Chemical Process Safety
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Lecture 18
Source Models for Gas

Welcome to this module, which is pertaining to the source model for various gases. Now, have a look about that what we have studied previously.

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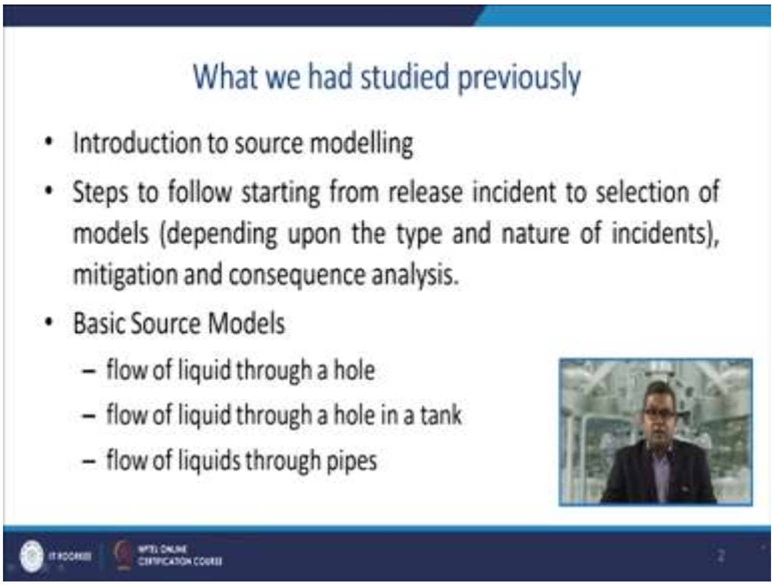

We have introduced that various aspect of source modelling. We have discussed about this step to follows starting from release incident to selection of model depending upon the type and a nature of various incidents, mitigation and a consequences analysis. We have discussed about the basic source models like flow of liquid through a hole, flow of liquid through a hole in a tank and the flow of a liquids through pipe.

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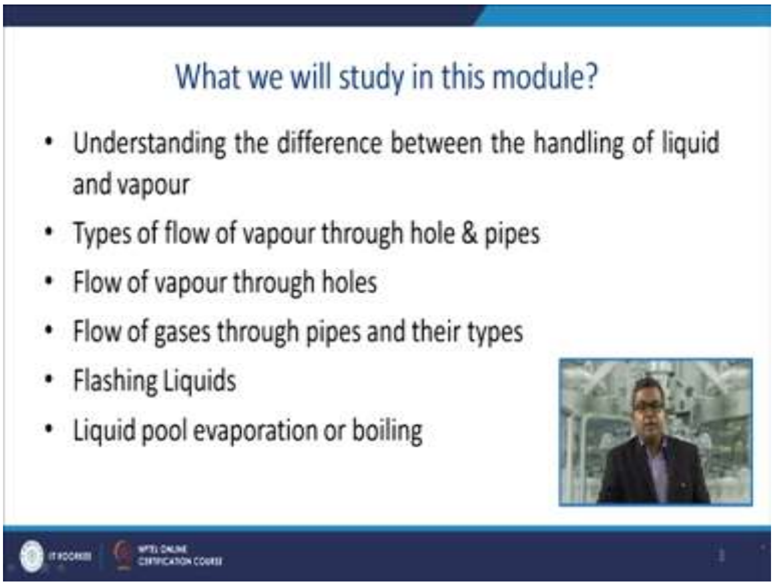

Now, in this particular module we are going to discuss about what is the understanding aspect, the difference between the handling of liquid and vapour, type of flow of vapour through hole and pipes, flow of vapours through different type of holes, flow of gases through pipe and their types, flashing liquids, liquid pool evaporation or boiling.

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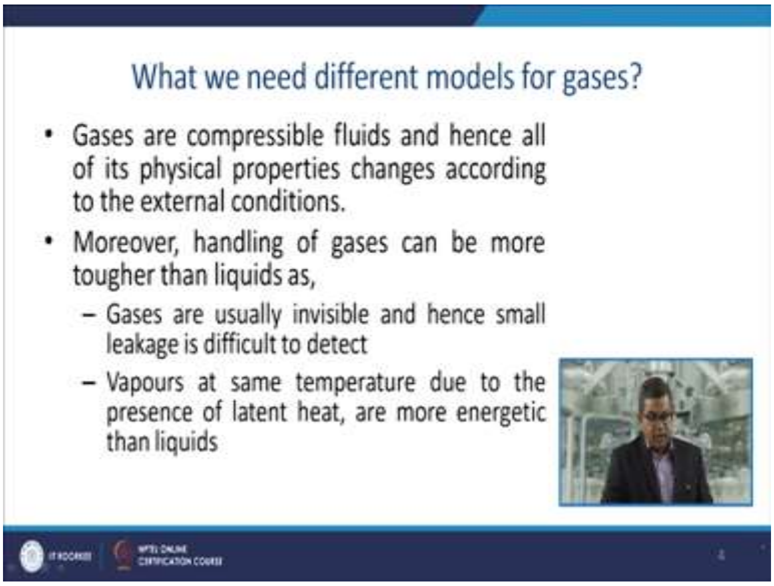

Now, what we need in different models for gases. Now, usually gases are compressible fluids and hence all of its physical properties changes according to the external conditions, these external conditions may be pressure, may be temperature, etcetera. Moreover handling of gases can more tougher than liquid. Now, because of the variety of reasons because one reason is, the gases they are usually invisible hence a small leakage is difficult to detect unless otherwise you are able to recognize through its order.

