

**Polymer Reaction Engineering**  
**Prof. Shishir Sinha**  
**Department of Chemical Engineering**  
**Indian Institute of Technology Roorkee**

**Lecture - 07**  
**Molecular Weight and It's Distribution**

Welcome to the molecular weight and its distribution under the head of polymer reaction engineering. In the previous lecture, we started about the things related to the molecular weight and we discussed about the importance of a molecular weight as per this particular slide that all molecular weight plays a very vital role in deciding the mechanical and chemical thermal properties of the polymer.


**(Refer Slide Time: 00:38)**

**Molecular weight and its distributions**

□ **Molecular weight**

It has prime importance in polymer synthesis and applications. As the **mechanical, chemical and thermal properties** of the polymer depend upon molecular weight.

- Some of the properties increase with molecular weight to a maximum value and then decrease on further an increase in the molecular weight.
- The problem with higher molecular weight polymer is the processability as the viscosity become high and melt flow difficult.
- .



2

And moreover, its change or this molecular weight also affects the processability of the polymeric system. So, we go ahead with this particular molecular weight approach in this particular segment. And we discuss about the various kind of molecular weight, how it distributed among the different type of system, we will discuss in the due course of time.

**(Refer Slide Time: 01:05)**

## Molecular weight and its distributions

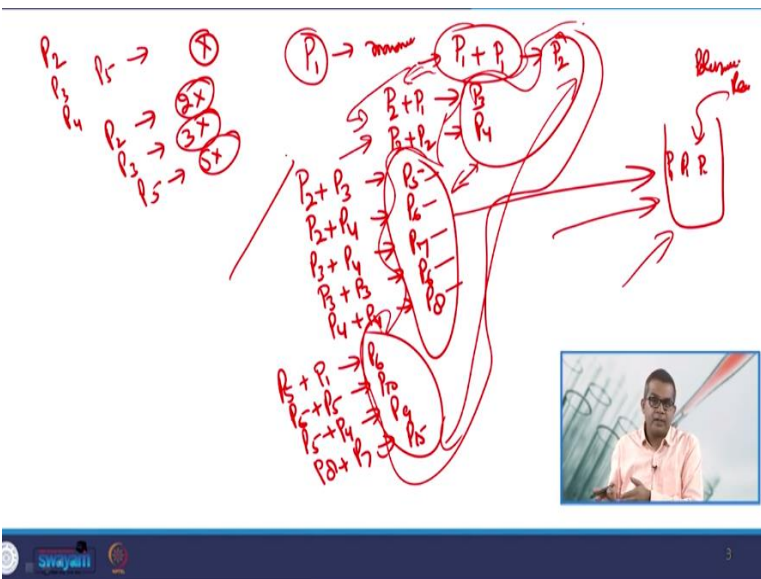
- The synthesis of higher molecular weight polymer is not necessary, but polymer with high, specified, and improvised molecular weight is required in many applications.

- $P_1 + P_1 \rightarrow P_2$
- $P_2 + P_1 \rightarrow P_3$



Now, the synthesis or synthesis of higher molecular weight polymer is always not necessary. But polymers with high specified and improvised molecular weight is required for the many application. Now, why I am saying the polymers they do not possess any kind of fixed molecular weight. The reason is that let us go to the this particular whiteboard and we will see that lets this  $P_1$  represents the monomer.

**(Refer Slide Time: 01:45)**



So, when we start the polymeric system or polymerization process, so, this is one repeating unit. Now, this  $P_1$  at the start at the outset, it may get combined with this  $P_1$  to give you  $P_2$ . Now, this is my reactor polymerization reactor. Now, here now, if you see that initially the  $P_1$ ,  $P_1$ , etc., they were present but when the reaction was started or the polymerization process initiated.

In that case, there may be this formation of  $P_2$  that means 2 repeating units are combined. Now, at the carry on this particular polymerization little further then the reaction mass may have  $P_2$  may get combined with  $P_1$  to give you  $P_3$ . Similarly,  $P_2$  may have an option to combined with  $P_2$  to give you  $P_4$ . And obviously, this option is always there. Now, you see that in the reaction mass you are having  $P_2$ ,  $P_3$ ,  $P_4$  apart from  $P_1$ .

