

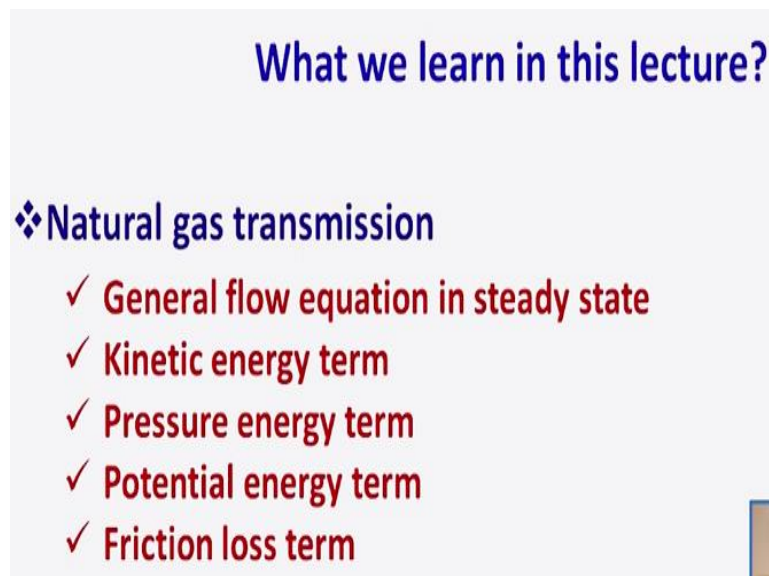
Chemical Process Utilities
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Lecture – 39
Natural Gas Transmission-I

Welcome to the new concept of natural gas transmission under the aegis of chemical process utilities. Now, before we go into the detail, let us have a brief outlook about that what we studied in the previous lecture. We discussed about the various elements of pipeline design. Under this, we discussed about the fluid properties, we discuss about the impact of environment, we discussed about the effect of temperature and pressure.

We discussed about the typical flow equations for liquid and gas and gave you an idea about the various codes and standards. We discussed the different societies those who design those codes and standards. And apart from this, we discussed about the environmental and hydrological consideration. Now, in this particular chapter we are going to discuss about the natural gas transmission.

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What we learn in this lecture?

- ❖ **Natural gas transmission**
- ✓ **General flow equation in steady state**
- ✓ **Kinetic energy term**
- ✓ **Pressure energy term**
- ✓ **Potential energy term**
- ✓ **Friction loss term**

Here we will discuss the general flow equation in steady state. We will discuss about the kinetic energy term. We will discuss about the pressure energy term. We will have an idea about the potential energy term and then we will discuss about the frictional loss term.

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Natural gas transmission

General Flow Equation in Steady State

Herein, general flow equation for compressible fluids in a pipeline at steady state condition is derived.

Consider a pipeline that transports a compressible fluid (natural gas) between points 1 and 2 at steady state condition.

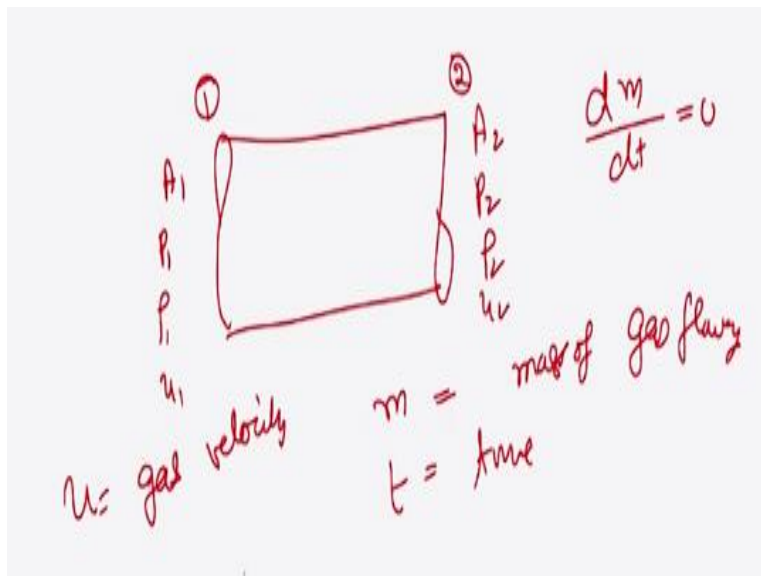
$$\frac{dm}{dt} = 0$$

So, let us talk about the general flow equation in the steady state. Now, the general flow equation for compressible fluid in pipeline at steady state condition is usually derived. Now, let us consider a pipeline that transports compressible fluid from point, maybe this is a natural gas from point number 1 to point number 2 at a steady state condition. So, the equation can be written as $\frac{dm}{dt} = 0$.

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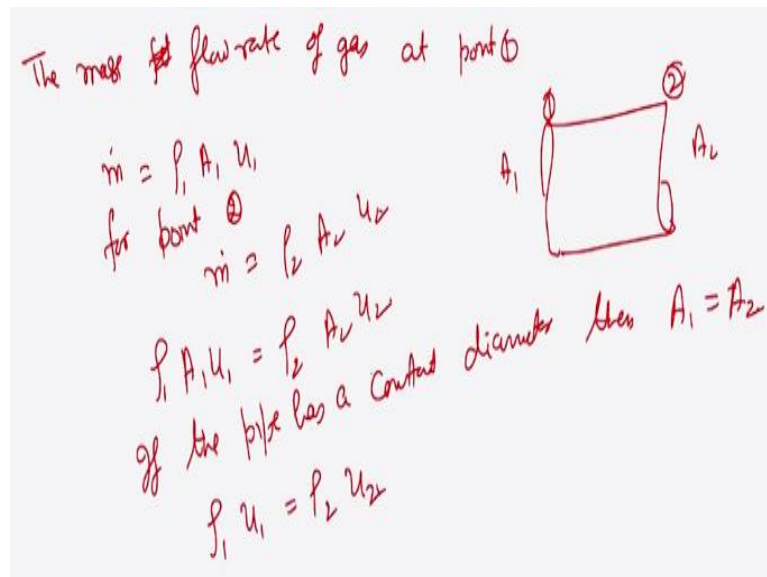
At steady state condition:

$$\frac{dm}{dt} = 0$$



Now, let us give you an idea about the pipeline. Now, this is the station number 1 and this is the station number 2. Now, here the A_1 , P_1 , ρ_1 , and u_1 . This is the area of cross section A , P is the pressure, ρ is the density and u_1 is the velocity that is the gas velocity and here this is the A_2 , P_2 , ρ_2 , u_2 . So, u is the gas velocity, m is the mass of gas flowing and t is the time for this equation dm over dt is $= 0$.