Vapours at same temperature due to the presence of latent heat they are more energetic than liquid.

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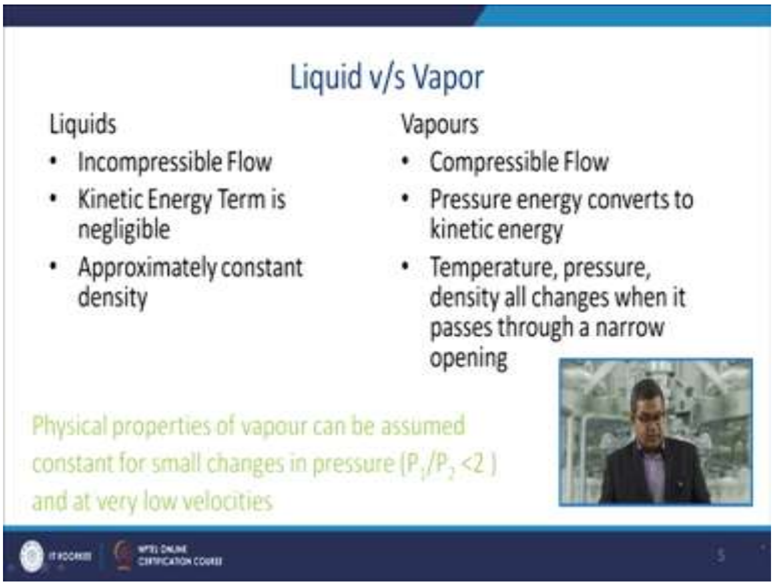

Now, here we have a comparison of liquid versus vapour, liquid usually they are compressible flow whereas vapours they are the compressible one. In liquids, the kinetic energy term is negligible whereas in vapours the pressure energy converts to kinetic energy and the liquids approximately constant density however the vapours, the temperature, pressure, density all changes when it passes through a narrow opening.

So, you can imagine that how critical is the scenario for the modelling in vapours. Now, physical properties of vapours can be assumed constant for small changes in pressure like P1, P2. P3 or etcetera and a very low velocities.

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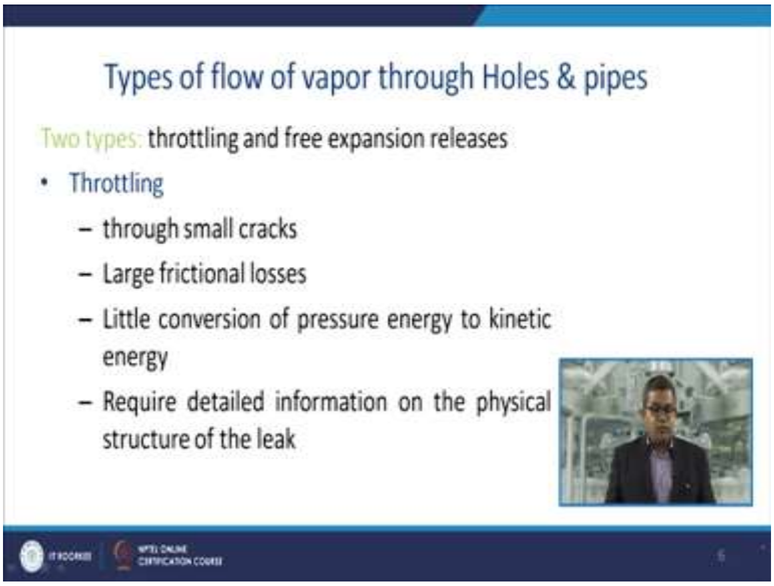

Now, let us have a look about the types of flow of vapour through holes and pipe. Now, there are two type of things, one is throttling and other one is the free expansion release. Now, throttling we all know may be through a small cracks, there is a large frictional losses, a little conversion of pressure energy to kinetic energy, they require the detailed information on the physical structure of the leaf.

So, you must have all these type of things with you while you are considering the throttling behaviour.

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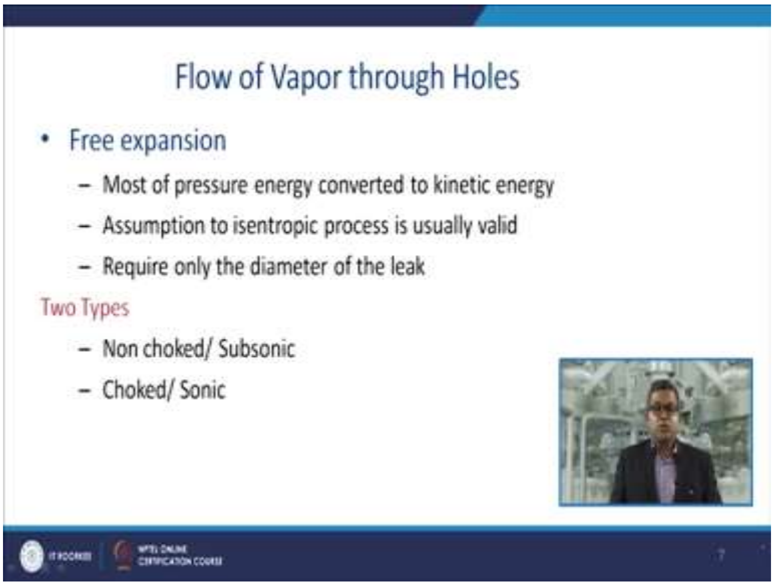

