

Process Equipment Design
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Lecture –36
Design of Evaporator-3

Hello everyone. I welcome you all in the 8th week of the course Process Equipment Design and here we are in 36th lecture of this course and in this lecture we will discuss design of evaporator. And if you remember this topic we have already started in 7th week where 34th and 35th lectures are devoted to this and we are continuing the design of evaporator in 8th week also. So, let us start the design of evaporator.

If you remember the 35th lecture there we have discussed the multiple effect evaporator functioning as well as different points which are related to triple effect evaporator in every detail. However, we have not derived the governing equation which are related to multiple effect evaporator and especially triple effect evaporator. So, in this lecture we are going to derive the governing equation. We will solve this equation to design triple effect evaporator.

(Refer Slide Time: 01:31)

Design of Multiple Effect Evaporator

The design of evaporators means determination of the heat transfer area and the steam consumption required for a specified separation at a specified set of steady-state operating conditions.

Effect 1:

Enthalpy Balance

$$F h_F + Q_1 - V_1 H_1 - L_1 h_1 = 0$$

$$F h_F + V_0 \lambda_0 - (F - L_1) H_1 - L_1 h_1 = 0$$

$$F (h_F - h_1) + V_0 \lambda_0 - (F - L_1) \lambda_1 = 0$$

Rate Equation

$$U_1 A (T_0 - T_1) = V_0 \lambda_0$$

So, as far as triple effect evaporator is concerned this is the schematic where feed is entering into the first effect and concentrated liquor is entering from first to second effect and then to third effect and finally we have concentrated product from third effect. And similarly we can have the movement of vapour where steam is entering into the first effect and vapour generated in first effect is entering as a heating media in second effect.

And so on from second to third effect and from third effect to condenser and the whole assembly is connected to the vacuum pump. So, this is the schematic so let us start deriving the governing equation associated to triple effect evaporator. So, first of all we will focus on effect 1. Now, if you remember the 35th lecture of this course there we have derived the governing equation based on material and energy balance for single effect.

So, in the same line we will derive the governing equation for triple effect evaporator also. So, let us focus on effect 1. First of all, we will make the enthalpy balance and this is basically the enthalpy balance of all streams and if you see the first effect as far as streams are concerned we have total five stream F , V_0 , V_1 and L_1 and along with this we have this condensate also which is again with the flow V_0 .

So, while making the balance we can consider enthalpy of feed that is F into $h_F + Q_1$, Q_1 is basically the heat associated with this stream minus V_1 . Vapour is generated which is having enthalpy h_1 . So, usually we represent enthalpy of vapour by capital H and enthalpy of liquid by small h and that is the usual nomenclature. So, further we have minus L_1 which is containing h_1 amount of heat.

So, in this case you see I am not considering this condensate and that will be clear in next equation when we will elaborate it further. So, I am having $F h_F + V_0 \lambda_0$ because here I am considering the latent heat only and after supplying latent heat this condensate exits. So, if I consider this condensate I should also consider enthalpy of vapour with this V_0 . So, you see V_0 when I am considering it is basically $H_0 - \text{small } h_0$.

So, that will be nothing, but $V_0 \lambda_0$. So this we have already considered over here. So, finally we have V_1 as $F - L_1$ and here if you see we have shown this L_0 that should be capital F please make a correction over here. So, $F - L_1$ should be V_1 and further I am having L_1 as it is H_1 as it is. Further, if I need to rearrange this equation we are considering this capital H as $\lambda_1 + \text{small } h_1$ because that is the enthalpy of vapour.

So, considering this as well as this equation we can rearrange and found that F into $h_F - h_1$. So that h_1 will come from here when it will be associated with $F + V_0 \lambda_0$ and then we can have $F - L_1$ into λ_1 . So in this way I can obtain the enthalpy balance in first

effect and further I can obtain the rate equation and what is rate equation $Q = UA \Delta T$ mean.

And in this evaporator mean temperature is basically the heating media temperature minus effects temperature. So, if I focus on first effect temperature difference should be $T_0 - T_1$. So, $U_1 A (T_0 - T_1)$ should be equal to $V_0 \lambda_0$ because this is the total heat duty of an effect. So, if you see as far as design is concerned what we want to calculate? We need to calculate the area of effect and steam consumption and here we are considering that area of each effect is equal. So, here I am having the governing equation for effect 1.

