Polymer Reaction Engineering
Prof. Shishir Sinha
Department of Chemical Engineering
Indian Institute of Technology-Roorkee

Lecture-20
Multiple Reactor System

Welcome to the multiple reactor system under the edge of polymer reaction engineering. Now before we go into detail of this concept of multiple reactor system let us have a look that what we have discussed in the previous lecture, we have discussed the basic concept of reactor design in the previous lectures.

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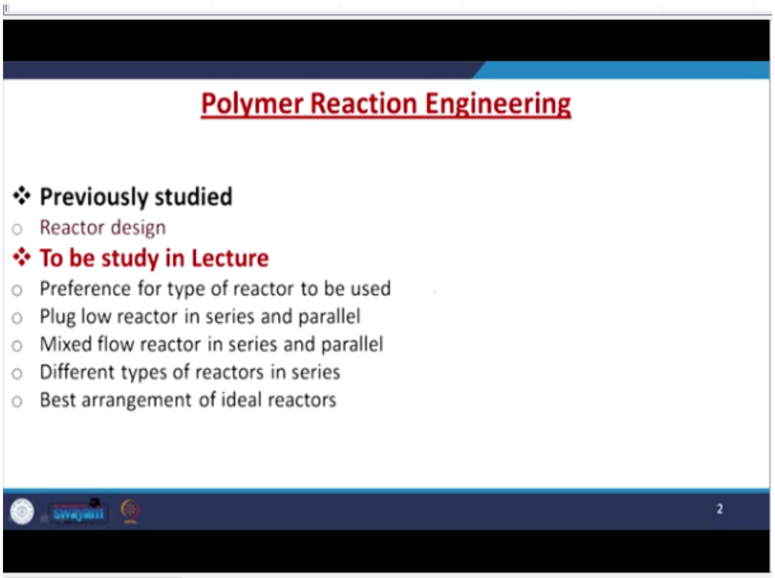

Now in this particular lecture we are going to study with the performance aspect of different type of reactor which are being used in the polymer processing, polymer synthesis. Then we will discuss about the plug flow reactor in series and parallel type of system. We will discuss about the mixed flow reactor in series and parallel. We have looked about the different type of reactors in the series, sometimes the combination of the reactors etc. And then we will have discussion about the best arrangement of ideal reactor system.

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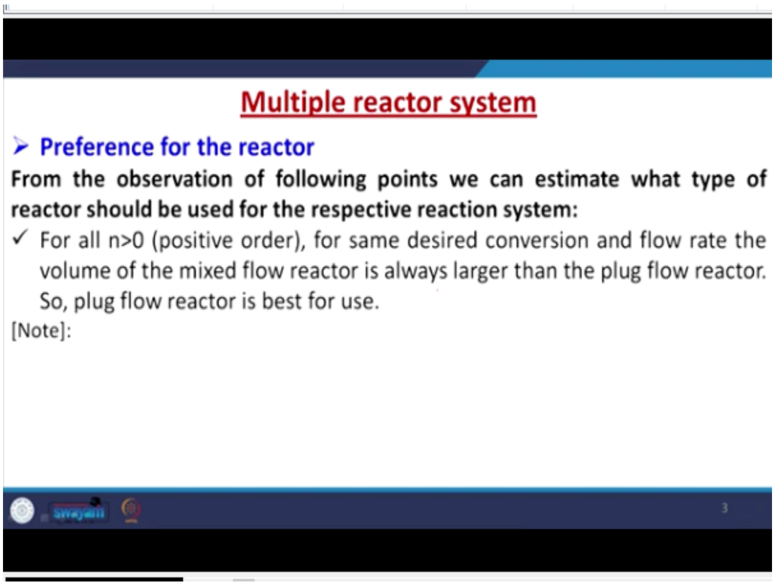

Now when we talk about the preference for the reactor so, from the observation of different points we can estimate that what type of reactor should be used for respective reaction system. Now if we take for all n is greater than 0 that means positive order. So, for some desired conversion and the flow rate the volume of the mixed flow reactor is always larger than the plug flow reactor. So, we can say the plug flow reactor is best for use.

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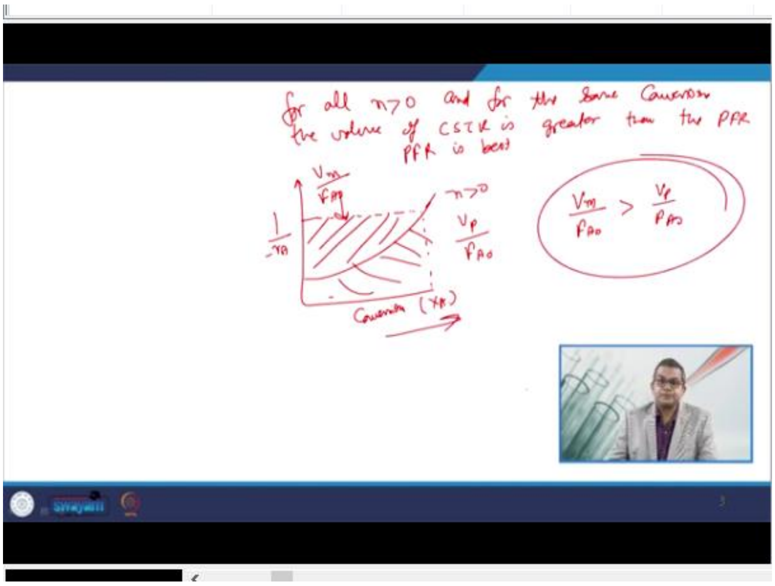

Let us have a look in this pictorial diagram that for all positive order reactors and for the same conversion the volume of CSTR is greater than the PFR. So, therefore based on the economics of the reactor the PFR is termed to be the best one. Now you see in these plots if we say the conversion versus 1 upon -rA. Now here if you see that for the shaded reason if you see that there are 2 type of shaded region one here and second one is here.

Now here V_P upon F_{A0} and this one is n is greater than 0. Now here this V_m upon F_{A0}. So, if you see that the performance etc. in this particular reactor system V_m upon F_{A0}, that is the initial feed rate is greater than V_p upon F_{A0} and that is why it is said to be that the PFR is the best one.

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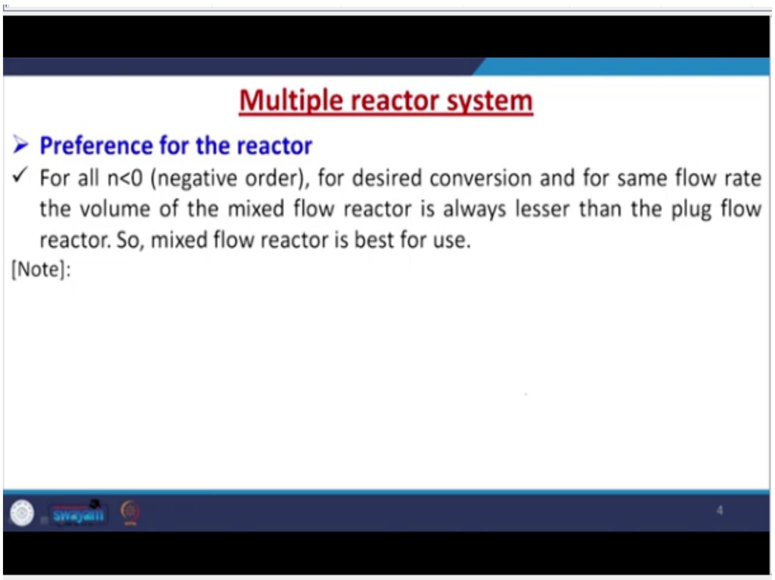

Now for sometimes people may ask that what may happen to the negative order reactor system. So, n is less than 0 for the desired conversion and the same flow rate the volume of the mixed flow reactor is always lesser than the plug flow reactor. So, MFR or mixed flow reactor is termed to be the best for the use of such kind of a system. Let us have a look about this particular thing.

