

Mass transfer phenomenon in polymers: Mass transfer coefficient and dimensionless numbers

Welcome to the mass transfer coefficient and dimensionless numbers. We are going to discuss about the mass transfer phenomena in polymer section. Now, we have covered the steady state diffusion through the constant area in the previous lecture apart from this we discussed about the non-diffusing components and we discussed about the steady state diffusion through the variable area. We had several problems to solve numerical problems for your convenience. Then we discussed about the diffusion from a sphere and equimolar counter diffusion.

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In this particular segment, we are going to discuss about the mass transfer coefficient and different types of mass transfer coefficient and we will discuss about the equimolar counter diffusion for 2 components and we will discuss about the relation between the mass transfer coefficient and mass transfer coefficient and film thickness and we will describe about the dimensionless numbers in mass transfer.

Mass transfer coefficient

The mass transfer coefficient is defined as:

- **The rate of mass transfer** is **proportional to the concentration driving force** that is the difference in concentration.
- **The rate of mass transfer** is **proportional to the area of contact** between the phases.
- If $W_A =$ Rate of mass transfer (kmol/s) of the solute A
 $\Delta C_A =$ Concentration driving force between two points
 $a =$ area of mass transfer

Where, $W_A \propto a\Delta C_A$ $W_A = k_C a\Delta C_A$
 k_C the **proportionality constant** known as **mass transfer coefficient**

$$W_A \propto a\Delta C_A$$

$$W_A = k_C a\Delta C_A$$

Where,

k_C the **proportionality constant** known as **mass transfer coefficient**

Now, the mass transfer coefficient this is defined as the rate of mass transfer which is proportional to the concentration driving force and that is a difference in the concentration. Apart from this, the rate of mass transfer is again proportional to the area of contact between the phases.

Mass transfer coefficient

- Consider, N_A is **the molar flux which is expressed in kmol/m².s**, we may write:

$$W_A = aN_A = k_C a\Delta C_A \dots (1)$$

- Mass transfer coefficient :

$$k_C = \frac{N_A}{\Delta C_A} = \text{Molar flux/concentration driving force}$$

- **The inverse of mass transfer coefficient** is a measure of the mass transfer resistance.
- If the driving force is expressed as **the difference in concentration** that is **kmol/m³**

the molar flux which is expressed in $\text{kmol/m}^2 \cdot \text{s}$

$$W_A = aN_A = k_C a \Delta C_A$$

Mass transfer coefficient

$$k_C = \frac{N_A}{\Delta C_A}$$



So, if we represent W_A as a rate of mass transfer of a solute A and the concentration driving force between the 2 points can be given as ΔC_A and area of mass transfer then mathematically we can represent like W_A is equal to $k_C \Delta C_A$ where k_C is the proportionality constant and this is known as the mass transfer coefficient. Now, if we consider this N_A is the mole flux which is expressed in kilo mole per meter square we may write that W_A is equal to $a N_A$ into $k_C \Delta C_A$ is equal to $k_C \Delta C_A$ which is equation number 1 and the mass transfer coefficient can be represented as k_C is equal to N_A over ΔC_A that is the mole molar flux over concentration driving force.

So, the inverse of mass transfer coefficient is a measure of the mass transfer resistance and if the driving force is expressed as a difference in the concentration and that can be represented in the units of kilo mole per meter cube. Now, the unit of mass transfer coefficient that would be meter per second which is the unit of velocity, if the mass transfer coefficient is expressed as a ratio of the local flux and the local driving force then it is called the local mass transfer coefficient.

Mass transfer coefficient

- **The unit** of mass transfer coefficient would be **m/s**, which is the unit of velocity.
- If the mass transfer coefficient is expressed as **the ratio of the local flux** and **the local driving force**, then it is called the local mass transfer coefficient.

Local mass transfer coefficient = Local flux/Local driving force

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So, the local mass transfer coefficient is equal to the local flux over local driving force. Now, when it is expressed as a ratio of the average flux over surface and the average driving force then it is known as the average mass transfer coefficient and average mass transfer coefficient can be represented mathematically is the average flux over average driving force.

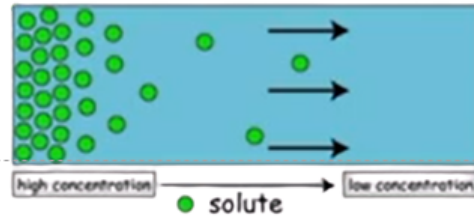
Types of mass transfer coefficient

- **Convective mass transfer** can occur in a **gas or in a liquid medium**, but it **does not occur in the solid medium**.

- A few choices of the driving force can be:

- a) **Difference in concentration**
- b) **Difference in partial pressure**
- c) **Difference in molar fraction**

- But in case of **the heat transfer the temperature difference is the only driving force**.



Now, there are different type of mass transfer coefficients, one is the convective mass transfer, this can occur in a gas or in a liquid medium, but it does not occur in the solid media. There are few choices of the driving force, this can be the difference in the concentration, the difference in the partial pressure and difference in the molar fraction. But in case of heat transfer, the temperature difference is the only driving force. Now, the different type of mass transfer coefficient have been defined, this depends on whether the mass transfer occur in a gas phase or in a liquid phase, then the choice of driving force, then whether it is a case of diffusing of component A through a non-diffusing B or whether it is an accumular counter current diffusion or counter diffusion.

Types of mass transfer coefficient

- **Different type of mass transfer coefficient** have been defined depending upon:

- a) **whether the mass transfer** occur in the gas phase or in the liquid phase.
- b) **choice of driving force**
- c) **whether it is a case of diffusing of component A** through non diffusing B or whether it is a equimolar counter current diffusion or the counter diffusion.