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Now, the mass flow rate of gas at point 1, again I am drawing this can be defined as $\rho_1 A_1 u_1$ and for point 2 $\rho_2 A_2 u_2$. So, if the above equation we can take $\rho_1 A_1 u_1 = \rho_2 A_2 u_2$. If the pipe has a constant diameter, then obviously A_1 will be equal to A_2 , so in that case $\rho_1 u_1 = \rho_2 u_2$.

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The mass flow rate of gas at point 1 can be defined as;

$$\dot{m} = \rho_1 \cdot A_1 \cdot u_1$$

Similarly, mass flow rate at point 2 is;

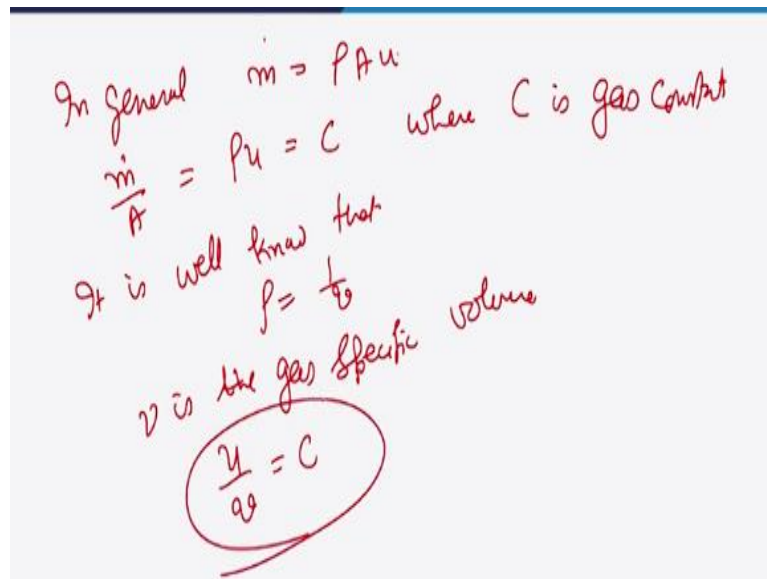
$$\dot{m} = \rho_2 \cdot A_2 \cdot u_2$$

From the above equation we have;

$$\rho_1 \cdot A_1 \cdot u_1 = \rho_2 \cdot A_2 \cdot u_2$$

If the pipe has a constant diameter, then

$$\rho_1 \cdot u_1 = \rho_2 \cdot u_2$$



So, in general if we say that $\dot{m} = \rho A u$, now \dot{m} upon $A = \rho u$ and that is equal to C where C is gas constant. Now, it is well known that $\rho = 1$ upon v , now v is the gas specific volume. Therefore, u upon $v = C$ in this particular equation.

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In general

$$\dot{m} = \rho \cdot A \cdot u$$

$$\frac{\dot{m}}{A} = \rho u = C$$

Where, C is gas constant

It is well known that;

$$\rho = \frac{1}{v}$$

Where v is gas specific volume, so;

$$\frac{u}{v} = C$$

from Newton's law of motion for a particle of gas moving in a pipeline

$$dF = a \, dm$$

where $\frac{du}{dt} = a$ (acceleration)

$$dF = \frac{du}{dt} \, dm = \frac{du}{dt} \rho A \, dy = \rho A \, du \frac{dy}{dt}$$

at & $\frac{dy}{dt} = u$

Therefore $dF = \rho A u \, du$

Now from Newton's law of motion for a particle of gas moving in a pipeline, then $df = a \, dm$ where $du \text{ over } dt = a$ that is acceleration. So, $df = du \text{ over } dt \, dm$ or $du \text{ over } dt \, \rho A \, dy$ and that is $\rho A \, du \, dy \text{ over } dt$ and $dy \text{ over } dt = u$. Therefore $df = \rho A u \, du$.

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From Newton's law of motion for a particle of gas moving in a pipeline

$$dF = a \, dm$$

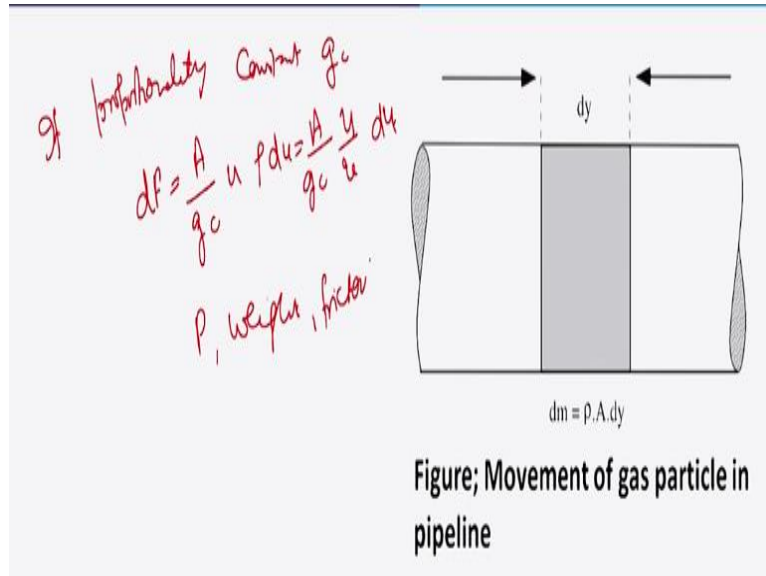
Where, $\frac{du}{dt} = a$ is the acceleration;

$$dF = \frac{du}{dt} \, dm = \frac{du}{dt} \rho \cdot A \cdot dy = \rho \cdot A \cdot du \cdot \frac{dy}{dt}$$

