Basic Environmental Engineering and Pollution Abatement
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Lecture 09
Transmission of Pollutants in Environment - 2

Hello everyone. Now, we will discuss on the topic Transmission of Pollutants in Environment, Part 2. In the Part 1, we have seen that pollutants get entry into the environment in water, in air, in soil and these are transmitted from one place to other place. And particularly gets entry into the human body and any other living organisms and creates different types of impacts. And now, we will discuss on the transmission of groundwater contaminants, hydraulic gradient and groundwater flow, retardation and transmission of surface water contaminants.

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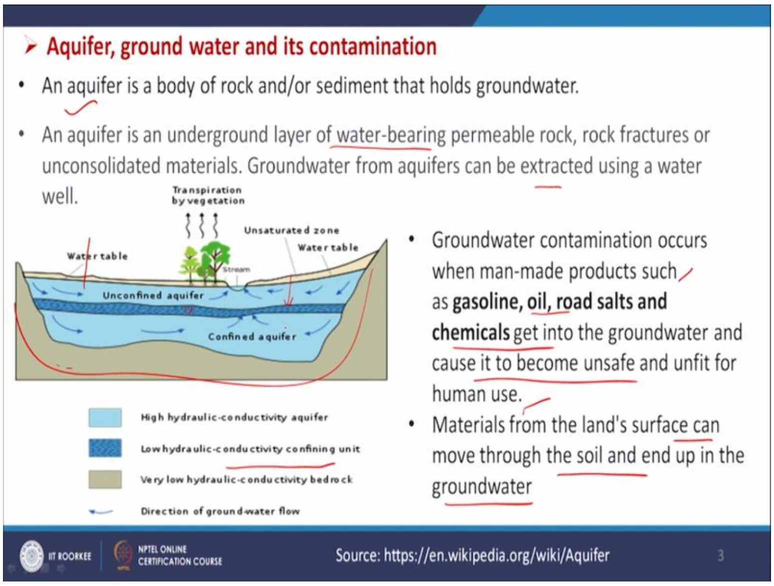

Aquifer, ground water and its contamination

- An aquifer is a body of rock and/or sediment that holds groundwater.
- An aquifer is an underground layer of water-bearing permeable rock, rock fractures or unconsolidated materials. Groundwater from aquifers can be extracted using a water well.
- Groundwater contamination occurs when man-made products such as **gasoline, oil, road salts and chemicals** get into the groundwater and cause it to become unsafe and unfit for human use.
- Materials from the land's surface can move through the soil and end up in the groundwater

Source: https://en.wikipedia.org/wiki/Aquifer

So, now, we will see what is aquifer and how the ground water is contaminated and how the contaminants transfer from one place to other place. So, an aquifer is a body of rock and or sediment that holds groundwater. So, you see the figure here. So, this is unconfined aquifer and this is confined aquifer and in between we have one low hydraulic conductivity-confining unit.

So, and this is very low hydraulic conductivity bedrock. So, this is our bedrock and we are having here some layer. That is your confining layer and we are showing here unconfined aquifer and confined aquifer. An aquifer is an underground layer of water bearing permeable rock, rock fractures or unconsolidated materials. Groundwater from aquifers can be extracted using a water well.

So, here we can anywhere we can put in well. So, that water can be extracted from that place. So, this is a definition of aquifer and which provides water and water can be extracted to the well. Now, the contaminants may enter into this ground water. How? Through the percolation.

So, groundwater contaminants occur when man made products such as gasoline oil, road salts and chemicals get into the ground water and cause it to become unsafe and unfit for the human use. And materials from the land surface can move through the soil and end up in the ground water. So, percolation takes place. So, hydraulic conductivity of the soil play role to allow the water to go from the surface to the aquifer through the surface layer.

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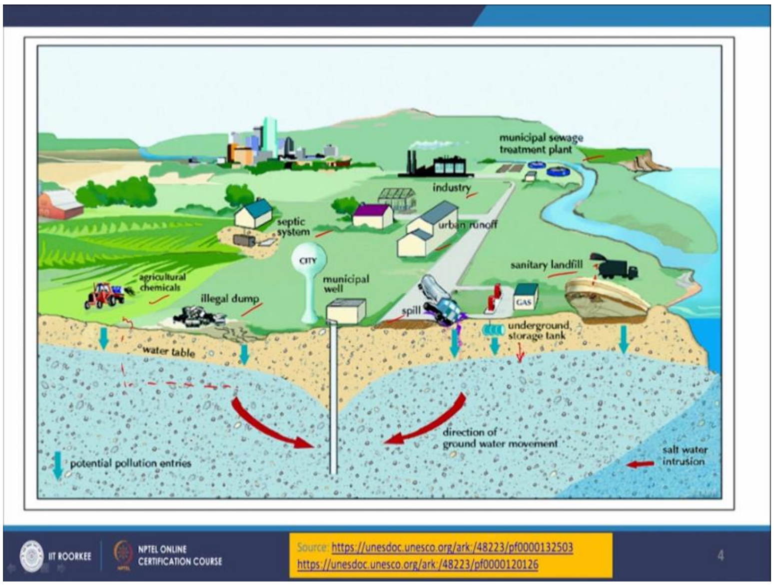

Source: https://unesdoc.unesco.org/ark:/48223/pf0000132503
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And these are the different activities. So, oil spills, urban runoff, industry, municipal sewage, sanitary landfills and then illegal dumping, agricultural, chemicals and septic systems, all are responsible for the creation of pollutants, which can enter into the ground water through this percolation process.

So, water is penetrated through the soil bed and then reaches to here. And once it is reaching here, so, some pollutants is coming in this place. So, it may reach to this place first and from this to this it will be moved. So, that that transmission will be taking place. Or it may also if this is our water table, so, it can be at the above of that water table sub surface soil will be getting there also the water can transmit from one to other place, the pollutants can transmit from one place to other place.

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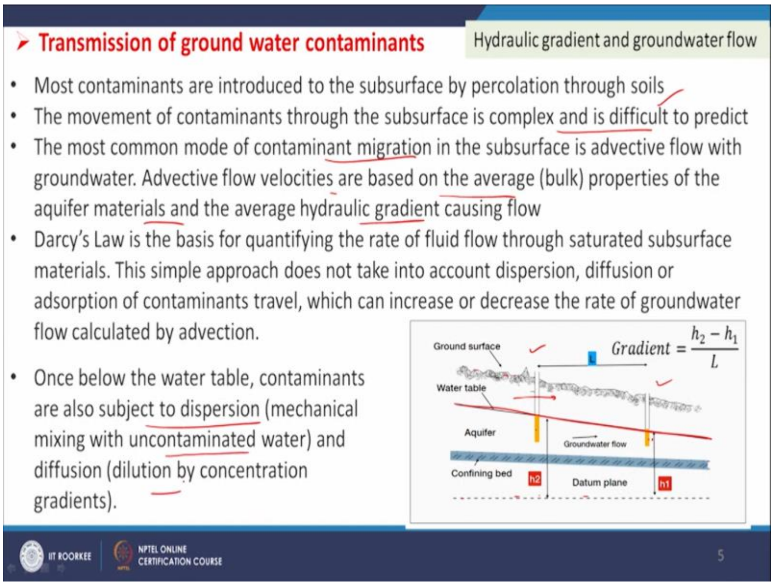

Transmission of ground water contaminants

Hydraulic gradient and groundwater flow

- Most contaminants are introduced to the subsurface by percolation through soils
- The movement of contaminants through the subsurface is complex and is difficult to predict
- The most common mode of contaminant migration in the subsurface is advective flow with groundwater. Advective flow velocities are based on the average (bulk) properties of the aquifer materials and the average hydraulic gradient causing flow
- Darcy's Law is the basis for quantifying the rate of fluid flow through saturated subsurface materials. This simple approach does not take into account dispersion, diffusion or adsorption of contaminants travel, which can increase or decrease the rate of groundwater flow calculated by advection.
- Once below the water table, contaminants are also subject to dispersion (mechanical mixing with uncontaminated water) and diffusion (dilution by concentration gradients).

$$Gradient = \frac{h_2 - h_1}{L}$$

So, here it is very clear. You see, this is our datum plane and say this is one well another well is here, or we can say this is our water table this one, so, from the datum plane we are getting different height. So, if we take two, consider two points, say point one and point two in between this height is more here, so, water will be flowing through this.