- If the transport of mass occur through a stagnant film of thickness δ , then:

$$\text{Flux} = \text{mass transfer coefficient} \times \text{driving force}$$

Diffusion of A through non-diffusing B

- Mass transfer in gas phase:**

$$N_A = k_G(p_{A1} - p_{A2}) = k_y(y_{A1} - y_{A2}) = k_C(C_{A1} - C_{A2}) \dots\dots(2)$$

- k_G , k_y and k_C are the gas phase mass transfer coefficient
- The unit of mass transfer coefficient k_y** is calculated from these flux equation, which is **$\text{kmol/m}^2 \cdot \text{s} \Delta y$**
- Δy stands for the driving force in mole fraction unit.

- Mass transfer in liquid phase:**

$$N_A = k_x(x_{A1} - x_{A2}) = k_L(C_{A1} - C_{A2}) \dots\dots(3)$$

- k_x and k_L are the liquid phase mass transfer coefficient, subscript 1 and 2 are the two positions



Mass transfer in gas phase

$$N_A = k_G(p_{A1} - p_{A2}) = k_y(y_{A1} - y_{A2}) = k_C(C_{A1} - C_{A2})$$

- k_G , k_y and k_C are the gas phase mass transfer coefficient
- The unit of mass transfer coefficient k_y** is calculated from these flux equation, which is **$\text{kmol/m}^2 \cdot \text{s} \Delta y$**
- Δy stands for the driving force in mole fraction unit.

Mass transfer in liquid phase

$$N_A = k_x(x_{A1} - x_{A2}) = k_L(C_{A1} - C_{A2})$$

- k_x and k_L are the liquid phase mass transfer coefficient, subscript 1 and 2 are the two positions

Now, if transport of mass occur through a stagnant film of thickness say delta, then flux is equal to mass transfer coefficient into the driving force. So, if we talk about the diffusion of A through non-diffusing B, the mass transfer in gas phase can be given as per this equation number 2, that is N_A is equal to $k_G p_{A1} - p_{A2}$ and that is equal to k_y into $y_{A1} - y_{A2}$ and that is equal to $k_C C_{A1} - C_{A2}$ and k_G , k_y and k_C are the gas phase mass transfer coefficient and a unit of mass transfer coefficient k_y is calculated from these flux equation and which is equal to the kilo mole per meter square second delta y and delta y stands for the driving force in the mole fraction unit and the mass transfer in the liquid phase can be given in the as per the equation number 3, where k_x and k_L are the liquid phase mass transfer coefficient and subscript A and 1 and 2 are the 2 positions.

Diffusion of A through non-diffusing B

- **If the gas phase is ideal:** the concentration term of eq 2 is given by:

$$C_A = p_A/RT$$

Where, p_A is the partial pressure of A

- Suppose that the distance between the two locations 1 and 2 is δ (the film thickness). The expression of mass transfer coefficient can be obtained by comparing equation 2 with:

$$N_A = \frac{D_{AB}P_t}{RT(x_2 - x_1)p_{BLM}}(p_{A1} - p_{A2}) \dots(4)$$



If gas phase is ideal

$$C_A = p_A/RT$$

Where, p_A is the partial pressure of A

The expression of mass transfer coefficient can be obtained by comparing equation 2

$$N_A = \frac{D_{AB}P_t}{RT(x_2 - x_1)p_{BLM}}(p_{A1} - p_{A2})$$

Now, if gas phase is ideal, the concentration term of this particular equation 2 can be given by C_A is equal to p_A over RT where p_A is the partial pressure of A. Now, suppose that the distance between the 2 location A and B is δ and the film that is the film thickness, the expression of mass transfer coefficient can be obtained by comparing the equation 2 with this N_A is equal to $D_{AB} P_1$ over RT into $x_2 - x_1$ p_{BLM} into $p_{A1} - p_{A2}$ that is equation number 4.

Diffusion of A through non-diffusing B

- The expression of mass transfer coefficient can be obtained by comparing equation 3 with

$$N_A = \frac{D_{AB} \left(\frac{\rho}{M_{avg}} \right)}{l \cdot X_{BLM}} (x_{A1} - x_{A2}) \quad \dots (5)$$

Where,

X_{BLM} = Logarithmic mean molar fraction of species B

$$X_{BLM} = \frac{(x_{B2} - x_{B1})}{\ln \left(\frac{x_{B2}}{x_{B1}} \right)}$$

Expression of mass transfer coefficient

$$N_A = \frac{D_{AB} \left(\frac{\rho}{M_{avg}} \right)}{l \cdot X_{BLM}} (x_{A1} - x_{A2})$$

Where,

X_{BLM} = Logarithmic mean molar fraction of species B

$$X_{BLM} = \frac{(x_{B2} - x_{B1})}{\ln \left(\frac{x_{B2}}{x_{B1}} \right)}$$

Now, the expression of mass transfer coefficient can be obtained by comparing this equation number 3 with this particular equation which is equation number 5. Now, here this X_{BLM} that is a logarithmic mean molar fraction of a species B and this can be represented by this mathematical representation. Now, if we talk about the gas phase, the mathematical expression for different mass transfer coefficient is given by KG, KY and KC and these are presented as equation number 6.

Diffusion of A through non-diffusing B

- Gas phase :** $k_G = \frac{D_{AB} P_t}{RT \delta \cdot p_{BLM}}, k_y = \frac{D_{AB} P_t^2}{RT \delta \cdot p_{BLM}}, k_C = \frac{D_{AB} P_t}{\delta \cdot p_{BLM}}$ (6)
- The relation among the three types of gas phase mass transfer coefficient that is k_G, k_y and k_C among these 3 can easily be obtained from eq (6)
- Liquid phase:** $k_x = \frac{D_{AB} (\rho/M)_{avg}}{\delta \cdot X_{BLM}}, k_L = \frac{D_{AB}}{\delta \cdot X_{BLM}}$ (7)
- The relation between the two types of liquid phase mass transfer we can obtained from k_x and k_L relations
- $k_C = RTk_G \quad k_y = P_t k_G \quad k_x = (\rho/M)_{avg} k_L$

