

**Physico-Chemical Processes for Wastewater Treatment**  
**Professor V.C. Srivastava**  
**Department of Chemical Engineering**  
**Indian Institute of Technology, Roorkee**  
**Lecture - 29**  
**Filtration – II**

Good day everyone and welcome to these lectures on Physico-Chemical Treatment of Waste Waters. So, in this lecture series, we are understanding the treatment of wastewater by various unit operations which are used in the, any treatment plant. So, previously we have already studied in detail, the flow localization basin, the aeration basins and thereafter, we studied the coagulation and flocculation and then further on sedimentation.

So, now, we are continuing with the filtration unit operation, and in the previous lectures we studied regarding the filtration, including the constant pressure filtration, constant volume filtration, we studied some of the little bit mechanisms of the filtration, but now, we will be continuing and we will try to understand the basic theories and the equations which have been developed and which are used in the filtration.

So, we are going to study these things in detail in today's lecture. And, and also we will try to solve and use these equations to get some of the parameters for design with respect to filtration. So, the basic filtration during any filtration what happens that there is a cake formation. So, we know that there is a filter media and after some time when the filtration goes on, there is a cake which gets formed on the filter medium and many times we use the sand gravels, etc in a bed and these sand gravels, etc, when they are actually in the bed they form a porosity and so, that is a porous bed.

So, when they filtration happens in the porous bed, we have cake formation which happens and the flow through the filtrate, the flow of the filtrate through this cake can be considered as similar to the fluid flow through a packed bed of granular solids. So, this is there and this is the basic idea which is adopted in the filtration.

So, there are different equations which have been developed to determine the pressure drop across this cake and those equations can be used for design things as well. And so, these, one of these equations is called Kozeny's equation and this equation has been developed based on the assumption of capillary bundle theory, so, what is that?

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### Filtration Equation

- During filtration, the cake forms a porous bed over the filter medium, the flow of filtrate through the accumulated cake is analogous to fluid flow through a packed bed of granular solid.
- Kozeny's equation is used to determine the pressure drop across this cake.
- It has been developed based on the "capillary bundle theory".
- We assume that the total void space of the packed bed is equivalent to a bundle of capillary tubes.
- Hagen-Poiseuille's equation for laminar flow through straight conduits can be used here after replacing the radius of the tube by the hydraulic radius  $R_H$ .

Narayanan and Bhattacharyya, 2014 2

Here in this theory, we assume that the total void space of the packed bed is equivalent to a bundle of capillary tubes. So, this is there and so, this is equivalent to a bundle of capillary tube and then Hagen-Poiseuille's type of equation for laminar flow through state conduits can be altered by replacing the radius of the tube with hydraulic radius, and using this we can derive the basic equation and then further modify it and use it as per our necessity.

So, we will try to understand that how the basic filtration equation is obtained. And the basic theory of this whole thing is related to flow through a packed bed, porous packed bed. So, fluid flow through a porous packed bed, that theory is further developed for filtration.

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### Cont....

- The velocity of fluid  $U'$  through the bed can be written as:

$$U' = \frac{(-\Delta P)}{K \mu_f L} R_H^2 \quad \checkmark$$

where,  
K = Kozeny's constant for random packed particles of definite size and shape  
= 25/6 ✓  
 $U'$  = Superficial velocity of fluid  
 $(-\Delta P)$  = Pressure drop across the bed  
L = Height (or thickness) of the bed  
 $R_H$  = Hydraulic radius = (flow area/wetted perimeter) ✓

Narayanan and Bhattacharyya, 2014 3

The velocity of fluid  $U'$  through the bed can be written as:

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where,

$K$  = Kozeny's constant for random packed particles of definite size and shape = 25/6

$U'$  = Superficial velocity of fluid

$(-\Delta P)$  = Pressure drop across the bed

$L$  = Height (or thickness) of the bed

$R_H$  = Hydraulic radius = (flow area/wetted perimeter)

Now, using this theory, we can easily get this particular equation from the literature, and in this equation the superficial velocity of the fluid through the packed bed can be written by this equation. And in this equation  $U$  dash is equal to minus delta  $P$  by  $L$  and into  $R_H$  square divided by  $K \mu_f$  and where  $K$  is the Kozeny's constant and it is for random packed particles of definite size or shape something, it can be taken as 25 by 6. So, remember this.

Similarly, the  $U$  dash is the superficial velocity, delta  $P$  is the pressure drop across the bed and  $L$  is the height or thickness of the bed and  $R_H$  is the hydraulic radius that already we have studied in the previous lectures. It is defined as the ratio of flow area per unit wetted perimeter. So, this already we have used earlier as well. So, this is well known. Now,  $R_H$  again using for the same equation same definition has been further been used here for dividing the equation with respect to packed bed.

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### Cont....

It is assumed that, all the voids are completely filled with the liquid, and all particles have been uniformly wet with the liquid,

$$R_H = \frac{\text{Void Volume}}{\text{Total surface area of particles}}$$

$$R_H = \frac{\varepsilon}{S_p(1-\varepsilon)}$$

Where,  $\varepsilon$  = The void fraction of the bed

$\varepsilon$  = Void volume /total volume

$S_p$  = Specific surface of the particle.

Hence, the equation becomes:

$$U' = \frac{(-\Delta P)}{K\mu_t L} \frac{\varepsilon^2}{S_p^2(1-\varepsilon)^2}$$



$$R_H = \frac{\varepsilon}{S_p(1-\varepsilon)}$$

Where,  $\varepsilon$  = The void fraction of the bed

$\varepsilon$  = Void volume /total volume

$S_p$  = Specific surface of the particle.

Now, it is assumed that all the voids are completely filled with liquid. So, now, in the bed if we assume that all the voids so, we have like this packed bed is there, and in between, so, this is totally filled this packed bed is totally filled. So, all the void space like this one, so, any void space which is there, this is totally filled with liquid and all the particles have been uniformly wetted with the liquid. So, under that condition the hydraulic radius is like can be defined as void volume divided by total surface area of the particle.

Now, if we assume this we can further get this particular equation, where  $S_p$  is the specific surface area of the particle,  $\varepsilon$  Epsilon is the void fraction of the bed or the porosity or it can also be divided as void volume divided by total volume.

So, this we can find out using the trick that we did in the previous lecture itself we tried to find out the porosity. So, we understood already that. So, if we replaced this  $R_H$  in the previous equation with respect to superficial velocity, so, this will be the equation that we will be getting. So, we can see here only we have added the  $R_H$  term. So, this is there.

