Polymer Process Engineering
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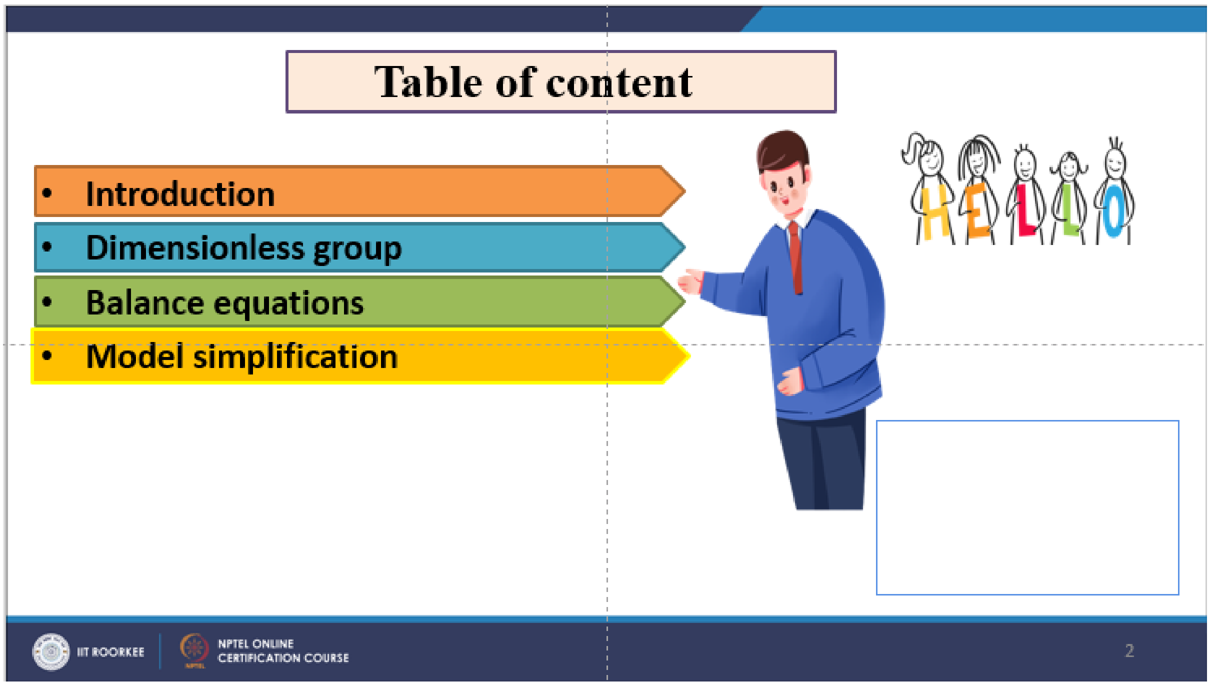
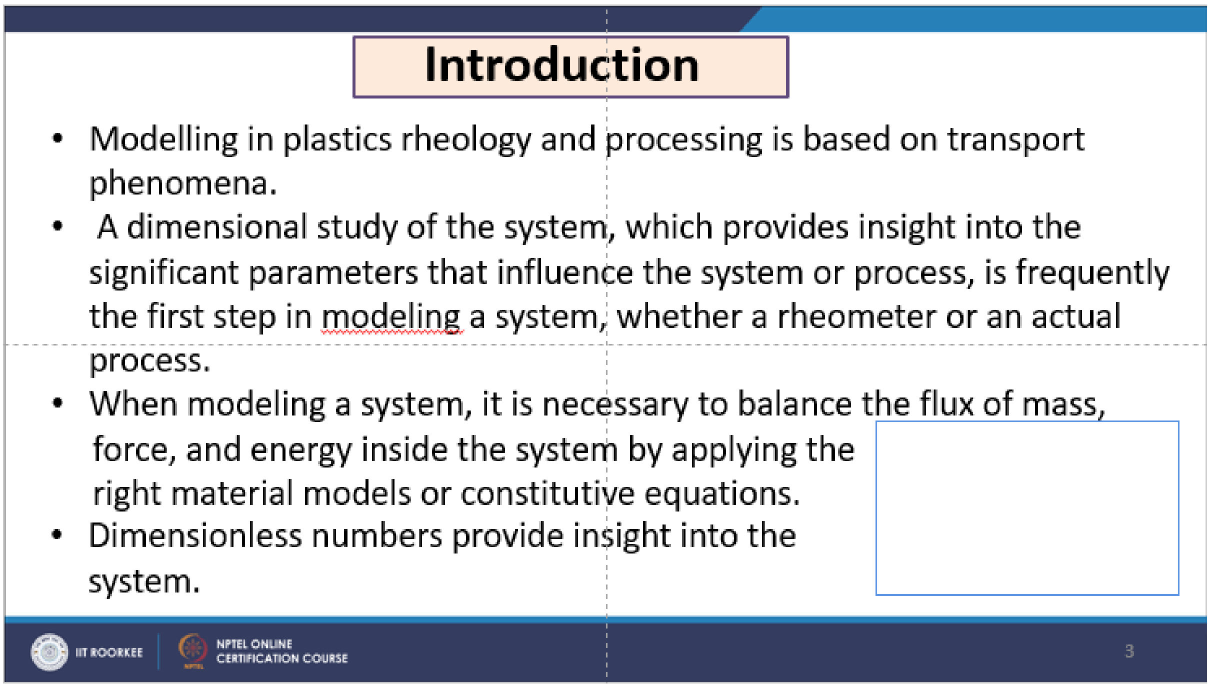
Lecture – 14
Applied polymer rheology: Transport phenomena

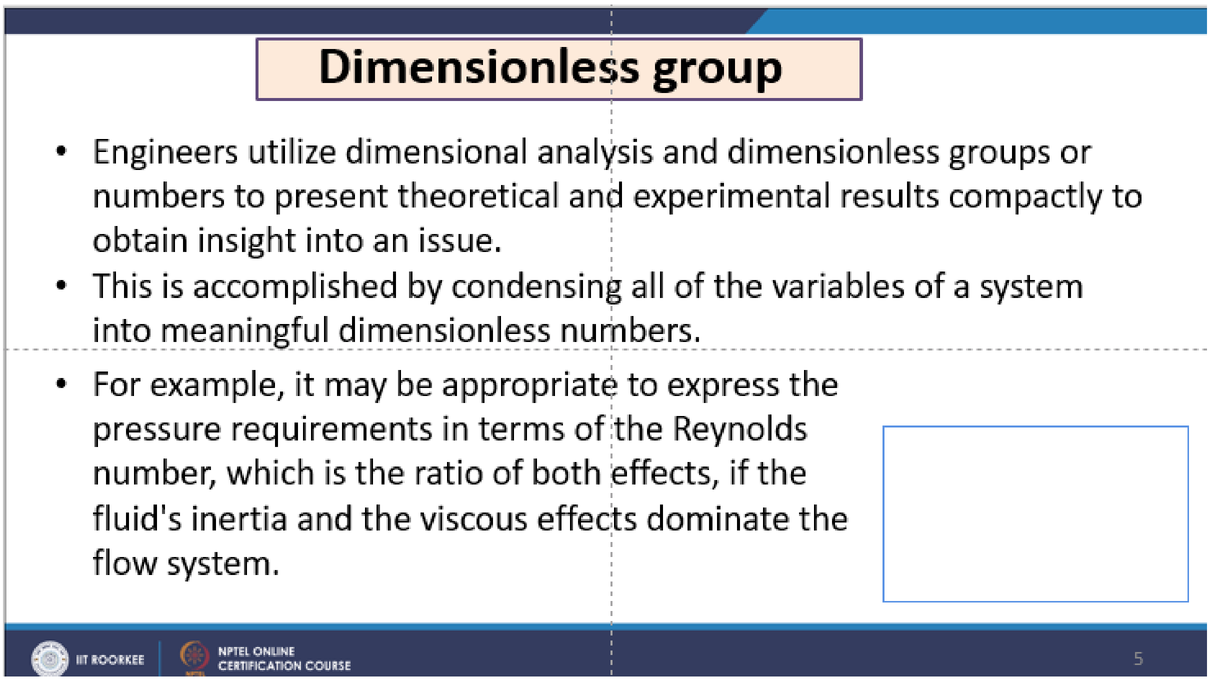
Hello friends, welcome to the applied polymer rheology and this aspect you are going to discuss about the transport phenomena under the areas of polymer process engineering. So, in this particular lecture, we are going to discuss the various dimensionless groups. We will discuss the balance equation and a model simplification aspect of the polymeric system.

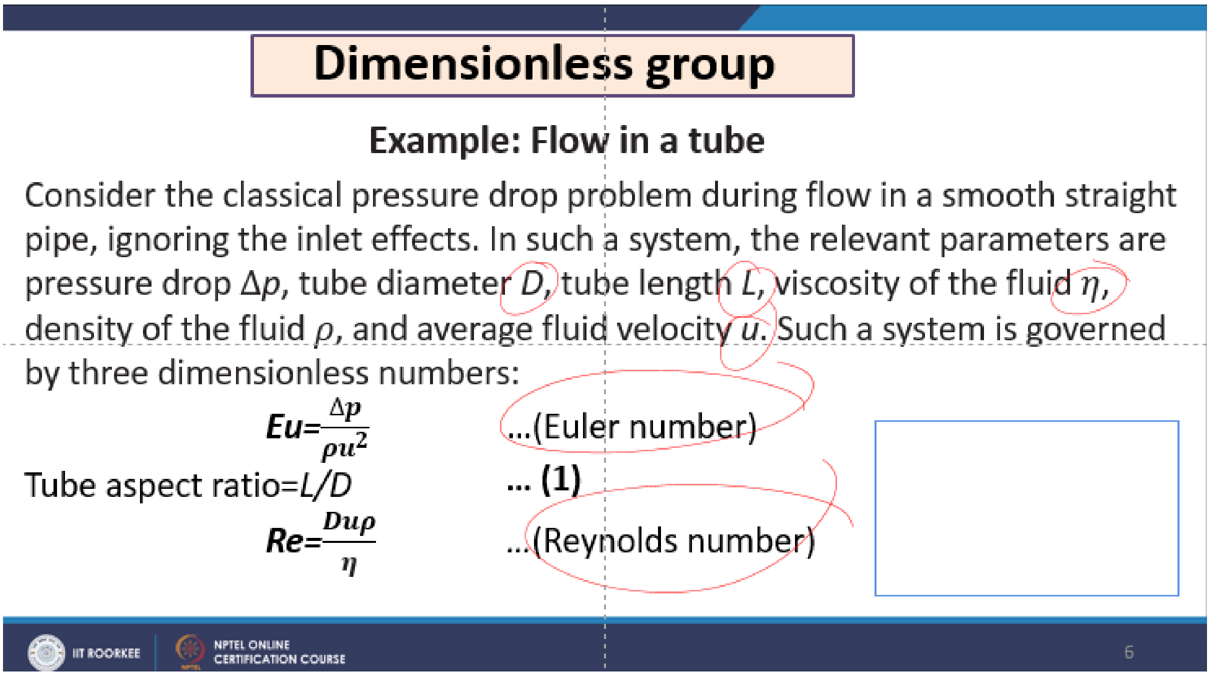


Now, modelling in polymer rheology or plastic rheology in processing is purely based on the transport phenomena. Now, we cannot overlook the importance of rheology because whenever we go for a useful product manufacturing, especially in the polymeric system, then ultimately, we need to go for certain sort of viscosity behaviour and some the thermodynamic concept. Based on this, we need to have a piece of proper knowledge about polymer rheology, and processing is purely based on that particular rheological aspect, which is why the integral part of this processing and plastic rheology is the transport phenomena.