Now, if you proceed further see the complexity of this particular system. If you proceed further, then  $P_2$  may get combined with  $P_3$  to give you  $P_5$ . Similarly,  $P_2$  may combine with  $P_4$  which is earlier developed  $P_6$ . Similarly,  $P_3$  may have a chance to combine with  $P_4$  to give you  $P_7$ . Similarly, the  $P_3$  may have a chance to combine on its own gave you  $P_6$ .

Similarly,  $P_4$  may have an option to combine on its own. So, give you  $P_8$ . Now, if you see this profiling apart from this, these products available in the reaction mass, there are additional polymeric chains having 5 repeating units, having 6 repeating units, having 7 repeating units and another one is the having the 8 repeating units. So, as a whole if we talk about after some time, then this polymer system may have this type of a polymeric chains.

Now, if you proceed further, then you see that the complexity may be like this  $P_5$  may combine to give you  $P_6$ . Similarly,  $P_5$  may combine with  $P_5$  to give you  $P_{10}$ .  $P_5$  may have an option to combine with  $P_4$  to give you  $P_9$ . Similarly,  $P_8$  may have an option to combine with  $P_7$  to give you  $P_{15}$ . So, by this way, the chains of the polymer may have an ability to enhance themselves with respect to the different type of monomeric units or repeating units.

Now, if you see, just after say Stage 3 or 4, now you see that how much different polymer chains are available in the reaction mass. Now see every chain like whether it is  $P_2$ ,  $P_3$ ,  $P_4$ ,  $P_5$ , etc., it represents the different number of monomeric unit. Now, let us have an example of suppose one monomer unit is having the molecular mass of  $x$ . So,  $P_2$  will have the molecular mass of  $2x$ ,  $P_3$  will have the molecular mass of  $3x$ ,  $P_5$  may have the molecular mass of  $5x$  and so on.

So, this is the molecular weight. Now, at the outset, there are 2 options, either you truncate the polymerization reaction at the desired level of viscosity as the desired level of the function or you keep on polymerizing different segments for the longer period of time. So, if you keep on

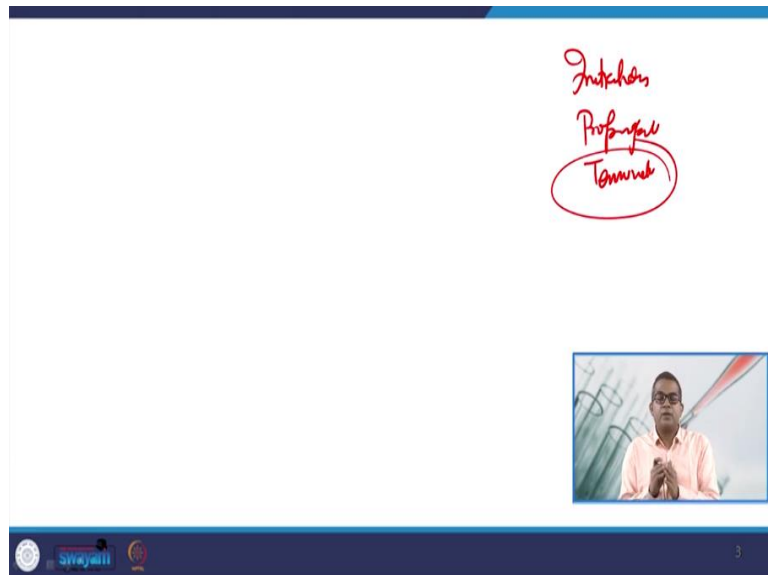
setting this particular polymerization process for a longer application, then there may be a chance that higher molecular weight chain may get processed.

In that case, the molecular mass of that particular polymer would be on the higher side, it may create the problem of processability as I discussed in the previous slide. Now, in that case, you are compelled to truncate or this particular polymerization process or terminate that particular polymerization process at some juncture.

So, when you truncate this particular polymerization process, there may be a chance that your polymer mass may consist of a different chain with the different chain lengths, maybe  $P_2$  maybe  $P_3$ ,  $P_4$ ,  $P_5$ ,  $P_6$ ,  $P_7$  up to  $P_n$  and so on. So, in that case, you may not have an opportunity to analyze that how many numbers of chains are there, how many repeating units are there, etc.