Gas phase

$$k_G = \frac{D_{AB} P_t}{RT \delta \cdot p_{BLM}}, k_y = \frac{D_{AB} P_t^2}{RT \delta \cdot p_{BLM}}, k_C = \frac{D_{AB} P_t}{\delta \cdot p_{BLM}}$$

Liquid phase

$$k_x = \frac{D_{AB} (\rho/M)_{avg}}{\delta \cdot X_{BLM}}, k_L = \frac{D_{AB}}{\delta \cdot X_{BLM}}$$

k_x and k_L relations

$$k_C = RTk_G \quad k_y = P_t k_G \quad k_x = (\rho/M)_{avg} k_L$$

So, the relation among 3 type of a gas phase mass transfer coefficient that is k_G, k_y and k_C among 3, these 3 can easily be obtained from this particular equation. Now, if we talk about the liquid phase, the mass transfer coefficient k_x can be given on k_L can be represented mathematically as per this particular equation number 7. So, the relation between the 2 types of liquid phase mass transfer we can obtain from k_x and k_L relations. So, this can be mathematically obtained by k_C is equal to RTk_G and k_y is equal to $P_t k_G$ likewise. So, when we talk about the equimolar counter diffusion of component to set of notation for the mass transfer coefficient they are used here with the sign of prime.

Equimolar counter diffusion of components

- The set of notations for mass transfer coefficient are used here with a sign of prime(').

- Gas phase: $N_A = k'_G(p_{A1} - p_{A2}) = k'_y(y_{A1} - y_{A2}) = k'_C(C_{A1} - C_{A2})$ (8)

- Liquid phase: $N_A = k'_x(x_{A1} - x_{A2}) = k'_L(C_{A1} - C_{A2})$ (9)

- Comparing eq 8 and 9 for gas-phase transport, we get

$$N_A = \frac{D_{AB}P_t}{R.T.l}(y_{A1} - y_{A2}) = \frac{D_{AB}}{R.T.l}(p_{A1} - p_{A2}) \dots (10)$$

For gas phase

$$N_A = k'_G(p_{A1} - p_{A2}) = k'_y(y_{A1} - y_{A2}) = k'_C(C_{A1} - C_{A2})$$

For liquid phase

$$N_A = k'_x(x_{A1} - x_{A2}) = k'_L(C_{A1} - C_{A2})$$

Gas phase transport

$$N_A = \frac{D_{AB}P_t}{R.T.l}(y_{A1} - y_{A2}) = \frac{D_{AB}}{R.T.l}(p_{A1} - p_{A2})$$

So, gas phase N_A is equal to $k'_G p_{A1} - p_{A2}$ is equal to $k'_y y_{A1} - y_{A2}$ and this is equal to $k'_C C_{A1} - C_{A2}$ this is equation number 8. And if we talk about the liquid phase, then it can be mathematically represented as per the equation number 9. Now, if we compare both the equations for the gas phase transport, we get this equation that is N_A is equal to $\frac{D_{AB}P_t}{R.T.l}(y_{A1} - y_{A2})$ and that is equation number 10. Now, if we compare these equation 8 and 9 for liquid phase transport, so, we can get this N_A equation and if we follow the different type of expression for the mass transfer coefficient. So, the for the gas phase, the k'_G prime and k'_y prime and k'_C prime can be given like this and for liquid phase, the k'_x prime is given as per this mathematical representation.

Equimolar counter diffusion of components

- Comparing eq 8 and 9 for **liquid-phase transport**, we get

$$N_A = \frac{D_{AB} \left(\frac{\rho}{M_{avg}} \right)}{l} (x_{A1} - x_{A2}) \quad \text{Here, } l = \delta \text{ (thickness of film)} \quad \dots (11)$$

- Therefore, following are the expressions for mass transfer coefficient:

- Gas phase: $k'_G = \frac{D_{AB}}{RT\delta}, k'_y = \frac{D_{AB}P_t}{RT}, k'_C = \frac{D_{AB}}{\delta}$

- Liquid phase: $k'_x = \frac{D_{AB} \left(\frac{\rho}{M} \right)_{avg}}{\delta}, k'_L = \frac{D_{AB}}{\delta}$



Liquid phase transport

$$N_A = \frac{D_{AB} \left(\frac{\rho}{M_{avg}} \right)}{l} (x_{A1} - x_{A2})$$

The expressions for mass transfer coefficient

For gas phase

$$k'_G = \frac{D_{AB}}{RT\delta}, k'_y = \frac{D_{AB}P_t}{RT}, k'_C = \frac{D_{AB}}{\delta}$$

For liquid phase

$$k'_x = \frac{D_{AB} \left(\frac{\rho}{M} \right)_{avg}}{\delta}, k'_L = \frac{D_{AB}}{\delta}$$

Equimolar counter diffusion of components

- If the concentration of A is expressed in mole ratio unit, the mass transfer coefficient k_Y and k_X are expressed as:

Conversion: $N_A = k_Y (Y_{A1} - Y_{A2})$ for gas phase
and

$N_A = k_X (X_{A1} - X_{A2})$ for the liquid phase

Where,

Y_A and X_A are the concentration of A in the gas or in the liquid phase in mole ratio unit,

Note that

$$Y_A = \frac{y_A}{1 - y_A}, X_A = \frac{x_A}{1 - x_A}$$



$N_A = k_Y (Y_{A1} - Y_{A2})$ for gas phase

$N_A = k_X (X_{A1} - X_{A2})$ for the liquid phase

$$Y_A = \frac{y_A}{1 - y_A}, X_A = \frac{x_A}{1 - x_A}$$

Now, if the concentration of A is expressed in mode ratio unit, the mass transfer coefficient K_Y and K_X they are expressed as a conversion and this mathematically can be represented like this. Now, here the Y_A and X_A they are the concentration of A in the gas or in the liquid phase in the mode ratio unit. So, therefore, Y_A is equal to y_A into 1 minus y_A into X and X_A is equal to x_A over X 1 minus x_A .

