Polymer Process Engineering

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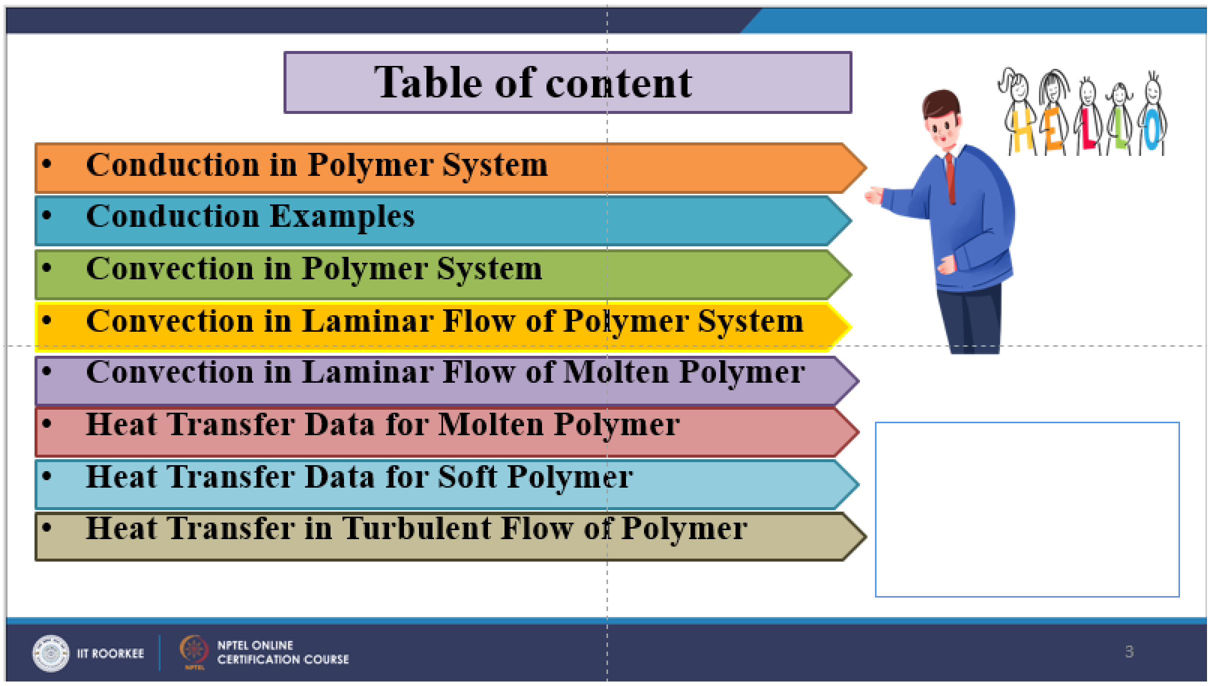
Lecture – 19

Heat Transfer Phenomenon in polymer systems: Conduction and Convection

Dear friends, welcome to the conduction and convection aspect of heat transfer phenomena in the edges of polymer process engineering. Now, here is a brief outlook that what we discussed in the previous segment. We discussed about the melting temperature, glass transition temperature, thermal conductivity, thermal conductivity in polymer and composites. We discussed about the conduction heat transfer polymeric system. Then heat transfer example were given in brief in that particular segment. In this particular segment, we are going to discuss about the conduction in polymeric system in detail with a couple of examples.



Then we will discuss the convection in polymeric systems and convection in the laminar flow of polymeric systems, then the laminar flow of molten polymers. We will discuss the heat transfer data for molten polymers heat transfer data for soft polymers, and heat transfer in the turbulent flow of polymers. Now, let us take up the conduction in polymeric systems, the cylindrical coordinates. So, heat transfer studies can be carried out for any system boundaries such as spheres, rectangles, cylindrical, and or any non-uniform shape.

Conduction in polymer system: Cylindrical Coordinates

- The het transfer studies can be carried out for any system boundary, such as, spheres, rectangles, cylinders or any non-uniform shape.
- However, most of the het transfer studies for polymers are carried out in cylindrical systems.
- For example, if we want to study the conduction phenomenon of a specific polymer system which is being synthesized in a injection molding system or in a CSTR, then these type of assembly are considered as cylinder for the convenience.

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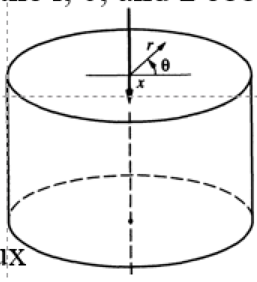
So, most of the heat transfer studies of the polymers are carried out in cylindrical systems. For example, suppose we want to study the conduction phenomena of a specific polymer system, which is being synthesized in the injection molding system in a CSTR.

Conduction in polymer system: Cylindrical Coordinates

For a cylindrical coordinate system (as shown in fig), the terms used are:

where:

- r, θ, z = coordinate directions
- V_r, V_θ, V_z = velocity components in the r, θ, and z coordinates
- $\tau_{r\theta}, \tau_{rz}, \tau_{\theta z}$ = shear stresses
- $\tau_{rr}, \tau_{\theta\theta}, \tau_{zz}$ = normal stresses
- ρ = density
- P = pressure
- T = point temperature
- q_r, q_z, q_θ = components of energy flux
- C_v = constant-volume specific heat
- A_o = a heat -generation term



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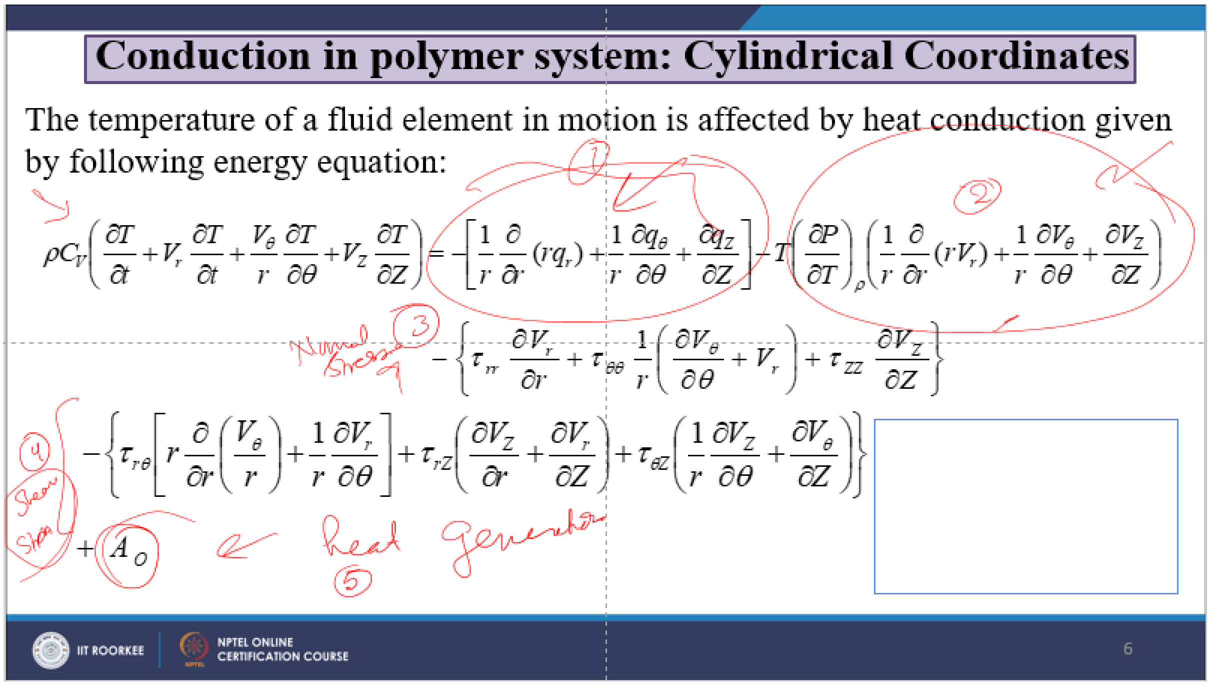
In that case, these types of assemblies are considered as a cylinder for convenience. Now, here you see that for the cylindrical coordinate system, which is represented in this particular figure, the terms used are where r theta and z are the coordinate directions, vr, v theta, vz are the velocity components in the r theta and z coordinates all 3 coordinates. Tau these are the shear stresses, then the normal stresses, and rho is the density, p is the pressure and t is the point temperature q they are the different components of energy flux and c is the constant volume specific heat. In contrast, a naught is the heat generation term. So, the temperature of a fluid element in motion is affected by the heat conduction given in this particular equation. So, these equations if you see, now this there are various terms like here, this particular term is represented at this particular part indicates the expansion effect due to the heating into the material.



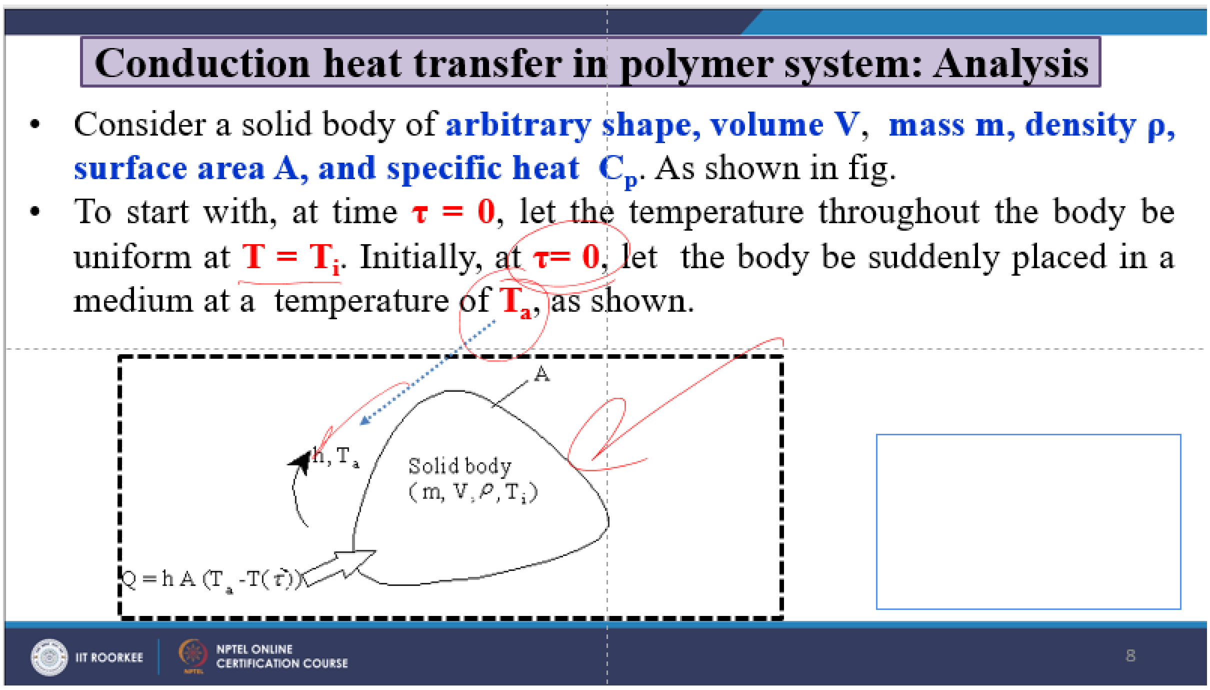
$$\rho C_V\left(\frac{\partial T}{\partial t}+V_r\frac{\partial T}{\partial t}+\frac{V_\theta}{r}\frac{\partial T}{\partial \theta}+V_Z\frac{\partial T}{\partial Z}\right)=-\left[\frac{1}{r}\frac{\partial}{\partial r}(rq_r)+\frac{1}{r}\frac{\partial q_\theta}{\partial \theta}+\frac{\partial q_Z}{\partial Z}\right]-T\left(\frac{\partial P}{\partial T}\right)_\rho\left(\frac{1}{r}\frac{\partial}{\partial r}(rV_r)+\frac{1}{r}\frac{\partial V_\theta}{\partial \theta}+\frac{\partial V_Z}{\partial Z}\right)$$

$$-\left\{\tau_{rr}\frac{\partial V_r}{\partial r}+\tau_{\theta\theta}\frac{1}{r}\left(\frac{\partial V_\theta}{\partial \theta}+V_r\right)+\tau_{ZZ}\frac{\partial V_Z}{\partial Z}\right\}$$

