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# Lecture-17 Interpretation of Batch Reactor Data-IV

Welcome to the next phase of the interpretation of batch reactor data for reactor design. Now if you recall that previously we have studied about the interpretation pattern of batch reactor data. (**Refer Slide Time: 00:35**)



Under this head we discussed about the second order reaction, both for equimolar and nonequimolar feed. We developed the mathematical equation for  $n^{th}$  order reaction for the prediction of batch reactor data. Now in this particular lecture we will discuss about the half-life order and varying volume reaction system. Both the things are very much important in relation with the batch reactor data and designing of polymer reaction reactors with respect to the product profiling. Now for half-life method for reactions, now it is as the basic line suggests that it is the time required to achieve 50% of conversion or time required to reduce the concentration value of the reactant to reach just half the initial value. So, if we take the example of our previous reaction that is A + B =

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Interpretation of batch reactor data
• For half life method for reactions It is the time required to achieve 50% of conversion or time required to reduce the concentration value of the reactant to reached just half of the initial value i.e. $\bigcirc = \bigcirc = \bigcirc = \ d + - \ k$
> Half life for nth order reaction The kinetic expression for nth order reaction i.e. $ \begin{bmatrix} 1 & -1 \\ C_{R} & C_{R^{2}} \end{bmatrix} = (n-1) k+1 $
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Now if we talk about the half-life pattern or half-life method then it may become  $C_A$  concentration of reactant  $A = C_{A0}$  by 2.

The rate equation for  $n^{th}$  order reaction is written as:

$$-\mathbf{r}_A = -\frac{\mathrm{d}\mathbf{C}_A}{\mathrm{d}\mathbf{t}} = \mathbf{k}\mathbf{C}^n_A$$

So, at we can say the time t = t, this one, so this is the initial concentration and this is the concentration after t half. So, the kinetic expression for n<sup>th</sup> order reaction will become 1 upon C<sub>A</sub><sup>n-1</sup> - 1 upon C<sub>A0</sub><sup>n-1</sup> = (n-1)kt.

$$\frac{1}{C^{n-1}{}_{A}} - \frac{1}{C^{n-1}{}_{A_{O}}} = (n-1)kt$$

This equation is valid for  $n \neq 1$ 

So, this equation is representing the n<sup>th</sup> order reaction this is the kinetic expression for this halflife prediction.

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Now on putting the value of half-life of the reactant concentration in this equation and try to rearrange, then we found that this equation becomes 1 upon  $C_{A0}$  by 2 to the power n - 1 upon  $C_{A0}$  n - 1 = n - k t half. So, if we go for the rearrangement 0.5 1 - n - 1 upon n - 1 k  $C_{A0}$  1 - n. So, this is I can say my general equation for predicting this particular half-life pattern of this data.

$$\Rightarrow \frac{1}{\binom{C_{A_o}}{2}^{n-1}} - \frac{1}{C^{n-1}_{A_o}} = (n-1)kt_{1/2}$$
$$\Rightarrow t_{1/2} = \frac{[(0.5)^{1-n} - 1]}{(n-1)k}C^{1-n}_{A_o}$$

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Interpretation of batch reactor data

This is required expression for half life of the n<sup>th</sup> order reaction. It is clear from the half life expression that the half life is the function of initial concentration and valid for n except 1.



Now this required expression for half-life of  $n^{th}$  order reaction, now it is from the half-life expression that the half-life is the function of initial concentration and valid for all kind of n except n is equal to 1. If you see that if you put n = 1, then the entire equation will spoil. So, the half-life of zero order reaction can find out on putting the values of say n = 0 in zero order rate equation and to find out the value of t half at C<sub>A0</sub> upon 2k.

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Similarly for second order reaction if try to find out the t half that will be 1 upon  $C_{A0}$  k. And for the first order reaction if we put the values then  $C_A = C_{A0}$  upon 2 in kinetic expression for first order reaction. And if you found that t half = 0.693 upon k, this is a very important result. Now let us come to the last phase of this interpretation of batch reactor data, that is the varying volume or reactor system. So, when volume V that is usually expressed in terms of a unit of a liter.