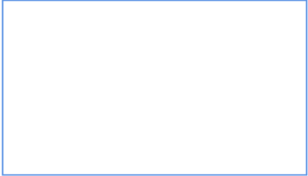
Different types of mass transfer coefficient

- The former mass transfer coefficient (k_G, k_Y, k_C, k_X and k_L) are inherently associated with the log mean concentration of the other species B which is non-diffusing.
- Accordingly, this type of mass transfer coefficient has a dependence on concentration because of the term D_{BLM} or X_{BLM} . This dependence can however, be ignored at low concentration of component.
- On the contrary, the coefficient k_G', k_Y', k_C', k_X' and k_L' do not have dependence on concentration.
- The second type of mass transfer coefficient or like k_C' is called Colburn Drew mass transfer coefficient.



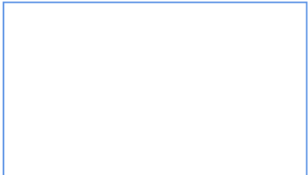
Relation between k_G and k_y

$$\begin{aligned}
 Y_{A1} - Y_{A2} &= \frac{P_{A1}}{P_t - P_{A1}} - \frac{P_{A2}}{P_t - P_{A2}} \\
 &= \frac{P_t P_{A1} - P_{A1} P_{A2} - P_t P_{A2} + P_{A1} P_{A2}}{(P_t - P_{A1})(P_t - P_{A2})} = \frac{P_t (P_{A1} - P_{A2})}{P_{B1} P_{B2}} \\
 P_{A1} - P_{A2} &= (Y_{A1} - Y_{A2}) \frac{P_{B1} P_{B2}}{P_t}
 \end{aligned}$$



Relation between k_G and k_y

$$\begin{aligned}
 N_A &= k_G (P_{A1} - P_{A2}) = \frac{D_{AB} P_t}{RT \delta_{\text{film}}} (P_{A1} - P_{A2}) \\
 &= \frac{D_{AB} P_t}{RT \delta_{\text{film}}} (Y_{A1} - Y_{A2}) \frac{P_{B1} P_{B2}}{P_t} = k_y (Y_{A1} - Y_{A2}) \\
 N_A &= k_y (Y_{A1} - Y_{A2}) \\
 k_y &= \frac{D_{AB} P_{B1} P_{B2}}{RT \delta_{\text{film}}} > k_G \frac{P_{B1} P_{B2}}{P_t}
 \end{aligned}$$



Relation between k_G and k_Y

$$\begin{aligned}
 p_{A1} - p_{A2} &= (y_{A1} - y_{A2}) \frac{(p_{B1} p_{B2})}{P} \\
 N_A &= k_G' (p_{A1} - p_{A2}) = \frac{D_{AB}}{RT \delta} (p_{A1} - p_{A2}) \\
 &= \frac{D_{AB}}{RT \delta} (y_{A1} - y_{A2}) \frac{p_{B1} p_{B2}}{P} = k_Y' (y_{A1} - y_{A2}) \\
 N_A &= k_Y' (y_{A1} - y_{A2}) \\
 k_Y' &= \frac{D_{AB} p_{B1} p_{B2}}{RT \delta P} \\
 &= \frac{k_G' p_{B1} p_{B2}}{P}
 \end{aligned}$$

Now, there are different type of mass transfer coefficients, a former mass transfer coefficient like K_G , K_Y , K_C , K_X and K_{Lr} inherently associated with the log mean concentration of the other species B which is non-diffusing and as per this particular type of mass transfer coefficient, they has a dependency on the concentration because the term P BLM or X BLM this dependency can however be ignored at a low concentration of the component. Now, on the contrary, the coefficient K_G prime, K_Y prime, K_C prime, K_X prime and K_L prime do not have a dependency on the concentration and the second type of mass transfer coefficient K_C prime that is called the Coulburn-Dreu mass transfer coefficient.

So, the driving force in case of mole ratio unit Y between the two points can be written as Y_{A1} minus Y_{A2} is equal to $\frac{p_{A1}}{P_T}$ minus $\frac{p_{A1}}{P_T}$ minus $\frac{p_{A2}}{P_T}$ minus $\frac{p_{A2}}{P_T}$ and that is equal to $\frac{P_T p_{A1} - p_{A1} p_{A2} - P_T p_{A2} + p_{A1} p_{A2}}{P_T - p_{A1} - P_T + p_{A2}}$. This is $\frac{P_T (p_{A1} - p_{A2}) - p_{A1} p_{A2} + p_{A1} p_{A2}}{P_T - p_{A1} - P_T + p_{A2}}$. This is $\frac{P_T (p_{A1} - p_{A2})}{P_T - p_{A1} - P_T + p_{A2}}$ and that comes out to be $\frac{p_{A1} - p_{A2}}{1 - \frac{p_{A1} + p_{A2}}{P_T}}$ that is $Y_{A1} - Y_{A2}$ that is $\frac{p_{B1} p_{B2}}{P_T}$. Now, we have N_A is equal to $K_G (p_{A1} - p_{A2}) \frac{dA_b P_T}{RT \delta P}$, $\frac{p_{A1} - p_{A2}}{1 - \frac{p_{A1} + p_{A2}}{P_T}}$ which is $\frac{dA_b P_T}{RT \delta P} \frac{p_{B1} p_{B2}}{P_T} (Y_{A1} - Y_{A2})$ and that is $\frac{p_{B1} p_{B2}}{P_T}$ that is $K_Y (Y_{A1} - Y_{A2})$. Now, since N_A is equal to $K_Y (Y_{A1} - Y_{A2})$, we may write K_Y is equal to $\frac{dA_b p_{B1} p_{B2}}{RT P}$ BLM which is equal to $K_G \frac{p_{B1} p_{B2}}{P}$. So, similar relation between the K_G prime and K_Y prime in case of equimolar counter diffusion this applies.

