Chemical Process Safety
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Lecture 41
Review of Probability Theory

Now welcome to this module of Review of Probability Theory, now as far as the reassessment our safety aspects are concerned we cannot overlook the importance of probability and how this particular theory is applicable for any kind of risk analysis we are going to discuss in this particular module.

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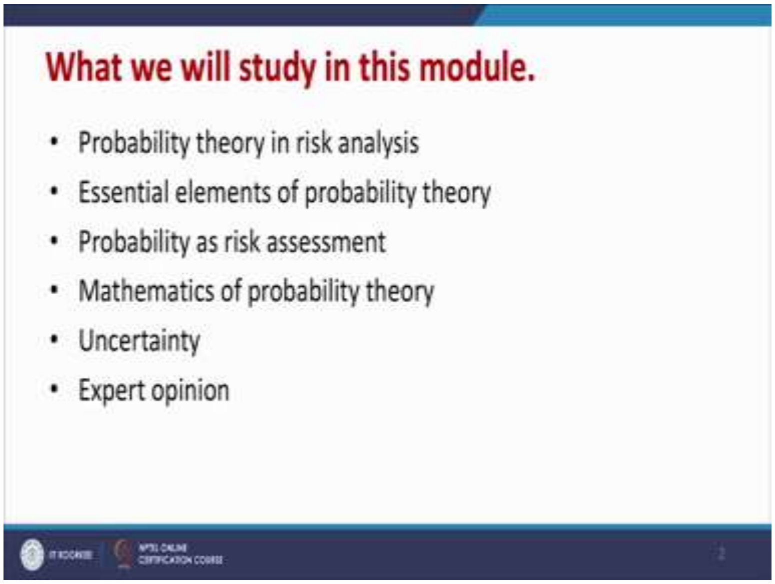

While discussing this risk analysis aspect related to the probability theory. We will discuss about the essential elements those who are involved in the probability theory remember we are not going to deep of this probability theory we will discuss only things which are applicable for the risk analysis. Then, what is the various aspects associative with respect to risk assessment we will go for various mathematical approach of probability theory, discuss about the uncertainty in the probability theory related to the risk assessment oblique analysis. Then we will go for various expert opinion. So, let us have look that how this probability is important in risk analysis.

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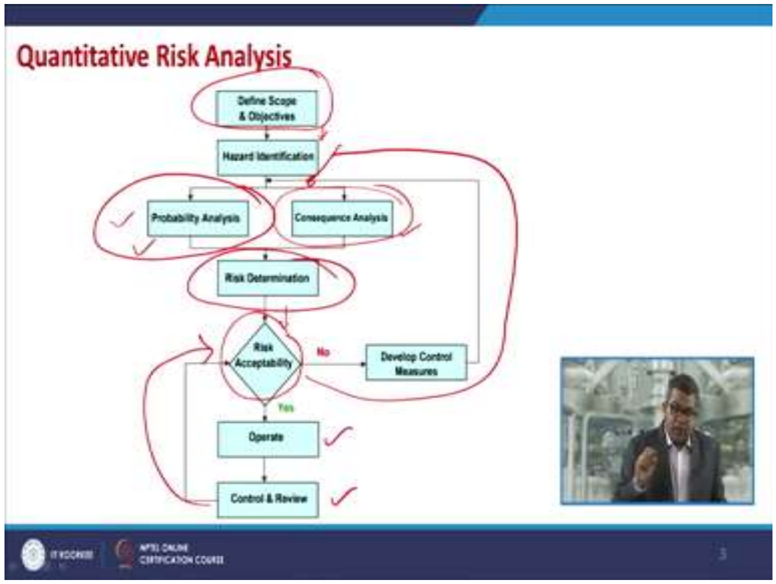

So, let us have a chart of this quantitative risk analysis, so while calculating the risk analysis we define the scope and objectives of our risk analysis approaches, then we find out that what are the hazards present in the work place or the area in question which is applicable then we calculated two aspect, one is the calculated that what is the probability of those hazards at workplace?

So we perform the probability analysis we need to perform other we need to perform the probability analysis and we need to perform the consequence analysis that is the what is the impact of those hazards on two different species those who are applicable at the workplace? So based on these two analysis we perform the risk determination analysis or we determine the risk. Now then we analyze the risk acceptability whether this risk is acceptable?

Now if this is not acceptable then again we need to go for this aspect to develop the control measures because the probability analysis and the consequences analysis these two approaches gives you an opportunity to modify your process based on the knowledge available to you or phase modify the process conditions etc.

Now if it is acceptable (though) the risk acceptable within the limit of the regulatory body guidelines or within the level of the approaches as suggested by the administrative persons then if it is acceptable then operate and again you need to go for the control and review. Remember do not overlook the importance of safety review so again in case any failure then again you need to perform the acceptability analysis of risk. So while going through this quantitative risk analysis flow sheet we are now concentrating towards this probability analysis.

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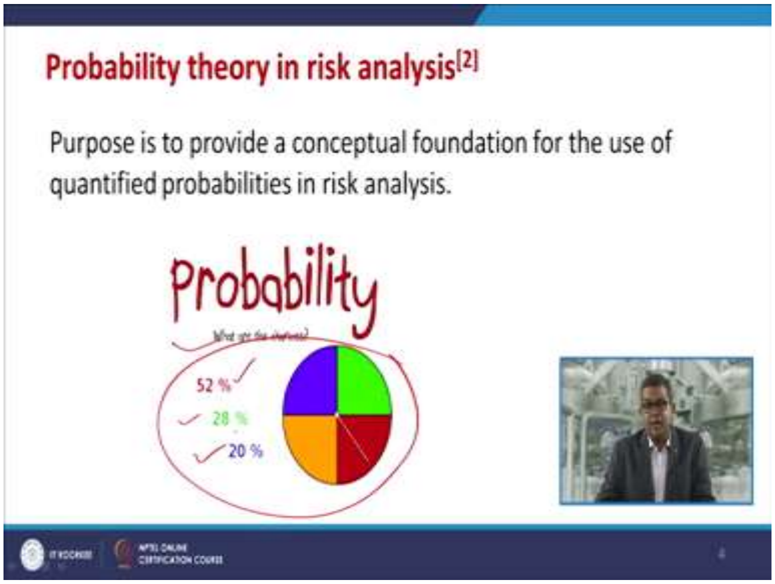

So probability the purpose of probability theory is to provide a conceptual foundation for the use of qualified probabilities in risk analysis now it gives you a idea it gives you an analytical approach to the plant person or the process modifiers or engineers that what is the problems in the particular system and how this problems may get populated in due course of time?

So this probability theory gives you an information about that what are the chances of any kind of failure or what are the chances of any kind of hazard population? You may draw a particular figure through which you can analyze that chances are on the say 52 percent you are say for on 28 percent approach you are in under the moderate conditions at 20 percent so it gives them measure impetus about the probability approach.

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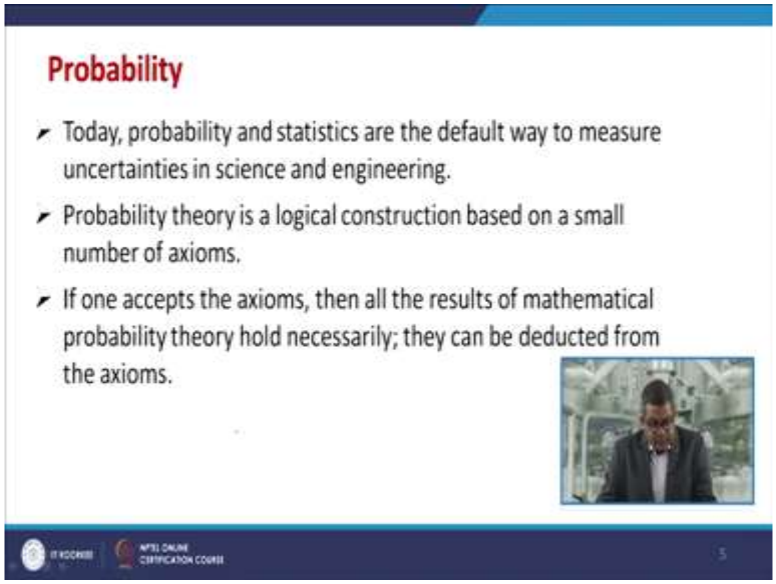

Now, today probability and statistical approaches they are the default way to measure the uncertainty is in science and engineering. Now probability theory usually a logical construction based on a small number of axioms, now, if one accepts those axioms then the result of mathematical probability theory hold necessarily and they can be deducted from those axioms. So we are not going into detail that what is the chances and other things we are just concentrating towards our approach for the analyzing the risk.

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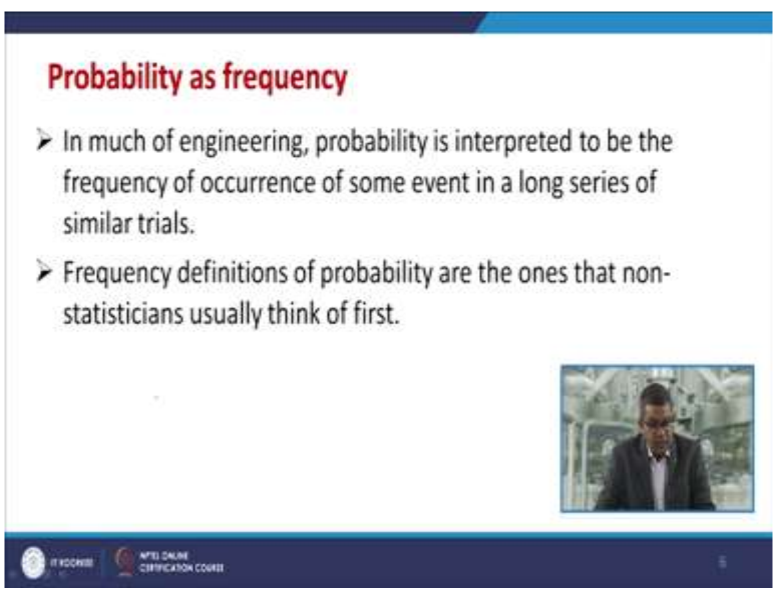

Now sometimes probability as a frequency much of engineering the probabilities interpreted to be the frequency of occurrence of some event in a longer series of similar trails. Now, frequency definition of probability are the ones that non-statisticians approach usually think of first.

