Physico-Chemical Processes for Wastewater Treatment

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Lecture 22

Settling and Sedimentation – I

Good day everyone and welcome to this lecture on Physico Chemical Processes for Wastewater Treatment. And in the previous lectures we studied regarding some of the treatment operations which are there like flow equalization then we studied aeration in lot of detail. Further we studied coagulation and flocculation in the previous four lectures in detail and we came across that coagulation coagulants can help in the bigger flock formation or denser flock formation which ultimately settle down easily as compared to if we do not use any coagulants so they will not settle down.

So, once we have reached the condition where lot of solids can settle down it is very essential that we determine the how, what should be the size of the settling unit or sedimentation unit. So, we will study these aspects in detail now onwards ah in a few lectures starting from today. And so like objective of the coagulation and flocculation process was to enhance the size of the particles so that they will settle in a reasonable period of time. So, once that objective has been reached so we have to see that that we are able to remove most of the satellitable solids which are there in the water either by gravitational settling in a sedimentation basin. So, that is one of the first objectives which is there for settling.

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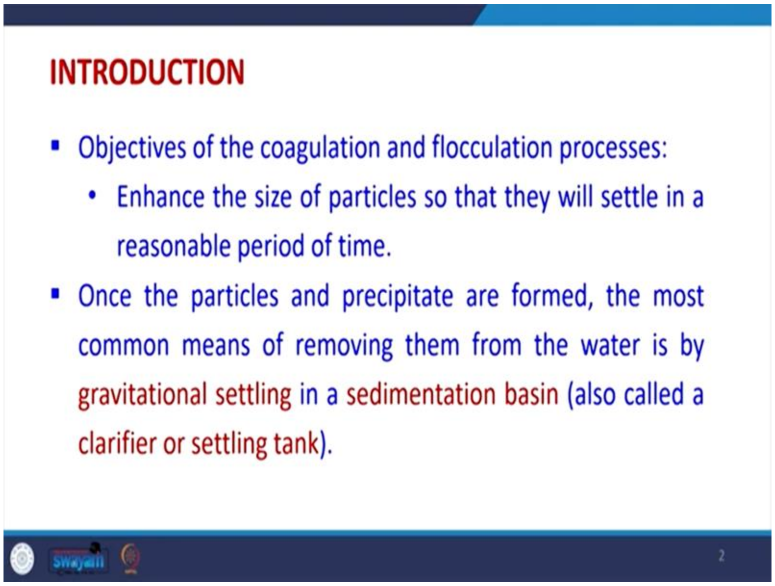

Now, these basins may are also called as clarifier or settling tank et cetera so that is there. So, we most often use gravitational settling there is a possibility of using some centrifugation or otherwise but most often it is only gravitational settling that we use in the water treatment. Before starting the design of the system and other things which should understand the theory behind the sedimentation first. So, today will be only concentrating on the different types of theories and some of the basics of sedimentation.

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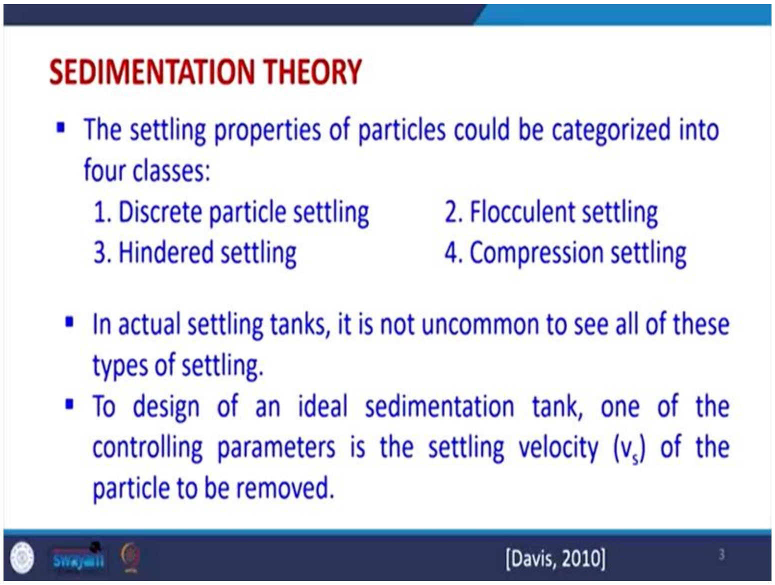

The settling property the particles if they settle down that can be the overall settling properties can be classified into four different groups. So, first thing is called discrete particle settling that means the particles are like individual particles. Then we have flocculant settling where some particles are coming together and they are settling down. Then we have hindered settling and compression settling so we will study all these thing in detail. In actual settling tanks it is not uncommon to see all these type of settling happening in sequence so this is also possible and individually also they are possible to see all these things. So, we will continue with the same so we will study these different types of settlings little bit and then further will study in detail these settling behaviors.