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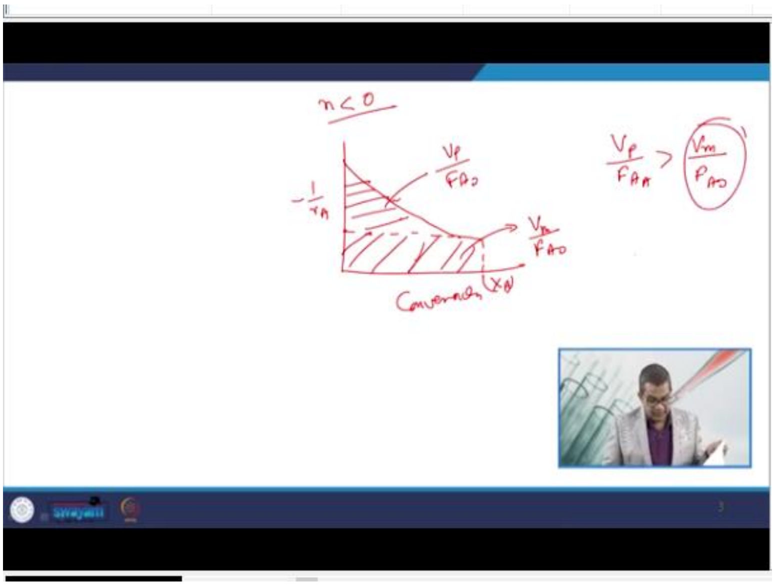

So, let us assume that we are dealing with the same conversion and here the n is less than 0. So, if we talk about the volume of the system then in that case again if we see or if we plot 1 upon $-r_A$ versus conversion, then if you see that there are 2 shaded regions, now this is represented by this one and this is represented by this one. So, here V_P upon A was greater than V_M upon F_{A0}. Now V_P is the volume of plug flow reactor and V_M is the volume of mixed flow reactor. So, you can say that at this approach the mixed flow reactor is termed to be the best one.

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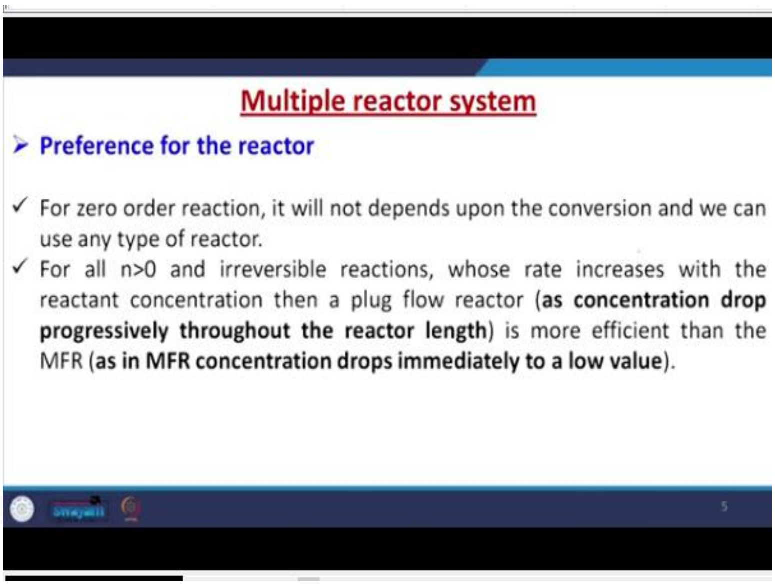

If we talk about the preference for the reactor. Now sometimes for zero order reaction it will not depend upon the conversion. So, we can use any type of reactor. So, for all n is greater than 0 that means the positive order reactor the irreversible reactions if we take into account whose rate increases with the reactant concentration, then a plug flow reactor usually as a concentration drop progressively throughout the reactor length is more efficient than the mixed flow reactor.

So, as in the case of mixed flow reactor or MFR concentration drops immediately to a low volume. Now when we talk about the PFR reactor in series, now if plug flow reactors for different volumes they are connected in series the product stream out from the first reactor will become the feed stream for the next reactor connected in series and so on. Now if one reactor volume is not sufficient for the required conversion then such type of series or a parallel connection may require, may be used for having the better conversion or enhanced conversion or desired conversion.

Now let us assume that n plug flow reactors they are operated in series with a different volume each. So, the fresh feed connected to the first reactor we will discuss this pictorial diagram. So, the fresh feed when it is connected to the first reactor having the initial reactant concentration with no conversion. Let us have a look about this one.

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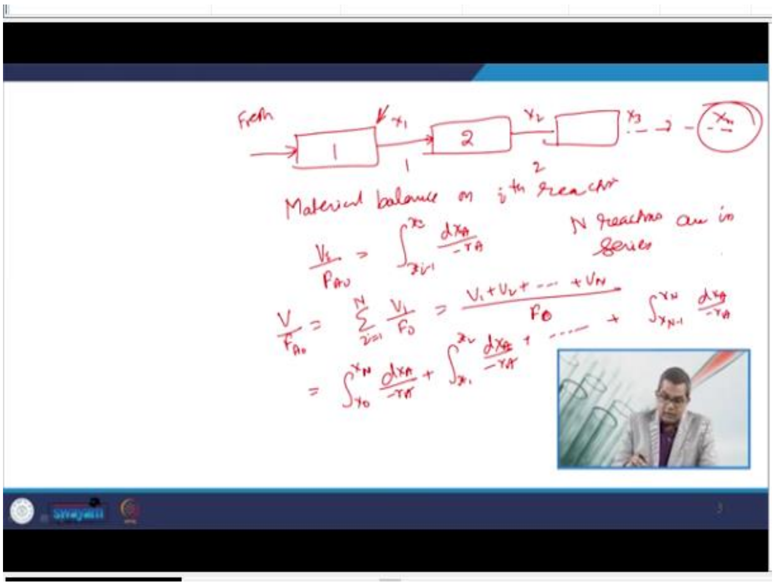

Now here you see that these plug flow reactors they are connected in series right. So, here you may say that the conversion is X_1, here you can say the conversion is X_2, here we say that conversion is X_3 up to X_N.

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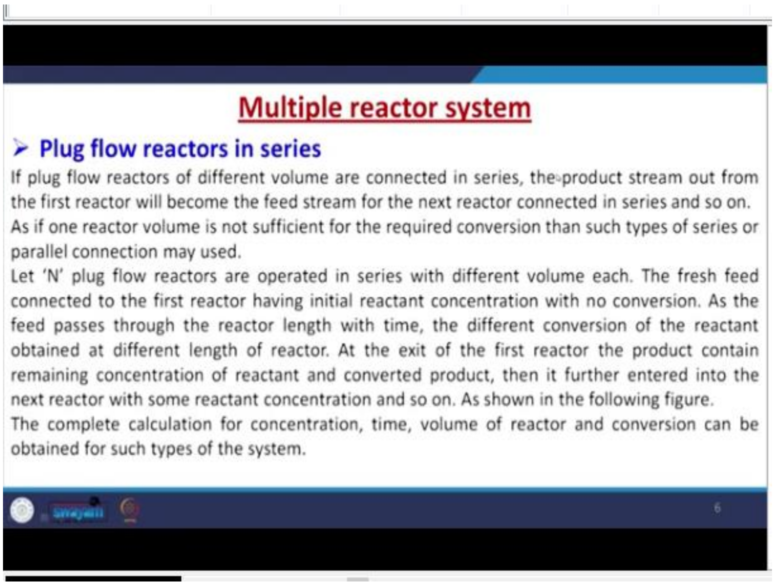

So, there are n plug flow reactor as you see in the figure, they are connected in series and the fresh feed we are supplying to the very first reactor this having the initial reactant concentration without any conversion, that is sometimes referred as F_{A0}. Now as the feed passes through the reactor length with the time the different conversion of the reactant obtained at a different length of reactor.