And, $\frac{dy}{dt} = u$

Therefore,

$$dF = \rho \cdot A \cdot u \cdot du$$

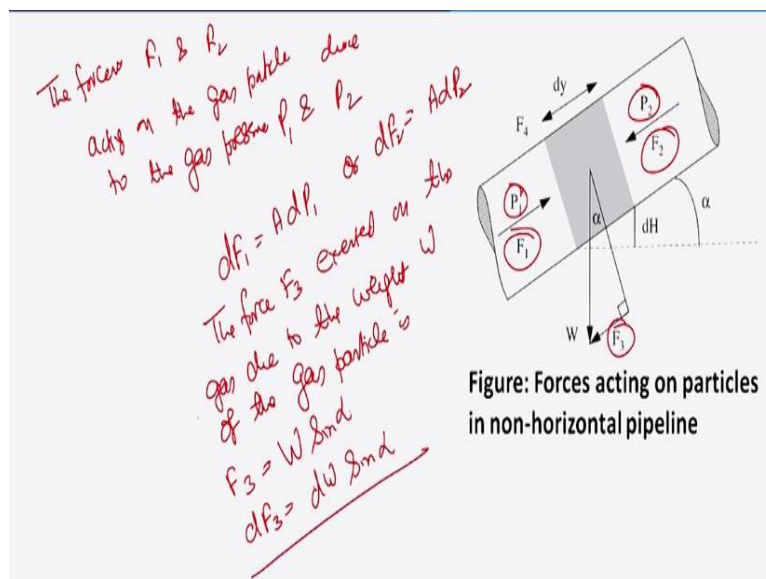


Now, if by using the proportionality constant g_c , if we use the proportionality constant g_c so we can write this equation $dF = \frac{A}{g_c} \rho u du = \frac{A}{g_c} \rho u v du$. So, the impact of all existing forces that is pressure, weight, friction exerted on a particle of gas in a non-horizontal pipeline this can be considered in different equation.

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By using proportionality constant g_c above equation can be written as;

$$dF = \frac{A}{g_c} \cdot \rho \cdot u \cdot du = \frac{A}{g_c} \cdot \frac{u}{v} \cdot du$$



Now, let us take the forces f_1 and f_2 . This is the f_2 and f_1 acting on the gas particle due to the gas pressure P_1 and P_2 . This can be defined as $df_1 = A dP_1$ or $df_2 = A dP_2$. So, this is the force acting on particles in non-horizontal pipeline. Now, the force f_3 exerted on the gas due to the weight W of the gas particle is $f_3 = W \sin \alpha$ or we can write in the differential form that is $df_3 = dW \sin \alpha$.

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The forces F_1 and F_2 acting on the gas particle due to the gas pressure P_1 and P_2 can be defined as;

$$dF_1 = AdP_1 \text{ or, } dF_2 = AdP_2$$

The force F_3 exerted on the gas due to the weight W of the gas particle is

$$F_3 = W \cdot \sin \alpha \text{ or, in differential form; } dF_3 = dW \cdot \sin \alpha$$

where the weight of the gas is
 $dW = \frac{\rho L}{g_c} A dy$
 where ρ is local acceleration of gravity
 $\sin \alpha = \frac{dH}{dy}$
 where dH is the change in elevation on substitution for both dW & $\sin \alpha$
 $dF_3 = \frac{\rho L}{g_c} A dy \frac{dH}{dy}$ or
 $dF_3 = \frac{\rho L}{g_c} A dH$

Now where the weight of the gas $dW = \rho L$ over g_c $A dy$ rho, now where ρ is local acceleration of gravity. Now, $\sin \alpha = dH$ upon dy where dH is the change in elevation on substitution for both dW and $\sin \alpha$. So, $df_3 = \rho L$ over g_c A rho dH or $dF_3 = \rho L$ over g_c A over v dH .

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Where, the weight of the gas is

$$dW = \frac{g_L}{g_c} \cdot A \cdot dy \cdot \rho$$

Where, g_L is local acceleration of gravity,

Furthermore;

$$\sin \alpha = \frac{dH}{dy}$$

Where, dH is the change in elevation,

on substitution for both dW and $\sin \alpha$

$$dF_3 = \frac{g_L}{g_c} \cdot A \cdot \rho \cdot dH \text{ or } dF_3 = \frac{g_L}{g_c} \cdot \frac{A}{v} \cdot dH$$

finally the friction force is defined as
 $df_4 = \pi D dy \tau$
 where $\pi D dy$ is the surface area and τ is shear stress
 The summation of all the forces acting on the element
 of the gas should be equal to zero
 $\frac{A}{g_c} \frac{v}{g_0} du + A dp + \frac{g_L}{g_c} \frac{A}{v} dH + \pi \cdot D dy \rho \tau = 0$
 This is Bernoulli's equation
 Numerical values of g_L & $g_0 = 1$

Now, finally the friction force is defined as $df_4 = \pi D dy \tau$ where $\pi D dy$ is the surface area and τ is shear stress. Now, the summation of all the forces acting on the element of the gas

should be equal to 0. So, therefore $\frac{A}{g_c} \frac{u}{v} du + A dP + \frac{A}{g_c} \frac{dH}{v} + \pi D dy \rho \tau = 0$. This is the generalized form of Bernoulli's equation. Now, in most cases the numerical values of g_L and g_c it is equal to 1.

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Finally, the friction force is defined as;

$$dF_4 = \pi \cdot D \cdot dy \cdot \tau$$

Where, $\pi \cdot D \cdot dy$ is the surface area and τ is the shear stress.

The summation of all the forces acting on the element of the gas should be equal to zero, therefore;

equal to zero, therefore;

$$\frac{A}{g_c} \frac{u}{v} du + A dP + \frac{A}{g_c} \frac{dH}{v} + \pi \cdot D \cdot dy \cdot \rho \cdot \tau = 0$$

This is general form of Bernoulli equation. In most cases the numerical values of g_L and g_c are equal to 1.

the numerical values of g_L and g_c are equal to 1.

Then $\frac{A}{g_c} \frac{u}{v} du + A dP + \frac{A}{g_c} \frac{dH}{v} + \pi D dy \rho \tau = 0$
 then $\frac{v}{A} \frac{1}{g_c} u du + v dp + dH + \frac{\pi D dy v}{A} \tau = 0$
 where $u \cdot du =$ Kinetic energy
 $v dp =$ Pressure Energy
 $dH =$ Potential Energy
 $\frac{\pi D dy v}{A} \tau =$ friction or losses

Now if you take this value then $\frac{A}{g_c} \frac{u}{v} du + A dP + \frac{A}{g_c} \frac{dH}{v} + \pi D dy \rho \tau = 0$. Now, if we multiply both sides by $\frac{v}{A}$, then $\frac{1}{g_c} u du + v dp + dH + \pi D dy v \rho \tau = 0$, now where $u \cdot du$ is equal to kinetic energy, $v dp$ is equal to pressure energy, dH is equal to potential energy and $\pi D dy v \rho \tau$ is equal to friction or losses.