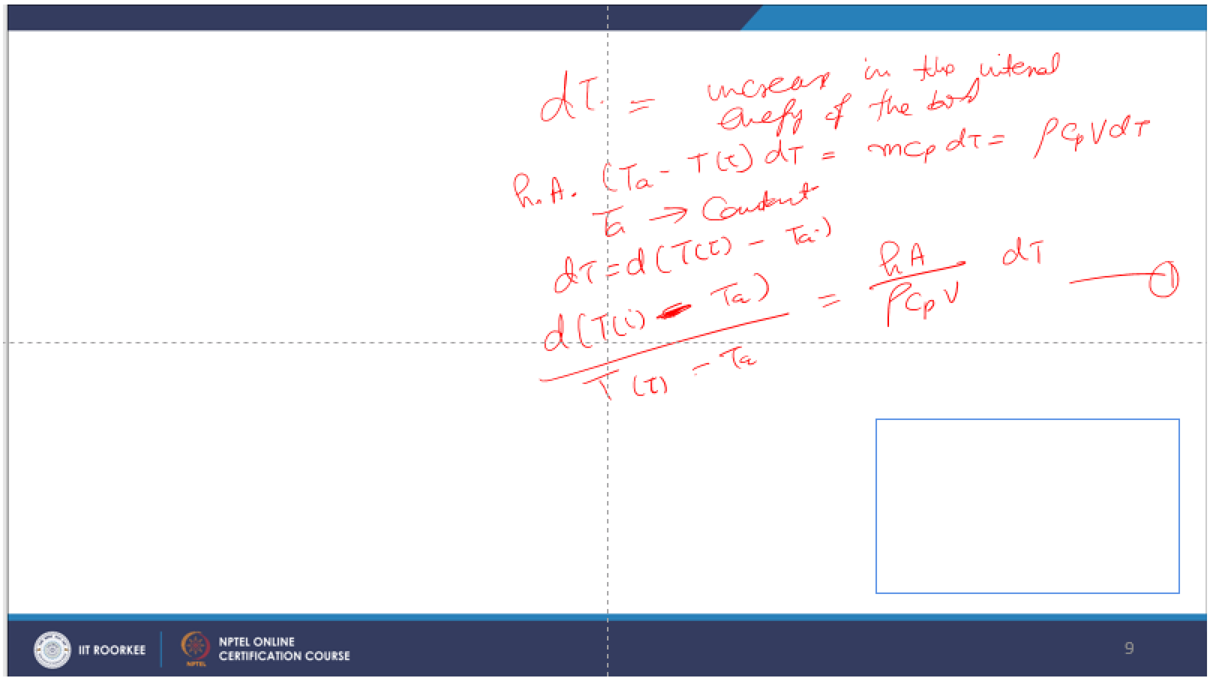
$$-\left\{\tau_{r\theta}\left[r\frac{\partial}{\partial r}\left(\frac{V_\theta}{r}\right)+\frac{1}{r}\frac{\partial V_r}{\partial \theta}\right]+\tau_{rZ}\left(\frac{\partial V_Z}{\partial r}+\frac{\partial V_r}{\partial Z}\right)+\tau_{\theta Z}\left(\frac{1}{r}\frac{\partial V_Z}{\partial \theta}+\frac{\partial V_\theta}{\partial Z}\right)\right\}$$

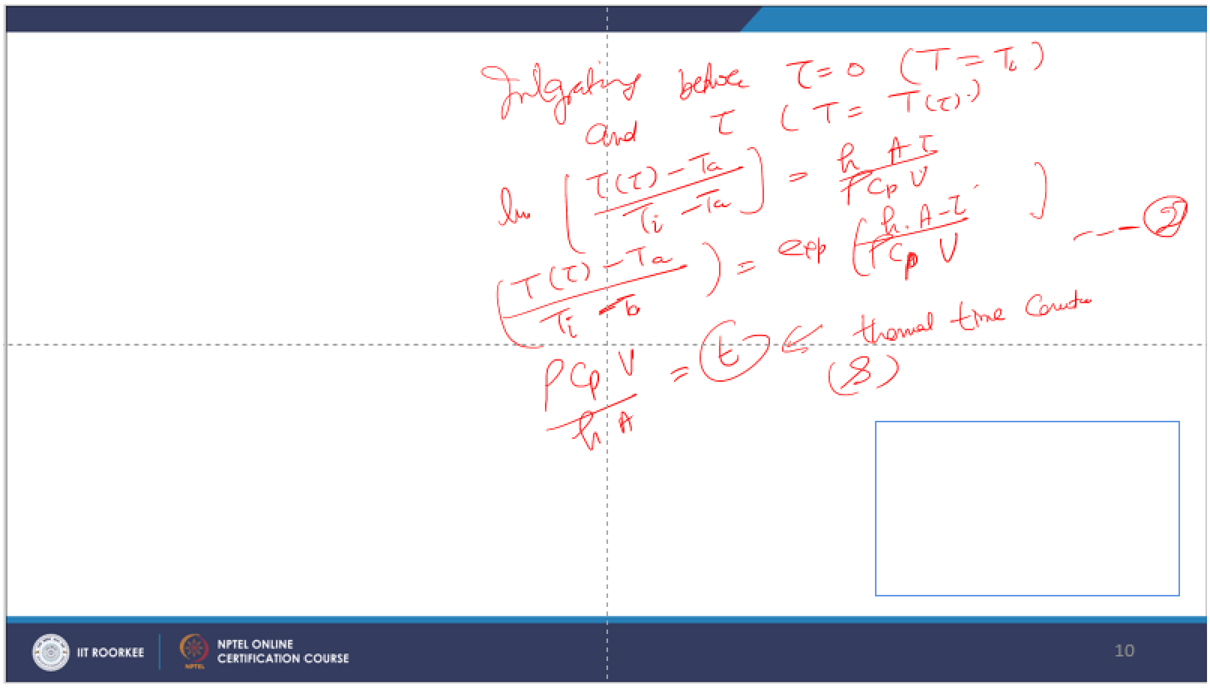
The temperature of a fluid element in motion is affected by the heat conduction, this is given by this particular equation. Now here these various terms being indicative and you see that this is this particular term is the indicates the expansion effect due to the heating into the material. However, this term this particular term this is multiplied by the first function at constant density, this indicates the viscous heating and similarity with the second term displays the viscous dissipation in the solution. Now the third term which is the normal stress term, this is a similarly these functions related to the normal stress term. Now when we talk about another term that is this is the shear stress term, these values account for the shear stress and finally, this a naught this indicates that all type of heat generation such as the phase change, chemical source and electrical source.

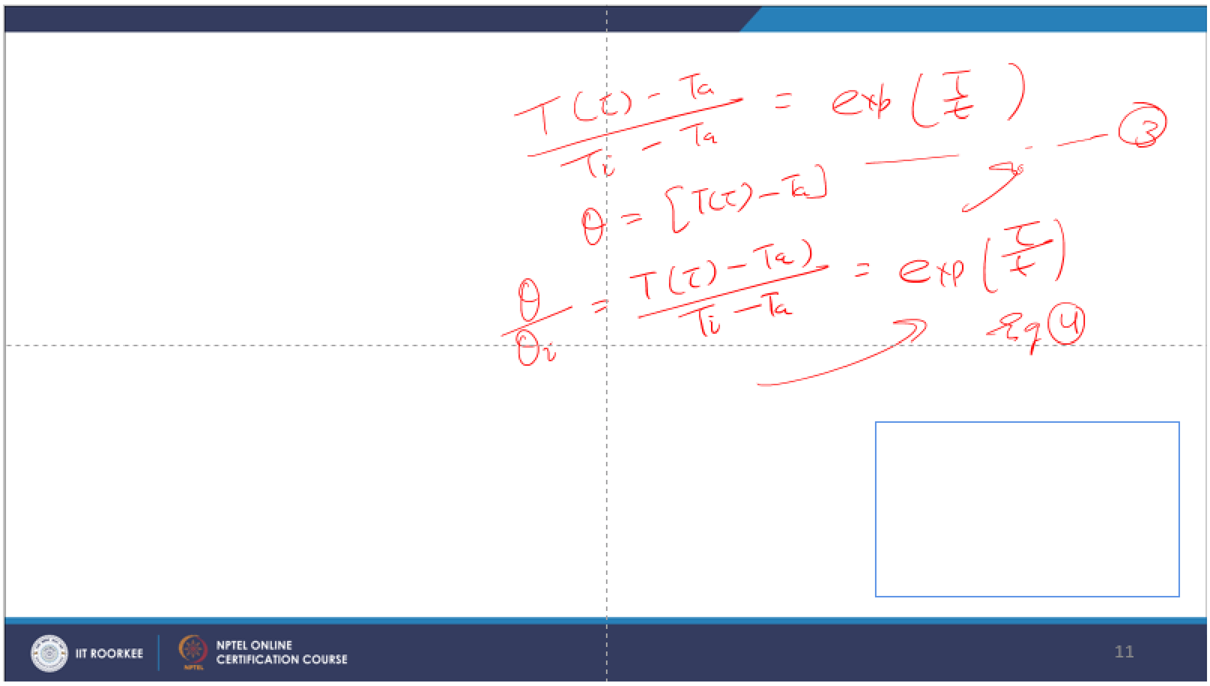
So, these are the various terms. Now the viscous dissipation effect, this takes place in all fluids because of the energy used to move the fluid, this becomes dissipated. The size of effect this is related to the both velocity gradient and the fluid apparent viscosity. Hence the fluid with the large apparent viscosity such as molten polymers in this effect can be quite sizable. Now let us consider a solid body of arbitrary shape which is having the volume V, mass m, density rho and the surface area A and specific heat C p.



This is as per this particular figure. To start with the time the tau t is equal to 0, let us temperature throughout the body is uniform that is t is equal to t initial and at tau is equal to 0, let the body be suddenly placed in the medium at a temperature rho of t A which is as per this figure. Now amount of heat transferred into the body in time interval say d tau that is the increase in the internal energy of the body that is H dot A t minus t tau d t is equal to m C p d t that is equal to rho C p v d t. Now since t A is constant so we can write d t is equal to d t tau t A. Therefore, d t tau is minus t A over t tau minus t A this is equal to H A over rho C p v d t.







This is equation number 1. Now integrating between tau is equal to 0 that is t is equal to t i and any tau that is t is equal to tau. So, ln t tau minus t A over t i minus t A is just equal to H A tau rho C p v and t tau t i minus t A which is equal to exponential H A tau over rho C p v this is equation number 2. Now let us take rho C p v over H A is equal to t where t is known as thermal time constant and has a unit of time unit has time that is s. Therefore, this equation number 2 can be rewritten as t tau minus t A over t i minus t A that is tau over t and that is equation number 3.

Conduction heat transfer in polymer system: Analysis

Therefore, eq (4) gives **the temperature distribution in a solid polymer as a function of time**, when the internal resistance of the solid for conduction is negligible compared to the convective resistance at its surface.

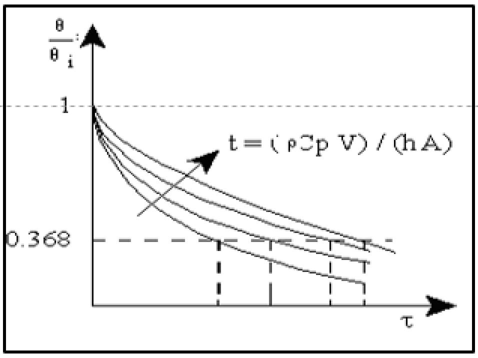


Fig 1 shows temperature variation with time in polymer system

So now denoting theta is equal to t tau minus t A we can write equation this particular equation compactly as theta over theta i t tau minus t A over t i minus t A is just equal to e to the power tau over t and that is equation number 4. So, therefore equation 4 this gives the temperature distribution in a solid polymer as a function of time when the internal resistance of the solid for the conduction is negligible compared to the convective resistance at surface. This shows this particular figure shows the temperature variation with time in various polymeric system. Now consider a plane slab this is shown in the figure. Let the surface on the left is maintained at temperature t 1 and the surface on the right is the temperature t 2 as a result of heat being lost to the fluid at temperature t A the flowing with the heat transfer coefficient h.

Conduction heat transfer in polymer system: Criteria

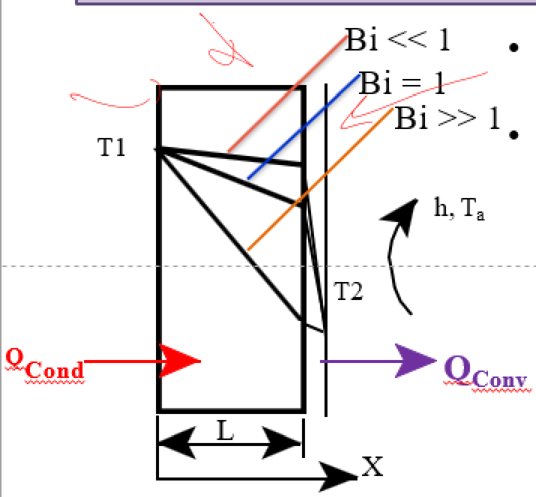
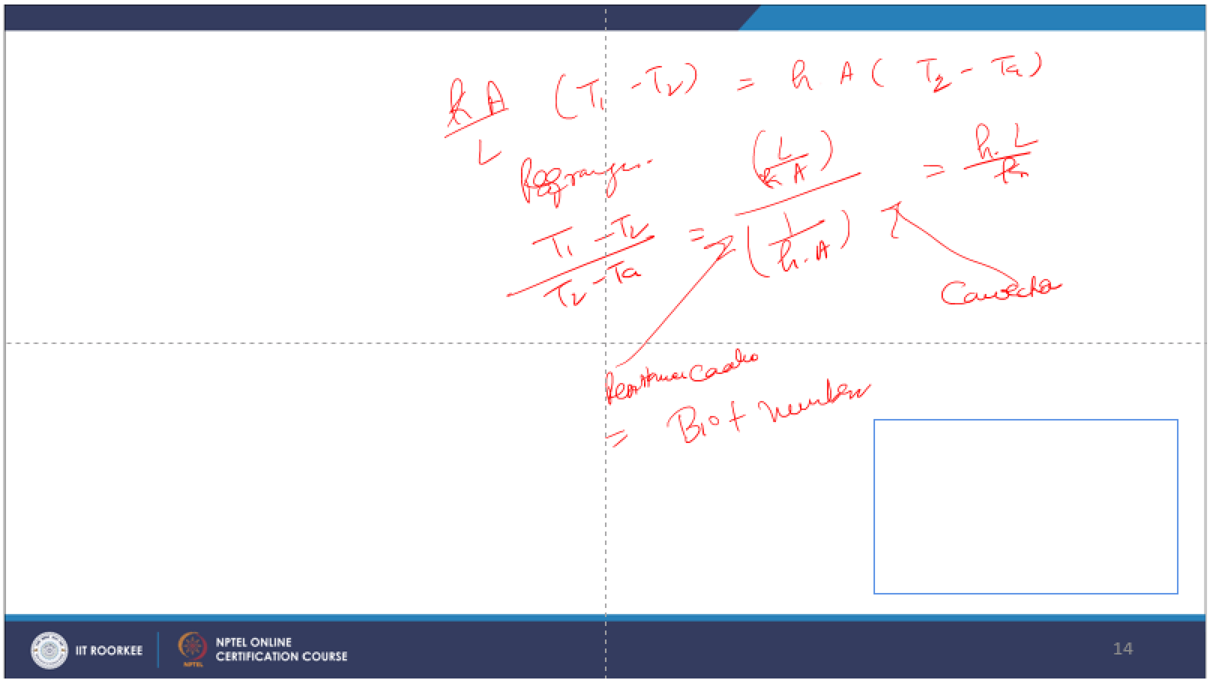