So, at the exit of the first reactor the product contains remaining concentration of a reactant and converted product. Then it is further entered into the next reactor with some reactant concentration and so on. So, if we see this particular figure here, we are supplying the fresh feed and here if you carry out the product analysis or the outlet analysis, here you will find that there are some unreacted reactants and then there are some sort of a product. That may become the inlet stream of this second reactor and subsequently it goes into the different direction.

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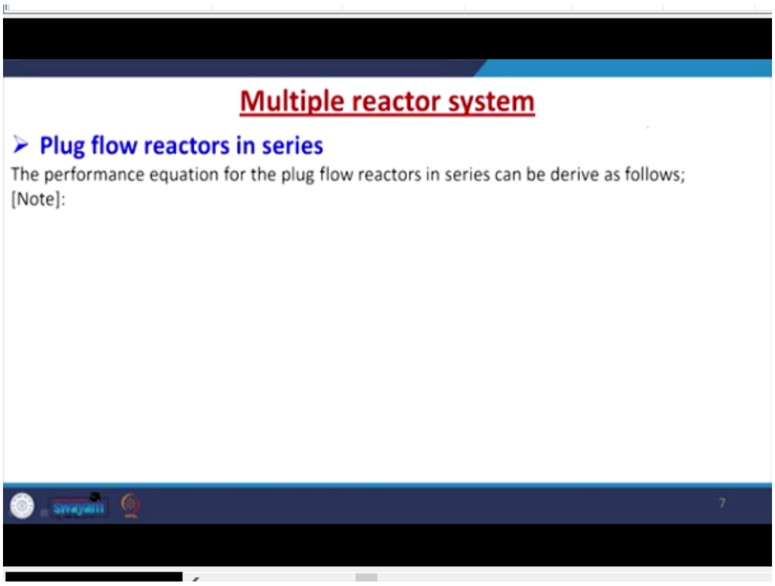

Now sometimes we need to take the material balance. So, in that case you need to calculate the performance equations. Now when we carry out the performance equation the first and foremost thing is that you need to carry out the material balance on this reactor. So, let us say that this is one of the reactors that is i_{th} reactor. So, if we talk about the material balance on say i th reactor, then it may be represented as V_i upon F_{A0} that is the initial condition x_{i-1} to x_i dX_A upon - r_A.

$$\frac{V_i}{F_{A_o}} = \int_{X_{i-1}}^{x_i} \frac{dX_A}{-r_A}$$

$$\frac{V}{F_{A_o}} = \sum_{i=1}^{N} \frac{V_i}{F_o} = \frac{V_1 + V_2 + ---V_N}{F_o}$$

$$\frac{V}{F_{A_o}} = \int_{X_o}^{X_1} \frac{dX_A}{-r_A} + \int_{X_1}^{X_2} \frac{dX_A}{-r_A} + ---- \int_{X_{N-1}}^{X_N} \frac{dX_A}{-r_A}$$

$$\Rightarrow \frac{V}{F_{A_o}} = \int_{X_o=0}^{X_N} \frac{dX_A}{-r_A}$$

Now if we say that there are n reactors as we can see over here, there are n reactor in series then it can be represented as F_{A0}. Let us write this there and reactors are in series. So, you can see this is F_0 $V_1 + V_2$ + and so on + Vn upon F_0. Now this can be represented as X_A - r_A + and this can be X_{N-1} x_N dX_A upon - r_A.

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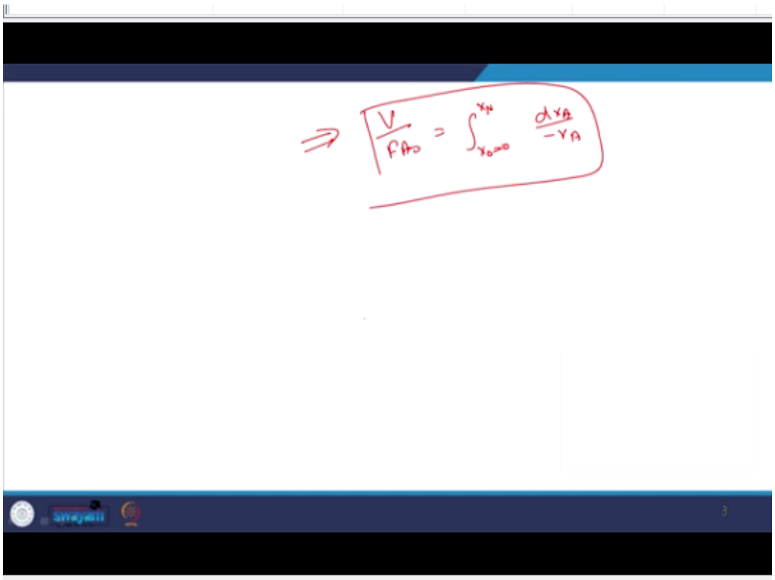

Or it can be represented as V upon $F_{A0} = X_0 = 0$, that is the mole conversion at the start would be 0, then it goes to N dX_A upon - r_A. So, this is one of my desired equation for the things.

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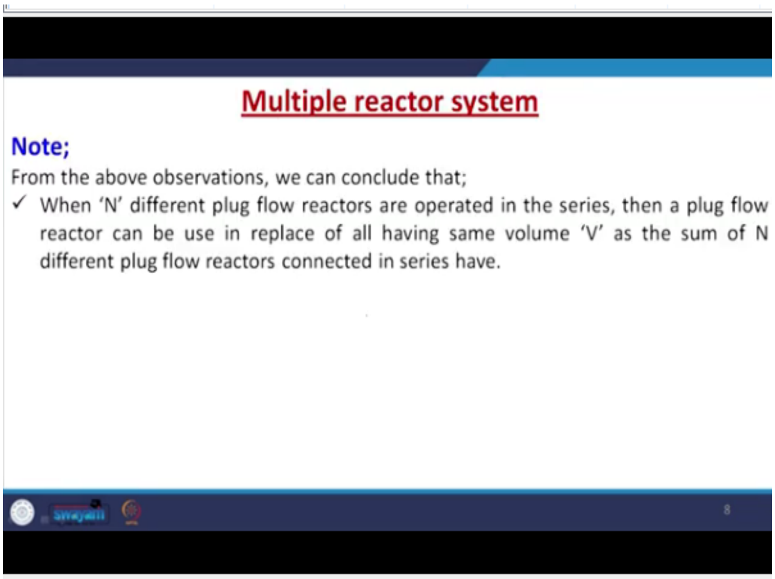

Now if we see that from this observation, we can conclude that when N different plug flow reactors are operated in series, then the plug flow reactor can be used in place of all having the same volume V as the sum of N different plug flow reactors connected in series. Now sometimes you may ask a question that if we have utilized this PFR concept in series then why not in the parallel?

Because for the optimization purpose of the performance you may need to look into all the best options available to you. So, let us have a look about the plug PFR in parallel. So, if the plug flow reactors are operated in parallel; then the fluid system having the same composition in each. In that case the reactors in parallel may have the same volume upon feed ratio or tau in each parallel line. Now let us try to develop the equation for this particular thing.