And that is why there is a usually a range of molecular weight for every kind of a polymeric system. And this may be from say 10 to 10 lakhs etcetera. So, it all depends on that how much the physical and chemical property you require in this particular polymerization process. So, it is a very crucial thing. And that is why if you recall that I discussed that initially, the polymeric systems they are having the 3 different steps, initiation, propagation and termination.

**(Refer Slide Time: 07:47)**



Now, initiation as I told you in previously, this this is your initiation step. Now, subsequent processes when you started this initiation process, the subsequent processes are termed as propagation. Now, when you achieve the desired property based on your requirement with

respect to the thermal, chemical, mechanical or the processability aspect or viscosity aspect then you try to truncate.

You can you cannot carry out this particular polymerization process for infinite period because I told you that it may create a bulky mass. So, in that case, you prefer the termination. So, at the outset, when there is a termination process in that case, you need to calculate the molecular weight because, one time we are saying that molecular weight plays a very vital role for the deciding the properties.

So, the determination of this molecular weight is extremely important to carry out the further processing of any kind of polymeric system. So, based on this particular approach, there are different type of molecular weight things are given. And they discuss about the different type of distribution pattern for the molecular weight. I told you that from P if at any point of time, your molecular mass may consist of  $P_1$  to say  $P_n$ .

**(Refer Slide Time: 09:09)**

### Molecular weight and its distributions

- Generally, polymers are the mixer of molecules of different molecular weight. Therefore **average molecular weight and exact distribution of different molecular weight** is usually required to characterized the polymers.
- There are needs of **control on the average molecular weight and molecular weight distribution** for desire physical properties in the polymers.



So, that is why there is a distribution pattern for these molecular weights. So, based on this particular nothing, we already discussed that polymers, usually they are a mixture of molecules of a different molecular weight, just because attributed to the different polymeric chains having the different number of monomers.

So, therefore, the average molecular weight and exact distribution of a different molecular weight is usually required to characterize the polymer rather than the specific molecular weight of those polymers. So, there are need as I discussed, there is a need to control on the average

molecular weight because of the processability because of the thermal, chemical and mechanical properties.

So, that is why we are always looking for the proper molecular weight distribution to carry out the desired physical properties in those polymers. So, usually this concept of average molecular weight is biased towards the larger and smaller sized molecules. So, various methods are used to determine the average molecular weight of polymers, sometimes colligative properties, sometimes light scattering things, sometimes the viscosity measurements etc.

**(Refer Slide Time: 10:31)**

### Molecular weight and its distributions

- The average molecular weight is biased towards the larger and small-sized molecules.
- Various methods are used to determine the average molecular weight of the polymers, like colligative properties, light scattering, viscosity measurements etc.



So, based on your requirement, you may choose the different protocols for calculating the molecular weight distribution. But before we discuss this, the protocol for the determination of these molecular weight. Let us have that how we can classify these molecular weight distributions. So, there are different methods or different protocols are available for this molecular weight distribution.

**(Refer Slide Time: 11:06)**

## Molecular weight and its distributions

### ➤ Number average molecular weight ( $\overline{M}_N$ )

- The number average molecular weight is defined as the total weight of polymer divided by the total number of molecules.
- It is experimental methods which count the number of molecules in the polymer. In this method, measuring the colligated properties of the solution is measuring of  $\overline{M}_N$

Which is represented by the equation;

$$\overline{M}_N = \frac{W}{\sum N_i} = \frac{\sum N_i M_i}{\sum N_i}$$

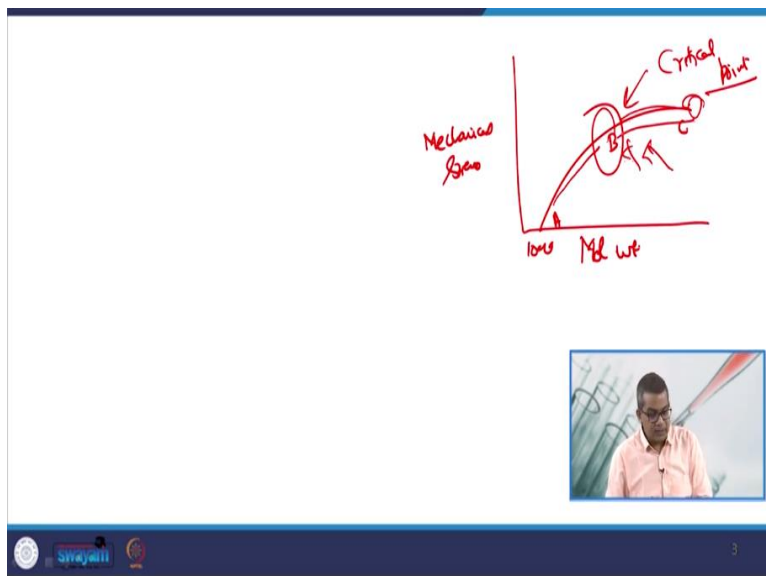
$M_i \rightarrow$  molecular weight of molecule  $X \rightarrow 1 \text{ to } \infty$