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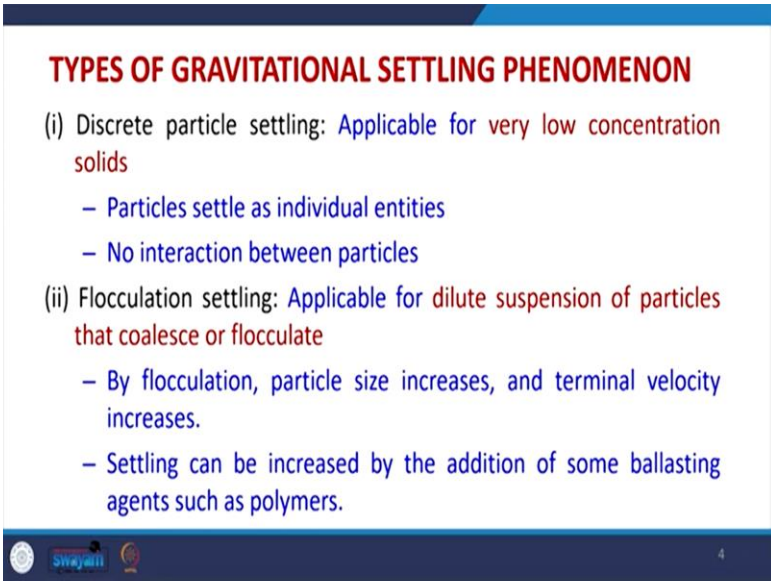

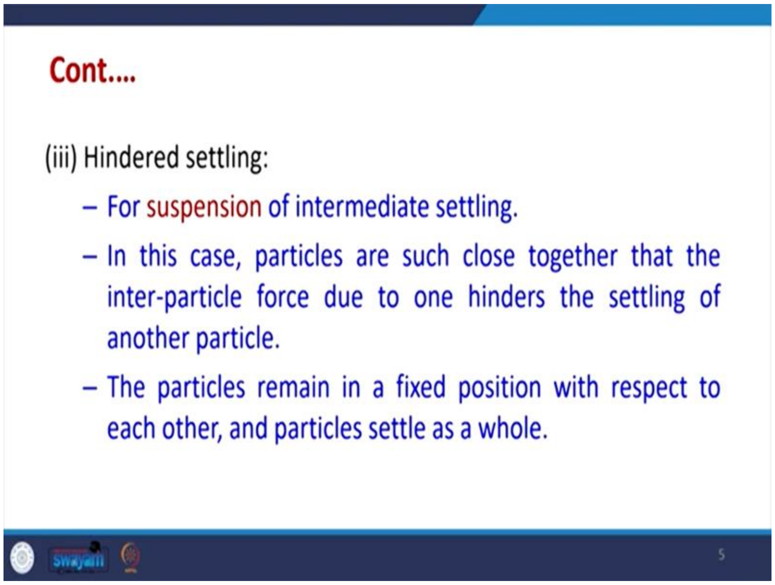

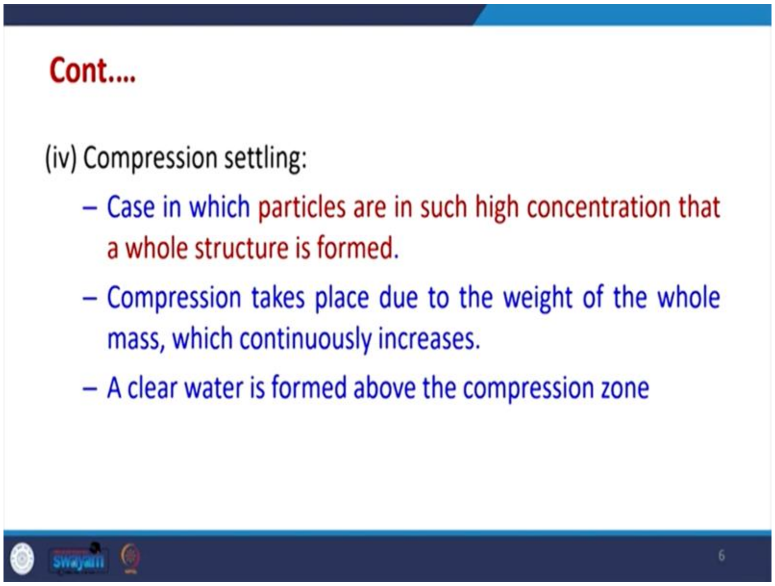

So, in the discrete particle settling the it is only applicable for very low concentration solid. So, very individual particles are there and they are settling down. So, they settle down as individual entities only and there is virtually no interaction among the particles and so this is the discrete particle settling. Now, in the flow class settling this is applicable for dilute suspensions of particle that may flocculate or coils together so this is possible. So, by flocculation the particle size actually increases and the terminal velocity also increases as compared to in the discrete particle settling case.

Now, settling can be increased by addition of some other agents such as polymers that we have studied in the previous case so it is possible and that will come under the flocculant settling case. Now, there is third settling which is called as hindered settling in the handled settling generally it is used for suspension where they are settling at the intermediate rate. And in this case the particles are such close together that the inter particle force due to one hinders the settling of another particle so it is possible at that case.

The particles remains in a fixed position with respect to each other. So, because the settling is being hindered by one another it is possible that whole bunch is settling down. So, they are in a fixed position with respect to each other and particles settle down as a whole so this is called hindered settling. And then we have lastly the compression settling.

So, case in which the particles are in such very high concentration that whole structure is formed. So, compression takes place due to the weight of the whole mass which continuously increases.

So, the structure volume may go become less but the mass will go on increasing so this is there and a clear water will be formed above the compression zone. So, these are the four different zones of settling so and it is possible to observe all these four zones within a settling tank also so that is also possible. Now, what we will do is that we will try to understand each of these things all the four cases in further detail. So, today we will be only concentrating on discrete particle settling.