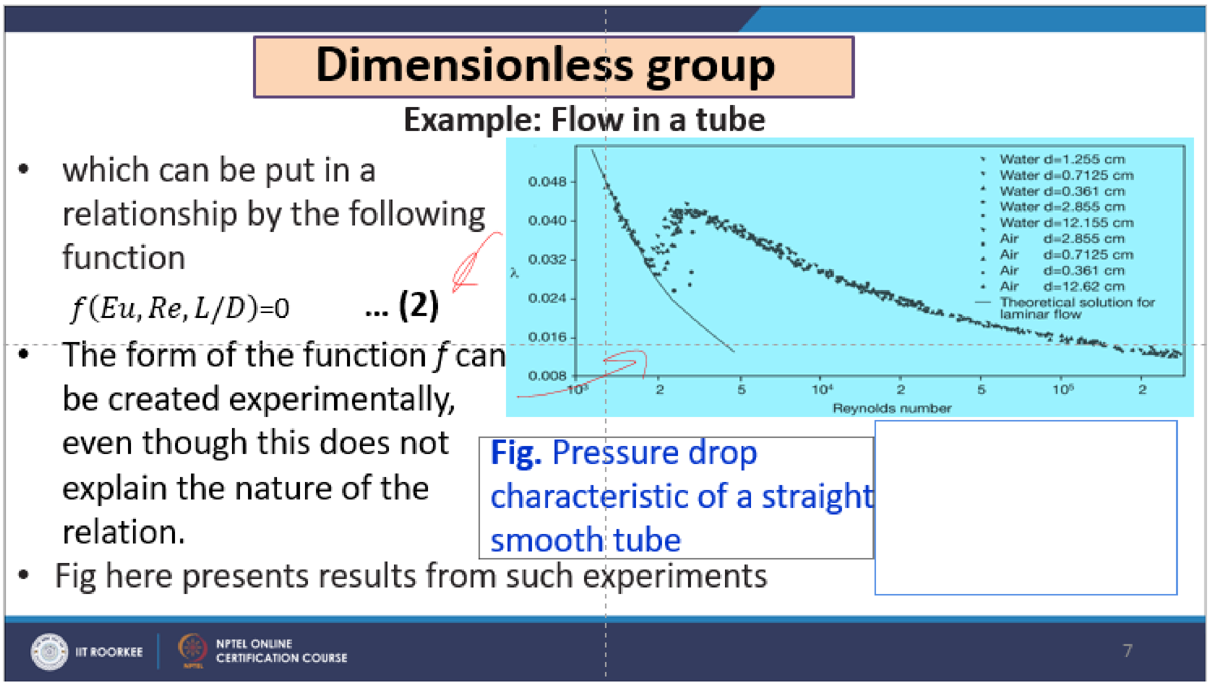


So, usually, a dimensional study of the system which provides insight into the significant parameter that influence the system or a process is frequently the first step in modelling a system where whether a rheometer or an actual process likewise. When modelling a system, it is necessary to balance the flux of mass force and energy inside the system by applying the right material models or constitutive equations and dimensional dimensionless numbers provide insight into the system. The most significant dimensionless groups pertinent to the plastic rheology and plastic processing we are going to discuss here. The balance equations, they are then derived along with the simple constitutive or rheological model that enabled the modelling of rheometric flows and polymer processes.





To simulate the fundamental system as flow in a tube, even partially flow or pressure flow between the parallel plates, the flow between the two rotating concentric cylinders give it flow. The model and analyse the rheometer and of course, the processes to improve them, one can utilise these straightforward system or combination of them. The engineers utilise dimensional analysis and dimensionless groups and they are having a very good importance and very well-versed importance in all kind of rheological studies. So, we are going to discuss the dimensional groups. Now, these groups or a number is present the theoretical and experimental result compactly to obtain insight into that particular aspect or rheological behaviour.



Now, this is accomplished by condensing all of the variables of a system into the meaningful dimensional dimensionless numbers. For example, it may be appropriate to express the pressure requirement in terms of Reynolds number, which is the ratio of both effects if the fluids inertia and the viscous effect dominate the flow system. Let us take example of flow in a tube. Now, consider a classical pressure drop during the flow in a smooth straight pipe, we can ignore the inlet effects in such a system the relevant parameters are pressure drop delta p, the tube diameter d at the tube length and viscosity of the fluid neta and the density of fluid rho and average fluid density u. Now, such a system is governed by three dimensional less numbers.

Dimensionless group

Table: Dimensionless Groups in Plastics Rheology and Polymer Processing

Name	Symbol	Definition	Meaning
Biot	Bi	Hl/k	Convection from surface/ Conduction through body
Brinkman	Br	$\eta u^2/k\Delta T$	Viscous heating/ Conduction
Capillary	Ca	$\tau R/\sigma_s$	Deviatoric stresses/ Surface tension stresses
Damköhler	Da	$c\Delta H_r/\rho C_p \Delta T$	Reaction energy/ Internal energy
Deborah	De	$\frac{\lambda}{t}$ or λ_ω	Relaxation time/ Process time

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Dimensionless group

Table: Dimensionless Groups in Plastics Rheology and Polymer Processing

Name	Symbol	Definition	Meaning
Fourier	Fo	$\frac{\alpha t}{L^2}$	Process time/ Thermal diffusion time
Giacomin	G_n	De + i We	Measure of non-Newtonianness
Graetz	Gz	$\left(\frac{d}{L}\right)\frac{uL}{\alpha}$	Lengthwise convection/ Transverse conduction
Manas-Zloczower	M_z	$\frac{\dot{\gamma}}{\dot{\gamma}+\omega}$	M_z= 0.0-No deformation M_z= 0.5-Shear flow M_z= 1.0-Elongational flow
Nahme-Griffith	Na	a ΔT Br	Effect of viscous heating on flow

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One is the Euler number that is Eu is equal to delta p over rho u square, then second is the Reynolds number, that is Re is equal to d u rho over neta if the tube aspect ratio is equal to L by d. Now, this can be put in a relationship by this particular function, f is a function of Euler number Reynolds number and L by d is equal to 0. Now, the form of the function f can be created experimentally even though this does not explain the nature of the relation. So, this figure represents the pressure drop characteristics of a straight smooth tube and this results from those experiments. Now, this particular plot shows the lambda is equal to 2 Eu d over L as a function of a Reynolds number.

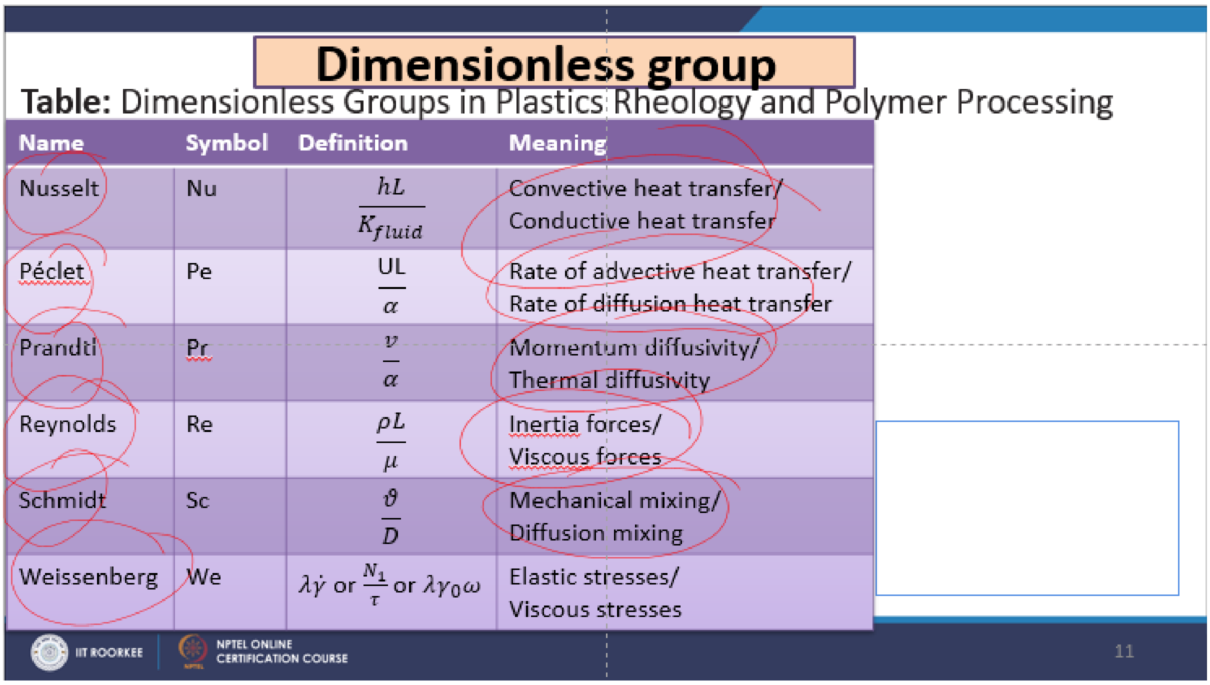
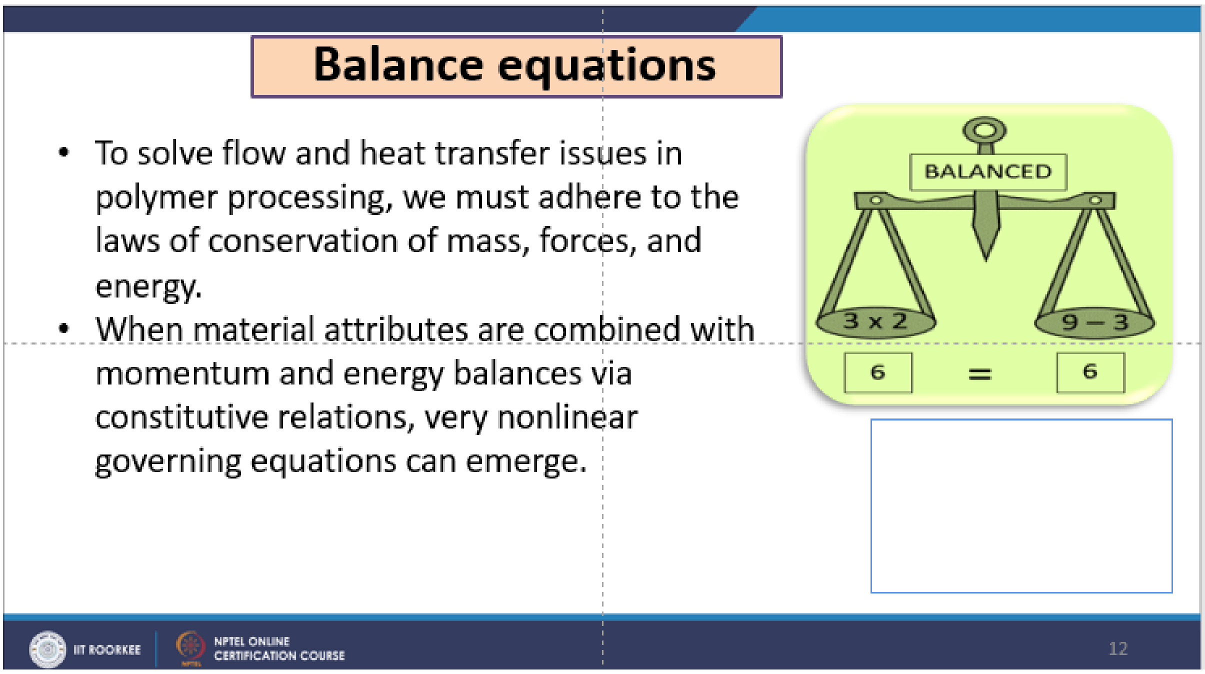