So, if you focus on these equation we can have this is the final enthalpy balance and this is the rate equation. So, one effect two governing equation. In the similar line, if I ask you to write the equation for second effect how you can write it. You can further write in the same line like $L_1 h_1 - h_2$. Here instead of V_0 , you can write V_1 and that should be equal to $F - L_1$ and that should be multiplied by λ_1 when it is entering into the second effect.

And minus we can have $L_1 - L_2 \lambda_2$. In the similar line, I can write this also $U_2 A (T_1 - T_2)$ should be equal to $V_1 \lambda_1$ where V_1 is this. So, in this way you can represent each effect with two equation.

(Refer Slide Time: 07:42)

Design of Multiple Effect Evaporator

Effect 2:

Enthalpy Balance ✓
 $L_1 (h_1 - h_2) + (F - L_1) \lambda_1 - (L_1 - L_2) \lambda_2 = 0$

Rate Equation ✓
 $U_2 A (T_1 - T_2) = (F - L_1) \lambda_1$

Effect 3:

Enthalpy Balance ✓
 $L_2 (h_2 - h_3) + (L_1 - L_2) \lambda_2 - (L_2 - L_3) \lambda_3 = 0$

Rate Equation ✓
 $U_3 A (T_2 - T_3) = (L_1 - L_2) \lambda_2$

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So, here we can have equation for second effect this is basically enthalpy balance of second effect and this is the rate equation. In the similar line, I can find equation for third effect also and we can have enthalpy balance like this and rate equation like this.

(Refer Slide Time: 08:04)

Design of Multiple Effect Evaporator

<p>Effect 1:</p> <p>Enthalpy Balance</p> $F(h_F - h_1) + V_0 \lambda_0 - (F - L_1) \lambda_1 = 0$ $\checkmark F C_p(T_F - T_1) + \checkmark V_0 \lambda_0 - \checkmark (F - L_1) \lambda_1 = 0$ <p>Rate Equation</p> $\checkmark U_1 A(T_0 - T_1) = \checkmark V_0 \lambda_0$	<p>Effect 2:</p> <p>Enthalpy Balance</p> $\checkmark L_1 C_p(T_1 - T_2) + \checkmark (F - L_1) \lambda_1 - \checkmark (L_1 - L_2) \lambda_2 = 0$ <p>Rate Equation</p> $\checkmark U_2 A(T_1 - T_2) = \checkmark (F - L_1) \lambda_1$
<p>Effect 3:</p> <p>Enthalpy Balance</p> $\checkmark L_2 C_p(T_2 - T_3) + \checkmark (L_1 - L_2) \lambda_2 - \checkmark (L_2 - L_3) \lambda_3 = 0$ <p>Rate Equation</p> $\checkmark U_3 A(T_2 - T_3) = \checkmark (L_1 - L_2) \lambda_2$	<p>Specifications: $\checkmark F, \checkmark X_F, \checkmark T_F, \checkmark T_0, \checkmark P_0, \checkmark P_3$ \checkmark (or T_3), $\checkmark X_3$ (or L_3), $\checkmark U_1, \checkmark U_2, \checkmark U_3$, Equal Areas, Forward feed</p> <p>To Find: $\checkmark V_0, \checkmark T_1, \checkmark T_2, \checkmark L_1, \checkmark L_2, \checkmark A$</p>

So, if I combinedly see equations of all effect we can observe that each effect is shown with two equation. We have total equation as 1, 2, 3, 4, 5 and 6 and here if you see we have further elaborated h_F or h_1 what is the enthalpy of liquid that should be C_p of the liquid into temperature. So, considering that we have elaborated equation that is the enthalpy balance equation of all effects.

Now, if I ask you what are the known parameter over here? So, known parameter we also called that as specified parameter and these parameters are F that is the feed rate X_F that is the concentration of the feed, T_F temperature of the feed, T_0 P_0 P_3 because last effect pressure or temperature you usually know. So, once I know P_3 I can find out T_3 so T_3 is usually known to me.