Relation between k_G' and k_y'

- Conversion among the gas phase mass transfer coefficient, we can write:

$$F = k_G p_{BLM} = k_y \frac{p_{BLM}}{P_t} = k_G' P_t = k_y' = k_C' \frac{P_t}{RT}$$

- Conversion among the liquid phase mass transfer coefficient we can write:

$$F = k_x X_{BLM} = k_L X_{BLM} C = k_L' C = k_L' \frac{\rho}{M} = k_x'$$



Conversion among the gas phase mass transfer coefficient

$$F = k_G p_{BLM} = k_y \frac{p_{BLM}}{P_t} = k_G' P_t = k_y' = k_C' \frac{P_t}{RT}$$

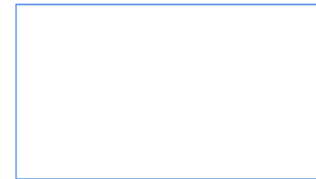
Conversion among the liquid phase mass transfer coefficient

$$F = k_x X_{BLM} = k_L X_{BLM} C = k_L' C = k_L' \frac{\rho}{M} = k_x'$$

So, we know that this $PA_1 - PA_2$ that is equal to $YA_1 - YA_2$ into $PB_1 - PB_2$ over PT . So, now we have NA is equal to k_G prime $PA_1 - PA_2$ which is equal to $dAb RT \Delta P A_1 - P A_2$ this is P sorry dAb over $RT \Delta P A_1 - YA_2 - PB_1 - PB_2$ over PT which is equal to KY prime $YA_1 - YA_2$. Now, since NA is equal to KY prime $YA_1 - YA_2$ we may write KY prime is equal to $dAb PB_1 - PB_2$ over $RT \Delta P$ and that is k_G prime is $PB_1 - PB_2$ over PT . So, now the conversion among the gas phase mass transfer coefficient we can write all these things like this F is equal to k_G BLM is equal to $KY P - BLM - PT$ is equal to k_G prime PT K is equal to KY prime equal to k_C prime PT over RT and if it conversion among the liquid phase mass transfer coefficient we can write like this.

Values of mass transfer coefficient and film thickness

- For gas phase mass transfer coefficient, $k_C \sim 10^{-2}$ m/s and the film thickness $\delta \sim 1$ mm.
- For liquid phase mass transfer coefficient, $k_C \sim 10^{-5}$ m/s and the film thickness $\delta \sim 0.1$ mm.

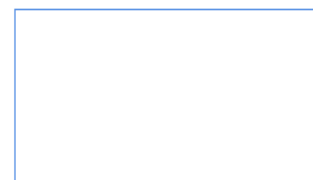


Now, for gas phase mass transfer coefficient K_C is almost equal to 10 to the power minus 2 meter per second and the film thickness is δ is about 1 mm.

For liquid phase the mass transfer coefficient K_C is around 10 to the power minus 5 meter per second and the film thickness is around δ thickness is 0.1 mm.

Problem-1

Question: Large volume of nitrogen gas (N_2) at atmospheric pressure is flowing over a pool of liquid of methanol, which is evaporating. Nitrogen is assumed to be insoluble in liquid. The gas phase mass transfer coefficient of methanol which is k_G is 2×10^{-5} kmol/m².s kPa. Assume vapour pressure of methanol at 298 K is 10 kPa. Calculate k_y , k_C , k_Y and F .



$$\begin{aligned}
 P_t &= 1 \text{ atm} = 101.3 \text{ kPa} \\
 R &= 8.314 \\
 T &= 298 \text{ K} \\
 K_y &= K_G P_t = 2 \times 10^{-5} \left(\frac{\text{kmol}}{\text{m}^2 \text{ s kPa}} \right) (101.3 \text{ kPa}) = 2.03 \times 10^{-3} \frac{\text{kmol}}{\text{m}^2 \text{ s}} \\
 K_c &= K_G RT = 2 \times 10^{-5} \times 8.314 \times 298 \\
 &= 0.0495 \text{ mol/s} \\
 P_{A1} &= 10 \text{ kPa} \quad P_{B1} = P_t - P_{A1} = 101.3 - 10 = 91.3 \text{ kPa} \\
 P_{A2} &= 0 \quad P_{B2} = P_t = 101.3 \text{ kPa}
 \end{aligned}$$

$$\begin{aligned}
 K_y &= K_G \frac{P_{B1} P_{A2}}{P_t} = 2 \times 10^{-5} \frac{91.3 \times 101.3}{101.3} \\
 &= 1.83 \times 10^{-3} \frac{\text{kmol}}{\text{m}^2 \text{ s}} \\
 F &= K_G P_{A2} = 2 \times 10^{-5} \times \frac{P_{B2} - P_{B1}}{\ln \left(\frac{P_{B2}}{P_{B1}} \right)} \\
 R &= 2 \times 10^{-5} \times \frac{101.3 - 91.3}{\ln \left(\frac{101.3}{91.3} \right)} = 1.92 \times 10^{-3} \text{ kmol/m}^2 \text{ s}
 \end{aligned}$$

Now, let us take up another problem and that is a large volume of nitrogen gas at atmospheric pressure is flowing over a pool of liquid methanol which is evaporating and nitrogen is assumed to be insoluble in liquid and the gas phase mass transfer coefficient of methanol which is K_G is 2×10^{-5} kilo mole per meter square second kilo Pascal and assuming the vapor pressure of methanol at 298 Kelvin is 10 kilo Pascal you need to calculate the K_y , K_c , K and F . So, in this case the diffusion of methanol occurs through a non-diffusing N_2 . So, given that P_t is equal to 1 atmosphere that is 101.3 kilo Pascal, R is equal to 8.314, T is equal to 298 Kelvin. So, K_y is equal to $K_G P_t$ and that is equal to 2×10^{-5} kilo mole kilo Pascal into 101.3 kilo Pascal and which comes out to be 2.03×10^{-3} kilo mole meter square delta y.