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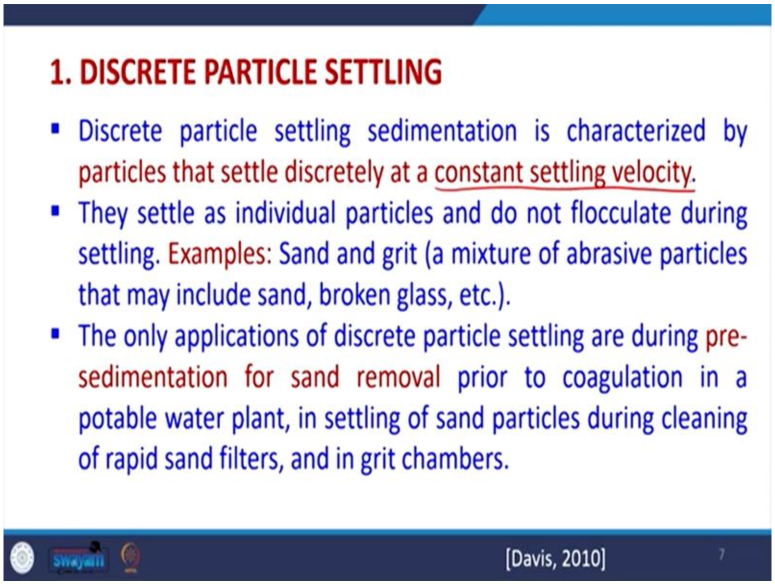

So, discrete particle settling is characterized by particles that settle discretely that means they are not affected by each other and they will settle down after sometime with a constant settling velocity which is also called as the terminal velocity. Now, since in the discrete particle settling each of the particles behave as individual particles and they do not flocculate. So, there are less chances of observation but sand and grid are common example a mixture of abrasive particles that may include sand broken glass etcetera so they may settle down as individual particle.

And this case is possible in pre-sedimentation unit like for suppose we want to remove the sand beforehand going for coagulation etcetera, so there this is possible. Now, will go further with the particle settling theory that how to find out the terminal velocity of any particle having a particular diameter and having a particular density and which is settling in a fluid which is having a particular density. So, there are few consideration that what is the density of the particle what is the density of the fluid and what is the particle size that we are considering so these are very very important parameters that affect the particle settling.

So, when particles actually settle discretely the particle settling velocity can be calculated and the basin design is based upon this particle settling velocity and we always design any basin based upon a specific size particle. So, we target that we want to remove all particles up to this size may be below that size they may be carried over but we always decide the specific size which we are targeting for removal. So, the theory can be further be elaborated and we are going to discuss.

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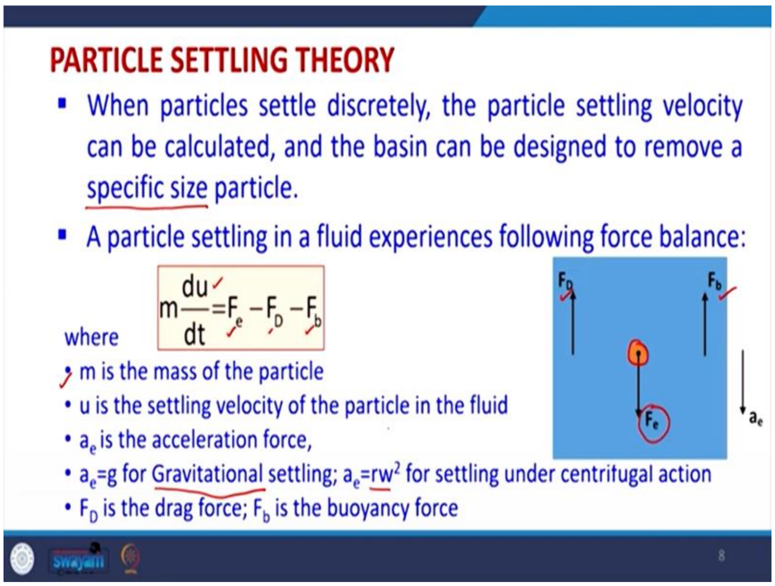