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### Cont....

The superficial velocity of the liquid  $U_{\text{sup}}$  is defined as the volumetric flow rate of the liquid divided by the total cross-sectional area and can be related to  $U'$  as:

$$\frac{U_{\text{sup}}}{U'} = \frac{\text{Void area}}{\text{Total area}} = \frac{\text{Void volume}}{\text{Total Volume}} = \varepsilon$$

Hence the equation will become: 
$$U_{\text{sup}} = \frac{(-\Delta P)}{K \mu_f L} \frac{\varepsilon^2}{S_p^2 (1-\varepsilon)^2}$$

Since the volumetric flow rate of filtrate is  $(dV/dt)$  and if  $A$  is the area of filtration, therefore, we can write

$$U_{\text{sup}} = \frac{1}{A} \left( \frac{dV}{dt} \right) = \frac{(-\Delta P)}{K \mu_f L_c} \frac{\varepsilon^2}{S_p^2 (1-\varepsilon)^2}$$

$L_c = \text{thickness of the cake}$



The superficial velocity of the liquid  $U_{\text{sup}}$  is defined as the volumetric flow rate of the liquid divided by the total cross-sectional area and can be related to  $U'$  as:

$$\frac{U_{\text{sup}}}{U'} = \frac{\text{Void area}}{\text{Total area}} = \frac{\text{Void volume}}{\text{Total Volume}} = \varepsilon$$

### Cont....

It is assumed that, all the voids are completely filled with the liquid, and all particles have been uniformly wet with the liquid,

$$R_H = \frac{\text{Void Volume}}{\text{Total surface area of particles}}$$

$$R_H = \frac{\varepsilon}{S_p (1-\varepsilon)}$$

Where,  $\varepsilon$  = The void fraction of the bed

$\varepsilon$  = Void volume / total volume

$S_p$  = Specific surface of the particle.

Hence, the equation becomes: 
$$U' = \frac{(-\Delta P)}{K \mu_f L} \left[ \frac{\varepsilon^2}{S_p^2 (1-\varepsilon)^2} \right]$$



Now, going further the superficial velocity of the liquid that is  $U$  superficial is defined as the volumetric flow rate of the liquid divided by the total cross-sectional area and it can be referred to as  $U$  dash. So, superficial velocity divided by the  $U$  dash is equal to void area divided by total area. So, if we take  $L$ , so, it will it can also be defined as void volume divided by total volume and this is basically epsilon. So, this is as per definition this is epsilon or the porosity.

So, hence the superficial velocity of the liquid can be found out as  $U$  superficial is equal to  $U$  dash into epsilon. So, this is what has been done here. So, this is there and in this equation this was the basic equation so, we multiply by, actually this will be  $E$  cube, so, and here also it will be  $E$  cube. So, this is there hence the equation will become this.

Now, since the volumetric flow rate of the filtrate is  $dV$  by  $dt$  and if  $A$  is the area of filtration, therefore, we can write  $V$  superficial is equal to  $dV$  by  $dt$  divided by  $A$  and hence, this equation can be written as this, where  $L_c$  is the thickness of the cake in place of the thickness of the bed. So, that is length of the bed in place of that we are using the thickness of the cake for applying this equation for filtration. So, this is one of the equation which has been developed.

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**Cont....**

Since,  $K = 25/6$

$$U_{\text{sup}} = \frac{(-\Delta P)}{\frac{25}{6} \mu_f L} \frac{\epsilon^2}{(6/D_p)^2 (1-\epsilon)^2}$$

$$\Rightarrow U_{\text{sup}} = \frac{(-\Delta P)}{\frac{25}{6} \mu_f L} \frac{D_p^2 \epsilon^2}{(36)(1-\epsilon)^2} \quad S_p = 6/D_p$$

$$\Rightarrow U_{\text{sup}} = \frac{(-\Delta P)}{150 \mu_f L} \frac{D_p^2 \epsilon^2}{(1-\epsilon)^2}$$

Narayanan and Bhattacharyya, 2014 6

$$\Rightarrow U_{\text{sup}} = \frac{(-\Delta P)}{150 \mu_f L} \frac{D_p^2 \epsilon^3}{(1-\epsilon)^2}$$

### Cont....

The superficial velocity of the liquid  $U_{sup}$  is defined as the volumetric flow rate of the liquid divided by the total cross-sectional area and can be related to  $U'$  as:

$$\frac{U_{sup}}{U'} = \frac{\text{Void area}}{\text{Total area}} = \frac{\text{Void volume}}{\text{Total Volume}} = \varepsilon$$

Hence the equation will become: 
$$U_{sup} = \frac{(-\Delta P)}{K \mu_f L} \frac{\varepsilon^3}{S_p^2 (1 - \varepsilon)^2}$$

Since the volumetric flow rate of filtrate is  $(dV/dt)$  and if  $A$  is the area of filtration, therefore, we can write

$$U_{sup} = \frac{1}{A} \left( \frac{dV}{dt} \right) = \frac{(-\Delta P)}{K \mu_f L_c} \frac{\varepsilon^3}{S_p^2 (1 - \varepsilon)^2} \quad L_c = \text{thickness of the cake}$$



### Cont....

- The velocity of fluid  $U'$  through the bed can be written as:

$$U' = \frac{(-\Delta P)}{K \mu_f L} R_H^2$$

where,

$K$  = Kozeny's constant for random packed particles of definite size and shape  
= 25/6

$U'$  = Superficial velocity of fluid

$(-\Delta P)$  = Pressure drop across the bed

$L$  = Height (or thickness) of the bed

$R_H$  = Hydraulic radius = (flow area/wetted perimeter)



Now, in this equation there is further development. Now,  $K$  in this equation  $K$  earlier we know Kozeny's constant, so, it is equivalent to 25 by 6. So, here it is 25 by 6, we put 25 by 6, similarly, specific surface area can be defined as this the 6 by  $D_p$ , so, square of that, so, that will be there and then we go further ahead and if you solve this particular equation,  $U$  superficial, I think every place it will be cube. So, this is there, so,  $U$  superficial velocity can be given by this particular equation  $D_p$  square into  $\varepsilon$  cube divided by  $1 - \varepsilon$  square. So, this is the equation  $150 \mu_f L$ , so this is there.