Whereas in a free expansion, the most of the pressure energy converted into kinetic energy, assumptions to isentropic process is usually valid because while you consider this free expansion you may have to take certain assumptions. So, one assumption is to the isentropic process which is usually valid in this case. Now, they require only a diameter of a leak, so this is a bit easier.

Now, there are two type of things which we need to consider under the head of the free expansion one is, the non-choked or subsonic another one, is the choked or sonic one.

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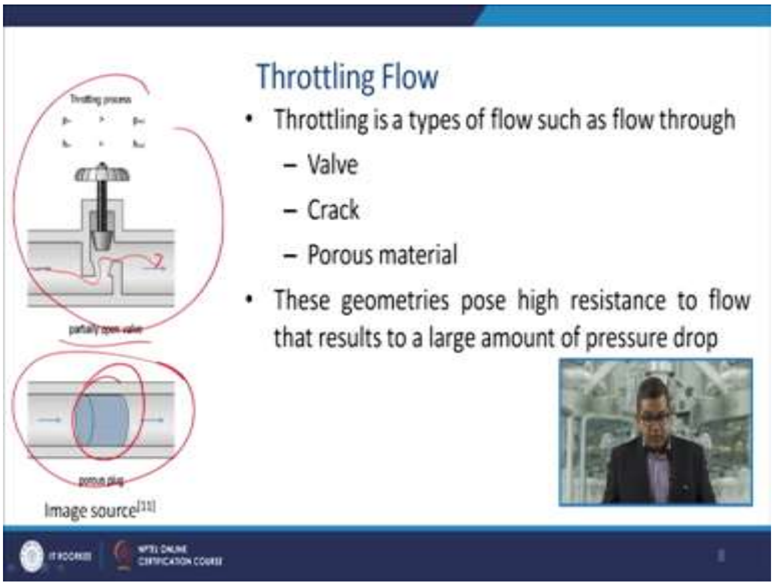

So, while considering the throttling flow, throttling is a type of flow such as a flow through valve like in this figure you can see, the flow through a valve or a crack or within the porous material. So, these geometries oppose high resistance to flow that results to a large amount of a pressure drop. Now, you can see in these figures this is the partially open valve and you can see the throttling flow.

Whereas in the second figure, you can see this is this is a porous plug and you can see the flow is through this porous plug under the throttling behaviour. So, while considering the throttling flow, we can seek the help of first law of thermodynamics.

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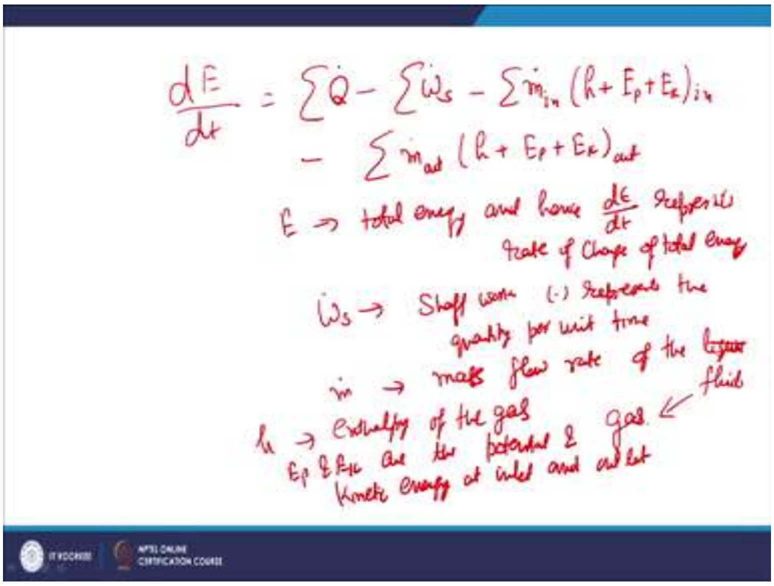

Now, first law of thermodynamics says

$$\frac{dE}{dt} = \sum \dot{Q} - \sum \dot{W}_s - \sum \dot{m}_{in} \left(h + E_p + E_k\right)_{in} - \sum \dot{m}_{out} \left(h + E_p + E_k\right)_{out}$$

Now, here E is the total energy and hence "dE/ dt" represents rate of change of total energy, "W_s" is the shaft work and this dot over this represents the quantity per unit time, m represents the mass flow rate of the liquid, in case sorry, mass flow rate of fluid. Now, here fluid is gas. "h" is enthalpy of the gas, "E_P" and "E_K" are the potential and kinetic energy at inlet and outlet. So, we may take different assumptions for this particular equation like the potential and kinetic energy effects are neglected. We may assume their steady flow, adiabatic conditions and we may assume the shaft work is zero.

