Chemical Process Utilities
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Module No # 12
Lecture No # 60
Heat transfer in Insulation Materials

Now let us talk about the heat transfer in insulation material in this particular lecture under the areas of chemical process utilities. Now before we go into the detail of this particular content let us have a brief outlook that what we covered in the previous lecture. We discussed about the brief history of thermal insulation. We had a brief discussion about the purposes of thermal insulation. We talked about the temperature ranges for insulation application in this. We discussed about the various type of insulating materials.

Apart from this we discussed about the heat transfermodes like conduction, convection, radiation. Now in this particular lecture, we are going to cover about the heat transfer aspect through the cylindrical insulating material. We will discuss about the heat transfer through spherical insulating materials. We will discuss about the criticalthickness of insulation especially for cylinder and for spheres. So, let us talk about the heat transfer in cylindrical insulating material.

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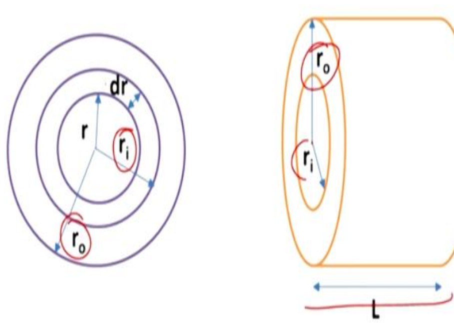
Heat transfer in cylindrical insulating materials

- **Cylinders**
  - Let us consider a long cylinder of length L, inside radius r_i and outside diameter r_o as shown in the following figure.



Now cylinders they are very common in the chemical engineering operation. So, let us consider a long cylinder of length L. Now inside radius is r_1 and outside radius is r naught and it is you can see this is a side view and this is the front view of this one this is the r naught r_i and the total radius is r. Now this is the differential radius small amount of radius.

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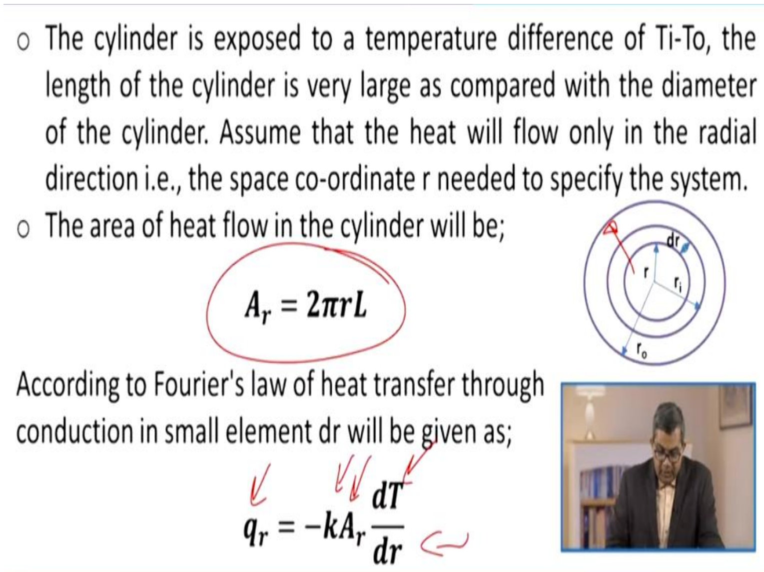

Now cylinder is exposed to the temperature difference of Ti to t naught, the length of the cylinder is very large compared with the diameter of the cylinder. So, assume that the heat will flow only in the radial direction that is the space coordinates r need to specify the system. So, the area of heat flow in the cylinder will be $A_r = 2\Pi rL$. Now according to the Fourier's law of heat transfer through conduction in small element this dr will be given as $q_r = -k A_r$ dT over dr.

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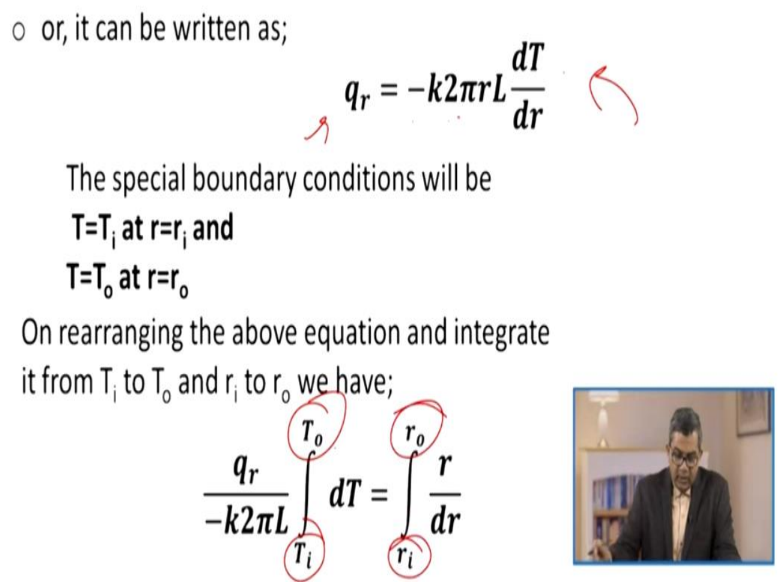
The area of heat flow in the cylinder will be;

$$A_r = 2\pi rL$$

According to Fourier's law of heat transfer through conduction in small element dr will be given as;

$$q_r = -kA_r \frac{dT}{dr}$$



Or, it can be written as q_r =- k2ΠrL because already we have given this area so qr = -k2ΠrL dT over dr. Now the spatial boundary conditions will be T = Ti and r = ri and T = t naught at r = r naught. So, if we rearrange this particular equation and integrate it from Ti to T naught, we can write this equation with respect to the qr over - k2ΠL integration d T from Ti to T naught. Or, it can be represented as this is equal to integration r over dr from r_i to r naught.

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or, it can be written as;

$$q_r = -k2\pi rL\frac{dT}{dr}$$

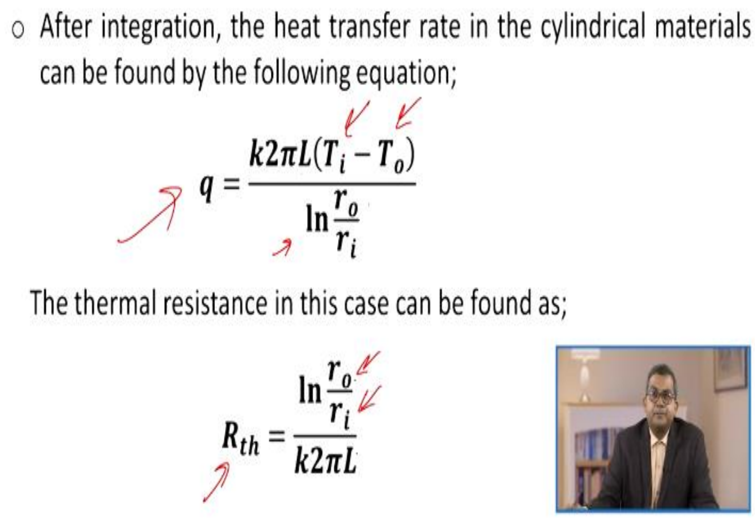
The special boundary conditions will be

$T=T_i$ at $r=r_i$ and

$T=T_o$ at $r=r_o$

On rearranging the above equation and integrate it from T_i to T_o and r_i to r_o we have;

$$\frac{q_r}{-k2\pi L}\int_{T_i}^{T_o} dT = \int_{r_i}^{r_o} \frac{r}{dr}$$



Now after integration, the heat transfer rate in the cylindrical material this can be found by this particular equation. That is q = k2ΠL Ti -T naught over ln r naught over r_i. Now the thermal resistance in this particular case can be written as R_th = ln r naught over r_i over k2ΠL.

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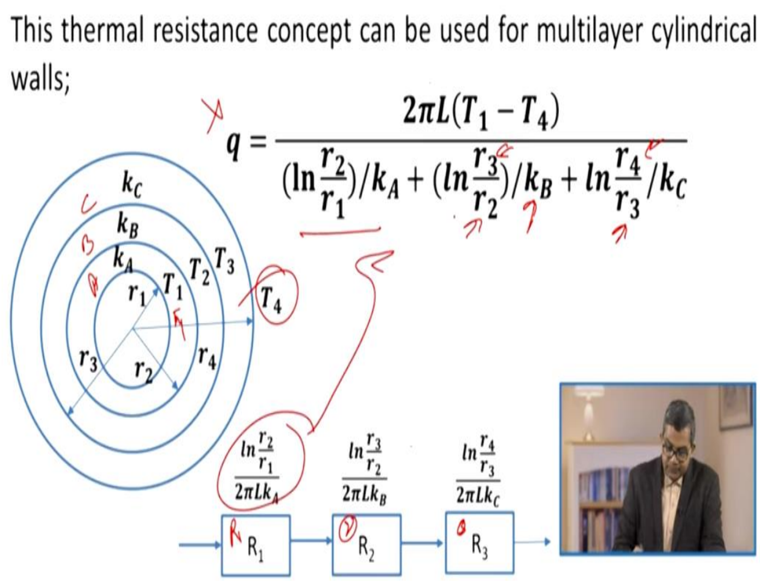
After integration, the heat transfer rate in the cylindrical materials can be found by the following equation;

$$q = \frac{k2\pi L(T_i - T_o)}{\ln\frac{r_o}{r_i}}$$

The thermal resistance in this case can be found as;

$$R_{th} = \frac{ln\frac{r_o}{r_i}}{k2\pi L}$$



Now this thermal resistance concept can be used for multilayer cylindrical walls. Now here you see that, this is the typical figure of a multilayer cylindrical wall, here we are having different layers. Now here if you see that r_1, r_2, r_3, r_4. So, there are 4 different radius apart from these different materials like A, B, and C. So, the generic equation can be written as q = 2ΠL T_1 -T_4. This is the T_1 the inner temperature and this one is the T_4.