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### Cont....

It is known that, Fanning friction factor is defined as "the drag force per wetted surface unit area (shear stress at the surface) divided by the product of density times the velocity head"

$$f = \frac{D}{4} \frac{(-\Delta P)}{L} \frac{1}{\frac{1}{2}\rho v^2} \quad \checkmark$$

In similar manner, bed friction factor ( $f_b$ ) could be defined as:

$$f_b = \frac{D_p}{4} \frac{(-\Delta P)}{L} \frac{1}{\frac{1}{2}\rho U_{sup}^2} \Rightarrow f_b = \frac{(1-\varepsilon)}{\varepsilon^3} \frac{75}{Re_b} \quad \text{where, } Re_b = \frac{D_p U_{sup} \rho_f}{\mu_f (1-\varepsilon)}$$



It is known that, Fanning friction factor is defined as "the drag force per wetted surface unit area (shear stress at the surface) divided by the product of density times the velocity head"

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Bed friction factor ( $f_b$ ) could be defined as:

$$\Rightarrow f_b = \frac{1-\varepsilon}{\varepsilon^3} \frac{75}{Re_b}$$

### Cont....

Since,  $K = 25/6$  ✓

$$U_{sup} = \frac{(-\Delta P)}{\frac{25}{6} \mu_f L} \frac{\varepsilon^2}{(6/D_p)^2 (1-\varepsilon)^2}$$

$$\Rightarrow U_{sup} = \frac{(-\Delta P)}{\frac{25}{6} \mu_f L} \frac{D_p^2 \varepsilon^2}{(36)(1-\varepsilon)^2}$$

$$S_p = 6/D_p \quad \checkmark$$

$$\Rightarrow U_{sup} = \frac{(-\Delta P)}{150 \mu_f L} \frac{D_p^2 \varepsilon^2}{(1-\varepsilon)^2} \quad \checkmark \quad \text{--- (A)}$$



Now, what we do is that it is we define another friction factor which is called as bed friction factor. So, this particular equation which is given here it can be reduced in terms of bed

friction factor, and how we can go ahead? So, in that there is a Fanning friction factor which is used in the fluid mechanics, fluid dynamics and that is the drag force per wetted surface area, per wetted surface unit area and like shear stress at the surface divided by the product of the density times the velocity head.

So, this is what is done and so,  $F$  is given by this particular equation. So, this particular equation can be modified by replacing  $V$  with respect to superficial velocity and we have this definition and which is there for, the, which is called as bed friction factor. Now, if we use this equation this definition, so, and this particular equation which is given here, so, if we call it  $A$  and if we can call it  $B$ , so, combining  $A$  and  $B$  we can get this equation.

So, this will be bed friction factor is equal to  $1 - \epsilon$  divided by  $E \epsilon^3$  into  $75$  divided by  $Re_b$ , so,  $Re_b$  is further defined which is called as bed Reynolds number and so, this particular equation can be obtained. So, overall what we have done is that, we have reduced the equation to this condition and where  $Re_b$  is called the bed Reynolds number and it is defined by this particular equation, which is given here and which is  $D_p$  into  $U$  superficial into the density of the fluid divided by viscosity of the fluid into  $1 - \epsilon$ , which is the porosity, so this is there.

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**Cont....**

**Laminar Zone:** ✓  
 Modified frictional factor  $f_b = \frac{2\varepsilon^3}{(1-\varepsilon)} f_b = \frac{150}{Re_b}$  ✓  
 Blake-Kozeny eq. valid for  $Re_b < 10$ ;  $\varepsilon < 0.5$  ✓  
 $Re_f = \text{Bed frictional factor} = \frac{D_p V_0 \rho_f}{\mu_f (1-\varepsilon)}$  ✓  $f_b = \frac{150}{Re_b} \times \frac{(1-\varepsilon)}{2\varepsilon^3}$  ✓

**Turbulent Zone:** ✓  $f_b = 1.75$  for  $Re_b > 1000$  ✓  
 Burke-Plummer eq.

**Transition Zone:** ✓  $f_b = \frac{150}{Re_b} + 1.75$  ✓  
 Ergun's eq.  $10 < Re_b < 1000$

Narayanan and Bhattacharyya, 2014 8

**Cont....**

It is known that, Fanning friction factor is defined as "the drag force per wetted surface unit area (shear stress at the surface) divided by the product of density times the velocity head"

$$f = \frac{D_p (-\Delta P)}{4 L} \frac{1}{\frac{1}{2} \rho V^2}$$

In similar manner, bed friction factor ( $f_b$ ) could be defined as:

$$f_b = \frac{D_p (-\Delta P)}{4 L} \frac{1}{\frac{1}{2} \rho U_{sup}^2}$$

$$\Rightarrow f_b = \left( \frac{1-\varepsilon}{\varepsilon^3} \right) \frac{75}{Re_b}$$

where,  $Re_b = \frac{D_p U_{sup} \rho_f}{\mu_f (1-\varepsilon)}$

Narayanan and Bhattacharyya, 2014 7

Now, what has been done is that, these equations can be applied under certain conditions and those certain conditions are with respect to flow rate etc. So, we have to cross check that what are the ranges of these Reynolds number under which we have to apply. So, similar to Hagen-Poiseuille or other types of equations which are valid for laminar condition, turbulent condition or transition zone. Here also we have to define these zone. So, these were studied in detail and then after that certain conditions have been evolved.

So, there is a Blake-Kozeny equation which is valid for the Reynolds number less than 10 or bed porosity less than 0.5. So, this is the condition and this will be the condition generally we will be finding in our case in the filtration. So, we have to cross check that whether Reynolds

number is less than 10 or not. So, if it is less than 10 under that condition, there is a modified friction factor that was defined and for easiness it was defined as  $150 \text{ by } Re_b$ .

So, it is very simple  $f_b$  dash is equal to  $150 \text{ by } Re_b$  and this is the modified friction factor and here we cross-checked with respect to  $Re_b$  that what is the range. Now, if it is less than 10 this is called as laminar zone and we can use this particular equation for finding out the modified friction factor or we can find out the friction factor also from here we can find out the friction factor also, this will be like  $150 \text{ upon } Re_b \text{ into } 1 \text{ minus } \epsilon \text{ divided by } 2 \text{ epsilon cube}$ . So, this will be the equation for finding out  $f_b$ .