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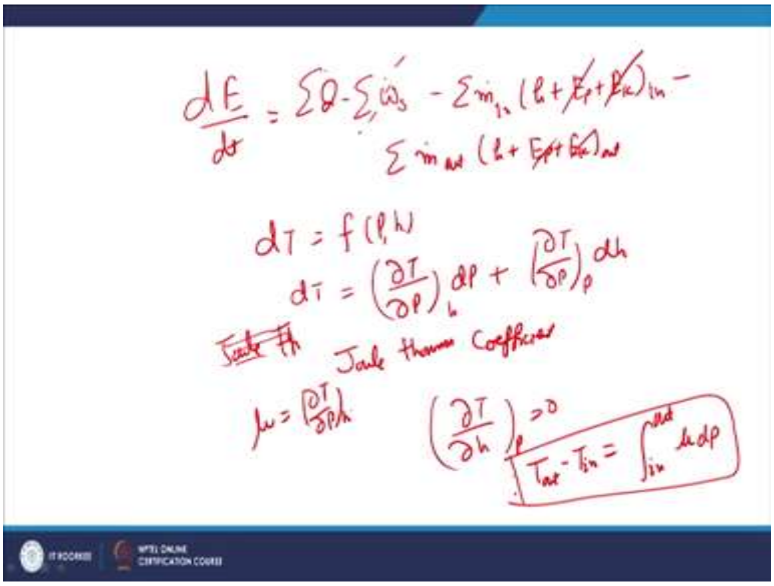

So, by employing these assumptions in our system we can get the this equation

$$\frac{dE}{dt} = \sum \dot{Q} - \sum \cancel{\dot{W_s}} - \sum \dot{m}_{in}\,(h + \cancel{E_p} + \cancel{E_k})_{in} - \sum \dot{m}_{out}\,(h + \cancel{E_p} + \cancel{E_k})_{out}$$

dE over dt is equal to Ws which is cut because we have assumed the shaft work negligible then summation mi h plus EP plus EK in, now this is again cancelled out then m out h plus EP plus EK for the outlet, now again this is a cancel out. So, hence isenthalpic flow can be consider in this particular case.

So, if we take that

dT = *f (P, h)*

then we can take the partial derivative like this

$$dT = (\frac{\partial T}{\partial P})_h\, dP + (\frac{\partial T}{\partial P})_P\, dh$$

Now, if define the Joule-Thomson coefficient, this is the Joule, Joule-Thomson coefficient, now

$$\mu \equiv \left(\frac{\partial T}{\partial P}\right)_h$$

So, if we consider the isenthalpic flow then in that particular case

$$(\frac{\partial T}{\partial h})_P = 0$$

and if we integrate then

$$T_{out} - T_{in} = \int_{in}^{out} \mu dP$$

Now, the most gases have positive Joule-Thomson coefficient, so as the pressure drops, the temperature drops.

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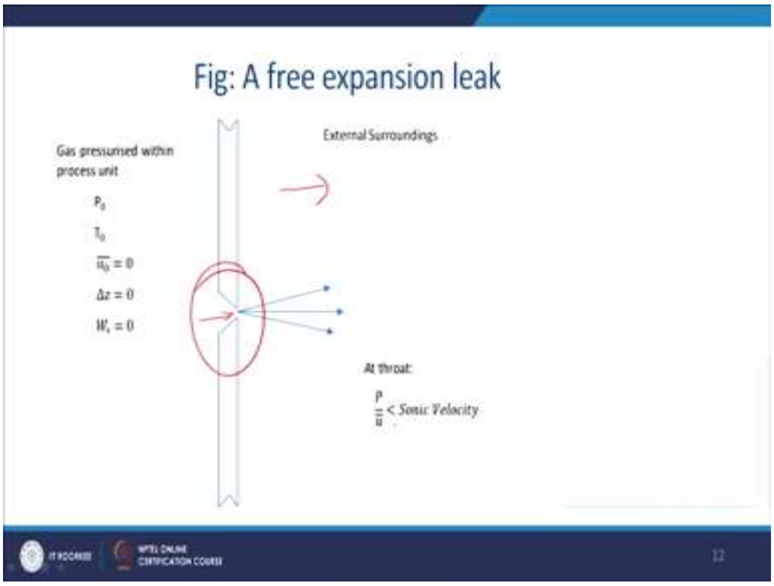

Now, let us have a look about a free expansion leak. Now, here we can visualize this figure you are having a gas pressurized within a process unit and it leaks through this particular point. So, initial condition if we say that P_0, T_0, u_0 is equal to 0, Δz is equal to 0 and shaft work is equal to 0 and here the things are at the external surrounding. So, here you can have a throat where P is

$$\frac{P}{\overline{\overline{u}}} < Sonic\ Velocity$$

So, if we consider this particular aspect through hole then we can write the mechanical energy balance for flow.