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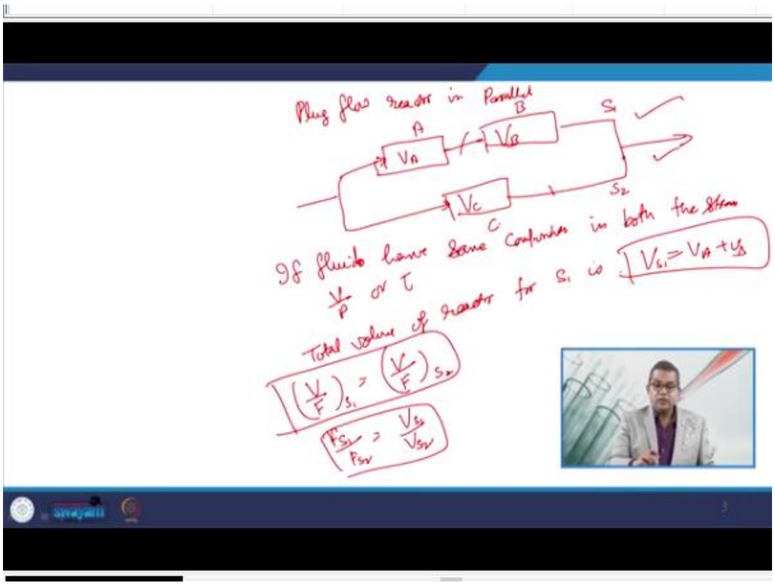

That is the plug flow reactor in parallel. Now here these are the reactors you can say this is the basic arrangement of parallel stream. Now here this is the A, this is the B and this is the C reactor and this is the stream 1 and stream 2. Now if a fluid have same composition in both the stream, then the reactors in parallel have same V upon F or tau which we have represented in each parallel line. So, these are the parallel lines.

So, the total volume of reactor for S_1 is V_{S1} is equal to $V_A + V_B$. Now we can write for the plug flow reactor that is in parallel, so we can write that V upon F S_1 = V upon F S_2. Similarly, if we can write like this F S_1 upon F S_2 is equal to VS_1 upon VS_2, from this relation we can calculate what amount of feed must be fed to the system either S 1 or S 2. So, this is again a very much important thing which we need to look into.

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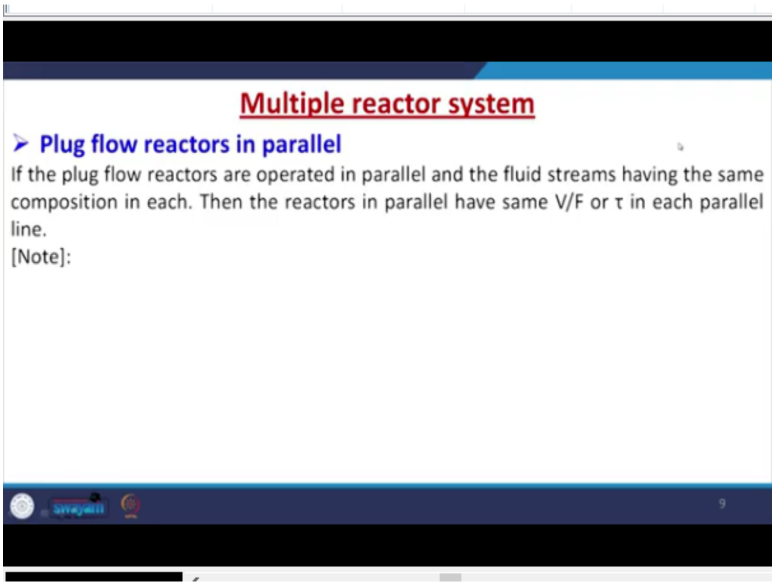

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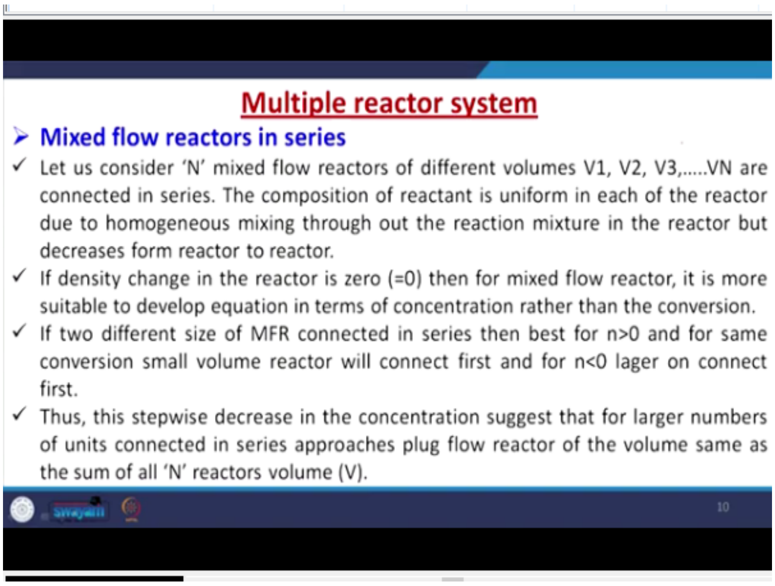

Now let us talk about the mixed flow reactors in series. Now mixed flow reactors in series again a very good concept and let me draw a basic figure for this one first.

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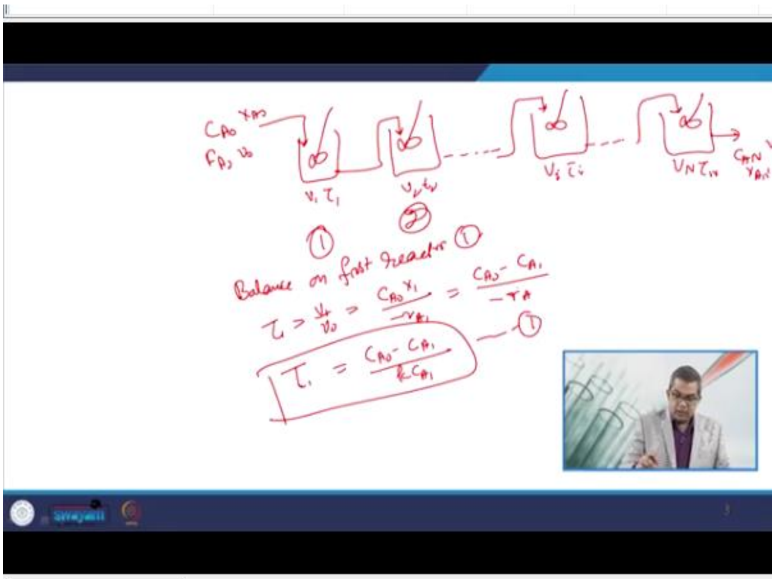

Now this is your basic battery of your mix flow reactor, here this is the basic concept there are several reactors in conjunction with the other one. Now let us have the coordinates of each reactors, this one is C_{A0} X_{A0} F_{A0} nu 0 V_1 tau 1 V_2 tau 2 and all these reactors are having, then this is Vi tau i and this is V_N tau N and here we are having the C_{AN} X_{AN} nu.

So, our main focus of attention will be to these reactors. Now if you see that we considered N mixed reactors in different volumes and you see here we have used the different volumes V_1 V_2 V_3 V_i up to V_N. Now they are connected in series and the composition of reactant is uniform in each of the reactor due to the homogeneous mixing throughout the reaction mixture in the reactor, but decreases from reactor to reactor.

Now if density change in the reactor is 0, then for the mixed flow reactor it is more suitable to develop equation in terms of concentration rather than conversion. Now if we are taking the 2 different size of mixed flow reactor connected in series, then best will be for where you may observe the positive order, that is n is greater than 0 and for the same conversion small volume reactor will connect first.

And for if we take the order to the negative that is n is less than 0, then larger reactor will be connected first. So, that is for the 2 dissimilar volume condition. Now if this step wise decrease in the concentration sometimes suggests that for the larger number of units connected in series approaches plug flow reactor of the volume same to the sum of all and reactor volumes.