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And the volumetric flow rate, let us say that having this small v liter per minute of the reacting material is varying with the time in a reactor pattern or reactant system with the progress of the reaction. Then such type of a system they are referred as the varying volume reactor type of system. And it is applicable for both batch and the continuous type of reaction mechanism.

So, I hope that this particular thing is understandable that you are carrying out a reaction in this particular any kind of a reactor. And the volumetric flow rate and volume V, they are into the consideration. And during the progress of the reaction the V and both the volumetric flow rate both are varying due course of time. So, we need to find out the various equation associated with this type of approach.

So, let us have this thing to the consideration for the batch reactor type of system. So, we express the volume as a function of convergent in the previous equation by using the equation of a state for varying volume batch reactor system. Now the equation of a state which attracts our focus that is a  $PV = N_T ZRT$ . Here other things are let us put as equation number 1, pressure, volume,  $N_T$  is total number of moles, Z is the compressibility factor, R is universal gas constant or sometimes it is referred as a characteristic gas constant and T is the temperature of the system. Equation of state;

$$PV = N_T ZRT$$
(1)

Where,

- P- total pressure (**kPa**)
- V- volume of the reactor at time 't' (litre)
- $N_T$  total number of moles at time 't'
- Z- compressibility factor
- R- gas constant (8.314 J/mol.K)

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So, this now if see that if we try to find out the situation for time t = 0 the equation of a state we can write as  $P_0V_0 = N_{T0}Z_0RT_0$ . Remember the P here we are representing P as a total pressure which is represented in equal Pascal, V is the volume of the reactor at time t and already mentioned that it is having the unit of liter. Then  $N_T$  is the total number of moles at time t and Z is the compressibility factor and we can put the value of R = 8.314 joule per mole Kelvin.

So, this not represents the initial condition, now if we this is my equation number 2 and if you recall here, we represented the equation number 1. So, if we divide equation 1 by equation 2, then we may have this PV upon  $P_0V_0 N_T ZRT$  upon  $N_{T0}Z_0RT$ . So, if we cancel out these R, so  $V = V_0 N_T$  upon  $N_{T0}$  this is a subscript T upon  $T_0P_0$  upon  $P_Z$  upon  $Z_0$ , so this is the generalized equation. To write more clearly it can be represented that  $V_0NT$  upon  $N_{T0}$  T upon  $T_0$ , initial temperature and the final temperature  $P_0$  upon  $P_Z$  upon  $Z_0$ . Let us refer at this is equation number 3.

Let for time t=0 the equation of state can be written as:

$$P_o V_o = N_{T_o} ZRT_o \qquad (2)$$

On dividing previous equation (1) by above equation (2) and rearranging

$$V = V_o \frac{N_T}{N_{T_o}} \frac{T}{T_o} \frac{P}{P_o} \frac{Z}{Z_o}$$
(3)

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Now if we want to express the volume in terms of a conversion, so the total number of moles at any time of the reactor can be given as  $N_T = N_{T0}$  change in the number of moles  $N_{A0}X_A$ , let us put as equation number 4.

$$N_T = N_{To} + \delta(N_{Ao}X_A) \tag{4}$$

So, if we divide this equation 4 with  $N_{T0}$  to both sides, then  $N_T$  upon  $N_{T0}$  1  $N_{T0}XA$ . Now here if you see that  $N_{A0}$  upon  $N_{T0}$  is  $Y_{A0}$ , that is the initial mole fraction, not the mole, initial mole fraction of reactant A.

$$\Rightarrow \quad \frac{N_T}{N_{\rm To}} = 1 + \delta \cdot \frac{N_{\rm Ao}}{N_{\rm To}} X_A \quad (5)$$

Now N<sub>T</sub> upon N<sub>T0</sub> thus become 1 plus change in this pattern X<sub>A</sub> or in other words you can see that N<sub>T</sub> upon N<sub>T0</sub> = 1 + epsilon A X<sub>A</sub>. Let us represent this particular equation as equation number 5.

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Now here if you see that we use the term epsilon, now this epsilon is this delta  $Y_A$ , where epsilon can be expressed as the ratio of change in total moles of reactant at full conversion to the initial moles of reactant feed to the reactor. So, this is again we can say it is a very crucial thing and it is very useful for determining the efficacy of any particular reaction.