So, we can do it or for easiness it is very easy to remember that  $f_b$  dash is equal to  $150 \text{ by } Re_b$  and where this Reynolds number, bed Reynolds number is defined by this, this is usual Reynolds number definition only difference is with respect to multiplication by  $1 \text{ minus } \epsilon$ , so that is there.

Now, if suppose the value is greater than 1000, at any condition the Reynolds number is greater than 1000. So, this is considered as a turbulent zone and here the value is fixed the  $f_b$  dash the modified frictional factor is equal to 1.75, so this is there. Now, if there is a transition zone that  $Re_b$  is less than 1000 but it is greater than 10. So, under that condition  $f_b$  dash can be given by combining both, so, we have  $150 \text{ by } Re_b \text{ plus } 1.75$ .

So, any of these equations, so, we have to cross-check whether which zone is there, so, whether it is laminar, turbulent or transition, so, we should cross check this. So, the usual method is that first find out the  $Re_b$  at the condition then we can find out the frictional factor, either modified frictional factor or the frictional factor, bed frictional factor directly, and once this is known, we can go on finding other parameters like pressure drop, etc, because the average particle size will generally be known and the velocity will also be known and density also known. So, we can find out the pressure drop.

Opposite way also we can do that, if the pressure drop is fixed or up to what limit so, we can find out the velocity etc also. Any of these can be. So, three things are important in this, three important parameters are find out the  $Re_b$  Reynolds number, this is one thing that should be well known to us. Second is  $f_b$ , once  $f_b$  is known, and then we can use the equation B to find out many of the things because,  $f_b$  is known and we can find out other parameters. So, this is how they are combined together. So, this is the basic equation which can be used in the filtration also for finding out pressure drops and other things.

And we can find out other parameters also using this particular equation, which has been developed or being used from the flow through a porous packed bed. So, this is like that. Now, the equation which has been defined earlier can further be extended to develop the filtration equation, how it is done, it is shown here.

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**Cont....**

Since the cake is deposited parallel to the filter medium, the cross-sectional area of the cake and that of filter medium can be taken to be the same and equal to the filtration area (A). Then the equation will become:

$$\frac{1}{A} \left( \frac{dV}{dt} \right) = \frac{(-\Delta P_c)}{\alpha \mu_f L_c} \quad \text{--- (C)}$$

where,

$$\alpha = \frac{K S_p^2 (1-\varepsilon)^2}{\varepsilon^3}$$

The factor  $\alpha$  is called the specific cake resistance and is measure of the resistance offered by the cake to the flow of filtrate. It can be seen that the dimension of  $\alpha$  is  $m^2$ . The equation will become:

$$(-\Delta P_c) = \frac{\alpha \mu_f L_c}{A} \frac{dV}{dt} \quad \text{--- (D)}$$

Narayanan and Bhattacharyya, 2014 9

**Cont....**

The superficial velocity of the liquid  $U_{sup}$  is defined as the volumetric flow rate of the liquid divided by the total cross-sectional area and can be related to  $U'$  as:

$$\frac{U_{sup}}{U'} = \frac{\text{Void area}}{\text{Total area}} = \frac{\text{Void volume}}{\text{Total Volume}} = \varepsilon$$

Hence the equation will become:

$$U_{sup} = \frac{(-\Delta P) \varepsilon^3}{K \mu_f L S_p^2 (1-\varepsilon)^2}$$

Since the volumetric flow rate of filtrate is  $(dV/dt)$  and if A is the area of filtration, therefore, we can write

$$U_{sup} = \frac{1}{A} \left( \frac{dV}{dt} \right) = \frac{(-\Delta P) \varepsilon^3}{K \mu_f L_c S_p^2 (1-\varepsilon)^2} \quad \text{--- (E)}$$

$L_c$  = thickness of the cake

Narayanan and Bhattacharyya, 2014 5

So, since the cake deposited parallel to the filter medium, it always gets deposited parallel on the top of the filter medium, the cross-sectional area of the cake and that of the filter medium both can be taken as same. So, for the cake also whatever is the cross-sectional area and whatever is the cross-sectional area the filter medium both can be taken as same and equal to some filtration area, which is like A.

So, now, the equation will become so, what we are doing is that we are trying to use the earlier equation which was defined here you can see, so, this particular equation can be easily converted into this particular equation where alpha has been defined, a new term has been defined and which is called as a specific cake resistance. So, this alpha term as defined and everything else remains the same and alpha is  $K S_p^2 \frac{1 - \epsilon}{\epsilon^3}$  and it is the measure of the resistance of the cake which is offered by the cake with respect to flow of the filtrate, so that is there.

And it can be seen that the dimensions of the alpha is per meter square. So, if you actually solve it, you will be finding that it is per meter square. So, and if we replace alpha we can further modify this particular equation, we call it C and we can get the equation D. Now, this is with respect to cake only. So, this is for pressure drop across the cake will be because of this alpha and all these parameters which are given here. So, they have already been defined. Now, what we can do is that we can add this is the pressure drop because of the cake. Now, similarly, there will be pressure drop because of the filter medium also.

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**Ultimate Filtration Equation**

$$(-\Delta P) = (-\Delta P_c) + (-\Delta P_m) = \frac{\mu_f}{A} \left[ \frac{\alpha V}{A} + R_m \right] \frac{dV}{dt}$$

**Conditions:**

**Incompressible Cakes**

1.  $\alpha \equiv \text{constant}$   $\alpha = \alpha_0$   $S = 0$

**Compressible Cakes**

1.  $\alpha = \alpha_0 (-\Delta P)^S$   $[S = 0.2 \text{ to } 0.8 \equiv \text{Compressibility coefficient}]$

2.  $\alpha = (\text{constant}) \times \left[ \frac{1}{A} \frac{dV}{dt} \right]$

Narayanan and Bhattacharyya, 2014
10

### Cont....

Since the cake is deposited parallel to the filter medium, the cross-sectional area of the cake and that of filter medium can be taken to be the same and equal to the filtration area (A). Then the equation will become:

$$\frac{1}{A} \left( \frac{dV}{dt} \right) = \frac{(-\Delta P_c)}{\alpha \mu_f L_c} \quad \text{--- (C)}$$

where,

$$\alpha = \frac{K s_p^2 (1 - \varepsilon)^2}{\varepsilon^3}$$

The factor  $\alpha$  is called the specific cake resistance and is measure of the resistance offered by the cake to the flow of filtrate. It can be seen that the dimension of  $\alpha$  is  $\text{m}^2$ . The equation will become:

$$(-\Delta P_c) = \frac{\alpha \mu_f L_c}{A} \frac{dV}{dt} \quad \text{--- (D)}$$

The specific cake resistance

$$(-\Delta P_c) = \frac{\alpha \mu_f L_c}{A} \frac{dV}{dt}$$

So, we can add for the filter medium, so, and this is the further filter medium. So, overall pressure drop will be because of the cake and because of the filter medium both combined together. So, this will be there and once both are there the overall definition is given here, and we can easily see that only the  $\mu_f$  by A has been added like it has been taken out and other things are like  $R_m$  is the filter medium resistance and this is the  $dV$  by  $dt$ .