So, if we try to write the balance equations for all kind of a reactor, then balance on the first reactor can be written as first reactor, then we can write the performance equation of CSTR that is tau 1 = V_1 upon nu 0 = C_{A0} X_{A1} - r_{A1} = C_{A0} - C_{A1} upon - r_{A1}. So, if we say that if we try to rearrange the things then it may become the C_{A0} - C_{A1} upon kC_{A1}. This is my equation number 1.

$$\tau = \frac{V_1}{\upsilon_o} = \frac{C_{A_o}X_1}{kC_{A_1}} = \frac{C_{A_o} - C_{A_1}}{-r_A}$$

$$\tau_1 = \frac{C_{A_o} - C_{A_1}}{kC_{A_1}} \qquad (1)$$

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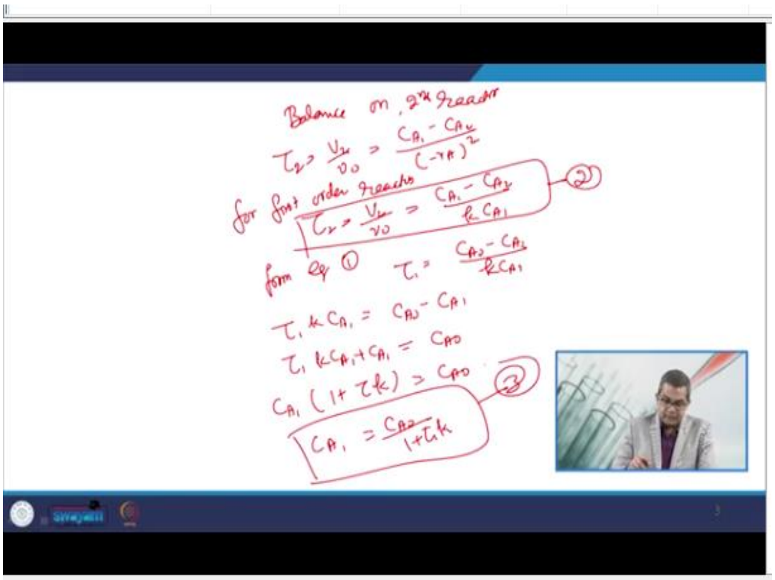

Now if we take the balance on second reactor then tau 2 = V_2 upon nu C_{A1} - C_{A2}. So, for first order reaction nu 0 on k C_{A1}. This may be termed as equation number 2. Now if we say that from equation number 1, if you recall that equation number 1 then tau 1 = C_{A0} - C_{A1} upon k C_{A1}. So, it can be tau k C_{A1} = C_{A0} - C_{A1} or tau kC_{A1} + C_{A1} = C_{A0} and C_{A1} into 1 + tau k C_{A0} or C_{A1} = C_{A0} upon 1 + tau 1 k. Now this becomes the equation number 3.

Balance on second reactor

$$\tau_2 = \frac{V_2}{\upsilon_o} = \frac{C_{A_1} - C_{A_2}}{-r_{A2}}$$

For first order reaction

$$\tau_2 = \frac{V_2}{\upsilon_o} = \frac{C_{A_1} - C_{A_2}}{kC_{A1}} \quad (2)$$

Now from equation (1)

$$\tau_1 = \frac{C_{A_0} - C_{A_1}}{kC_{A_1}}$$

$$\Rightarrow \tau_1 k C_{A_1} = C_{A_0} - C_{A_1}$$

$$\Rightarrow \tau_1 k C_{A_1} + C_{A_1} = C_{A_0}$$

$$\Rightarrow C_{A_1}(\tau_1 k + 1) = C_{A_0}$$

$$\Rightarrow C_{A_1} = \frac{C_{A_0}}{(\tau_1 k + 1)} \qquad (3)$$

Now from equation (2) we have

$$\tau_2 = \frac{C_{A_1} - C_{A_2}}{kC_{A_2}}$$

$$\Rightarrow \tau_2 k C_{A_2} = C_{A_1} - C_{A_2}$$

$$\Rightarrow \tau_2 k C_{A_2} + C_{A_2} = C_{A_1}$$

$$\Rightarrow C_{A_2}(\tau_2 k + 1) = C_{A_1}$$

$$\Rightarrow C_{A_2} = \frac{C_{A_1}}{(\tau_2 k + 1)} \qquad (4)$$

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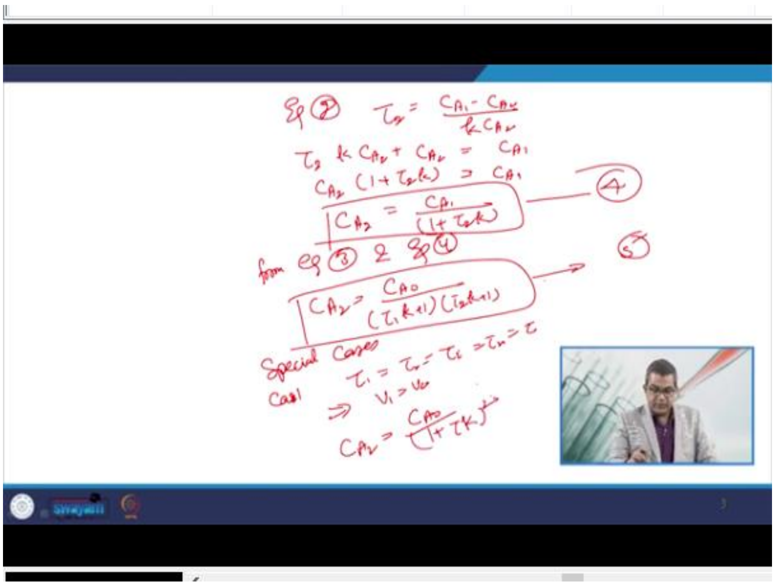

Now if we take the equation 2 into cognitions then it is tau 2 = C_{A1} - C_{A2} upon kC_{A2} tau 2 kC_{A2} + C_{A2} = C_{A1} it will imply C_{A2} (1 + tau 2 k) = C_{A1}. Now we if we write the equation with respect to the C_{A2} then it becomes C_{A1} upon 1 + tau 2 k. Now this becomes the equation number 4. So, from equation 3 and equation 4 we can have C_{A2} is equal to C_{A0} + upon tau 1 k + 1 tau 2 k + 1 and that becomes the equation number 5.

Now sometimes we may take some special cases. So, these special cases that is referred as like case 1, if we talk about the case 1. Then there may be if tau 1 = tau 2 = say tau i = tau n and let us say tau and V_1 = V_2 then C_{A2} will become the C_{A0} upon (1 + tau k) whole square.

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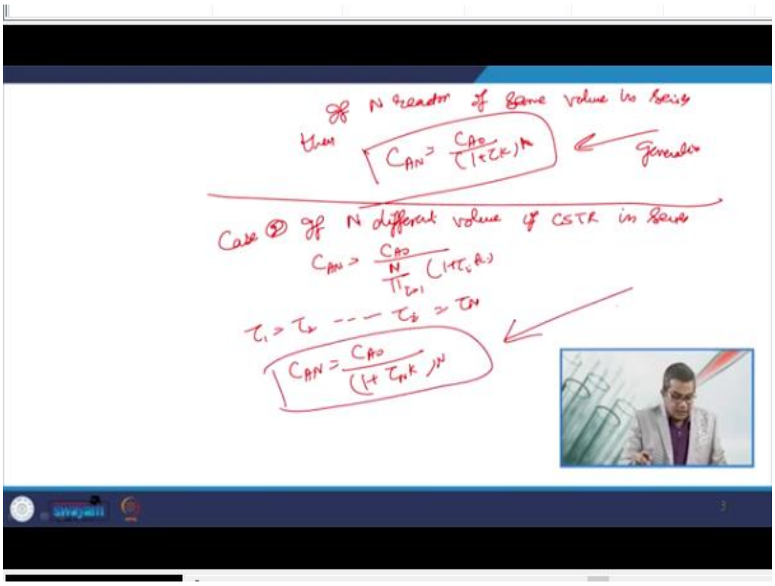