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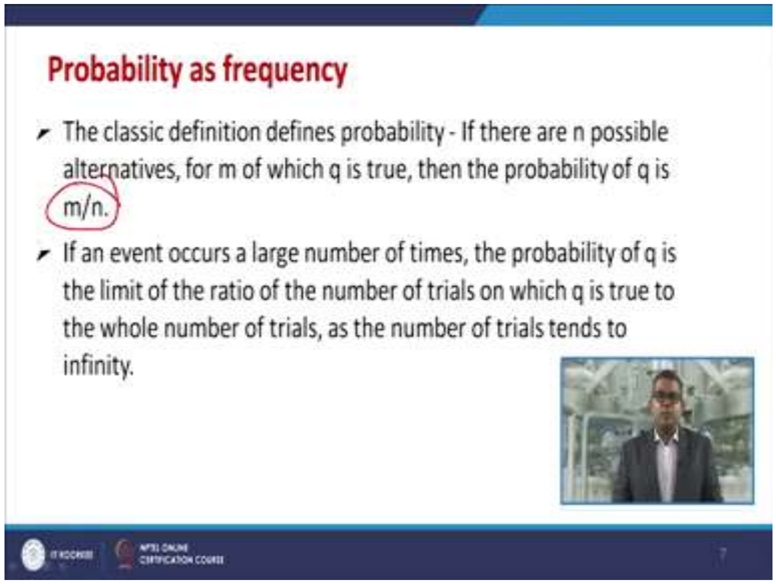

Now, the classical definition this defines probability if there are n possible alternatives, for m of which q is true then the probability of q is m by n. now this is a very small or simpler approach. Now, if an event occurs a large number of times the probability of q is the limit of the ratio of the number of trials on which q is true to whole number of trials as a number of trial tends to infinity.

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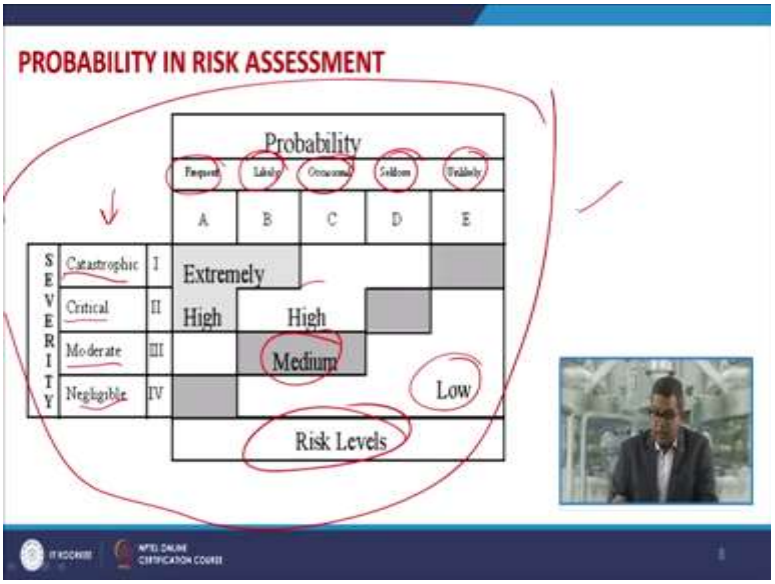

So you can apply that particular approach to the risk assessment. Sometimes there are four different severities one is catastrophic, critical, moderate and negligible, so you need to analyze that whether the probability of any system is frequent, likely, occasional, seldom or unlikely so based on your probability approach or based on your knowledge you may experience that sometimes the frequency of that the probability of particular incident may be extremely high in term with respect to the, the frequent or a likely.

Sometimes it may be on the higher side sometimes it may be at lower side and sometimes it is on the extremely lower so these are the risk levels. So while performing all kind of this probability approach for the risk assessment you need to find out or you need to construct this type of chart so that you can analyze the things more in a more proper way.

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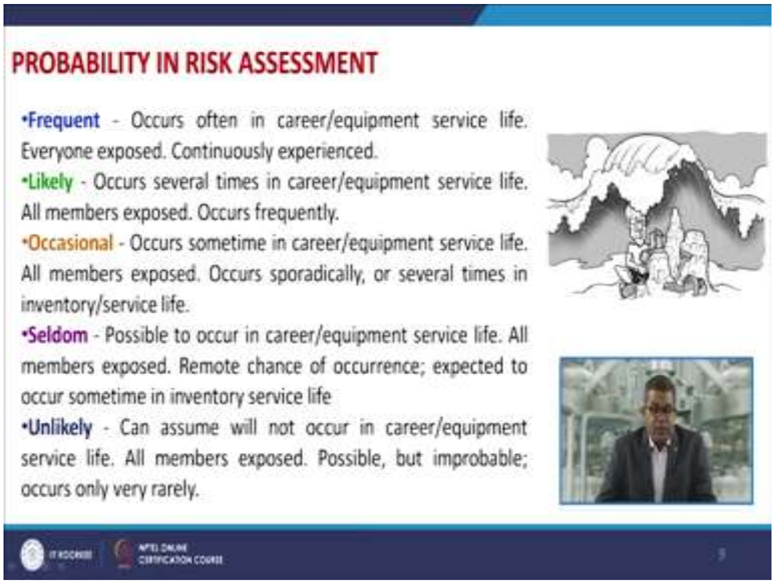

Now, sometimes we may encounter several definitions in this particular approach like frequent this occurs often in carrier oblique equipment service life everyone exposed this is continuously experienced approach etc. then likely occurs several times in the equipment service life and sometimes all members are exposed and occurs frequently. Occasional, this occur sometimes in carrier or equipment service life so all members they are usually exposed occurs seldomly or several times in the inventory service life.

Seldom, is possible to occur in the equipment service life all members they are exposed, remote chances of occurrence expected to occur sometimes in inventory service life, unlikely this can assume will not occur in the equipment service life all members maybe exposed to in case of any eventuality possible but improbable occurs only very rarely, so sometimes (())(09:08) feels so they are unlikely.

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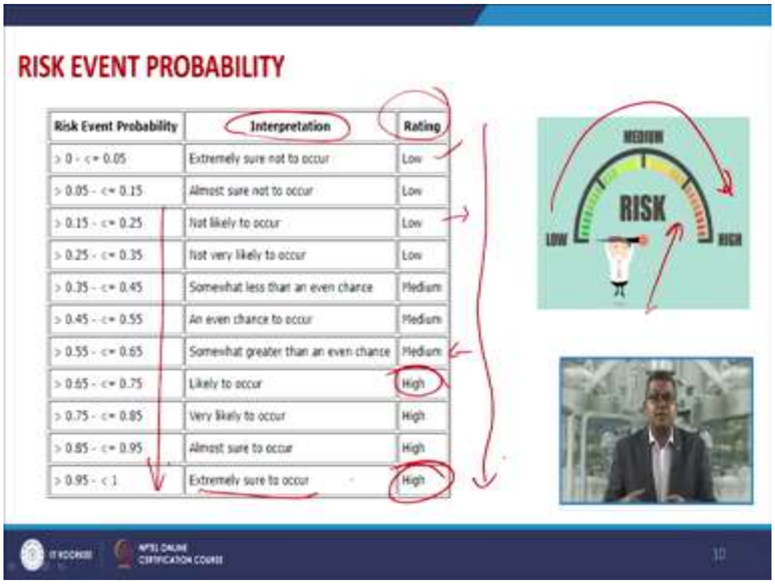

So based on this particular knowledge you can put all this risk event probability under one head then you go for the interpretation of those probability and then you give the rating. So because based on this particular approach or a knowledge you can devise or you can adopt or you can discuss the flowsheet which had discussed in the first couple of slides like probabilities within say 0.5 then extremely sure but not to occur the rating is very low, if the probability is in between 0.15 to say 0.25 not likely to occur then it is very low, sometimes if the probability lies from say between 0.65 to 0.75 likely to occur then the rating is very high.

So you can go for this risk protocol that is very high or risk meter that is very high so it goes from low to high. Now, if you go on increasing the probability, risk event probability then it goes from low, medium to higher one. So sometimes you may interpret like if it is per say 0.55 to 0.65 somewhat greater than an even chance so the risk is on the medium side etc. Now if it is say 0.95 to say 1 then extremely sure to occur then the risk is on the higher sides so you need to analyze the things mathematically.

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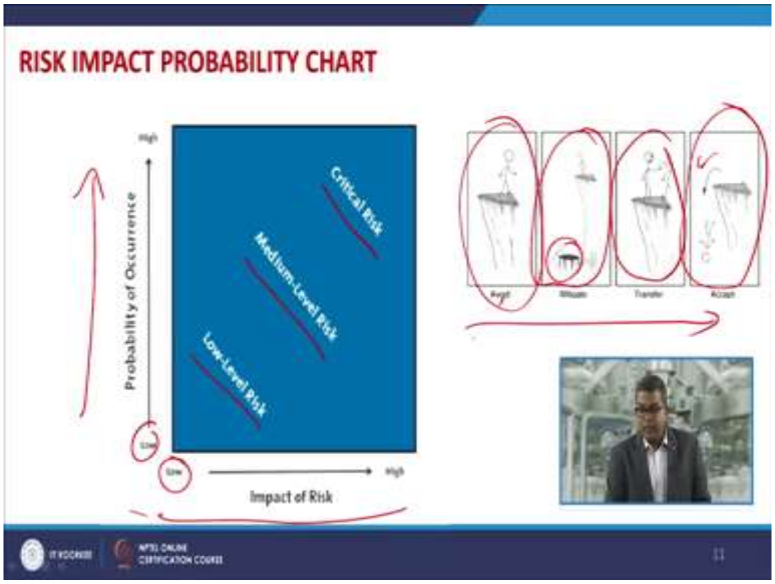

Then you may need to construct the risk impact probability chart and it is as depicted in this particular cartoon like you need to avoid the things then sometimes when you are compelled to go for this one then there may be chances that you may be in a position to mitigate the things like this the support is there so once if it is intend to fall then there is a mitigation that means you are having the safety relief system with you. Then sometimes you need to transfer the things if accordingly then again you are having various alternatives like substitute, attenuation, isolation etc.