Now, KC is equal to $KG RT$ which is equal to 2 into 10 to the power minus 5 into 8.314 into 298 which comes out to be 0.0495 meter per second. Now, $PA1$ is equal to 10 kilo Pascal and PB is equal to PT minus $PA1$ that is 101.3 minus 10 which is 91.3 kilo Pascal and $PA2$ is equal to 0 that is $PB2$ is equal to PT which is 101.3 kilo Pascal. So, KY is equal to $KG PB1, PB2$ over PT that is 2 into 10 to the power minus 5 into 91.3 into 101.3 over 101.3 and that is 1.83 into 10 to the power minus 3 kilo mole delta y . Now, F is equal to $KG PBLM$ which is 2 into 10 to the power minus 5 into $PB2$ minus $PB1$ over $\ln PB2$ over $PB1$. This is equal to 2 into 10 to the power minus 5 into 101.3 minus 91.3 over $\ln 101.3$ over 91.3 , 1.92 into 10 to the power minus 3 kilo mole per meter square second and this is our answer.

Dimensionless numbers in mass transfer

- **The transport coefficient** and other important parameters such as **fluid properties, velocity** etc. can be expressed in terms of meaningful **dimensionless groups**.

Examples:

a) **The heat transfer coefficient h** is often expressed in terms of the **Nusselt number, Reynolds number and Prandtl number**.

b) **Experimental forced convection heat transfer** data are frequently correlated as:

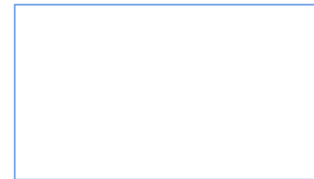
$$Nu = \phi(Re, Pr)$$

Now, there are so many dimensionless numbers in mass transfer operation, the transport coefficient and other important parameters like fluid, properties, velocity, these can be expressed in terms of meaningful dimensionless groups. For example, the heat transfer coefficient H is often expressed in terms of a Nusselt number, Reynolds number and Prandtl number.

Dimensionless numbers in mass transfer

- **The resulting correlation** may be used to estimate **the heat transfer coefficient** for any other set of process conditions and system parameters.
- The most important equations which relates **the Nusselt number with the Reynolds number and Prandtl number** is the **Dittus-Boelter equation**.
- Here we have **two most important dimensionless group**:

- a) **The Sherwood number** which is the mass transfer analogue of the **Nusselt number**.
- b) **The Schmidt number** which is the mass transfer analogue of the **Prandtl number**



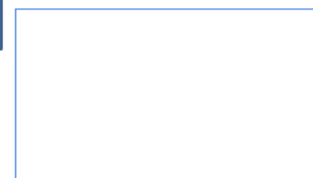
Now experimental forced convection heat transfer data, they are frequently correlated with the Nusselt number is proportional to the Reynolds number and Prandtl number. Now, the resulting correlation may be used to estimate the heat transfer coefficient for any other set of process condition and system parameters. The most important equation which relates the Nusselt number with the Reynolds number and Prandtl number is the Dittus-Boelter equation. Now, here we have two most important dimensionless group, one is the Sherwood number which is the mass transfer analogous of Nusselt number and second one is the Schmidt number which is the mass transfer analogous of Prandtl number. Now, the origin of Sherwood and Schmidt can be traced by the analogy with the Nusselt and Prandtl number respectively.

Dimensionless numbers in mass transfer

- The origin of **Sherwood and Schmidt can be traced by analogy with Nusselt and Prandtl number** respectively.
- In heat transfer the Nusselt number is

$$Nu = \frac{\text{Convective heat flux}}{\text{Heat flux for conduction}}$$
 through a stagnant medium of thickness l for the same ΔT

$$Nu = \frac{h\Delta T}{(k/l)\Delta T} = \frac{hl}{k}$$
 where k = thermal conductivity



$$Nu = \frac{h\Delta T}{(k/l)\Delta T} = \frac{hl}{k}$$

Where, k is thermal conductivity

In heat transfer, the Nusselt number is equal to the convective heat transfer over heat flux for conduction through a stagnant medium of thickness l for the same delta t and this can be mathematically represented like Nusselt number H is equal to H delta t over k over L into delta t and that is H L k over k, where k is the thermal conductivity.

Dimensionless numbers in mass transfer

- Similarly, **in mass transfer the Sherwood number is :**

$$Sh = \frac{\text{Convective mass (molar) flux}}{\text{Mass or molar flux for molecular diffusion through a stagnant medium of thickness l under the driving force of } \Delta p_A}$$
- So, if we considered a **gas phase**, mass transfer of A through a binary mixture of A and B in which we considered B is not diffusing.

Convective mass flux = $k_G \Delta p_A$

Convective mass flux = $k_G \Delta p_A$

Similarly, the mass transfer of Sherwood number is the convective mass transfer or mass or molar flux over mass or molar flux at molecular diffusion through a stagnant number of thickness l under the driving force of delta p A.