Now if we say that n reactors of same volume in series if N reactor of same volume in series, then C_{AN} C_{A0} upon (1 + tau k) raise to the power n. So, this is the more generalized equation. Now let us take another case that is case 2. Now if we say that N different volume of CSTR in series then C_{AN} = C_{A0} upon product of (1 + tau i k) from i=0 to ni. Now again we take some special case tau 1 = tau 2 = tau i = tau N. Then it can become C_{AN} = C_{A0} upon 1 + tau N k to the power N. Now this is again one of the more generalized equation.

$$\Rightarrow C_{A_N} = \frac{C_{A_o}}{(\tau_N k + 1)^n}$$

If N different volume of CSTR connected in series then

$$C_{A_N} = \frac{C_{A_o}}{\prod_{i=1}^{N}(1 + \tau_i k)}$$

If $\tau_1 = \tau_2 = \tau_i = \tau_N$ then it will become

$$\Rightarrow C_{A_N} = \frac{C_{A_o}}{(\tau_N k + 1)^n}$$

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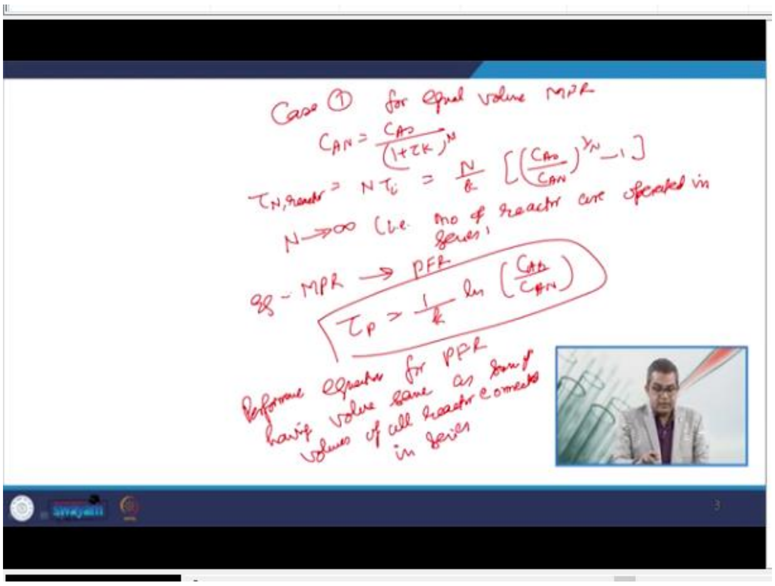

Now if we take the case 1 again for equal volume MFR, case 1 for equal volume MFR, then it can become C_{AN} = C_{A0} upon 1 + tau k to the power N. Now tau N reactor this is can be represented as tau i and upon k C_{A0} upon C_{AN} to the power 1 upon N - 1. Now if there are various cases, now if say N tends to infinity that is number of reactors are operated in series. Then this equation for the mixed flow reactor approaches to the plug flow reactor.

So, MFR the equation attributed to MFR approaches to PFR. So, in that case tau p = 1 upon k ln C_{A0} upon C_{AN}. So, this equation will become. Now this is the performance equation for PFR having volume same as sum of volumes of all reactors connected in series. So, by this way we can generate the performance equation which are query applicable for various system.

For equal volume MFR

$$\Rightarrow C_{A_N} = \frac{C_{A_o}}{(\tau_N k + 1)^N}$$

$$\tau_N = N\tau_i = \frac{N}{k}[\left(\frac{C_{A_o}}{C_{A_N}}\right)^{1/N} - 1]$$

If N→∞ i.e. N no of reactor are operated in series

$$\tau_P = \frac{1}{k} ln\left(\frac{C_{A_o}}{C_{A_N}}\right)$$

This is the performance equation for PFR having N different PFR of same volume are operated in series.

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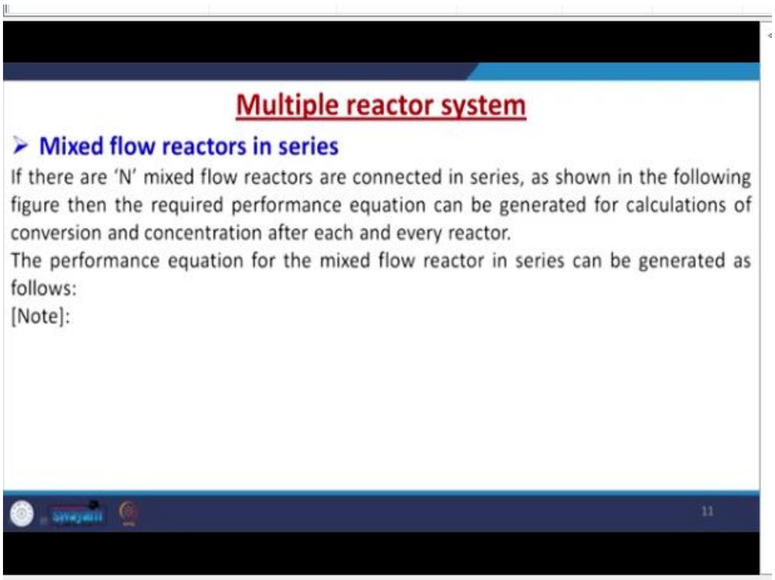

Now if we talk about mixed flow reactors those connected in series, then the required performance equation can be generated for the calculation of conversion and concentration after each and every reactor. Now let us have a look about the different type of reactors in series, because this is one of the most common things in various industrial approaches. Now if there are 3 different size and type of reactors are operated in series, let us have a look about the figure first.

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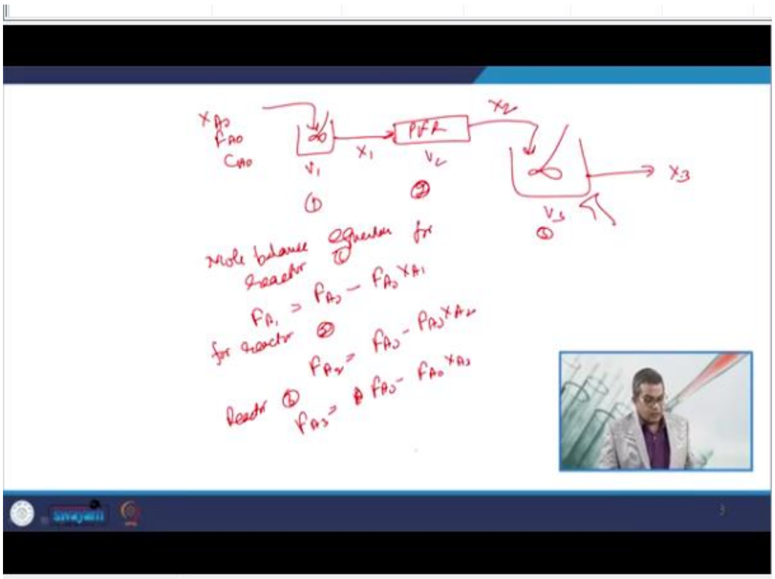

Like this there are 3 different type of reactor you see that this one is MFR, this one is PFR. So, V_1 V_2 V_3 and here you are feeding with X_{A0} F_{A0} C_{A0}, then you may experience some conversion over here, then the output of this reactor is subjected to this reactor having this X_2 and here you are having the X_3. So, 1, 2 and 3 systems are in series.