So, it may be below the water table or maybe above the water table. So, this is your ground water surface. This is water table. So, here also the transmission of the water will be there, water pollutants will be there. Here also the transmission of the water pollutants will be there. So, the gradient, the difference in the head is giving the driving force for the transmissions of the pollutants in this water or in the subsoil layers also. But subsoil transmission is not so easy.

It is a complex process. And the structure of the soil will influence along with this gradient. So, most contaminants are introduced to the surface by percolation through soils that just I have told you. And most contaminants are introduced to the subsurface by percolation through soils and the movement of contaminants through the sub surface is complex and is difficult to predict.

The most common mode of contaminant migrations in subsurface is advective flow. Just like your transmissions in atmosphere, here also, advection or advective flow play significant role for the transmissions of the pollutants in the subsurface also. And then the advective flow velocities are based on the average properties of the sub layer or aquifer materials and the average hydraulic gradient causing the flow. Just Δh and the structure of the sub layer.

And when the pollutants are in the water then also there may be some mechanical mixing or diffusion etc. So, once below the water table, contaminants are also subjected to dispersion. There is mechanical mixings with uncontaminated water and diffusion or variations in the concentration. So, these are the basically the driving force which helps the transmissions of the contaminants in the ground water.

Now, how will mathematically represent these phenomena. Just like say in case of transmission of pollutants in atmosphere, we have developed some mathematical relationship. The people have developed and we have discussed on it. So, similarly in this case we will also make some discussion, how the pollutants are transmitted in the aquifer.

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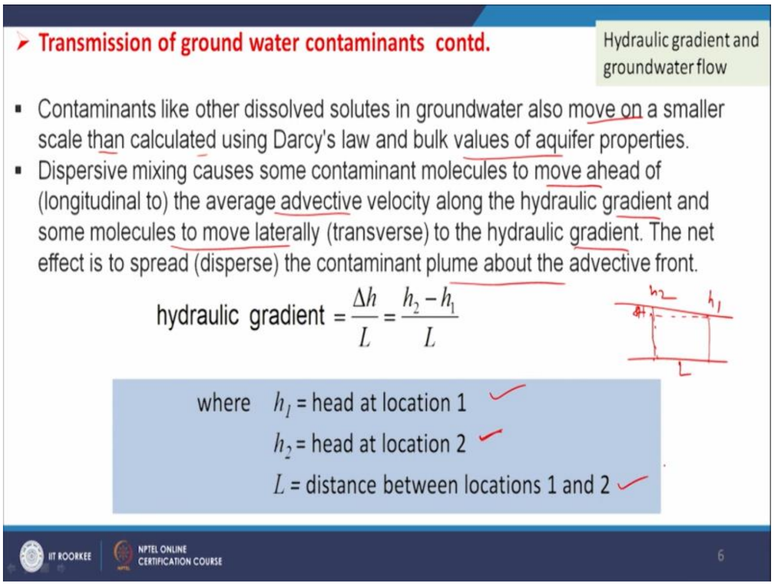

Transmission of ground water contaminants contd.

Hydraulic gradient and groundwater flow

- Contaminants like other dissolved solutes in groundwater also move on a smaller scale than calculated using Darcy's law and bulk values of aquifer properties.
- Dispersive mixing causes some contaminant molecules to move ahead of (longitudinal to) the average advective velocity along the hydraulic gradient and some molecules to move laterally (transverse) to the hydraulic gradient. The net effect is to spread (disperse) the contaminant plume about the advective front.

$$\text{hydraulic gradient} = \frac{\Delta h}{L} = \frac{h_2 - h_1}{L}$$

where h_1 = head at location 1

h_2 = head at location 2

L = distance between locations 1 and 2

So, contaminants like other dissolved solutes, in ground water also move on its smaller scale than calculated, using Darcy's law and bulk values of aquifer properties. So, when we think about the sub soil, so, here it is a porous bed. So, flow through porous bed, the Darcy's equation is applicable.

So, for groundwater flow that equation can be applicable, but what happens you see, dispersive mixing causes some contaminant molecules to move ahead of this longitudinal to the average advective velocity along the hydraulic gradient and some molecules to move laterally to the hydraulic gradient.

So, the net effect is the spread that disperse the contaminant plume about the advective front. That means that will be just like a plume structure which you have discussed in the transmission in case of atmosphere. So, here the similar type of plumes are also formed. And we can define hydraulic gradient as

hydraulic gradient = $\Delta h/L = (h_2 - h_1)/L$.

So, we have a datum line. So, here if I have say water table if it is, so, h_2 and this is h_1. So, from this place it is h_2, from this place it is h_1. So, this h_2 minus h_1 is the Δh. So, this is $\Delta h/L$, this is the distance. So, this is the hydraulic gradient where L is the head at location one and h_1 is the head at location one, h_2 is the head at location two and L the distance between one and two. So, this is the hydraulic gradient which is responsible for the transmission.

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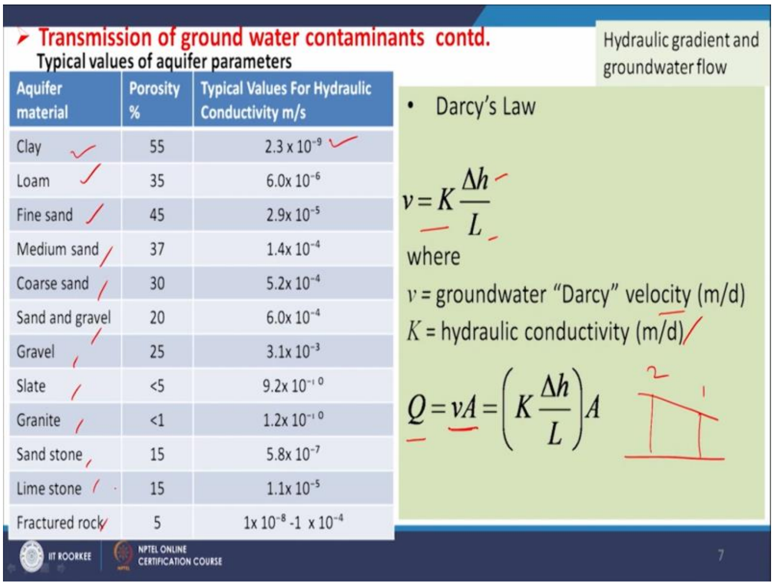

Transmission of ground water contaminants contd.

Typical values of aquifer parameters

Aquifer material	Porosity %	Typical Values For Hydraulic Conductivity m/s
Clay	55	2.3×10^{-9}
Loam	35	6.0×10^{-6}
Fine sand	45	2.9×10^{-5}
Medium sand	37	1.4×10^{-4}
Coarse sand	30	5.2×10^{-4}
Sand and gravel	20	6.0×10^{-4}
Gravel	25	3.1×10^{-3}
Slate	<5	9.2×10^{-10}
Granite	<1	1.2×10^{-10}
Sand stone	15	5.8×10^{-7}
Lime stone	15	1.1×10^{-5}
Fractured rock	5	$1 \times 10^{-8} - 1 \times 10^{-4}$

Hydraulic gradient and groundwater flow

- Darcy's Law

$$v = K\frac{\Delta h}{L}$$

where

v = groundwater "Darcy" velocity (m/d)

K = hydraulic conductivity (m/d)

$$Q = vA = \left(K\frac{\Delta h}{L}\right)A$$

And Darcy's law, we know that

$$v = K\frac{\Delta h}{L}$$

So, the K is called hydraulic conductivity and v is the ground water Darcy velocity. So, what will be the flow rate? What will be the flow rate?

So, $Q = vA = (K\frac{\Delta h}{L})A$

So, if we have once a cross sectional area is ‘A’ in the aquifer and v is the velocity of water then v*A is the flow rate in this direction. So, if we have say like this. So, one and two. So, this direction. So, what is the A in this and multiplied by v that will be our Q from this to this. So, higher head to lower head.