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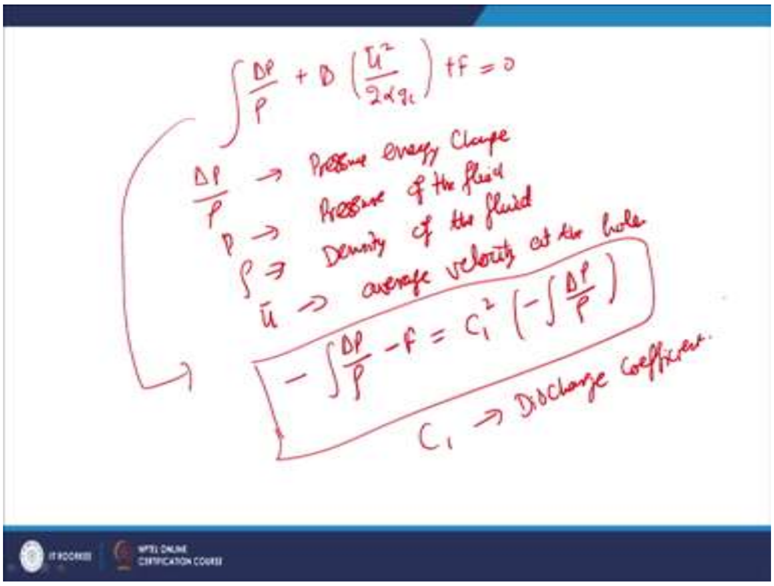

And that is the

$$\int \frac{\Delta P}{\rho} + \Delta\left(\frac{\overline{u}^2}{2\alpha g_c}\right) + F = 0$$

Where delta P over delta rho is the pressure energy change, P is the pressure of the fluid of the fluid, ρ is the density of the fluid, $\overline{u}$ is the average velocity at the hole. So, we need to define the discharge coefficient C_1 at this juncture, now this is defined as

$$-\int \frac{\Delta P}{\rho} - F = {C_1}^2\left(-\int \frac{\Delta P}{\rho}\right)$$

So, but at this point of time we can define the discharge coefficient, now C_1 is the discharge coefficient.

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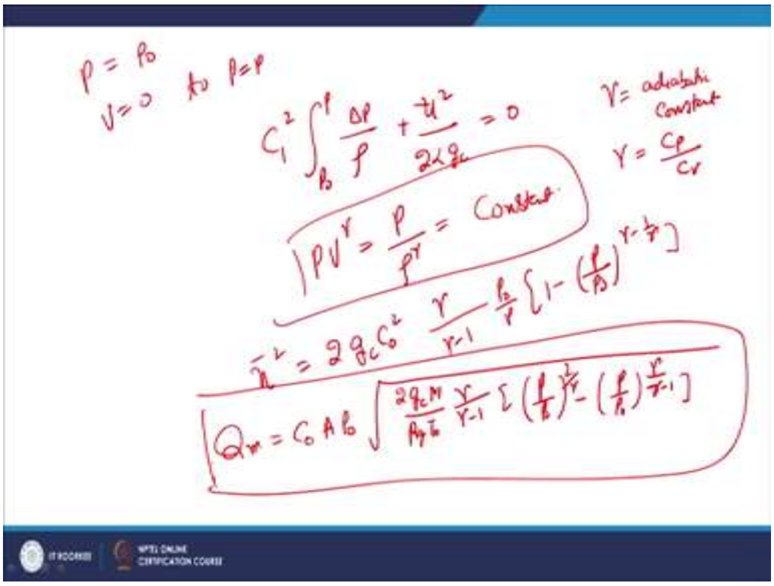

Now, if we combine the both the equation and integration from integrate from P is equal to P_0, V is equal to 0 to P is equal to P then we will have

$$C_1{}^2 \int_{P_0}^{P} \frac{\Delta P}{\rho} + \frac{\overline{u}^2}{2\alpha g_c} = 0$$

So, for any ideal gas undergoing isentropic expansion, we are having this universal equation

$$PV^{\gamma} = \frac{P}{\rho^{\gamma}} = Constant$$

This is the well-known thermodynamic equation.

So, velocity of a fluid any at any point during the isentropic expansion is given by

bar u square 2 gc C 0 square gamma, gamma is a adiabatic constant and usually it is represented by C_P/ C_V.

$$\overline{u}^2 = 2g_c C_0{}^2 \frac{\gamma}{\gamma - 1} \frac{P_0}{P} [1 - \left(\frac{P}{P_0}\right)^{\gamma - \frac{1}{\gamma}}]$$

Now, therefore the mass flow rate of any time during the isentropic expansion can be given by

$$Q_m = C_0 A P_0 \sqrt{\frac{2g_c M}{R_g T_0} \frac{\gamma}{\gamma - 1} [\left(\frac{P}{P_0}\right)^{\frac{2}{\gamma}} - \left(\frac{P}{P_0}\right)^{\frac{\gamma}{\gamma - 1}}]}$$

So, by this way you can calculate the mass flow rate at any time during the isentropic expansion, so this is the general equation. Now, sometimes velocity of a gases gas increases with the decrease in downstream pressure or increase in upstream pressure until it reaches a critical velocity, the choked the pressure "P_{choked}" is the maximum downstream pressure resulting in maximum flow through the hole or a pipe.