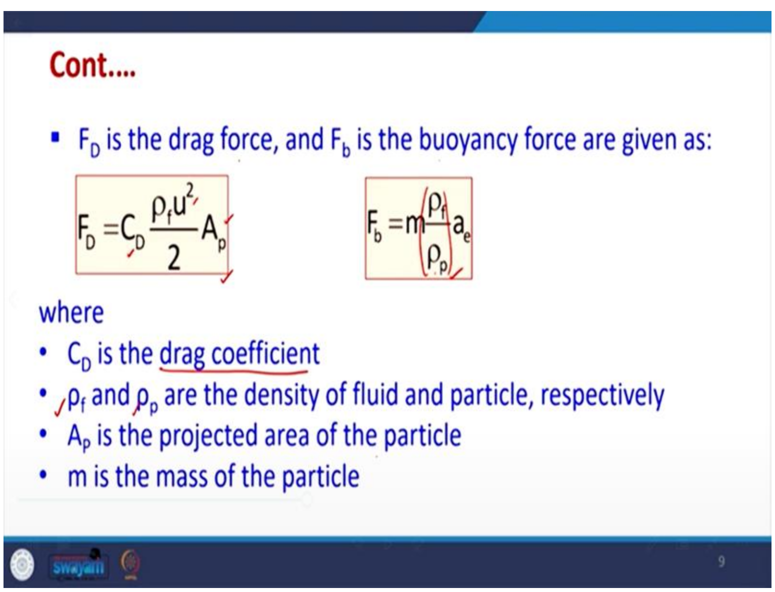

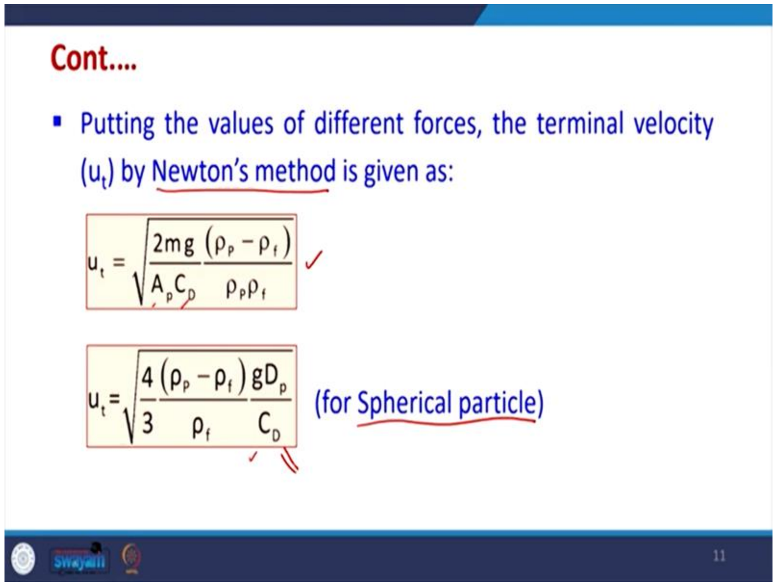

A particle settling in a fluid experiences following force balance:

$$m\frac{du}{dt} = F_e - F_D - F_b$$

where

- m is the mass of the particle
- u is the settling velocity of the particle in the fluid
- a_e is the acceleration force,
- a_e=g for Gravitational settling; a_e=rw^2 for settling under centrifugal action
- F_D is the drag force; F_b is the buoyancy force

$$F_D = C_D \frac{\rho_f u^2}{2} A_p$$

$$F_b = m\frac{\rho_f}{\rho_p} a_e$$

where

- C_D is the drag coefficient
- ρ_f and ρ_p are the density of fluid and particle, respectively

- A_P is the projected area of the particle
- m is the mass of the particle

For spherical particles having diameter (D_P), the value of A_P and m is given as:

$$A_P = \frac{\pi D_P^2}{4}, \quad m = \frac{\pi D_P^3}{6}\rho_p$$

The terminal velocity (u_t) by Newton's method is given as:

$$u_t = \sqrt{\frac{2mg}{A_p C_D}\frac{(\rho_P - \rho_f)}{\rho_P \rho_f}}$$

$$u_t = \sqrt{\frac{4}{3}\frac{(\rho_P - \rho_f)}{\rho_f}\frac{gD_p}{C_D}} \qquad \text{(for Spherical particle)}$$

So, suppose there is a particle like here and it is settling down so and its like it is going downwards in the so its acceleration is towards downward. So, this is the this is the force which is there and because of which it is going down and that force may be the gravitational force up. So, generally will be considering only gravitational force it is possible that the centrifugal force may also be applied some cases but we are only considering gravitational force. So, net acceleration in the downward direction will be dependent upon the net the force by which it is moving downwards and when the particle is moving downwards it will be actually be hindered by two forces which will be acting in the opposite direction.

And one will be the drag force and another will be the buoyancy force so these drag force and buoyancy force they will act opposite to the movement because the particle is settling down so certainly buoyancy force only will be upwards and drag force is always opposite to the direction of the movement. So, we have two forces so net we have this external force or gravitational force minus the drag force and the buoyancy force. Now, going further I will elaborate it further m is the mass of the particle and u is the settling velocity of the particle in the fluid so this is the m into a this is the acceleration net acceleration which is there.

Now, further expanding it if F is the drag force and Fb is the buoyancy force so they can be calculated using this equation so this is well known and drag force is can be calculated using this equation where Ap is the Fb the Cd is called the drag coefficient and rho is the fluid density rho p is the density of the particle which will be used here in the buoyancy force and Ap is the projected area of the particle. Assuming it to be the it may possible to assume it to be spherical so and m is the mass of the particle. So, drag force can be calculated from here u is the terminal velocity.

Similarly, are the velocity of the particle buoyancy force is Fm into a and where we re like it is the ratio of fluid density to particle density. So, we can see here so this is the very common buoyancy force we know very well. So, we now substitute these things in the first equation so and Ap here in this case the projected area if you are assuming the particle to be having a spherical particle with diameter Dp. So, in this case a p becomes equal to pi Dp square by 4 and the mass of the particle is like particle density into volume so this volume has been multiplied together so we have this.

Now, for particles settling with terminal velocity if suppose the particle is settling with a particular terminal velocity which will not increase further. So, under that condition the rate of change of the velocity will be equal to 0. And under the gravitational force then actual Ae value is always equal to g. So, if we substitute all these four conditions the equation of drag force equation of buoyancy force then replace the Ap value the projected area assuming the particle to be spherical.

Similarly mass also we can find out particle to be spherical so if we do this we can calculate the terminal velocity. So, terminal velocity can be calculated without replacement of Ap and Cd so we can calculate like this. But if we assume the particle to be spherical then we can substitute the value of Ap in this particular equation and the equation will become like this. So, this is the terminal velocity as calculated by newton's method so this is the terminus velocity which is given. Now, in this case the drag coefficient which is there this particular thing is very important.