So, this will be the flow of the water along with the pollutants as well. So, these are the different values of the porosity and the hydraulic conductivity. As you mentioned that the porous structure of the aquifer materials will be responsible for the flow of the water as well as the pollutants and similarly the hydraulic conductivity here we have discussed that is also responsible. And this table shows the values, typical values for different types of materials present in the aquifer.

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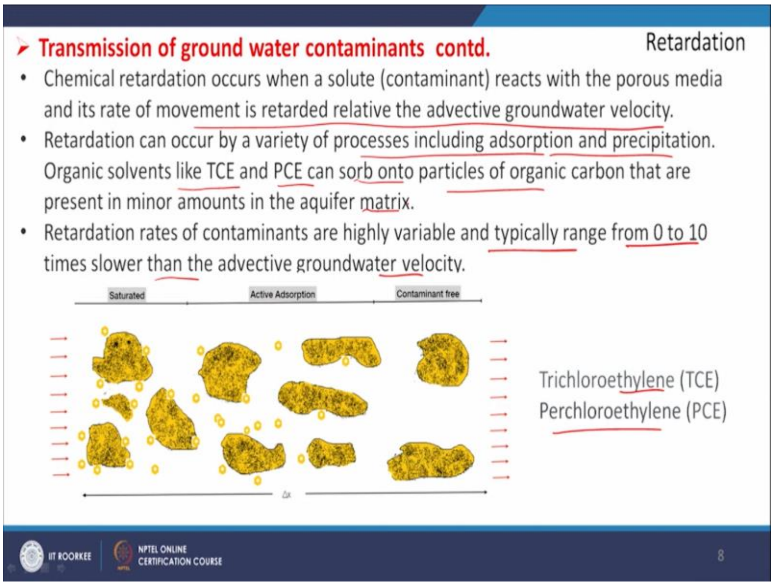

Retardation

Transmission of ground water contaminants contd.

- Chemical retardation occurs when a solute (contaminant) reacts with the porous media and its rate of movement is retarded relative the advective groundwater velocity.
- Retardation can occur by a variety of processes including adsorption and precipitation. Organic solvents like TCE and PCE can sorb onto particles of organic carbon that are present in minor amounts in the aquifer matrix.
- Retardation rates of contaminants are highly variable and typically range from 0 to 10 times slower than the advective groundwater velocity.

Trichloroethylene (TCE)
Perchloroethylene (PCE)

So, now, we see that when the water moves then pollutants present in it may be equally move with the velocity or may not be able to move equally with the velocity of the water or water molecules. Because there will be some interactions between the aquifer materials with the pollutants.

So, if there is no interactions that is theoretically the water velocity and the velocity of the pollutants will be same and we see here, if we have here the water is going on, so, these are the pollutants and these are the materials. So, these materials may have some interactions and if there is no interactions, so, all will go. There will be no variations. All water and pollutants will go.

But if there is some interactions, the pollutants will be adsorbed by this materials present in the aquifer. So, the front head if we see the pollutants may be the amount of materials passing through this will be reduced. So, that is called the retardation.

The chemical retardation occurs when a solute reacts with the porous media and its rate of movement is returned relative to the advective groundwater velocity. And retardation can occur by a variety of processes including adsorption and precipitation. So, retardation rates of contaminants are likely variable and typically range from 0 to 10 times slower than the advective groundwater velocity.

So, these are some typical velocity that sometimes it may be 0 and sometimes it may be up to 10. And it has been said that some solvents like say TCE and PCE which are trichloroethylene and perchloroethylene, they can solve onto particles of the organic carbon present at the aquifer matrix. So, that is the reason why the flow of the pollutants will be lower the velocity of the pollutants, may be lower than the advective velocity of the water and deterioration can occur. Then how to quantify this retardation?

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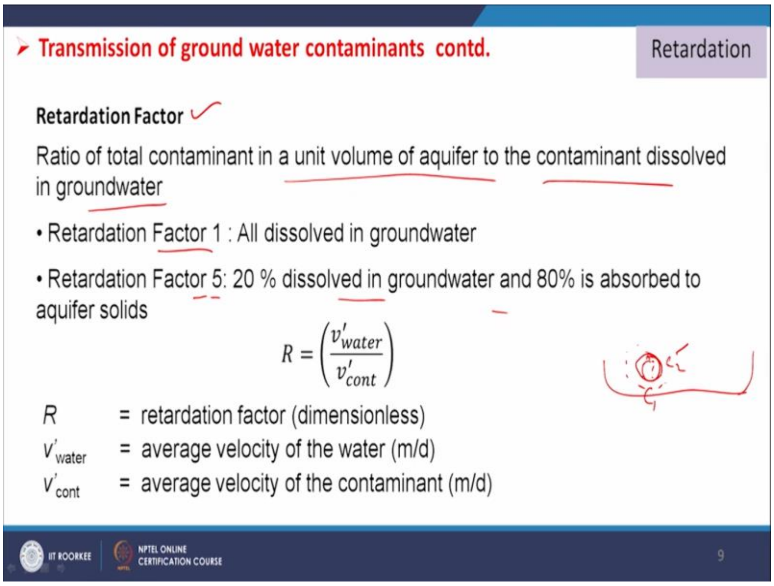

➢ Transmission of ground water contaminants contd.

Retardation

Retardation Factor

Ratio of total contaminant in a unit volume of aquifer to the contaminant dissolved in groundwater

- Retardation Factor 1 : All dissolved in groundwater
- Retardation Factor 5: 20 % dissolved in groundwater and 80% is absorbed to aquifer solids

$$R = \left(\frac{v'_{water}}{v'_{cont}}\right)$$

R = retardation factor (dimensionless)
V'_{water} = average velocity of the water (m/d)
V'_{cont} = average velocity of the contaminant (m/d)

We have used one factor that is called retardation factor. So, what is that? The ratio of total contaminant in a unit volume of aquifer to the contaminant dissolved in ground water. So, you have some aquifer. So, per unit volume we take. Then what is the concentration and what is the concentration in the whole water.

So, dissolve material concentration in the water c_2 and c_1 is the concentration of the pollutants in the unit volume of the aquifers. So, here water and materials will also be available. So, if it is 1, the ratio is 1 that means all the contaminants are dissolved in the water.

But if it is 5 that means all pollutants or contaminants are not dissolved in the water, some are associated with the materials, aquifer materials. So, if it is 5 then 20% 20 is dissolved in the ground water and 80% of the pollutants are dissolved in this material. So, this is the concept of retardation factor and more the value of retardation factor lesser the solubility of the pollutants in the water. So, the availability of the water in the downstream will be lower. And in this the relation

$$R = \left(\frac{v'water}{v'contamination}\right)$$

that is equal to velocity of water and then velocity of contaminant. So, that way also we can define. That means certainly the if it is captured by the materials, this velocity will be reduced. So, that velocity of water by velocity of contaminants this ratio also give us the R-value. Now, we will discuss the transmission of surface water contaminants.

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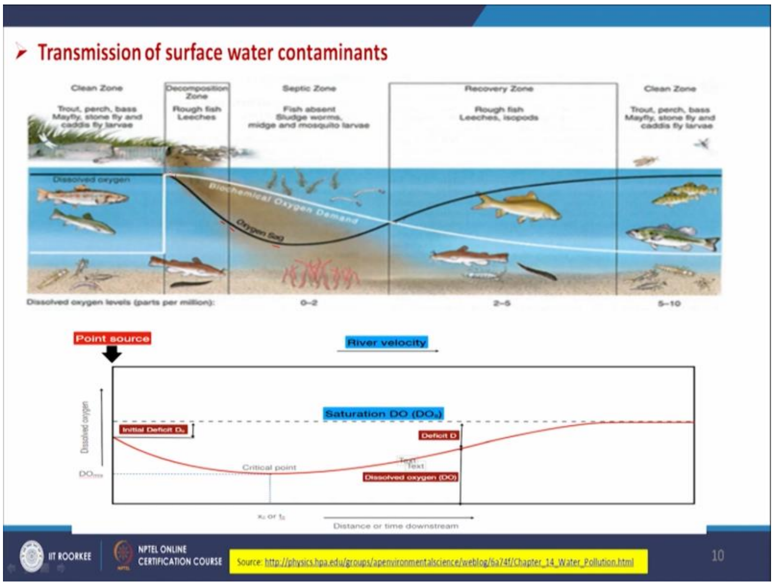

Transmission of surface water contaminants

Source: http://physics.hpa.edu/groups/apenvironmentalscience/weblog/6a74f/Chapter_14_Water_Pollution.html

So, in the previous class, we have seen that when the pollutants get entry into the surface water through some drain say the pollutants are getting into the river then immediately here the pollutants BOD values increases and microbes start to work on it and DO concentration reduces and it reaches its minimum DO values and again the DO value increases because there are two processes re-aeration and deoxygenation.