 $N_{Ao}/N_{To} = y_{Ao}$  initial mole fraction of reactant A

$$\Rightarrow \quad \frac{N_T}{N_{\text{To}}} = 1 + \delta . y_A X_A \quad (6)$$

As,  $\varepsilon_A = \delta . y_A$ 

$$\Rightarrow \quad \frac{N_T}{N_{\rm To}} = 1 + \varepsilon_A X_A$$

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Now if you see that from earlier equation number 3, let us go back this is the equation number 3 and equation number 5 which we discussed over here. So, if you see that equation number 3 and 5 for varying volume the above these equations. And if you put  $Z = Z_0$  for the gaseous system, the compressibility factor does not change during the reaction at the temperature and pressure, so in that case we have  $V = V_0 1 + epsilon A X_A T_0 P_0$  upon P.

$$V = V_o(1 + \epsilon_A X_A) \frac{T}{T_o} \frac{P}{P_o}$$

Now for constant T and P, and if you see that always we are looking for the extent of a reaction, so epsilon A is represented as the extent of reaction, so  $V = V_0 1 + \text{epsilon A } X_A$ . Now here the T = T<sub>0</sub> and P<sub>0</sub> = P, so you see that these things are case sensitive. So, we need to take either certain assumptions or we need to assume the case maybe the gaseous segment maybe for some other segments.

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Now as far as the concentration is in question, so for concentration of reactant A at any time you may get the putting the values of varying volume in the concentration expression  $C_A = N_A$  upon V. So, this particular equation if you see that  $C_A$  can be represented as  $N_{A0} (1 - X_A) T_0 P$  upon V<sub>0</sub>  $1 + epsilon A X_A TP_0$  plus.

$$C_{A} = \frac{N_{A_{0}}(1 - X_{A})}{V_{0}(1 + X_{A})} \frac{T_{0}}{T} \frac{P}{P_{0}}$$

$$\Rightarrow \qquad C_{A} = C_{A_{0}} \frac{(1 - X_{A})}{(1 + \varepsilon_{A}X_{A})} \frac{T_{0}}{T} \frac{P}{P_{0}}$$

Now this is the general equation or we can represented as  $N_A = N_{A0} (1 - X_A)$  or  $V = V_0 1 + epsilon$ A X<sub>A</sub>T upon T<sub>0</sub>P<sub>0</sub> upon P. Now similarly the concentration of reactant B or product C and D if you are having the equation like this.

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This can be written as  $C_B = N_B$  upon V that is  $C_{B0} - b$  upon a  $C_{A0}$  X<sub>A</sub> upon (1 + epsilon A X<sub>A</sub>)T<sub>0</sub> upon T P upon P<sub>0</sub>, here you see that N<sub>B0</sub> upon V<sub>0</sub> = C<sub>B0</sub>. Similarly, if we try to write the equation for concentration of C, that is N<sub>C</sub> upon V this is equal to N<sub>C0</sub> + c upon a C<sub>A0</sub> X<sub>A</sub> upon 1 + epsilon A X<sub>A</sub>) T<sub>0</sub> upon T P upon P<sub>0</sub>. Here N<sub>C0</sub> upon V<sub>0</sub> = C<sub>co</sub> concentration at zeroth time.

And last if we try to write the C concentration of D, this is represented as the number of moles of D divided by volume,  $C_{D0}$  initial number of moles of component d, d upon a, a is the limiting reactant  $C_{A0}$   $X_A$  upon 1 + epsilon A  $X_A$  To/T P upon P<sub>0</sub>, needless to mention that  $N_{D0}$  upon  $V_0 = C_{D0}$ .

$$C_{B} = \frac{N_{B}}{V} = \frac{C_{Bo} - b/a (C_{Ao}X_{A})}{(1 + \varepsilon_{A}X_{A})} \frac{T_{o}}{T} \frac{P}{P_{o}}$$
$$C_{C} = \frac{N_{C}}{V} = \frac{C_{Co} + c/a (C_{Ao}X_{A})}{(1 + \varepsilon_{A}X_{A})} \frac{T_{o}}{T} \frac{P}{P_{o}}$$
$$C_{D} = \frac{N_{D}}{V} = \frac{C_{Do} + d/a (C_{Ao}X_{A})}{(1 + \varepsilon_{A}X_{A})} \frac{T_{o}}{T} \frac{P}{P_{o}}$$

So, this if you see that we are referring this particular reaction like this b upon a C upon a C + d upon a D, this is based on this particular equation. So, based on this particular approach you can predict the things accordingly. Now if I wish to predict the rate of reaction.

