy. Now, where vy this is the velocity of the fluid at the position and therefore the equilibrium thus the equilibrium we can write delta minus Z into W dy rho g sin theta is equal to mu dv y over dz W dy. Now since there will be normally no slip between the liquid and a surface where vy is equal to 0 when Z is equal to 0. So, we can write the integrated integration like from 0 to vy dv y is equal to rho g sin theta over mu and 0 to Z delta minus Z into dz which can be modified to vy is equal to rho g sin theta over mu into del Z delta Z minus 1 over Z square.

$$\int_0^{V_y} dV_y = \frac{\rho.g.\sin\theta}{\mu}\int_0^{z}(\delta - z)dz$$

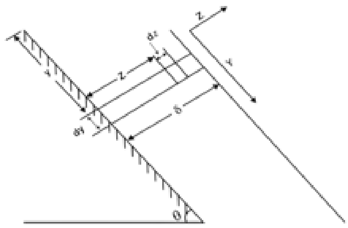
$$V_y = \frac{\rho.g.\sin\theta}{\mu}\left(\delta z - \frac{1}{2}z^2\right)$$

Laminar falling film in an inclined surface

- Since there will be normally no slip between the liquid and the surface, then **$V_y = 0$ when $z = 0$:**

$$\int_0^{V_y} dV_y = \frac{\rho.g.\sin\theta}{\mu}\int_0^{z}(\delta - z)dz$$

$$V_y = \frac{\rho.g.\sin\theta}{\mu}\left(\delta z - \frac{1}{2}z^2\right)$$



Flow of liquid over a surface

Laminar falling film in an inclined surface

- **The mass rate of flow** that is $\dot{m}$ of liquid down the surface can be calculated as:

$$\dot{M} = \int_0^{\delta}\frac{\rho.g.\sin\theta}{\mu}w\left(\delta z - \frac{1}{2}z^2\right)\rho dz$$

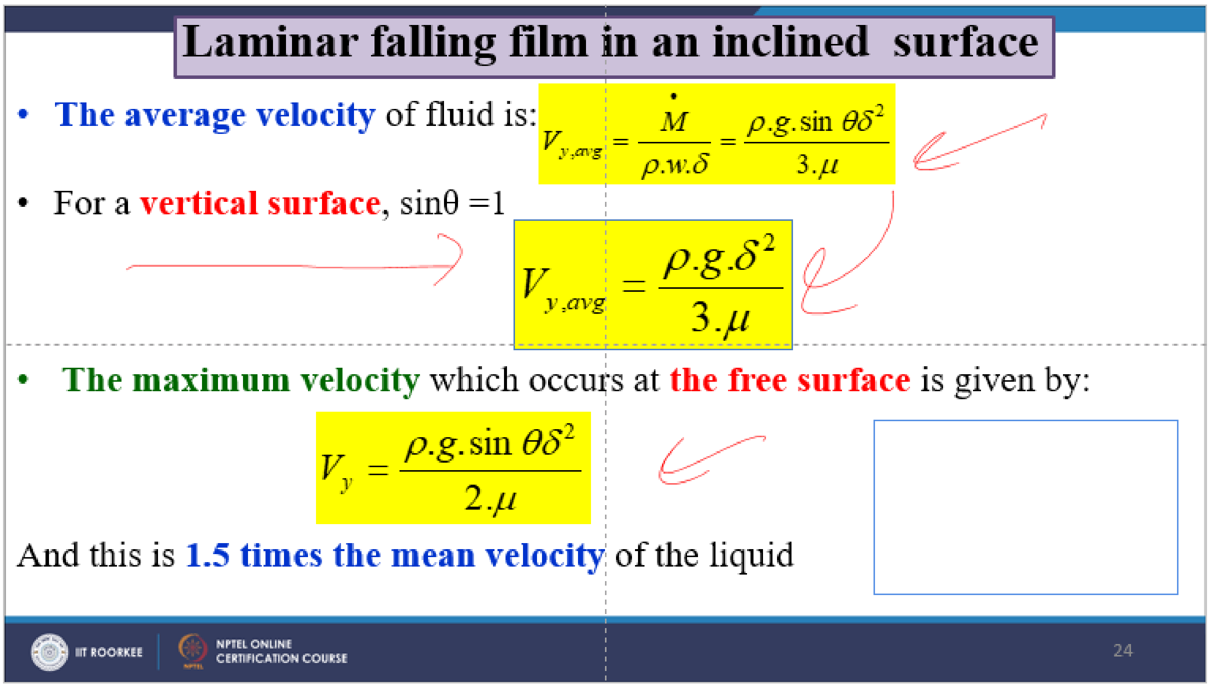
$$\dot{M} = \frac{\rho^2 g.\sin\theta}{\mu}w\left(\frac{\delta^3}{2} - \frac{\delta^3}{6}\right)$$

$$\dot{M} = \frac{\rho^2 g.\sin\theta.w.\delta^3}{3.\mu}$$

$$\dot{M} = \int_0^{\delta}\frac{\rho.g.\sin\theta}{\mu}w\left(\delta z - \frac{1}{2}z^2\right)\rho dz$$

$$\dot{M} = \frac{\rho^2 g.\sin\theta}{\mu}w\left(\frac{\delta^3}{2} - \frac{\delta^3}{6}\right)$$