Table: Dimensionless Groups in Plastics Rheology and Polymer Processing

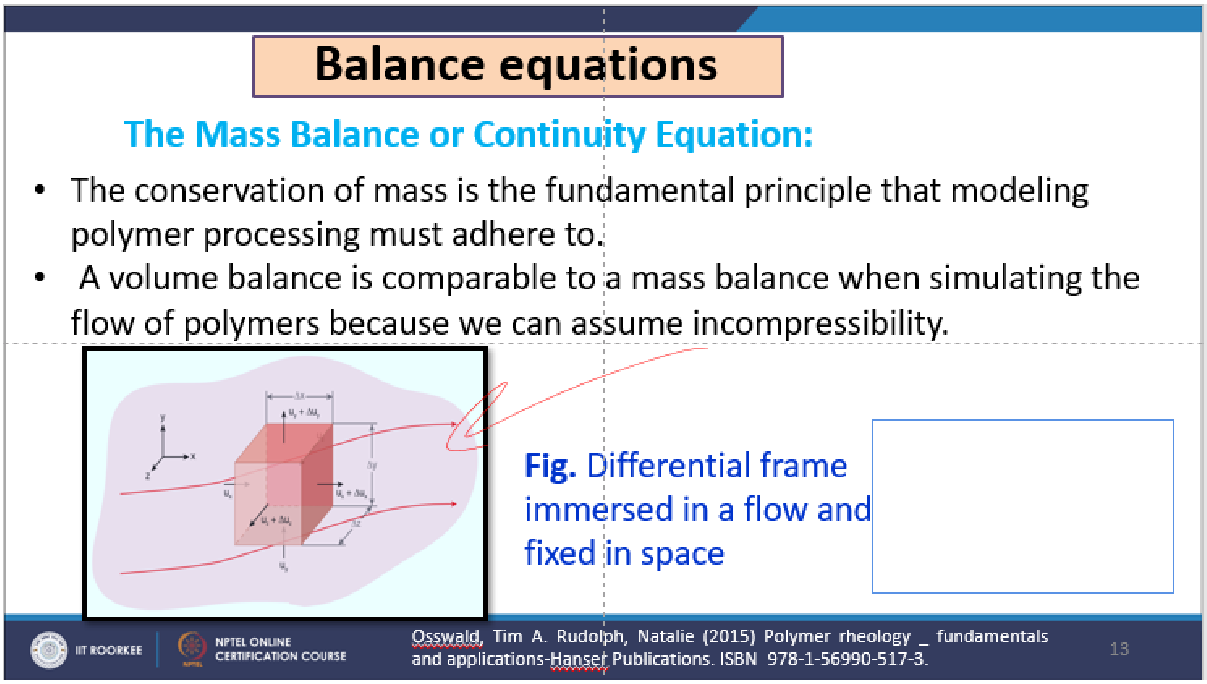
Name	Symbol	Definition	Meaning
Nusselt	Nu	$\frac{hL}{K_{fluid}}$	Convective heat transfer/ Conductive heat transfer
Péclet	Pe	$\frac{UL}{\alpha}$	Rate of advective heat transfer/ Rate of diffusion heat transfer
Prandtl	Pr	$\frac{\nu}{\alpha}$	Momentum diffusivity/ Thermal diffusivity
Reynolds	Re	$\frac{\rho L}{\mu}$	Inertia forces/ Viscous forces
Schmidt	Sc	$\frac{\vartheta}{D}$	Mechanical mixing/ Diffusion mixing
Weissenberg	We	$\lambda\dot{\gamma}$ or $\frac{N_1}{\tau}$ or $\lambda\gamma_0\omega$	Elastic stresses/ Viscous stresses

The value of the dimensionless number is clearly shown in this particular plot. The line representing the laminar flow is exemplified by a pressure flow in a tube derived further. Now, this particular table represents a dimensionless group in plastic rheology and polymer process like Biot number, the symbol is Bi represented as Hi over k that is a convective from the surface and conduction through body, then Brinkman number represented as Br and this is represented as neta u square over k delta t. This is the meaning of the viscous heating and conduction, then capillary, Damkohler, Da Broglie number and all these having the different type of a definition and their meanings. Similarly, the Fourier number represented by Fo definition alpha t over L square this is the process time and thermal diffusion time, then Giacomin, Gn, De plus i We where we have given this Ge over here, and this is a measure of non-Newtonian stress, then Grate's number, then Manus-Jacquard number and Nern-Griffith number.

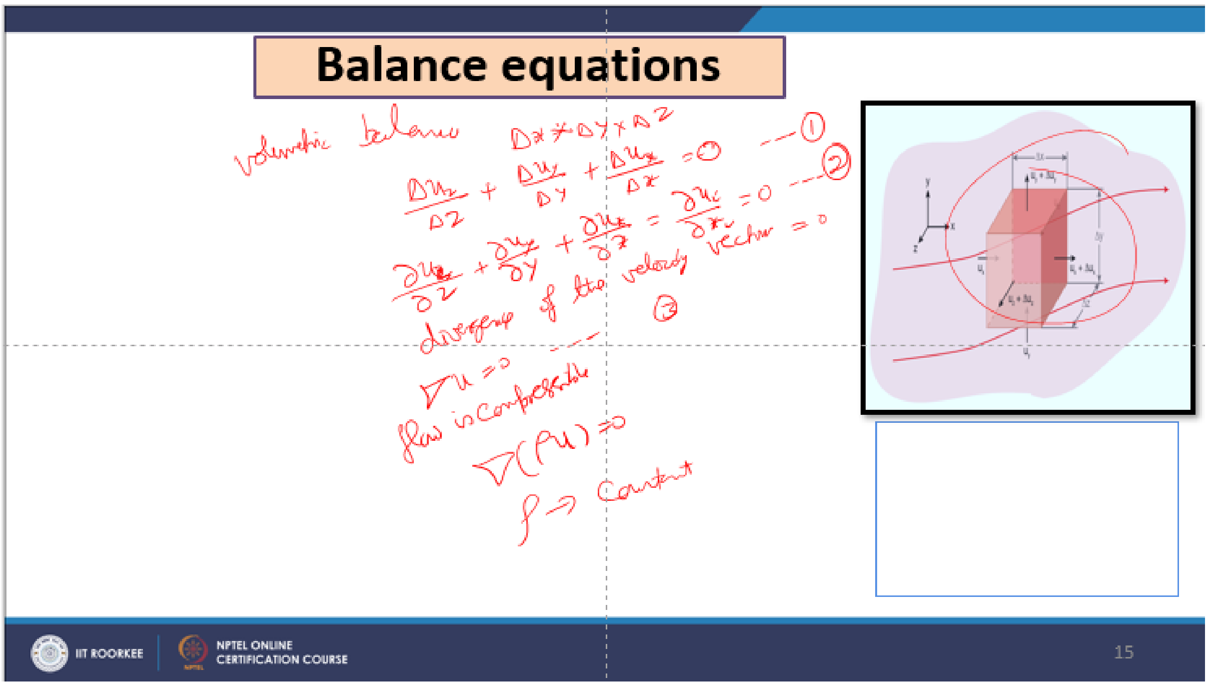
So, all these numbers they represent similarly, the Nusselt number, the Peclet number, the Prandtl number, the Reynolds number, the Schmidt number and then Weissenberg number. So, all these numbers, they are the dimensional group, dimensionless groups in the rheology and they all having the major impetus in describing the polymer rheology and the polymer processing, like sometimes require the convective heat transfer then should have considered the Nusselt number, then rate of advective heat transfer, then Peclet number, then if we talk about the momentum diffusivity and the thermal diffusivity, then the Prandtl number in force and then if we talk about the inertia forces, then viscous or viscous forces, then Reynolds number all these things are there. And if we talk about the mechanical mixing or a diffusion mix mixing, then Schmidt numbers, these numbers in force. Now, let us talk about the balance equation. Now, to solve the flow and a heat transfer issue in polymer processing, we must adhere to the laws of conservation of mass, force and energy.



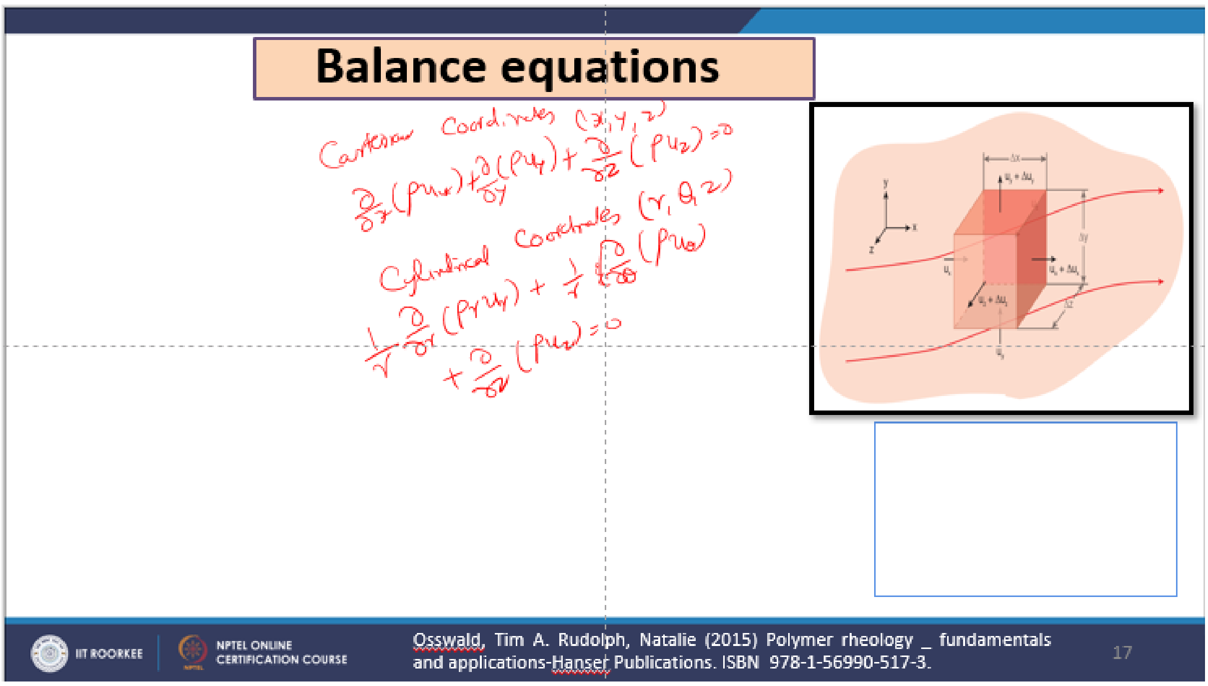
Now, when material attributes they are combined with momentum and energy balances via constitutive relations, a very nonlinear governing equation, these can emerge. So, let us talk about the mass balance or a continuity equation, the conservation of mass usually is a fundamental principle that modelling polymer processing must adhere to. A volume balance is comparable to mass balance when simulating the flow of polymer, because we can assume incompressibility. Now, this is the figure, the differential frame immersed in the flow and fixed in space. Now, the continuity equation is the name to given to the resulting equation.