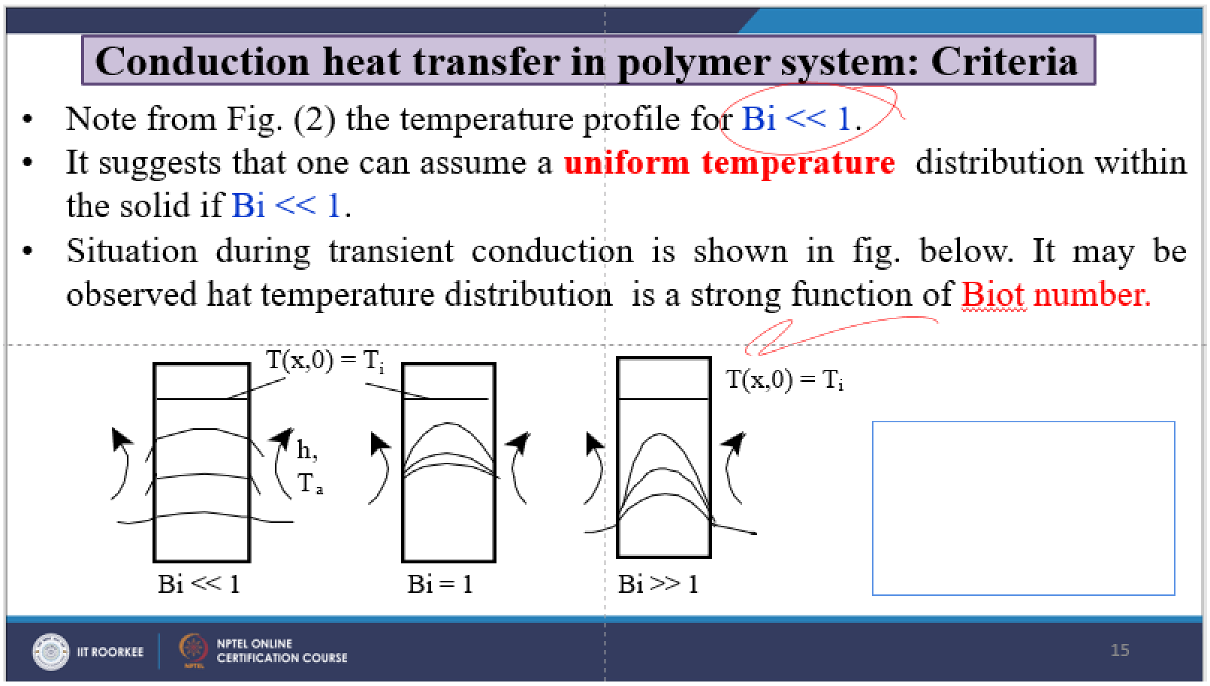
Fig 2: Biot number and temp. distribution in a plane wall

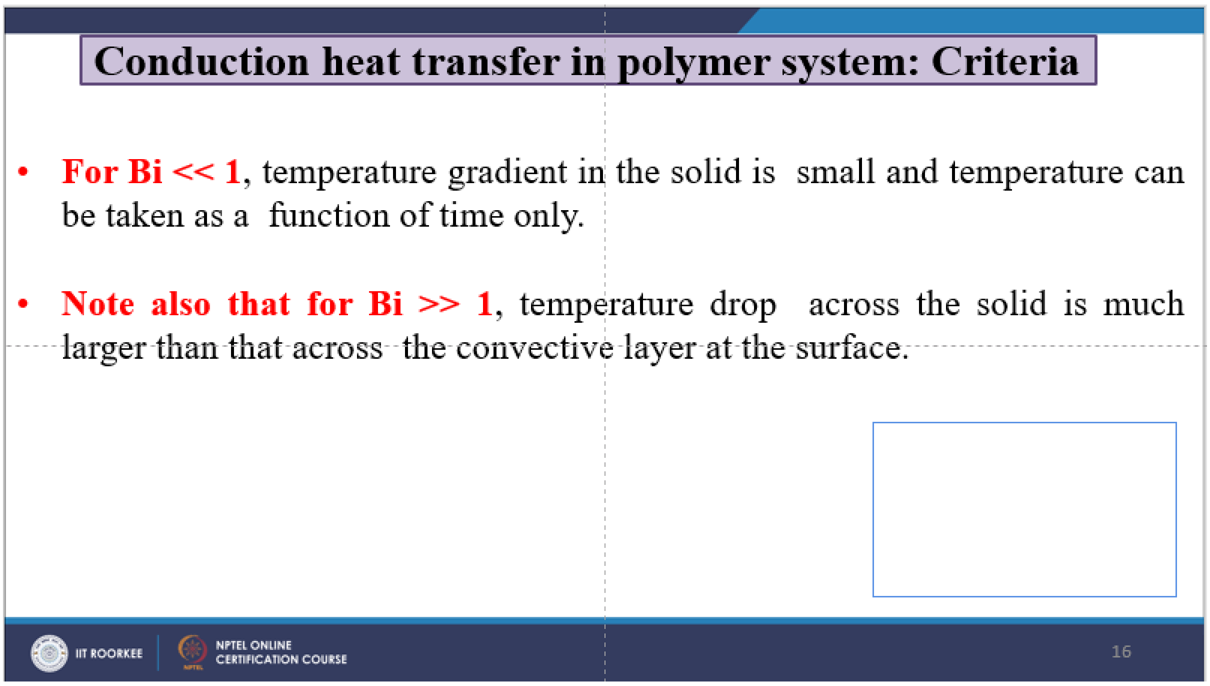
- Consider a plane slab as shown in fig.
- Let the surface on the left be maintained at temperature T_1 and the surface on the right is at a temperature of T_2 as a result of **heat being lost to a fluid** at temperature T_a flowing with a **heat transfer coefficient h.**

**Eiermann, K., Kunststoffe 55, 335 (1965).



Now this once we write the energy balance at the right-hand side surface which can be represented as k A l t 1 minus t 2 which is equal to h A t 2 minus t A. So, if you rearrange the things it will become t 1 minus t 2 over t 2 minus t A that is l over k A into 1 h A that is equal to h l k. Now here this is the resistance conduction, this is the resistance convection and which is equal to Biot number. What is the criteria in the conduction heat transfer in the polymeric system? Now if we recall this previous figure here the temperature profile for this is less than B i less than less than 1 this is the Biot number. Now it suggests that one can assume a uniform temperature distribution within the solid.

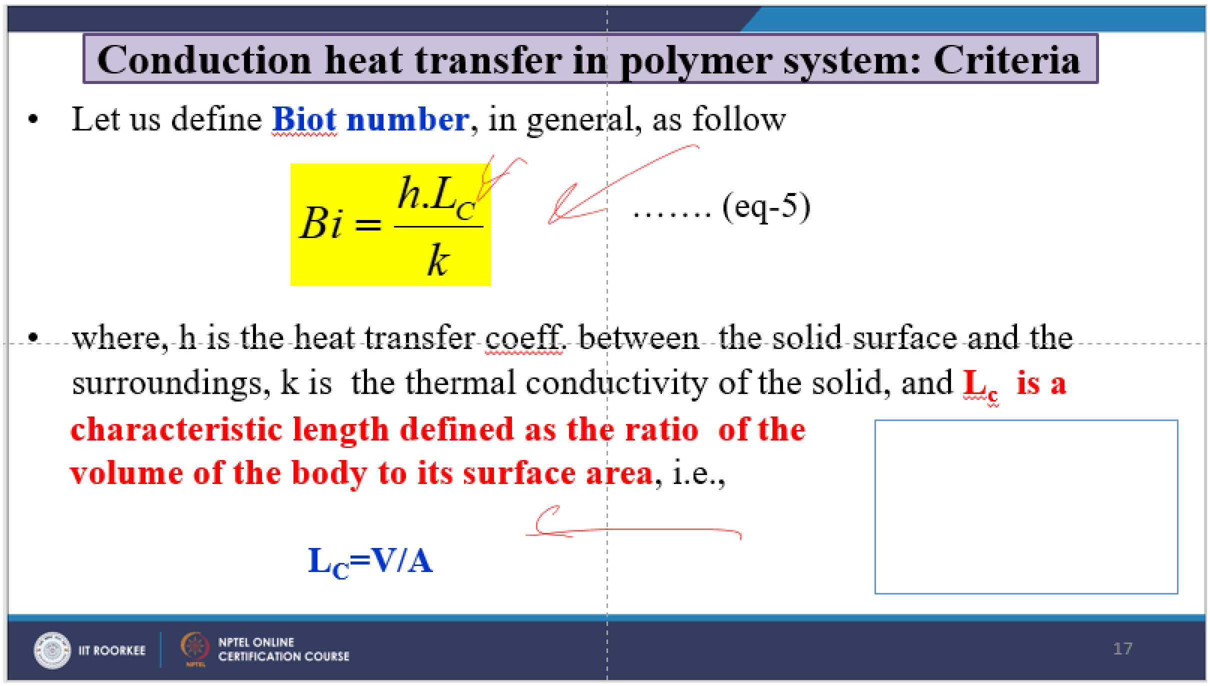


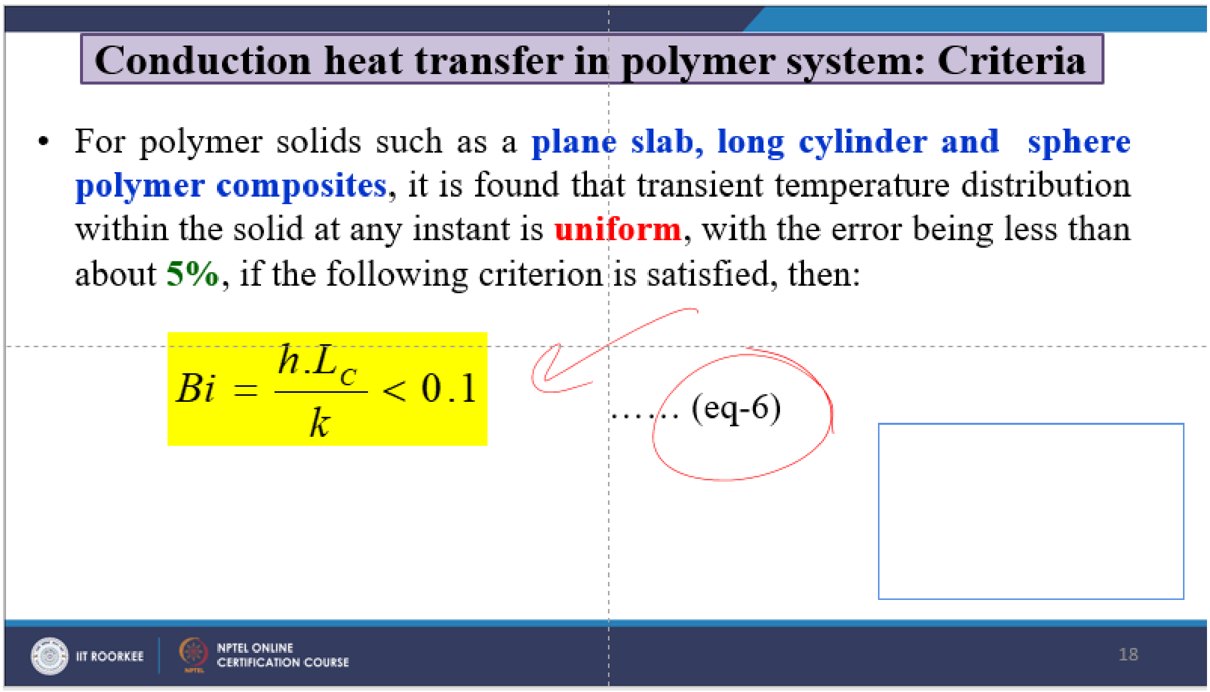


Now situation during the transient conduction which is shown in the figure this particular figure it may be observed that temperature distribution is a strong function of Biot number. Now for Biot number less than less than 1 the temperature gradient in the solid is a small and the temperature can be taken as a function of time only. Now also for if Biot number is greater than greater than 1 then the temperature drops across the solid is much larger than that across the convective layer of at the surface. So, let us define the Biot number it is in general is given as B i is equal to h l c over k where h is the heat transfer coefficient between the solid surface and the surrounding and k is the thermal conductivity of the solid and l c is this one is a characteristic length defined as the ratio of the volume of the body to its surface area that is l c is equal to v over a. For polymer solids such as the plane slab long cylinders and sphere polymer composite it is found that the transient temperature distribution within the solid at any instant is uniform with the error being less than about say 5 percent if this particular criterion is satisfied that is Biot number is equal to h l c over k is less than 0.