$$\dot{M} = \frac{\rho^2 g.\sin\theta\,.w.\delta^3}{3.\mu}$$

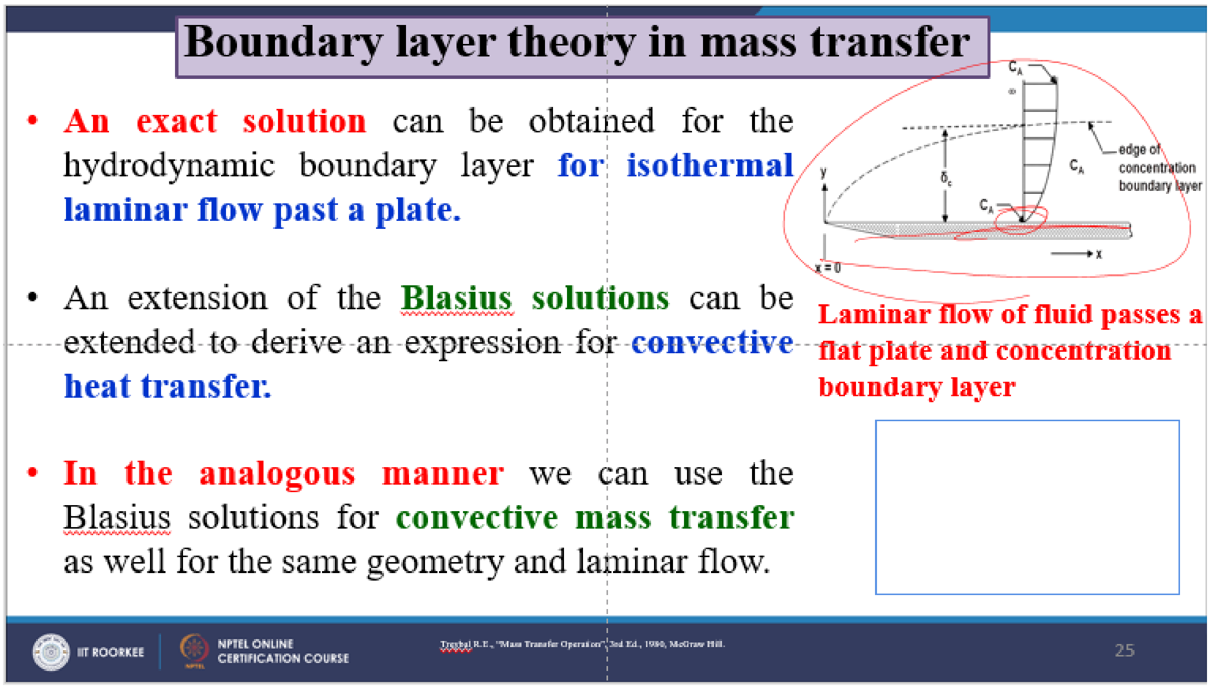


$$V_{y,avg} = \frac{\dot{M}}{\rho.w.\delta} = \frac{\rho.g.\sin\theta\,\delta^2}{3.\mu}$$

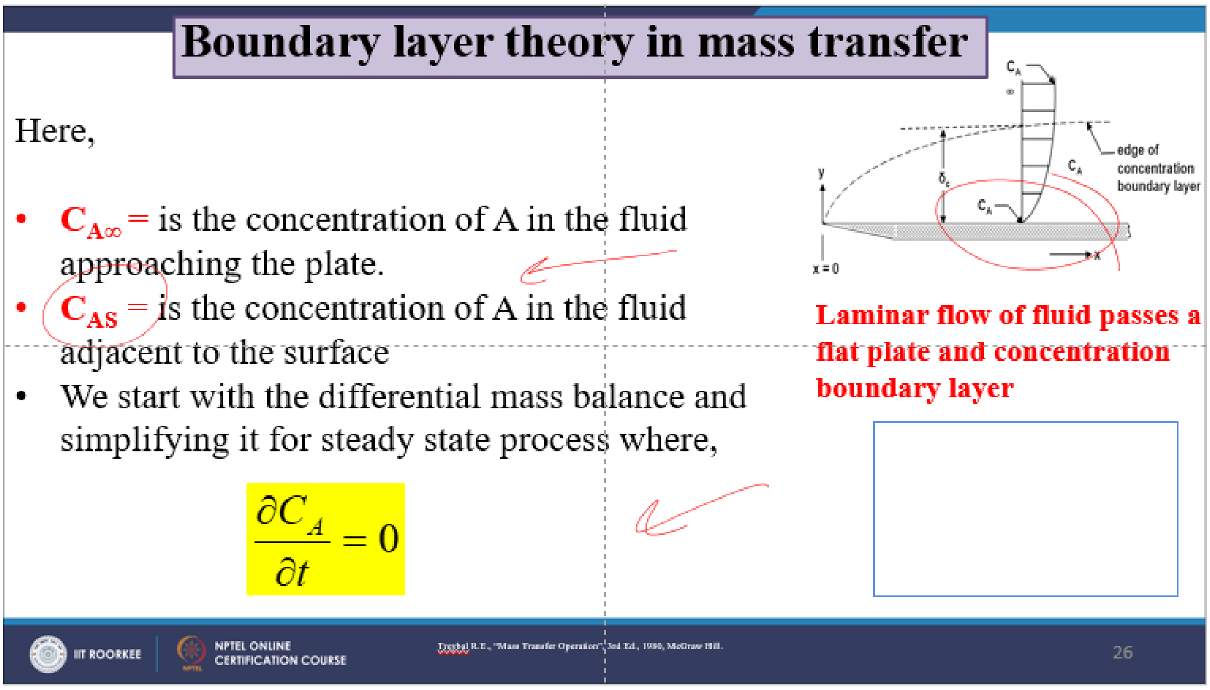
$$V_{y,avg} = \frac{\rho.g.\delta^2}{3.\mu}$$

$$V_y = \frac{\rho.g.\sin\theta\,\delta^2}{2.\mu}$$

So, the mass rate of the flow, that is the m of the liquid or mass of the liquid down to the surface, can be calculated as dot m is equal to 0 to delta rho g sin theta over mu into W delta Z minus 1 over 2 Z square into rho Z which can be after simplification it can be written as dot m is equal to rho square g sin theta over mu into W into del 3 delta 3 over 2 minus delta 3 over Z and which can be further modified to this final shape. Now, if we talk about the laminar falling film in an inclined surface, so average velocity of the fluid can be represented as vy average is equal to dot m over rho W delta is equal to rho g sin theta delta square over 3 mu and for vertical surface sin theta is equal to 1. So, the equation can be modified as v is equal to rho g delta square over 3 mu the maximum velocity that occurs at the free surface is given by vy is equal to rho g sin theta delta square over 2 mu and this is almost 1.5 times the mean velocity of the fluid. Now, let us talk about the boundary layer theory in mass transfer.



The exact solution can be obtained for the hydrodynamic boundary layer for isothermal laminar flow usually past a plate. Now, this is the typical laminar flow of the fluid that passes a flat plate, this is the flat plate and the concentration boundary layer. So, at this level this is CA. Now, an extension to the Blasius solution can be extended to derive an expression for the convective heat transfer. Now, in the analog manner, we can use the Blasius solution for convective mass transfer as well as the same geometry and laminar flow.