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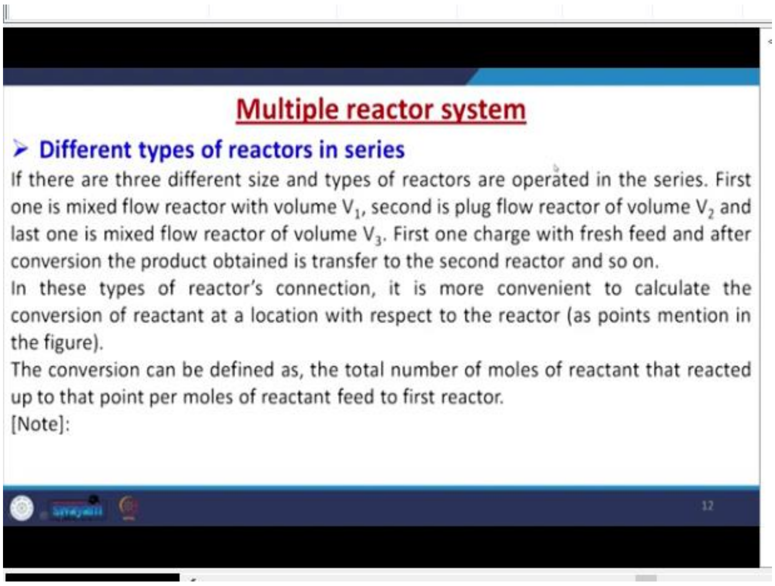

So, now first you can say that one is the mixed flow with the volume 1, the second is the plug flow reactor with the volume 2 and the last one is again the mixed flow reactor of having the volume 3 V_3. So, first one charge with the fresh feed in this particular first reactor then the reactant or the product stream is the reactant line or inlet of this PFR, then again, the outlet of this PFR is the inlet line of this MFR.

So, this particular type of approaches are very common in the industry. So, in this type of reactors connection it is more convenient to calculate the conversion of reactant at a location with respect to the reactor. Now as point mentioned which we have already mentioned in this type of thing in the previous segment, now the conversion can be defined as the total number of moles of reactant that reacted up to the point per mole of the reactant feed to the first reactor.

Now let us have this the mole balance equation for reactor 1. So, mole balance equation for reactor 1. Now F_{A1} equal to $F_{A0} - F_{A0} X_{A1}$. Now if we talk about for reactor 2 then it may become $F_{A2} = F_{A0} - F_{A0} X_{A2}$. Now if we talk about the reactor 3 for this reactor. So, for reactor 3, $F_{A3} = F_{A0} - F_{A0} X_{A3}$. So, these are the mole balance equation.

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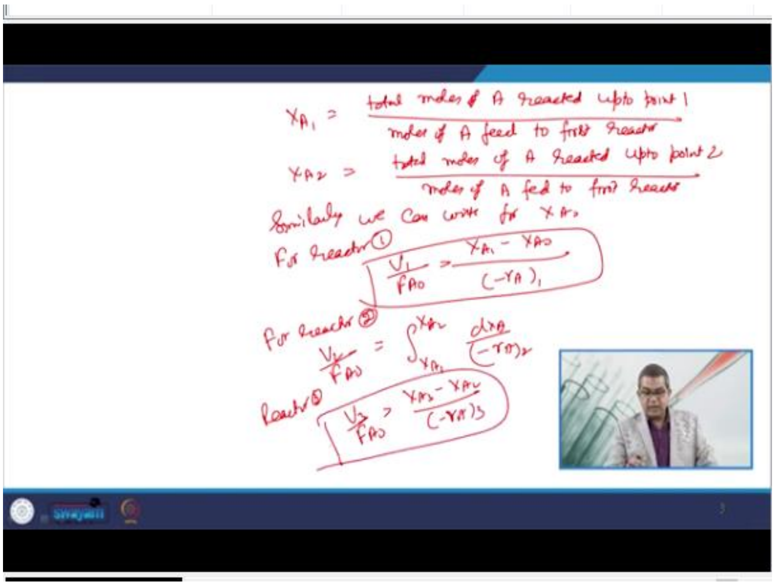

Now where if we say that what is X_{A1}? That is the total moles of a reacted up to point 1. That means up to here divided by moles of A fed to first reactor. Similarly, we can write X_{A2} that is the total moles of A reacted up to point 2, that means up to here divided by moles of a fed to first reactor. So, similarly we can write for X_{A3}, now for reactor 1 if we talk about the reactor 1 that may be represented as V_1 upon F_{A0} = X_{A1} - X_{A2} upon - r_{A1}.

Similarly, if we try to write the equation similar type of reaction for reactor 2 then it can be represented as V_2 upon F_{A0} = integration form X_{A1} to X_{A2} dX_A upon - r_{A2}. Now if we try to write the equation for reactor 3 then it may become V_3 upon F_{A0} X_{A3} - X_{A2} upon - r_{A3}. So, sometimes graphical representation usually helps in this approach.

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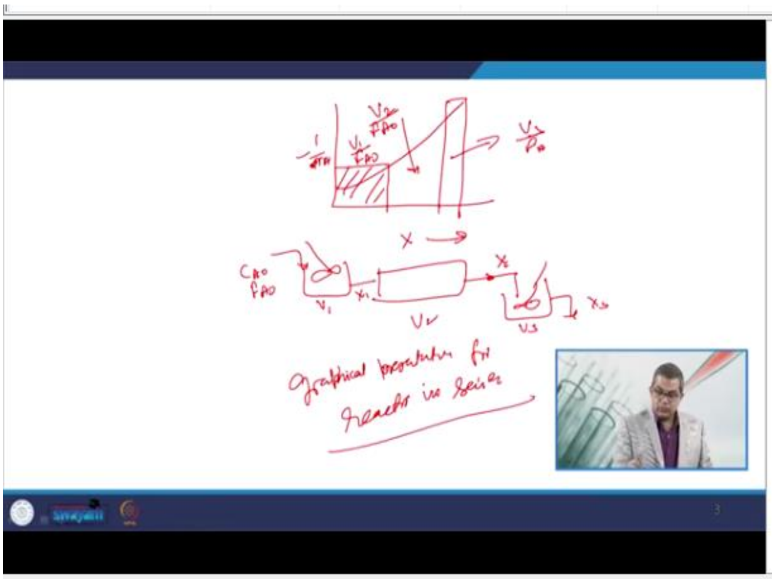

And you can draw the graph with the conversion. So, here this is the conversion you may have such type of a graph where you may represent the first reactor conversion like this with respect to 1 upon - r A, that is V_1 upon F_{A0}. Similarly, you may represent another mix flow reactor that is V_3 upon F_{A0} and here this may be represented as for V_2 upon F_{A0}. For your convenience I am again giving you the battery of that particular reactor system.

Now here you are getting X_3 and here you are having the C_{A0} F_{A0}, here V_1 that is X_1 V_2, then this is V_3 and here you may have this X_2, this is coming out and there X_3. So, this is the graphical representation for reactor in series. So, this is the best way to look into this the performance of these different types of reactors in series.

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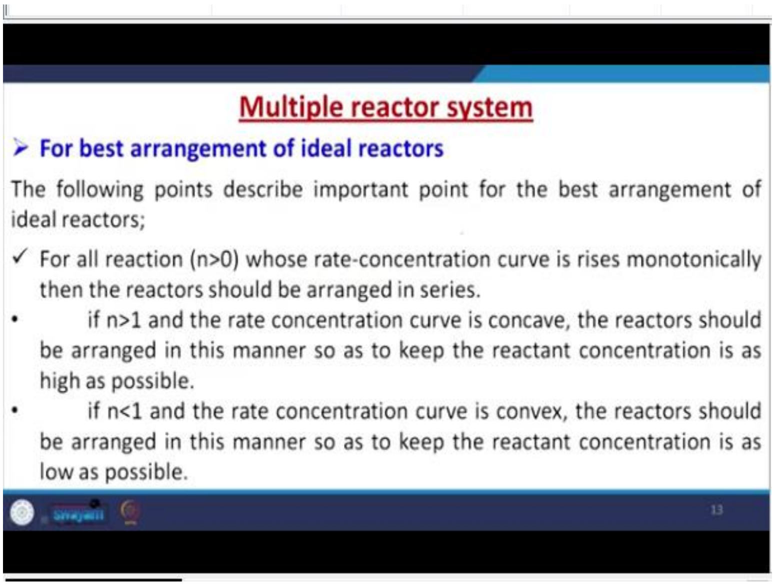

Now sometimes people may ask that what should be the best arrangement for ideal reactors? So, there are several points which we need to describe and we need to take some important point for the best arrangement of these ideal reactor for all reaction if we are having the positive order that is n is greater than 0 whose rate concentration curve is usually rises monotonically, then the reactor should be arranged in series.