Biot Number $Bi = \frac{h.L_C}{k}$

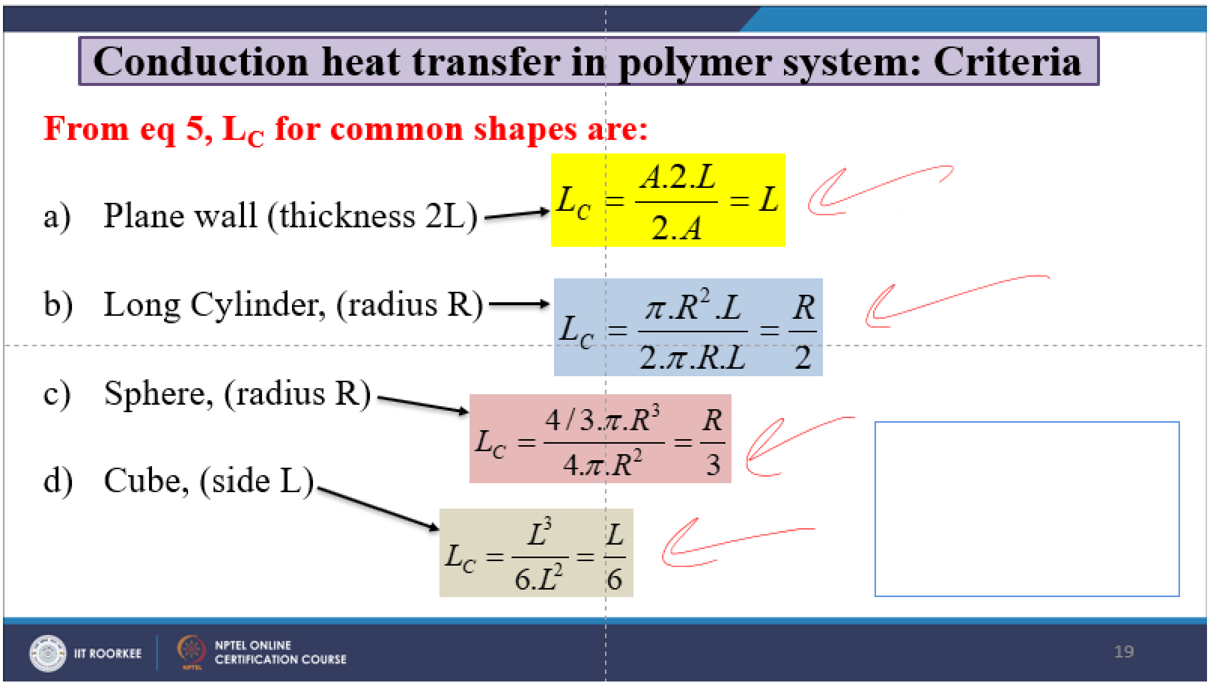
Characteristic length (L_C)**=V/A**





$$Bi = \frac{h.L_C}{k} < 0.1$$

1 that is called the equation number 6. So, if we recall the equation 5 the l c for the common shapes are like plane wall thickness having the 2 l that is l c is equal to a 2 l over 2 a is equal to l. Similarly, if a long cylinder that is having the radius r in the sphere if they are having the radius r and cube they are having the side l. So, we are having various common shape which are represented in the l c.



Pane wall

$$L_C = \frac{A.2.L}{2.A} = L$$

Long cylinder

$$L_C = \frac{\pi.R^2.L}{2.\pi.R.L} = \frac{R}{2}$$

Sphere

$$L_C = \frac{4/3.\pi.R^3}{4.\pi.R^2} = \frac{R}{3}$$

Cube

$$L_C = \frac{L^3}{6.L^2} = \frac{L}{6}$$

Now, here they are putting the value of t that is rho is equal to C p v over h a this equation number 4 if you recall.

Conduction heat transfer in polymer system: Criteria

Putting the value of t i.e $\frac{\rho.C_P.V}{h.A} = t$ in eq 4, we get

Eq: $$\frac{\theta}{\theta_i} = \frac{T(\tau) - T_a}{T_i - T_a} = \exp{\frac{h.A.\tau}{\rho.C_P.V}}$$

- **Its application to a given problem is very simple** and solution of any transient conduction problem must begin with examining if the criterion, **Bi < 0.1** is satisfied to see if eqn. (7) could be applied.

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$$\frac{h.A.\tau}{\rho.C_P.V} = \left(\frac{h.L_C}{k}\right)\left(\frac{k.\tau}{\rho.C_P.L_C^2}\right) = \left(\frac{h.L_C}{k}\right)\left(\frac{\alpha.\tau}{L_C^2}\right)$$

$\alpha = \frac{k}{\rho.C_p}$,

$Fo = \frac{\alpha.\tau}{L_C^2}$

So, theta over theta i is equal to t tau minus t a over t i minus t a that is e to the power h a tau over rho C p v. So, application to a given problem is very simple and a solution for any transient conduction problem must begin with the examining of the criteria less than Biot number less than 0.1 is satisfied to see if the equation previous equation number 7 is could be applied. Now, if we go to the equation number 7 the term h a tau over rho C p v this can be written as per this particular equation. In this the alpha is a thermal diffusivity and mathematically alpha can be represented as k over rho C p and the Fourier number this is represented as f o is equal to alpha tau over l l c square.

Conduction heat transfer in polymer system: Criteria

- Now, in eq 7, the term (h.A.τ)/(ρ.C_P.V) can be written as follows:

Eq:

$$\frac{h.A.\tau}{\rho.C_P.V} = \left(\frac{h.L_C}{k}\right).\left(\frac{k.\tau}{\rho.C_P.L_C^2}\right) = \left(\frac{h.L_C}{k}\right)\left(\frac{\alpha.\tau}{L_C^2}\right)$$

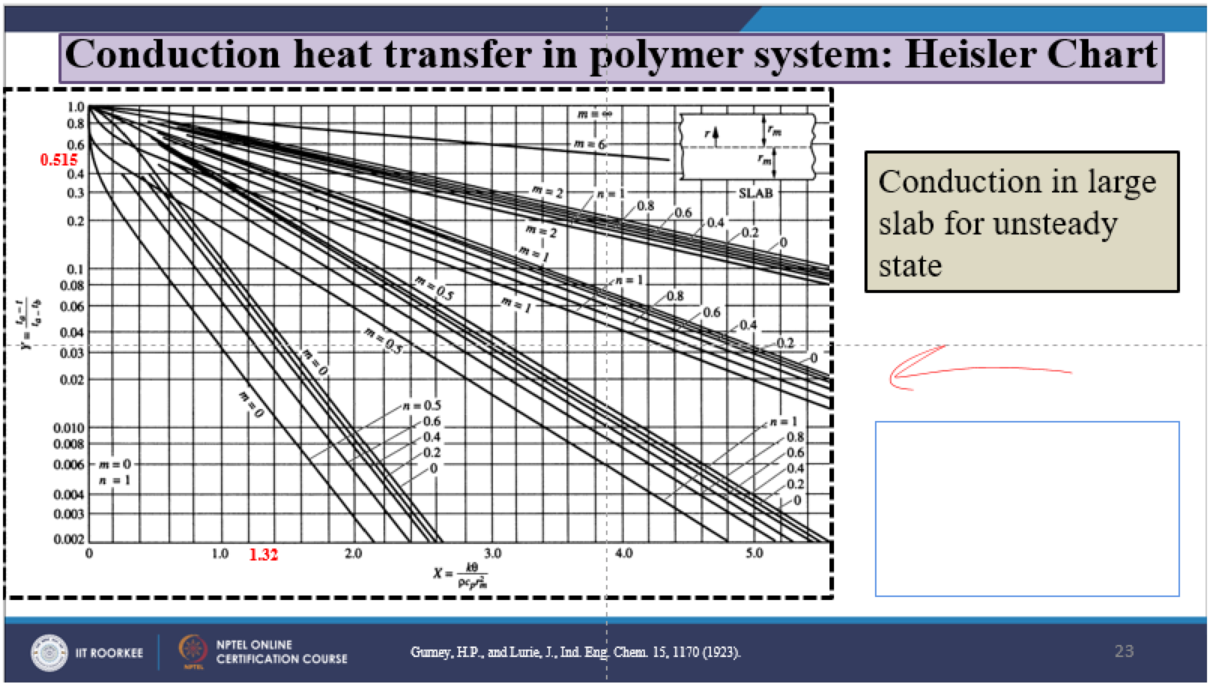
Where, **α is the thermal diffusivity** and mathematically, $\alpha = \frac{k}{\rho.C_p}$. Also, Fourier number $Fo = \frac{\alpha.\tau}{L_C^2}$

Conduction heat transfer in polymer system: Examples

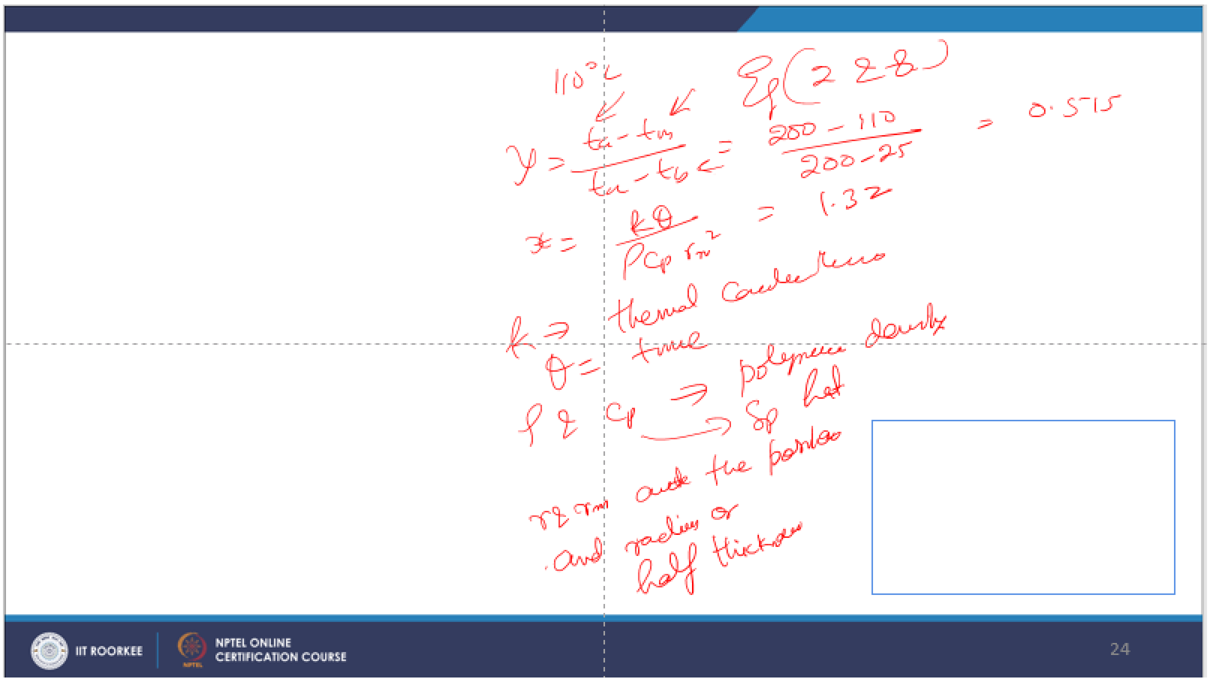
Question 1: Two sheets of glass-reinforced polyester are to be bonded together with an adhesive that fuses at 110°C. The press used to heat the system has platens capable of attaining 200°C. How long should it take to bond the sheets if each is 2 cm thick? (Given n=0, m=1/Bi=1; α (thermal diffusivity)=2.6 x 10^{-3} cm^2/sec)