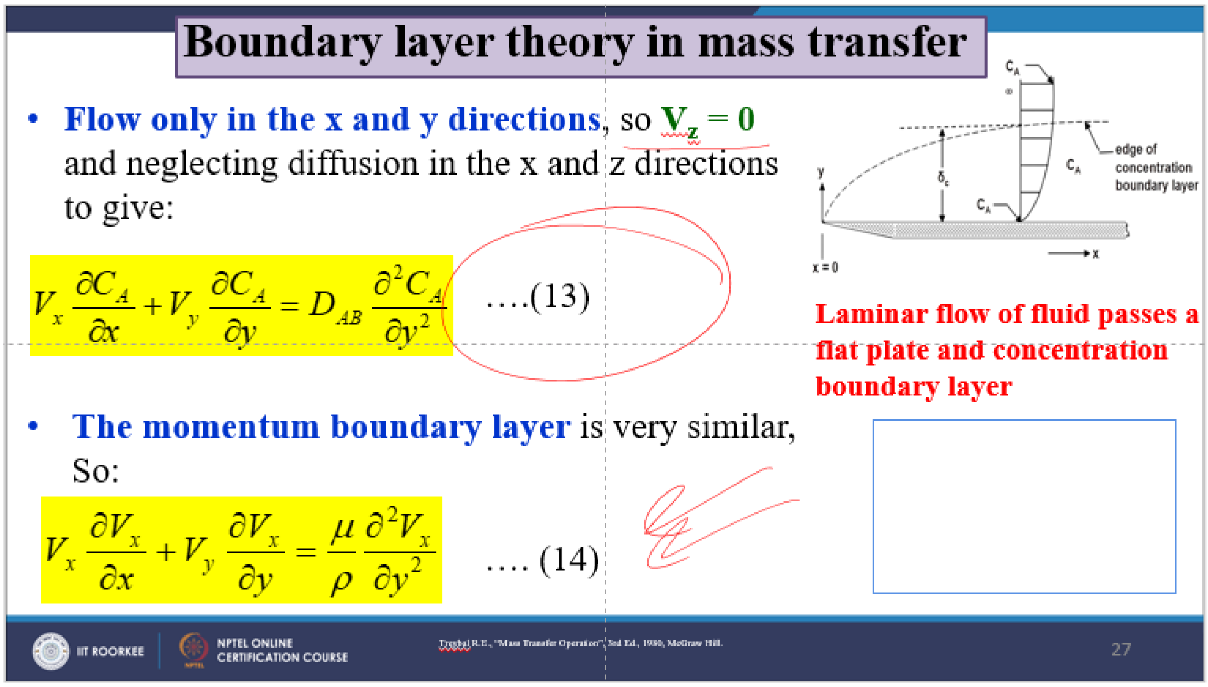


$\frac{\partial C_A}{\partial t} = 0$

$$V_x \frac{\partial C_A}{\partial x} + V_y \frac{\partial C_A}{\partial y} = D_{AB} \frac{\partial^2 C_A}{\partial y^2}$$

$$V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 V_x}{\partial y^2}$$

Now, here CA infinite is the concentration of A in the fluid approaching the plate. Now, CAS is the concentration of A in the fluid adjacent to the surface. So, in other words, we can say like this. So, if we start with the differential mass balance and simplify it for steady-state process where Del C over Del t is equal to 0. Now, flow only direction flow in the direction of x and y.



So, if so, vz is equal to 0 and neglecting diffusion in x and z directions. So, it can be given as vx is into Del C over Del x plus vy into Del CA over Del y is equal to D AB Del 2 CA over Del y2 and this is can be represented as equation number 13. So, the momentum boundary layer is very similar. Therefore, this equation can be modified as vx to Del vx over Del x plus vy is equal to Del vx over Del y plus is equal to Mu over Rho Del 2 vx over Del y2 and this can be represented as equation number 14.

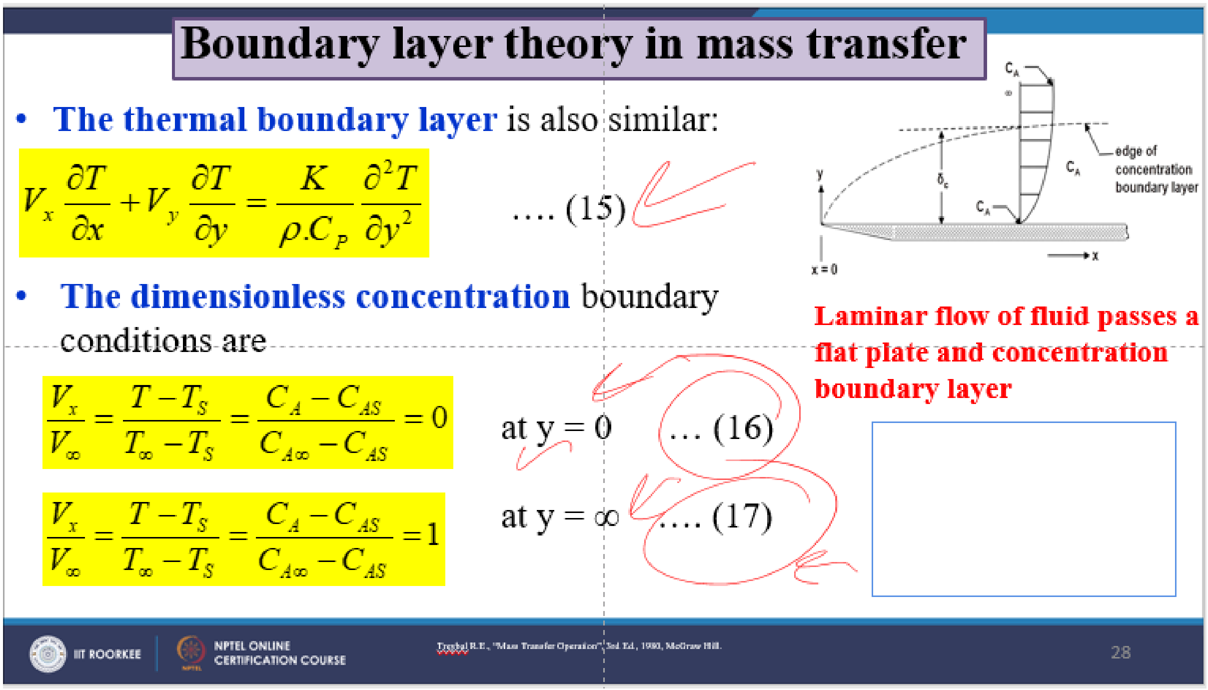
The thermal boundary layer is also similar

$$V_x \frac{\partial T}{\partial x} + V_y \frac{\partial T}{\partial y} = \frac{K}{\rho . C_P} \frac{\partial^2 T}{\partial y^2}$$

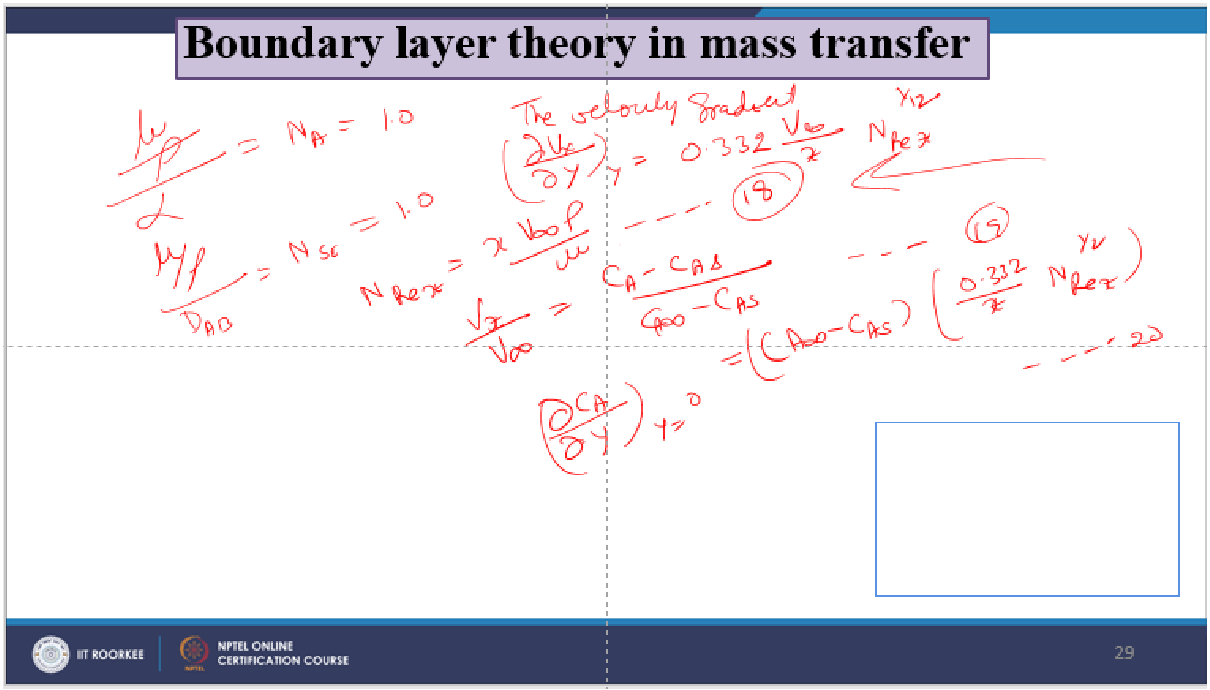
Dimensionless concentration boundary conditions are;