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Then,

$$\frac{A}{g_c} \cdot \frac{u}{v} \cdot du + A dP + \frac{A}{v} \cdot dH + \pi \cdot D \cdot dy \cdot \rho \cdot \tau = 0$$

Multiplying both sides by $\frac{v}{A}$

$$\frac{1}{g_c} \cdot u \cdot du + v dP + dH + \frac{\pi \cdot D \cdot dy \cdot v}{A} \cdot \tau = 0$$

Where, $u \cdot du$ =kinetic energy; $v dP$ =pressure energy; dH =potential energy;
 $\frac{\pi \cdot D \cdot dy \cdot v}{A} \tau$ =friction or losses.

The friction term or losses created by moving a fluid in a pipeline is defined by Fanning equation

$$dF_{\text{Fanning}} = \frac{2 f u^2}{g_c D} \cdot dL$$

where u = average gas velocity
 f = friction factor
 D = pipeline diameter
 L = pipeline

Substitute the Fanning equation for losses in the general energy equation

$$\frac{1}{g_c} u du + v dp + dH + \frac{2 f u^2}{g_c D} dL$$

The friction term or losses created by moving a fluid in a pipeline is defined by Fanning equation and this is $dF_{\text{Fanning}} = \frac{2 f u^2}{g_c D} dL$ where u is equal to average gas velocity, f is equal to friction factor, D is pipeline diameter, L is pipeline. Now, if you substitute the Fanning equation for losses in the general energy equation this equation will result like $\frac{1}{g_c} u du + v dp + dH + \frac{2 f u^2}{g_c D} dL$ this is equal to 0.

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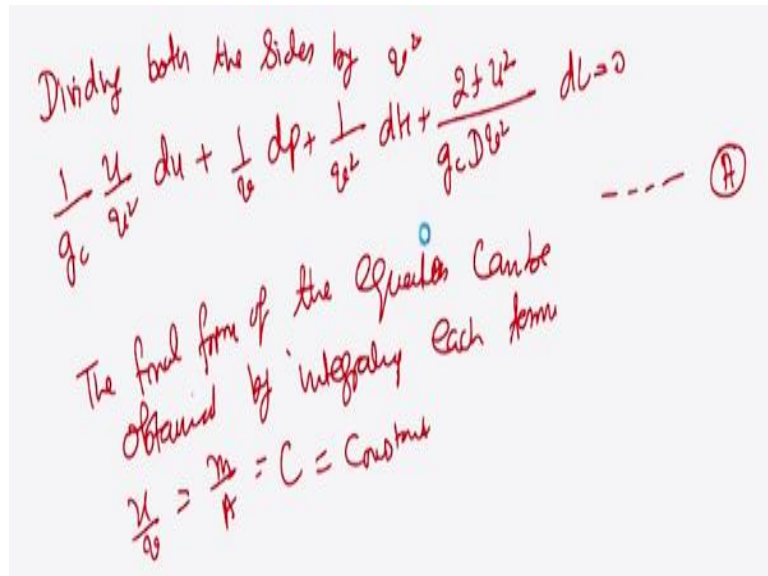
The friction term or losses created by moving a fluid in a pipeline is defined by the fanning equation as follows:

$$dF_{Fanning} = \frac{2fu^2}{g_c \cdot D} \cdot dL$$

Where, u = average gas velocity, f = friction factor; D = pipeline diameter; L = pipeline;

On Substituting the fanning equation for losses in the general energy equation will results in;

$$\frac{1}{g_c} \cdot u \cdot du + v dP + dH + \frac{2fu^2}{g_c \cdot D} \cdot dL = 0$$



Now, if we divide both the side by v square then equation becomes 1 upon g_c u upon v square $du + 1$ upon v $dp + 1$ upon v square $dH + 2f$ u square upon g_c D v square dL that is equal to 0 . Let us say that this is our equation number A . The final form of the equation can be obtained by integrating each term by assuming u upon $v = m$ upon $A = C$ and that is constant. **(Refer Slide Time: 17:49)**

Dividing both sides of the equation by v^2 ;

$$\frac{1}{g_c} \cdot \frac{u}{v^2} \cdot du + \frac{1}{v} dP + \frac{1}{v^2} dH + \frac{2f}{g_c \cdot D} \frac{u^2}{v^2} \cdot dL = 0$$

Note:

The final form of the equation can be obtained by integrating each term, assuming $\frac{u}{v} = \frac{m}{A} = C = \text{constant}$.

Kinetic energy term

$$\int_1^2 \frac{C}{g_c} \frac{dy}{v} \frac{y}{v^2} = \int_1^2 \frac{dy}{v^3}$$

Since $v = \frac{u}{C} \Rightarrow \frac{C}{g_c} \int_1^2 \frac{dy}{\frac{u}{C}} = \frac{C^2}{g_c} \int_1^2 \frac{dy}{u}$

$$\Rightarrow \text{Kinetic energy} = \frac{C^2}{g_c} \ln \frac{u_2}{u_1}$$

So, the kinetic energy term that can be written as integration from 1 to 2 C upon g c du over v u over v square and that is equal to 1 to 2 du over v. Since v = u upon C this can be represented as C or g c integration from 1 to 2 du u c and that is C squared upon g c integration 1 to 2 du u and that is the kinetic energy = C square g c ln u 2 over u 1.

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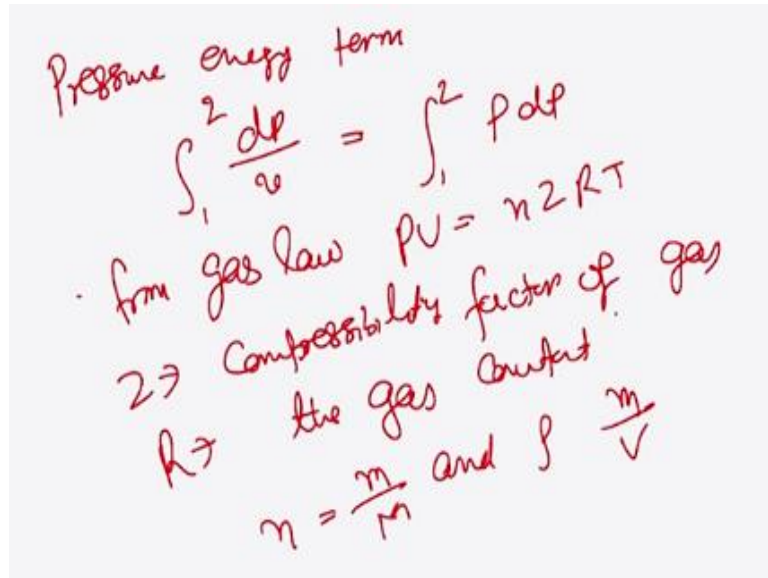
Kinetic energy term;

$$\int_1^2 \frac{C}{g_c} \cdot \frac{du}{v} \frac{u}{v^2} = \int_1^2 \frac{du}{v^3}$$

Since;

$$v = \frac{u}{C} \Rightarrow \frac{C}{g_c} \int_1^2 \frac{du}{\frac{u}{C}} = \frac{C^2}{g_c} \int_1^2 \frac{du}{u}$$

$$\Rightarrow \text{Kinetic energy} = \frac{C^2}{g_c} \ln \frac{u_2}{u_1}$$



Let us talk about the pressure energy term. So, it is from 1 to 2 dp upon v 1 to 2 rho dp. Now, from gas law $P V = n Z R T$, Z is the compressibility factor of gas and R is the gas constant. So, $n = \frac{m}{M}$ and $\rho = \frac{m}{V}$.