This over ln r_2 over r_1 over k_A+ ln r_3 over r_2 over k_B+ ln r_4 over r_3 over k_C. Now these are in the series if you recall that in previous lecture, we have already studied about this one. So, the resistance to the first layer, second layer and third layer and this can be like this.

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This thermal resistance concept can be used for multilayer cylindrical walls;

$$q = \frac{2\pi L(T_1 - T_4)}{(\ln\frac{r_2}{r_1})/k_A + (ln\frac{r_3}{r_2})/k_B + ln\frac{r_4}{r_3}/k_C}$$

Problems

Question 1: A thick wall tube made up of stainless steel with inner diameter of 4cm and outer diameter of 8 cm. The thermal conductivity of the stainless steel is 19 W/m.K. It is covered with a insulation of thickness 4cm (k= 0.17 W/m.K). The temperature at outside of the insulation was 200ºC. The inside wall of the pipe is maintained at 550ºC temperature. Calculate the heat loss per unit length of the pipe and tube insulation interface temperature?

Now let us discuss first problem now here a thick wall tube made up of stainless steel with inner diameter of 4 centimeter and outer diameter of 8 centimeter. The thermal conductivity of the stainless steel is 19 watt per meter Kelvin. It is covered within insulation of a thickness 4 centimeter and that is k = 0.17 watt per meter Kelvin. The temperature at outside of the insulation is around the 200 degree Celsius.

The inside wall of the pipe is maintained at 550-degree Celsius temperature. Now here you need to calculate the heat loss per unit length of the pipe and tube insulation interface temperature.

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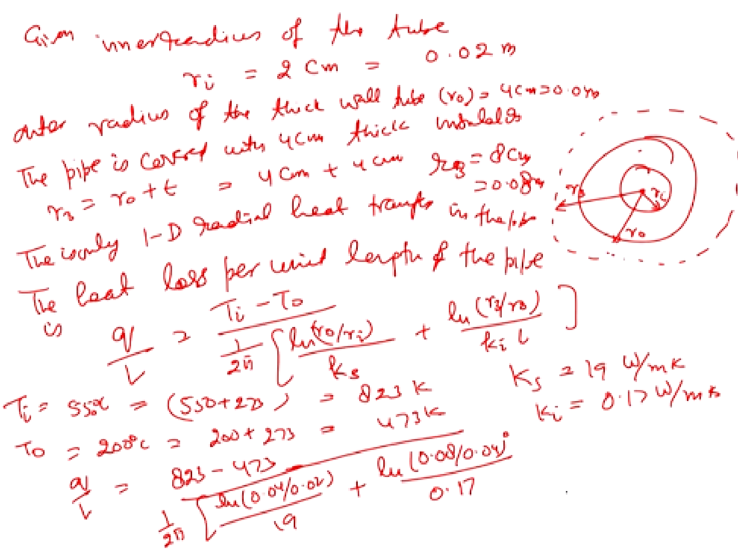

Now it is given that inner radius of the tube r_i = 2 centimeter and that is equal to 0.02 meter the outer radius of the thick wall tube r naught = 4 centimeter 0.04 meter. Now let us draw the figure this is r naught and this one is r_i. Now the pipe is covered with 4 centimeter thick insulation. So, r 3= r naught + t and that is equal to 4 centimeter + 4 centimeter and r_3 = 8 centimeter which is 0.08 meter.

Now there is only one directional radial heat transfer in the pipe. The heat loss per unit length of the pipe is q over L=T_i-T naught upon 1 upon 2 Π ln r naught over r_i upon +ln r_3 over r naught over ki L. Now in this particular equation Ti = 550 degree Celsius this is equal to 550 + 273 it is 823 Kelvin. T naught is 200 degree Celsius which is 200+ 273 that is 473 Kelvin k_s = 19 watt per meter Kelvin and k_i= 0.17 watt per meter Kelvin. So, q upon l = 823 - 473 upon 1 upon 2 Π L n 0.04 0.02 upon 19 + L n 0.08 over 0.04 upon 0.17.

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Given;

The inner radius of the tube

r_i = 2cm = 0.02 m

outer radius of the thick wall tube (r_o) = 0.04 m

the pipe is covered with 4 cm thick insulation materials

then r_3= r_o + t = 4+4= 8 cm

there is only 1-D radial heat transfer in the pipe

the heat loss per unit length of the pipe is given as;

$$q = \frac{2\pi L(T_i - T_o)}{(\ln\frac{r_o}{r_i})/k_s \quad + (ln\frac{r_3}{r_o})/k_i}$$

T_i = 550 $_o$C = (550+273) = 823 K

Ks = 19 W/mK

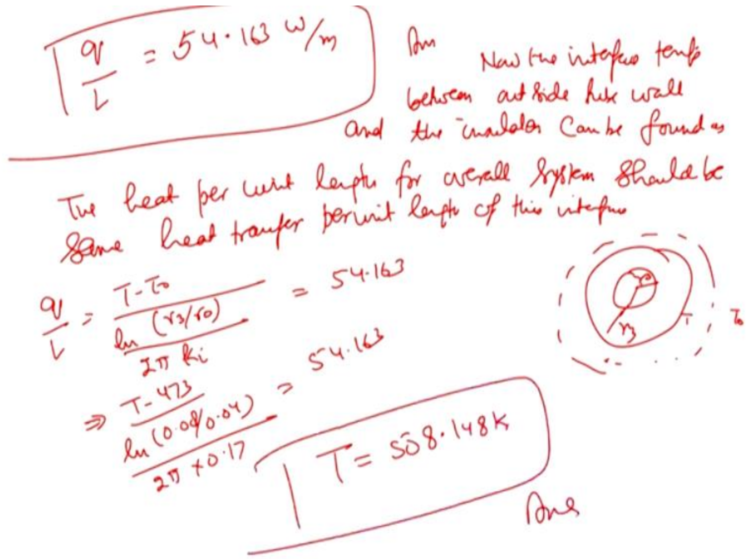
To = 200oC = (200+273) = 473K

Ki = 0.17 W/mK

On putting all these values in the above equation we have;

$$\frac{q}{L} = \frac{2\pi(823 - 473)}{(\ln\frac{0.04}{0.02})/19 \quad + (ln\frac{0.08}{0.04})/0.17}$$

$$\frac{q}{L} = 54.163\ W/m$$



And q upon L is equal to which is comes out to be 54.163 watt per meter that is the answer. Now the interface temperature between outside tube wall and the insulation can be found as the heat per unit length for overall system should be same as heat transfer per unit length of this interface. So again, coming back to the figure, this is t t naught this is r now q over L= t - t naught upon ln r_3 over r naught upon 2 Π k_i and that is 54.163.

Which is we have already obtained and that is T - 473 upon ln 0.08 0.04 upon 2 Π into 0.17 and that is 54.163. So, while calculating then we got this t = 508.148 Kelvin and that is our answer.

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Now the interface temperature between outside tube wall and the insulation can be found. As the heat per unit length for overall system should be same heat transfer per unit length of the interface.

$$\frac{q}{L} = \frac{2\pi(T - T_o)}{(\ln\frac{r_3}{r_o})/k_i}$$

On putting the value of the know parameter we have;

$$54.163\ W/m = \frac{2\pi(T - 473)}{(\ln\frac{0.08}{0.04})/0.17}$$

On solving this equation, we have
T = 508.148 K

Problems

Question 2: What will be the thickness of insulation, if the temperature of inner side is 300°C and temperature at outer side is 38°C. The thermal conductivity of the insulation is k=0.057 W/m.K and inner radius is given as r_1=0.035 m. if the heat transfer per unit length of the cylinder (q/L) is 60 W/m?

Now let us talk about the second question. Now here what will be the thickness of insulation, if the temperature of inner side is 300 degree Celsius and temperature at outer side is 38 degree Celsius? The thermal conductivity of the insulation is k this is which is K = 0.57 watt per meter Kelvin and the inner radius is given as r_1 = 0.035 meter. Now if heat transfer per unit length of the cylinder that that is q L is 60 watt per meter.