$\frac{V_x}{V_\infty} = \frac{T - T_S}{T_\infty - T_S} = \frac{C_A - C_{AS}}{C_{A\infty} - C_{AS}} = 0$, at y=0.

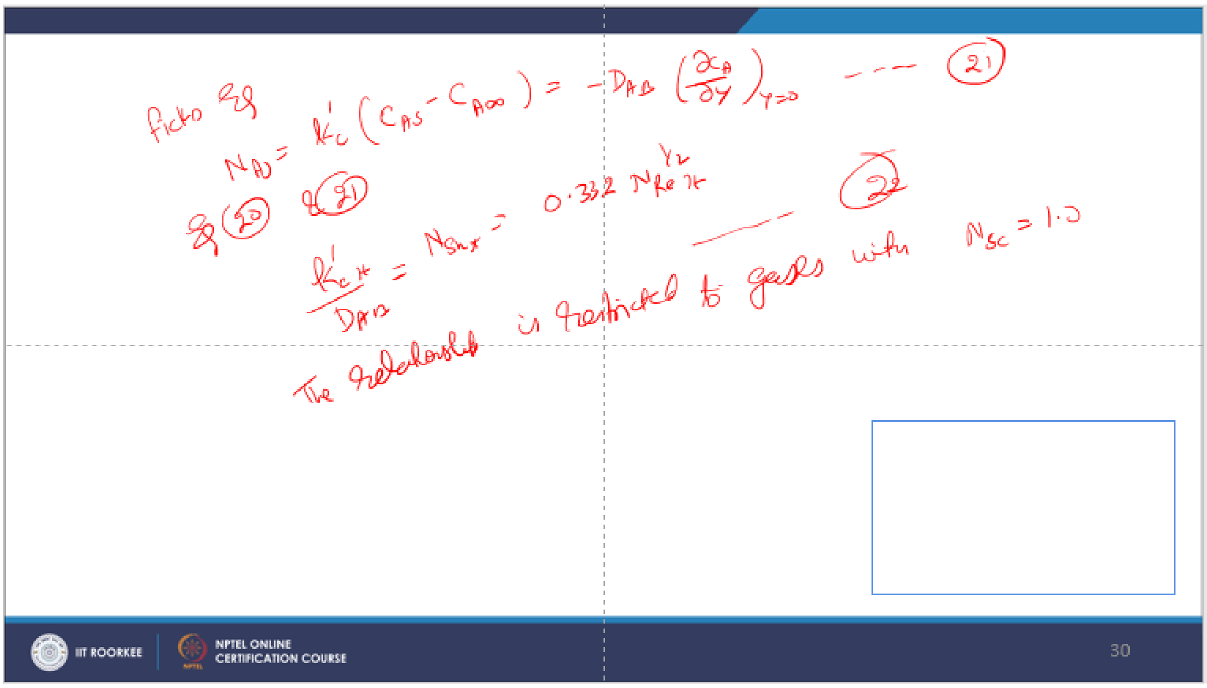
$\frac{V_x}{V_\infty} = \frac{T - T_S}{T_\infty - T_S} = \frac{C_A - C_{AS}}{C_{A\infty} - C_{AS}} = 1$, at y is at infinity



So, the thermal boundary layer if we talk about is also similar. So, this can be represented as vx into Del t over Del x plus vy is equal to Del t over Del y is equal to k over Rho CP Del 2 t over Del y2, and this is equation number 15. And if we adopt the dimensionless correct concentration boundary conditions, then at y is equal to 0, the vx over v infinite is equal to t minus TS over t infinite minus TS is equal to CA minus CAS over C infinite minus CAS is equal to 0. So, this can be represented as equation number 16. And on simplification, if we go for y is equal to infinite, this equation can be further modified, which is represented here, that is vx over v infinite is equal to t minus TS over t finite minus TS is equal to CA minus CAS over CA infinite minus CAS is equal to 1. So, this is equation number 17.



Now, the Blasius equation is applied to convective heat transfer in that case Mu is Mu over Rho over alpha is equal to n a, which can be represented as 1. Now, we use the same type of solution for the laminar convective mass transfer, where Mu over Rho over D AB is equal to n SC that is Schmidt number 1 minus 0, and the velocity gradient, the velocity gradient is derived in the fluid mechanics and that is d vx over del vx over del y at y is equal to 0 that is 0.0332 v infinite over x n Reynolds number to the power half. So, where n Re that is Reynolds number at x, x v infinite Rho Mu, this is equation number 18. Now, if we recall equation number 17 here, then vx over v infinite is equal to CA minus CAS over C infinite A infinite minus CAS, this is equation number 19.