Now, if you are not in a position to adopt all these things then definitely you need to accept that the accident will occur. So based on this particular aspect you need to construct the chart that is the impact of risk, what is the impact of risk and then what is the probability of occurrence? So based on this thing low then you may adopt the low level risk go for the medium level risk and last one is the critical risk. So like this you are moving form avoidance to the acceptance level.

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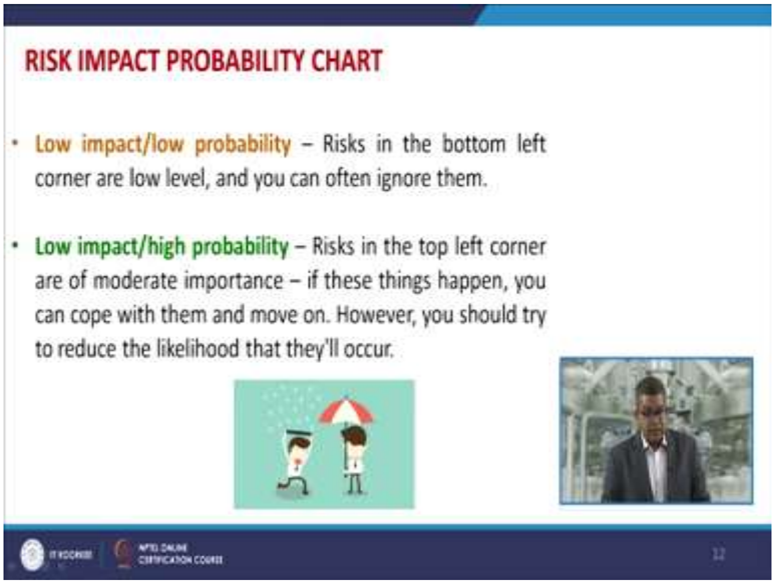

So when we are considering the low impact or low probability risk in the bottom left corner like this, this one are low level and you can often ignore them because if you are giving much attention to those scenarios then definitely you may lose precious time and you may lose the proper attention to other important activities. Then there are certain things like low impact and high impact probability the risk on the top corner this one are the moderate importance and these are happen you can cope up with them and move on however you should try to reduce the likelihood that they will occur.

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Another one is that high impact or a low probability the risk this one high impact and sometimes low probability then risk in the this particular right hand corner are the high importance and they do occur but they are very unlikely to happen. For these however you should do what you can do to reduce the impact and they will have if they do occur and you should have a contingency plan in just you should have a contingency plan in place just in case of they do. So you need to be practically aware about this risk analysis it is likely it is shown in this particular cartoon.

Now there are certain possibilities of high impact and high probability that is this high impact somewhere here in this zone, there risk towards the top right corner this is the critical importance these are this should be or this are your top priorities and are risk that you may pay close attention to that means you must devise the things in a accordance with this aspect. So we had discussion that the probability plays a vital role while deciding the risk assessment methodology, so let us have the look about the basic approach of a probability. Let us do for one experiment toss a coin twice.

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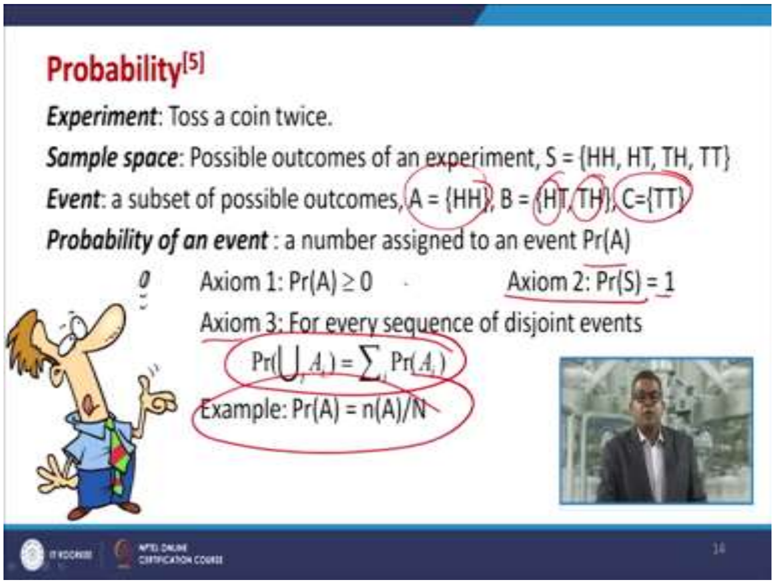

Now there are two things in sample space the possible outcome of an experiments are either you may have twice head, you may have a head and tail, you may have a tail and head and you may have a tail-tail. Now, event that is a subset of four all possible outcome like A, A may have a both the heads, B head and tail, tail and head and C that is both the tails, so probability of an event that is number assigned to an event is say Pr(A) that axiom that Pr(A) is greater than equal to 0, and the second is that Pr(S) that is equal to 1 and third one is for every sequence of disjoint event. You are having this one or example Pr(A) is equal to n(A) upon capital N.

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There may be certain independent events now to test whether two events A and B are independent this two events A and B are independent now you calculate the probability of A and a probability of B and probability of A subset B, now check whether P probability of A subset B is equals to probability of AB if they are equal then A and B are independent if not they are dependent. So a set of event Ai is independent case. So, for example independent event they are not affected by the previous events so like a coin does not know it came up heads before and each toss of a coin is a perfect isolated things. So it does not have any impact of the previous event.

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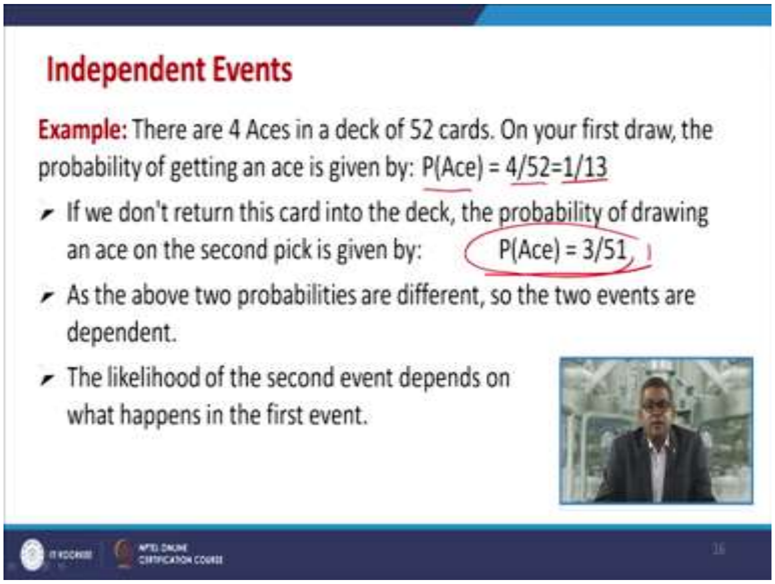

Now, (example) another example; now there are four aces in a deck of 52 cards so on your first draw the probability of getting an ace is given by the probability of ace 4 by 52 is equal to 1 by 13 now if we do not return this card into the deck the probability of the drawing of an ace on the second pick is given by 3 by 51. Now as the above two probabilities are different so the two events are dependent like because this event is dependent whether you are putting this card into the deck or not. So the likelihood of the second event depends on what happened in the first event.

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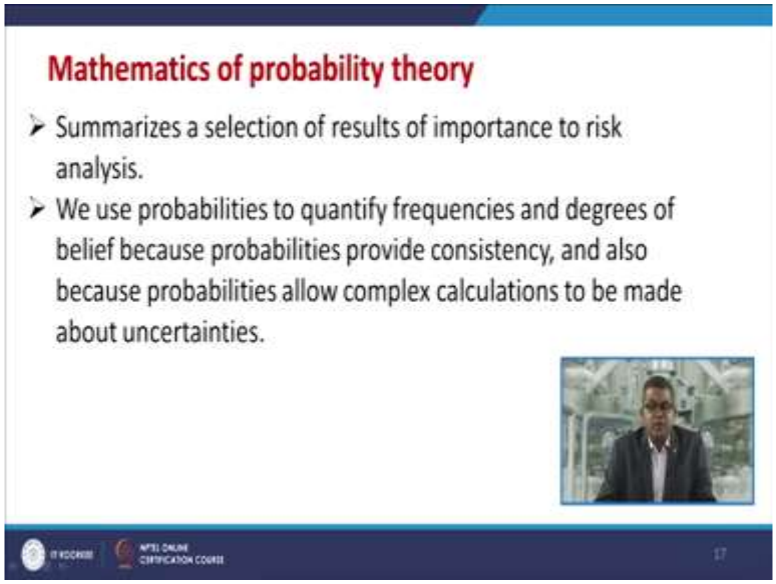

So summarize now whenever we are going for the mathematics of a probability theory the summarizes a selection of result of importance to risk analysis. So we use the probabilities to quantify the frequencies and degree of believe because probabilities provide consistencies and also because the probabilities allow complex calculations to be made about uncertainties.