So, this equation is called Ultimate filtration equation and it can be modified in a number of ways to solve many filtration problem. So, this equation is called as ultimate rotation equation. And there are lots and lots of possibility of solving this equation using various cases, etc. Now, incompressible there is a term which is alpha here, this alpha if it is the cake is incompressible. So, we have already studied the compressible cakes and incompressible cake.

So, compressible cake where the cake is fluffy in nature and its density can be varied so little bit with pressure, so, all those things if it is there. So, alpha can be represented here by this equation for compressible cake alpha is equal to alpha 0 pressure drop raise to s where s may vary from 0.2 to 0.8, so, this is called compressibility coefficient and it can be obtained for various types of cake experimentally.

Similarly, alpha can also be related by this type of equations, so, many possibilities are there, but generally this equation is used most often for compressible cakes and if it is

incompressible, so,  $s$  is equal to 0. So, under that condition  $\alpha$  becomes equal to  $\alpha_0$ . So, this is for incompressible cake. So, this is the equation. Now, when we are using compressible for compressible cake here minus  $\Delta P$ , so, there is some modification in the equation happens.

So, that is so, while integrating, we have to remember whether what is the value of  $s$ . So, we always try to integrate to get that how much filtrate volume we can obtain in such time maybe 20 minutes, 1 hour, 5 hour, 10 hours. So, during those integration equations that we are obtained after integrating this particular equation. So,  $\alpha$  if it is raised to  $\alpha$ ,  $\alpha$  is equal to  $\alpha_0$  where  $\Delta P$  raised to  $s$ . So, actually that  $\Delta P$  will go this side so, there is a some variation happens, so, that we have to take care.

(Refer Slide Time: 21:48)

### Question: 1

A plate and frame press filtering a slurry, gave a total of  $25 \text{ m}^3$  of filtration in 30 minutes and  $35 \text{ m}^3$  in 60 minutes when filtration was stopped. Estimate the washing time in minutes if  $10 \text{ m}^3$  of wash water are used. The resistance of the cloth can be neglected and a constant pressure is used throughout.

Given: For constant pressure filtration with negligible cloth resistance,  <sup>$R_m$  neglected</sup> the basic equation is as follows:

$$\frac{dV}{dt} = \frac{A^2 \Delta P}{2C_v V}$$

$$\Delta P = \frac{1}{2} \frac{\alpha V}{A} V \cdot \frac{dV}{dV}$$

### Ultimate Filtration Equation

$$(-\Delta P) = (-\Delta P_c) + (-\Delta P_m) = \frac{\mu_t}{A} \left[ \frac{\alpha V}{A} V + R_m \right] \frac{dV}{dt}$$

#### Conditions:

##### Incompressible Cakes

1.  $\alpha \equiv \text{constant}$   <sup>$\alpha = \alpha_0$   $s=0$</sup>

##### Compressible Cakes

1.  $\alpha = \alpha_0 (-\Delta P)^s$  [ $s=0.2$  to  $0.8 \equiv$  Compressibility coefficient]
2.  $\alpha = (\text{constant}) \times \left[ \frac{1}{A} \frac{dV}{dt} \right]$

## Ultimate Filtration Equation

$$(-\Delta P) = (-\Delta P_c) + (-\Delta P_m) = \frac{\mu_f}{A} \left[ \frac{\alpha V}{A} V + R_m \right] \frac{dV}{dt}$$

Now, some questions related to this. A plate and frame press is filtering a slurry. So, this is given and it is giving a total of 25 meter cube of filtration is being done in 30 minutes and only 35 meter cube little bit more in 60 minutes, when the filtration was stopped. So, we have two data, one data in 30 minute we are getting 25 meter cube of volume and in 60 minute we are getting 35 meter cube of volume, when the filtration was ultimately stopped.

So, estimate the washing time in minutes, if 10 meter cube of wash water has to be used or filtration has to be done. The resistance of the cloth can be neglected. Here remember, the resistance of the cloth can be neglected and a constant pressure is used. So, two things are there, pressure is constant and the  $R_m$  that means, the  $R_m$  has been neglected. So, in this particular equation, if you go previously the equation was this.

So, we can use the same equation but  $R_m$  has been dropped. So, the equation will become like this. So, if we just remember the previous equation we can like  $\Delta P$  is equal to from here sorry from here  $\mu_f A$ , this  $\mu_f A$  and then  $\alpha V$  by  $A$ , so, this is another term which is there and then another  $V$  and into  $\Delta V$  by  $\Delta t$ , so, this is the equation. Now, we can modify this equation, so,  $dV$  by  $dt$  is equal to this and this all terms have been modified a little bit  $A^2$  is there, so, and all have been clubbed together in one. So, this has been done.

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**Solution:**

Assume  $\frac{A^2 \Delta P}{C_v}$  as a constant  $M$

Now Integrating it:  $\frac{dV}{dt} = \frac{M}{2V}$

$$\int_0^V 2V dV = \int_0^t M dt \Rightarrow V^2 = kt$$

12

### Cont....

For  $t=30$  min,  $V$  is  $25 \text{ m}^3$   $M = \frac{0.25 \text{ m}^3}{30 \text{ min}}$

Filtration was complete at  $t=60$  min and  $V=35 \text{ m}^3$

Final rate of filtration is:  $\left(\frac{dV}{dt}\right)_{\text{final}} = \frac{M}{2V} = \frac{0.25}{30 \times 2 \times 35} = 0.298 \text{ m}^3 / \text{min}$

**For plate and frame filter press:**

Rate of washing =  $0.25 \times$  Final rate of filtration =  $0.25 \times 0.298 = 0.0745 \text{ m}^3 / \text{min}$

Washing time =  $\frac{\text{Volume of wash water}}{\text{Rate of washing}} = \frac{10}{0.0745} = 134.2 \text{ min}$   
 $\approx 135 \text{ min}$



13

### Question: 1

A plate and frame press filtering a slurry, gave a total of  $25 \text{ m}^3$  of filtration in 30 minutes and  $35 \text{ m}^3$  in 60 minutes when filtration was stopped. Estimate the washing time in minutes if  $10 \text{ m}^3$  of wash water are used. The resistance of the cloth can be neglected and a constant pressure is used throughout.