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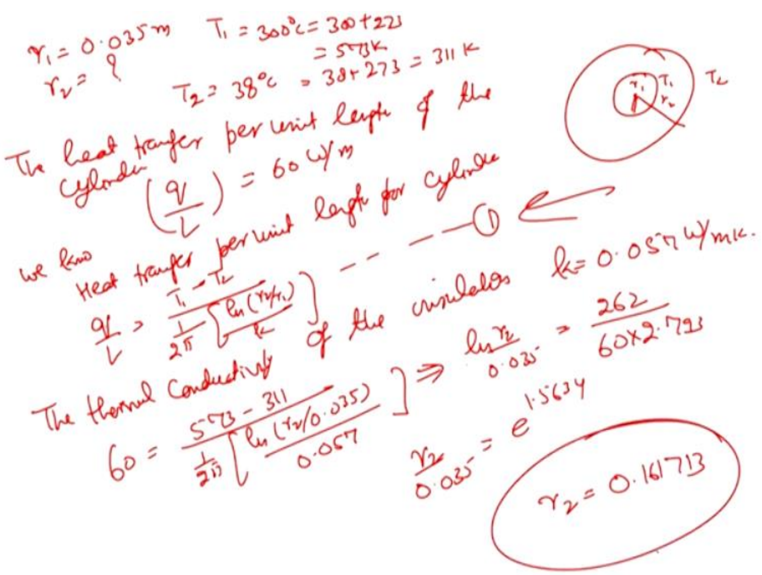

Now let us solve this particular problem now here it is given that r_1= 0.035 meter. We do not have any clue about r_2 T_1 = 300 degree Celsius and that is equal to 300+273 and that comes out to be 573 Kelvin. So let us draw the figure first that is r_1, r_2, T_1 and T_2. So, T_2 is equal to it is given that

38 degree Celsius so 38 + 273 it comes out to be 311 Kelvin. Now the heat transfer per unit length of the cylinder that is q over L = 60 watt per meter.

So, we know the heat transfer per unit length for cylinder so q upon L= $T_1 - T_2$ upon 1 upon 2 Π ln r_2 over r 1 upon k that is equation number 1. The thermal conductivity of the insulation that is k is equal to which is given at k = 0.057 watt per Kelvin. Now if you substitute these values in this particular equation, we will have 60 = 573 - 311 upon 1 upon 2 Π into L n r 2 over 0.035 upon 0.057 and this is ln r_2 over 0.035 = 262 upon 60 into 2.793 and r_2 = 0.035 that is equal to e to the power 1.5634 and 2 comes out to be 0.161713.

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Given;

r_1= 0.035 m

T_1 = 300_oC = 300 + 273= 573K

Heat transfer per unit length of the cylinder

q/L = 60 W/m

as we know heat transfer per unit length of the cylinder can be written as;

$$\frac{q}{L} = \frac{2\pi(T - T_o)}{(\ln\frac{r_2}{r_1})/k}$$

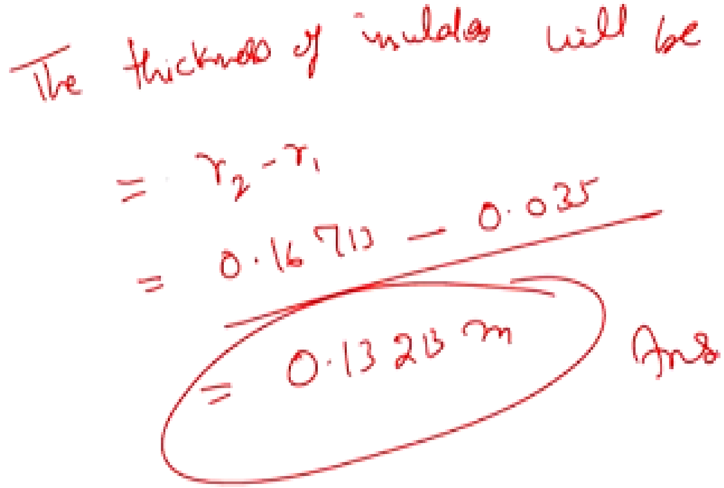
The thermal conductivity of the insulator is k = 0.057 W/mK

On putting above values in the above formula we get;

$$60 = \frac{2\pi(573 - 311)}{(\ln\frac{r_2}{0.035})/0.057}$$

On solving we get;

r_2 = 0.161713 m



The thickness of insulation will be r_2 - r_1 and that is 0.16713 - 0.035 and that is 0.13213 meter and that is our desired answer.

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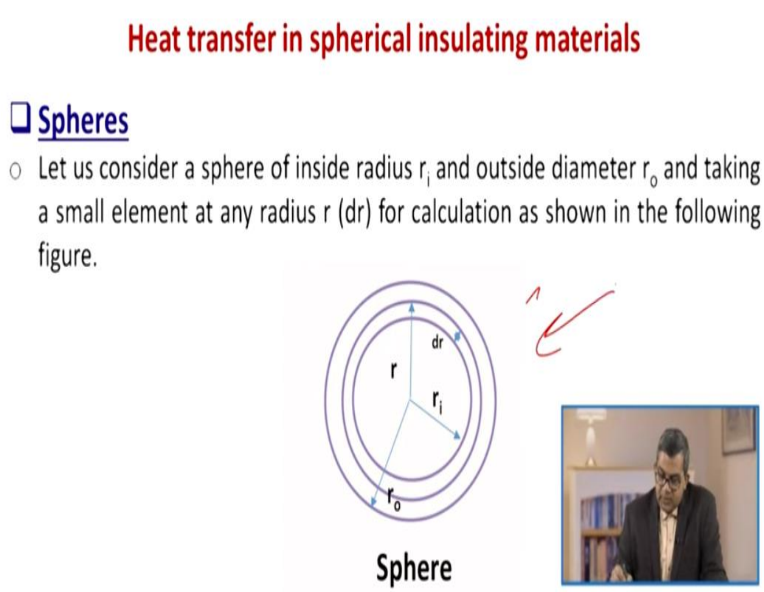
The thickness of the insulation will be = r_2-r_1
= 0.16173-0.035
= 0.13213 m



Now let us talk about the heat transfer in spherical insulating material. So first in this category is the sphere. So let us consider a sphere this is the sphere of inside radius is given as r_i and outside diameter or radius is given as r naught and taking a small element of any radius and that is d r this one for calculation as per this particular figure.

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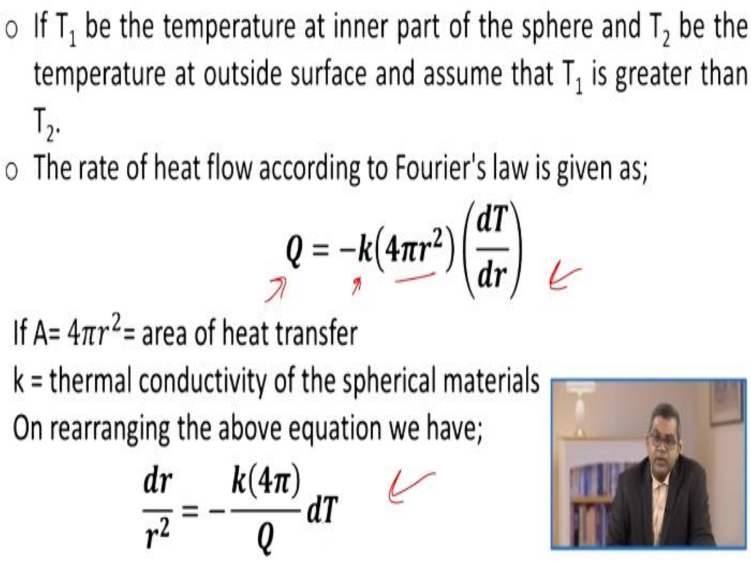

Now if T_1 be the temperature at the inner part of this sphere and T_2 be the temperature of the outside surface and assume that T_1 is greater than T_2 that is the pi flow the rate of heat flux according to the Fourier law is given as q = - k 4 Π r square d T over dr where a is we can write at a is equal to 4 Π r square and that is the area of heat transfer k is the thermal conductivity of the spherical material. So, if we rearrange this particular equation then we will have d r over r square =- k into 4 pi upon Q dT.

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For Sphere;

The rate of heat flow according to Fourier's law is given as;

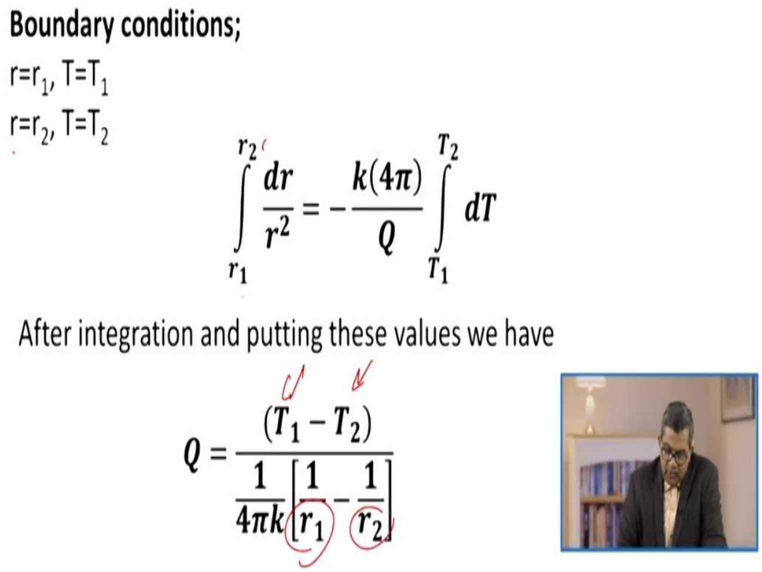
$$Q = -k(4\pi r^2)\left(\frac{dT}{dr}\right)$$

If A=$4\pi r^2$= area of heat transfer

k = thermal conductivity of the spherical materials

On rearranging the above equation we have;

$$\frac{dr}{r^2} = -\frac{k(4\pi)}{Q}dT$$



So let us adopt the boundary condition r = r_1 and T= T_1 and r = r_2 and T= T_2. So, if we integrate d r over r square from the limits r_1 to r_2 then it is equal to -k into 4 Π upon q integration from T_1 to T_2 d T. Now after integration and putting the values we have Q = T_1 – T_2 over 1 upon 4 Π k into 1 upon r_1–1 upon r_2.