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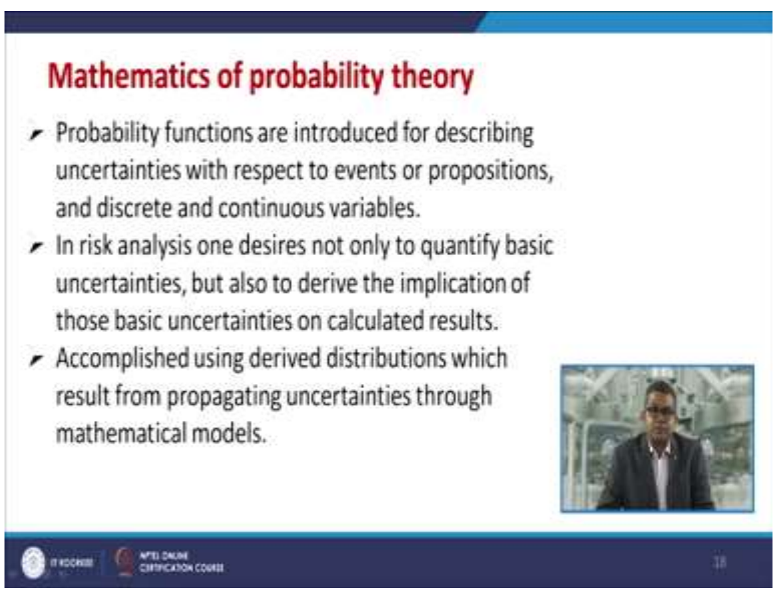

Now the probability functions they are introduced for describing uncertainties with respect to event or propositions and discrete and continuous variables. Now in risk analysis if one desires not only to quantify basic uncertainties but also to derive the implications of those basic uncertainties on calculated results. Now this is usually accomplished using a derive distributions which result from propagating uncertainties through mathematical models.

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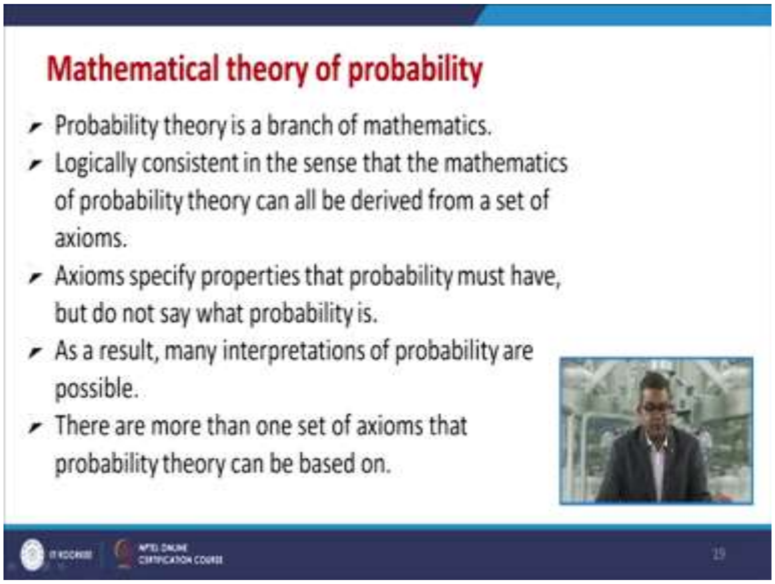

So probability theory is a branch of a mathematics logically consistent in sense that the mathematics of probability theory can all be derived from a set of axioms. Axioms specify properties that probability must have but do not say what probability is. So as a result many interpretations of probabilities, probability are possible. Now this are more than one set of axioms that probability theory can be based on.

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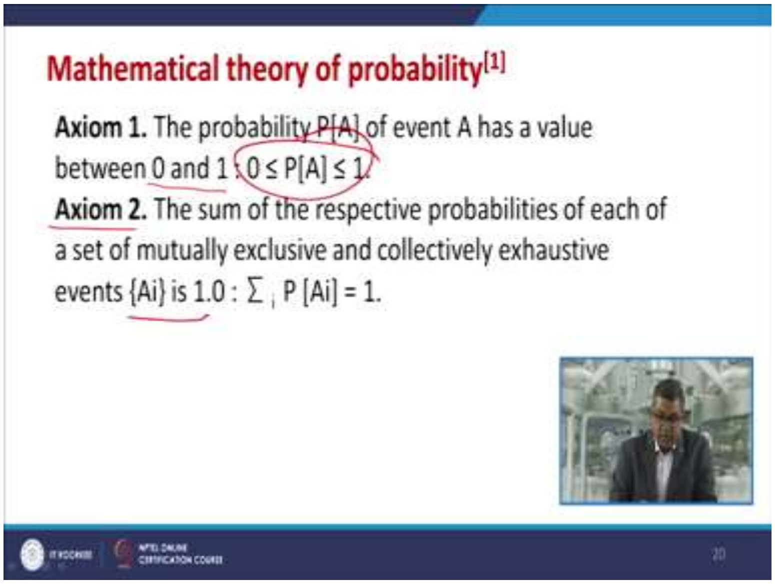

Like one axioms the probability of A event has a value between 0 and 1 so this is may be probability of A occurrence of probability A is greater than equal to 0 or less than equal to 1. Another axiom says that the sum of the respective probabilities of each of set of mutually exclusive and collective exhaustive event like Ai is 1.0 that is the summation i P and all the probabilities A i is equal to 1.

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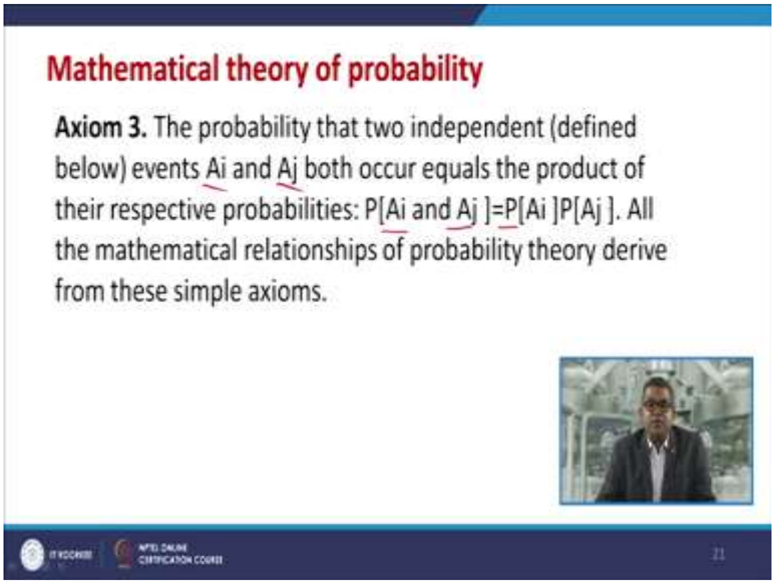

Third one the probability that two independent defined below that is event (A y) A i and A j both occurs equals the product of their respective probabilities that is probability of A i and A j equal to probability A i and the probability of A j. So, all the mathematical relationship of probability theory derive from these simple axioms.

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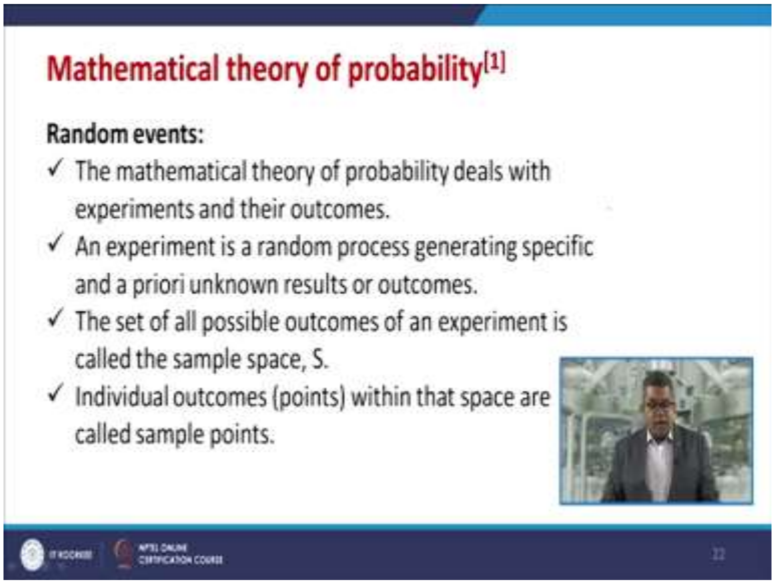

Now, there are certain random events like the mathematical theory of probability deals with the experiments and their outcomes. An experiment is a random process generating specific and a priori an unknown result of an outcome. So the set of all possible outcome of an experiment is called the sample space S and a individual outcome that is point within the spaces space are called the sample points like this.

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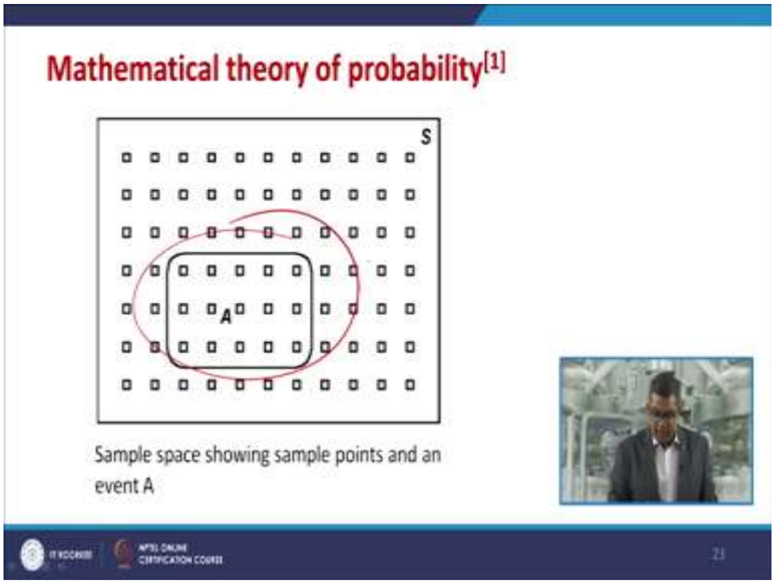

So these are the sample space showing sample points and an event A.