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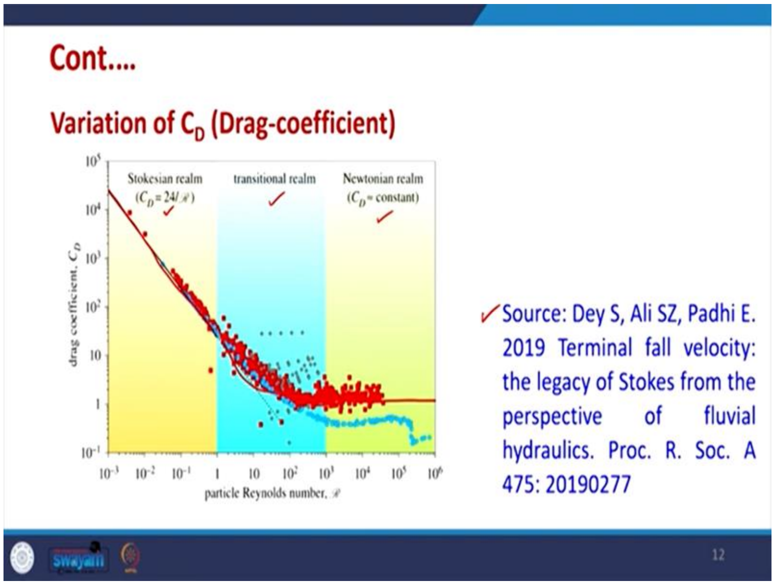
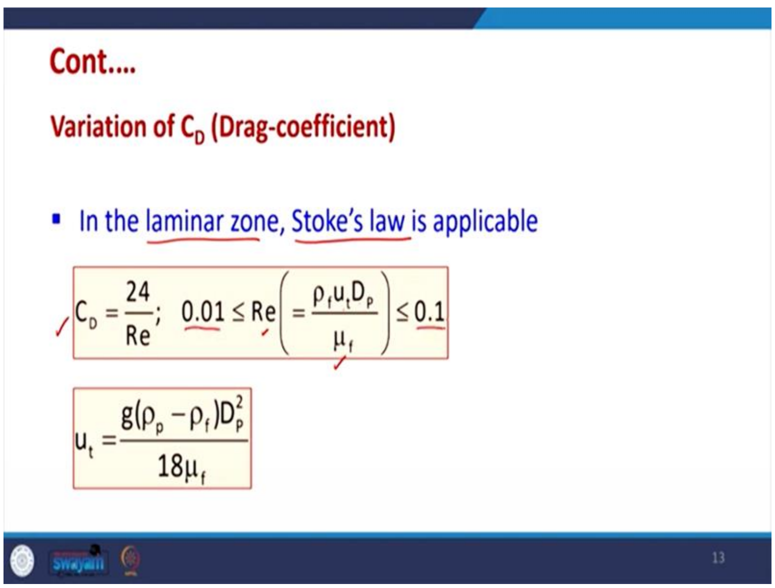
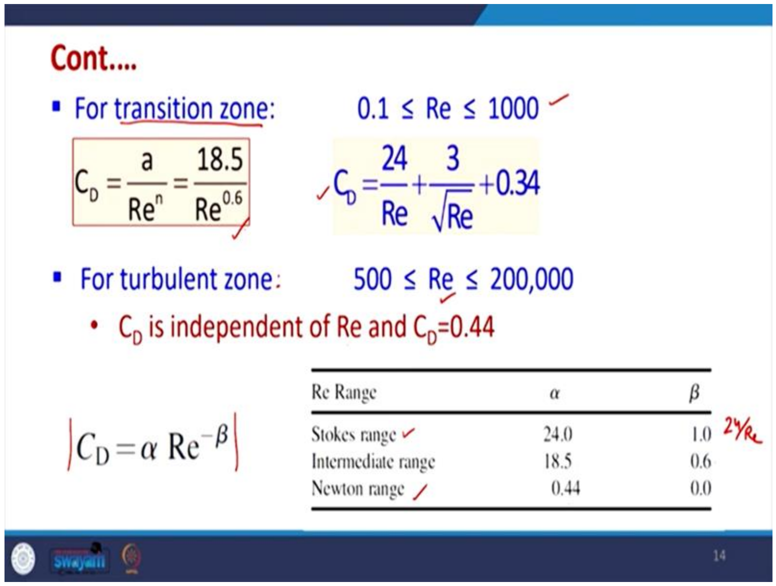




In the laminar zone, Stoke's law is applicable

$$C_D = \frac{24}{\text{Re}}; \quad 0.01 \le \text{Re}\left(= \frac{\rho_f u_t D_P}{\mu_f} \right) \le 0.1$$

$$u_t = \frac{g(\rho_p - \rho_f) D_P^2}{18\mu_f}$$



For transition zone:

$$C_D = \frac{a}{\mathrm{Re}^n} = \frac{18.5}{\mathrm{Re}^{0.6}}$$

And drag coefficient actually varies depending upon various parameters and variation of drag coefficient is very important and it has been shown in classical fluid dynamics. And so we study in detail so you will be referred to any of this source also you can refer so you can understand further. So, drag coefficient behavior is given like this and for different types of systems. So, we have three zones one is called Stokes law reason another is called transition region and another is Newtonian region.

So, Newtonian region where Cd is a constant value so it is becoming virtually constant to a particular value. Similarly, in the first case it is only decreasing at a linear rate but in between it is changing continuously. So, we have three zones so for three zones we have different equations which can be used for finding out the value of that coefficient. So, these are given like this in the laminar Stokes law region it is also called as a laminar region so in this case the Cd is given by 24 by Re where Re is the Reynolds number and which is defined as rho F into u t terminal velocity into Dp the diameter of the particle divided by the viscosity of the fluid so that is there.

And here in this case the Reynolds number is in the range of 0.01 to 0.1 some little variations are also reported sometime it is used up to one also so it is possible. Now, if we substitute the Cd by

24 by Re and further where Re is equal to rho F u t Dp by nu F. So, the original u t equation which is here so here we are what we are doing is that we are substituting by 24 by Re and Re is further being replaced by this term. So, if you solve it we have u t here Re in the Re also the Reynolds number so and if you solve it this will be the final equation which will be there for the Reynolds number so with respect to stokes law reason.