1 is that number average molecular weight. Now, that number average molecular weight is usually defined as the total weight of polymer divided by the total number of molecules. Now, it is usually the experimental method, which usually count the number of molecules in the polymer. So, in this method, the measurement of the colligative properties of the solution is measuring by  $M_N$ .

Now, this particular approach is represented by this particular equation. Now, before we go into the detail of this number average molecular weight, let us give you a typical plot of mechanical strength versus molecular weight distribution. There is a useful plot because, by this particular plot, you will be able to learn more and more specifically about the importance of this molecular weight distribution.

**(Refer Slide Time: 12:17)**



So, let us say that this is the molecular weight and here this is the mechanical strength. Now, here this plot is having 2, 3 different sub segments. Now, A is the minimum molecular weight of polymer. This may be say 1000 etc. Above A, usually the strength increases rapidly up to the point B. After this point, the this increases a bit slowly and reach to the limiting value of C.

Now, this shows the critical point correspond to the minimum molecular weight of a polymer to exhibit the sufficient strength. Now, this particular concept is important. Because there are certain limitations and these limitations you cannot carry out as I told you cannot carry out the experimentation or polymerization process for a longer time. Because of the efficiency of a process you require the optimum strength optimum property versus desired molecular weight.

So, the critical point approach is extremely important in this case. No doubt, you may have a higher molecular weight and sometimes a slight enhancement in the mechanical strength. But, if you carry on up to from point B to point C, certain quantum of energy economics, these (()) (14:10) may get involved in this process. So, sometimes you may not get the feasibility to carry out this particular process from B to C.

So, that is why the important point to carry out to analyze is the critical point. Now, this critical point is always useful while deciding the feasibility of any kind of a polymerization process. Now, go back to the number average molecular weight. Now, we discussed this particular approach in this particular slide where we are discussing about the number average molecular weight.

$$\overline{M}_n = \frac{W}{\sum N_x} = \frac{\sum N_x M_x}{\sum N_x}$$

$$\overline{M}_n = \sum f_x M_x$$

$f_x \rightarrow$  molar fraction of molecules

Where  $x \rightarrow 1$  to  $\infty$

Now, here this the equation where or sometimes it is referred as now that here  $N_x$  is the number of moles whose weight is usually the  $M_x$ . This is the summation over the overall different size of a polymer molecules from say  $x$  to extends from 1 to infinity. So, in other words, if we carry



out this particular approach with respect to the mole fraction, it is represented as  $\bar{M}_n$  where this is the mole fraction of molecules.

**(Refer Slide Time: 15:55)**


### Equation $\bar{M}_n$

**Molecular weight and its distributions**

The most common methods for measuring the number average molecular weight are vapor pressure osmometry (used to measure Mol. wt. below 10,000 to 15,000) and membrane osmometry (above 20,000 to 30,000 and below 50,000).

Generally it is experimentally determined by:

- **Vapor Pressure Osmometry**
  - It works by taking advantage of the decrease in vapor pressure that occurs when solutes are added to pure solvent.



7

So, this type of approach is again important. Now, the most common method for measuring the number average molecular weight are the vapor pressure osmometry. Usually measures the molecular weight below 10,000 to 15,000. And another approach is the membrane osmometry for above say to 20,000 to 30,000 and some below 50,000. So, generally, it is determined with the help of experimental tools.

And let us have a brief look about these vapor pressure osmometry and a membrane osmometry. Vapor pressure osmometry, it works by taking advantage of the decrease in vapor pressure that occurs when solute are added to your solvent. It is a very common phenomena of a chemistry.