Dimensionless numbers in mass transfer

- The mass flux due to molecular diffusion of A through non-diffusing B we have derived as :

$$\frac{D_{AB} P_t}{R.T.l.p_{BLM}} \Delta p_A$$

Then,

$$Sh = \frac{k_C \Delta p_A}{\frac{D_{AB} P_t}{R.T.l.p_{BLM}} \Delta p_A} = \frac{k_C p_{BLM} R.T.l}{D_{AB} P_t} = \frac{k_C l.p_{BLM}}{D_{AB} P_t}$$

- If we considered transport of A in a liquid solution at a rather low concentration ($X_{BLM} = 1$),

$$\text{Convective mass flux, } N_A = k_L \Delta C_A$$



Mass flux due to molecular diffusion of A through non-diffusing B we have

$$Sh = \frac{\frac{D_{AB} P_t}{R.T.l.p_{BLM}} \Delta p_A}{\frac{D_{AB} P_t}{R.T.l.p_{BLM}} \Delta p_A} = \frac{k_C p_{BLM} R.T.l}{D_{AB} P_t} = \frac{k_C l.p_{BLM}}{D_{AB} P_t}$$

Convective mass flux, $N_A = k_L \Delta C_A$

So, if we consider the gas phase mass transfer of A through the binary mixture of A and B in which we consider B is not diffusing to convective mass transfer flux can be represented as $k_g \Delta p_A$. Now, the mass flux due to the molecular diffusion A through non diffusing B, we have derived as $\frac{D_{AB} p_t}{R.T.l.p_{BLM}} \Delta p_A$ and the Sherwood number can be represented as $\frac{k_C \Delta p_A}{\frac{D_{AB} p_t}{R.T.l.p_{BLM}} \Delta p_A}$ and which is in general can be represented as $\frac{k_C l.p_{BLM}}{D_{AB} p_t}$. Now, if we consider the transport of A in a liquid solution and at a rather low concentration that is X_{BLM} is equal to 1.

Dimensionless numbers in mass transfer

- The diffusive flux of A through a stagnant liquid layer of thickness l is:

$$\frac{D_{AB}}{l} \Delta C_A$$

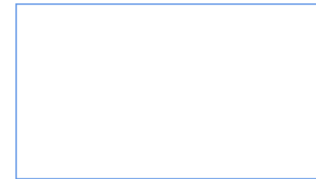
- Then, the Sherwood number is :

$$Sh = \frac{k_L \Delta C_A}{\left(\frac{D_{AB}}{l} \right) \Delta C_A} = \frac{k_L l}{D_{AB}}$$

Here, l is the characteristic length

- The commonly used characteristics lengths are:

- For a sphere: diameter, d
- For a cylinder: diameter, d
- For a flat plate: distance from the leading edge, x



$$Sh = \frac{\frac{D_{AB}}{l} \Delta C_A}{\left(\frac{D_{AB}}{l} \right) \Delta C_A} = \frac{k_L l}{D_{AB}}$$

So, convective heat convective mass flux can be represented as $n A$ is equal to $k L \Delta c A$. Now, the diffusive flux of A through a stagnant liquid layer of thickness l can be represented as D_{AB} over l into Δc , then the Sherwood number can be represented like this. Here l is the characteristics length. Now, commonly used characteristic lengths are for sphere the diameter D , for cylinder the diameter D and for flat plate the distance from the leading edge that is represented as X and the Schmidt number is the mass transfer analogues of the Prandtl number and we define the Prandtl number as momentum diffusivity over thermal diffusivity and mathematically Prandtl number can be represented like Pr is equal to $C_p \mu$ over k . Now, analogically we can define a Schmidt number like momentum diffusivity over molecular diffusivity and which can be represented as μ over ρ into D_{AB} and μ over ρD_{AB} which is equal to μD_{AB} .

$$Pr = \frac{\mu / \rho}{k / \rho \cdot C_p} = \frac{C_p \cdot \mu}{k}$$

$$Sc = \frac{\mu / \rho}{D_{AB}} = \frac{\mu}{\rho \cdot D_{AB}} = \frac{\nu}{D_{AB}}$$

Dimensionless numbers in mass transfer

- The Schmidt number is the mass transfer analogue of Prandtl number.
- We define Prandtl number as :

$$\text{Pr} = \frac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}}$$

$$\text{Pr} = \frac{\mu/\rho}{k/\rho C_p} = \frac{C_p \cdot \mu}{k}$$

- Analogously we can define the Schmidt number as:

$$\text{Sc} = \frac{\text{Momentum diffusivity}}{\text{Molecular diffusivity}}$$

$$\text{Sc} = \frac{\mu/\rho}{D_{AB}} = \frac{\mu}{\rho \cdot D_{AB}} = \frac{\nu}{D_{AB}}$$

Dimensionless numbers in mass transfer

- Schmidt number** also represents **the relative order of magnitude** of the thickness of concentration boundary layer in comparison with that of the velocity boundary layer.
- Taking the case of **gas phase mass transfer** for flow past a sphere, 2 cm in diameter, at low partial pressure of the solute (i.e., $P_{BLM}/P_t \sim 1$)

- The Sherwood number is:

$$\text{Sh} = \frac{k_c d \cdot p_{BLM}}{D_{AB} P_t} = \frac{(10^{-2} \text{ m/s}) \times (2 \times 10^{-2} \text{ m})}{10^{-5} \text{ m}^2/\text{s}}$$

$$\text{Sh} \sim 20$$

Sherwood number

$$\text{Sh} = \frac{k_c d \cdot p_{BLM}}{D_{AB} P_t} = \frac{(10^{-2} \text{ m/s}) \times (2 \times 10^{-2} \text{ m})}{10^{-5} \text{ m}^2/\text{s}} \text{ therefore, } \text{Sh} = 20$$

Now, Schmidt number also represents the relative order of magnitude of the thickness of concentration boundary layer in comparison with that of velocity boundary layer. Now, taking the case of gas phase mass transfer for flow past a sphere 2 centimeter in a diameter of a low partial pressure of the solute. So, p_{BLM} over p_t is almost equal to 1, then the Sherwood number can be represented as $k_c d \cdot p_{BLM}$ over $D_{AB} p_t$ and if put the numerical value then it comes out to be nearly 20. Now,

the Schmidt number may be found to be like this and therefore, if we say that Schmidt number is almost equal to 1, so for common gases the Prandtl number is almost equal to the Schmidt number which is representing 1.