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Boundary conditions;

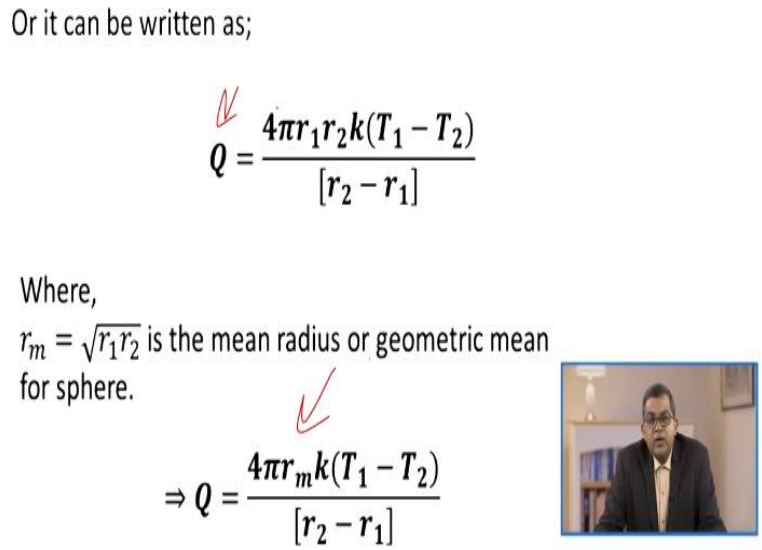
r=r_1, T=T_1

r=r_2, T=T_2

$$\int_{r_1}^{r_2} \frac{dr}{r^2} = -\frac{k(4\pi)}{Q} \int_{T_1}^{T_2} dT$$

After integration and putting these values, we have

$$Q = \frac{(T_1 - T_2)}{\frac{1}{4\pi k}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]}$$



Our conduction heat transfer through a sphere can be written as Q = T_1 =T_2 over 1 upon 4 Π k into 1 upon r_1 – 1 upon r_2. Now it is the ratio of temperature difference to the thermal resistance to heat transfer between the radius r_1 and r_2. Or, you can write as Q = 4 Π r_1 r_2 k into T_1 - T_2 upon r_2 - r_1. Now where r_m = square root of r_1, r_2. This is the mean radius or geometric mean for sphere. So if we write this particular thing into this equation it can become the Q = 4 Π r_m k into T_1 – T_2 upon r_2 - r_1.

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Or, conduction heat transfer through the sphere can be written as;

$$\boldsymbol{Q = \frac{(T_1 - T_2)}{\frac{1}{4\pi k}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]}}$$

It is the ratio of temperature difference to the thermal resistance to heat transfer between the radius r_1 and r_2.

Or it can be written as;

$$\boldsymbol{Q = \frac{4\pi r_1 r_2 k(T_1 - T_2)}{[r_2 - r_1]}}$$

Where,

$r_m = \sqrt{r_1 r_2}$ **is the mean radius or geometric mean for sphere.**

$$\Rightarrow Q = \frac{4\pi r_m k(T_1 - T_2)}{[r_2 - r_1]}$$

Problems

Question 3: If a spherical ball of steel of inner radius 2cm and outer radius of 4cm is filled with hot matter and the temperature of it was 275°C. The thermal conductivity of the steel is 7.68 W/m.K. If it is exposed to ambient temperature (25°C), the heat loss will be?

The convective heat transfer coefficient is 0.20 $W/m^2.K$.

Now let us discuss another problem which is related to the spherical issue. Now if a spherical ball of steel of inner radius 2 centimeter and outer radius of 4 centimeter is filled with hot matter and the temperature of it was say 275 degree Celsius. The thermal conductivity of the steel is 7.68 watt per meter Kelvin. Now if it is exposed to ambient temperature 25 degree Celsius the heat loss would be calculated and the convective heat transfer coefficient is given as 0.20 watt per meter square Kelvin.

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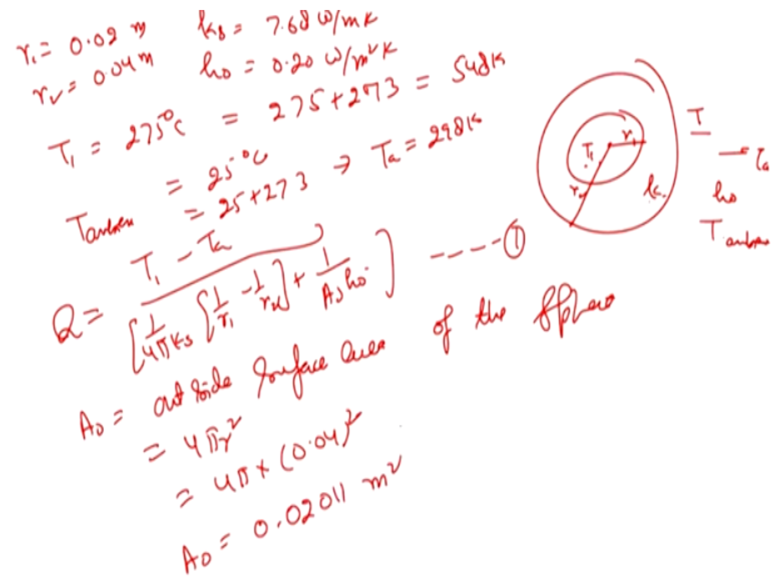

So, let us take up this take this problem so this is your sphere where we are having the r_1 r_2 and t 1 t and this one is k and h naught and t ambient. So, this is a sphere now it is given that r_1 = 0.02 meter, r_2 = 0.04 meter, ks is sphere = 7.68 watt per meter, Kelvin h naught = 0.20 watt per meter square Kelvin, T 1 = 275 degree Celsius. So, if we convert it into the Kelvin, it can it do it needs to be added with 273 and then T1 comes out to be 548 Kelvin.

T ambient = 25 degree Celsius, if 25 + 273 it comes out to be Ta = 298 Kelvin. Now the heat transfer from inner surface at T_1 to the outer surface at T will be through the conduction and from outside from our surface at T to m b and t a will be transferred by the convection so both will take place simultaneously. So let us write the equation $T_1 - T_a$ upon 1 upon 4 Π k s into 1 upon r_1- 1 upon r_2+ 1 upon a naught h naught and that is equation number 1.

Now A naught is outside surface area of the sphere which is equal to 4 Π r_2 square which is equal to 4 Π into 0.04 square and A naught = 0.02011 meter square.

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Given;

r_1 = 0.02 m

r_2= 0.04 m

k_s= 7.68 W/mK

h_o = 0.20 W/m^2K

T_1= 275°C = 275 + 273 = 548K

$T_{ambient}$ = 25+273 = 298K

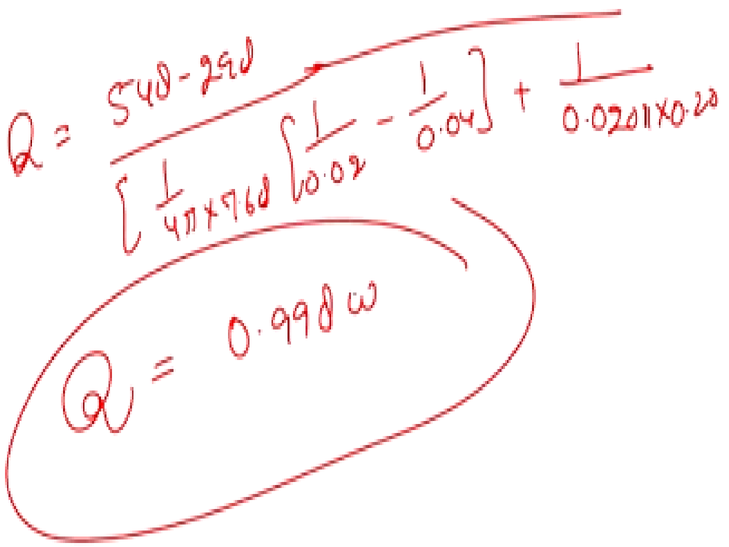
$$Q = \frac{(T_1 - T_a)}{\frac{1}{4\pi k_s}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] + \frac{1}{A_o h_o}}$$

Ao = outside surface area of the sphere
= $4\Pi r^2$
= $4\Pi(0.04)^2$
= 0.02011 m^2

On putting all values in the above heat transfer equation, we will get;

$$\boldsymbol{Q = \frac{(548 - 298)}{\frac{1}{4\pi\times 7.68}\left[\frac{1}{0.02} - \frac{1}{0.04}\right] + \frac{1}{0.02011\times 0.20}}}$$

$$\boldsymbol{Q = 0.998\, W}$$



Now if we substitute all given values in equation given equation then Q = 548 - 298 upon 1 upon 4 pi into 7.68 into 1 upon 0.02 – 1 upon 0.04 + 1 upon 0.02011 into 0.20 and that comes out to be Q = 0.998 watt and that is our desired answer.