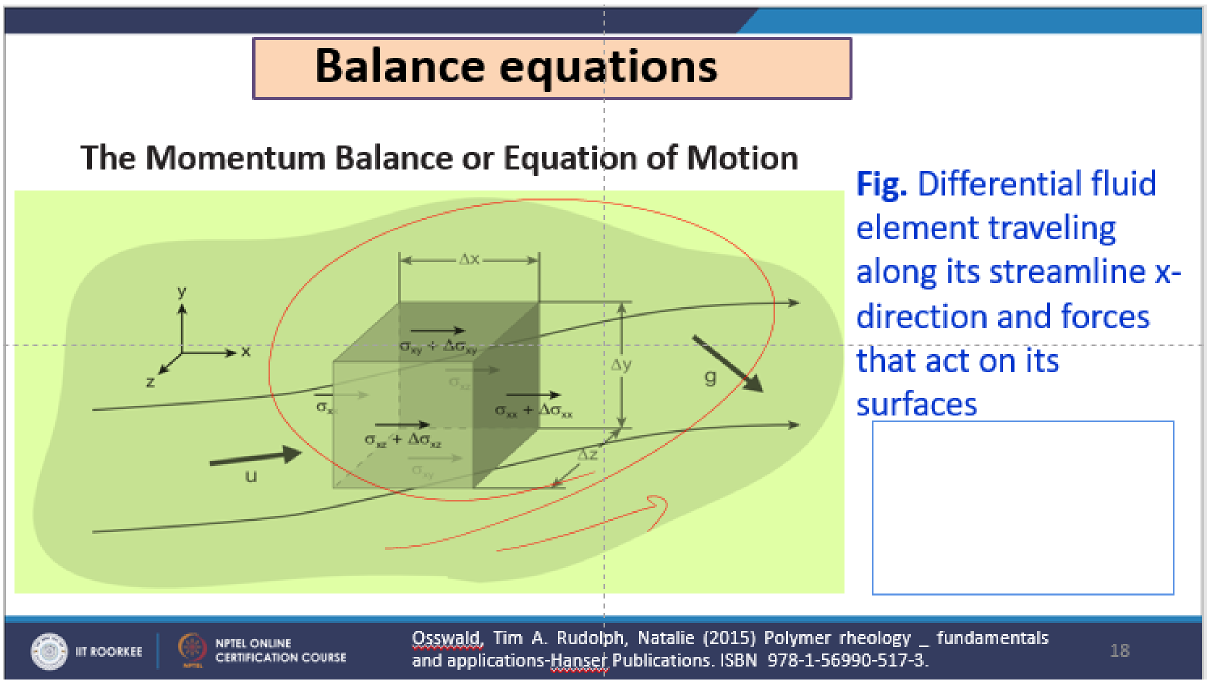
Now, if we see this particular diagram or a figure, we insert an imaginary wire frame with the dimensions of delta X into delta Y into delta Z inside a flowing system to calculate the continuity equation. Now, if we take the using the notation introduced in that in this particular figure, we can perform a volumetric balance. Now, in and out of the differential element in the volume is specified a specific form by dividing the balance by the elements volume delta X into delta Y into delta Z. So, this can be delta U Z over delta Z plus delta U Y over delta Y plus delta U X over delta X is equal to 0. Let us take this is equation number 1.



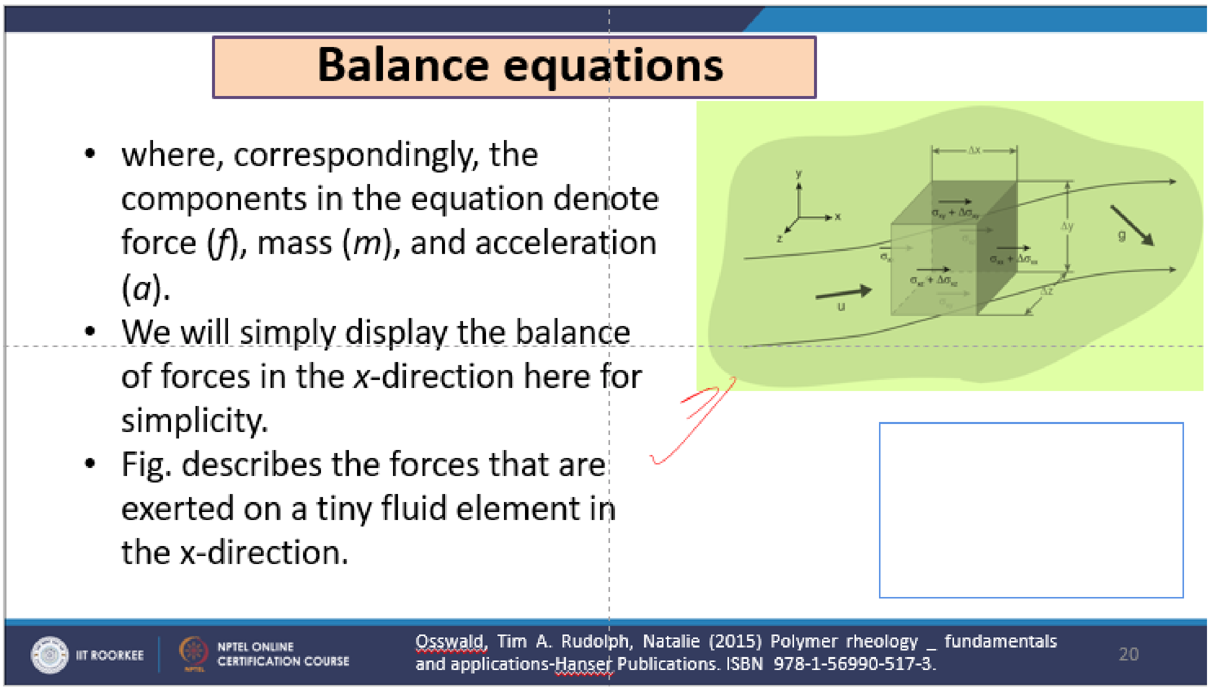
Now, letting the size of a differential element go to 0, this results del U Z over del Z plus del U Y over del Y plus del U X over del X is equal to del U I over del X I that is equal to 0. Let us say this is equation number 2. Now, this is stating the divergence of the velocity vector must equal to 0 when the mass or volume is conserved. So, we can write this equation as is equal to 0, this is equation number 3. Now, when the flow is compressible, variable density has to be taken into account and the continuity equation must be written as U is equal to 0.

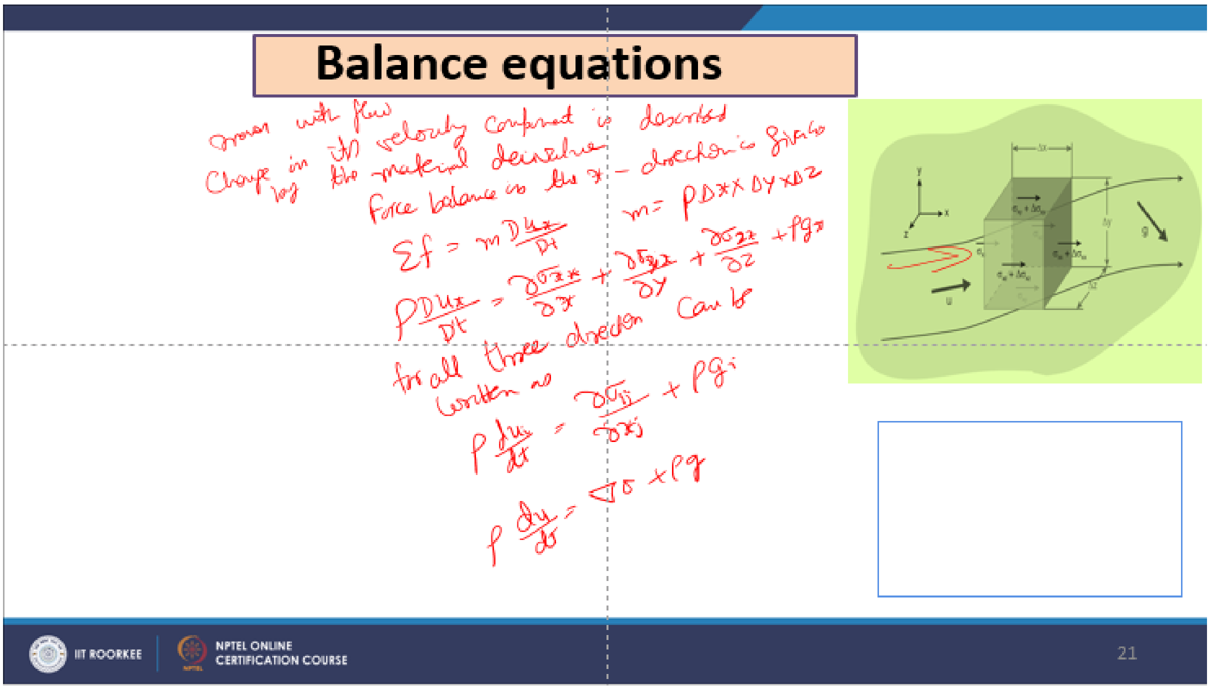


Now, this can be the continuity equation in the Cartesian and the cylindrical coordinate system. Now, this the most cases the density rho is constant and can be dropped from the equation. Now, here the Cartesian coordinate if we talk about the Cartesian coordinate X, Y, Z, so, we can write del over del X rho U X plus del over del Y rho U Y plus del over del Z rho U Z is equal to 0. Now, the cylindrical coordinates r, theta and z, this can be written as 1 over r del over del r rho r U r plus 1 over r del over del theta rho U theta plus del over del Z rho U Z is equal to 0. Now, the momentum balance or equation of motion can be depicted with this particular figure, where the differential fluid elements they are travelling along the streamline X direction and forces that acts on its surface.



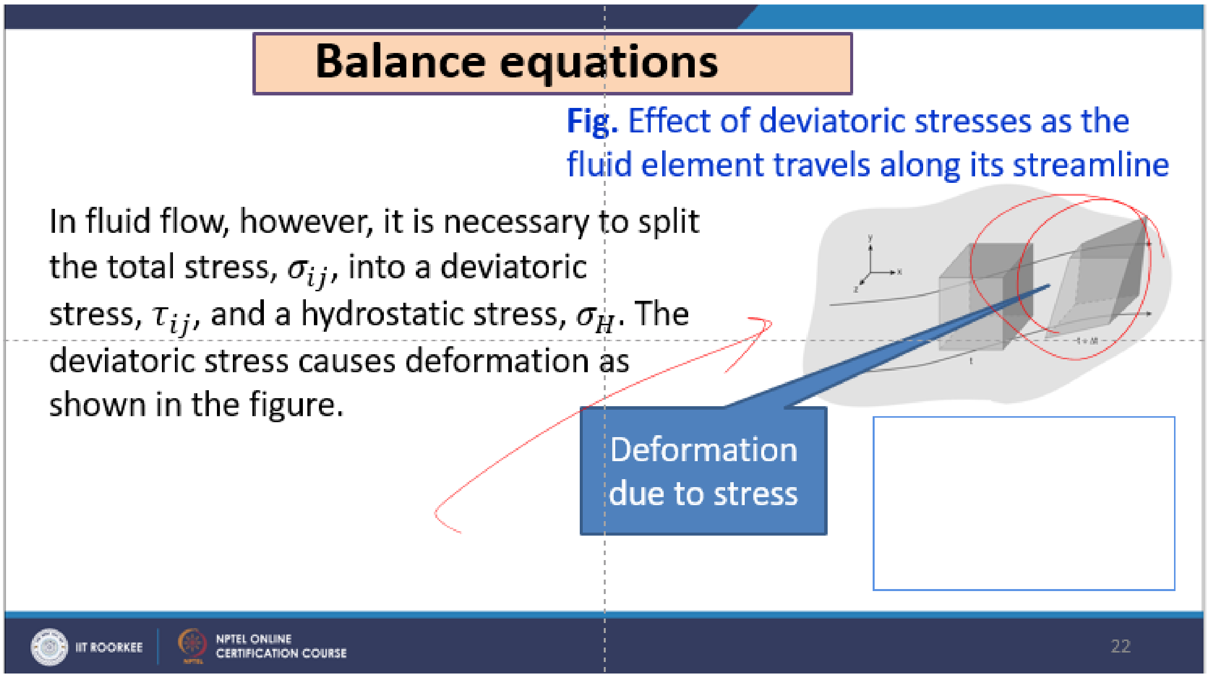
Now, in order to calculate the momentum balance, we use the figure. Now, instead of submerging a hypothetical frame into the melt, we use a real fluid element with the dimensions of delta X into delta Y into delta Z and calculate a force balancing using the force acting on its surface. The force balance can be written as summation of force is equal to m a. Now, where the correspondingly the component in the equation denotes the force F, mass m and acceleration a, we will simply display the balance of forces in the X direction here for simplicity. Now, this figure describes the forces that are exerted in a tiny fluid element in the X direction.