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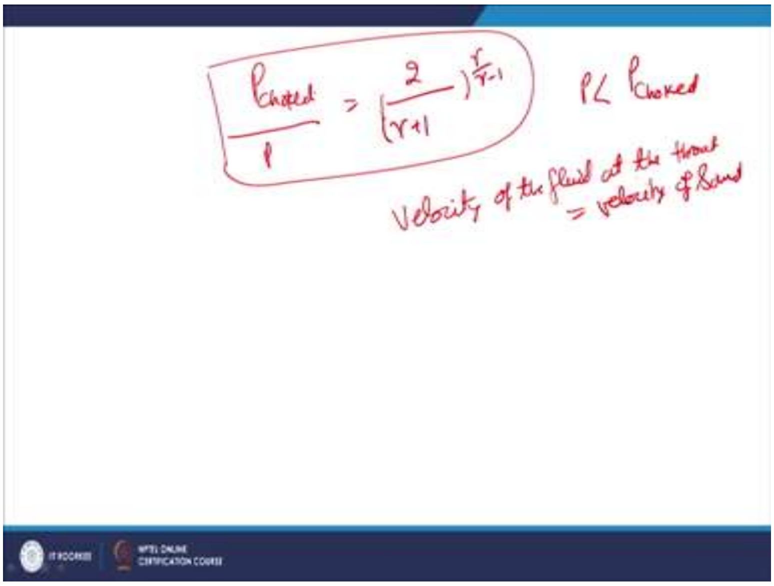

So, if we wish to calculate the P_{choked} then it is represented as

$$\frac{P_{choked}}{P} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}}$$

to be more precise you can get this particular P_{choked} from this particular equation.

Now, for if we have P is less than P_{choked}, the velocity of the fluid at throat is equal to the velocity of sound, velocity of the fluid at the throat is equal to velocity of sound and the velocity and mass flow rate cannot be increased further and third point is that become the flow become independent of the downstream condition, so this type of a flow is called choked, critical or sonic flow.

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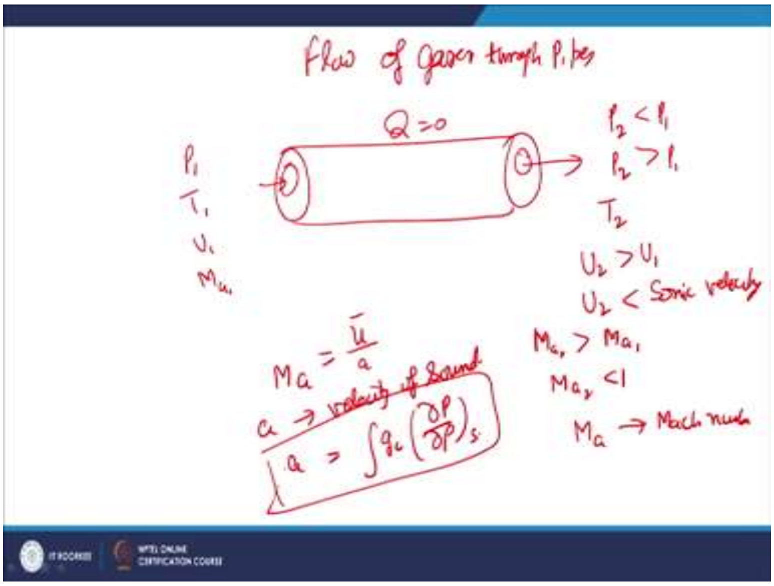

Now, in the next aspect, we will consider the flow of gas through pipe, flow of gases through pipes. Now, here we are having a pipe like this, here the initial conditions are P_1, T_1, U_1 Ma_1 and Q is equal to 0, here the final condition are P_2 is less than P_1, P_2 is the greater than P_1, these two are the conditions T_2, U_2 is greater than U_1 or U_2 is less than sonic velocity Ma_2 is greater than Ma_1 and Ma_2 is less than 1.

Now, Ma is the Mach number, (here Ma is the Mach number), so Ma is represented as

$$Ma = \frac{\overline{u}}{a}$$

Now "a" is the velocity of sound, so "a" is represented as

$$a = \int g_c \left(\frac{\partial P}{\partial \rho}\right)_s$$

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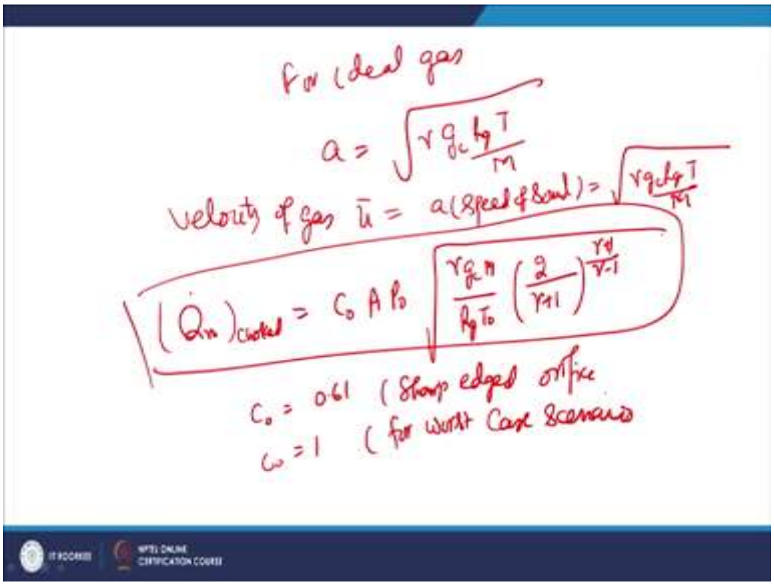