So, re-aeration from air oxygen enters into the water and deoxygenation means the oxygen is consumed by the microorganism to degrade the organic compounds. So, in the water two opposite phenomena goes on and gradually the requirement of oxygen or consumption of oxygen by the microorganisms reduced. And ultimately but the reaeration rate remains constant.

So, gradually ultimately reaches to the saturated DO level. So, that we have already discussed. Now, we will be discussing how can we predict the DO level at any location downstream of this entry of the pollutants in the river stream. How these pollutants are distributed or how these pollutants are transmitted?

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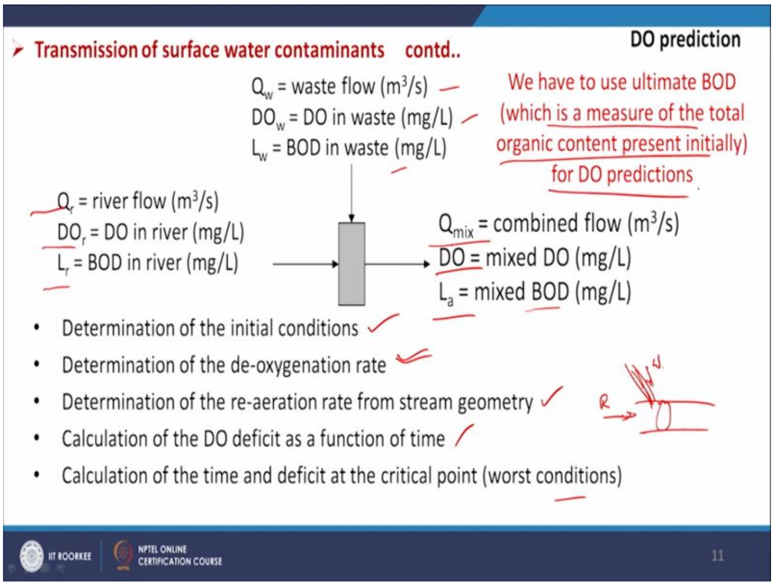

Transmission of surface water contaminants contd..

DO prediction

Q_w = waste flow (m^3/s)
DO_w = DO in waste (mg/L)
L_w = BOD in waste (mg/L)

We have to use ultimate BOD (which is a measure of the total organic content present initially) for DO predictions

Q_r = river flow (m^3/s)
DO_r = DO in river (mg/L)
L_r = BOD in river (mg/L)

Q_{mix} = combined flow (m^3/s)
DO = mixed DO (mg/L)
L_a = mixed BOD (mg/L)

- Determination of the initial conditions
- Determination of the de-oxygenation rate
- Determination of the re-aeration rate from stream geometry
- Calculation of the DO deficit as a function of time
- Calculation of the time and deficit at the critical point (worst conditions)

Now, we see, we had a river. So, another point, from discharge point from municipal sources, it is coming here say, and we have a river one. This is our waste stream and this is our reverse stream. So, after that immediately it is mixing, so, our initial concentration we are getting this, after mixing. Here the microbial activity will start and gradually those will be the pollutants will be converted.

So, here different types of phenomena are going on. Organic compounds are being degraded by the microorganisms. So, concentration will vary may be precipitation etc. So, that way we will see how this phenomenon is going on. Now, say two stream we have, one is your waste stream another is your river stream. So, this river stream is having a Qr flow rate and then DOr, dissolve oxygen in the river stream and Lr is the BOD in the river stream, river stream and then Qw is the waste flow and DOw is the DO in waste and Lw is the BOD in waste.

So, after this, when it is mixing here, so, ultimately what we are getting? Qmix. What is Qmix here? Qmix = Qw+Qr and DO_{mixed} and La_{mixed} BOD We can calculate this by mass balance. So, we can predict the DO at any downstream location or at any time. So, for that we need to determine the initial conditions. These are the initial conditions. And then determinations of the deoxygenation rate.

As we know there are two major phenomena, one is deoxygenation another re-aeration. So, we have to measure both rates. So, determination of re-aeration rate from the stream geometry. So, river stream geometry will help us to get the value of the re-arration constant. And by doing some BOD test we can get the value of deoxygenation rate. And calculation of the DO deficit as a function of time that we have to do.

As a function of time what will be the deficit, DO deficit and calculation of the time and deficit at the critical point worst conditions. So, when the DO will be the minimum. So, that way we can calculate. Now, we have to use ultimate BOD that which is measure of the total organic content present initially for DO predictions.

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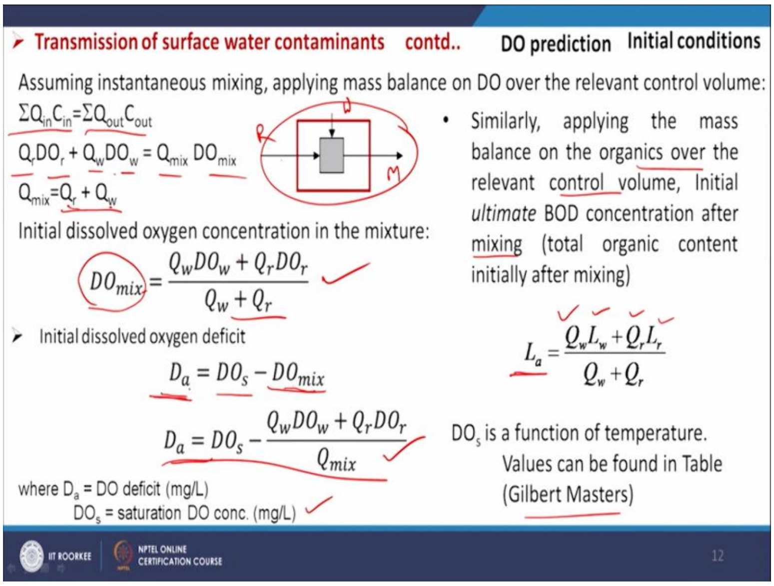

Transmission of surface water contaminants contd.. **DO prediction** **Initial conditions**

Assuming instantaneous mixing, applying mass balance on DO over the relevant control volume:

$\Sigma Q_{in} C_{in} = \Sigma Q_{out} C_{out}$

$Q_r DO_r + Q_w DO_w = Q_{mix}\, DO_{mix}$

$Q_{mix} = Q_r + Q_w$

Initial dissolved oxygen concentration in the mixture:

$$\boxed{DO_{mix}} = \frac{Q_w DO_w + Q_r DO_r}{Q_w + Q_r} \checkmark$$

- Initial dissolved oxygen deficit

$$D_a = DO_s - DO_{mix}$$

$$D_a = DO_s - \frac{Q_w DO_w + Q_r DO_r}{Q_{mix}} \checkmark$$

where D_a = DO deficit (mg/L)
DO_s = saturation DO conc. (mg/L) ✓

- Similarly, applying the mass balance on the organics over the relevant control volume, Initial *ultimate* BOD concentration after mixing (total organic content initially after mixing)

$$L_a = \frac{Q_w L_w + Q_r L_r}{Q_w + Q_r}$$

DO_s is a function of temperature. Values can be found in Table (Gilbert Masters)

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Now, you see the same figure shown here. So, this is waste, this is river and this is mixed. So, what will be the mass balance? The sum of the inputs is equal to some of the output since there is no reactions and no loss. So, this is the condition. So, what we are getting? Qr DOr with respect to dissolve oxygen balance.

So, this is for this stream and this is Qw DOw, it is equal to Qmix DOmix. So,

Qmix = Qr+Qw. So, what will be the DOmix?

DOmix = (Qr DOr + Qw Dow)/Qmix

and Qmix is nothing but Qw + Qr. So, put it here. So, we are getting the DOmix equal to this expression.