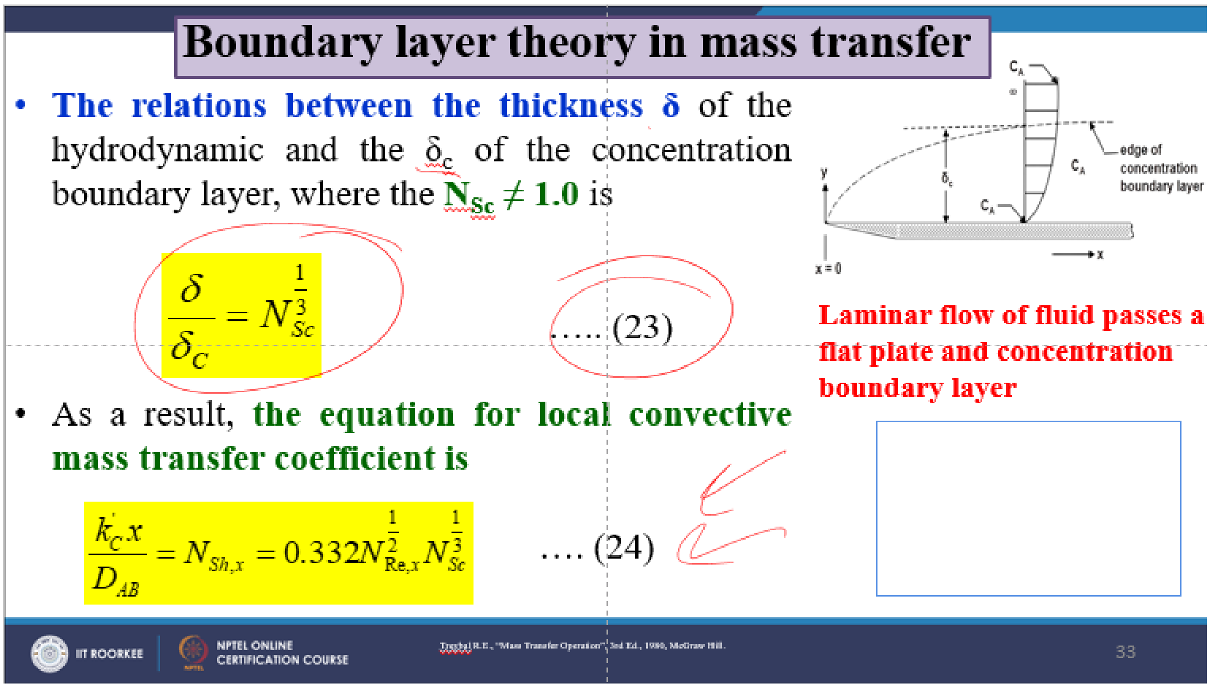
Now, if you differentiate equation 19 and combine the result with equation 18, then it can be D written as d CA over del CA over del y at y is equal to 0, this is equal to CA infinite minus CAS 0.332 over x n that is Reynolds number over half. This is equation number 20. So, the convective mass transfer equation can be written as and also, if we relate with Fick's law, Fick's equation, then n A y is equal to k C CAS minus CA infinite is equal to minus D AB del CA over del y, y is equal to 0, this is equation number 21. So, from equations 20 and 21, we get k dash C x over D AB is equal to Sherwood number 0.332 n that is Reynolds number to the power half x and this is equation number 22. So, the relationship, this relationship is restricted to gas, gases with a Schmidt number is equal to 1.0.

Relation between thickness and concentration boundary layer

$$\frac{\delta}{\delta_C} = N_{Sc}^{\frac{1}{3}}$$

Equation for local convective mass transfer coefficient

$$\frac{k_C' x}{D_{AB}} = N_{Sh,x} = 0.332 N_{\mathrm{Re},x}^{\frac{1}{2}} N_{Sc}^{\frac{1}{3}}$$



Now, the relationship between the thickness delta of the hydrodynamic and delta C, the concentration boundary layer where Schmidt number is not equal to 1 is delta C delta over delta C is equal to Schmidt number to the power 1 by 3 and that is equation number 23. Now, as a result, the equation for local convective mass transfer coefficient is given as k prime C x over D AB is equal to Sherwood number the x direction is equal to 0.332 Reynolds number to the power half and Schmidt number to the power 1 by 3 and this can be represented as equation number 24. So, we can obtain the mass transfer coefficient k c prime from x is equal to 0 to L for a plate with B by integrating this particular equation. So, this can be the k c prime is equal to B over B L 0 to L integration k c prime d x and this is equation number 25. So, we can represent the integration like k c prime over into L over D AB is equal to Schmidt number is equal to 0.664 Reynolds number to the power half and Schmidt number to the power 1 by 3.

Mean mass transfer coefficient

$$\overline{k_C'} = \frac{b}{b.L}\int_0^L k_C' dx$$

Or

$$\frac{\overline{k_C'}.L}{D_{AB}} = N_{Sh,L} = 0.664 N_{\mathrm{Re},L}^{\frac{1}{2}} N_{Sc}^{\frac{1}{3}}$$

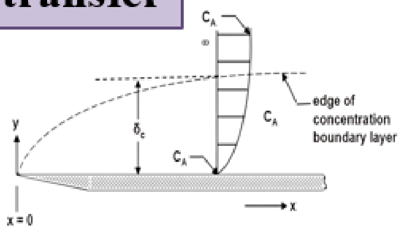
Boundary layer theory in mass transfer

- We can obtain the **mean mass transfer coefficient** k_C' from **x = 0 to x = L** for a plate of width b by integrating as follows:

$$\overline{k_C'} = \frac{b}{b.L}\int_0^L k_C' dx \qquad \text{..... (25)}$$

- **The result is**

$$\frac{\overline{k_C'} L}{D_{AB}} = N_{Sh,L} = 0.664 N_{Re,L}^{\frac{1}{2}} N_{Sc}^{\frac{1}{3}} \qquad \text{..... (26)}$$

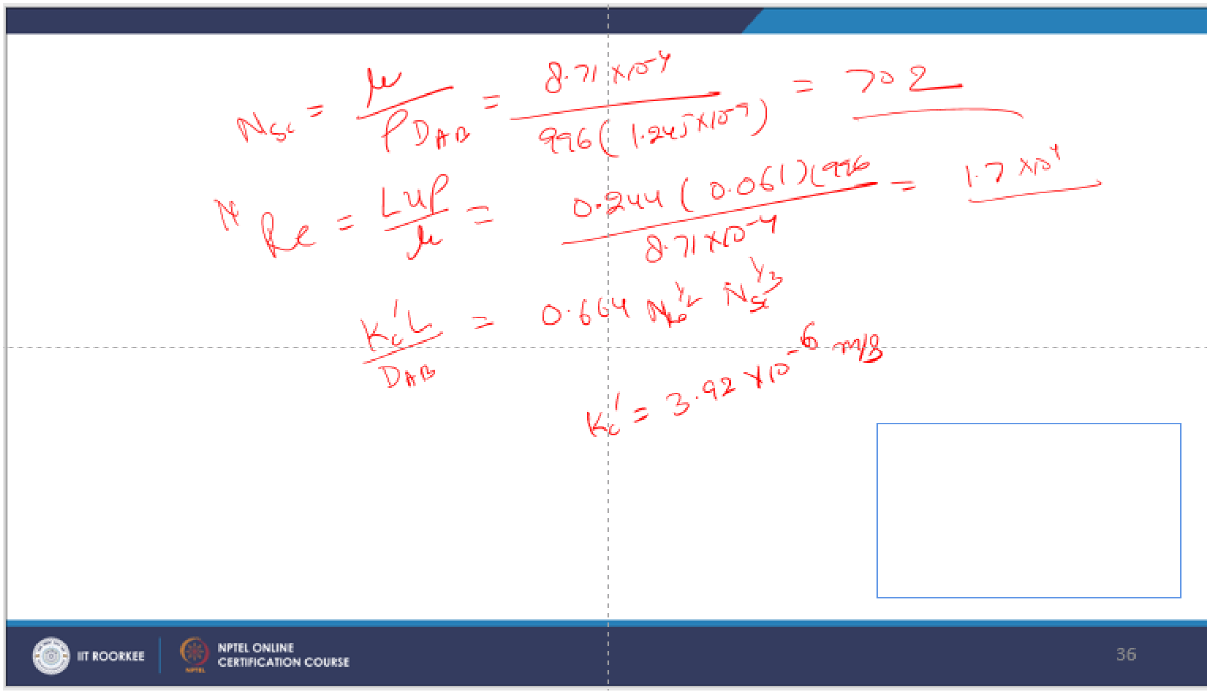


Laminar flow of fluid passes a flat plate and concentration boundary layer

Now, let us take another problem, a large volume of pure water at 25 degrees Celsius is flowing parallel to a flat surface of solid benzoic acid where L is 0.244 meter in the direction of the flow and the water velocity is 0.061 meter per second, and the solubility of the benzoic acid in water is given that is 0.2948 kilo mole per meter cube and the diffusivity of the benzoic acid is 1.245 into 10 to the power minus 9 meter square per second.