So, for ideal gas,

$$a = \sqrt{\frac{\gamma g_c R_g T}{M}}$$

So, by this way you can calculate the velocity of a sound, so velocity of a gas if you wish of gas is represented as bar u is equal to a that is a speed of sound is equal to

$$\bar{u} = a\,(speed\ of\ sound) = \sqrt{\frac{\gamma g_c R_g T}{M}}$$

So, the mass flow rate at the choked condition would be Qm that is choked is equal to

$$(\dot{Q_m})choked = C_o A P_o \sqrt{\frac{\gamma g_c M}{R_g T_o}\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

Now, C_0 is equal to 0.61 for sharp edged orifice and C_0 is equal to 1 for worst case scenario. So, these are the two conditions which may need to encounter during the course of calculation.

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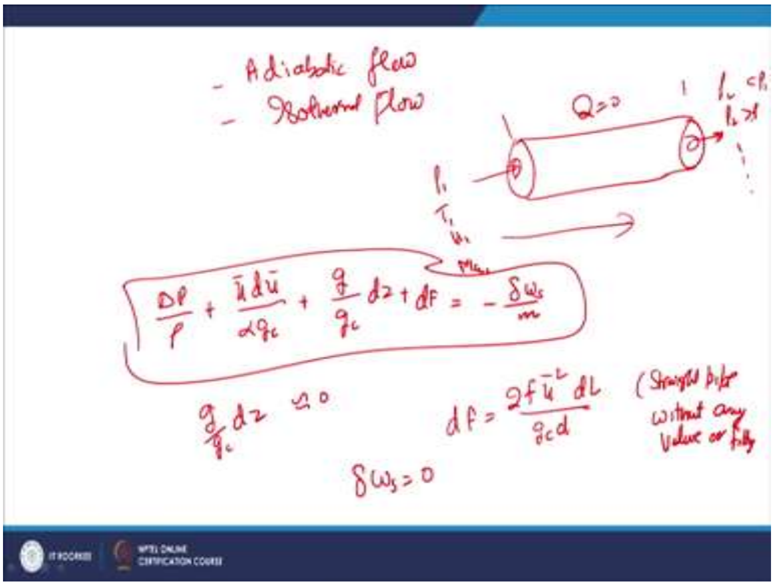

Now, there are two types of flow, one is the adiabatic flow, adiabatic flow and second one is the isothermal flow. Now, in adiabatic flow the well-insulated valve there is no energy loss to the surrounding and in isothermal flow the constant valve temperature that is the submerged pipe. So, let us take the example of adiabatic flow, now please recall the previous figure which we have and for your the convenience I am redrawing it, that was the pipe flow, initial conditions were fixed at P_1, T_1, U_1, Ma_1 and the final conditions are P_2 is less than P_1, P_2 is greater than P_1 and so and so on, you may recall the previous figure.

Now, as shown in the figure, the gas is flowing from left to right direction due to the presence of a pressure gradient across the two ends of the pipe. Now, as the gas moves forward, it is starts expanding due to the decrease in the pressure because P2 is less than P1 in the direction of the flow,this results to increase in velocity and the kinetic energy of gas. The increase in kinetic energy is compensated from the decrease in thermal energy of the gas.

Now, the gas starts cooling down due to the decrease in thermal energy, so the friction force between the gas molecules and the valve of container also plays a significant role in the change in temperature of the gas as well as towards the pressure drop. Now, these frictional forces increases or increase the temperature of the gas hence both cooling or heating of the gas during the flow is possible.

So, through this way we can write the mechanical energy balance equation

$$\frac{\Delta P}{\rho} + \frac{\bar{u}d\bar{u}}{\alpha g_c} + \frac{g}{g_c}dz + dF = -\frac{\delta W_s}{m}$$

Now, the following assumption that is

$$\frac{g}{g_c}dz \approx 0$$

is valid for this particular case. So, assuming the straight pipe without any valve or fitting although this is purely an assumption, so we will have our dF is equal to

$$dF = \frac{2f\bar{u}^2 dL}{g_c d}$$

Now this is for the straight pipe without any valve or fitting and no mechanical leakage, this is again the purely assumption that if no mechanic leakage are present then