Now, this DOmix, we can represent it in terms of deficit. So, what is that?

$$\mathrm{D_a} = \mathrm{DO_s} - \mathrm{DO_{mix}}$$

$$D_a = DO_s - \frac{Q_w DO_w + Q_r DO_r}{Q_{mix}}$$

So, this expression we are getting here as Da. So, DOs we know that saturated value and it will depend upon the temperature and conditions. So, that we can get from any reference. So, like say Gilbert Masters book, we can refer to get the value of this Da. Normally, it is say around 9 ppm. So, at normal condition.

And similarly, applying the mass balance on the organics over the relevant control volume, initial ultimate BOD concentration after mixing that is La we can calculate.

La*(Qw + Qr) = (Qw Lw + Qr Lr). So, that way you can get the value of La and also you can get value of Da, initial value determination is done.

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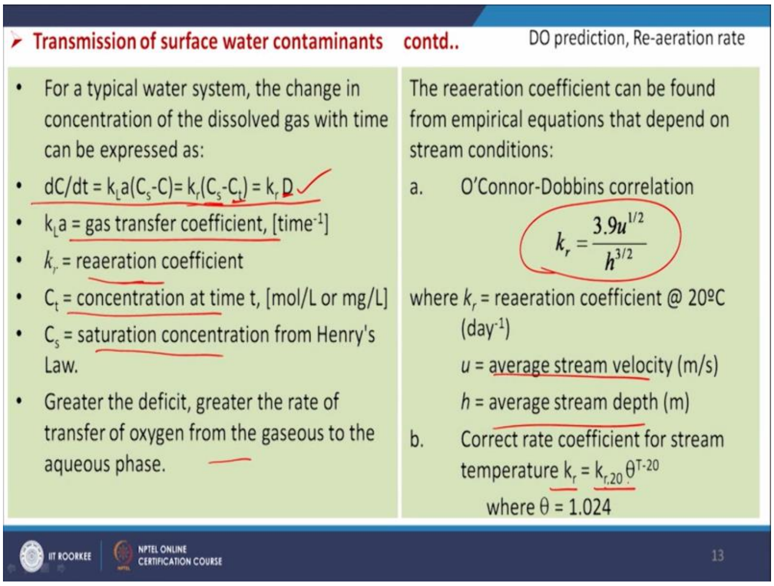

Transmission of surface water contaminants contd.. DO prediction, Re-aeration rate

- For a typical water system, the change in concentration of the dissolved gas with time can be expressed as:
- $dC/dt = k_La(C_s-C)= k_r(C_s-C_t) = k_r D$
- k_La = gas transfer coefficient, [$time^{-1}$]
- k_r = reaeration coefficient
- C_t = concentration at time t, [mol/L or mg/L]
- C_s = saturation concentration from Henry's Law.
- Greater the deficit, greater the rate of transfer of oxygen from the gaseous to the aqueous phase.

The reaeration coefficient can be found from empirical equations that depend on stream conditions:

a. O'Connor-Dobbins correlation

$$k_r = \frac{3.9u^{1/2}}{h^{3/2}}$$

where k_r = reaeration coefficient @ 20ºC (day^{-1})

u = average stream velocity (m/s)

h = average stream depth (m)

b. Correct rate coefficient for stream temperature $k_r = k_{r,20}\theta^{T-20}$

where $\theta = 1.024$

Now, we have to determine the value of re-aeration rate and deoxygenation rate. Now, re-aeration rate always, oxygen is coming from the atmosphere to the water. So, in that case, the governing expression is

$dC/dt = k_La *(Cs – C) = k_r *(Cs – C_L) = k_rD$

where k_La is the gas transfer coefficient, oxygen is being transferred from atmosphere to water, because concentration of oxygen is more in case of atmosphere.

dC/dt = kr D. So, greater the deficit, greater the rate of transfer of oxygen from the gaseous to the aqueous phase. Now, what will be the value of kr? kr value depends upon the geometry of the streams. If we know the velocity that is average stream velocity if we know in meter per second and if we know the average stream depth in meter then this formula can be used that is your O'connor-Dobbins correlation.

$$k_r = \frac{3.9u^{1/2}}{h^{3/2}}$$

So, by that way we can calculate the value of kr. And temperature may vary. So, kr correction is needed. So, kr = $k_{r\,20}\,\Theta^{T-20}$

When k_{r20} is the re-aeration constant at 20 ^{o}C and ϴ theta value is equal to 1.024.

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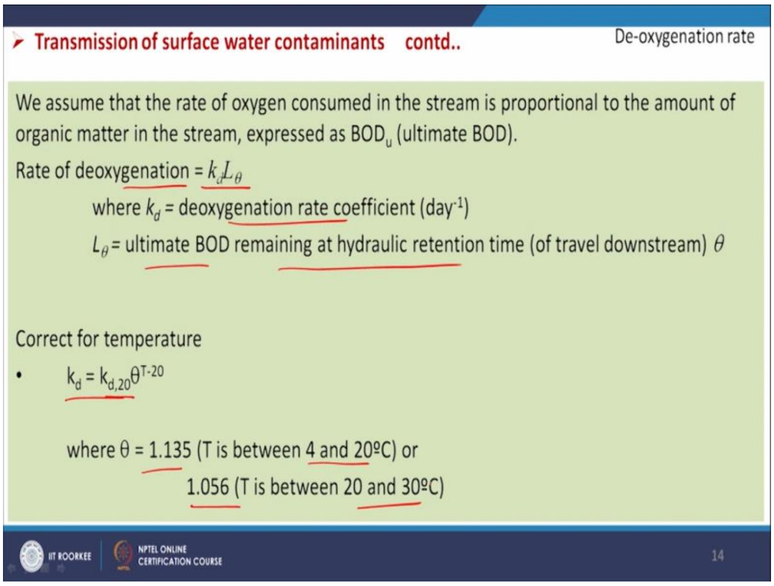

Transmission of surface water contaminants contd..

De-oxygenation rate

We assume that the rate of oxygen consumed in the stream is proportional to the amount of organic matter in the stream, expressed as BOD_u (ultimate BOD).

Rate of deoxygenation = $k_d L_\theta$

where k_d = deoxygenation rate coefficient (day^{-1})

L_θ = ultimate BOD remaining at hydraulic retention time (of travel downstream) θ

Correct for temperature

- $k_d = k_{d,20} \theta^{T-20}$

where θ = 1.135 (T is between 4 and 20ºC) or

1.056 (T is between 20 and 30ºC)

Then deoxygenation rate. So, deoxygenation rate will be proportional to the BOD available. More the BOD available or more organics available more will be the degradation. So, rate of deoxygenation is $k_d * L_\Theta$. Where, L_Θ is the ultimate BOD remaining at hydraulic retention time theta and kd is the deoxygenation rate coefficient per day.

And here also this kd value we can make some correction for temperature. So,

$k_d = k_{d\,20}\, \Theta^{T-20}$.

So, here kd 20 is the value of deoxygenation rate constant at 20 °C. And Ө value is 1.135 when the T is between 4 and 20 °C and 1.056 when T is between 20 and 30 °C. So, these are people have already worked on it and they have given this recommendations and this is mostly used for the calculation purpose.