Now, because the element in this particular figure is a fluid particle that moves with the flow, moves with the flow and the change in its velocity, the change in its velocity component is described by the material derivative. So, therefore, the force balance in the X direction is given by summation F is equal to m d u x over dt, where m is equal to rho delta X into delta Y into delta Z. So, after adding the forces divided by elements volume and letting the volume go to 0, the force balance in the X direction results like rho d u x over dt del X X del X plus del small sigma Y X del Y plus del small sigma Z X del Z plus rho g X. Now, which for all 3 dimensions directions for all 3 directions can be written as rho d u i over dt is equal to del small sigma i j del X j plus rho g i and rho d u over dt is equal to rho g. Now, this particular thing represents a deformation due to stress.

Now, this is the effect of deviatoric stresses as the fluid element travels along the streamline. Now, in fluid however, it is necessary to split the total stress into the deviatoric stress tau and hydrostatic stress, deviatoric stress causes the deformation as per this particular figure, the deformation due to the stress. Now, the hydrostatic stress is described by the pressure. Now, here you see the hydrostatic stress acting on a differential element. So, we can write sigma i j is equal to sigma H delta i j plus tau i j, where delta i j is the Kronecker delta as the above this particular equation reveals the hydrostatic stress only that can act in the normal direction of the surface and it is equal, it is equal in all 3 directions.



Hence, we can write sigma is equal to sigma H is equal to minus p, where p is the pressure, the negative pressure reflects the fact that a positive pressure causes a compressive stress. So, the total stress can be written as a sigma i j is equal to minus p delta i j plus tau i j. So, using the definition of total stress, the momentum balance can be written as can be rho du i over dt is equal to minus del p over del xi plus del tau over del x plus rho gi and rho du over dt is equal to Kronecker delta p, this is p plus tau plus rho g. Now, the momentum equations which can be written as with respect to the Cartesian coordinates and in terms of tau that can be written as rho del ux over del t plus ux del ux over del x plus uy del ux over del y plus uz is equal to del ux over del z, this is equal to minus del rho over del x plus del tau xx over del x plus del tau yx over del y plus del tau zx over del z plus rho gx. And similarly, rho del uy over del t plus ux del uy over del x plus uy over del uy over del y plus uz del uy over del z, this is equal to minus del p over del y plus del tau xy over del x plus del tau yy del y plus del tau zy over del z plus rho gy or rho del uz over del t plus ux del uz over del x plus uy del uz over del y plus uz del uz over del z, this can be represented as minus del p over del z plus del tau xz over del x plus del tau yz plus del y plus del tau zz over del z plus rho gz.

Balance equations

The hydrostatic stress is described by pressure:

$\sigma_{ij} = \sigma_H \delta_{ij} + \tau_{ij}$

it is equal in all three directions

$\sigma_H = -p$ $p \rightarrow$ pressure

The negative pressure reflects the fact that a positive pressure causes a compressive stress

$\sigma_{ij} = -p\delta_{ij} + \tau_{ij}$

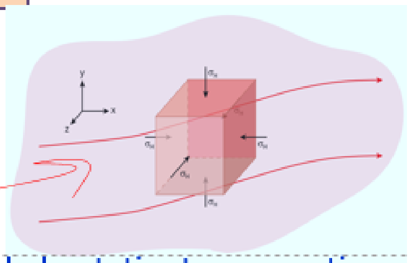


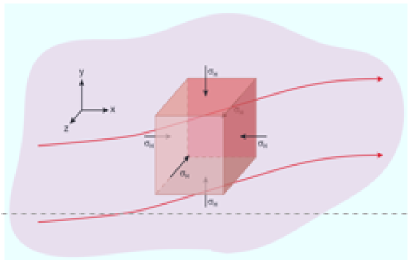
Fig. Hydrostatic stresses acting on a differential element

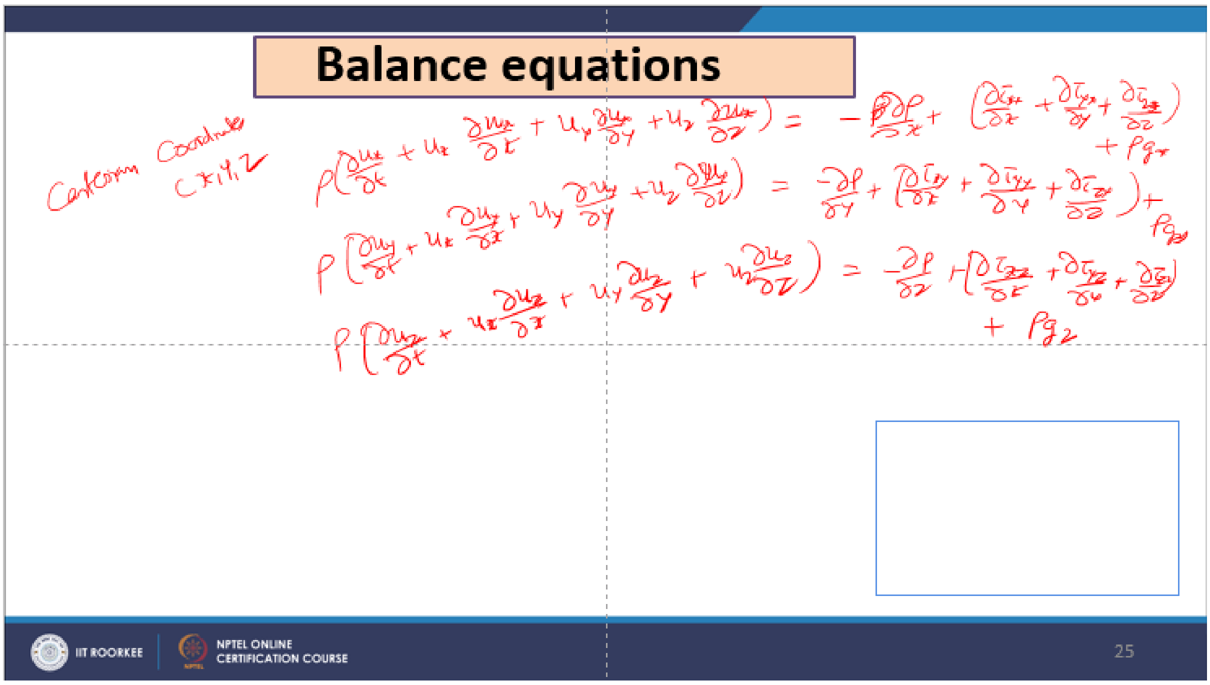
Balance equations

Momentum balance can be

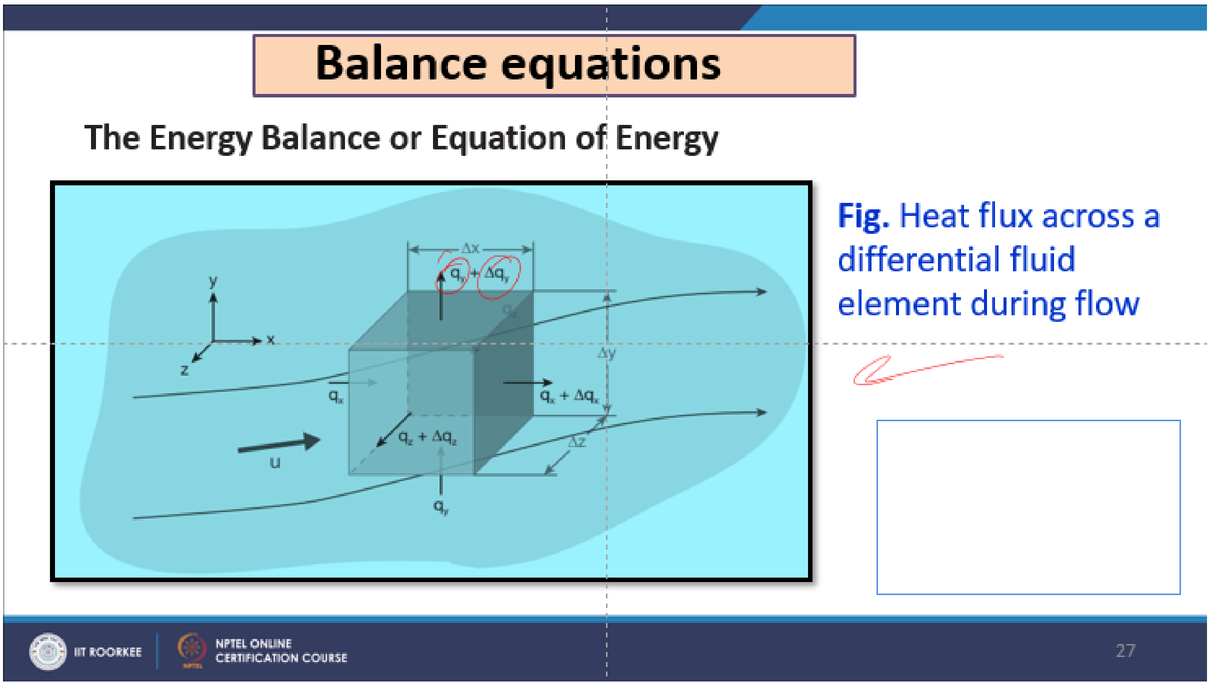
$$\rho \frac{du_i}{dt} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x} + \rho g_i$$

$$\rho \frac{du}{dt} = -\nabla p + \nabla \tau + \rho g$$

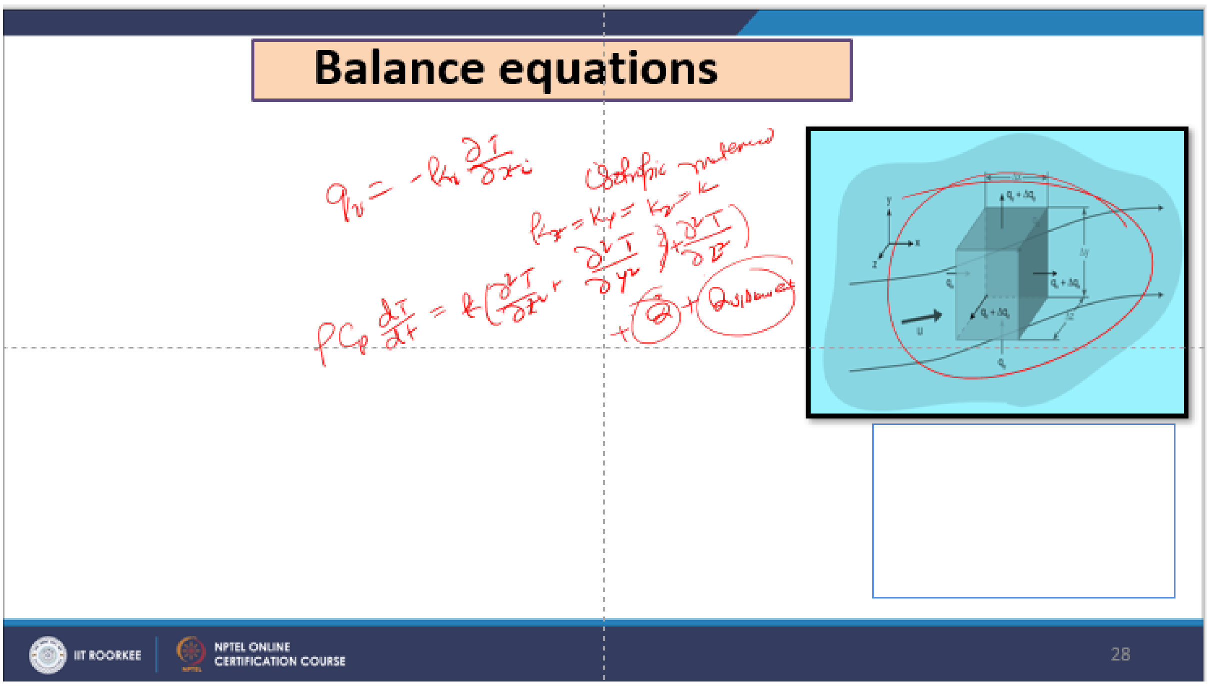




So, these are the Cartesian coordinate x, y, z. Now, if we try to write with the help with the cylindrical coordinates r theta z. So, rho del ur over del t plus ur del ur over del r plus u theta over r del ur over del theta minus u theta square over r plus uz over del u r over del z, this is equal to minus del p over del r plus 1 over r del r tau plus 1 over r del tau r theta del theta minus tau theta plus del tau iz del z plus rho gi or rho del u theta plus del t plus ur del u theta plus del r plus u theta over r del u theta over del theta plus ur u theta over r plus uz del u theta over del z, which is equal to minus 1 over r del rho, del p over del theta plus 1 r square del r r square tau r theta plus 1 over r del tau theta plus rho g theta or rho del uz over del t plus u r del uz over del r plus u theta over r del u z over del theta plus u z del u z over del z, which is equal to minus del p over del z plus 1 over r del r r tau r z plus 1 over r del tau theta z plus del tau zz del z plus rho gz. Now the energy balance or equation of energy, now this is the heat flux across a differential fluid element during the flow. Now Q represents the heat aspect and the different direction.