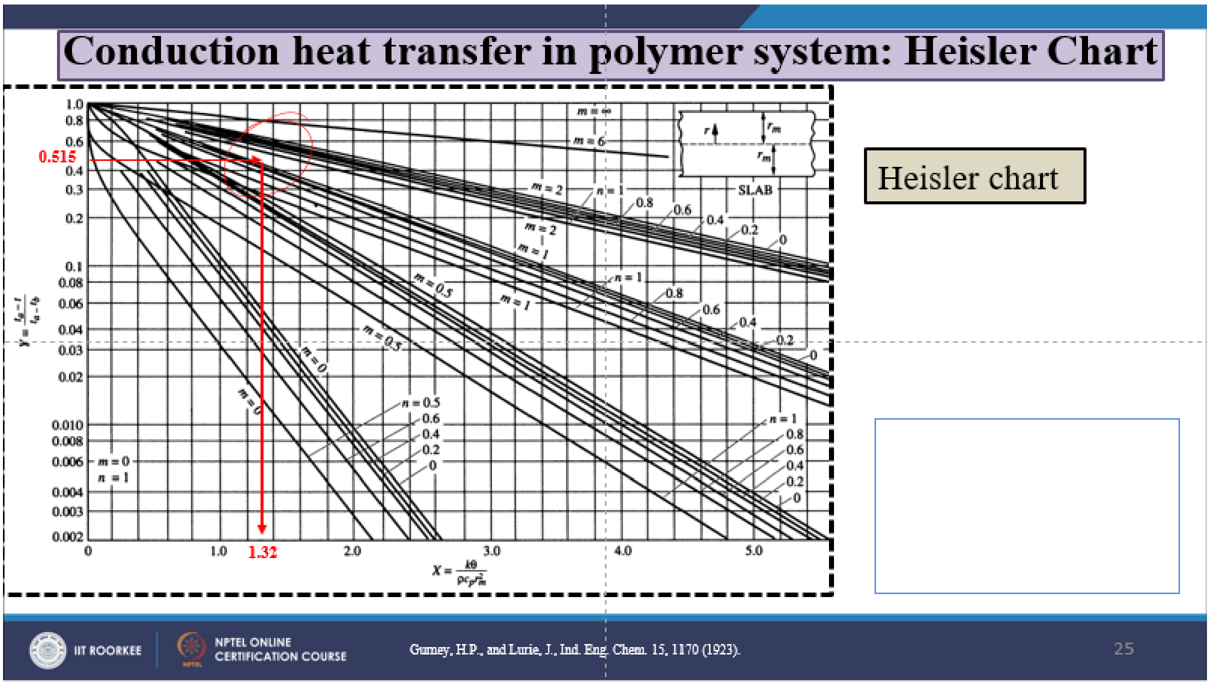
Hint: Use Heisler chart. Also remember that t_b that is the original point temperature is always taken as 25°C

Now, let us take up another question. Now, this example that 2 sheets of a glass reinforced polyester are to be bonded together with an adhesive that fuses at 110 degree Celsius. The press used to heat the system has the platens capable of attending 200 degree Celsius. So, how long should it take to bond the sheets of if each is 2 centimeter thick. So, you require n is equal to 0 m is equal to 1 Biot number 1 alpha that is a thermal diffusivity is given as 2.

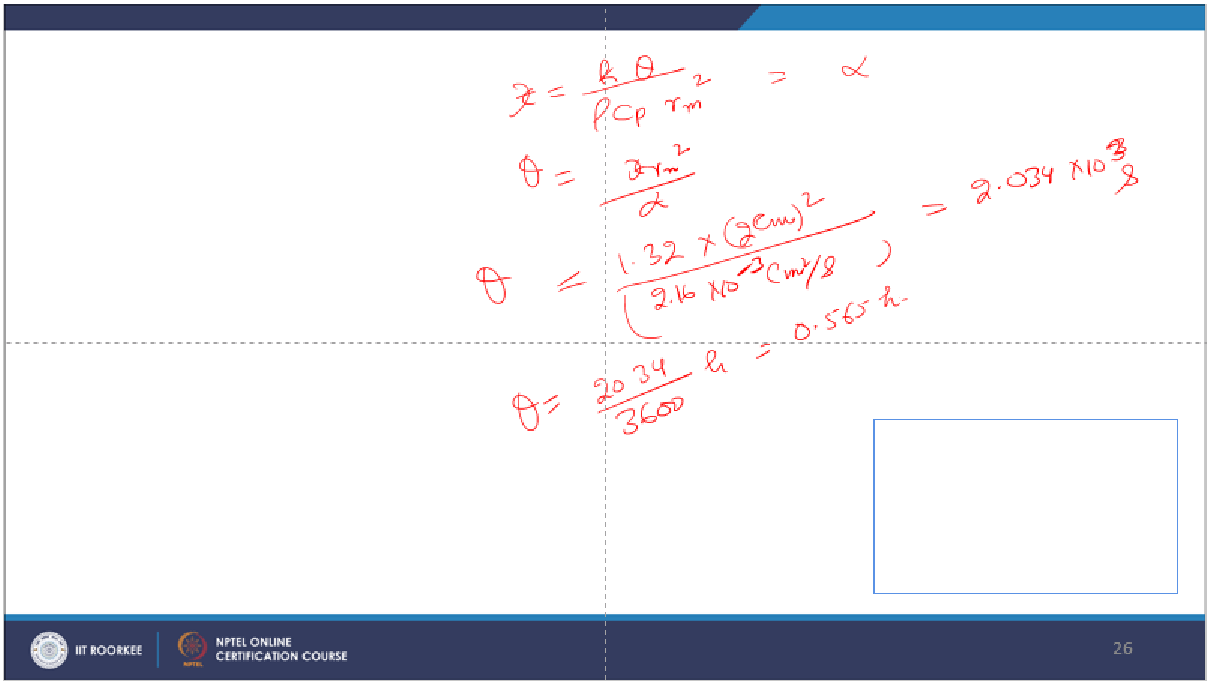


6 into 10 to the power minus 3 centimeter square per second. Now, you can use the previous chart as your chart which is given here and also remember that T b that is the original point temperature is always taken as 25 degree Celsius. So, this is the Heisler chart this is conduction in a large for unsteady state. Now, let us take in this case we assume that the interface which is the adhesive must reach 110 degree Celsius to bond. So, if we use the Heisler chart and equation 2 and 8 for reference and if you modify the equation to y is equal to T a minus T m over T a minus T b and that is 200 minus 110 over 200 minus 25 and that is 0.





515 and x is equal to k theta over rho C p R m square that is equal to 1.32 where T a this one T a T m and T b they are the ambient mid plane and original temperatures and k is the polymer thermal conductivity and theta is time and rho and C p are polymer density and specific heats and R and R m are the positions and radius or half thickness. Now, if let us plot the y and x values in the Heisler graph and mark the that the line where m is equal to 1 and n is equal to 0 is given. So, you will find this particular thing. Now again let us take that x is equal to k theta over rho C p R m square this is equal to alpha or theta is equal to x R m square alpha or theta is equal to 1.



32 into 2-centimetre square that is R m and 2.16 into 10 to the power minus 3-centimetre square per second which comes out to be 2.034 into 10 to the power 3 and theta is comes out to be 2034 over 3600 H which is 0.565 H.

Conduction heat transfer in polymer system: Examples

Question: A nylon 610 material is charged to a screw extruder whose barrel and screw walls are heated to 200°C. If the depth of the helical flow channel is 1.2 cm, what is the maximum time required for the midplane temperature to reach 175°C ? (Given, α=9.55 x 10^{-5} cm^2/s)

Hint: In this case, the granular solids can be represented by a bed of the material. The maximum required time will occur when heat is supplied only by the extruder barrel and screw walls.

So, another let us take another example and this is a nylon 610 material is charged to a screw's extruder whose barrel and a screw walls are heated to 200 degree Celsius.

Now if the depth of a helical flow channel is 1.2 centimetre what should be the maximum time required for the mid plane temperature to reach 175 degree Celsius. We are provided the values of alpha that is equal to 9.55 into 10 to the power minus 5-centimetre square per second. In this case the granular solid this can be represented by a bed of the material the maximum required time will occur when heat is supplied only by the extruder barrel and a screw wall.

Conduction heat transfer in polymer system: Examples

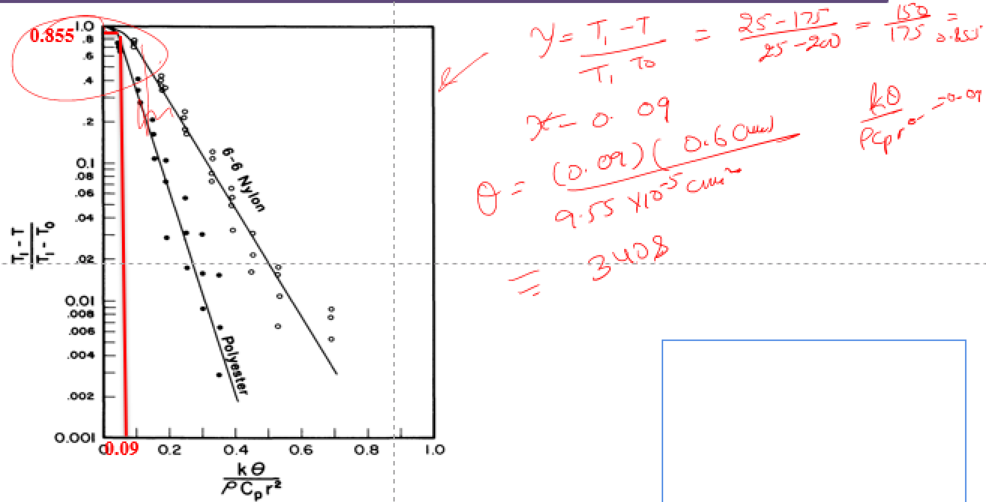
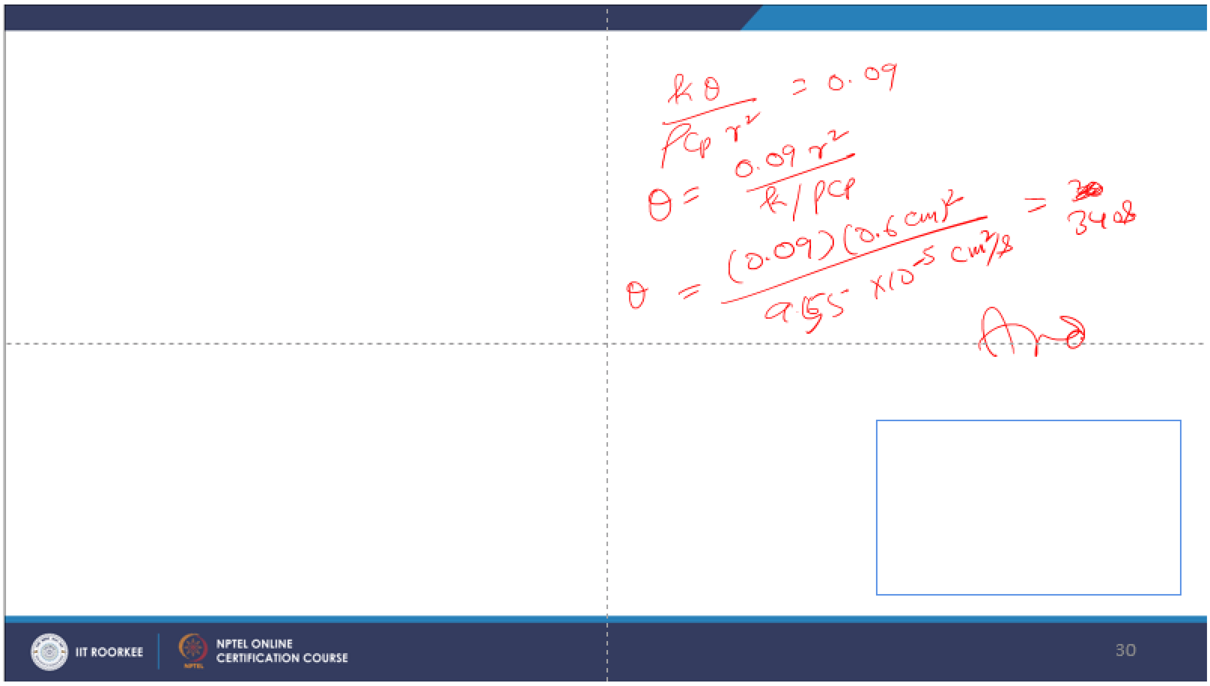


Fig: Unsteady-state heat-transfer to polymer chips.

Let us solve this problem with the help of this unsteady state heat transfer for polymer chips graph. Now here y is equal to t 1 minus t over t 1 minus t naught which is as per the problem 25 minus 175 upon 25 minus 200 which comes out to be 150 to 107 over 175 which is 0.855. So, if we plot the line of a y values to get the x values of nylon 66 the x value corresponding to this nylon 66 is x is equal to 0.

09. Now here you see so theta is equal to 0.09 into 0.6 over 9.55 into 10 to the power minus 5 centimetre square and k theta over rho C p r square is equal to 0.



09. So, theta comes out to be 340 s. Now again if we see this k theta over rho C p r square is equal to 0.09 and theta is equal to 0.09 r square over k rho C p.

So, therefore, theta is equal to 0.09 into 0.6 centimetre square over 9.55 into 10 to the power minus 5 centimetre square over and this comes out to be 340 second and that is our answer. Now let us talk about the convection heat transfer in polymeric system for circular conduits. Now mostly slurry is suspensions dispersions solution of a polymeric materials and melt exhibits the complex flow behaviour which cannot be described by the Newtonian law of viscosity that is tau is equal to gamma where tau is the shear stress and gamma is the shear rate and the constant of proportionality eta is the material property that is called the viscosity.

Convection heat transfer in polymer system for circular conduits

- Most **slurries, suspensions, dispersions, solutions of polymeric materials** and melts exhibit complex flow behavior which **cannot be described** by Newton's law of viscosity $\tau = \eta\gamma$, where **τ is the shear stress**, **γ is the shear rate** and **the constant of proportionality η** is the material property called **viscosity.**
- **Convective heat transfer** to such fluids depends on the **fluid rheology, geometric configuration of the flow** domain as well as the **flow regime** (e.g., laminar, turbulent, etc).