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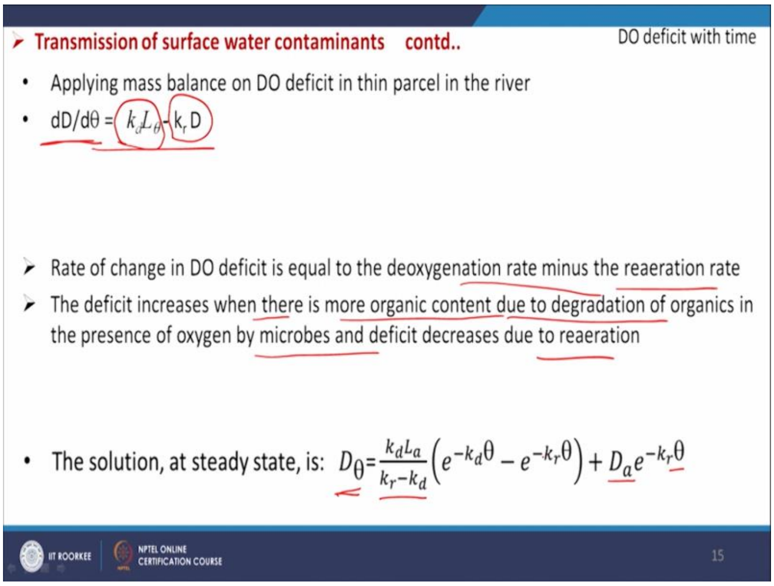

➤ **Transmission of surface water contaminants contd..**

DO deficit with time

- Applying mass balance on DO deficit in thin parcel in the river
- $dD/d\theta = k_d L_\theta - k_r D$

➤ Rate of change in DO deficit is equal to the deoxygenation rate minus the reaeration rate

➤ The deficit increases when there is more organic content due to degradation of organics in the presence of oxygen by microbes and deficit decreases due to reaeration

- The solution, at steady state, is: $D_\theta = \frac{k_d L_a}{k_r - k_d}\left(e^{-k_d\theta} - e^{-k_r\theta}\right) + D_a e^{-k_r\theta}$

And then if we do the mass balance now, we have dD/dϴ . How the deficit is changing? So, one method that is your deoxygenation, another method is re-aeration. So, deoxygenation is reducing the concentration and reaeration is increasing the concentration of oxygen in the water. So, the difference, basically, will be dD/dϴ.

$$\frac{dD}{d\theta} = k_d L_\theta - k_r D$$

When we have more value of deoxygenation then we will be our D, deficit will be increasing and then when this value is less then deficit will be decreased and ultimately saturation value can be achieved. So, this is the expression, governing expression which can be used to calculate the deficit.

Now, rate of change in DO deficit is equal to the deoxygenation rate minus the reaction rate, I have just discussed and the deficit increases when there is more organic content due to degradation of organics in the presence of oxygen by microbes and deficit decreases due to reaeration. Just we have discussed.

And if we get the solution of this, the solution will be like this. Dϴ, at time ϴ, the deficit

$$D\theta = \frac{k_d L_a}{k_r - k_d}\left(e^{-kd\theta} - e^{-kr\theta}\right) + D_a e^{-kr\theta}$$

So, all these terms we have already defined. So, knowing the values of this term, we can get the value of Dϴ.

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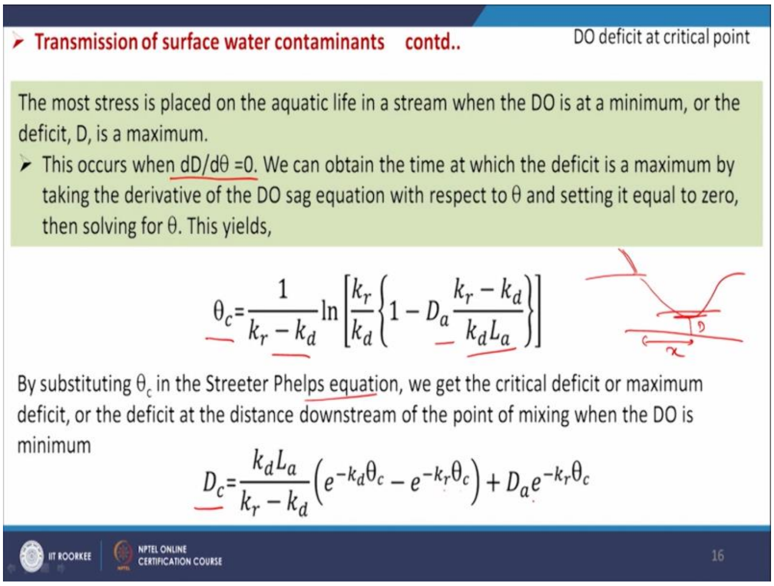

➤ Transmission of surface water contaminants contd..

DO deficit at critical point

The most stress is placed on the aquatic life in a stream when the DO is at a minimum, or the deficit, D, is a maximum.

➤ This occurs when $dD/d\theta = 0$. We can obtain the time at which the deficit is a maximum by taking the derivative of the DO sag equation with respect to θ and setting it equal to zero, then solving for θ. This yields,

$$\theta_c = \frac{1}{k_r - k_d} \ln\left[\frac{k_r}{k_d}\left\{1 - D_a \frac{k_r - k_d}{k_d L_a}\right\}\right]$$

By substituting θ_c in the Streeter Phelps equation, we get the critical deficit or maximum deficit, or the deficit at the distance downstream of the point of mixing when the DO is minimum

$$D_c = \frac{k_d L_a}{k_r - k_d}\left(e^{-k_d \theta_c} - e^{-k_r \theta_c}\right) + D_a e^{-k_r \theta_c}$$

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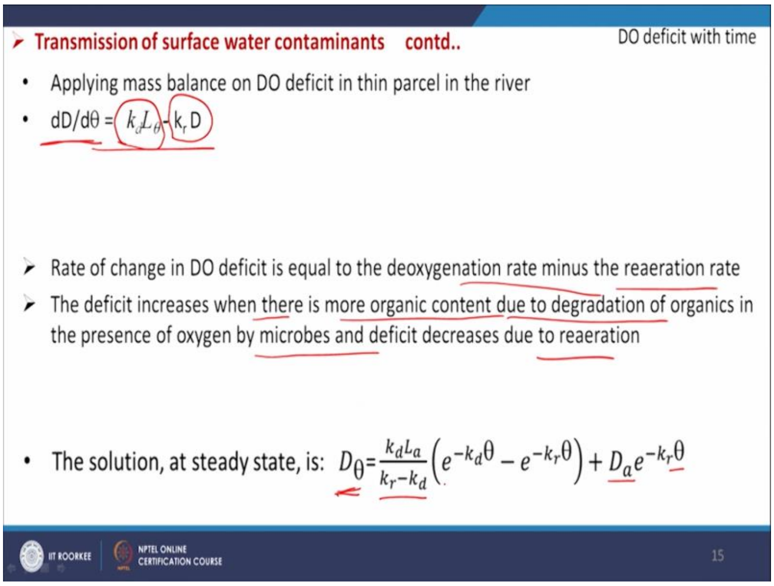

➤ Transmission of surface water contaminants contd..

DO deficit with time

- Applying mass balance on DO deficit in thin parcel in the river
- $dD/d\theta = k_d L_\theta - k_r D$

➤ Rate of change in DO deficit is equal to the deoxygenation rate minus the reaeration rate

➤ The deficit increases when there is more organic content due to degradation of organics in the presence of oxygen by microbes and deficit decreases due to reaeration

- The solution, at steady state, is: $D_\theta = \frac{k_d L_a}{k_r - k_d}\left(e^{-k_d \theta} - e^{-k_r \theta}\right) + D_a e^{-k_r \theta}$

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Now, we need to calculate the critical value. Because we have seen in case of river, so, when the stream is coming and river is flowing, so, in that case the DO reaches maximum and then again attains the saturated value. So, this point at the minimum point. So, what is the this minimum point? At what distance this will happen? And what will be the D value at this point?