Given: For constant pressure filtration with negligible cloth resistance, the basic equation is as follows:

$$\frac{dV}{dt} = \frac{A^2 \Delta P}{2C_v V}$$

$$\Delta P = \frac{k}{A} \frac{\alpha V}{A} V = \frac{dV}{dE}$$



Patil K.D., 2002

11

So, we can get this equation. Now,  $\Delta V$  by  $\Delta t$  is equal to  $M$  by  $2V$  and if we actually integrate this is capital  $M$ . So, if we integrate it will be  $V$  square is equal to  $k t$ . So, this is the basic equation that we will be getting and from this basic equation for we can use the data which is already being given.

So, for  $t$  is equal to 30 minute  $V$  has been given as 25 meter cube. So, this is the first data, so, we can use in this particular equation to find out, that now, we have to cross check what is the value of  $M$  so, which is the constant so, we get the  $m$  to be 625, this is 25 divided by 30. So, this is the  $M$  value.

Similarly, in the 60 minutes we have 35, so, under that condition, if we solve it, it will be like this, we can easily solve it and overall we can if we can solve it will be 0.298 meter cube per minute. Now, for plate and frame filter press, the rate of washing is equal to 0.25 into final

rate of filtration, because remember we have to find out the rate of washing, not filtration, estimate the washing time in minutes, if only 10 meter cube of wash water has to be used.

So, under that condition, we are assuming that the rate of washing is around one fourth of the final rate filtration, so, final rate of filtration is this much meter cube per minute. So, it is 0.25, so, this will be the meter cube per minute will be the rate of washing. So, overall volume of wash water which has to be used is 10 and rate of washing is 0.0745. So, if we divide we will be finding that, we require approximately 135 minutes for washing this filter press.

So, from here we get an idea that washing time is large. And so, that means, as for sedimentation basin, we always here also generally we can have two or more filtration units in parallel. So, if three are working one maybe in washing mode. So, this way, we can always have continuous filtration which is going on, because out of the few units one has to be washed. So, this is also another design criteria that has to be taken care of whenever we are designing any filtration unit. So, washing time is larger.

So, that is why if suppose four filtration units are operating in parallel, so that means three will be actually be working and one will be in the washing mode. So, after some time, this will become now operational. So, it will become operational and another filtration unit which may have been operating earlier that may go into the washing mode. So, this is how it is planned.

(Refer Slide Time: 27:33)

## Porous Bed Hydraulics

**Kozeny's equation:** It has been developed based on the "capillary bundle theory"

Superficial velocity =  $\frac{\text{Volume flow rate of fluid}}{\text{Empty cross section area of packed column}}$

$$U' = \frac{(-\Delta P)}{K \mu_t L} R_H^2$$

Where,

$U'$  = Superficial velocity of fluid

$K$  = Kozeny's constant  $[(25/6)]$ , for random packed particles of definite size and shape

$(-\Delta P)$  = Pressure drop across the bed

$L$  = Height (or thickness) of the bed

$R_H$  = Hydraulic radius = (flow area/wetted perimeter)

$$R_H = \frac{\text{Void Volume}}{\text{Total surface area of particles}}$$

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14

## Porous Bed Hydraulics

- The hydraulics of flow through porous beds, which applies to clean filters and to granular activated carbon and ion exchange beds is usually described by empirical relationships given by **Carman Kozeny**.

$$\frac{\Delta P}{L} = \left[ \frac{E(1-\epsilon)}{\epsilon^3} \right] \times \frac{v^2}{gD_p S_p}$$

$h/l$  is the head loss per unit depth of bed

$\epsilon$  = porosity of bed

$S$  is the particle shape factor ( $S=1$  for sphere and for sand grains it is 0.70 to 0.90)

$v$  = Filtration rate, m/s

$E = [150(1 - F)/R] + 1.75$

$R$  = Reynold's Number =  $(dv\rho/\mu)$

$\mu$  = Kinematic viscosity of fluid

Carman Kozeny equation as follows:

$$\frac{\Delta P}{L} = \left[ \frac{E(1-\epsilon)}{\epsilon^3} \right] \times \frac{v^2}{gD_p S_p}$$

$h/l$  is the head loss per unit depth of bed

= porosity of bed

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$v$  = Filtration rate, m/s

$E = [150(1 - F)/R] + 1.75$

$R$  = Reynold's Number =  $(dv\rho/\mu)$

$\mu$  = Kinematic viscosity of fluid

Now, further going ahead earlier already we have defined this Kozeny equation and all the parameters were gained. So, based upon this we have already derived this equation also. So, we can use these equations to find out the pressure drop across the porous bed and this is very very important that how much is the head loss which is happening, and determination of head loss with respect to velocity and all other parameters in the filter bed is very, very important. And through this we can find out, most often the velocity is fixed because we have the same amount of water which can be taken.

So, what we can do is that we can still adjust the superficial velocity by adjusting the diameter of the bed, length of the bed, the type of material that we are going to use in the bed.

So, we can modify all those things, so that we can keep the pressure drop lowest possible. So, this is there.

(Refer Slide Time: 28:37)

### Question: 1

A filter bed is composed of 800 mm of unit-size spherical sand of 0.5 mm diameter with a porosity of 45%. Calculate the head loss when the clean bed is operated at rate of 145 m/d. Kinematic viscosity of water at 20°C is  $1.01 \times 10^{-6} \text{ m}^2/\text{s}$ .

$Re_d$   
 $\frac{V_s d_p}{\nu}$

So, this is another question which is given here. A filter bed is composed of 800 millimeter of unit-size spherical sand, and like the bed is of 800 millimeter which is having sand of 0.5 millimeter diameter and overall the bed is having a porosity of 45 percent. Calculate the head loss when the clean bed is operated at a rate of 145 meter per day, so, that is velocity is given then the kinematic viscosity of the water is given at 20 degrees centigrade and which is already  $1 \times 10^{-6} \text{ m}^2/\text{s}$ .