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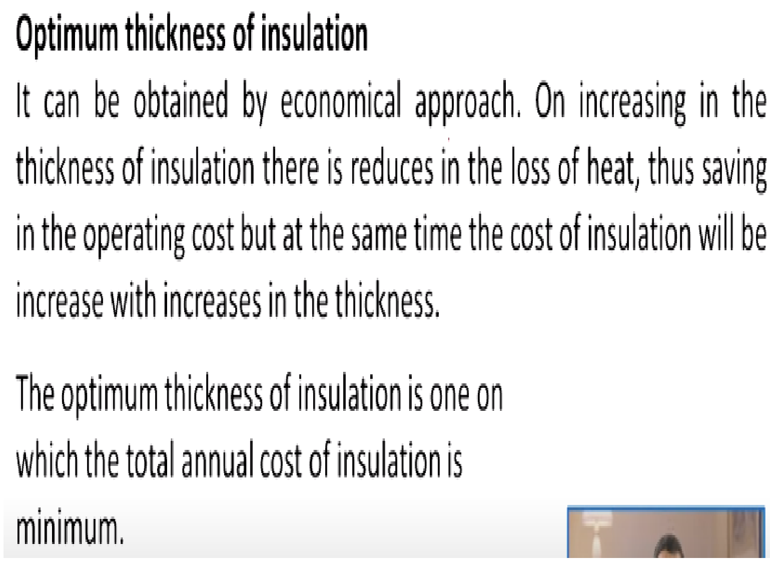

Now let us talk about the optimum thickness of insulation. Now it can be obtained by economical approach on increasing in the thickness of insulation there is a reduction in the loss of heat. Thus, saving in the operating cost but at the same time the cost of insulation will be increased with increase in the thickness. The optimum thickness of insulation is one on which the total annual cost of insulation is minimum.

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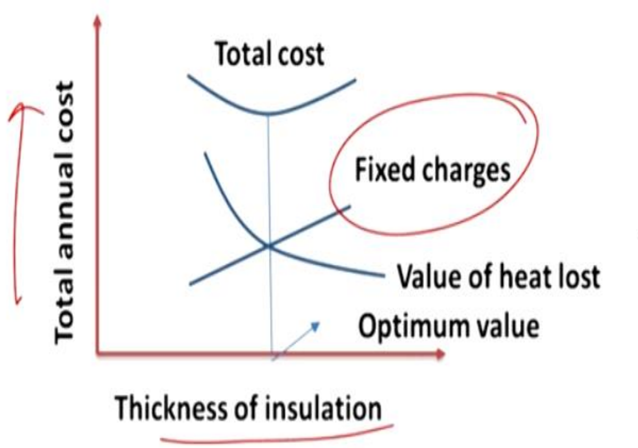

Figure showing optimum thickness of insulation

Now here you see that the optimum insulation thickness now here the total annual cost and the thickness of insulation. So, you need to find out the optimum value based on the fixed charge and the value of heat loss and this is again a well-known phenomenon.

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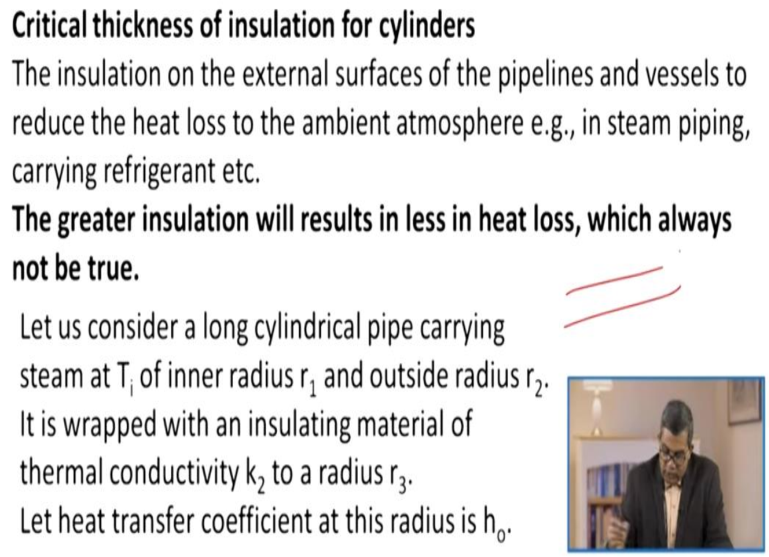

Let us talk about the critical thickness of insulation for cylinder. Now for the insulation on the external surface of the pipeline and the vessel to reduce the heat loss to the ambient temperature and that is in the steam piping carrying the refrigerant etc. So, the greater insulation will result in less in heat loss which always not be true. So let us consider a long cylinder pipe carrying the steam at T_1 of the inner radius r_1 and outside the radius r_2. It is wrapped with an insulating material of the thermal conductivity k_2 to a radius r_3. So let heat transfer coefficient at this radius is h naught.

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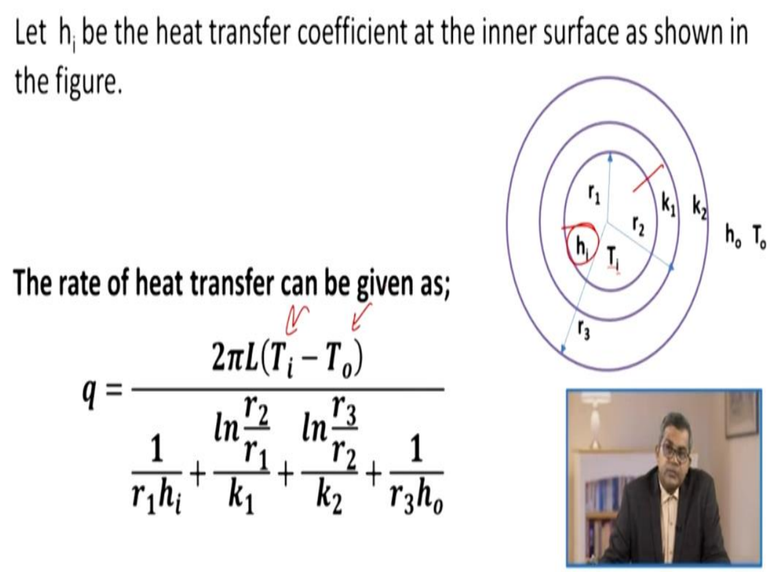

Now h_i be the heat transfer coefficient at the inner surface which is as reflected in this particular figure. Now the rate of heat transfer can be given as q = 2 Π L into T_i- T naught over 1 upon r_1 h_i + ln r_2 over r_i over k_1 +ln r_3 over r_2 over k_2 + 1 over r_3 h naught.

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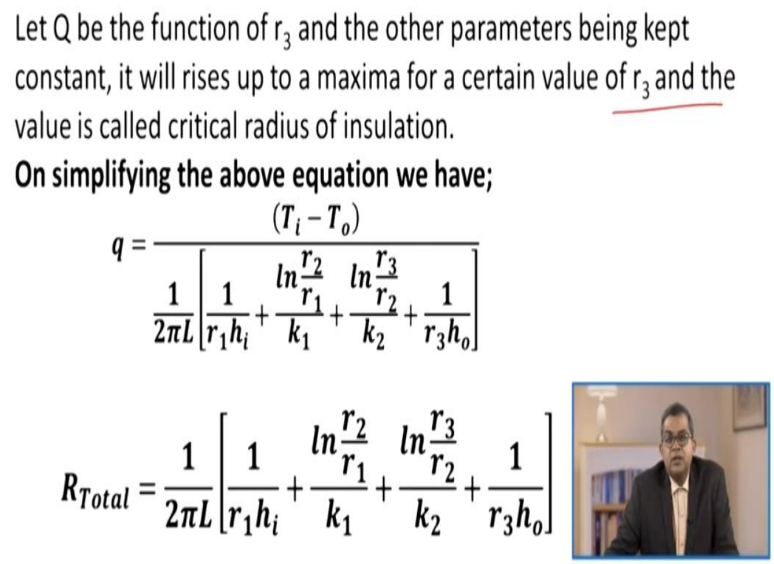
Let h_i be the heat transfer coefficient at the inner surface as shown in the figure.

The rate of heat transfer can be given as;

$$q = \frac{2\pi L(T_i - T_o)}{\frac{1}{r_1 h_i} + \frac{ln\frac{r_2}{r_1}}{k_1} + \frac{ln\frac{r_3}{r_2}}{k_2} + \frac{1}{r_3 h_o}}$$



So let Q be the function of r_3 and the other parameter being kept constant it will rise up to a maxima of a certain value of r_3 and the value is called the critical radius of insulation. So, if we simplify this particular equation, we will have q = T_1 –T naught over 1 upon 2 Π L into 1 upon r_1 h_i + ln r_2 over r_1 over k_1 + ln r_3 over r_2 over k_2 + 1 over r_3 h naught and by this way we can calculate the R total and that comes out to be our total is equal to 1 upon 2 Π L into 1 upon r_1 h_i + ln r_2 over r_1 + over k_1 + ln r_3 over r_2 over k_2 + 1 over r_3 h naught.