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In that case when usually the epsilon is not equal to 0. So, the rate of a reaction of a chemical reaction you can write as  $-r_A = -1$  upon VdN<sub>A</sub>/dt.

$$-\mathbf{r}_A = \frac{1}{V} \frac{\mathrm{dN}_A}{\mathrm{dt}}$$

So, at constant temperature and pressure and for the varying volume system, reaction system where we are considering that is epsilon is not equal to 0. Then  $-r_A = 1$  upon V<sub>0</sub> (1 + epsilon A X<sub>A</sub>) d(N<sub>A0</sub> (1 - X<sub>A</sub>) upon dt.

$$-\mathbf{r}_{\mathrm{A}} = -\frac{1}{\mathbf{V}_{o}(1+\varepsilon_{\mathrm{A}}\mathbf{X}_{\mathrm{A}})} \frac{\mathrm{d}\left(\mathbf{N}_{\mathrm{A}_{o}}(1-\mathbf{X}_{\mathrm{A}})\right)}{\mathrm{d}t}$$

So, if we differentiate this particular equation with the respect to t, then we may have -  $r_A$  and  $N_{A0}$  upon  $V_0$  (1 + epsilon A  $X_A$ ) dX<sub>A</sub> upon dt or you can write -  $r_A = C_{A0} / (1 + epsilon A X_A dX_A upon dt. This is my final equation and this is the required performance equation for batch reactor system.$ 

$$-\mathbf{r}_{A} = \frac{\mathbf{N}_{A_{0}}}{\mathbf{V}_{o}(1 + \varepsilon_{A}\mathbf{X}_{A})} \frac{\mathbf{d}\mathbf{X}_{A}}{\mathbf{d}\mathbf{t}}$$
$$-\mathbf{r}_{A} = \frac{\mathbf{C}_{A_{0}}}{(1 + \varepsilon_{A}\mathbf{X}_{A})} \frac{\mathbf{d}\mathbf{X}_{A}}{\mathbf{d}\mathbf{t}}$$

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Now again another thing is that if we have the varying volume flow rate for flow reactor. So, this type of similar equation for concentration term is formed for varying volume system, for flow system as in the we observed in the batch reactor system. So, if you have the varying volume in liter per second, this is the ratio of a molar flow rate F that is mole per second to the concentration moles per liter.

volumetric flow rate (v) = 
$$\frac{F}{C_i}$$

So, sometimes this volumetric flow rate is represented as F upon Ci, i is the i<sup>th</sup> component. Now if the total concentration of  $C_T$  is in question, so the total concentration  $C_T$  can be then in the form of a volumetric flow rate and the molar flow rate. And the relate with the gas law, so we can find that total concentration at any time T this is  $C_T = F_T$  upon v = P/ZRT, let us put this thing as equation number 1, so this is the total concentration at any time.

$$C_{\rm T} = \frac{F_{\rm T}}{\nu} = \frac{P}{ZRT}$$

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Now, similarly the total concentration at the entrance of the reactor in the flow system, you can write like this because at the entrance of the reactor some of the concentrations maybe put as 0 or not. So, we can write the total concentration at the entrance of the reactor  $C_{T0} = F_{T0}$  or  $P_0$  upon  $Z_0$  RT<sub>0</sub>.

$$C_{T_o} = \frac{F_{T_o}}{v_o} = \frac{P_o}{Z_o R T_o}$$

let us refer this particular equation as equation number 2. Now if we divide these two equations and putting the negligible change in the compressibility factor, that is represented as  $Z = Z_0$ .