And further we have concentration X_3 so X_3 is basically product concentration. So, when you define any evaporator problem you know a priori that the concentration of the feed should be this much when we obtain the product. So, feed should be concentrated from this level to that level. So, you already know the product concentration along with the feed concentration.

Along with this we have U_1 , U_2 and U_3 here you should consider U_2 . So, all these values are known to me I am considering equal area and forward feed I have considered. So, whatever governing equations we are discussing these equations are specifically for forward feed. If I change the feed sequence these equation will be changed. So, you should understand

how to make balance in each effect so that you can make the governing equation depending upon different feed sequences.

So, here we are saying that specified parameters are given and if I ask you that what should be the unknown parameter. If you observe these 6 equations what are the unknown if I focus on this unknown is basically T_1 unknown is V_0 , unknown is L_1 . In this equation, T_1 and V_0 along with the area. Second effect if I focus on L_1 , T_2 , L_1 , L_2 all lambdas are usually known to us.

And we have area and T_2 , T_1 and L_1 . So, if we focus on third effect equation we have L_2 unknown T_2 unknown, T_3 is known to me L_1 and L_2 . L_3 now it is unknown, but you know this while making the component balance this we will see in the solution of these equations and here I am having unknown as area and T_2 and L_1 and L_2 . So, if you see these 6 equations of three effects the unknowns are V_0 , T_1 , T_2 , L_1 , L_2 and A .

So 1, 2, 3, 4, 5, 6 unknowns are there and how many equations I am having 6 equation. So, 6 equation 6 unknown so unique solution must exist. I understand that you know about this unique solution that is basically we have only one solution and so if you solve this problem how you can solve this because some equation among these are nonlinear in nature. For example, if you focus on this rate equation A is unknown to me and T_1 is unknown to me.

And similarly if I focus on this L_1 is unknown to me T_2 is unknown to me so this will become nonlinear term. In the similar line, this will become non linear term and this term also becomes non linear this term also becomes nonlinear like this. So, if I am having the 6 equation even a single equation is nonlinear in nature you have set of nonlinear equation and you have to solve these equation accordingly. So, to solve these equation we will use a specific method that is Badger McCabe.

(Refer Slide Time: 13:22)

Design of Multiple Effect Evaporator

Badger McCabe Method

This method consists of the following steps:

- Assume values of temperatures for first and second effect ✓
- Determine evaporation rates V_1 , V_2 , and V_3 ✓
- Use the heat transfer rate equations to compute the heating surface A_1 , A_2 and A_3 ✓

So as far as this method is concerned we should consider some steps and these steps are we have to assume temperature of first and second effect like T_1 and T_2 . How we have to assume this that we will discuss. We need to determine V_1 , V_2 and V_3 . So, instead of that we can say that we need to determine L_1 , L_2 and L_3 . If I know all these I can automatically calculate V_1 , V_2 , V_3 .

Third step is we have to use the rate equations to find out heat transfer area A_1 , A_2 and A_3 and if you remember we have consider heat transfer area equal for all effects, so that should be A . Now if I am observing the variation in area like A_1 , A_2 , A_3 and these areas are not same we can further redistribute the temperature in such a way so that area should be equal. So this method is basically iterative method. So, let us see all these steps one by one with the help of example so that method is more clear to you.

(Refer Slide Time: 14:41)

Design of Multiple Effect Evaporator

Badger McCabe Method

It is desired to design a triple-effect evaporator system to concentrate the solute from a 10% solution (feed) to 50% by weight. The feed rate is 50000 lb/h and it enters the first effect at 100°F. Forward feed is to be used. Saturated vapor at 250 °F is available as heating medium. The third effect is to be operated at absolute pressure corresponding to boiling point of 125°F. Neglect boiling point elevation.