So, that is our point of concern now. So, if we want to get, we have one mathematical expression with time and D in the water. Now, if you going to get the this point then dD/dϴ should be 0. So, by that we will be simplifying, we will be getting the derivation, dD/dϴ and then we will put it equal to 0 and the corresponding theta value is ϴc and ϴc will be

$$\theta_c = \frac{1}{k_r - k_d} \ln\left[\frac{k_r}{k_d}\left\{1 - D_a \frac{k_r - k_d}{k_d L_a}\right\}\right]$$

So, this is the expressions, we are getting for ϴc for the critical time which will be required to reach the DO value at the minimum position or minimum value. So, by substituting ϴc in the Streeter Phelps equation, then the D which will get that will be the Dc that is the critical distance. So, what will be the distance x where the D will be the, deficit will be the maximum. So, that we can calculate. So,

$$D_c = \frac{k_d L_a}{k_r - k_d}\left(e^{-k_d \theta_c} - e^{-k_r \theta_c}\right) + D_a e^{-k_r \theta_c}$$

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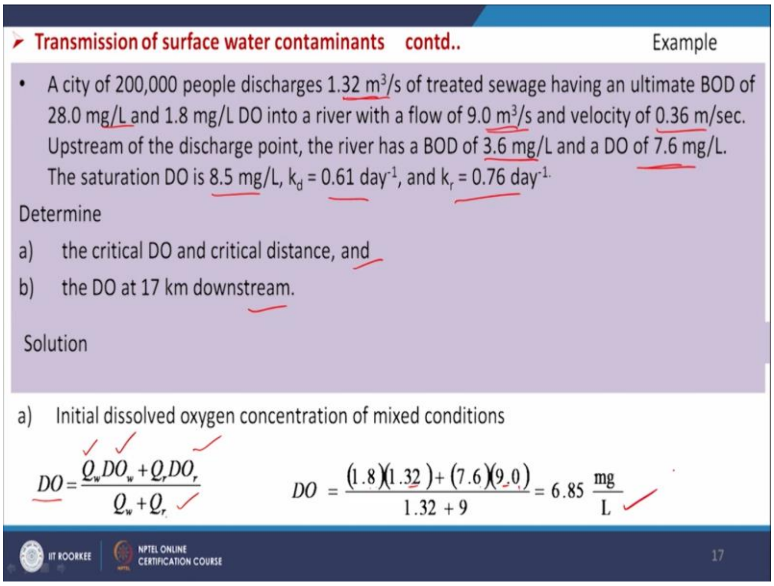

Transmission of surface water contaminants contd..

Example

- A city of 200,000 people discharges 1.32 m^3/s of treated sewage having an ultimate BOD of 28.0 mg/L and 1.8 mg/L DO into a river with a flow of 9.0 m^3/s and velocity of 0.36 m/sec. Upstream of the discharge point, the river has a BOD of 3.6 mg/L and a DO of 7.6 mg/L. The saturation DO is 8.5 mg/L, $k_d = 0.61\ day^{-1}$, and $k_r = 0.76\ day^{-1}$.

Determine

a) the critical DO and critical distance, and

b) the DO at 17 km downstream.

Solution

a) Initial dissolved oxygen concentration of mixed conditions

$$DO = \frac{Q_w DO_w + Q_r DO_r}{Q_w + Q_r} \qquad DO = \frac{(1.8)(1.32) + (7.6)(9.0)}{1.32 + 9} = 6.85\ \frac{mg}{L}$$

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So, that way you can calculate the value of theta c and Dc. Now, one numerical problem we have. A city of two lakhs people discharges 1.32 m^3/s of treated sewage having an ultimate BOD of 28 mg/Land 1.8 mg/L DO into a river with a flow of 9 m^3/s and velocity of 0.36 m/s.

Upstream of the discharge point, the river has a BOD of 3.6 mg/L and DO of 7.6 mg/L. The saturation DO is 8.5mg/l, kd is 0.61 per day and kr 0.76 per day. Determine the critical DO and critical distance and the DO at 17 km downstream. So, we have to solve this problem.

Now, what we need to do? We have to calculate after mixing what will be the initial DO or initial deficit. So, that we have to calculate. So, DO after mixing, we have calculated, we have seen in the previous slide that

$$DO = \frac{Q_w DO_w + Q_r DO_r}{Q_w + Q_r}$$

$$DO = \frac{(1.8)(1.32) + (7.6)(9.0)}{1.32 + 9}$$

DO = 6.85 mg/L

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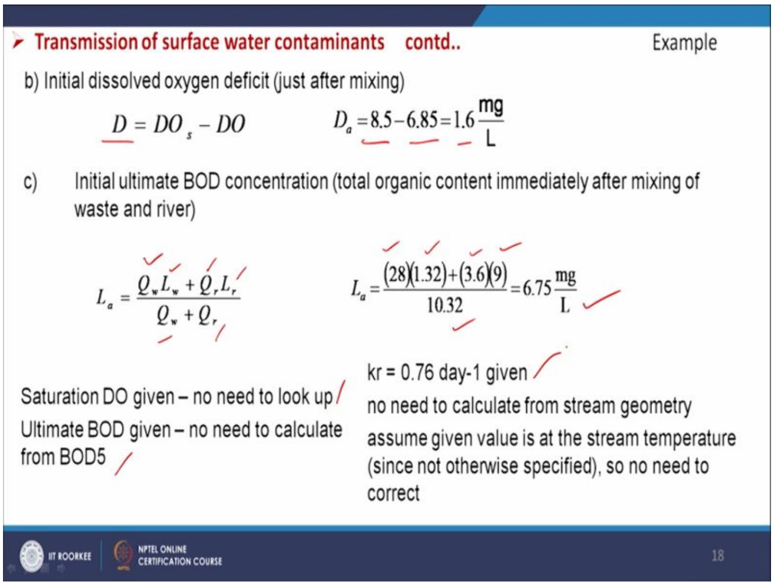

Transmission of surface water contaminants contd..

Example

b) Initial dissolved oxygen deficit (just after mixing)

$$D = DO_s - DO \qquad D_a = 8.5 - 6.85 = 1.6\,\frac{\text{mg}}{\text{L}}$$

c) Initial ultimate BOD concentration (total organic content immediately after mixing of waste and river)

$$L_a = \frac{Q_w L_w + Q_r L_r}{Q_w + Q_r} \qquad L_a = \frac{(28)(1.32) + (3.6)(9)}{10.32} = 6.75\,\frac{\text{mg}}{\text{L}}$$

Saturation DO given – no need to look up

Ultimate BOD given – no need to calculate from BOD5

kr = 0.76 day-1 given

no need to calculate from stream geometry

assume given value is at the stream temperature (since not otherwise specified), so no need to correct

So, deficit will be

$$D = DO_s - DO$$

$$D_a = 8.5 - 6.5 = 1.6\ \text{mg/L}$$

So, we are getting the initial DO value. So, initial ultimate BOD, how much here?

$$L_a = \frac{Q_w L_w + Q_r L_r}{Q_w + Q_r}$$

$$L_a = \frac{(2.8)(1.32) + (3.6)(9.0)}{1.32 + 9}$$

La = 6.75 mg/L

So, this is our initial ultimate BOD after mixing. So, saturation DO is given. So, no need to look up. and ultimate BOD given, no need to calculate from BOD_5. And kr value is given. We do not need to calculate it from the geometry of the stream.

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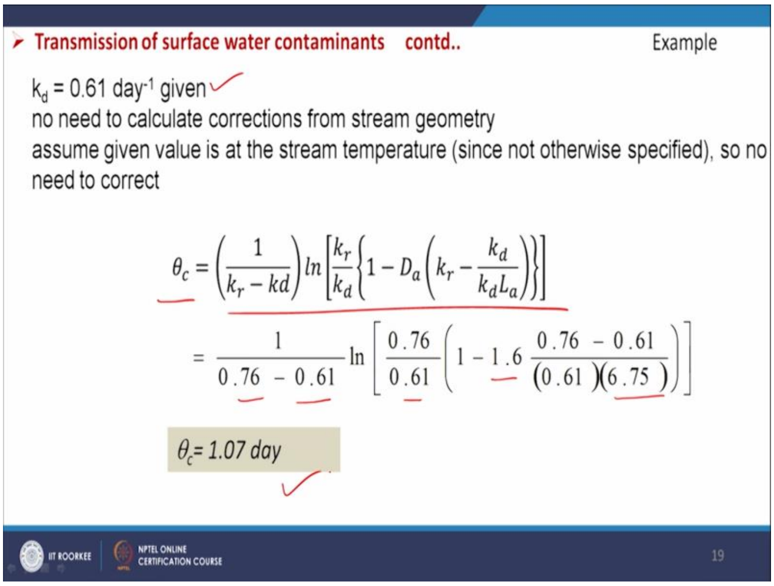

Transmission of surface water contaminants contd..