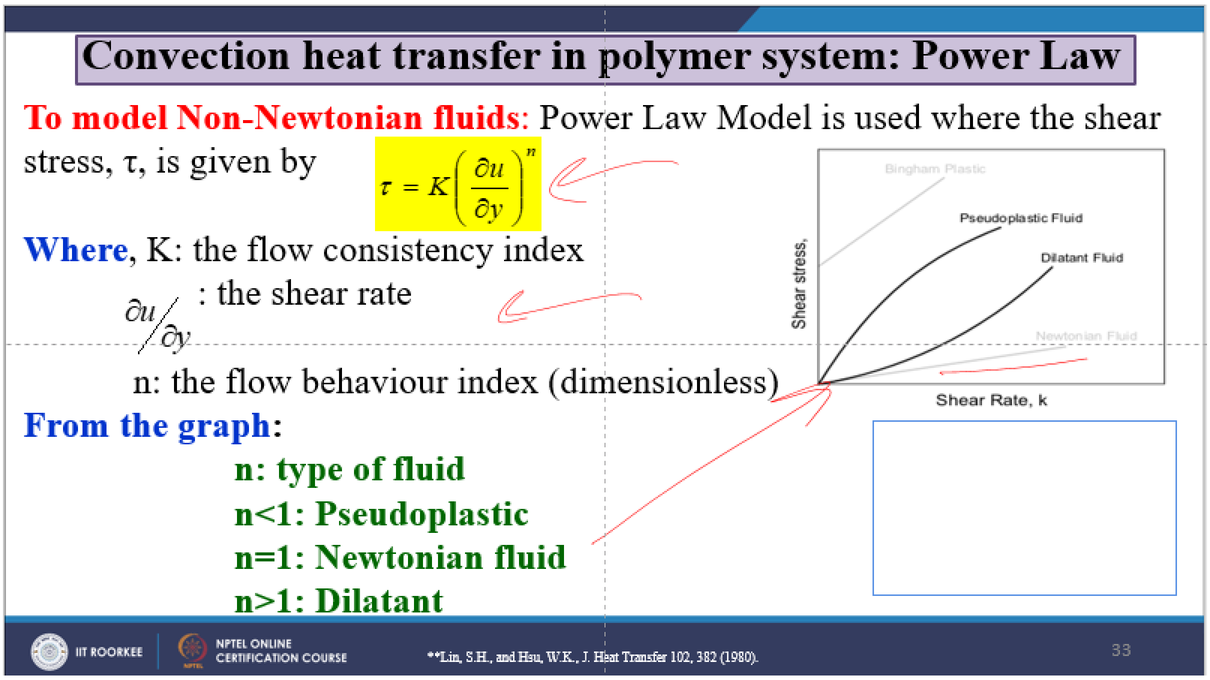
So, the convective heat transfer to such fluids depend upon the fluid rheology geometric configuration of the flow domain as well as the flow regime maybe laminar, turbulent whatever. The apparent viscosity of non-Newtonian fluid that is eta a is equal to tau gamma this is not a material property as in the case of Newtonian fluid, but may depend on the rate of shear and previous flow history of fluid. So, the convective heat transfer to the pseudo plastic or dilatant fluids described by the well-known Oswald D well power law model.

Convection heat transfer in polymer system for circular conduits

- **The apparent viscosity** of Non-Newtonian Fluids, $\eta a = \tau/\gamma$, is not a material property (as is the case for Newtonian Fluids) but may depend on the rate of shear and previous flow history of the fluid.
- Convective heat transfer to pseudoplastic or dilatant fluids described by the well known **Ostwald-de-Waele power law model**.

Now this power law model this the model of non-Newtonian fluid the power law model is used where the shear stress tau is given by k is equal to del u over del y to the power n where k is the flow consistency index and del u over del y is the shear stress and n is the flow behaviour index that is dimensionless. So, if we see this particular plot you find that n this is the type of a fluid if n is less than

1 this is the pseudo plastic if n is equal to 1 that is the Newtonian fluid and if n is greater than 1 that is the dilatant.



Non- Newtonian Fluids

$$\tau = K\left(\frac{\partial u}{\partial y}\right)^n$$

Now here there are various governing equations are represented now here you see that this is the continuity equation and if we talk about the x momentum and r momentum these are the governing equations which can be used for the reference to find out the convective convection heat transfer in the polymeric system. So, if we see these governing equations in the cylindrical coordinates with x r and theta denoting the stream wise radial and tangential coordinates u and v this denoting the stream wise and the radial viscosity velocity component rho is the fluid density g x and g f they are the component of acceleration of gravity vector and p is the pressure. So, the fluid total extra stress is the sum of a Newtonian solvent contribution having a solvent viscosity and the polymer additive stress contribution. So, this is your energy equation under the edge of governing equation. Now in this particular equation k is the fluid thermal conductivity, T is the temperature and C is the specific heat of the fluid.

$$\frac{1}{r}\frac{\partial}{\partial r}(rv) + \frac{\partial u}{\partial x} = 0$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rx})\right)$$

$$\rho\left(v\frac{\partial v}{\partial r} + u\frac{\partial v}{\partial x}\right) = -\frac{\partial p}{\partial r} + \rho g_r + \left(\frac{\partial \tau_{xr}}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}^T) - \frac{\tau_{OO}}{r}\right)$$

Convection heat transfer in polymer system: Governing equations

- Continuity equation: $\frac{1}{r}\frac{\partial}{\partial r}(rv) + \frac{\partial u}{\partial x} = 0$
- x-momentum: $\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rx})\right)$
- r-momentum:

$$\rho\left(v\frac{\partial v}{\partial r} + u\frac{\partial v}{\partial x}\right) = -\frac{\partial p}{\partial r} + \rho g_r + \left(\frac{\partial \tau_{xr}}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}^T) - \frac{\tau_{OO}}{r}\right)$$

Convection heat transfer in polymer system: Governing equations

From the governing equations:

In cylindrical coordinates with **x, r and θ** denoting the stream wise, radial and tangential coordinates

u and v denoting the stream wise and radial velocity components.

ρ is the fluid density,

g_x and g_r are components of the acceleration of gravity vector

p is the pressure.

The **fluid total extra stress (τ_{ij}^T)** is the sum of a Newtonian solvent contribution having a solvent viscosity and a polymer/additive stress contribution.

Convection heat transfer in polymer system: Governing equations

- Energy equation:

$$\rho cu\frac{\partial T}{\partial x}+\rho cv\frac{\partial T}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\tau_{rx}\left(\frac{\partial u}{\partial r}+\frac{\partial v}{\partial x}\right) \quad \ldots\ldots \text{(eq-9)}$$

- In above eq-9, k is the fluid thermal conductivity, T is the temperature and c is the specific heat of the fluid. These equations are valid for developing and fully-developed pipe flow in the absence of swirl.

Convection heat transfer in polymer system: Governing equations

- Also, the energy eq. (9) is also used in a simplified form, for developing flow, if, $\partial v/\partial x << \partial u/\partial r$ i.e., that axial diffusion is far less important that radial diffusion and that radial heat convection is much weaker than axial heat convection.

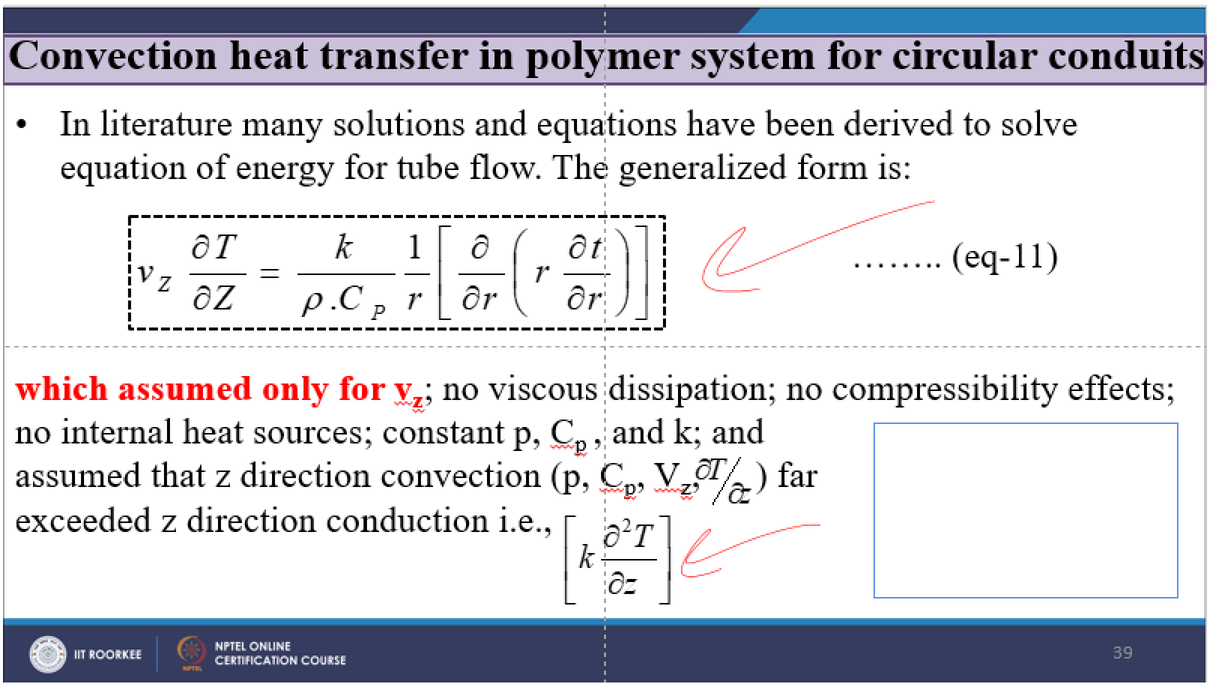
  Therefore, the energy equation becomes:

$$\rho cu\frac{\partial T}{\partial x}=\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\tau_{rx}\frac{\partial u}{\partial r} \quad \ldots\ldots \text{(eq-10)}$$

$$\rho cu\frac{\partial T}{\partial x}+\rho cv\frac{\partial T}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\tau_{rx}\left(\frac{\partial u}{\partial r}+\frac{\partial v}{\partial x}\right)$$

$$\rho cu\frac{\partial T}{\partial x}=\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\tau_{rx}\frac{\partial u}{\partial r}$$

These equations are valid for developing and fully developed pipe flow in absence of any kind of cell. Now also this energy equation which is developed in this particular slide is used in a simplified form for developing flow if del v over del x is less than less than to the del u over del r that is the axial diffusion is far less important than the radial diffusion and that the radial heat convection is much weaker than the axial heat convection. Therefore, the energy equation that is equation number 10 becomes like this. Now a fairly sizable technical literature has accumulated over the years in the area of convective heat transfer in polymeric system. Now this literature can be roughly divided into those effects that can mainly experimental and those are essentially solution to the equation of energy.



$$v_Z \frac{\partial T}{\partial Z} = \frac{k}{\rho . C_P} \frac{1}{r} \left[\frac{\partial}{\partial r} \left(r \frac{\partial t}{\partial r} \right) \right]$$

Now there are many solutions and equations have been derived to solve the equation of energy for tube flow and the generalized form of the equation is represented like this equation number 11. Now which assumes only for v and there is no viscous dissipation, no compressibility effects, no internal heat source, constant pressure C p and k and assume that z direction convection p C p and v and del t over del z for exceeded the z direction conduction and that is k is equal to del 2 t over del z. Now this is a tabular form which represents the literature search of various solution which is required in this type of study. Various authors they have enlisted, they developed the various equations and these equations are depend on like first equation that is Christian and Craig, the equation used in the form of a power law and these equations are temperature dependent to viscosity. Apart from this another equation which is used for the power and airing this temperature dependent viscosity and the third one Joshi and Bergles this is a uses a power line that constant heat flux temperature dependent properties.