Dimensionless numbers in mass transfer

- The Schmidt number may be found to be :

$$Sc = \frac{\nu}{D_{AB}} = \frac{10^{-5} m^2 / s}{10^{-5} m^2 / s}$$

Therefore, Sc ~ 1

- For common gasses, Pr ≈ Sc ≈ 1.0



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The Schmidt number

$$Sc = \frac{\nu}{D_{AB}} = \frac{10^{-5} m^2 / s}{10^{-5} m^2 / s} \text{ therefore } Sc \sim 1.$$

Now, for liquid phase mass transport system in a similar geometry the Sherwood number can be represented as $k_L d / D_{AB}$ that is almost equal to $10^{-2} \times 2 \times 10^{-2} / 10^{-9}$ and this is the Sherwood number is almost 200.

Dimensionless numbers in mass transfer

- For liquid phase mass transport** in a similar geometry:



$$Sh = \frac{k_L d}{D_{AB}} \approx \frac{(10^{-2} m / s) \times (2 \times 10^{-2} m)}{10^{-9} m^2 / s} \Rightarrow \text{Sh} \sim 200$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{10^{-6} m^2 / s}{10^{-9} m^2 / s} \Rightarrow \text{Sc} \sim 1000$$

- For common liquids except for liquid metals:**

$$10 < Pr < 10^2$$

$$400 < Sc < 10^4$$



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For liquid phase mass transport

$$Sh = \frac{k_L d}{D_{AB}} \approx \frac{(10^{-2} m/s) \times (2 \times 10^{-2} m)}{10^{-9} m^2/s} \text{ for } Sh \sim 200$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{10^{-6} m^2/s}{10^{-9} m^2/s} \text{ for } Sc \sim 1000$$

Now, if we put or if we compare with the Schmidt number this comes out to be nearly 1000. So, for common liquid except for the liquid metals the Prandtl number is in between 10 to 10 to 10 to 10 square and Schmidt number is between 400 to 10 to the power 4. The Stanton number for the mass transfer which is analogous of the Stanton number of for the heat transfer. Now, we define the Stanton number for the heat transfer is the convective heat flux over heat flux due to the bulk flow which can be represented as Nusselt number over Reynolds number and Prandtl number.

Dimensionless numbers in mass transfer

- **The Stanton number for the mass transfer** which is **analogue of the Stanton number for the heat transfer**.
- We define **the Stanton number for heat transfer as :**

$$St_H = \frac{\text{Convective heat flux}}{\text{Heat flux due to bulk flow}}$$

$$St_H = \frac{h\Delta T}{C_p \rho v \Delta T} = \frac{h.l/k}{(v.l.\rho/\mu)(C_p \mu/k)} = \frac{Nu}{Re.Pr}$$

$$St_H = \frac{h\Delta T}{C_p \rho v \Delta T} = \frac{h.l/k}{(v.l.\rho/\mu)(C_p \mu/k)} = \frac{Nu}{Re.Pr}$$

Now analogously we define the Stanton number for the mass transfer as convective mass flux over flux due to bulk flow of the medium and then it can be represented as a Sherwood number over Reynolds number and Schmidt number.

Dimensionless numbers in mass transfer

- Analogously, we define **the Stanton number for the mass transfer** as :

$$St_M = \frac{\text{Convective mass flux}}{\text{Flux due to bulk flow of the medium}}$$

$$St_M = \frac{k_L \Delta C}{v \Delta C} = \frac{(k_L l / D_{AB})}{(v l \cdot \rho / \mu)(\mu / \rho \cdot D_{AB})} = \frac{Sh}{Re \cdot Sc}$$

$$St_M = \frac{k_L \Delta C}{v \Delta C} = \frac{(k_L l / D_{AB})}{(v \cdot l \cdot \rho / \mu)(\mu / \rho \cdot D_{AB})} = \frac{Sh}{Re \cdot Sc}$$

The Peclet number for the mass transfer which is analogous of the Peclet number in case of heat transfer and we define the heat transfer Peclet number as a heat flux due to the bulk flow over the flux due to conduction across a thickness L.

$$Pe_H = \frac{C_p \cdot \rho \cdot v \cdot \Delta T}{(k/l) \Delta T} = \left(\frac{v \cdot l \cdot \rho}{\mu} \right) \left(\frac{C_p \cdot \mu}{k} \right) = Re \cdot Pr$$

Dimensionless numbers in mass transfer

- The **Peclet number for the mass transfer** which is **analogue of the Peclet number in case of the heat transfer**.
- We define **Peclet number in case of heat transfer** as:

$$Pe_H = \frac{\text{Heat flux due to bulk flow}}{\text{Flux due to conduction across a thickness } l}$$

$$Pe_H = \frac{C_p \cdot \rho \cdot v \cdot \Delta T}{(k/l) \Delta T} = \left(\frac{v \cdot l \cdot \rho}{\mu} \right) \left(\frac{C_p \cdot \mu}{k} \right) = Re \cdot Pr$$



Dimensionless numbers in mass transfer

- Analogously, we define **the Peclet number for the mass transfer** as:

$$Pe_M = \frac{\text{Flux due to bulk flow of the medium}}{\text{Diffusive flux across a thickness } l}$$

$$Pe_M = \frac{v \Delta C}{(D_{AB}/l) \Delta C} = (v \cdot l \cdot \rho / \mu) (\mu / \rho \cdot D) = Re \cdot Sc$$



$$Pe_M = \frac{v \Delta C}{(D_{AB}/l) \Delta C} = (v \cdot l \cdot \rho / \mu) (\mu / \rho \cdot D) = Re \cdot Sc$$

So, the Peclet number can be given as Reynolds number into Prandtl number and as per this particular analogy the Peclet number for the mass transfer can be defined as the Peclet number is equal to flux due to bulk flow of the medium over the diffusivity diffusive flux across the thickness. So, if we represent this in terms of the Peclet number this can be given as the multiplication of the Reynolds number over into Schmidt number.

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So, dear friends in this particular segment we discussed about the mass transfer operation and we discussed about the various dimensionless numbers which are useful for this particular segment. For your convenience we have been listed couple of references; you can utilize those as per your requirement. Thank you very much.