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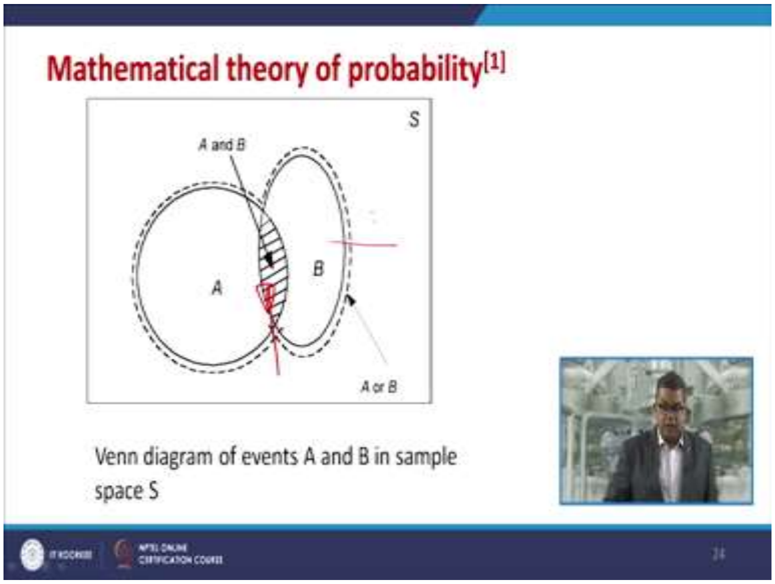

Now this is the Venn diagram of event A and B in the sample space and this is where both the event A and B both are occurs this is exclusively A and B so this is the set of A and B.

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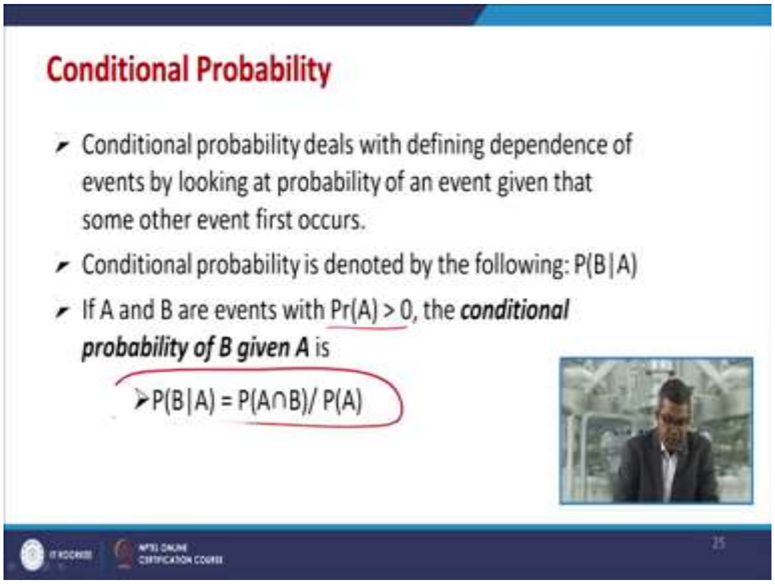

There are certain conditional probability. The conditional probability this deals with defining dependence of event by looking at probability of an event given that some other event first occurs. So the conditional probability is denoted by the probability of B A. Now, if A and B are event with the probability of A is greater than 0 the conditional probability of B given A is by this equation.

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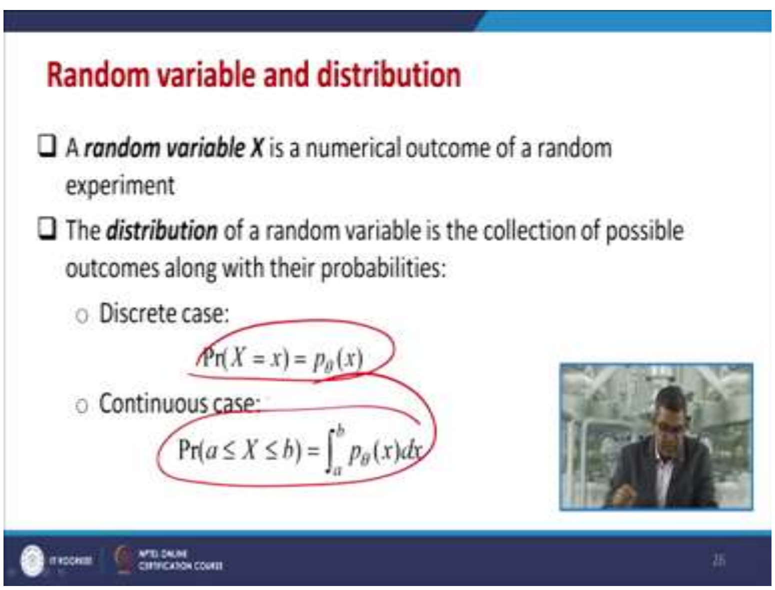

A random variable x is numerical outcome of a random experiment and the distribution of a random variable is the collection of a possible outcome along with their probabilities. So there are couple of cases like discrete case that is this one

$$\Pr(X = x) = P_\theta(x)$$

and the continuous case that is this one

$$\Pr(a \le X \le b) = \int_a^b P_\theta(x)dx$$

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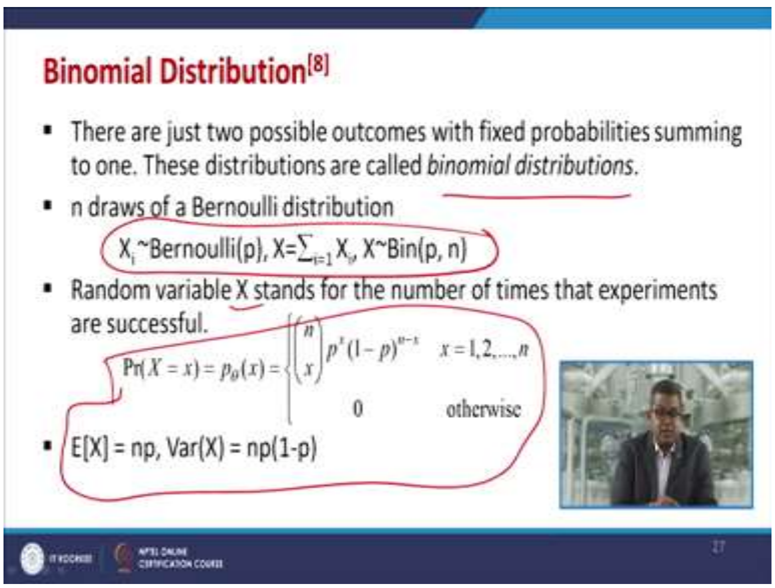

There are certain binomial distribution so these are the just two (possibilities out) possible outcome with the fixed probabilities summing to 1. Now this distributions are called the binomial distributions like n draws of a Bernoulli distribution is given by this particular equation

$$\Pr(X = x) = P_\theta(x) = \begin{cases} \binom{n}{x} P^x (1 - P)^{n-x}, & x = 1,2, \ldots\ldots, n \\ 0, & otherwise \end{cases}$$

and the random variable X, this stands for the number of times that experiments are successful. So it can be represented by this mathematical expression.

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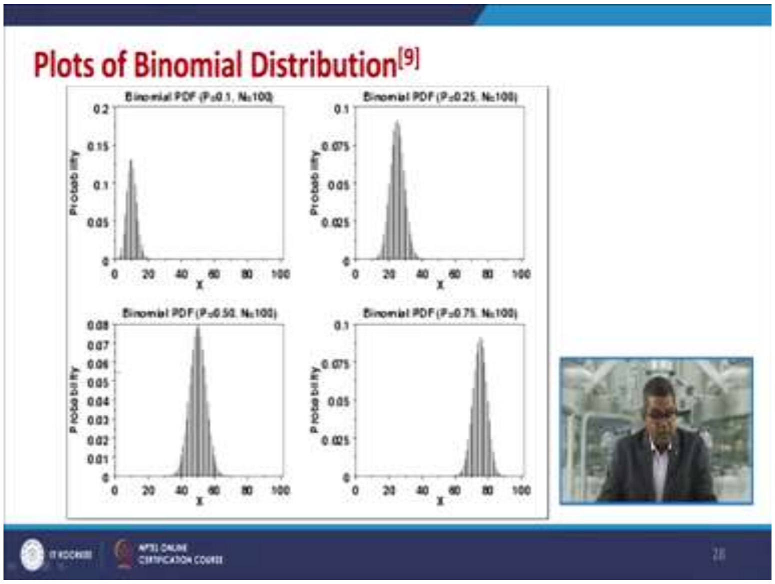

Now, once you have this binomial distribution you plot this binomial distribution with respect to probability and different type of events and sometimes this plots gives you very useful information about the critical zones.