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Pressure energy term;

$$\int_1^2 \frac{dP}{v} = \int_1^2 \rho \cdot dP$$

From real gas law;

$$PV = nZRT$$

Where Z is the compressibility factor of the gas and R is the gas constant for;

$$n = \frac{m}{M} \text{ and } \rho = \frac{m}{V}$$

The Equations for the density of gas is

$$\rho = \frac{P \cdot M}{Z R T} \quad M \rightarrow \text{average molecular weight of the gas}$$

$$\int_1^2 \rho dP \Rightarrow \int_1^2 \frac{P M}{Z R T} dP = \frac{M}{Z_{avg} T_{avg} R} \int_1^2 P dP$$

$$\Rightarrow \int_1^2 \frac{P M}{Z R T} dP = \frac{M}{Z_{avg} T_{avg} R} \frac{P_1^2 - P_2^2}{2}$$

Where T_{avg} is defined as

$$T_{avg} = \frac{T_1 + T_2}{2}$$

The equation for the density of a gas is $\rho = \frac{P M}{Z R T}$, M is the average molecular weight of the gas. So, if we substitute this into the integration $\int_1^2 \rho dP$ that it becomes $\int_1^2 \frac{P M}{Z R T} dP = \frac{M}{Z_{avg} T_{avg} R} \int_1^2 P dP$. This can be $\int_1^2 \frac{P M}{Z R T} dP$ that is equal to $\frac{M}{Z_{avg} T_{avg} R} \frac{P_1^2 - P_2^2}{2}$ where T_{avg} is defined as $T_{avg} = \frac{T_1 + T_2}{2}$.

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The equation for density of a gas is;

$$\rho = \frac{P \cdot M}{Z \cdot R \cdot T}$$

Where, M is the average molecular weight of the gas;

After substitution into $\int_1^2 \rho \cdot dP$

$$\Rightarrow \int_1^2 \frac{P \cdot M}{Z \cdot R \cdot T} \cdot dP = \frac{M}{Z_{avg} T_{avg} R} \int_1^2 P dP$$

$$\Rightarrow \int_1^2 \frac{P \cdot M}{Z \cdot R \cdot T} \cdot dP = \frac{M}{Z_{avg} T_{avg} R} \frac{P_1^2 - P_2^2}{2}$$

Where T_{avg} is defined as follows;

$$T_{avg} = \frac{T_1 + T_2}{2}$$

T_1 & T_2 are the upstream and downstream gas temperature

P_{avg} is obtained based on the relation

$$\Rightarrow P_{avg} = \frac{\int_1^2 P \cdot P \cdot dP}{\int_1^2 P \cdot dP}$$

$$\Rightarrow P_{avg} = \frac{2}{3} \left[P_1 + P_2 - \frac{P_1 \cdot P_2}{P_1 + P_2} \right]$$

where P_1 & P_2 are the upstream and downstream gas pressures

Now T_1 and T_2 are the upstream and downstream gas temperature and P_{avg} is obtained based on the relations 1 to 2 integration $P \cdot dP$ that is $P_{avg} = \frac{\int_1^2 P \cdot P \cdot dP}{\int_1^2 P \cdot dP}$ and that is $P_{avg} = \frac{2}{3} \left[P_1 + P_2 - \frac{P_1 \cdot P_2}{P_1 + P_2} \right]$ where P_1 and P_2 are the upstream and downstream gas pressures.

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T_1 and T_2 are the upstream and downstream gas temperature and P_{avg} is obtained based on the relation

$$\Rightarrow P_{avg} = \frac{\int_1^2 P \cdot P \cdot dP}{\int_1^2 P \cdot dP}$$

$$\Rightarrow P_{avg} = \frac{2}{3} \left[P_1 + P_2 - \frac{P_1 \cdot P_2}{P_1 + P_2} \right]$$

Where, P_1 and P_2 are the upstream and downstream gas pressure.

Cont...

T_{avg} and P_{avg} are obtained for the gas, now for compressibility factor or Z_{avg} , that can be obtained from Kay's rule and compressibility factor charts.

To calculate Z_{avg} , for a natural gas using Kay's rule, T_{avg} and P_{avg} of the gas are needed, and also pseudocritical pressure and temperature of the natural gas.

Pseudocritical values can be obtained with Kay's rule as follows.

$$T'_C = T_{CA} Y_A + T_{CB} Y_B + T_{CC} Y_C + \dots$$
$$P'_C = P_{CA} Y_A + P_{CB} Y_B + P_{CC} Y_C + \dots$$

Now, T average and P average they are obtained for the gas for the compressibility factor Z average that can be determined from the Kay's rule and the compressibility factor chart. TO calculate the Z average for a natural gas using Kay's rule T average and average P of the guests are needed and also pseudocritical pressure and temperature of the natural gas. So, if we talk about the pseudocritical values this can be obtained by the Kay's rule which is as follows T dash C = T CA Y A + T CB Y B + T CC Y C and P dash C = P CA Y A + P CB Y B + P CC Y C + and so on.

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Pseudocritical values can be obtained with Kay's rule as follows.

$$T'_C = T_{CA} \cdot Y_A + T_{CB} \cdot Y_B + T_{CC} \cdot Y_C + \dots$$

$$P'_C = P_{CA} \cdot Y_A + P_{CB} \cdot Y_B + P_{CC} \cdot Y_C + \dots$$

Cont...