**(Refer Slide Time: 16:40)**

## Molecular weight and its distributions

This technique can be used for polymers with a molecular weight of up to 20,000 though accuracy is best for those below 10,000

### ➤ Membrane Osmometry

- The membrane osmometry is a technique for the determination of molecular masses of polymers by means of osmosis. It carries out in membrane osmometer. Which consists of two chambers:
- One chamber contains pure solvent and other one with solution that contains a solution in which the solute is a polymer with an unknown



$M_n$



Now, this technique is used for the polymer with the molecular weight they may have up to 20,000. Though accuracy is best below as I told you in the previous slide that below 10,000. The next approach for determination of this is the membrane osmometry. Now, this membrane osmometry is a technique for the determination of molecular masses of polymer by means of osmosis membrane osmosis.

It carries out in a membrane osmometer. Usually, this membrane osmometer consists of 2 chamber, one chamber contains the pure solvent and other one is solution. And there is a barrier in between is that is attributed to the semi permeable membrane. Now, this the solution container contains a solution in which the solute is polymer with an unknown number average molecular weight.

Now, sometimes you may need to form the solution with the different type of a solvent like sometimes hexane, sometimes of a different hydrocarbon solvent being used for that different type of a dilution to carry out this membrane osmometric process. Now, by measurement of osmotic pressure across the semi permeable membrane, this gives the information about this number average molecular weight.

**(Refer Slide Time: 17:56)**

## Molecular weight and its distributions

- By measurement of osmotic pressure across the semi permeable membrane gives information about  $(M_n)$

➤ **Weight average molecular weight ( $\overline{M}_w$ )**

- It is based upon the light scattering by the polymer solution, and it is greater for larger sized molecules and smaller for small-sized molecules.
- This method is most accurate for higher molecular weight polymers. There is no upper limit for light scattering method to measure the molecular weight but the lower limit is 5,000 to 10,000.



Now, another segment of determination of molecular weight distribution that is called the weight average molecular weight is a very common practice for the determination of or representing the molecular weight distribution. Now, it is usually based on the light scattering by the polymer solution and it is a greater for large sized molecule and is smaller for small sized molecule.

So, this method is usually accurate for higher molecular weight polymers. Usually, we do not prefer to have any upper limit of light scattering method to measure the molecular weight but for lower limit it is 5000 to 10,000. Now, this weight average molecular weight referred as  $\overline{M}_w$ . This can be described as the product of weight fraction  $f_x$  of the molecule whose weight is  $M_x$ .

$$\overline{M}_w = \sum f_x M_x$$

$f_x \rightarrow$  weight fraction of molecules whose weight is  $M_x$

$$\overline{M}_w = \frac{\sum C_x M_x}{\sum C_x} = \frac{\sum C_x M_x}{C} = \frac{\sum N_x M_x^2}{\sum N_x M_x}$$

$C_x \rightarrow$  weight concentration of  $M_x$  molecule

$C \rightarrow$  total weight concentration of all polymer molecules

$$C_x = N_x M_x$$

$$C = \sum C_x = \sum N_x M_x$$

(Refer Slide Time: 18:53)

### Molecular weight and its distributions

Weight average molecular weight ( $\overline{M}_w$ ) can be described as the product of weight fraction ( $f_x$ ) of molecules whose weight is  $M_x$  i.e.

$$\begin{aligned} \overline{M}_w &= \sum f_x M_x \\ f_x &\rightarrow \text{weight fraction of molecules whose wt is } M_x \\ \overline{M}_w &= \frac{\sum C_x M_x}{\sum C_x} = \frac{\sum N_x M_x^2}{\sum N_x M_x} \\ C_x &\rightarrow \text{weight concentration of } M_x \text{ molecule} \\ C &\rightarrow \text{total weight concentration of all polymer molecule} \\ C_x &= N_x M_x \\ C &= \sum C_x = \sum N_x M_x \end{aligned}$$



Now, the mathematically it is represented as now, this  $f_x$  is the weight fraction of molecules whose weight is  $M_x$  or this  $\overline{M}_w$  is concentration. This  $C$  is the concentration of  $x$  component. And this is summation  $N_x M_x^2$  upon summation  $N_x M_x$ . Now here the  $C_x$  is the weight concentration of  $M_x$  molecule.  $C$  is the total weight concentration of all polymer molecules.

Now,  $C_x$  can see that is equal to  $N_x M_x$  and  $C$  is. So, this is the mathematical representation of weight average molecular weight how we can represent this weight molecule is respect to the polymeric system. And other the third approach is referred to as the viscosity average molecular weight.