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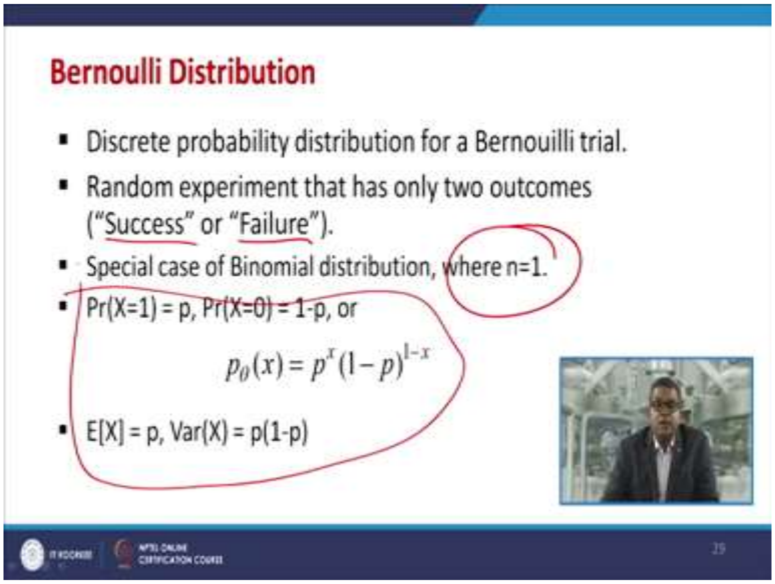

Now, this discrete probability distribution of this Bernoulli trial the random experiments that has only two outcome success or failure. So the special case of, you may consider the special case of binomial distribution where n is equal to 1 so these are the certain special cases of binomial distribution.

$$\Pr(X = 1) = p$$

$$\Pr(X = 0) = 1 - p$$

$$P_\theta(x) = P^x(1 - P)^{1-x}$$

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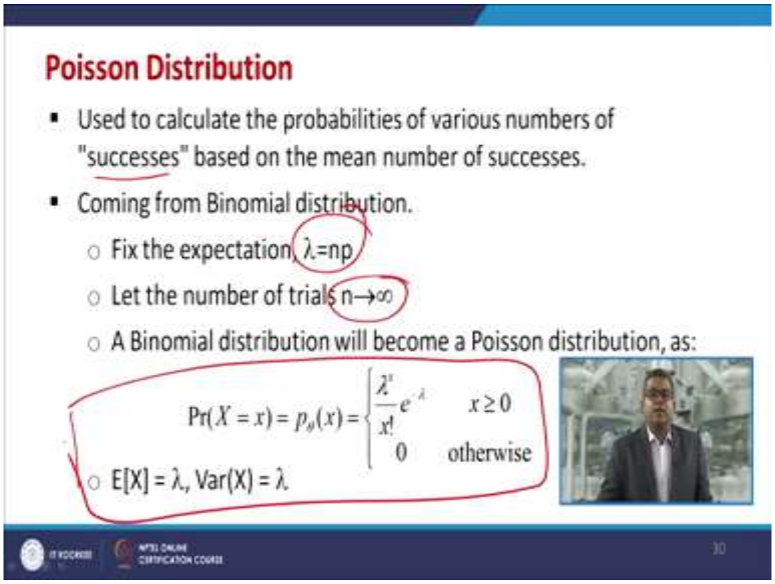

Let us have a look about the Poisson Distribution, this is used to calculate the probabilities of various numbers of success based on the mean number of success. So this coming from the binomial distribution you need to fix the expectations that is lambda np and now let the number of trials say n tends to infinity so a binomial distribution will become the Poisson Distribution and it is represented by this mathematical expression. So again you need to plot the Poisson Distribution with respect to the probability and you will get this plots for various lambda values like lambda 5, 15, 25, 35 etc. so you can identify that what are the critical zones for your process.

$$\Pr(X = x) = P_\theta(x) = \begin{cases} \frac{\lambda^x}{x!} e^{-\lambda}, & x \geq 0 \\ 0, & Otherwise \end{cases}$$

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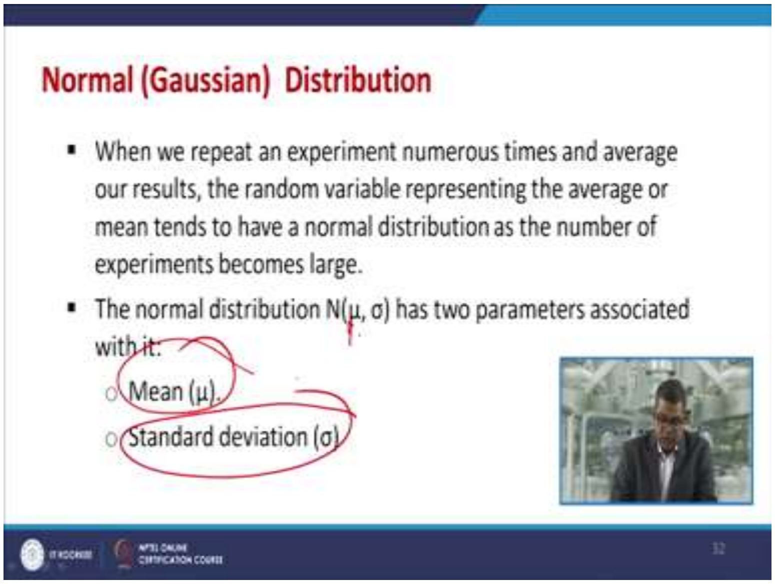

Then go for the normal or a Gaussian Distribution so when we repeat an experiment numerous times and average our result the random variable representing the average of mean tends to have a normal distribution as a number of experiment become very large. So the normal distribution has two parameters associated with it that is the Mu (mean) that is represented by Mu and standard deviation by this one.

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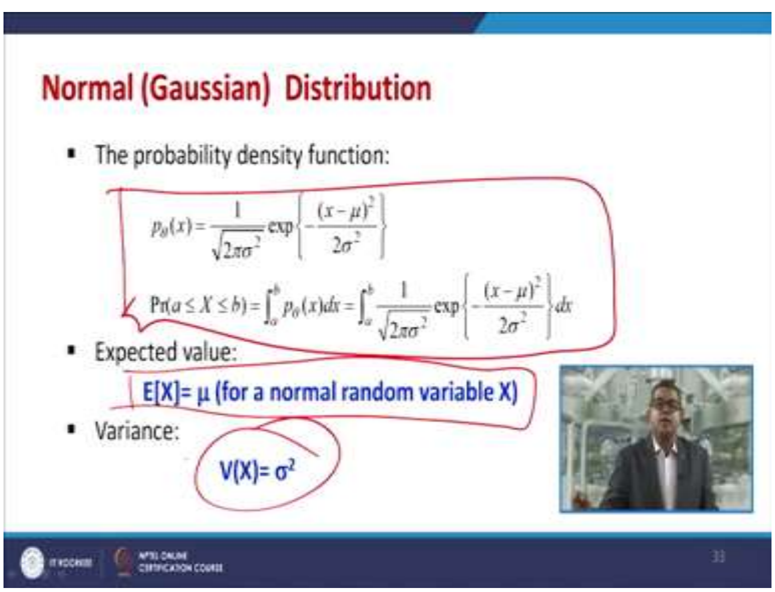

So the probability density function is given by this particular mathematical relationship

$$P_\theta(x) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$

$$\Pr(a \le X \le b) = \int_a^b P_\theta(x) = \int_a^b \frac{1}{\sqrt{2\pi\sigma^2}} exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\} dx$$

and the expected value is E [X]= μ for normal random variable X and a variance is given by this particular expression.

V(X)= σ^2

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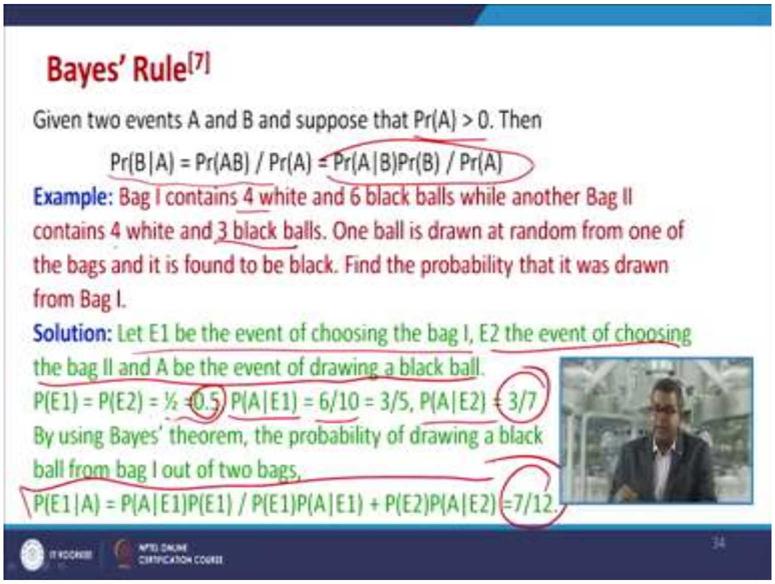

Another thing is that the Baye’s Rule this is given the two events A and B and suppose or you may assume that probability of A is greater than 0 then the

Pr (B| A) = Pr(AB/Pr(A) = Pr(A|B)Pr(B)/Pr(A)

Let us have a an example that bag 1 contains 4 white and a 6 black balls while another bag 2 contains 4 white and 3 black balls. One ball is drawn at random from one of the bag and it is found to be the black, find the probability that it was drawn from bag 1. So you can calculate easily by a certain assumption let have the look that which bag is it one? How many total number of balls are there?

So based on this particular corollary you can have the solution like E1 with the event of choosing the bag 1 and E2 is the event of choosing the bag 2 and A be the event of drawing a black ball. So probability of E1 is equal to probability of E2 is equal to half that is 0.5. Then the probability of A being picked from bag 1 is 6 /10 because it is having the 6 black balls and total number of balls are 10 so the probability of A being picked from bag 2 that is 3 /7 because it has the 3 black balls and total number of balls are 7, so by using the Baye's theorem the probability of drawing a black ball from the bag 1 out of two bags is 7 by 12, by this particular mathematical representation.