Where, T'_C =average pseudocritical temperature of the gas; P'_C = average pseudocritical pressure of the gas; T_{CA}, T_{CB}, T_{CC} =Critical temperature of each component; P_{CA}, P_{CB}, P_{CC} = critical pressure of each component; y_A, y_B, y_C = mole fraction of each component;

Finally, pseudo-reduced pressure and temperature can be obtained as follows;

$$T_r = \frac{T_{avg}}{T'_C} \quad \text{or} \quad P'_r = \frac{P_{avg}}{P'_C}$$

The obtained values of T'_r and P'_r can be used in Compressibility factor charts to calculate Z_{avg} .

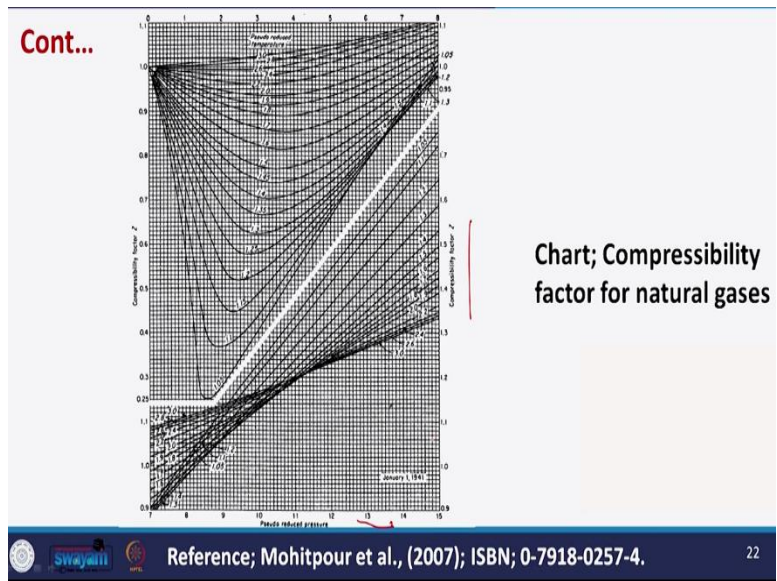
Now, where T dash C this is the average pseudocritical temperature of the gas. P dash C this is the average of pseudocritical pressure of the gas and T_{CA}, T_{CB}, T_{CC} these are the critical temperatures of each component. Where P_{CA}, P_{CB}, P_{CC} these are the critical pressures of each component; y_A, y_B, y_C these are the mole fraction of each component. So, finally pseudo-reduced pressure and temperature can be obtained as T dash r = average T upon T dash C or P dash r = average P upon P dash C. Now, to obtain the values of T_r dash and P_r this can be used in the compressibility factor chart to calculate Z average.

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Finally, pseudoreduced pressure and temperature can be obtained as follows;

$$T'_r = \frac{T_{avg}}{T'_C} \quad \text{or} \quad P'_r = \frac{P_{avg}}{P'_C}$$

The obtained values of T'_r and P'_r can be used in Compressibility factor charts to calculate Z_{avg} .



Now, this is the compressibility factor chart for natural gas and you can see here this is the compressibility factor and this is the pseudoreduced factor. So, by substituting the values you can get the values of either Z or pseudoreduced pressure whatever required.

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Potential Energy term
On integration of the Potential Energy term
of main equation

$$\int_1^2 \frac{1}{\rho^2} d\rho = \int_1^2 P^2 dH = \int_1^2 \left(\frac{P M}{Z R T} \right)^2 dH$$

$$= \frac{P_{avg}^2 M^2 2f}{Z_{avg}^2 R_{avg} T_{avg}} \Delta H = 0$$

$\Delta H = H_2 - H_1$

Now let us talk about a potential energy term. So, on integration of the potential energy term of main equation which we derived earlier, so it will result integration 1 to 2 $\int_1^2 \rho^{-2} d\rho = \int_1^2 P^2 dH = \int_1^2 \left(\frac{P M}{Z R T} \right)^2 dH$ and that is equal to average P square M square 2 f upon average Z square average R square average T square into delta H = 0 where delta H = H 2 – H 1.

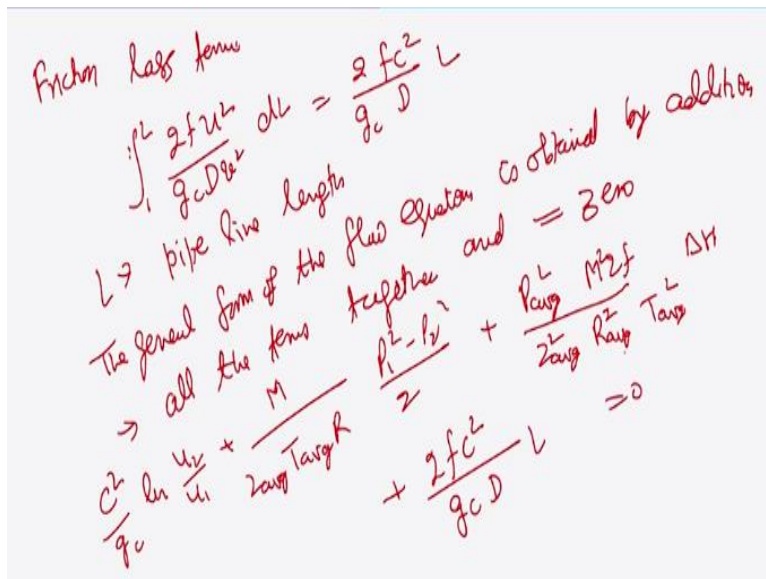
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Potential energy term

On integration of the potential energy term of main equation will result in;

$$\int_1^2 \frac{1}{v^2} dH = \int_1^2 \rho^2 dH = \int_1^2 \left(\frac{P \cdot M}{ZRT} \right)^2 dH = \frac{P_{avg}^2 \cdot M^2 \cdot 2f}{Z_{avg}^2 \cdot R_{avg}^2 \cdot T_{avg}^2} \cdot \Delta H = 0$$

Where, $\Delta H = H_2 - H_1$.



Now, let us talk about the friction loss term. So, if we integrate the energy loss energy losses, this can be evaluated as $\int_1^2 \frac{2f u^2}{g c D v^2} dL$ this is equal to $\frac{2f C^2}{g c D} L$, L is the pipeline length. So, the general form of the flow equation is obtained by adding all the terms together and equating them to zero. So, it is $\frac{C^2}{g c} \ln \frac{u_2}{u_1} + \frac{M}{2 Z_{avg} R_{avg} T_{avg}} \left(\frac{P_1^2 - P_2^2}{2} + \frac{P_{avg} M^2 f}{2 Z_{avg} R_{avg} T_{avg}} \Delta H \right) + \frac{2f C^2}{g c D} L = 0$.