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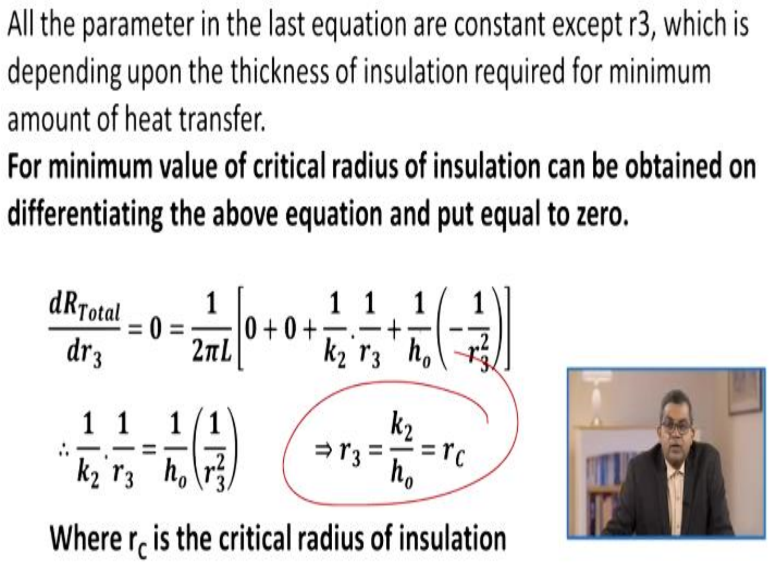
On simplifying the above equation, we have;

$$q = \frac{(T_i - T_o)}{\frac{1}{2\pi L}\left[\frac{1}{r_1 h_i} + \frac{ln\frac{r_2}{r_1}}{k_1} + \frac{ln\frac{r_3}{r_2}}{k_2} + \frac{1}{r_3 h_o}\right]}$$

$$R_{Total} = \frac{1}{2\pi L}\left[\frac{1}{r_1 h_i} + \frac{\ln\frac{r_2}{r_1}}{k_1} + \frac{\ln\frac{r_3}{r_2}}{k_2} + \frac{1}{r_3 h_o}\right]$$



So, the all the parameter in the last equation they are constant except r_3, which is depending upon the thickness of insulation and required for minimum amount of heat transfer. For minimum value of critical radius of insulation, it can be obtained on differentiating this particular equation and put equal to 0. So, by this way if we substitute all these things, it can become $r_3 = k_2$ over h naught = r_c where r_c is the critical radius of insulation. So, by this way you can calculate the critical radius of insulation.

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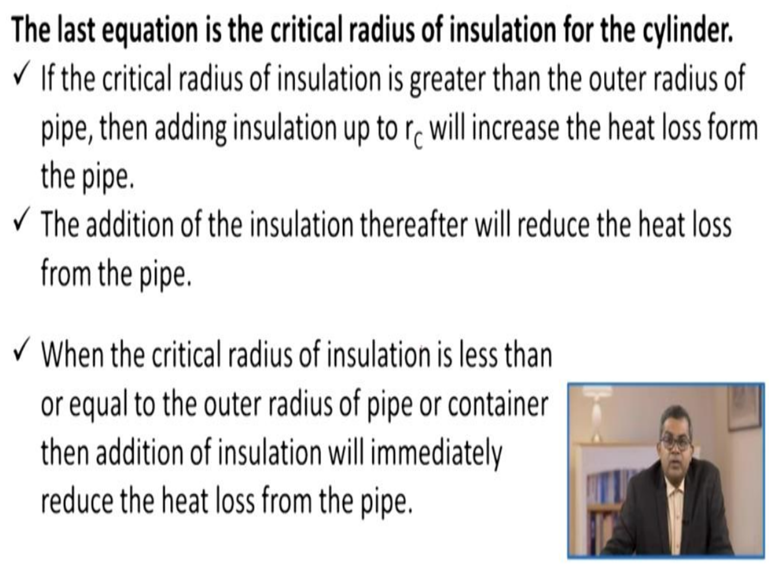
For minimum value of critical radius of insulation can be obtained on differentiating the above equation and put equal to zero.

$$\frac{dR_{Total}}{dr_3} = 0 = \frac{1}{2\pi L}\left[0 + 0 + \frac{1}{k_2}.\frac{1}{r_3} + \frac{1}{h_o}\left(-\frac{1}{r_3^2}\right)\right]$$

$$\therefore \frac{1}{k_2}.\frac{1}{r_3} = \frac{1}{h_o}\left(\frac{1}{r_3^2}\right) \qquad \Rightarrow r_3 = \frac{k_2}{h_o} = r_C$$

Where r_C is the critical radius of insulation



Now the last equation is the critical radius of insulation for the cylinder. Now if the critical radius of insulation is greater than the outer radius of the pipe then adding insulation up to rc will increase the heat loss from the pipe. The addition of the insulation there after reduces the heat loss from the pipe. When the critical radius of insulation is less than or equal to the outer radius of the pipe or container then addition of insulation will immediately reduce the heat loss from the pipe.

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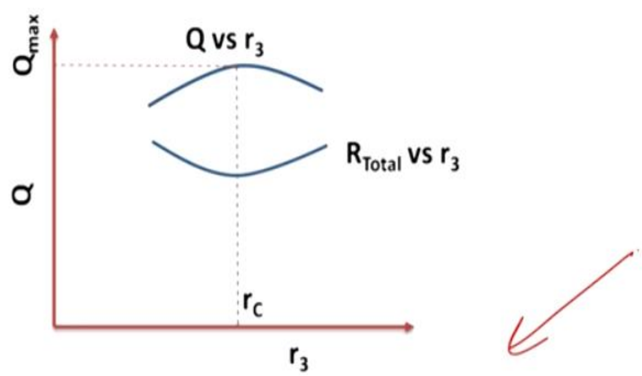

Figure showing relationship in thickness of insulation and Q and r_3.

Now this is the figure which reflects the relationship in thickness of insulation and Q and r_3.

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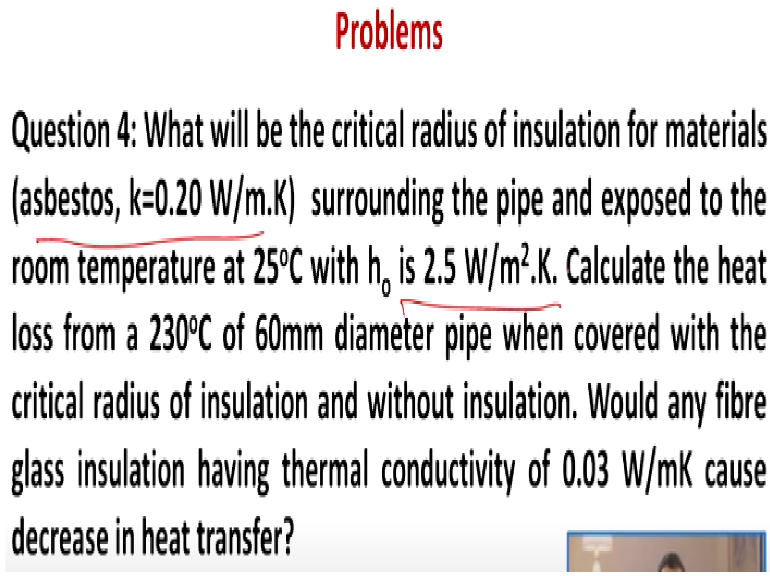

Now let us talk about problem now here the problem statement says that what will be the critical radius of insulation for material that is asbestos in this case where k = 0.20 watt per meter Kelvin. Surrounding the pipe and exposed to the room temperature at 25 degree Celsius with h naught is 2.5 watt per meter square Kelvin. Now you need to calculate the heat loss from a 230 degree Celsius of a 60 millimeter diameter pipe when covered with the critical radius of insulation and without insulation.

Would any; fiber glass insulation having the thermal conductivity of 0.03 watt per meter Kelvin cause decrease in heat transfer.

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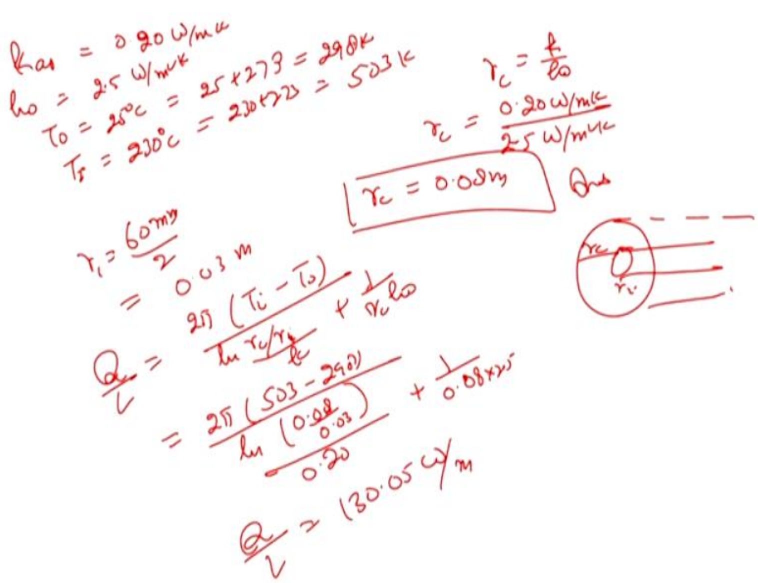


So let us solve this problem it is given that k for asbestosis 0.20 watt per meter Kelvin, h naught is given as 2.5 watt per meter square Kelvin. t naught is given as 25 degree Celsius and that comes out to be 273 that is equal to 298 Kelvin, T_i= 230 degree Celsius and that is 230 + 273 this comes out to be 503 Kelvin. Now we know the critical radius of insulation for the cylinder is given by r_c = k upon h naught. So, r_c = 0.20 watt per meter Kelvin upon 2.5 watt per meter square Kelvin and r_c = 0.08 meter that is one answer.