Now using the Fourier law of heat conduction Q i is equal to minus k i del t over del x i. Now assuming an isentropic material for this the k x is equal to k y is equal to k z is equal to k. Now an energy balance around the moving fluid element, this can be written as rho C p dT over d t k del 2 T over del x 2 plus del 2 T over del y 2 plus del 2 T over del z 2 plus Q dot plus Q viscous heating, where an arbitrary heat source Q dot and viscous dissipation term were included. Now this is the schematic flow of a simple shear flow system used to illustrate the viscous dissipation term in the energy balance system. Now as an illustration we will derive the viscous dissipation term in the energy balance using a simple shear flow system.



Balance equations

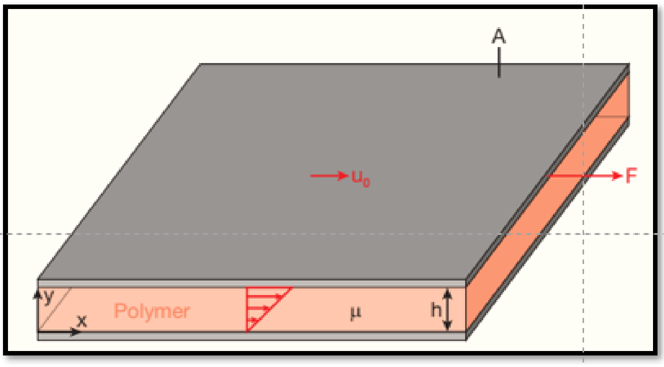


Fig. Schematic of a simple shear flow system used to illustrate viscous dissipation terms in the energy balance

Osswald, Tim A. Rudolph, Natalie (2015) Polymer rheology _ fundamentals and applications-Hanser Publications. ISBN 978-1-56990-517-3

Balance equations

The stresses within the system

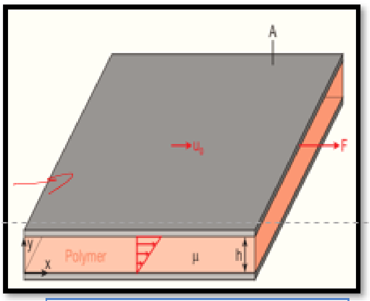
$\tau_{yx} = \mu \frac{\partial u_x}{\partial y}$

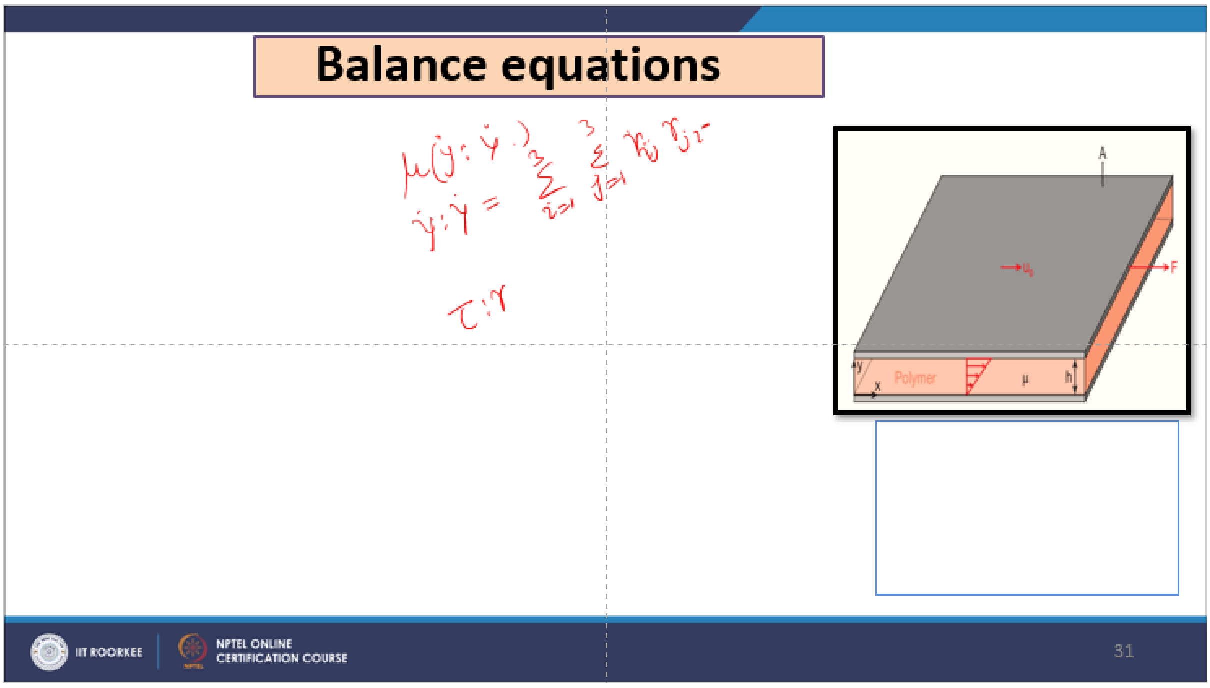
$\frac{F}{A} = \mu \frac{u_0}{h}$ ← plate speed ← height

$F u_0 = \mu \frac{u_0}{h} A u_0$ volume of plate Ah

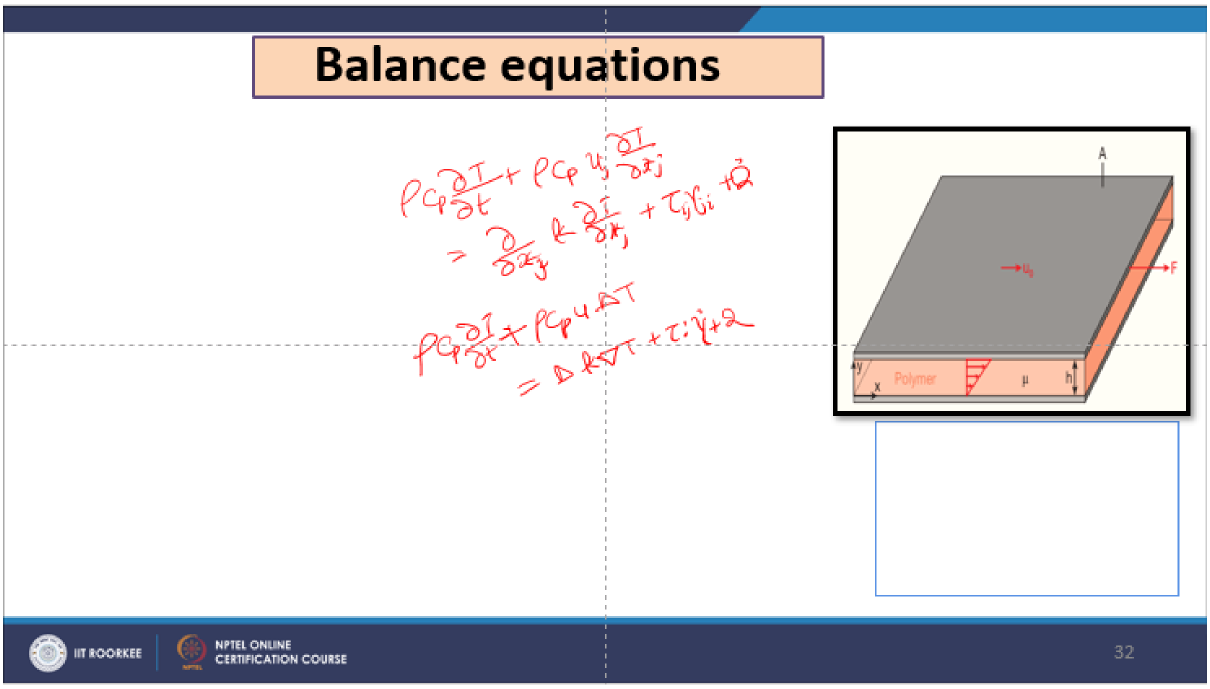
$\frac{F u_0}{Ah} = \mu \left(\frac{u_0}{h}\right)\left(\frac{u_0}{h}\right)$

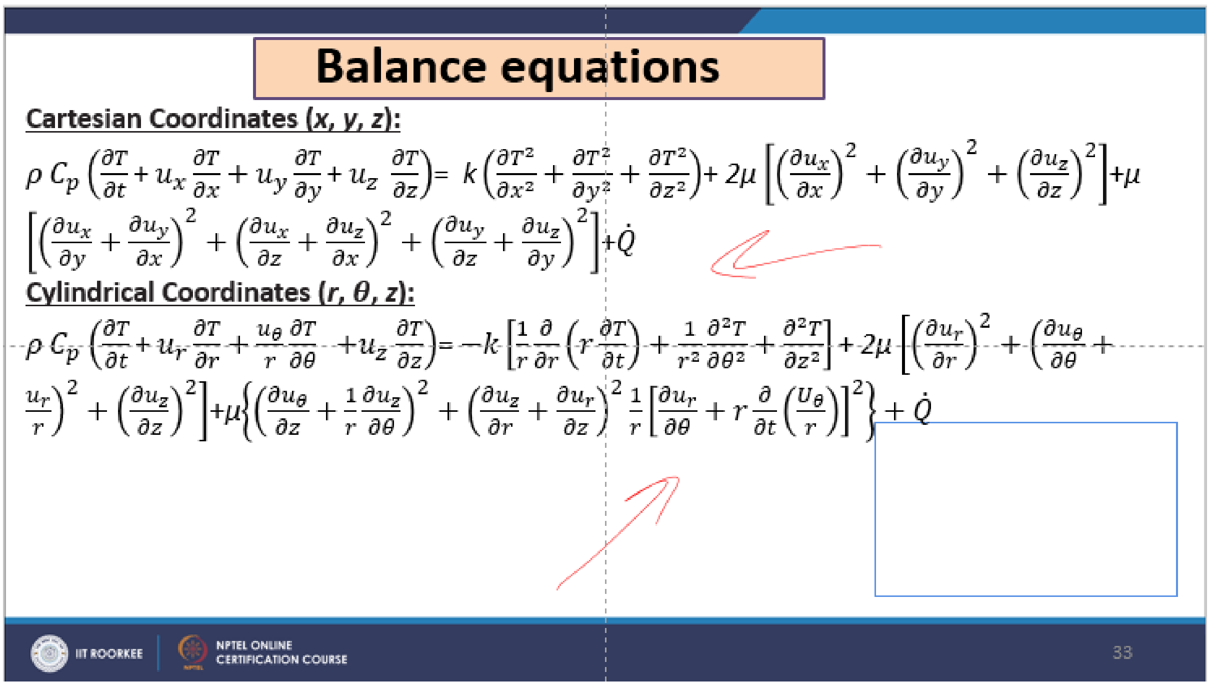
$\dot{Q}_{viscous\ heat} = \mu \left(\frac{\partial u_x}{\partial y}\right)\left(\frac{\partial u_x}{\partial y}\right)$