So, if the Reynolds number is only in this range we can directly apply this particular equation for finding out the terminal velocity of a discrete particle. Now, if suppose the reason is not this it is in the transition zone so in the transition zone different types of different types of equations have been given. So, the equation is this transition zone is valid from 0.1 to 1000 Reynolds number range. And this type of equation has been reported like 18.5 divided by Reynolds number raised to 0.6. Similarly, this type of expressions have been also been reported there are numerous other types of expressions also reported depending upon the type of particle type of fluid etcetera so these are dependent upon.

Then the finally the turbulent zone for the turbulent zone where the Reynold number varies from 500 to 2 lakh. So, under this condition the Cd is assumed to be independent of Re and it is constant like 0.44. So, there is a fixed value of Cd so that is why this equation is called as newton range and the first original expression if Cd is constant here then it may be called as newton's method also. So, newton's method Stokes region and then there is a transition region so these are there and this the dependence of Cd the drag coefficient with respect to the Reynold number can be expressed by this particular equation which is given here.

So, we can see here that for stokes range the alpha value is 24 and beta value is 1 so we have the expression like 24 by Re which was the first expression. Now, if we substitute this to be 18.5 and beta value to be 0.6 we get this particular equation for transition zone. And similarly, for third the beta value is 0 and the alpha value is 0.44. So, this way we can have a common equation also but this is how we can represent the dependence of drag coefficient with respect to Reynolds number.

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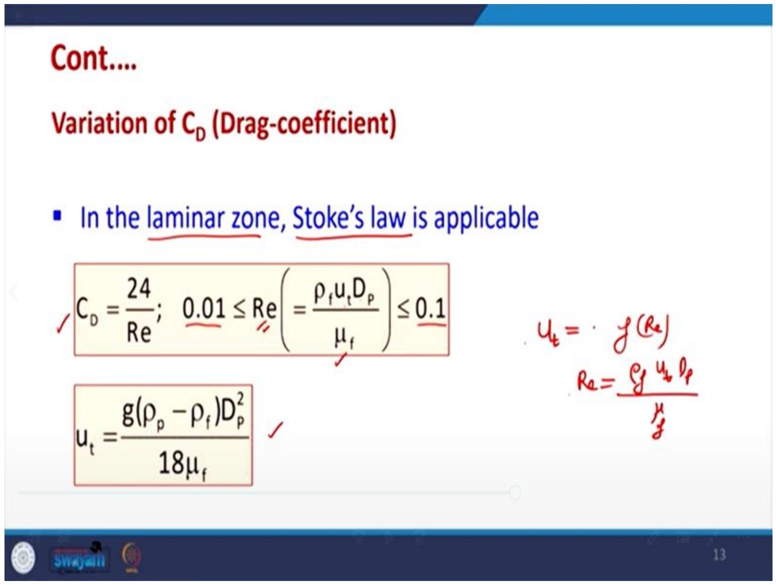

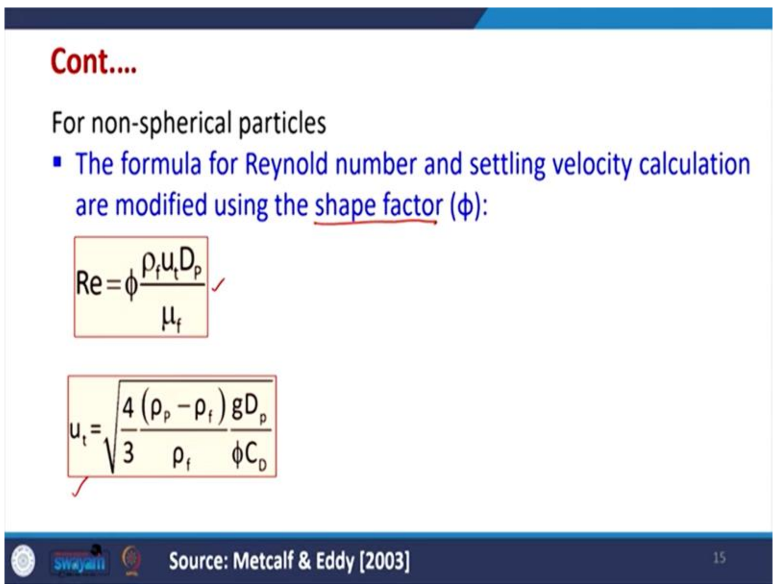

The formula for Reynold number and settling velocity calculation are modified using the shape factor (ϕ):

$$Re = \phi \frac{\rho_f u_t D_P}{\mu_f}$$

$$u_t = \sqrt{\frac{4}{3} \frac{(\rho_P - \rho_f)}{\rho_f} \frac{g D_p}{\phi C_D}}$$

So, now we if suppose we have to find out the terminal velocity of non-spherical particles so Reynolds number is changed a little this is given by incorporating the shape factor and the terminal velocity changes by this. So, this is there and this is the expression for this shape factor which is used and later on so we will solve some problem.

Now, in the this particular expression or any of these expressions what we see is that in this case we have to find out the terminal velocity. Now, we have to apply the equation as such that we should know the Reynolds number range. So, for each different Reynolds number range we have different expression. And the Reynolds number itself can be found out only when we have the velocity so there is two things Ut is a function we can find out by using this expression but it is dependent upon the Reynolds number.