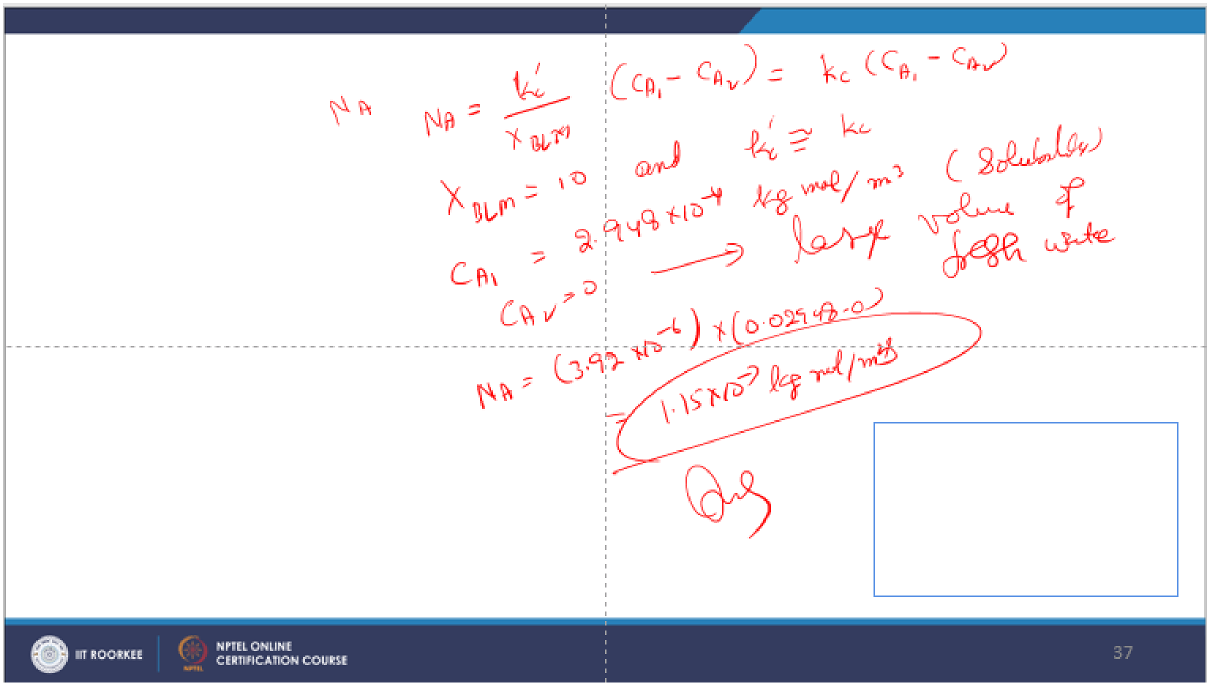
Problem-2

Question: A large volume of pure water at 25 ^oC is flowing parallel to a flat surface of solid benzoic acid, where L is 0.244 m in the direction of flow. The water velocity is 0.061 m/s. The solubility of benzoic acid in water is 0.02948 kmol/m^3. The diffusivity of benzoic acid is 1.245 x 10-^{9} m^2/s. Calculate the mass transfer coefficient $\overline{k}_C'$ and the flux N_A. Given that μ = 8.71 x 10^{-4} kg/m.s and ρ = 996 kg/m^3

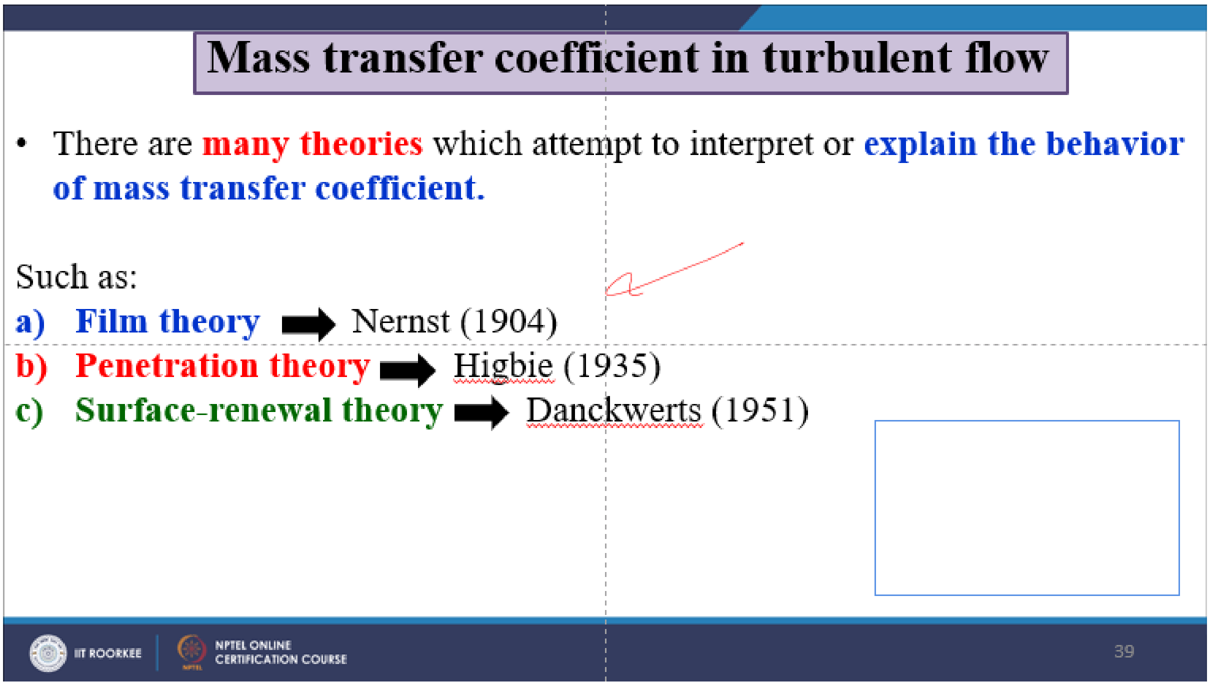


You need to calculate the mass transfer coefficient to k c prime and the flux and given is whatever given that is mu is equal to 8.71 into 10 to the power minus 4 kilograms per meter second and density rho is 996 kilogram per meter cube. So, let us solve this particular problem. So, the Schmidt number that is N SC that is mu is equal to rho D AB this is 8.71 into 10 to the power minus 4 996 into 1.245 into 10 to the power minus 9 that is equal to 702 and Reynolds number is given by this particular mathematical representation and if you substitute the value, then it can be into 996 over 8.71 into 10 to the power minus 4 and that is 1.7 into 10 to the power 4. So, k c prime over D AB is equal to 0.664 Reynolds number to the power half and Schmidt number to the power 1 by 3.