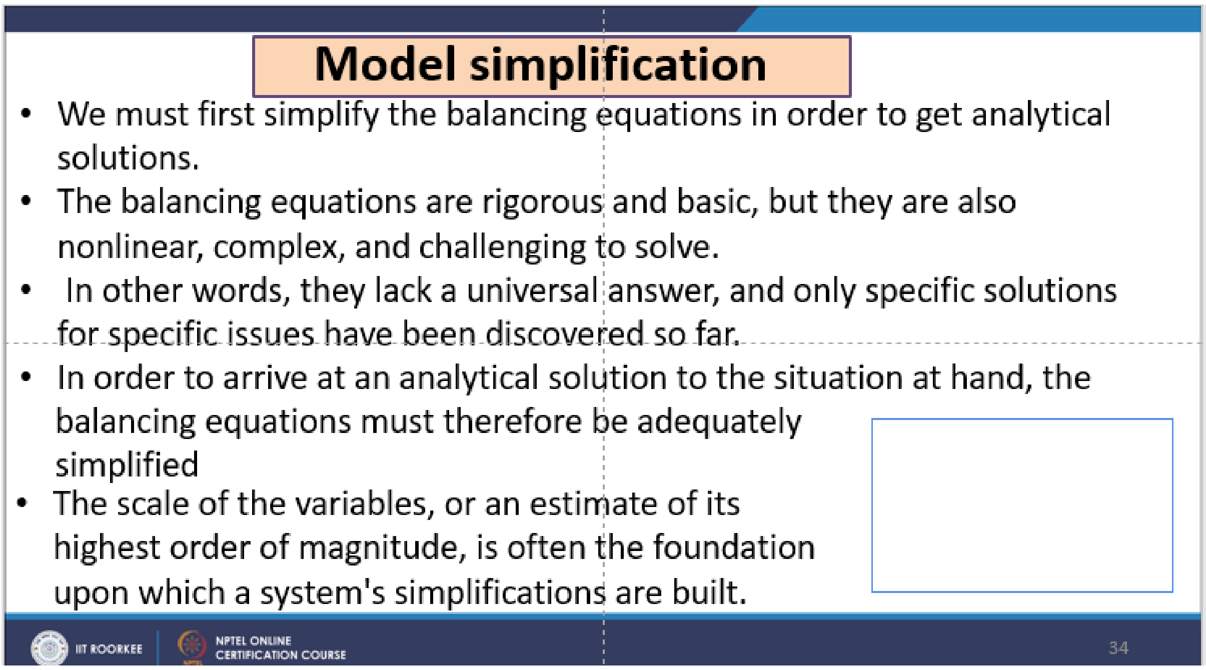
Now here the stresses within the system can be given as tau y x is equal to mu d u x over del u x over del y, which in terms of parameters depiction like force, area, gap, height, plate, speed u naught. This can be written as force over the area of cross section mu u naught, which is the plate speed over h which is the height. So, the system in the system rate of energy input is given by f u naught is equal to mu u naught over h A u naught. And if we divide the above this particular equation by the volume of the polymer of polymer A h, this is A h the rate of energy input per unit volume is given by f u naught over A h is equal to mu u naught over h then u naught over or q viscous heating q dot mu del u x over del y into del u x over del y. This equation can be deduced from the Newtonian fluid and the general term of the viscosity dissipation is given by mu dot gamma dot gamma, where dot gamma is equal to summation i is equal to 1, 2, 3 and summation z is equal to 1, 2, 3 gamma i j gamma j i.



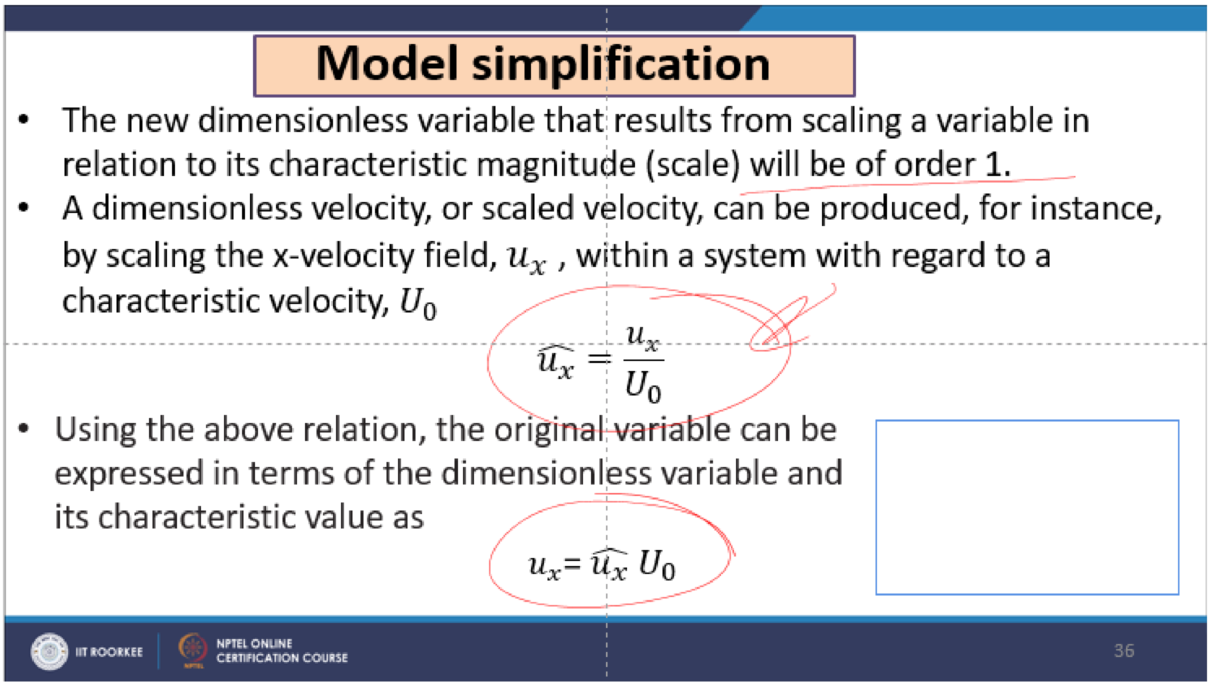


And for the Newtonian for the non-Newtonian material the viscous heating is written as tau gamma. Hence, the energy energy balance becomes rho C p d t del t over del t plus rho C p u j del t over del x j which is equal to del over del x j k del t over del x j plus tau i j gamma j i plus q dot or rho C p del t over del rho C p u delta t that is equal to k t plus tau dot gamma plus q. Now, the if we talk about the Cartesian coordinates this can be mathematically represented like rho C p del t over del t plus and so on like this and if we talk about the cylindrical coordinates this can be represented in this particular mathematical form. Now, we must first simplify the balancing equation in order to get the analytical solution. The balancing equations they are rigorous and basic but they are also non-linear complex and challenging to solve.

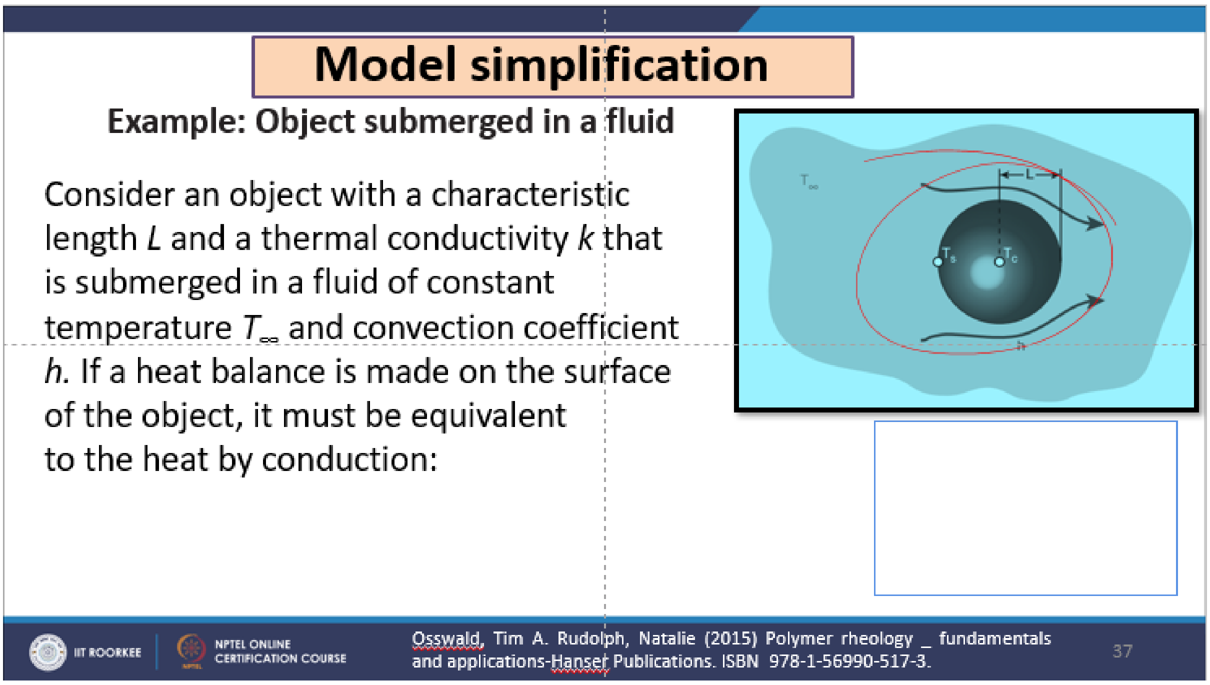
In other words, they lack a universal answer and only specific solutions for specific issues have been discovered so far. Now, in order to arrive at an analytical solution to the situation at hand the balancing equation must therefore be adequately simplified. The scale of variables or an estimate of its highest order of magnitude is often the foundation upon which the system simplifications are built. Now, in order to arrive an analytical solution to the situation at hand the balancing equations must therefore be adequately simplified and the scale of variable or an estimate of its highest order of magnitude is often the foundation upon which the system simplifications are built. Scaling is the process of determining the precise order of magnitude of the many unknowns as was discovered in different sections.