$C_p = 1 \text{ Btu/lb } ^\circ\text{F}$ for feed and all other liquid streams ✓

$\lambda_0 = \lambda_1 = \lambda_2 = \lambda_3 = \lambda = 1000 \text{ Btu/lb}$ ✓

$U_1 = 500, U_2 = 300, U_3 = 200$ ✓

So example is like it is desired to design a triple effect evaporator to concentrate the solute from 10% solution which is the feed to 50% by weight and this is for the product. The feed rate is 50,000 pound per hour and it enters the first effect at 100 degree Fahrenheit. We have to use forward feed and saturated vapour is available at 250 degree Fahrenheit and which is used as a heating media.

Third effect is operated at absolute pressure corresponding to boiling point of 125 degree Fahrenheit and this is corresponding to the vacuum which is generated in third effect and in this problem we have to neglect boiling point elevation. C_p value for simplicity we are considering as 1 Btu for all feed as well as liquid streams. All lambdas are equal and that should be 1,000 Btu per pound.

U_1, U_2, U_3 are known to me as 500, 300 and 200. So, this problem I have taken from ((15:56)) book and that is also given in Serth book. So, you can follow these books and it is also available in ((16:05)) book. So, here we are going to solve this problem using Badger McCabe method. Now, the first step is to find out unknown temperature that is T_1 and T_2 . Now, if I ask you what is the total driving force available in the system that should be $T_0 - T_3$.

And the complete driving force in each effect like ΔT_1 ΔT_2 and ΔT_3 will be distribution of this maximum driving force which is $T_0 - T_3$. So, considering this we have to find out unknown temperature that is T_1 and T_2 .

(Refer Slide Time: 16:47)

Design of Multiple Effect Evaporator

Badger McCabe Method

Step 1: To initiate the calculation procedure, the total temperature drop across the system ($250-125=125$) is distributed.

$$\begin{aligned} \Delta T_2 / \Delta T_1 &= U_1 / U_2 = 500 / 300 = 5/3 \\ \Delta T_3 / \Delta T_1 &= U_1 / U_3 = 500 / 200 = 5/2 \\ \Delta T &= 125 = \Delta T_1 + (\Delta T_2 / \Delta T_1) \Delta T_1 + (\Delta T_3 / \Delta T_1) \Delta T_1 \\ \Delta T_1 &= 24.1936 = T_0 - T_1 \\ \Delta T_2 &= 40.3226 \quad T_2 \\ \Delta T_3 &= 60.4839 \quad T_3 = 125 \end{aligned}$$

$$\begin{aligned} Q_1 &= Q_2 = Q_3 \\ U_1 \Delta T_1 &= U_2 \Delta T_2 \quad Q_1 = Q_2 \\ \frac{DQ_2}{DQ_1} &= \frac{U_1}{U_2} \quad \frac{DQ_3}{DQ_1} = \frac{U_1}{U_3} \end{aligned}$$

Now to find out unknown temperature we are taking another assumption that equal heat is transferred in each effect. So, if I consider Q_1 heat of first effect that should be equal to Q_2 and that should be equal to Q_3 . If I am considering these two and elaborating it what we can observe $U_1 \Delta T_1$ that should be equal to $U_2 \Delta T_2$. So, here we are assuming that each effect area is same.

So this factor will be cancelled out $\Delta T_2 / \Delta T_1$ will be equal to U_1 / U_2 as you can observe here. U_1 and U_2 value I know so that should be $5 / 3$. In the similar line, I can consider $Q_2 = Q_3$ or let us say $Q_1 = Q_3$ anything I can take. So that should be equal to $\Delta T_3 / \Delta T_1$ should be U_1 / U_3 . So, in that way I can obtain $\Delta T_3 / \Delta T_1$ which should be equal to $5 / 2$.

As we have already discussed that the maximum driving force available in the system it is distributed in all effects. So, we can say that ΔT which is the maximum driving force and that should be equal to $T_0 - T_3$. It is equal to $\Delta T_1 + \Delta T_2 + \Delta T_3$ and ΔT_1 is basically driving force in first effect and that is $T_0 - T_1$. In the similar line, I can consider ΔT_2 as $T_1 - T_2$ and ΔT_3 as $T_2 - T_3$.