Because this is a very complex type of system that is why this type of an advisory is always required for this to handle the situation. Now if n is greater than 1 the rate concentration curve is usually concave the reactor should be arranged in a manner so that to keep the reactant concentration **is** as high as possible. Now sometimes you may see that if n is less than 1 then the rate concentration curve is convex.

The reactor should be arranged in such a manner, so that the reactant concentration should be kept as low as possible. Now if a rate concentration curve passes through the maximum or minimum then the arrangement of the units depends upon the shape of curve desired conversation or desire conversion and number of units available. Now the examination of the curve between the rate inverse and the concentration is again very important factor for the best arrangement determination, that needs to be addressed.

Now if you take an example of a reactor arrangement as we discussed in the previous one that reactor of a different type in series, the arrangement can be made with respect to having the things V_1 is less than V_3 in our mind and n is greater than 1. Then if we are handling such type of sequence the sequence of the reactor is plug flow reactor and the small mixed flow reactor and at the last large mixed flow reactor will suffice.

So, the volume of those things play a very vital role and if you see that in the previous discussion we have discussed this thing V_1, V_2 and V_3, all three reactors they were having the different volumes. So, if they are V_1 is less than V_3 and n is greater than then you may have a different configuration you may have because this is quite obvious that which reactor should be having the volume on the higher side or sometimes people may say that the reactor may have all volumes equal.

So, this is a very debatable thing. So, you must have these things in your mind. Now for determining the arrangement of ideal reactor form the usually a graph is drawn between 1 upon - A versus X_A. So, the area should be minimum for the best possible arrangement.

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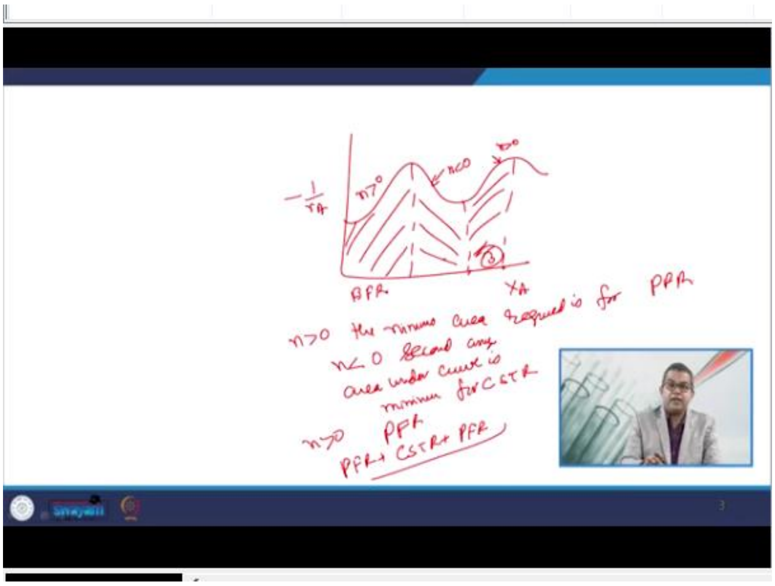

Now here you see that we have drawn a graph here 1 upon $-r_A$ and this is the X_A. So, you may have this type of curve where sometimes n is greater than 0, here you may have n is less than 0, here you may have n is greater than 0. So, the maxima and minima. So, you are having different zones. So, the PFR now for n is greater than 0 the minimum area required is for PFR if n is less than 0, that is the second arrangement.

The area under curve is minimum for CSTR and for n is greater than 0 for this third zone the area under the curve is possible only for the PFR. So, this is for the PFR. So, the arrangement would be PFR + CSTR + PFR. So, this would be the arrangement.

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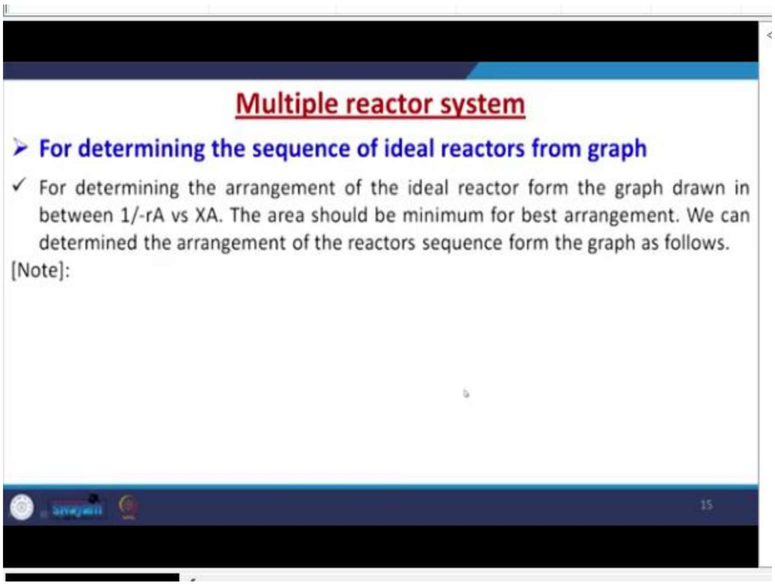

Now at the last we discussed about the multiple reactor system and we discussed the various configurations whether it is a mixed flow reactor or plug flow reactor and how we can arrange whether they are in parallel or they are in series, how we can discuss and how we can address the performance equation concept in this arena and we especially emphasized with the plots. So, that the area under curve you can calculate and then you can decide that which kind of arrangement is best suited for your system.

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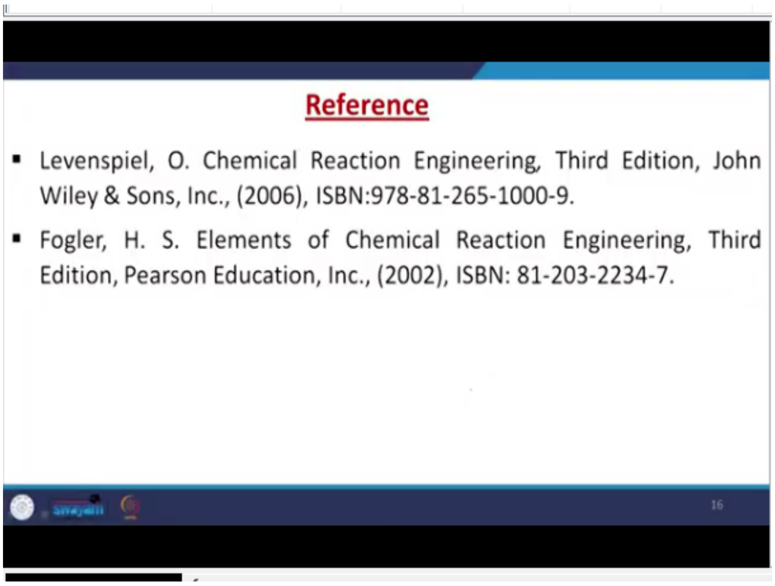

For your convenience we have enlisted couple of references you can utilize those references for the further study, thank you very much.