So, we have to find out the head loss when that clean bed is to be updated. Remember in the previous slides, we gave that we have three equations which are known to us one is that under what zone this bed is going to be operated. So we have to cross-check that means we should find out the Reynolds number. So, that is first, we should cross-check what is the Reynolds number in which range it is operation, whether it is laminar or turbulent or transition? So, once it is known, we can calculate in the second step  $f_b$  or  $f_b$  dash, so these any of these can be found out. So, these already we know the equation for any of the zones and once this is known, we can use the superficial velocity equation to find out the pressure drop. So, this is how it is done.

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### Solution:

Filtration rate 145 m/d ✓

$$145/(24 \times 60 \times 60) \text{ m/s}$$

$$= 1.68 \times 10^{-3} \text{ m/s} \checkmark$$

Reynold's number  $Re_b = \frac{D_p U_{sup} \rho_f}{\mu_f (1 - \varepsilon)}$  ✓

$$= \frac{(1.68 \times 10^{-3} \times 5 \times 10^{-4})}{(1 - 0.45)(1.01 \times 10^{-6})}$$

$$= 1.51$$

$< 10$   ~~$\frac{m \times m}{m \times s}$~~



### Question: 1

A filter bed is composed of 800 mm of unit-size spherical sand of 0.5 mm diameter with a porosity of 45%. Calculate the head loss when the clean bed is operated at rate of 145 m/d. Kinematic viscosity of water at 20°C is  $1.01 \times 10^{-6} \text{ m}^2/\text{s}$ .

$Re_b$   
 $\frac{D_p U_{sup}}{\nu}$



So, filtration rate in this case is 145 meter per day which is already given. So, we solve it in meter per second for our easiness. So, this has been given here, now, bed Reynolds number was earlier defined by this. So, in this case the filtration rate is already known, this is the diameter, which is already given to be 0.5 millimeter. So, that means, it is 0.5 into 10 raised to 4 meter.

One more thing that we should always use, while solving any problem the units must always be taken care of, here I have not given but they may be taken care of. So, like 1.68 this is meter per second, we should always write the units also along with the other numeric values. Similarly, for superficial velocity already the unit has been given for  $D_p$  we are taking the

meter and already the overall kinematic, since, we are directly using the kinematic viscosity, so, its unit is meter square per second which is already given.

So, this gives the idea that these units are cutting each other. So, if it is dimensionless and here 0.5, 0.45 is the porosity of the bed. So, 1 minus epsilon because 0.55 and if you solve it, it will come out as 1.51, the Reynolds number. Now, this Reynolds number is less than 10. So, this is there. So, that means, we can use the, for the laminar flow zone this we can find out the fb dash.

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**Cont....**

Blake-Kozeny eq. valid, since  $Re_b < 10$ ;  $\varepsilon < 0.5$

Therefore Laminar Zone is applicable

Modified fractional factor  $f'_b = \frac{2\varepsilon^3}{(1-\varepsilon)} f_b = \frac{150}{Re_b}$

$$\frac{(-\Delta P)}{L} = \frac{4f'_b}{D_p} \frac{1}{2} \rho U_{sup}^2 = \frac{4 \times 150}{(Re_b)} \frac{1}{D_p} \frac{1}{2} \rho U_{sup}^2$$

After solving  $\frac{(-\Delta P)}{L} = \frac{300}{D_p^2} \mu_f (1-\varepsilon) U_{sup}$

**Solution:**

Filtration rate 145 m/d ✓  
 $145 / (24 \times 60 \times 60) \text{ m/s}$   
 $= 1.68 \times 10^{-3} \text{ m/s}$  ✓

Reynold's number  $Re_b = \frac{D_p U_{sup} \rho_f}{\mu_f (1-\varepsilon)}$  ✓

$$= \frac{(1.68 \times 10^{-3} \times 5 \times 10^{-4})}{(1-0.45)(1.01 \times 10^{-6})}$$

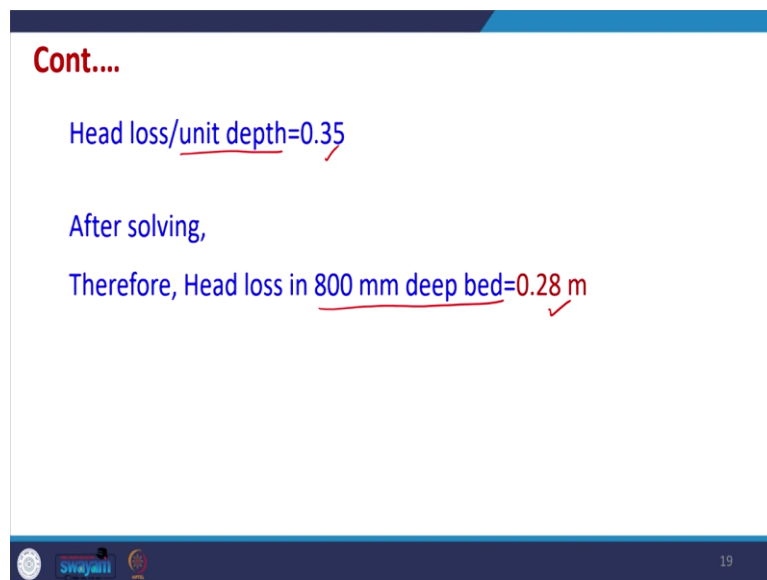
$$= 1.51$$

$< 10$  ✓

So, since the Blake-Kozeny equation is valid and since Reynolds number is less than 10 and epsilon is also less than 0.5 therefore, laminar zone is applicable here, we have find out the modified frictional factor which is given by 150 by Reb and Reb has already found out 1.51, we can easily solve it and we can use the earlier equation also the earlier equations which are given like  $\Delta P$  by with respect to definitions which are given, so, we can solve it.

So,  $\Delta P$  by  $L$  can be given by this particular equation. So, everything has taken care of only Reb is given if Reb is known, we can directly use this equation if we know that we can substitute the values or Reb here also, we can this equation also and indirectly, we can substitute all the parameters in this equation to get the value of pressure drop which is there.

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Cont....