So, I can write total driving force which is basically $250 - 125$ so 125 is available and I am considering $\Delta T_1 + \Delta T_2 + \Delta T_3$ and divide this by ΔT_1 and multiply this by ΔT_1 similarly I am considering here. So, considering all these points all these value like $\Delta T_2 / \Delta T_1$ and $\Delta T_3 / \Delta T_1$ are given here. So, we can finally obtain ΔT_1 as 24.1936 ΔT_2 as 40.3226 and ΔT_3 as 60.4839 .

And this you know already this is basically $T_0 - T_1$. So, you can find out T_1 from here and similarly you can find out T_2 and T_3 you can fix at 125. So, in this way we can find the unknown temperature, but that is based on the assumption that equal heat transfer is occurring in each effect. Now second step is to find out L_1 , L_2 and L_3 . So, first of all we will find out L_3 because in forward sequence final product exits from third effect.

If I consider backward sequence so here I should consider L_1 not L_3 . I hope you understand that. Whatever solute is available in feed the complete solute is now available in L_3 assuming that all vapours do not contain any solute.

(Refer Slide Time: 20:30)

Design of Multiple Effect Evaporator

Badger McCabe Method

Step 2: L_3 is computed directly using

$$L_3 = F \cdot x_F \cdot x_3 = 50000 \cdot 0.1 / 0.5 = 10000 \text{ lb/h}$$

Since $C_p = 1$ and $\lambda_0 = \lambda_1 = \lambda_2 = \lambda_3 = \lambda$



$$\left\{ \begin{array}{l} F(T_F - T_1) + V_0 \lambda - (F - L_1) \lambda = 0 \\ L_1(T_1 - T_2) + (F - L_1) \lambda - (L_1 - L_2) \lambda = 0 \\ L_2(T_2 - T_3) + (L_1 - L_2) \lambda - (L_2 - L_3) \lambda = 0 \end{array} \right.$$

Three unknowns: V_0 , L_1 and L_2

Last two equations contains L_1 and L_2 and they may be solved simultaneously.

Thus,

$$L_1 = 38194.4 \text{ lb/h}, \quad L_2 = 24848.7 \text{ lb/h}, \quad \text{So, } V_0 = 18095.9 \text{ lb/h}$$

So, here we can make the component balance such as $F \cdot X_F$ should be equal to $L_3 \cdot X_3$ and feed and product concentration you know already. So you can find out L_3 value as 10,000 pound per hour. Now next you have to compute L_1 and L_2 because if you remember you have 6 parameters in which T_1 and T_2 you have already obtained. Now we have to compute L_1 and L_2 .

And for that we should focus on all three enthalpy balance equation or we can say the enthalpy balance equation of all three effects and these equations are like this. Now, if you focus on these two equations what are the unknown over here? We have L_1 , T_1 , T_2 now I know F I know L_1 is unknown and L_2 is unknown. Similarly, L_2 is unknown T_2 I know T_3 I know L_1 , L_2 are unknown L_2 are unknown and L_3 is known to me.

So, if you consider last two equations you can find that the unknowns are only L 1 and L 2 and as we have two equations and two unknown we can simply solve these equation simultaneously to find out value of L 1 and L 2 and I hope you understand the meaning of solving the equation simultaneously. So, solving these equation we can find L 1 and L 2 like this.

And once I know the value of L 1 and L 2 I can put this values in first effect to find out V 0 so V 0 I can obtain like this. And now next step is to find out heat transfer area because 5 parameters you have already computed. So, to find out heat transfer area we should focus on rate equation.

(Refer Slide Time: 22:42)

Design of Multiple Effect Evaporator

Badger McCabe Method

Step 3: The areas are computed as

$$A_1 = V_0 \lambda / U_1 \Delta T_1 = 1495.93 \text{ ft}^2 \checkmark$$

$$A_2 = (F - L_0) \lambda / U_2 \Delta T_2 = 975.932 \text{ ft}^2 \checkmark$$

$$A_3 = (L_1 - L_2) \lambda / U_3 \Delta T_3 = 1103.24 \text{ ft}^2 \checkmark$$

Step 4: A new set of ΔT 's is obtained using:

$$A = \frac{A_1(\Delta T_1) + A_2(\Delta T_2) + A_3(\Delta T_3)}{\Delta T} = 1138.18 \text{ ft}^2 \checkmark$$