And Reynolds number itself is a function of Ut so that means there we have to iterate if we have to find out the velocity so suppose we any condition is given so first we have to assume that suppose this condition lies in the stokes law region. So, we apply the equation for stokes law region once the velocity is known we find out Reynolds number we cross check whether actually it is in the stokes law region or not so we iterate between these to solve the problem. So, this is given by this numeric which is given here.

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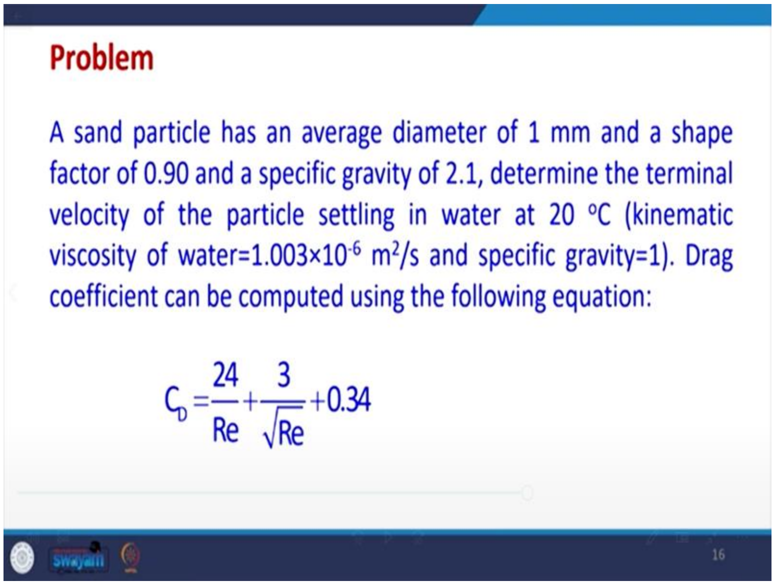

A sand particle has an average diameter of 1 millimeter and a shape factor of 0.9 and a specific gravity of 2.1. Determine the terminal velocity of the particle settling in water at 20-degree centigrade kinematic viscosity of the water has been given specific gravity has been given to be 1. The drag coefficient in the transition zone can be calculated by this expression it is given.

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Solution

Kinematic viscosity$(\nu) = \mu_f/\rho_f = 1.003\times10^{-6}$

$\mu_f = 1.003\times10^{-6}\times10^{3} = 1.003\times10^{-3}$ kg/ms

Settling velocity using stokes law is

$$u_t = \frac{g(\rho_p-\rho_f)D_P^2}{18\mu_f} = \frac{9.81\times((2.1\text{-}1)\times1000)\times(1\times10^{-3})^2}{18\times1.003\times10^{-3}} = 0.597 \text{ m/sec}$$

Cont....

- This iterative procedure is repeated until the initial assumed velocity is approximately equal to settling velocity calculated from Newton's equation.
- Initial drag coefficient is calculated as:

$$C_D = \frac{24}{Re} + \frac{3}{\sqrt{Re}} + 0.34 = 0.5142$$

$$u_t = \sqrt{\frac{4}{3}\frac{(\rho_p\text{-}\rho_f)}{\rho_f}\frac{gD_p}{\phi C_D}} = 0.1763 \text{ m/s} \longrightarrow Re$$

Cont....

- Now, iterative procedure is continued:

u_t (previous calculated)	Re	C_D	u_t	Difference
0.5977	536.3272	0.5143	0.1763	0.4214
0.1763	158.2037	0.7302	0.1480	0.0283
0.1480	132.7684	0.7811	0.1431	0.0049
0.1431	128.3690	0.7917	0.1421	0.0010
0.1421	127.5052	0.7939	0.1419	0.0002
0.1419	127.3315	0.7943	0.1419	0.0000

- Final settling velocity=0.1419 m/s

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Now, we have kinematic viscosity which is given by this so we are nu f by rho f is equal to this so this value is already given. So, kinematic viscosity is given so we can find out nu f also so this is multiplied by the density so specific gravity is already one so we can find out this. Now, settling velocity we can first what we are doing that we are assuming that stokes law reason is valid, so if this is valid so we find out the velocity using this. So, since all the parameters are known and specific gravity of particle is 2.1 and say factor is also given so we can find out the terminal velocity we can see here so it is 0.597 meter per second.

So, once the terminal velocity is there we should cross check whether actually it is in stoke law region or not so we go further and find out the Reynolds number. The Reynolds number is like 536.32 and which is beyond what is given here so for stoke law reason the Reynolds number should be at most 1 or it should be actually less than 0.1 but it is beyond 0.1. So, that means whatever we assumed it is not correct and Newton's law should be should not be used for finding out the velocity in the transition zone.

So, initial assumption which is given is not valid so what we do is that so we have to adopt the iterative procedure. Now, what we do is that we since Reynolds number is known now we find out the value of drag coefficient, so Cd is known so this Cd is found now we put in the terminal velocity and we find out the terminal velocity. And from here we have to find out the Reynolds number again and again put in this Cd equal expression to find out so this way we have to iterate.

So, now this iterative procedure will be like first we find out the u t from Ut we find out the Reynolds number from Reynolds number we find out the drag coefficient and once drag coefficient is known we find out the Ut value. So, when we cross check what is the difference which is there so if the difference is not much we can stop there. So, this Ut is found this Ut is put here again the procedure is repeated we find out Reynolds number drag coefficient again Ut found we found significant difference is there and we cannot solve the terminal velocity.

We can get the terminal velocity under those condition when the difference is virtually 0. So, we always look for this condition where we see that, that this was the terminal velocity and from this if we find the Reynolds number further find the drag coefficient and using the drag coefficient we find the terminal velocity and both are same. So, these are the final condition which are there and we can assume that final settling velocity or terminal velocity to be 0.1419. So, this is a iterative procedure which is complicated so there is a non-iterative method for determining terminal industry which is given in few books so that also we can refer to.