Example

$k_d = 0.61\ day^{-1}$ given

no need to calculate corrections from stream geometry

assume given value is at the stream temperature (since not otherwise specified), so no need to correct

$$\theta_c = \left(\frac{1}{k_r - kd}\right) ln\left[\frac{k_r}{k_d}\left\{1 - D_a\left(k_r - \frac{k_d}{k_d L_a}\right)\right\}\right]$$

$$= \frac{1}{0.76 - 0.61} \ln\left[\frac{0.76}{0.61}\left(1 - 1.6\frac{0.76 - 0.61}{(0.61)(6.75)}\right)\right]$$

$\theta_c = 1.07$ day

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And k_d value is also given. So, no need to calculate corrections from stream geometry. And we also do not need to do the temperature correction, I think. So, you also do not need to do any further correction. So, what will get Ɵc directly we will put the values in this expression which we have discussed.

$$\theta_c = \frac{1}{k_r - k_d} \ln\left[\frac{k_r}{k_d}\left\{1 - D_a\frac{k_r - k_d}{k_d L_a}\right\}\right]$$

$$\theta_c = \frac{1}{0.76 - 0.61} \ln\left[\frac{0.76}{0.61}\left\{1 - 1.6\frac{0.76 - 0.61}{(0.61)(6.75)}\right\}\right] = 1.07\ \text{day}$$

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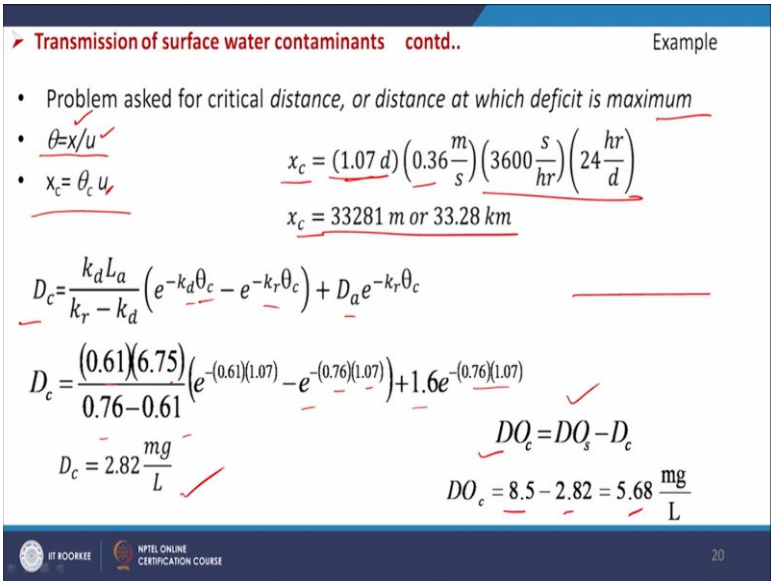

Transmission of surface water contaminants contd..

Example

- Problem asked for critical *distance, or distance at which deficit is maximum*
- $\theta = x/u$
- $x_c = \theta_c u$

$$x_c = (1.07\ d)\left(0.36\frac{m}{s}\right)\left(3600\frac{s}{hr}\right)\left(24\frac{hr}{d}\right)$$

$$x_c = 33281\ m\ or\ 33.28\ km$$

$$D_c = \frac{k_d L_a}{k_r - k_d}\left(e^{-k_d\theta_c} - e^{-k_r\theta_c}\right) + D_a e^{-k_r\theta_c}$$

$$D_c = \frac{(0.61)(6.75)}{0.76 - 0.61}\left(e^{-(0.61)(1.07)} - e^{-(0.76)(1.07)}\right) + 1.6e^{-(0.76)(1.07)}$$

$$D_c = 2.82\frac{mg}{L}$$

$$DO_c = DO_s - D_c$$

$$DO_c = 8.5 - 2.82 = 5.68\ \frac{\text{mg}}{\text{L}}$$

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So, problem ask for critical distance or distance at which deficit is maximum. So, deficit is maximum. So, then we will be getting when DO is minimum. So, residence time

$\Theta = x/u$

x is the distance, u is the velocity of the water. So, at critical condition x_c

$x_c = \Theta c * u$

And xc is equal to what?

It is already given that we have seen the critical point comes at Ѳc is 1.07 day. So, at 1.07 day, how long the water will flow, that we have to calculate. So, here this is our time 1.07 day, and this is our speed we have to put. So, 0.36 m/s this is our velocity.

$$x_c = (1.07\ \mathrm{d})\left(0.36\frac{\mathrm{m}}{\mathrm{s}}\right)\left(3600\frac{\mathrm{s}}{\mathrm{hr}}\right)\left(24\frac{\mathrm{hr}}{\mathrm{d}}\right)$$

$$x_c = 33281\ \mathrm{m\ or}\ 33.28\ \mathrm{km}$$

Then what will be the Dc now? We have got the xc, we have got the Ѳc.

$$D_c = \frac{k_d L_a}{k_r - k_d}\left(e^{-k_d \theta_c} - e^{-k_r \theta_c}\right) + D_a e^{-k_r \theta_c}$$

$$D_c = \frac{(0.61)(6.75)}{0.76 - 0.61}\left(e^{-(0.61)(1.07)} - e^{-(0.76)(1.07)}\right) + 1.6e^{-(0.76)(1.07)}$$

Dc = 2.82 mg/L

So, this is our critical deficit. So, what will be the critical DO concentration

$$DO_c = DO_s - D_c$$

DOc = 8.5 -2.82 = 5.68 mg/L

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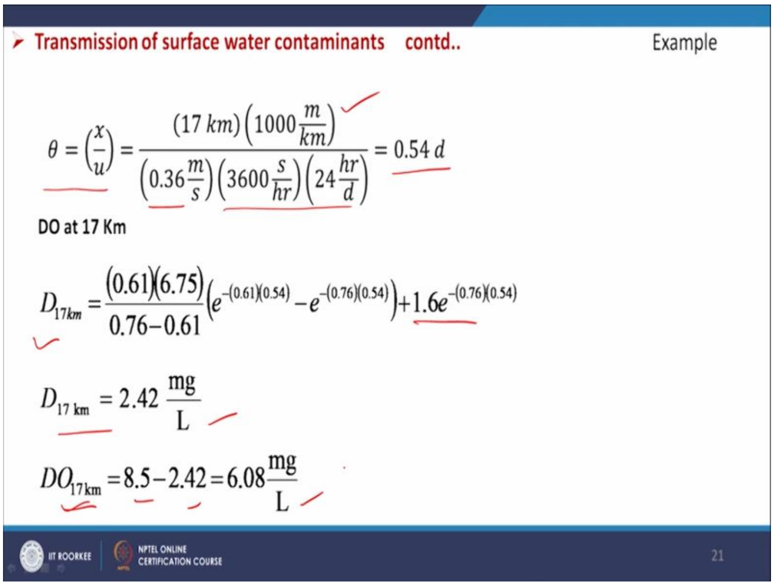

Transmission of surface water contaminants contd.. Example

$$\theta = \left(\frac{x}{u}\right) = \frac{(17\,km)\left(1000\frac{m}{km}\right)}{\left(0.36\frac{m}{s}\right)\left(3600\frac{s}{hr}\right)\left(24\frac{hr}{d}\right)} = 0.54\,d$$

DO at 17 Km

$$D_{17km} = \frac{(0.61)(6.75)}{0.76-0.61}\left(e^{-(0.61)(0.54)} - e^{-(0.76)(0.54)}\right) + 1.6e^{-(0.76)(0.54)}$$

$$D_{17\,km} = 2.42\,\frac{mg}{L}$$

$$DO_{17km} = 8.5 - 2.42 = 6.08\frac{mg}{L}$$

Then

$$\theta = \left(\frac{x}{u}\right) = \frac{(17\,\mathrm{km})\left(1000\frac{\mathrm{m}}{\mathrm{km}}\right)}{\left(0.36\frac{\mathrm{m}}{\mathrm{s}}\right)\left(3600\frac{\mathrm{s}}{\mathrm{hr}}\right)\left(24\frac{\mathrm{hr}}{\mathrm{d}}\right)} = 0.54\ \mathrm{d}$$

DO at 17 km.

$$D_{17} = \frac{(0.61)(6.75)}{0.76 - 0.61}\left(e^{-(0.61)(1.07)} - e^{-(0.76)(1.07)}\right) + 1.6e^{-(0.76)(1.07)}$$

D_{17} = 2.42 mg/L

D_{17} = 8.5 - 2.42 = 6.08 mg/L

After this in this class, thank you very much for your patience.