Now the diameter of the bare cylinder now here you see now the diameter of the bare cylinder is given as r_1 = 60mm upon 2 and that is equal to 0.03 meter. Now heat transfer with insulation that is Q over L = 2 Π T_i – T naught upon lnr_c over r_1 upon k + 1 over r_c over h naught and if you substitute the value 503 - 298 upon ln 0.08 over 0.03 over 0.20 + 1 upon 0.8 into 2.5. Now upon solving it comes out to be Q over L= 130.05 watt per meter. Now without using insulation now the heat will flow through the outer surface of the bare cylindrical pipe to the ambient atmosphere

(Refer Slide Time: 30:29)

Given;

k_{as} = 0.20 W/mK

h_o = 2.5 W/m2K

T_o = 25oC = 25+273= 298K

T_s = 230oC= 230+273= 503K

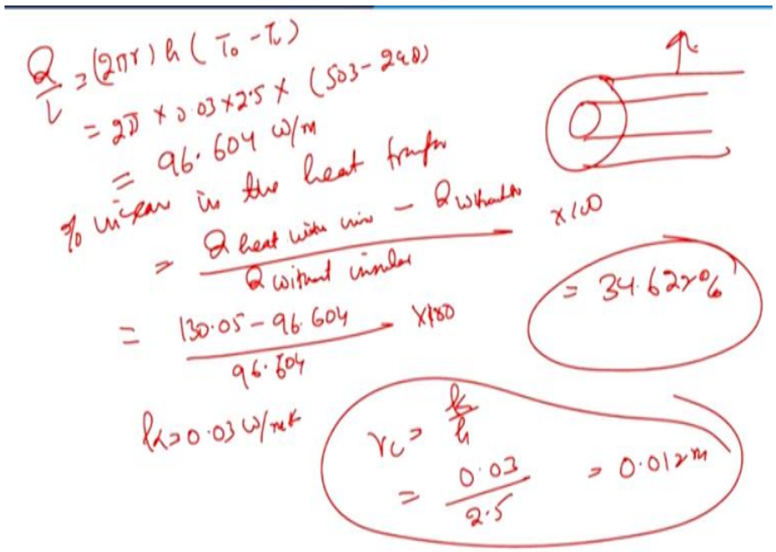
r_1= 60/2= 30mm = 0.03 m

r_c = k/ho = 0.20/2.5 = 0.08 m

$$q = \frac{(T_i - T_o)}{\frac{1}{2\pi L}\left[\frac{ln\frac{r_c}{r_1}}{k_{as}} + \frac{1}{r_o h_o}\right]}$$

$$q = \frac{(503 - 298)}{\frac{1}{2\pi L}\left[\frac{ln\frac{0.08}{0.03}}{0.20} + \frac{1}{0.08\times 2.5}\right]}$$

$$q = 130.05\ W/m$$



If you see here it will go to the ambient atmosphere at 25 degree Celsius. Now this is a case of convection convective heat transfer so in that case you can write 2 Π r into h into T naught– T_i which is 2 Π into 0.03 into 2.5 into 503 – 298 and that comes out to be 96.604 watt per meter. Now the percentage increase in the heat transfer that is equal to Q heat transfer within with insulation - Q without installation over Q without insulation multiplied by 100.

And that is 130.05 - 96.604 divided by 96.604 into 100 and this comes out to be 34.622% that is the answer. Now if a given fiber glass insulation, then k = 0.03 watt per meter Kelvin and the critical rate radius of insulation rc would be k over h and that comes out to be 0.03 upon 2.5 which is equal to 0.012 meter. Now the value of rc is less than the outside radius of the pipe and that is 0.03. So, addition of any fiber glass insulation would cause a decrease in heat transfer. So that is the issue.

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$$\frac{Q}{L} = (2\pi r)h(T_o - T_{am})$$

$$\frac{Q}{L} = 2\pi \times 0.03 \times 2.5 \times (503 - 298)$$

$$\frac{Q}{L} = 96.604\ W/m$$

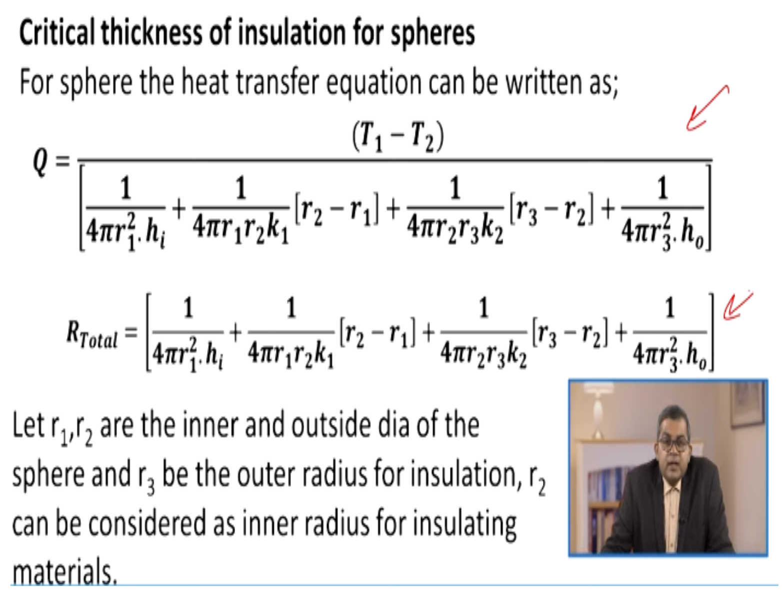
$$Percentage\ increase\ in\ the\ heat\ transfer = \frac{Q_{heat\ with\ insulation} - Q_{without}}{Q_{without\ insulation}}$$

$$Percentage\ increase\ in\ the\ heat\ transfer = \frac{130.05 - 96.604}{96.604} \times 100 = 34.62\%$$

K = 0.03 W/mK

r_C = k/h

= 0.03/2.5 = 0.012 m (answer)



Now let us talk about the critical thickness of insulation for spheres for spheres the heat transfer equation can be written as this particular equation. And when we talk about the r total this can be written as this particular equation number 2. Now let r_1 and r_2 are the inner and outside diameter of the sphere and r_3 be the outer radius of the insulation and r_2 can be considered as inner radius for insulating material.

(Refer Slide Time: 33:05)

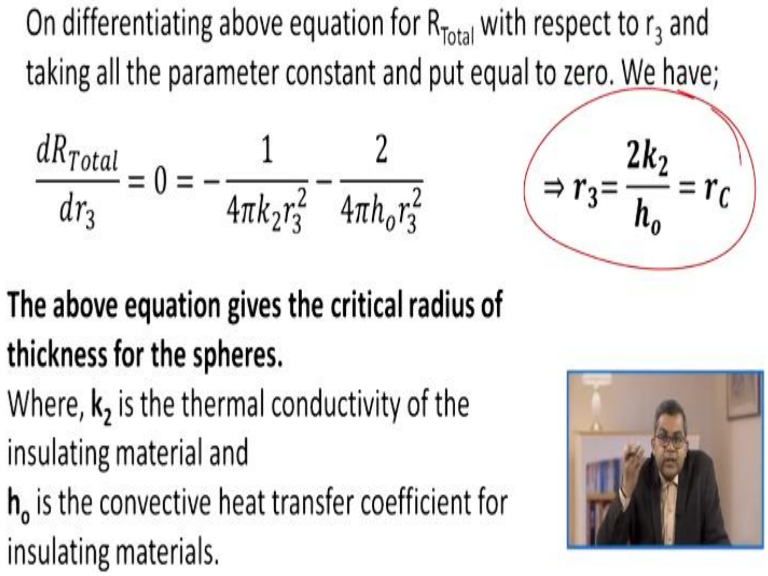
Critical thickness of insulation for spheres

For sphere the heat transfer equation can be written as;

$$Q = \frac{(T_1 - T_2)}{\left[\frac{1}{4\pi r_1^2 . h_i} + \frac{1}{4\pi r_1 r_2 k_1}[r_2 - r_1] + \frac{1}{4\pi r_2 r_3 k_2}[r_3 - r_2] + \frac{1}{4\pi r_3^2 . h_o}\right]}$$

$$R_{Total} = \left[\frac{1}{4\pi r_1^2 . h_i} + \frac{1}{4\pi r_1 r_2 k_1}[r_2 - r_1] + \frac{1}{4\pi r_2 r_3 k_2}[r_3 - r_2] + \frac{1}{4\pi r_3^2 . h_o}\right]$$

Let r_1,r_2 are the inner and outside dia of the sphere and r_3 be the outer radius for insulation, r_2 can be considered as inner radius for insulating materials.