So, if we substitute all the values that we calculated and D AB, which is given then k c prime comes out to be 3.92 into 10 to the power minus 6 meters per second. So, if we calculate N A we have N A is equal to k c prime x into CA 1 minus CA 2 which is equal to k c CA 1 minus CA 2. Now, if the solution is very dilute then this can be represented at k c prime is almost equal to k c.



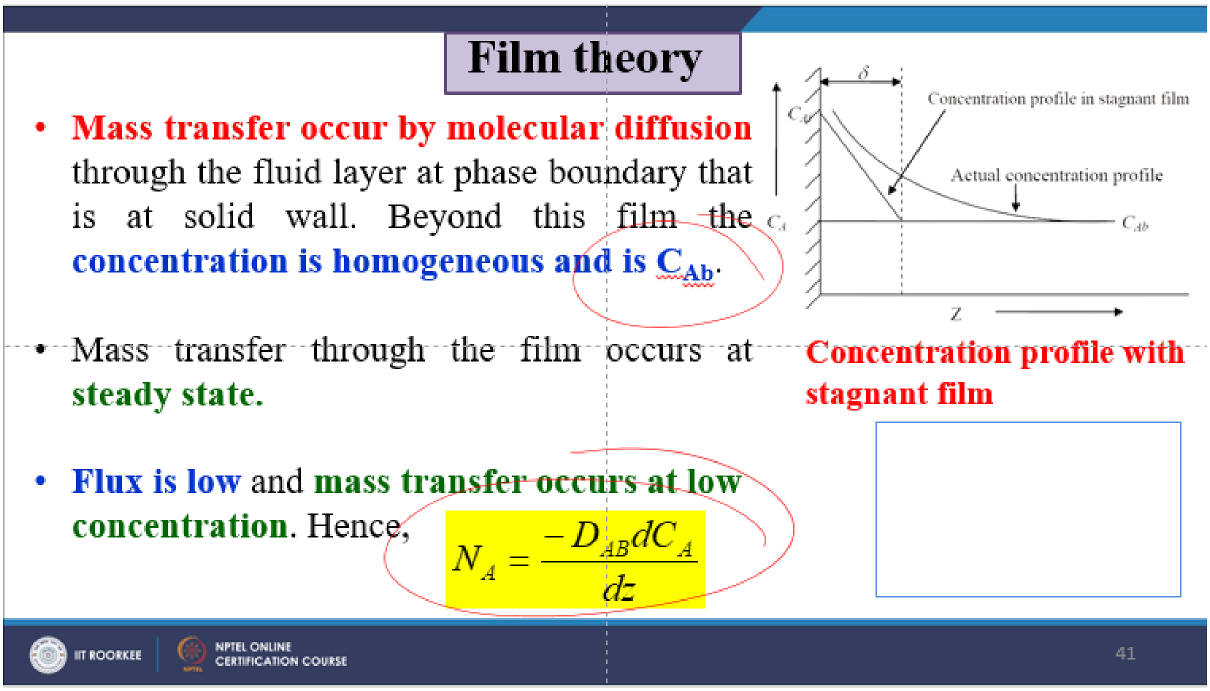
Also, initially it is given the solubility is given 2.948 into 10 to the power minus 4-kilogram mole per meter cube that is solubility and CA 2 is equal to 0 for a large volume of fresh water. So, if we substitute all these values it 3.9 N A is equal to 3.92 into 10 to the power minus 6 into 0.02948 minus 0. This comes out to be 1.15 into 10 to the power minus 7-kilogram mole per meter cube. This is our answer.



Now, if we talk about the turbulent flow, so there are many theories that attempt to interpret the explaining explain the behaviour of mass transfer coefficient like NIST given the film theory the penetration theory is given by Higbie in 1935 and the surface enable theory is given by Dankwerts in 1951. Now, NIST the film theory the NIST postulates that near the interface, there exists a stagnant film and the stagnant film, which is hypothetical since we really do not know the details of the velocity profile near the interface.

The basic concept is that the resistance to diffusion can be considered equivalent to that in the stagnant film of a certain thickness.

$$N_A = \frac{-D_{AB} dC_A}{dz}$$



The mass transfer occurs by the molecular diffusion through the fluid layer at a phase boundary that is a solid wall and beyond this particular film the concentration is homogeneous and it is given by C A B. So, the mass transfer through the film occurs at a steady state and flux is very low and the mass transfer occurs at low concentration. So therefore, this can be N A can be represented like this. Now, a steady state mass balance over the elementary volume thickness if we try to find out at delta z.

$$\lim_{\Delta Z \to 0} \frac{N_A|_Z - N_A|_{Z+\Delta Z}}{\Delta Z} = 0$$

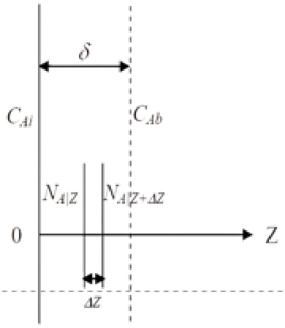
$$\frac{dN_A}{dz} = 0 \, \frac{d}{dz}\left(\frac{-D_{AB} dC_A}{dz}\right) = 0 - D_{AB}\frac{d^2 C_A}{dz^2} = 0 \, \frac{d^2 C_A}{dz^2} = 0$$

Film theory

- **The steady state mass balance** over an elementary volume of thickness ΔZ.
- **Rate of input of solute** at: $Z = N_A\big|_Z$
- **Rate of output of solute** at: $Z + \Delta Z = N_A\big|_{Z+\Delta Z}$
- **Rate of accumulation** = 0

  = rate of input-rate of output
- Therefore,

  At steady state: $N_A\big|_Z - N_A\big|_{Z+\Delta Z} = 0$



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Film theory

Consider

$$\lim_{\Delta Z \to 0} \frac{N_A|_Z - N_A|_{Z+\Delta Z}}{\Delta Z} = 0$$

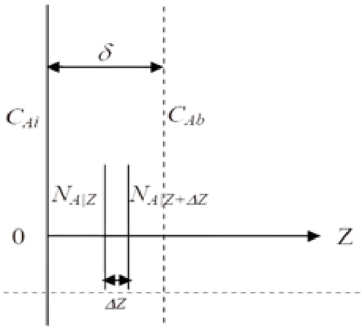
Then,

$$\frac{dN_A}{dz} = 0 \quad \ldots\ldots (27)$$

$$\frac{d}{dz}\left(\frac{-D_{AB}\, dC_A}{dz}\right) = 0 \quad \ldots\ldots (28)$$