Convection heat transfer in polymer system for circular conduits

- In this respect, more rigorous solutions of the equation of energy were developed by a number of authors.
- These authors assumed that convective Z direction $\rho . C_P . V_Z (\partial T / \partial z)$ heat transfer **exceeded conductive direction** $k(\partial^2 T / \partial z^2)$ **heat transfer** and normal stresses could be neglected.
- Then, the energy equation then becomes:

$$\rho . C_V \left(V_Z \frac{\partial T}{\partial z} \right) = -\left[\frac{1}{r} \frac{\partial}{\partial r} (r q_r) \right] - \tau_{rz} \left(\frac{\partial V_Z}{\partial r} \right)$$

….. (eq-12)

$$\rho . C_V \left(V_Z \frac{\partial T}{\partial z} \right) = -\left[\frac{1}{r} \frac{\partial}{\partial r} (r q_r) \right] - \tau_{rz} \left(\frac{\partial V_Z}{\partial r} \right)$$

$$r q_r = -k r \frac{\partial T}{\partial r} \quad \leftarrow \text{Eq } 13$$

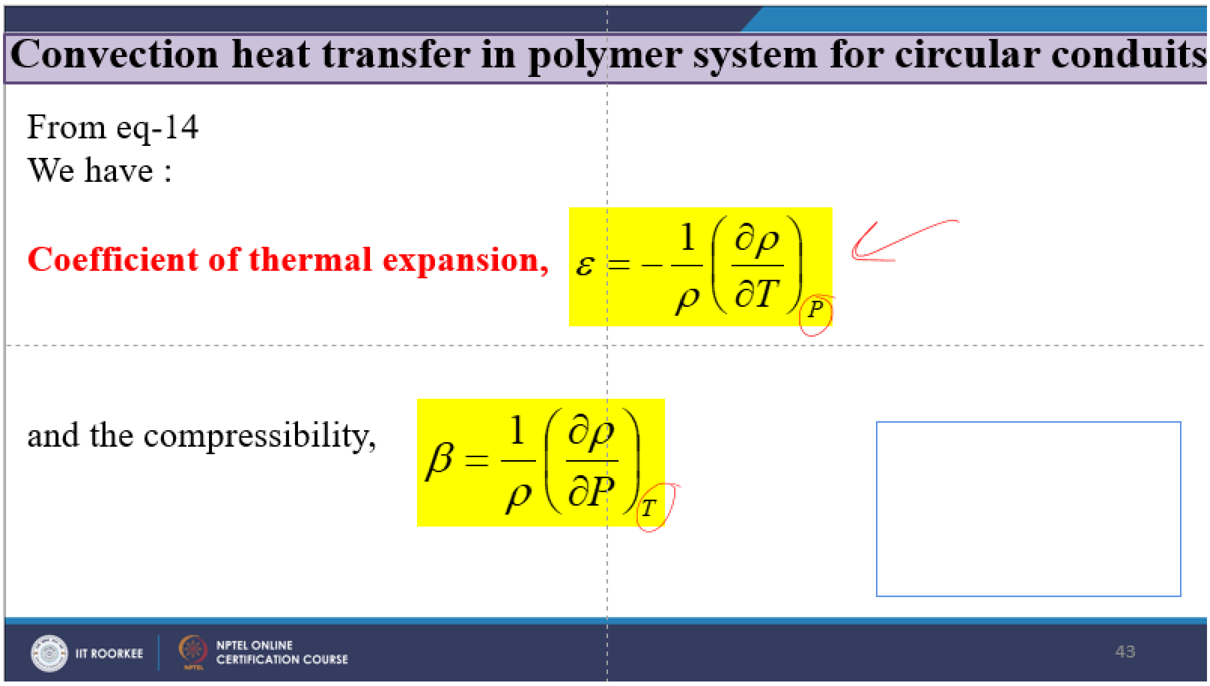
$$C_p \neq C_v$$

$$C_p - C_v = \frac{T \epsilon}{\rho \beta} \quad \text{---- (14)}$$

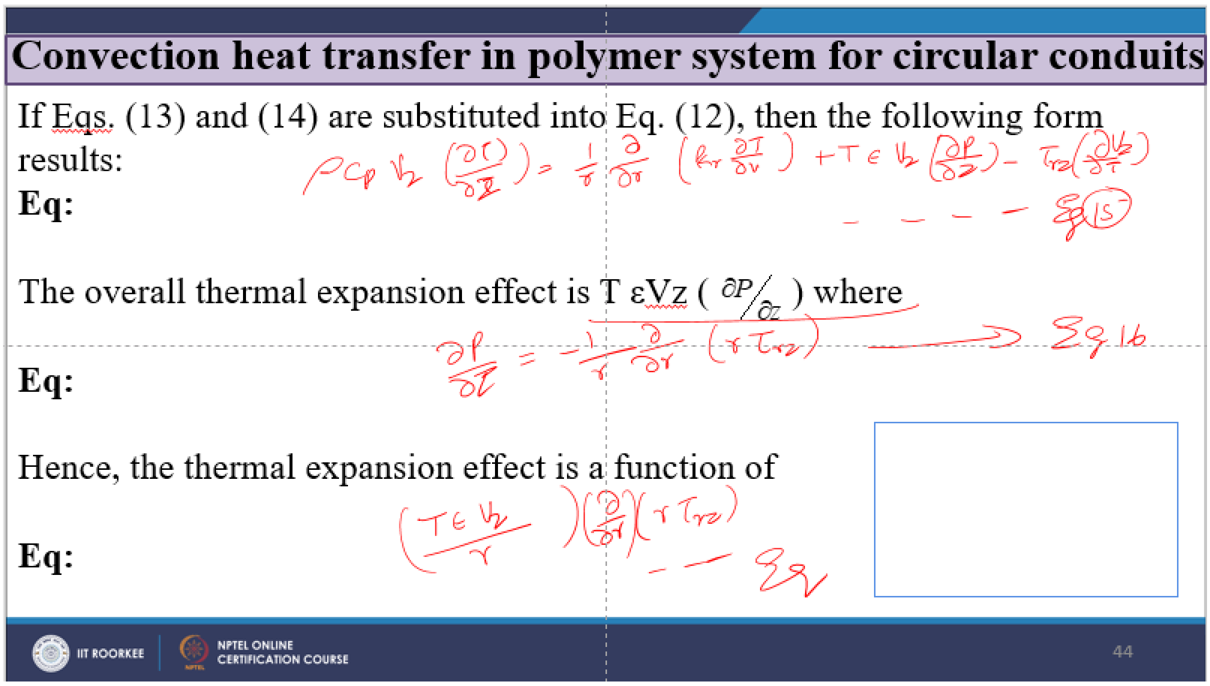
Now in this aspect when we talk we are talking about the circular conduits in this aspect more rigorous solution of the equation of energy is need to be developed by a number of authors. These authors assume that the convective z direction that is rho C p v z into del t over del z heat transfer exceeded the convective direction and that is k del 2 t over del z the heat transfer in normal stresses could be neglected. Then the energy equations which is represented as equation number 12 which can be represented like this. Now this particular equation can be transformed into more amenable form by several substitutions. The first of this is that from the Fourier's law that is r q r is equal to minus k r del t over del r and that is equation number 13.

$$\varepsilon = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$

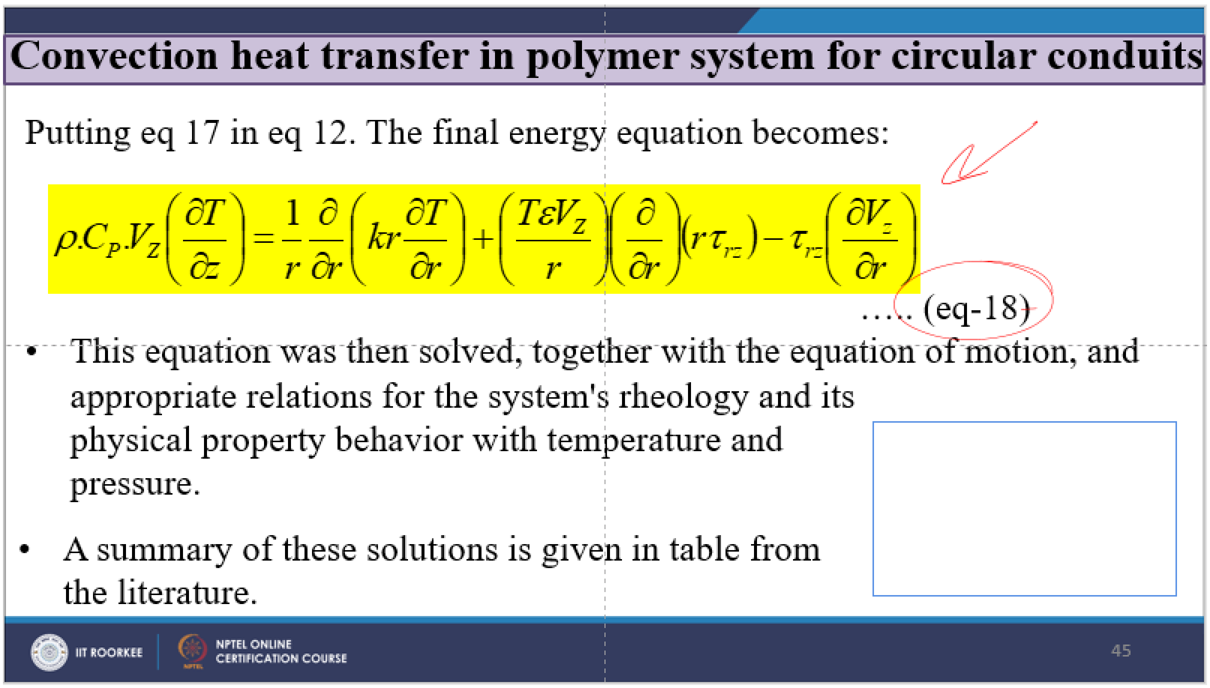
$$\beta = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial P}\right)_T$$



Also, since the system is now a compressible one so C p is not equal to C v and consequently the C p minus C v is equal to t epsilon over rho beta this is equation number 14. So, from equation number 14 we have the coefficient of thermal expansion that is epsilon is equal to minus 1 upon rho del rho over del t at constant pressure and for the compressibility beta is equal to 1 over rho into del rho over del p at constant temperature. So, if the equation 13 and 14 are substituted to the equation 12 the following form of equation can be resulted the overall thermal expansion Now if equation 13 and 14 are substituted into the equation 12 then the following form of the result can be represented like rho C p v z del t over del z is equal to 1 upon r del over del r k r del t over del r plus t sun v z del p over del z minus tau r z del v z over del r. So, this is the equation number 15. Now the overall thermal expansion effect is this one where del p over del z is equal to minus 1 upon r del over del r k r minus tau r z and this is the equation number 16.



So, the thermal expansion effect usually is a function of t epsilon v z over r into del del r r tau r z and this is the equation number 17.

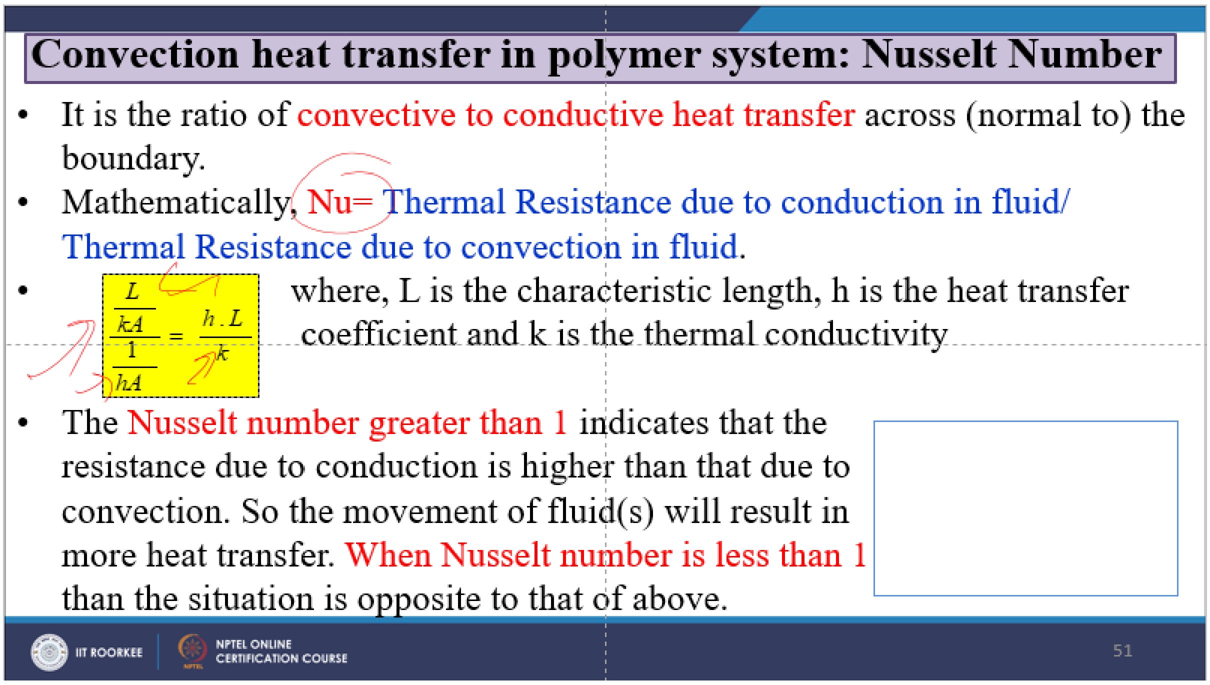


$$\rho.C_P.V_Z\left(\frac{\partial T}{\partial z}\right)=\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\left(\frac{T\varepsilon V_Z}{r}\right)\left(\frac{\partial}{\partial r}\right)(r\tau_{rz})-\tau_{rz}\left(\frac{\partial V_Z}{\partial r}\right)$$