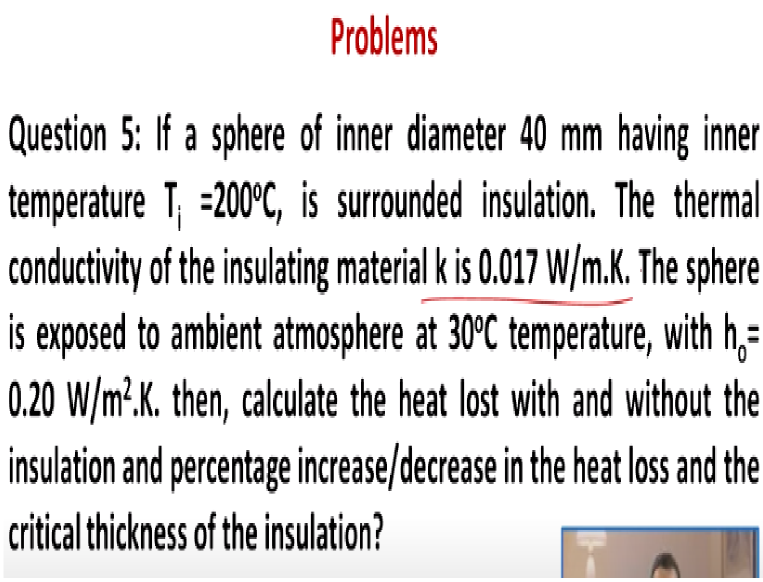
So, if you differentiate the above equation for r total with respect to r_3 and taking all the parameter constant and put equal to 0 then we have this r_3 is equal to our critical radius of thickness is equal to r_3 and that comes out to be 2 k_2 upon h naught. So, this equation this particular equation gives the critical radius of thickness for the sphere. Now where k_2 is the thermal conductivity of the insulating material and h naught is the convective heat transfer coefficient for the insulating material. Now let us discuss about another problem.

(Refer Slide Time: 33:42)

On differentiating above equation for R_{Total} with respect to r_3 and taking all the parameter constant and put equal to zero. We have;

$$\frac{dR_{Total}}{dr_3} = 0 = -\frac{1}{4\pi k_2 r_3^2} - \frac{2}{4\pi h_o r_3^2}$$

$$\Rightarrow \boldsymbol{r_3 = \frac{2k_2}{h_o} = r_C}$$



Now if a sphere of inner diameter having a 40 mm having the inner temperature T_i= 200 degree Celsius is surrounded by insulation. The thermal conductivity of the insulating material k is 0.017 watt per meter kelvin and the sphere is exposed to ambient atmosphere at 30 degree Celsius temperature, with h naught = 0.20 watt per meter square Kelvin. Then you need to calculate the heat loss with and without the insulation and the percentage increase and decrease in the heat loss and the critical thickness of the insulation.

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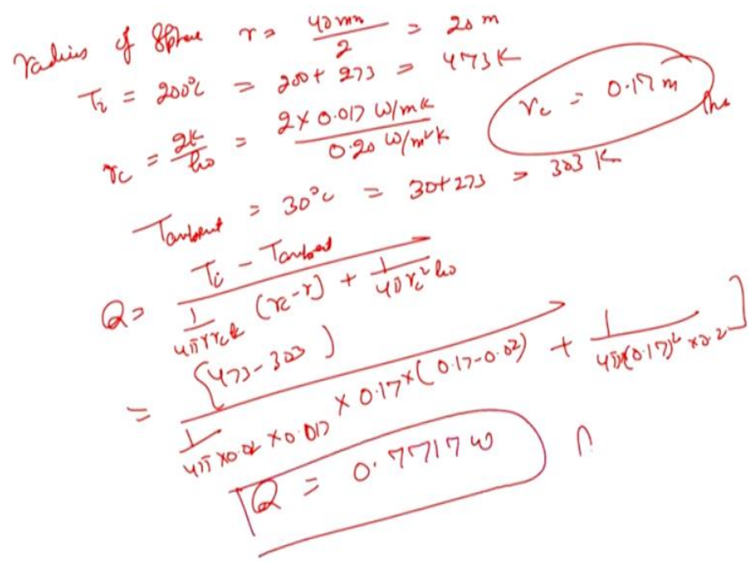

So let us solve this particular problem now here the radius of sphere is given r = 40 mm 0.2 = 20 mm. Now here T_i= 200 degree Celsius which is equal to 200 + 273 that comes out to be 473 Kelvin. Now the if sphere is surrounded with the insulating material then r_c 2 k upon h naught which is 2 into 0.017 watt per meter kelvin upon 0.20 watt meter square Kelvin and r_c = 0.17 meter that is the first answer.

Now T ambient is given as 30 degree Celsius or 30 + 273 that comes out to be 303 kelvin.So the rate of heat transfer in sphere with insulation that is Q = T_i- T ambient over 1 upon 4 Π r r_c k into r_c – r+ 1 upon 4 Π r_c square h naught and that is 473 -303 upon 1 upon 4 Π into 0.02 into 0.016 into 0.17 into 0.17- 0.02+ 1 upon 4 Π into 0.17 into 0.2 and the Q comes out to be 0.7717 watt and that is the answer.

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Given;

Radius of sphere (r) = 40mm/2 = 20 mm = 0.02 m

Ti = 200 oC = 200+273 = 473K

r_C= 2k/h_o = 2x0.017 W/mK/ (0.20 W/m2K)

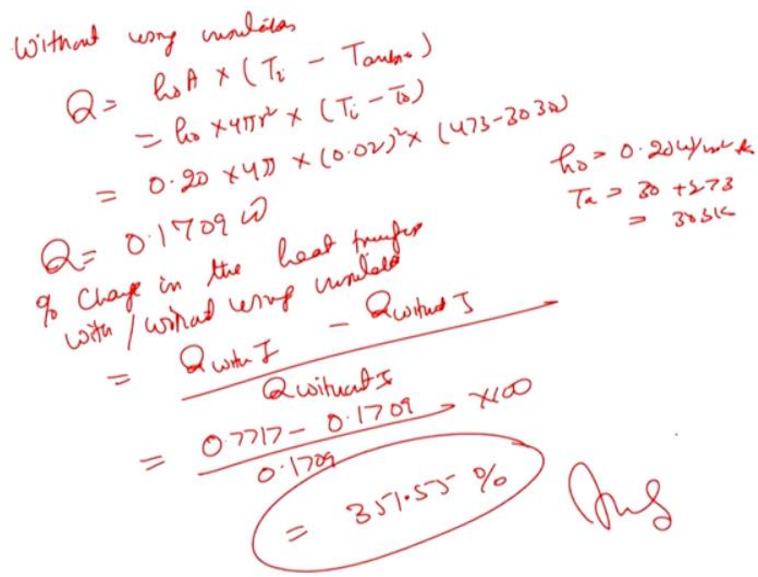
r_C = 0.17 m

Tambient = 30oC = 30+273 = 303K

On putting these values in the heat transfer equation

$$Q = \frac{(473 - 303)}{\left[\frac{1}{4\pi \times 0.02 \times 0.017 \times 0.17}[0.17 - 0.02] + \frac{1}{4\pi \times (0.17)^2 \times 0.2}\right]}$$

$$Q = 0.7717\,W$$



Now the heat transfer in sphere without using insulation so Q = h naught A into T_i- T ambient which is h naught into 4 Π r square into T_i – T naught and that is 0.20 into 4 Π into 0.02 square into 473 – 303 where h naught = 0.20 watt per meter square Kelvin T_a 30 + 273 which is equal to 303 Kelvin. So, Q = 0.1709 watt now the percentage change percentage change in the heat transfer with or without using insulation.

So, this is Q with insulation minus Q without insulation upon Q without insulation and that comes out to be 0.7717 - 0.1709 upon 0.1709 into 100 and that is equal to 351.55% and that is our desired answer. So, in this particular lecture we discussed about the various aspects of heat transfer, insulating material, and solved and we discussed the various cases like cylindrical aspect and then spherical aspect.

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Without any insulation:

$$Q = h_o A\,(T_i - T_{ambient})$$

$$Q = h_o \times 4\pi r^2 \times\ (T_i - T_{ambient})$$

On putting the values of the parameter, we have:

$$Q = 0.20 \times 4 \times \pi 0.02^2 \times\ (473 - 303)$$

$$Q = 0.1709\ W$$

% Change in the heat transfer with/without using insulation can be found by the following equations.

$$\%\ change\ in\ the\ heat\ transfer = \frac{Q_{With\ insulation} - Q_{without\ insulation}}{Q_{Without\ insulation}} \times 100$$

$$\%\ change\ in\ the\ heat\ transfer = \frac{0.7717 - 0.1709}{0.1709} \times 100$$

$$\%\ change\ in\ the\ heat\ transfer = 35.55\%\ (answer)$$

References

- Richard T. Bynum, Insulation Handbook, McGraw-Hill Companies, Inc., (2014), ISBN: 0-07-134872-7, DOI: 10.1036/0071414614.
- Alireza Bahadori, Thermal Insulation Handbook for oil, gas and petrochemical industries, Elsevier (2013), ISBN: 978-0-12-800010-6.

For your convenience we have included couple of references and if you wish you can have a look of all those references for further study. Thank you very much.