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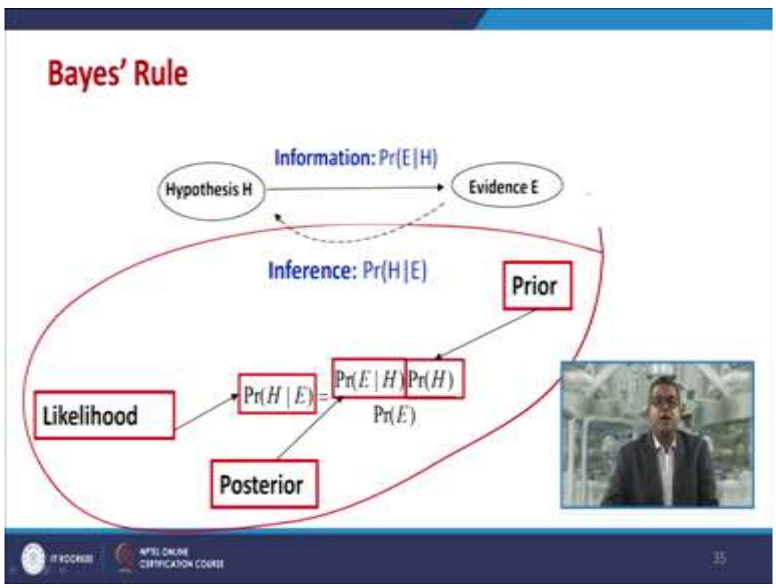

Now the hypothesis once you analyze all this things you must have an hypothesis and you must have an evidence so based on this things you can calculate or you can adopt this Baye's Rule.

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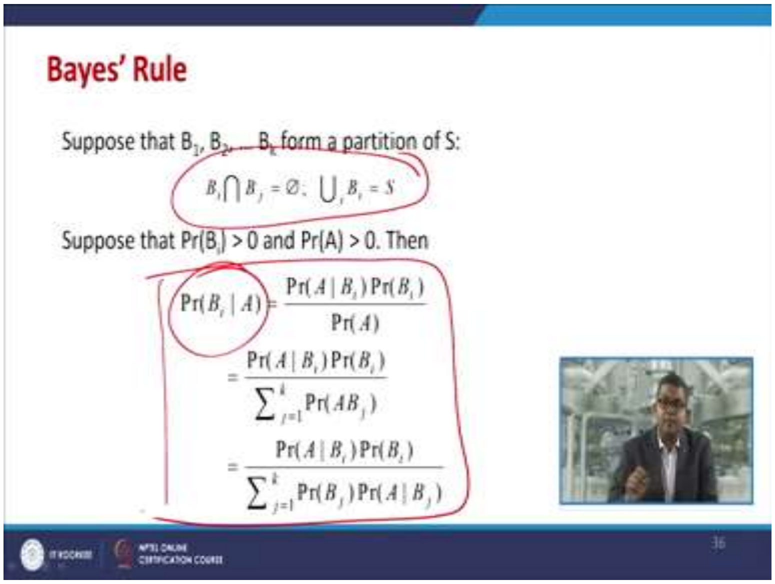

Now, suppose that the B1 B2 B k from the partition of s over here then you may have this representation

$$B_i \bigcap B_j = \emptyset;\ \bigcup_i B_i = S$$

and if the probability of B_i greater than 0 and the probability of A is again 0 then you can analyze this one by this mathematical equation.

$$\Pr(B_i|A) = \frac{\Pr(A|B_i)\Pr(B_i)}{\Pr(A)}$$

$$= \frac{\Pr(A|B_i)\Pr(B_i)}{\sum_{j=1}^{k}\Pr(AB_j)}$$

$$= \frac{\Pr(A|B_i)\Pr(B_i)}{\sum_{j=1}^{k}\Pr(B_j)\Pr(A|B_j)}$$

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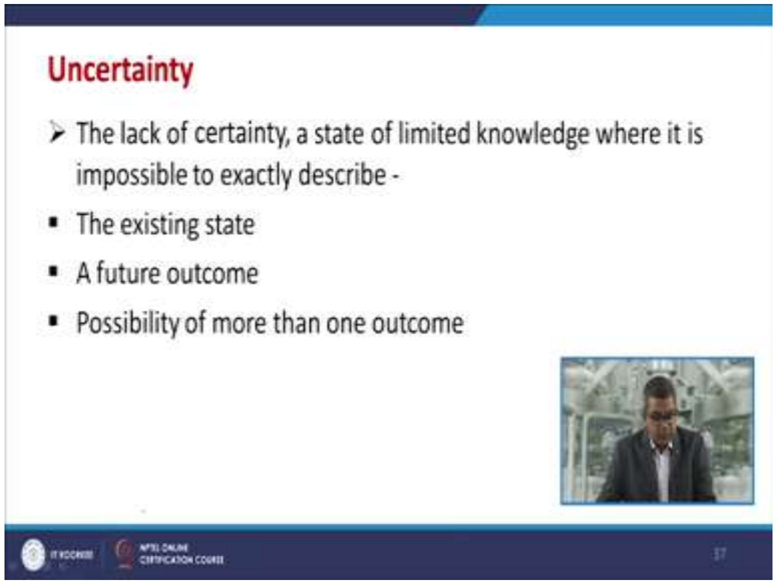

Now there is a concept of uncertainty now the lack of certainty state of limited knowledge where it is impossible to exactly describe so you must have the existing state you must have a knowledge about future outcome and the possibility of for more than one outcome this is very crucial aspect while calculating the risk analysis.

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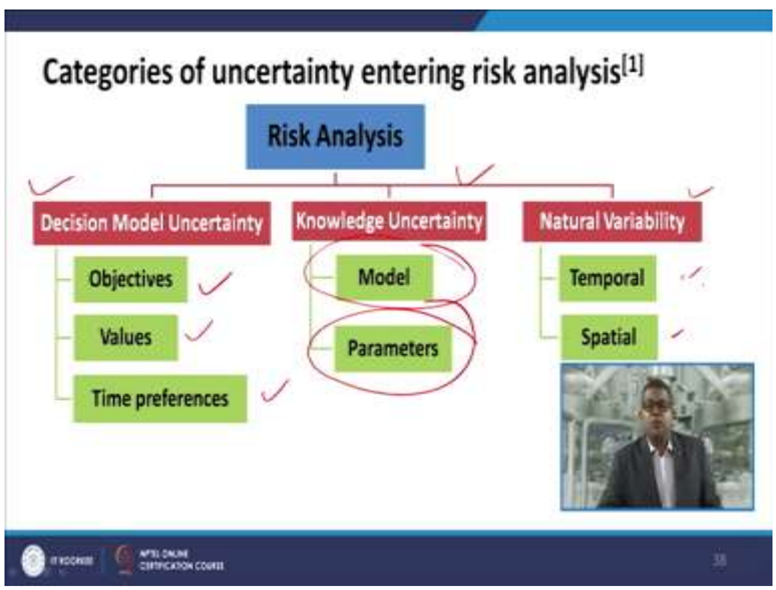

Now there are categories of uncertainty entering into the risk analysis like risk analysis, you are having three aspect decision model uncertainty then knowledge uncertainty then the natural variability. Now this are the subdivided into three aspect the decision of decision model uncertainty may be having the various objectives must be fit in with the values they are given the time preferences and the knowledge based is the model and different parameters. The natural variability is the temporal and special. So if you wish to have proper analysis or detail analysis you can look into this particular reference.

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Now there are certain expert opinion many important uncertainty is in the risk analysis are not amenable to quantitative estimation from the data. In some cases there are no (())(28:41) at all only for only the judgment of experts. Now these uncertainties have traditionally being treated using the expert opinion. So the knowledge of those expert is based on the intuition, the un-enumerated past experience the subjective theory and other important but the qualitative benefits. So there is question that how do people estimates subjective probabilities?

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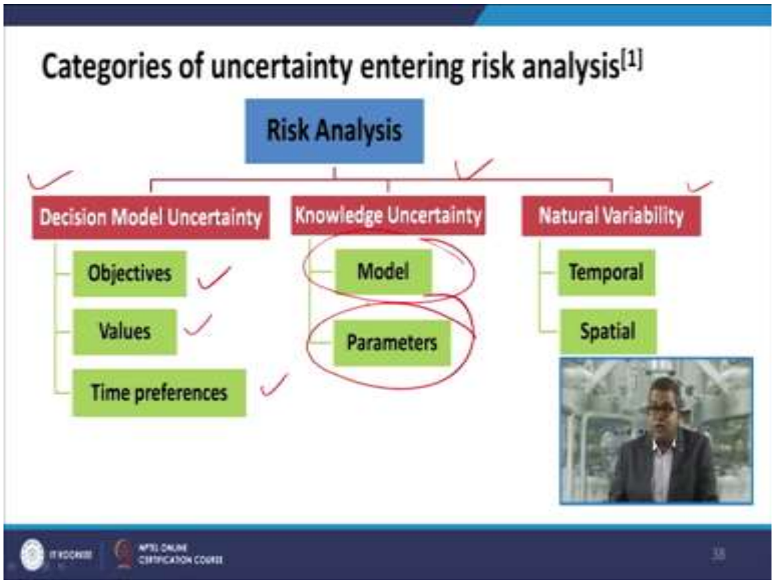

While it is evident that the subjective probability requires integration of integrating information of various kind of within a consistent framework it is less clear how people do this. Now the subjective probabilities they should be concordant with the probabilities theory they should have a sum of 1.0 and 1 would prefer they be calibrated to observe frequencies in the physical world and they should be they should have a predictive values.

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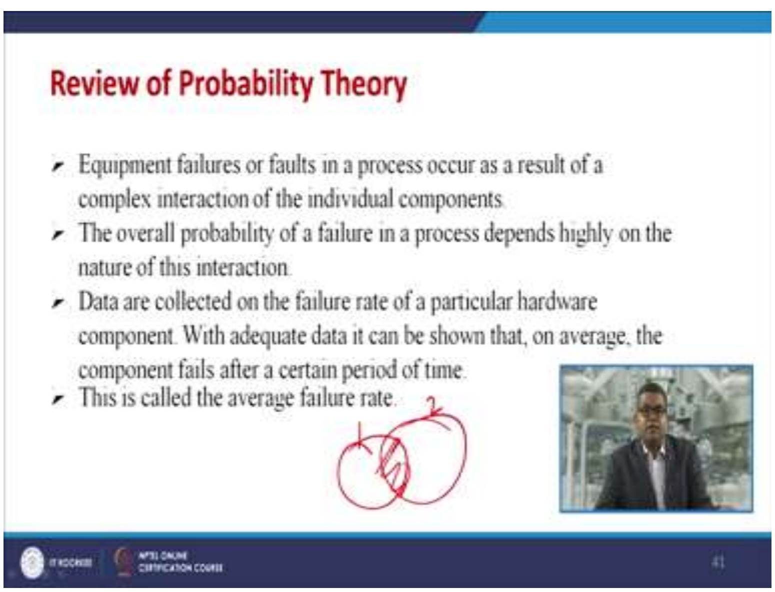

So let us have a review of this probability theory the equipment failure on fault in a process occur as result of complex interaction of individual components so there are variety of components involve in such kind of an incident. The overall probability of a failure in a process depends highly on the nature of the interaction.