And if we try to rearrange and then we find that this is  $F_T$  upon  $F_{T0} P_0$  upon P and T upon T<sub>0</sub>, so this is my equation and let us put this equation as equation number 3.

$$\nu = \nu_o \left(\frac{F_T}{F_{T_o}}\right) \left(\frac{P_o}{P}\right) \left(\frac{T}{T_o}\right)$$

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So, if we talk about the flow reacting system, the total flow rate if we sometimes we may require the total flow rate this can be written as like  $F_T = F_{T0} + \delta F_{Ao}X_A$ , this is equation number 4.

$$F_{\rm T} = F_{\rm To} + \delta(F_{\rm Ao}X_{\rm A})$$

Now here  $F_T$  is the total molar flow rate that is represented in liter per minute, then  $F_{T0}$  is the initial molar flow rate because we are changing this flow rate or varying flow rate we are considering into question.

Then delta is equal to d upon a + c upon a + - b upon a - 1, now here minus sign is put just to represent that both these reactants are being consuming in due course of time. And  $F_{A0}$  is the initial molar flow rate of reactant A into the reactor system.

$$\delta = \frac{d}{a} + \frac{c}{a} - \frac{b}{a} - 1$$

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Now we have already from these above equations, now if we divide the equation that is the total molar flow rate by say  $F_{T0}$  in both the side of this particular equation. Then the things become like  $F_T$  upon  $F_{T0} = F_{T0}$  upon + delta  $F_{A0}$  upon  $F_{T0} X_A$ . And this can be represented like this  $F_T$  upon  $F_{T0}$  1 + delta Y mole fraction of A0 into  $X_A$ .

On dividing the equation (84) by  $F_{To}$  in both side of the equation we have;

$$\frac{F_{T}}{F_{To}} = 1 + \delta. \frac{F_{Ao}}{F_{T}} X_{A}$$

As on putting  $y_A = F_{Ao}/F_{To}$  and  $\varepsilon = \delta y_A$  in the above equation we have;

$$\mathbf{F}_T = F_{\mathrm{To}}(1 + \varepsilon X_A)$$

And upon rearranging this  $F_T = F_{T0}$  (1 + epsilon A XA), we can put this as equation number 5. So, this here  $Y_{A0} = F_{A0}$  upon  $F_{T0}$  and epsilon A = delta  $Y_{A0}$ , so these are the some of the differencing things.

Now from this equation which we discussed and on rearrangement we can have this  $F_{T0}$  (1 + epsilon A X<sub>A</sub>) upon  $F_{T0}$  P<sub>0</sub> upon P T upon T<sub>0</sub>, so we can represent like this V<sub>0</sub> (1 + epsilon A X<sub>A</sub>) P<sub>0</sub> upon P T upon T<sub>0</sub>.

$$\nu = \nu_o \left( \frac{F_{T_o}(1 + \varepsilon X_A)}{F_{T_o}} \right) \left( \frac{P_o}{P} \right) \left( \frac{T}{T_o} \right)$$
$$\nu = \nu_o (1 + \varepsilon X_A) \left( \frac{P_o}{P} \right) \left( \frac{T}{T_o} \right)$$

Now this is the required equation for volumetric flow rate in varying volume system. So, this is again a very I can say this is very important relation which we developed in due course of time. Now if we see the concentration effect because we discussed about the varying volume, we discussed about the other factors. Now for the concentration terms can be described now from these relationships which we discussed in the previous slides. And similar way we can write the concentration term for all reactants and other product terms.

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So, if we would like to have the concentration of any species which can be written as for this particular equation, we are taking this particular equation into count A + b upon a B c upon a C + d upon D. Now here this is my master equation or sometimes it is referred as Ci Fi upon P<sub>0</sub> into T upon T<sub>0</sub>.

$$C_{i} = \frac{F_{i}}{\nu} = \frac{F_{io}}{\nu_{o}(1 + \varepsilon X_{A})\left(\frac{P_{o}}{P}\right)\left(\frac{T}{T_{o}}\right)}$$