(Refer Slide Time: 21:02)

### Molecular weight and its distributions

#### ➤ Viscosity average molecular weight ( $\overline{M}_v$ )

The viscosity is useful for measuring the molecular weight of the polymers. For larger molecules, the viscosity is larger and smaller for smaller molecules.

The solution viscosity measures the viscosity average molecular weight.

This can be written as;

[Note]



Now, as we know in the previous slide we discussed that viscosity plays a very vital role and if you carry on the polymerization process, sometimes viscosity may increase up to such a level that you cannot process that particular polymer. So, viscosity is a very useful thing for the measuring the molecular weight of polymer. So, for larger molecule viscosity is usually large and for a smaller molecule usually it is bit small.

Now, this solution viscosity usually measures the viscosity average molecular weight. Now, this solution average viscosity average molecular weight mathematically it can be represented as  $\bar{M}_v$ . This is the rate of x component  $M_x$ .  $N_x M_x$  upon  $1/a$ . Here a is constant. Now, viscosity and weight average molecular weights are equal when a is equal to 1.

$$\bar{M}_v = [W_x M_x]^{1/a} = \left[ \frac{\sum N_x M_x^{a+1}}{\sum N_x M_x} \right]^{1/a}$$

a → Constant

a=1 then viscosity and weight average molecular weight are equal

$\bar{M}_v < \bar{M}_w$  if  $0.5 < a < 0.9$

It is usually 20% of  $\bar{M}_w$

(Refer Slide Time: 21:52)

$\bar{M}_v = [W_x M_x]^{1/a} = \left[ \frac{\sum N_x M_x^{a+1}}{\sum N_x M_x} \right]^{1/a}$   
 a → Constant  
 a=1 Viscosity & wt average mol wt are equal  
 $\bar{M}_v < \bar{M}_w$  if  $0.5 < a < 0.9$   
 It is usually 20%  $\bar{M}_w$   
Polymer solvent & temperature

So, viscosity and weight average molecular weight are equal. Now, sometimes we may encounter this type of a scenario where viscosity average molecular weight is less than the weight average molecular weight. So, this scenario happens when a is in between 0.5 to 0.9. Now, it is usually 20% of weight average molecular weight and it varied with the polymer solvent and temperature.

So, you can say that the different controlling parameter for this viscosity average molecular weight is the polymer solvent and temperature. So, these are the governing factor for this viscosity average molecular weight. Now, another important part of this particular concept is the polydispersity index.

(Refer Slide Time: 23:42)

### Molecular weight and its distributions

#### □ Poly-Dispersity Index (PI)

- Polydispersity index (PDI) is used as a measure of broadness of molecular weight distribution.
- It is the ratio of weight average molecular weight to number average molecular weight. It can be written as;

[Note]

- It is usually one (for monodispersed polymer) or greater than one (actual polymer).



Now, this polydispersity index it offers a very vital role when we discuss about the polymerization process and the distribution of molecular weight etcetera. So, this is used as a measure of broadness of a molecular weight distribution, how large or how enhanced this one. Usually, it is the ratio of weight average molecular weight to the number average molecular weight.

$$Polydispersity\ Index = \frac{\overline{M}_w}{\overline{M}_n}$$

(Refer Slide Time: 24:21)

---

$$\text{Polydispersity Index (PI)} = \left( \frac{M_w}{M_n} \right)$$



Now, it can be written as polydispersity index, PI. This is now this ratio depend upon the breadth of the distribution curve. Now, sometimes this particular curve plays a very vital role we will discuss this particular curve later on. Now, this is usually 1 for mono dispersed polymer or greater than 1 for actual polymer. So, it depends on the class of polymer which are being questioned.

**(Refer Slide Time: 24:57)**

---

### Molecular weight and its distributions

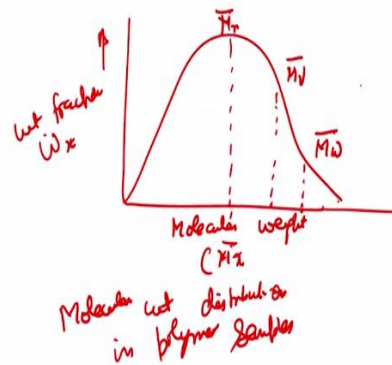
#### □ Poly-Dispersity Index (PI)

- The characterization of the polymer with the number average molecular weight alone without considering the poly-dispersity may mislead. The polymer properties are more dependent on larger molecule size than the smaller ones.
- Poly-dispersity index depends upon the breadth of the distribution curve (a graph in-between weight fraction and molecular weight) as shown in the following figure.