It is just like that you are having two process 1 and 2, so what is the interaction between these two processes? So sometimes this depends on this interactive parameters now data are collected on the failure rate of a particular hardware component with adequate data it can be shown that on average the component fails after a certain period of time and this is called the average failure rate.

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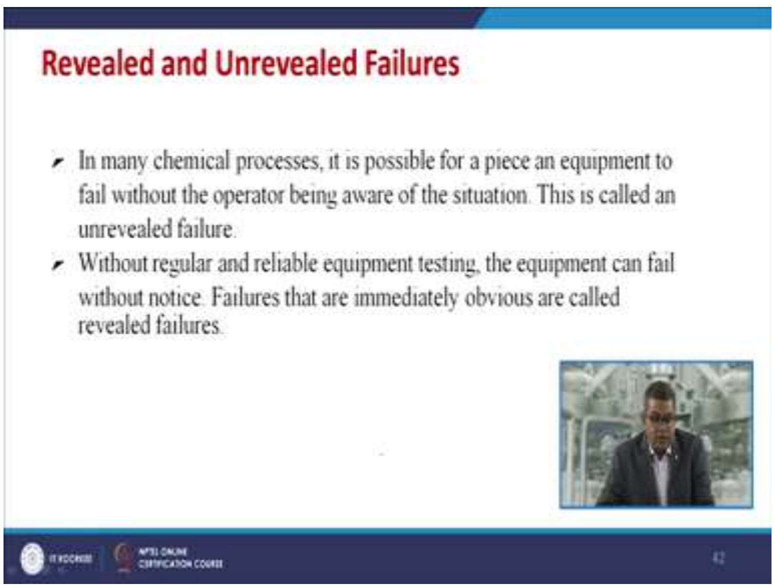

Now there are certain revealed and unrevealed failures in many chemical process it is possible for a piece of an equipment to fail without the operator being aware about the situation and that is called the unrevealed failures. Now without regular and reliable equipment testing the equipment can fail without notice the failure that are immediately obvious they are called the revealed failures.

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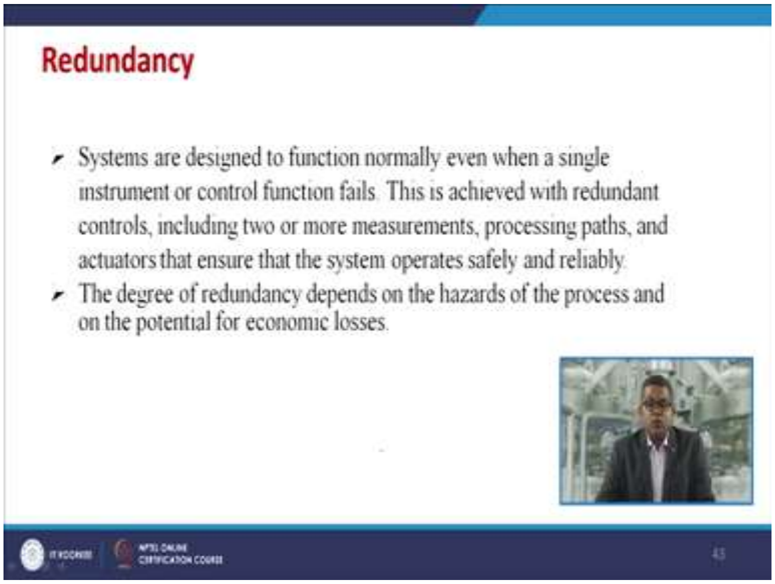

So we must know this type of approaches. Now redundancy the systems they are designed to function normally even when a single instrument or control function fails this is achieved with the redundant control including two or more measurements, processing paths, actuators that ensure that the system operates safely and reliably. Now the degree of redundancy depends on the hazards of the process and on the potential of the economic losses that is why this particular aspect should be kept always in your mind.

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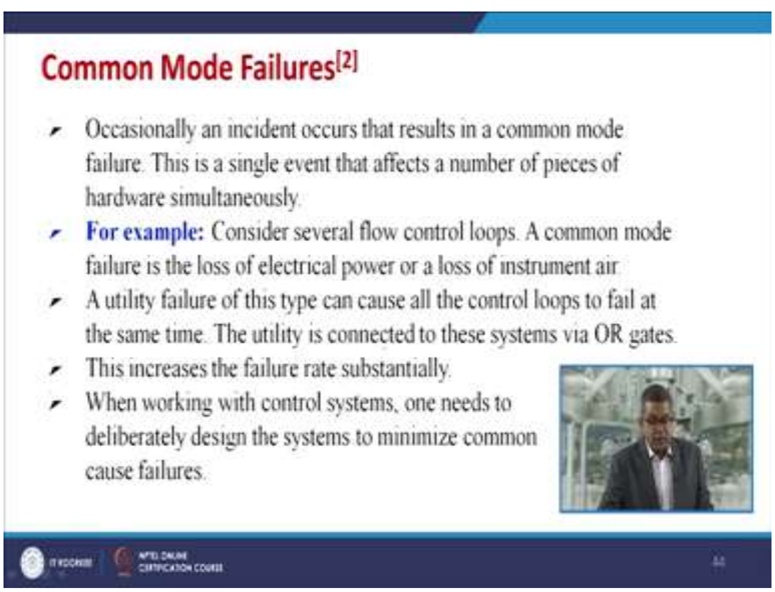

There are certain common mode failures occasionally and incident occurs their result in a common mode failure. Now this is a single event that affects a number of pieces of hardware simultaneously. For example consider several flow control loops a common mode failure is loss of electrical power or a loss of instrument air etc. Now a utility failure of such type can cause all control loops to fail at the same time. Now the utility is connected to those systems via OR-gate now this increases the failure rate substantially.

When working with control system one needs to deliberately design the system to minimize the common cause failure and sometimes it may lead because if temperature sensor fails then it may got to the thermal (())(32:44) reaction. So all things this increases the failure rate then with n number of times. So in this particular module we had a discussion about the basic approach of a probability theory, how it can apply how this particular probability theory knowledge we can apply for the risk assessment.

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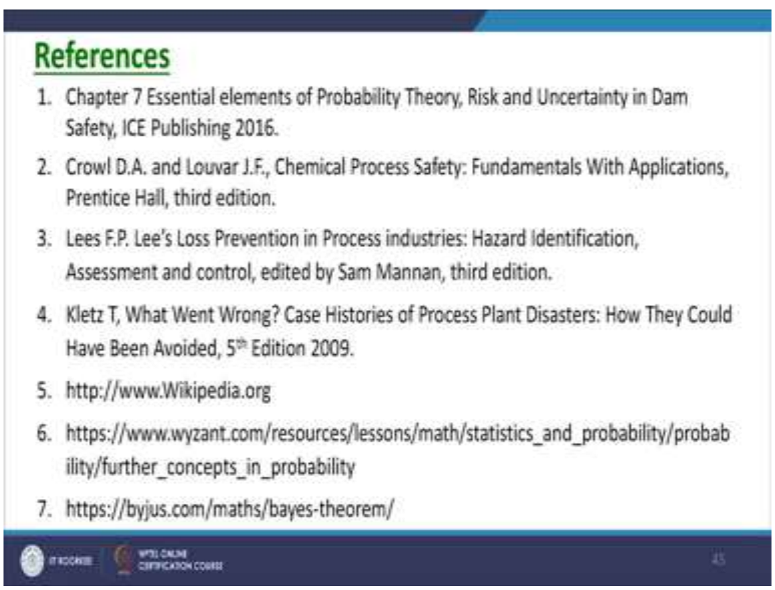

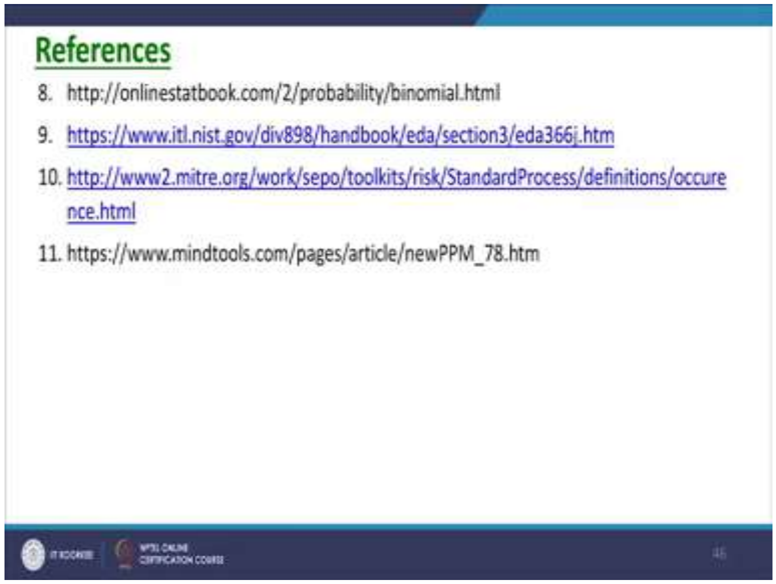

Now if you wish to have some other related the readings we have enlisted various references for your convenience you can go through you can gain more and more knowledge about the probability approach on the risk assessment, thank you.