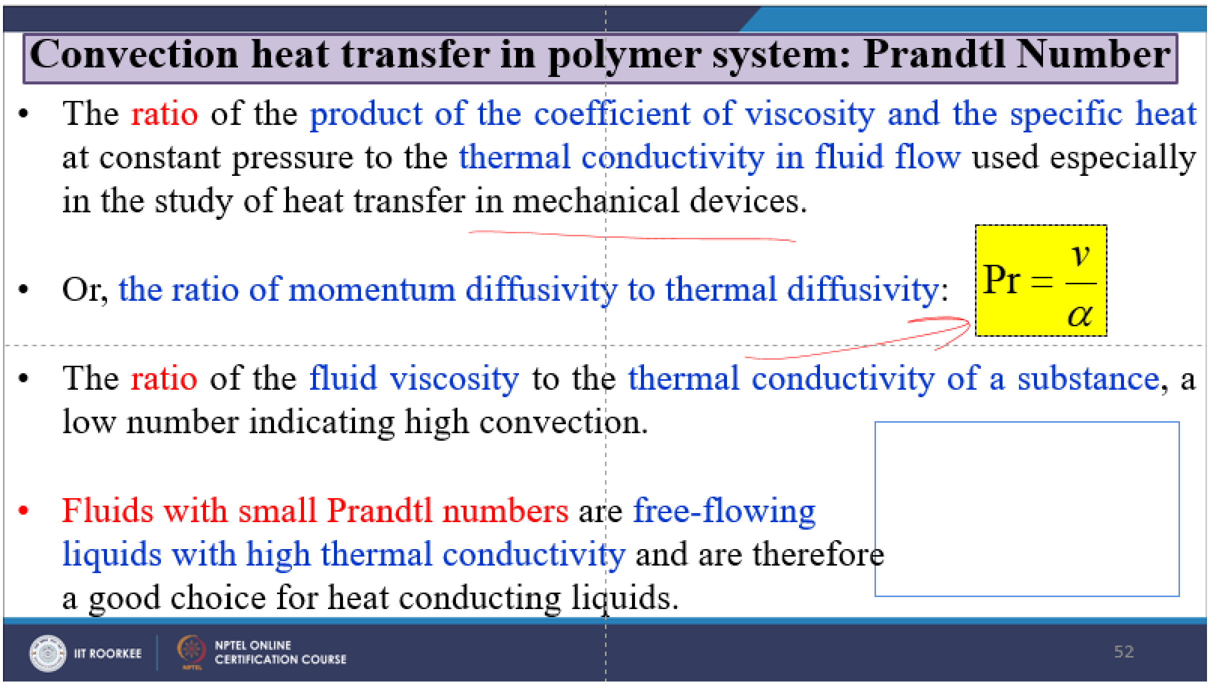
So, putting if you put the equation number 17 in equation 12 the final energy equation this becomes like this and it is represented as equation number 18. Now this equation is then solved together with the equation of the motion and appropriate relation of the for the systems rheology and its physical property behaviour with the temperature and pressure. So, summary of these solutions is given by these various authors in this particular table for the for your convenience we have enlisted all those things like tour they they they use the power law equation and they neglect the viscous dissipation assumes the constant physical properties and analytical solution. Similarly, again in the subsequent year he used the power law for the constant properties treated in that region and analytical solutions.

Apart from this various other author they used where they no effect of compressibility cooling was considered apart from this the temperature dependent physical properties and the computer solutions they have considered. Now, in moving from the centre to the wall the portion this is a centre from to the wall let the portion of the previous equation 18 that undergoes the greatest change that is 1 upon r where r over r is equal to 0 and 1 over r is equal to infinite over r then the wall r over r is equal to 1 and 1 over r is equal to 1 over r. In contrast the viscous dissipation term tau r z this depend directly on tau t z and velocity gradient. So, if we consider separately the effect of the tube centre and the tube wall regime we see the various effects in this particular table with respect to the location then 1 over r values and effect on this particular equation the result is represented in the tabular form. You can see over here these are the various results which we have enlisted for your convenience.

Now, when we plot a temperature profile in the non-Newtonian system with expansion effects, so, this shows the effect of thermal expansion and when n is equal to eta is equal to 0.25 with the negligible viscous dissipation and constant fluid properties. So, the various silence values represent the average value of temperature across the tube. So, it can be seen in this particular plot the effect of thermal expansion or compressibility cooling this is depressed the point temperature in the centre of the tube.



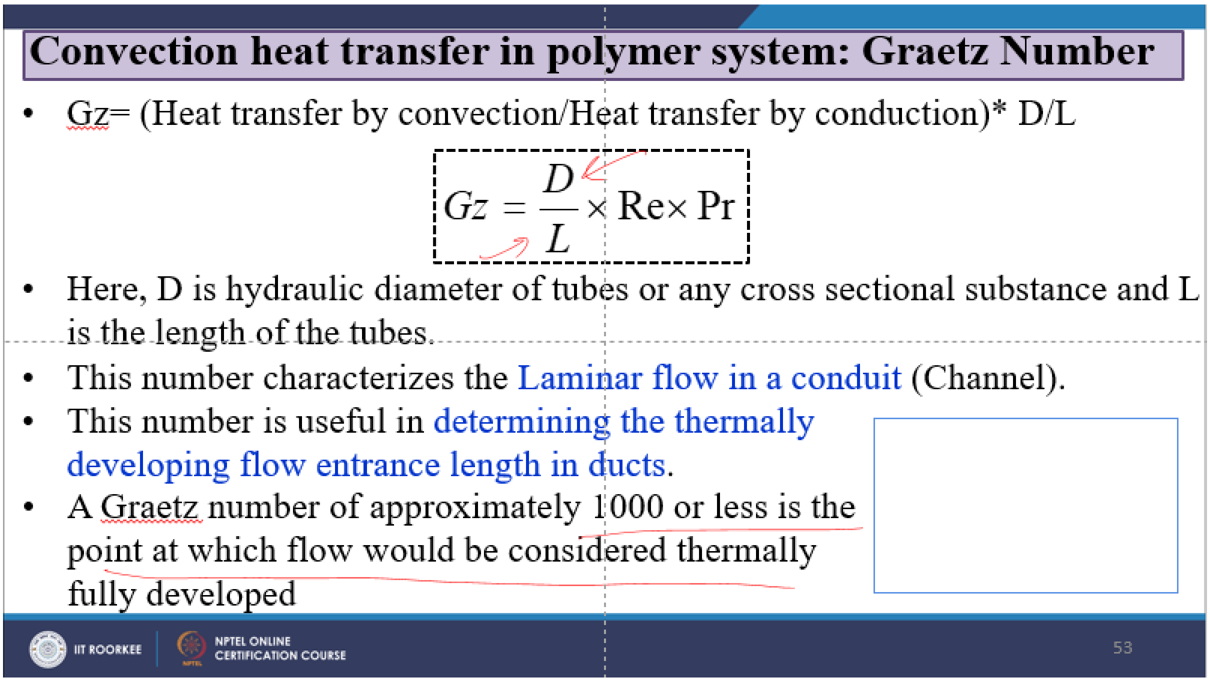
Now, let us talk about the Nusselt number this is the ratio of the convective to conductive heat transfer across normal to the boundary and mathematically this Nusselt number is equal to the thermal resistance due to the conduction in fluid and thermal resistance due to convection in fluid, and this can be represented in mathematical form like this where the L is the characteristics length, and H is the heat transfer coefficient and k is the thermal conductivity. So, the Nusselt number greater than 1 indicates that the resistance due to the conduction is higher than that due to convection and so the movement of the fluid will result in more heat transfer when the Nusselt number is less than 1 then the situation is opposite to the above. The ratio of the product of the coefficient of viscosity and specify a specific heat at constant pressure to the thermal conductivity in fluid flow used especially in the study of heat transfer in the mechanical devices. So, the ratio of momentum diffusivity to the thermal diffusivity is given at t r is equal to u over alpha and that is called the Prandtl number. The ratio of the fluid viscosity to the thermal conductivity this is of a substance a lower number indicating the high convection.



$$\text{Pr} = \frac{v}{\alpha}$$

So, the fluid with a small Prandtl number are free flowing liquid with a high thermal conductivity and therefore good choice for heat conducting liquids. Grades number this is given by d over L into Reynolds number into Prandtl number. Now this there d is the hydraulic diameter of the tube or any cross sectional number and L is the length of the tube.

$$Gz = \frac{D}{L} \times \text{Re} \times Pr$$



This number characterizes the laminar flow in a conduits and this number is useful for determining the thermally developing the flow entrance length index and Grades number is approximately 1000 or less is the point at which the flow would be considered thermally fully developed. Now the relationship between the Nusselt and Grades number for constant wall heat flux condition the neglecting longitudinal heat conduction and internal frictional heat dissipation.

The approximation is given to obtain the analytical expression for the local Nusselt number which is given by this particular equation and this is equation number 19 which is useful for determining the relationship between the Nusselt and the Grades number.

$$Nu_L = 0.650 \left(\frac{p.D}{u}\right)^{\frac{1}{3}} \left(\frac{Re.Pr}{\frac{L}{D}}\right)^{\frac{1}{3}}$$

Convection heat transfer: Relation between Nusselt-Graetz

- For constant wall heat flux conditions, neglecting longitudinal heat conduction and internal frictional heat dissipation, Bird, R. B. (1957) has used the Lyche, J. (1928) approximation to obtain an analytical expression for the local Nusselt number as

$$Nu_L = 0.650\left(\frac{p.D}{u}\right)^{\frac{1}{3}}\left(\frac{Re\,.\,Pr}{\frac{L}{D}}\right)^{\frac{1}{3}} \quad \ldots\ldots \text{(eq-19)}$$

Now if we are having 0 is less than eta less than equal to 1, then the most useful form of equation is represented like this. This is the Grades number. Now for the Newtonian case when eta is equal to 1 this equation reduced to like this. Now in this particular graph the Nusselt and Grades solution for the plug flow and a Newtonian fluid are compared to the power law fluid solution where eta is equal to half or 1 by 3 this is derived by the Lyche and the Bird in their studies.

Convection heat transfer: Relation between Nusselt-Graetz

- For, 0<n≤1, the most useful form if equation (19) is

$$Nu_L = 1.418\left(\frac{3n+1}{4n}\right)^{\frac{1}{3}}(Gz)^{\frac{1}{3}} \quad \ldots\ldots \text{(eq-20)}$$

- For the Newtonian case (n=1), the eq-20 reduces to,

$$Nu_L = 1.418\,(Gz)^{\frac{1}{3}} \quad \ldots\ldots \text{(eq-21)}$$

$$Nu_L = 1.418\left(\frac{3n+1}{4n}\right)^{\frac{1}{3}}(Gz)^{\frac{1}{3}}$$

$$Nu_L = 1.418(Gz)^{\frac{1}{3}}$$

Convection heat transfer: Relation between Nusselt-Graetz

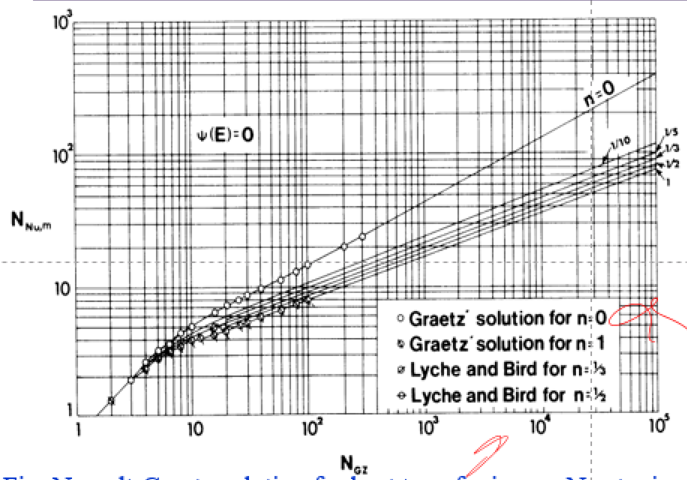


Fig: Nusselt-Graetz solution for heat transfer in non-Newtonian systems

- In graph, the Nusselt-Graetz solutions for plug flow and Newtonian fluids are compared to power law fluids solutions (n = 1/2, n = 1/3) as derived by Lyche and Bird.

**Lyche, B.C., and Bird, RB., Chem. Eng. Sci. 6, 35 (1957)

Another Nusselt-Grades relation is shown in this particular graph which uses the term Br which is the measure of the internal heat generation.

Convection heat transfer: Relation between Nusselt-Graetz

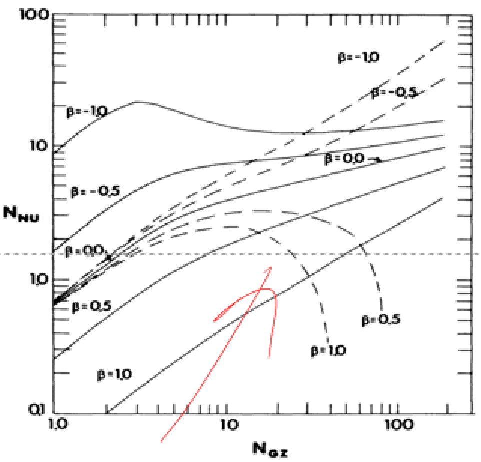


Fig: Temperature profiles in polymethyl methacrylate

- Another Nusselt-Graetz relation is shown in graph, which uses the term Br' as a parameter (**Br' is a measure of internal heat generation**). If Br' is small, heat generation can be neglected.
- Negative values indicate situations in which the fluid is being heated ($T_w > T_1$), and positive Br' values indicate a cooled fluid ($T_w < T_1$)

**Gee, R.E., and Lyon, J.B., Ind. Eng. Chem. 49, 956 (1957).

Now if Br is small the heat generation can be neglected and the negative value indicate the situation which is the fluid is being heated that is Tw is greater than T_1 and positive value indicates the cold fluid that is Tw is less than T_1.

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So dear friends in this particular segment we discussed the different aspects of convective heat transfer, conductive heat transfer, we developed the various relations and for your convenience we have enlisted the large number of references which can be utilized for the further studies. Thank you very much.