$$-D_{AB}\frac{d^2 C_A}{dz^2} = 0 \quad \ldots\ldots (29)$$

$$\frac{d^2 C_A}{dz^2} = 0 \quad \ldots\ldots (30)$$



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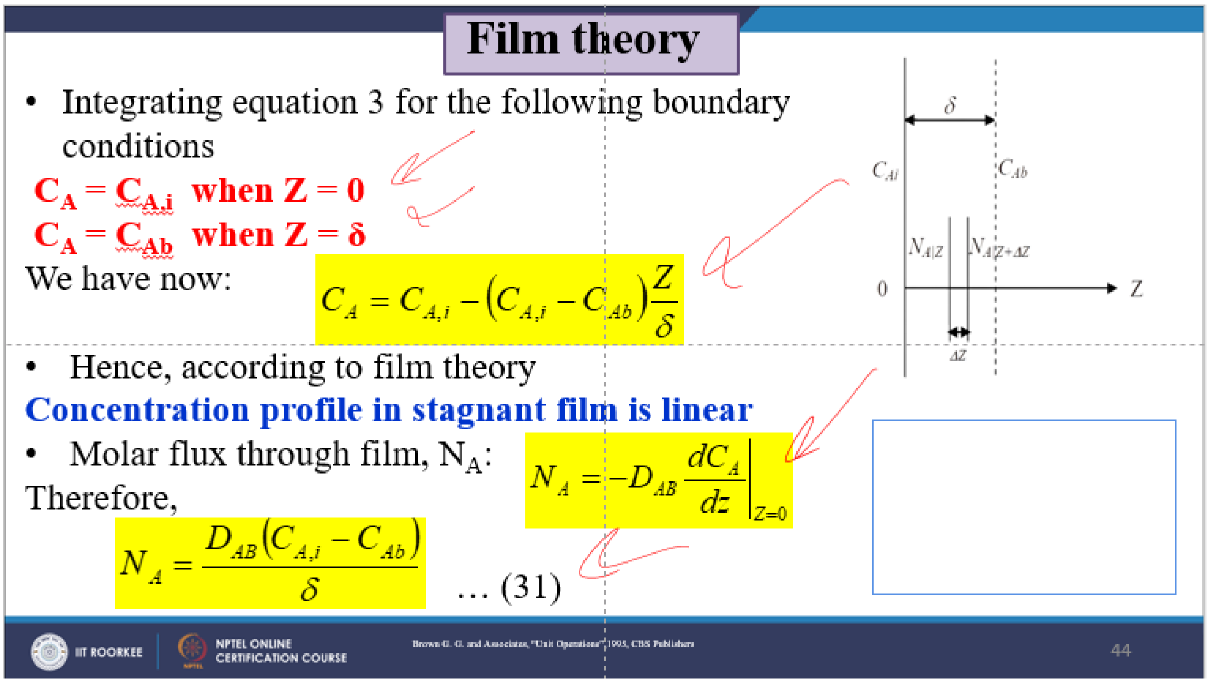
So, the rate of input solute at z is equal to N A at z and the rate of output can be given as like this. So, if we take the rate of accumulation is equal to 0 that is the rate of input minus the rate of output; therefore, at a steady state, mathematically it can be represented like this. Now, let us consider that if we go for z tends to 0, then N A z at z minus N A at z plus z delta z over delta z is equal to 0. So, upon simplification we can go we can obtain various equations like d N A over d z is equal to 0 and then d over d z minus d A B d C A over d z is equal to 0. So, we can obtain various equations which are useful in due course of time.

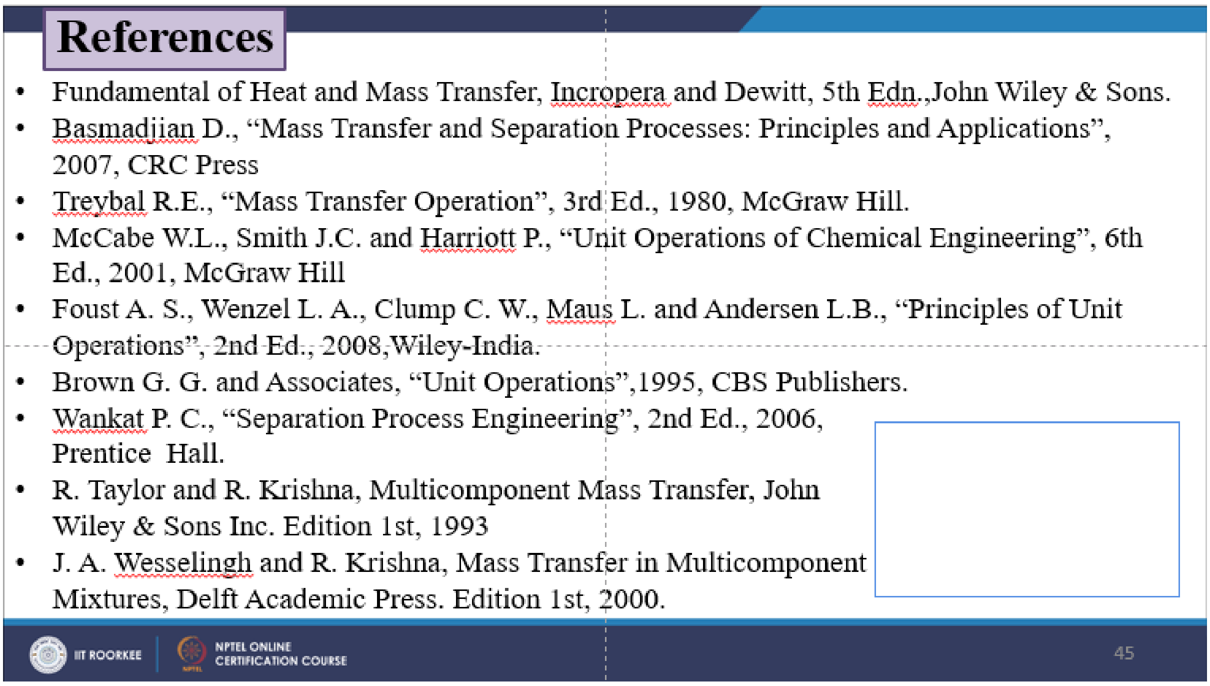
$$C_A = C_{A,i} - \left(C_{A,i} - C_{Ab}\right)\frac{Z}{\delta}$$

$$N_A = -D_{AB}\frac{dC_A}{dz}\bigg|_{Z=0}$$

$$N_A = \frac{D_{AB}(C_{A,i} - C_{Ab})}{\delta}$$



Now, if we integrate the previous equation, which we developed over like equation number 3, then with the different conditions, a different boundary layer condition that is C A is equal to C A I when z is equal to 0, and C A is equal to C A B where z is equal to the delta. So, we have now mathematically represented like this. Therefore, according to the film theory, the concentration profile in stagnant film is linear, and the molar flux through N A is given by these two equations.



So, dear friends, at the outset, we discussed the mass transfer operation in various aspects like falling film, thickness all those things, and for your convenience, we have enlisted a couple of references you can utilize as per your requirement. Thank you very much.