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Now here 1 + epsilon A  $X_A$  P upon P<sub>0</sub> T<sub>0</sub> upon T. Now if we see about the reactant concentration A then  $C_A = F_{A0} \ 1 - X_A \ 1 + epsilon A X_A P_0$  upon P T upon T<sub>0</sub> as we know that  $F_A = F_{A0} \ 1 - X_A$ . Now here then  $C_A = C_{A0} \ 1 - X_A$  upon 1 + epsilon A X<sub>A</sub> and that becomes the P<sub>0</sub> T<sub>0</sub> upon T.

$$C_{i} = \frac{F_{i}}{\nu} = \frac{F_{io} (1 - X_{A})}{\nu_{o} (1 + \varepsilon X_{A}) \left(\frac{P_{o}}{P}\right) \left(\frac{T}{T_{o}}\right)}$$
$$C_{A} = \frac{F_{A}}{\nu} = \frac{F_{Ao} (1 - X_{A})}{\nu_{o} (1 + \varepsilon X_{A}) \left(\frac{P_{o}}{P}\right) \left(\frac{T}{T_{o}}\right)}$$
$$C_{A} = \frac{C_{Ao} (1 - X_{A})}{(1 + \varepsilon X_{A})} \left(\frac{P}{P_{o}}\right) \left(\frac{T_{o}}{T}\right)$$

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Now similarly if we wish to have this type of equation for reactant B. So, let put this thing in this way  $F_B = F_{B0} - B$  upon A  $F_{A0} X_A$ , now  $C_B = b$  upon a  $F_{A0} X_A$  upon  $v_0$  (1 + epsilon A  $X_A$ ) P upon P<sub>0</sub> into T<sub>0</sub> upon T or C B =  $F_{A0} F_{B0}$  upon  $F_{A0}$  - b upon a X<sub>A</sub> and this is 1 + epsilon A X<sub>A</sub>. And P upon P<sub>0</sub> T<sub>0</sub> upon t or  $C_B = C_{A0} F_{B0}$ , all these things are with their usual interpretation b upon a X<sub>A</sub> into (1 + epsilon A X<sub>A</sub> P upon P<sub>0</sub> T<sub>0</sub> upon T.

$$F_{B} = F_{Bo} - \frac{b}{a} F_{Ao} X_{A}$$

$$\Rightarrow C_{B} = \frac{(F_{Bo} - \frac{b}{a} F_{Ao} X_{A})}{\nu_{o} (1 + \varepsilon X_{A}) \left(\frac{P_{o}}{P}\right) \left(\frac{T}{T_{o}}\right)}$$

$$\Rightarrow C_{B} = \frac{F_{Ao} \left(F_{Bo}/F_{Ao} - \frac{b}{a} X_{A}\right)}{\nu_{o} (1 + \varepsilon X_{A})} \left(\frac{P}{P_{o}}\right) \left(\frac{T_{o}}{T}\right)$$

$$\Rightarrow C_{B} = \frac{C_{Ao} \left(F_{Bo}/F_{Ao} - \frac{b}{a} X_{A}\right)}{(1 + \varepsilon X_{A})} \left(\frac{P}{P_{o}}\right) \left(\frac{T_{o}}{T}\right)$$

So, similarly we can write for concentration of C, this is  $C_{A0}$ . Now here we are putting the plus sign because the C is being generated in due course of time upon 1 + epsilon A X<sub>A</sub> multiplied by P upon P<sub>0</sub> T<sub>0</sub> upon T. Now here  $F_C = F_{C0} + c$  upon a  $F_{A0}$  X<sub>A</sub>. So, by this way we generated these mathematical equations for prediction of varying volume concept. Now at last in this particular

segment we discuss the varying volume concept and with the taking the concentration into cognizance. And we developed the various equation associated to this for the interpretation of batch reactor data.

$$\Rightarrow C_{C} = \frac{C_{Ao} \left(\frac{F_{Co}}{F_{Ao}} + \frac{c}{a} X_{A}\right)}{(1 + \varepsilon X_{A})} \left(\frac{P}{P_{o}}\right) \left(\frac{T_{o}}{T}\right)$$

As,

$$F_{\rm C} = F_{\rm Co} + \frac{c}{a} F_{\rm A_0} X_{\rm A}$$

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Now if you wish to have further reading, again we have listed couple of references in their reference slide and hope that this particular lecture you enjoyed a lot, thank you very much.