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Friction loss Term

On integration of energy losses can be evaluated as follows;

$$\int_1^2 \frac{2f u^2}{g c \cdot D v^2} \cdot dL = \frac{2f C^2}{g c \cdot D} L$$

Where, **L** is the pipeline length.

The general form of the flow equation is obtained by adding all the terms together and setting them equal to zero.

$$\frac{C^2}{g_c} \ln \frac{u_2}{u_1} + \frac{M}{Z_{avg} T_{avg} R} \frac{P_1^2 - P_2^2}{2} + \frac{P_{avg}^2 \cdot M^2 \cdot 2f}{Z_{avg}^2 \cdot R_{avg}^2 \cdot T_{avg}^2} \cdot \Delta H + \frac{2f C^2}{g_c \cdot D} L = 0$$

Handwritten derivation of the flow equation. The text reads: "If the K.E. is neglected". The equation shown is:
$$\frac{M}{Z_{avg} T_{avg} R} \frac{P_1^2 - P_2^2}{2} + \frac{P_{avg}^2 \cdot M^2 \cdot 2f}{Z_{avg}^2 \cdot R_{avg}^2 \cdot T_{avg}^2} \Delta H + \frac{2f C^2}{g_c D} L = 0$$
Substitutions shown are:
$$C = \frac{\dot{m}}{A}$$

$$C^2 = \left(\frac{\dot{m}}{A}\right)^2$$

$$A = \frac{\pi D^2}{4}$$

Now, if the kinetic energy term is neglected for all high-pressure gas transmission lines, the contribution of kinetic energy term compared to other terms is insignificant. So, we can write $\frac{M}{Z_{avg} T_{avg} R} \frac{P_1^2 - P_2^2}{2} + \frac{P_{avg}^2 \cdot M^2 \cdot 2f}{Z_{avg}^2 \cdot R_{avg}^2 \cdot T_{avg}^2} \Delta H + \frac{2f C^2}{g_c \cdot D} L$ that is equal to 0. Now, this equation can be further simplified upon if you take the following assumption or following substitution. $C = \dot{m} / A$, $C^2 = (\dot{m} / A)^2$ and $A = \pi D^2 / 4$.

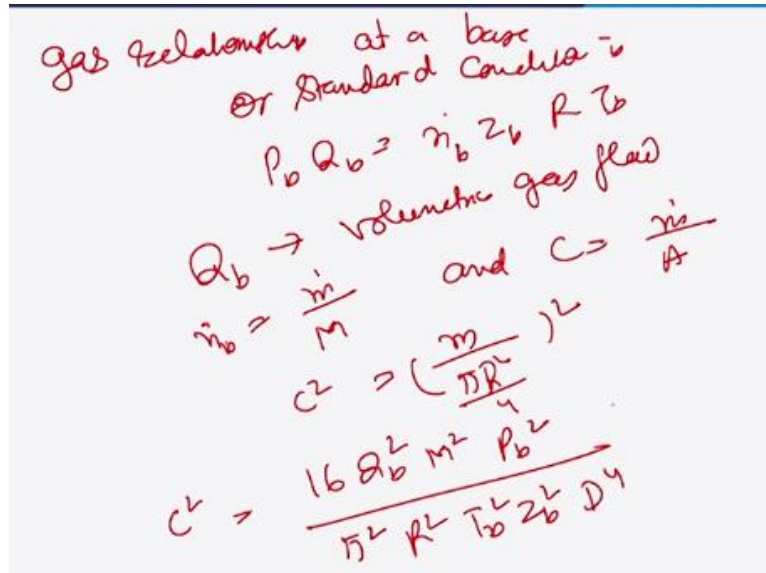
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If the kinetic energy term is neglected (for all high-pressure gas transmission lines, the contribution of the kinetic energy term compared to other terms is insignificant), so;

$$\frac{M}{Z_{avg} T_{avg} R} \frac{P_1^2 - P_2^2}{2} + \frac{P_{avg}^2 \cdot M^2 \cdot 2f}{Z_{avg}^2 \cdot R_{avg}^2 \cdot T_{avg}^2} \cdot \Delta H + \frac{2f C^2}{g_c \cdot D} L = 0 \dots \dots (2)$$

The above equation can be further simplified upon the following substitutions;

For pipe $C = \frac{\dot{m}}{A}$, $C^2 = \left(\frac{\dot{m}}{A}\right)^2$, $A = \frac{\pi D^2}{4}$



Now, the gas relationship at a base or the standard condition is $P_b Q_b = \dot{n}_b Z_b R T_b$. Q_b is the volumetric gas flow. So, if $\dot{n}_b = \frac{\dot{m}}{M}$ and $C = \frac{\dot{m}}{A}$, then $C^2 = \left(\frac{\dot{m}}{\frac{\pi D^2}{4}}\right)^2$ and $C^2 = \frac{16 Q_b^2 M^2 P_b^2}{\pi^2 R^2 T_b^2 Z_b^2 D^4}$.

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Gas relationship at a base or standard condition is;

$$P_b \cdot Q_b = \dot{n}_b \cdot Z_b \cdot R \cdot T_b$$

Where, Q_b is the volumetric gas flow

If $\dot{n}_b = \frac{\dot{m}}{M}$ and $C = \frac{\dot{m}}{A}$, $C^2 = \left(\frac{\dot{m}}{\frac{\pi D^2}{4}}\right)^2$

Then, $C^2 = \frac{16 Q_b^2 \cdot M^2 \cdot P_b^2}{\pi^2 \cdot R^2 \cdot T_b^2 \cdot Z_b^2 \cdot D^4}$

$G = \frac{M_{gas}}{M_{air}}$ where $M_{air} \approx 29$
 $Q_b^2 = \frac{\pi^2 R g_c Z_b^2 T_b^2}{32 P_b^2} \left\{ \frac{P_1^2 - P_2^2 - \frac{58 G \Delta H P_{avg}^2}{R T_{avg} Z_{avg}}}{58 Z_{avg} T_{avg} G L} \right\} \frac{D^5}{f}$

Now, gas gravity is defined as $G = M_{gas} / M_{air}$ where M_{air} is approximately 29. Now, if we substitute and rearrangement to solve Q_b , then the equation becomes $Q_b^2 = \frac{\pi^2 R g_c Z_b^2 T_b^2}{32 P_b^2} \left[\frac{P_1^2 - P_2^2 - 58 G \Delta H P_{avg} / (R T_{avg} Z_{avg})}{58 Z_{avg} T_{avg} G L} \right] \frac{D^5}{f}$.