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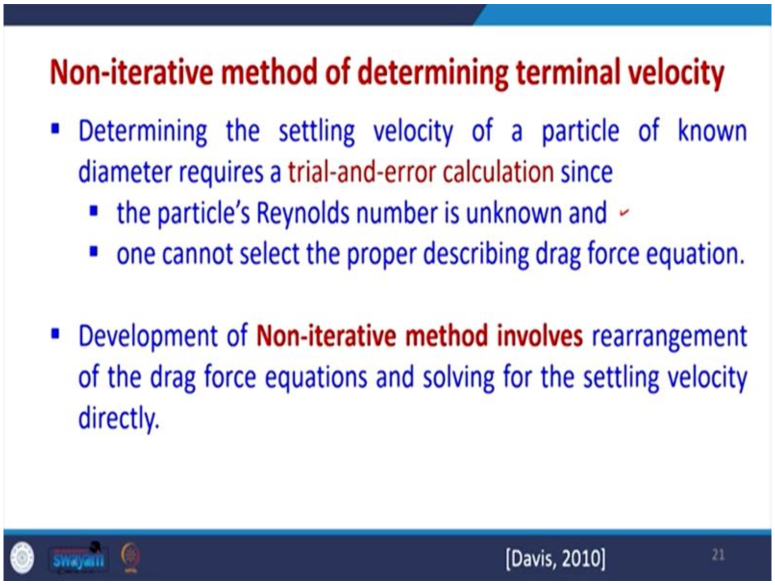

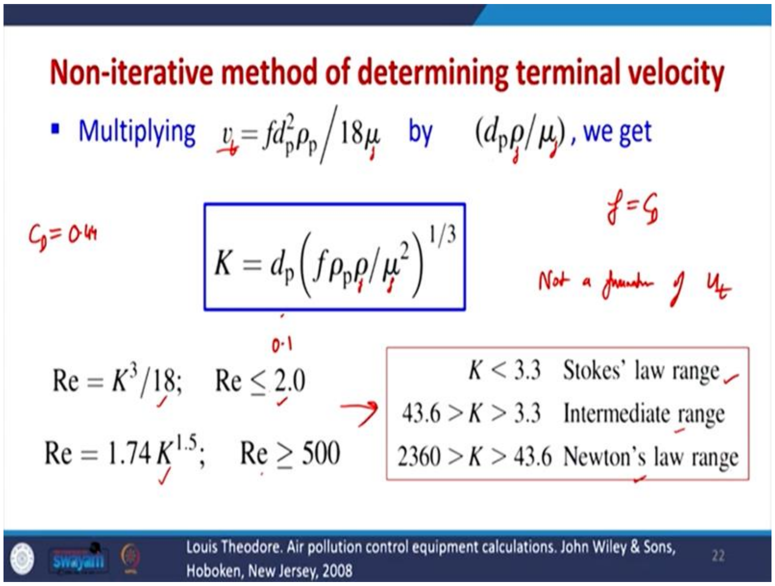

So, determining the settling velocity of a particle of known diameter by the previous method requires lot of trial and error. Since the particle Reynolds number is unknown initially and one cannot select the proper equation which can be used for drag force. So, that becomes a difficult situation so a non-iterative method actually was developed which requires some rearrangement of the drag force equations and solution with respect to settling velocity so it can be given by this method.

So, in this method what we do is that this is how this non-iterative method has been developed. So, what we do is that we find out if we multiply the terminal velocity expression like this is the Ut expression which was there 18 nu f if you remember. This is the expression with respect to and this is multiplied together by Dp rho by nu and this is for with respect to fleet. So, this is for fluid we were multiplying so wherever p is not written that is for fluid. Now, if we multiply it by this so this expression will be obtained Dp will be here f will be there the f or f is same as drag coefficient sorry so this is there.

And then we have rho p and rho of the fluid and nu f of the fluid, so if we multiply together this expression can be obtained which is which is the k that we take. Now, if we use this expression and we cross check with respect to stokes law region so suppose stoke law reason is considered in some of the books it is considered up to 2 also in some of the books it is considered up to 0.1. So, if the stokes law reason is considered up to 2 which is there in many books so the original expression can be written as the Reynolds number is equal to k ray k cube divided by 18. So, we can easily write this expression.

So, earlier remember the Cd value was given to be for this particular stroke lower region 24 by Re and this expression can be changed so we can write for Re and this expression can be changed with respect to this. So, Cd in place of Cd will have f and overall this expression can be obtained. And once this expression similarly for turbulent reason this expression can be obtained so sorry for anything beyond 500 so for turbulent region where the Cd value was constant Cd value was given to be 0.44.

So, for this case also we can find out, so this will be the expression and if we actually since Reynolds number range is known we can easily convert these things into this form. So, k if the k determined by this particular equation is found to be less than 3.3 it is stoke law region. And if it is in this range it is intermediate or Newton's law region so this is there. And what we do is that we find out k here and k is this expression is not a function of this is not a function of Ut.

So, there is no velocity term so that is why we in this expression there is no Ut term so k can be found out directly and once a k can be found out we can easily know the reason in which it lies and once we know the reason directly we can directly apply that particular expression. So, we can always solve using this particular non-iterative method for determining the terminal velocity.

We will now end this lecture so we have started with the basics how to find out the terminal velocity this expression is very important and knowing the terminal velocity helps in the design of the unit. So, we will continue with the settling section in the next lecture so as to understand the settling and sedimentation in detail thank you very much.