Now, the characterization of the polymer with the number average molecular weight alone without considering the polydispersity index may sometimes give you a misleading information. The polymer properties they are more dependent on a larger molecular size than the smaller one. Because then they are switches towards the monomer side. So, polydispersity index depends on the breadth of the distribution curve graph in between the weight fraction and the molecular weight.

(Refer Slide Time: 25:38)



Now, we can have a look about this particular graph, which we are describing here. Now, here this is the molecular weight and this is the weight fraction. Now, here this is the number average and this is the viscosity average and this one is the weight average. So, this is the molecular weight distribution in polymer samples.

Now, this particular graph is important in this context that sometimes, because of the complexity of the nature you may require all 3 type of molecular weight distribution pattern. So, for this particular plot or this particular curve is extremely important to have an analysis. Because sometimes you may require to have a vetting of a data during the polymerization process and this particular approach is extremely useful to carry out this analysis.

(Refer Slide Time: 27:05)

### Control of polymers synthesis

#### ➤ Thermodynamics and kinetic control

The extent of crystallization can be developed in a polymer by both kinetic and thermodynamics control but the melting temperature can only be thermodynamically controlled.

- Kricheldorf and his coworkers explain that step polymerization can be proceed with either thermodynamic or kinetic control.
- The equilibrium in between the cyclic and linear products can be attained by thermodynamic control in polymerization process and without kinetic control.





Now, when we discussed about this particular approach related to the molecular weight distribution, and I told you about the importance of this particular molecular weight concept approach and how sensitive this particular approach is. That is why we discussed the different type of molecular weight distribution pattern like number average, weight average and viscosity average.


So, you can sensitize the thing that based on the polymerization process, based on the desired property of the polymers, based on the processability aspect, the molecular weight distribution plays a very vital role. Because sometimes you may have a bulky chains then you may face a problem of processability, you may require certain other things sometimes the molecular mass becomes so thick that it may not be possible to remove that particular molecular polymeric mass from the reactor.

So, that is why the molecular weight distribution is extremely important. Now, how to carry out this molecular weight distribution effectively, and how to control this molecular weight distribution, we will discuss this particular thing in the control of control methodologies of polymer synthesis in the next lecture.

**(Refer Slide Time: 28:33)**

**References**

- G.Odian, Principles of Polymerization, Fourth Edition, John Wiley & Sons, Inc., (2004), ISBN:9780471478751 | DOI:10.1002/047147875X
- J.M. Asua, Polymer Reaction Engineering, Blackwell Publishing, (2007), ISBN: 978-1-4051-4442-1.
- M. Szwarc, Marcel Van Beylen (auth.) - Ionic Polymerization and Living Polymers, Springer Netherlands, (1993).
- Stephen L. Rosen. Fundamental principles of polymeric materials, An SPE textbook, John Wiley & Sons (1937), ISBN:0-471-08704-1.
- V. R. Gowariker, N. V. Viswanathan, J. Sreedhar. Polymer Science, New age international (P) limited, publishers (1986), ISBN-08-522-6307-4.
- Joel R. Fried. Polymer Science and Technology:, Prentice-Hall, Inc., (1995), ISBN-81-203-1458-1.



So, in this particular lecture, we have discussed about the molecular weight distribution, we have discussed about the efficacy of this molecular weight distribution to the polymeric system. We analyzed that how we can determine this molecular weight may be number average, may be weight average, may be viscosity average mathematically.

Because there are several experimental tools available as on date through which you can determine these molecular weights. And in the last slide, we discussed that sometimes you may require all 3 molecular weight distribution for in particular polymeric system for to carry out the feasibility of that particular polymerization process. So, we have gained the knowledge about these molecular weight distributions in this particular lecture.

In next slide, because, we have discussed about the efficacy of this molecular weight distribution, we will discuss that how we can control these molecular weight distribution pattern, how we can control the polymeric synthesis. So, we will discuss this particular approach in the next lecture, thank you very much.