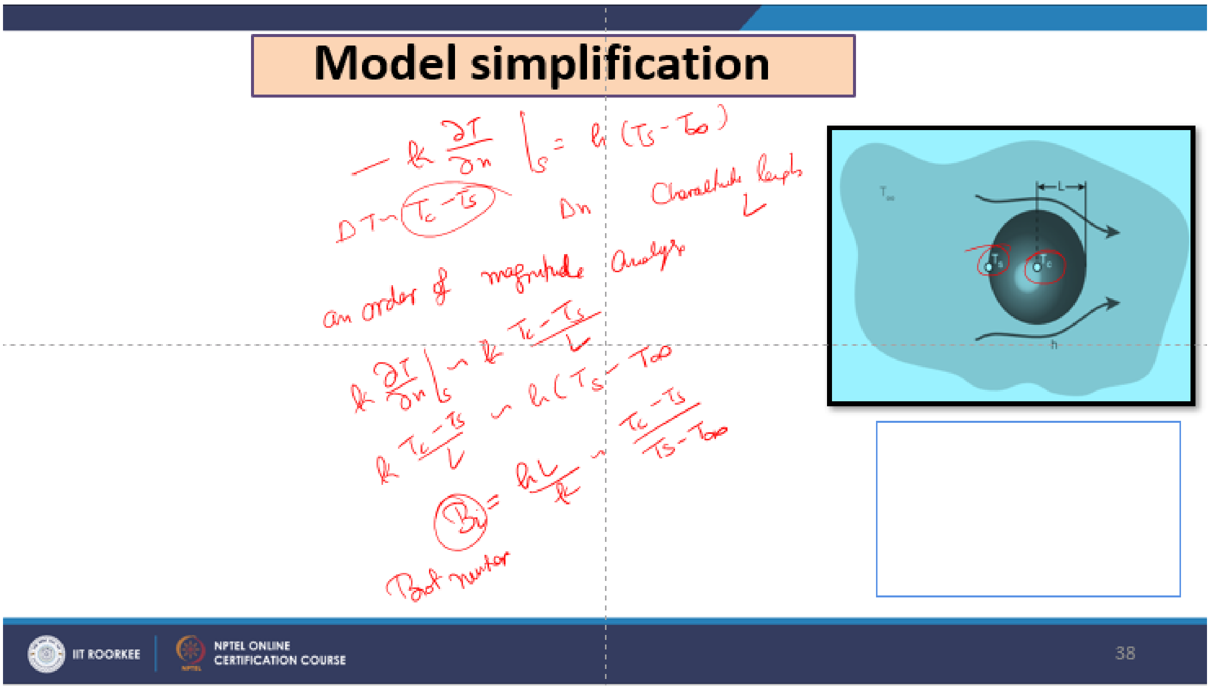


These values such as characteristic times, characteristic length these are the frequently referred to as a characteristic's magnitude. The new dimensionless variables that results from scaling a variable in relation to its characteristic magnitude scale will be of order 1. A dimensionless viscosity or scaled viscosity can be produced for instance by scaling the x velocity field u x within a system with regard to the characteristic velocity u naught. So, this can be represented mathematically like this. Now, using this particular relation the original variable can be expressed in terms of a dimensionless variable and its characteristic value like this u x is equal to bar u x over into u naught.



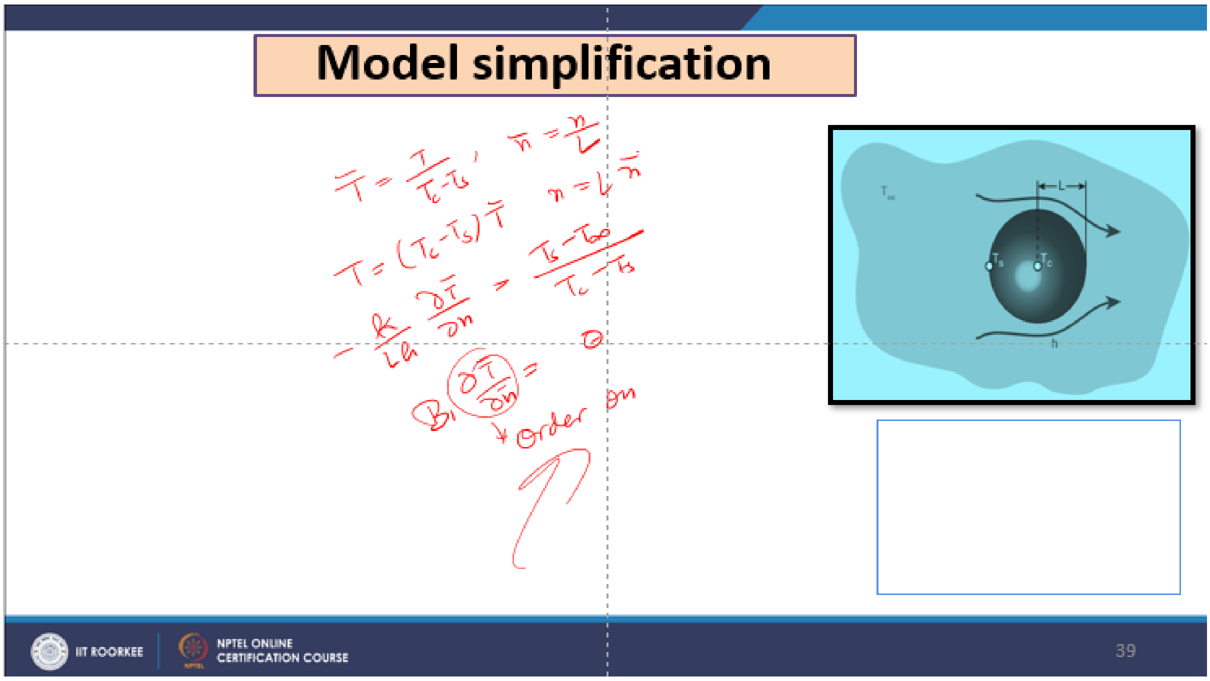
Now, let me give you an example that object submerged in fluid. Consider an object with the characteristic length L and the thermal conductivity k that is submerged in a fluid of constant temperature tau naught and the convection coefficient H. If a heat balance is made on the surface of the object it must be equivalent to the heat by conduction. Now, this can be represented like this minus k del T over del n H T s minus T infinity. The maximum value the possible for the temperature gradient must be the difference between the central temperature and the surface temperature.



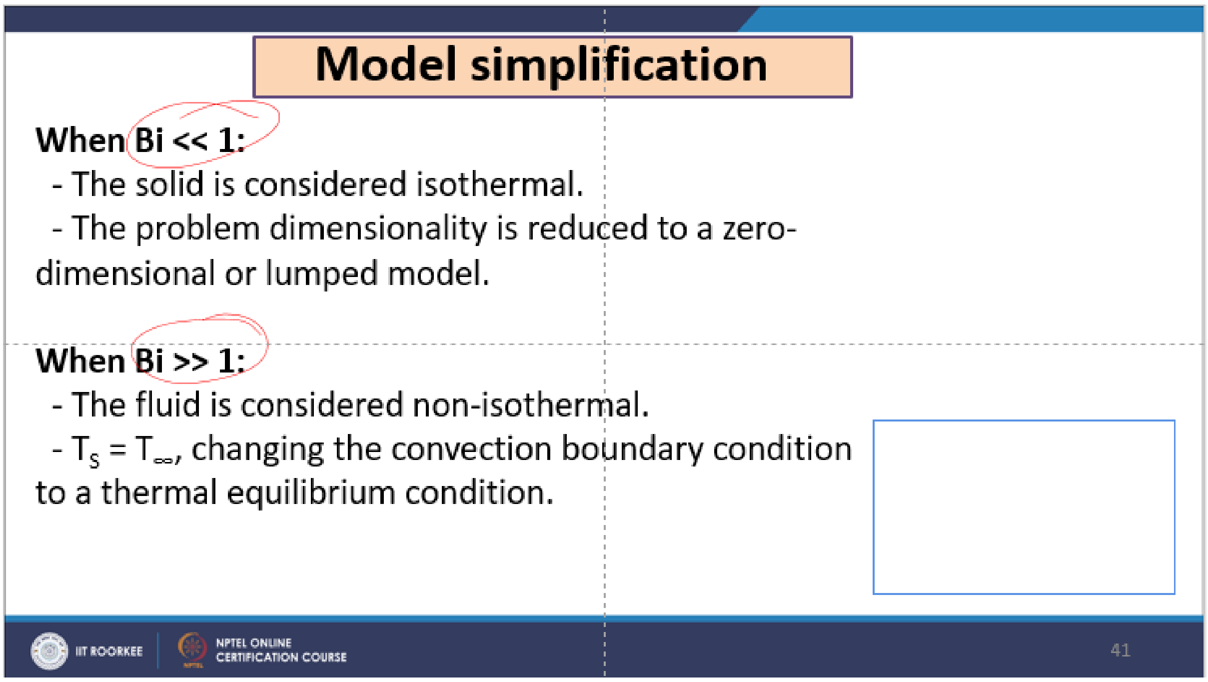


This is the central temperature and this is the surface temperature. So, delta T is almost equal to T c minus T s this which provides the characteristic temperature difference this is the characteristic temperature difference. Here the length variable is a normal distance delta n and has a characteristics length L. We can now approach to the scaling in two ways. The first and quickest is to simplify the substitute the variable into the original equation often referred as the order of an order of magnitude analysis.

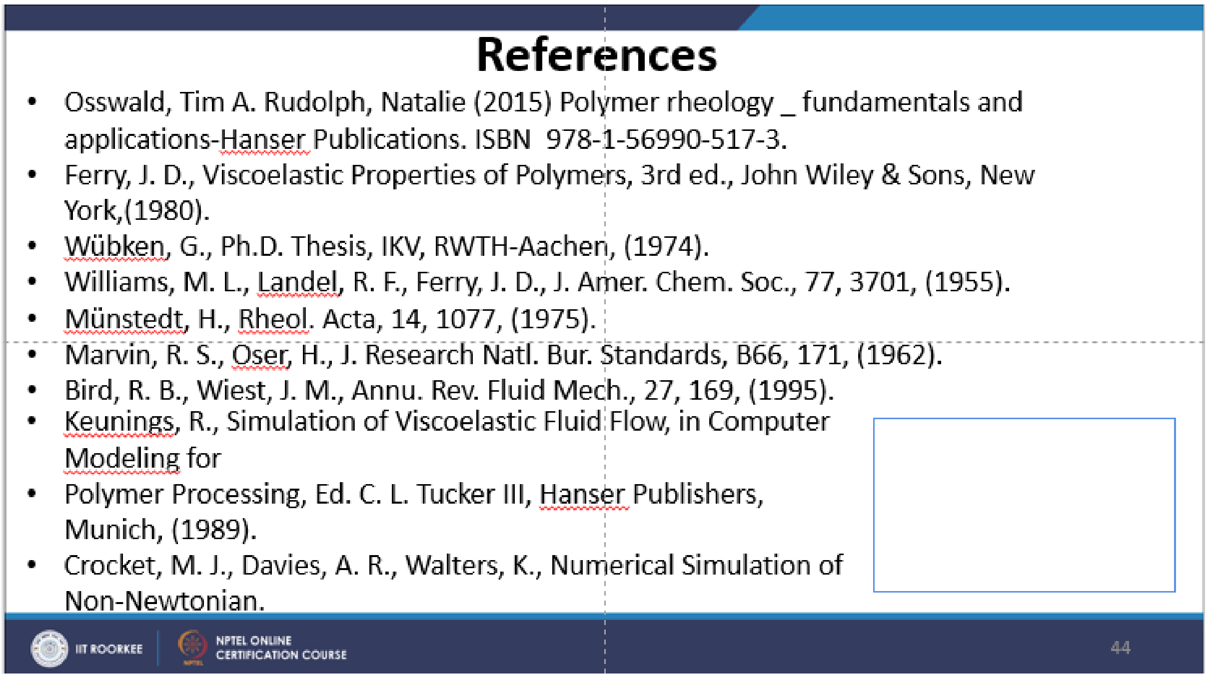
The second is to express the original equation in terms of dimensionless variable and the order of magnitude analysis result in the scale conduction is given by k del T over del n k T c minus T s over L. Now, reducing the problem to say k T c minus T s over L H T s minus T infinity or in a more convenient way B i is equal to H L. B i is the biot number. Now, this is the second and the third way. So, T cover k T c minus T s over T s minus T infinity.



So, the bar T is equal to T over T c minus T s and bar n is equal to n over L. This can be solved to give T is equal to T c minus T s bar T or n is equal to bar. Now, substituting these two in the original equation this results minus k L H del n T s minus T infinity over T c minus T s or if we represent in terms of biot number. So, B i del T bar over del n. Now, this this del T bar over this one is of order 1 and the same analysis done this which is applied over here.



So, let us take that if B i is less than less than 1 the solid is considered the isothermal and the problem dimensionality is reduced to a 0 dimensional or lumped model and when B i is greater than greater than 1 the fluid is considered the non-isothermal and the minus T s is equal to T infinity changing the convection boundary condition to a thermal equilibrium condition. The choice of characteristic value of for normal distance and the temperature allows the generation of dimensionless variables and scaling the problem and expressing the governing equation in dimensionless form.



So, dear friends in this particular segment we discussed the different rheological nature and developed the different models and for your convenience we have enlisted several references can be utilized as per the requirement. Thank you very much.