Head loss/unit depth=0.35 ✓

After solving,

Therefore, Head loss in 800 mm deep bed=0.28 m ✓

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So, head loss per unit depth will be coming out to be 0.35, so, this is per unit depth when  $L$  is taken as 1. Now, for 800 millimeter deep bed, the value will be coming out to be 0.28 meter. So, this is the head loss that we will be finding when we will be using this condition.

(Refer Slide Time: 33:36)

**Solution:**

Filtration rate 145 m/d ✓  
 $145/(24 \times 60 \times 60) \text{ m/s}$   
 $= 1.68 \times 10^{-3} \text{ m/s}$  ✓

Reynold's number  $Re_o = \frac{D_p U_{sup} \rho_f}{\mu_f (1 - \varepsilon)}$  ✓

$= \frac{(1.68 \times 10^{-3} \times 5 \times 10^{-4})}{(1 - 0.45)(1.01 \times 10^{-6})}$   
 $= 1.51$   
 $< 10$  ✓

17

**Ultimate Filtration Equation**

$(-\Delta P) = (-\Delta P_c) + (-\Delta P_m) = \frac{\mu_f}{A} \left[ \frac{\alpha V}{A} + R_m \right] \frac{dV}{dt}$  ← (A)

**Conditions:**

**Incompressible Cakes**  
 $\alpha = \alpha_o$  ✓  $S = 0$

**Compressible Cakes**

1.  $\alpha = \alpha_o (-\Delta P)^S$  [S = 0.2 to 0.8 ≡ Compressibility coefficient] ← (B)
2.  $\alpha = (\text{constant}) \times \left[ \frac{1}{A} \frac{dV}{dt} \right]$  ✓ ← (C)

$\Delta P =$

Narayanan and Bhattacharyya, 2014 10

So, we can cross check whether it is suitable for us or not otherwise, we have to modify some parameters, but if we are using the sand itself, so,  $\varepsilon$  will be fixed, we can modify the superficial velocity a little bit, this is the only parameter that we have. So, through this we can modify the cross sectional area. So, we can vary the diameter of the bed etc that will become a design condition.

So, through this we can solve it. So, in today's lecture, we tried to understand two basic things One was that we can apply the theory of flow through porous packed bed and use the equation to find out the pressure drop and other design parameters with respect to that.

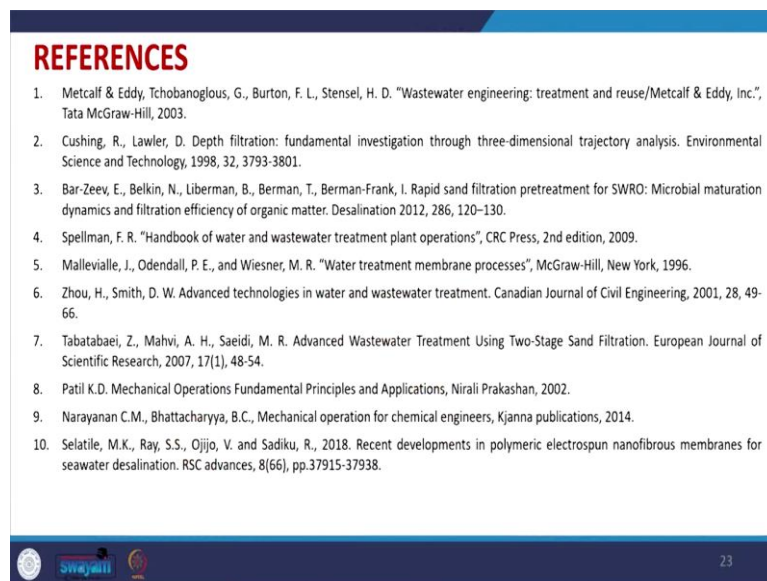
Then the second equation which is very important n number of problems can be solved, because of the for this filtration equation which has been given it has lot of uses and along

with the filter cake, this compressible cake, so, here alpha is there, so, this equation, I call it like A and along with this equation alpha is equal to  $\alpha_0 - \Delta P^s$ .

Both combined together can be used in a number of cases and they can be used combined together and we can solve and this  $\Delta P$  itself can be given by these equations which we have solved. So, we can solve by number of methods and through this combining all these three the  $\Delta P$  using flow through the porous bed, we can combine all through and design the proper filtration units.

So, all these three are very, very important equations and they can be used together to solve a number of problems. Now, in the next lecture, we will try to understand details of some of the filtration types of beds which are commonly used in the water and wastewater treatment. So, we will go ahead and understand those in the next lecture.

(Refer Slide Time: 35:49)



### REFERENCES

1. Metcalf & Eddy, Tchobanoglous, G., Burton, F. L., Stensel, H. D. "Wastewater engineering: treatment and reuse/Metcalf & Eddy, Inc.", Tata McGraw-Hill, 2003.
2. Cushing, R., Lawler, D. Depth filtration: fundamental investigation through three-dimensional trajectory analysis. Environmental Science and Technology, 1998, 32, 3793-3801.
3. Bar-Zeev, E., Belkin, N., Liberman, B., Berman, T., Berman-Frank, I. Rapid sand filtration pretreatment for SWRO: Microbial maturation dynamics and filtration efficiency of organic matter. Desalination 2012, 286, 120-130.
4. Spellman, F. R. "Handbook of water and wastewater treatment plant operations", CRC Press, 2nd edition, 2009.
5. Mallevalle, J., Odendall, P. E., and Wiesner, M. R. "Water treatment membrane processes", McGraw-Hill, New York, 1996.
6. Zhou, H., Smith, D. W. Advanced technologies in water and wastewater treatment. Canadian Journal of Civil Engineering, 2001, 28, 49-66.
7. Tabatabaei, Z., Mahvi, A. H., Saeidi, M. R. Advanced Wastewater Treatment Using Two-Stage Sand Filtration. European Journal of Scientific Research, 2007, 17(1), 48-54.
8. Patil K.D. Mechanical Operations Fundamental Principles and Applications, Nirali Prakashan, 2002.
9. Narayanan C.M., Bhattacharyya, B.C., Mechanical operation for chemical engineers, KJanna publications, 2014.
10. Selatile, M.K., Ray, S.S., Ojijo, V. and Sadiku, R., 2018. Recent developments in polymeric electrospun nanofibrous membranes for seawater desalination. RSC advances, 8(66), pp.37915-37938.

For today, we will close this lecture, these are some of the references which have been used for these equations, so you can refer to them these books and other things. So, thank you very much.