2nd iteration

$$\Delta T_1 = \frac{A_1(\Delta T_1)}{A} = 31.798^\circ\text{F} \checkmark$$

$$\Delta T_2 = \frac{A_2(\Delta T_2)}{A} = 34.575^\circ\text{F} \checkmark$$

$$\Delta T_3 = \frac{A_3(\Delta T_3)}{A} = 58.626^\circ\text{F} \checkmark$$

So, let us see the first effect rate equation so that should be equal to U 1 delta T 1 into A because now I am considering different area because area of each effect I have assumed as equal, but now we are considering different areas. So A 1 we can obtain as V 0 lambda 0 divided by U 1 delta T 1 so this value we can obtain like this. Similarly, I can obtain A 2 as well as A 3. So, area you can obtain like this.

So, what is the point to consider over here? Each effect has different area and we already assume that area of each effect is same. So, how I can find the average area of this and we will further redistribute the temperature difference based on the deviation of average area and the individual area. I hope you are getting the procedures because all these values are based on the assumption that equal heat transfer is occurring which is not possible in evaporators.

So, we have to carry out different iterations to reach to the final result and these iteration we will carry out based on different area and equal area which is basically the average area of effect. So, now we can find out the average area of each effect and that area we can weight it based on temperature difference of each effect. So, here I am having A into delta T and this should be equal to $A_1 \Delta T_1 + A_2 \Delta T_2 + A_3 \Delta T_3$.

So, average area you can find as 1,138 and if you observe these area these area differ significantly than this value. So, we have to deviate driving force of each effect like here I am having the second iteration. So, here I am having delta T 1 which is basically the revised value and that should be $A_1 \Delta T_1 / A$. How much area we have to obtain and how much we are deviating based on that we will recalculate delta T 1.

So, similarly we can find out delta T 2 and delta T 3 and values of these deltas we can obtain as 31.798 degree Fahrenheit, 34.575 degree Fahrenheit and 58.626 Fahrenheit. So, in this way you can find out the temperature difference and now you can find out T 1 and T 2 value as revised values because you can fix T 0 and T 3 you cannot change these, but T 1 T 2 you can change.

So, for the new values of temperatures you have to find out new values of L 1 and L 2 and so the V 0 and so A 1, A 2, A 3 and again you have to find out the average area. So, this practice should be kept on doing till I will find values of all 6 parameters equal in two consecutive iterations.

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Design of Multiple Effect Evaporator								
Badger McCabe Method								
On the basis of new set of ΔT 's, steps 2 to 3 are repeated till almost equal values of parameters as obtained:								
Trail no.	L_1	L_2	V_0	A	ΔT_1	ΔT_2	ΔT_3	
1	38194.4	24848.7	18095.9	1138.18	31.798	34.575	58.627	$= \frac{V_1 + V_2 + V_3}{3}$
2	38026.6	24738.5	17883.5	1137	31.459	35.104	58.438	
3	38038.8	24742.4	17888.2	1137	31.465	35.065	58.47	
4	38038.1	24742.4	17888.7	1137	31.466	35.067	58.467	
5	38038.1	24742.4	17888.5	1137	31.465	35.067	58.467	

And results I have summarized as. If you see here I am having number of trials or the iterations and here I am having L_1 value, L_2 value V_0 ΔT_1 , ΔT_2 and ΔT_3 . So, if you see in first iteration I have observed values like this. This is the average area of first iterations and these values we already have seen in first iteration. So, if I change these temperatures the driving force you can further revise like this.

And you can find out other parameters. So, in this way you can stabilize the result where if you observe last two iterations results like this values are equal in two consecutive iterations except value after decimal. So, in this way we can design a multiple effect evaporator. It means you can find out area of evaporator along with the steam consumption. Now once you are having the steam consumption, you can find out steam economy/

And that steam economy should be $V_1 + V_2 + V_3$ divided by V_0 in this way you can obtain the steam economy because once you know L_1 , L_2 , L_3 you can compute V_1 , V_2 and V_3 . So, in this way we can design multiple effect evaporators and here I am stopping this lecture. We will continue design of multiple effect evaporator with another method in next lecture and that is all for now. Thank you.