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Gas gravity is defined as

$$G = \frac{M_{gas}}{M_{air}} \quad \text{Where, } M_{air} \approx 29.$$

Upon substitution and rearrangement to solve for Q_b , the equation (2) becomes;

$$Q_b^2 = \frac{\pi^2 \cdot R \cdot g_c \cdot Z_b^2 \cdot T_b^2}{32 \cdot P_b^2} \left\{ \frac{P_1^2 - P_2^2 - \frac{58 G \cdot \Delta H \cdot P_{avg}^2}{R \cdot T_{avg} \cdot Z_{avg}}}{58 Z_{avg} \cdot T_{avg} \cdot G \cdot L} \right\} \cdot \frac{D^5}{f}$$

Now, after taking the square root of Q_b , the general flow equation of natural gas in pipeline can be $Q_b = \pi R g_c$ upon 1856 $Z_b T_b$ upon P_b whole square root P_1 square – P_2 two square – 58 $G \Delta H P_{avg}$ square upon $R T_{avg} Z_{avg}$ 58 $Z_{avg} T_{avg} G L$ upon $f D$ to the power 2.5.

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After taking square root of Q_b , the general flow equation of natural gas in a pipeline is;

$$Q_b = \pi \sqrt{\frac{R \cdot g_c}{1856}} \cdot \frac{Z_b \cdot T_b}{P_b} \sqrt{\frac{P_1^2 - P_2^2 - \frac{58 G \cdot \Delta H \cdot P_{avg}^2}{R \cdot T_{avg} \cdot Z_{avg}}}{58 Z_{avg} \cdot T_{avg} \cdot G \cdot L}} \cdot \sqrt{\frac{1}{f}} D^{2.5} \dots \dots (3)$$

Cont...

Note;

The above equation can be used in imperial or S.I. units; for any size of length of pipe; for laminar, partially turbulent or fully turbulent flow; and for low medium or high pressure system.


Now, this equation can be used in the imperial or you can say S.I. units for any size of length of pipe, for laminar, partially turbulent or fully turbulent flow and for low medium or high-pressure system.

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Cont...

Where, Q_b =gas flow rate at base conditions
 g_c =proportionality constant
 Z_b =compressibility factor at base condition
 T_b =temperature at base condition
 P_b =pressure at base condition
 P_1 =gas inlet pressure to pipeline
 P_2 =gas outlet pressure to pipeline
 G =gas gravity, dimensionless
 F = friction factor, dimensionless
 ΔH =elevation pressure

T_{avg} =average temperature
 P_{avg} =average pressure
 Z_{avg} =compressibility factor at P_{avg}
 T_{avg}
 L =length of pipeline
 $\sqrt{\frac{1}{f}}$ =transmission factor, dimensionless



Now, as far as the notations are in question then Q_b is the gas flow rate at base condition, g_c is the proportionality constant, Z_b is the compressibility factor at base condition, T_b is the temperature at base condition, P_b is the pressure at base condition. P_1 , P_2 these are the gas inlet and outlet pressure in the pipeline. G is the gas gravity which is dimensionless. F is the friction factor, again it is dimensionless. ΔH is the elevation pressure.

Average T is the average temperature, P_{avg} is the average pressure, Z_{avg} the average compressibility factor at P_{avg} and T_{avg} and L is the length of pipeline whereas $\frac{1}{\sqrt{f}}$ that is the transmission factor which is dimensionless.

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$$Q_b = C \sqrt{\frac{P_1^2 - P_2^2 - \frac{58G \Delta H P_{avg}^2}{R T_{avg} Z_{avg}}}{58 Z_{avg} T_{avg} G \cdot L}} \sqrt{\frac{1}{f}} D^{2.5}$$

$$P_1^2 - P_2^2 \gg \frac{58G \Delta H P_{avg}^2}{R T_{avg} Z_{avg}}$$

So, I am taking all constant as a C. So the previous equation becomes $Q_b = C \sqrt{P_1^2 - P_2^2 - \frac{58G \Delta H P_{avg}^2}{R T_{avg} Z_{avg}}}$ whole divided by $58 Z_{avg} T_{avg} G \cdot L$ upon $f D$ to the power 2.5. If the pipeline is horizontal ΔH is insignificant to compare the value of $P_1^2 - P_2^2$, so, $P_1^2 - P_2^2$ is greater than $\frac{58G \Delta H P_{avg}^2}{R T_{avg} Z_{avg}}$.

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On taking all constant as C, the equation (3) becomes;

$$Q_b = C \cdot \sqrt{\frac{P_1^2 - P_2^2 - \frac{58G \cdot \Delta H \cdot P_{avg}^2}{R \cdot T_{avg} \cdot Z_{avg}}}{58 Z_{avg} \cdot T_{avg} \cdot G \cdot L}} \cdot \sqrt{\frac{1}{f}} D^{2.5}$$

If the pipeline is horizontal or ΔH is insignificant compare to the value of $P_1^2 - P_2^2$

$$P_1^2 - P_2^2 \gg \frac{58G \cdot \Delta H \cdot P_{avg}^2}{R \cdot T_{avg} \cdot Z_{avg}}$$

$$Q_b = C \sqrt{\frac{P_1^2 - P_2^2}{Z_{avg} T_{avg} G.L.}} \sqrt{\frac{1}{f} D^{2.5}}$$

Then the elevation term is omitted and the final equation becomes $Q_b = C \sqrt{P_1^2 - P_2^2}$ upon $Z_{avg} T_{avg} G.L.$ upon $f D^{2.5}$. So, this equation shows the effect of transmission factor and the diameter of the pipeline on flow of gas in pipeline.

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Then the elevation term is omitted and the final equation becomes;

$$Q_b = C \cdot \sqrt{\frac{P_1^2 - P_2^2}{Z_{avg} \cdot T_{avg} \cdot G.L.}} \cdot \sqrt{\frac{1}{f}} D^{2.5}$$

The above equation shows the effect of transmission factor and diameter of pipeline on flow of gas in pipeline.

References

- M. Mohitpour, H. Golshan, A. Murray, PIPELINE DESIGN & CONSTRUCTION: A Practical Approach; Third Edition, American Society of Mechanical Engineers., (2007), ISBN 0-7918-0257-4.

Now, in this particular segment we discussed about the design equations and especially the impact of kinetic energy, potential energy, friction terms, etc. For reference we have listed a reference for your convenience. If you wish to have further reading, you can take the help of this particular reference. Thank you very much.