$$\delta W_s = 0$$

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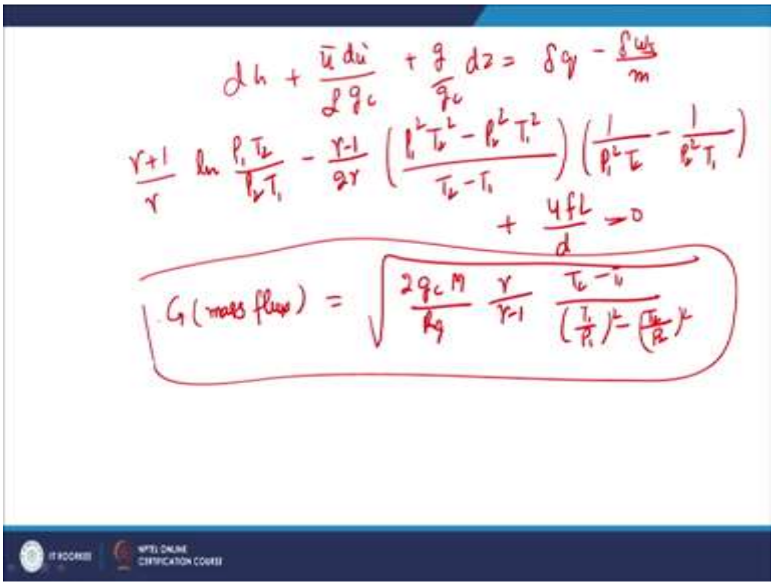

So, we may write the total energy balance equation like this

$$dh + \frac{\bar{u}d\bar{u}}{\alpha g_c} + \frac{g}{g_c}dz = \delta q - \frac{\delta W_s}{m}$$

this is the total energy balance equation. So, if we go ahead further then this is represented by

$$\frac{\gamma+1}{\gamma} ln\frac{P_1T_2}{P_2T_1} - \frac{\gamma-1}{2\gamma}\left(\frac{{P_1}^2{T_2}^2 - {P_2}^2{T_1}^2}{T_2 - T_1}\right)\left(\frac{1}{{P_1}^2T_2} - \frac{1}{{P_2}^2T_1}\right) + \frac{4fL}{d} = 0$$

Now, let us take the G which is a mass (flus) flux is equal to

$$G = \sqrt{\frac{2g_cM}{R_g}\frac{\gamma}{\gamma-1}\frac{T_2-T_1}{(\frac{T_1}{P_1})^2-(\frac{T_2}{P_2})^2}}$$

this is the mass flux.

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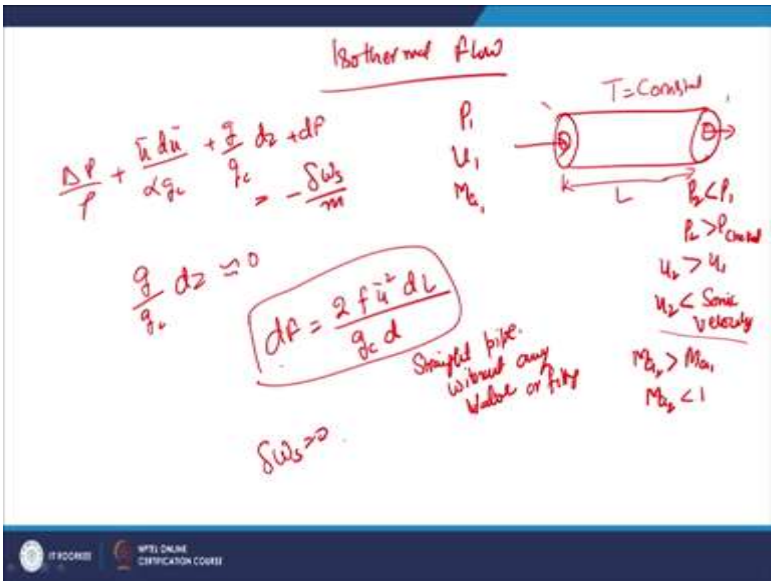

Now, in the other example we may take the isothermal flow. Again considering this, the figure of the pipe flow P_1, U_1, Ma_1, P_2 is less than P_1, P_2 is greater than P choked u_2 is greater than u_1, u_2 is less than sonic velocity Ma_2, Ma_1 or Ma_2 is less than 1. So, if we consider this type of aspect then the isothermal flow of a gas, we have considered in this case the T is constant, let us have the thing that this we are considering the L length.

Now, considering the case when the velocity of the gas is well below the sonic velocity of the gas, like this, now gas is flowing from left to right direction due to the presence of pressure gradient across the two ends of the pipe, these are the two ends of the pipe. Now, gases starts expanding due to the decrease in pressure in the directional of direction of flow. Now, as the gas expends the velocity must increase to maintain the same mass flow rate.

So, the pressure at the end of the pipe is equal to the pressure of the surrounding. So, the temperature is constant across the entire pipe length we have taken (all) already taken this assumption. So, we may write the mechanical energy balance equation that is

$$\frac{\Delta P}{\rho} + \frac{\overline{u}d\overline{u}}{\alpha g_c} + \frac{g}{g_c}dz + dF = -\frac{\delta W_s}{m}$$

Now we may take the following assumption for this particular case that is

$$\frac{g}{g_c}dz \approx 0$$

Now, we may assume that the straight pipe without any valve or a fitting. So,

$$dF = \frac{2f\overline{u}^2 dL}{g_c d}$$

this is for the straight pipe, pipe without any valve or fitting. So, if there is no mechanic leakage then this is

$$\delta W_s = 0$$

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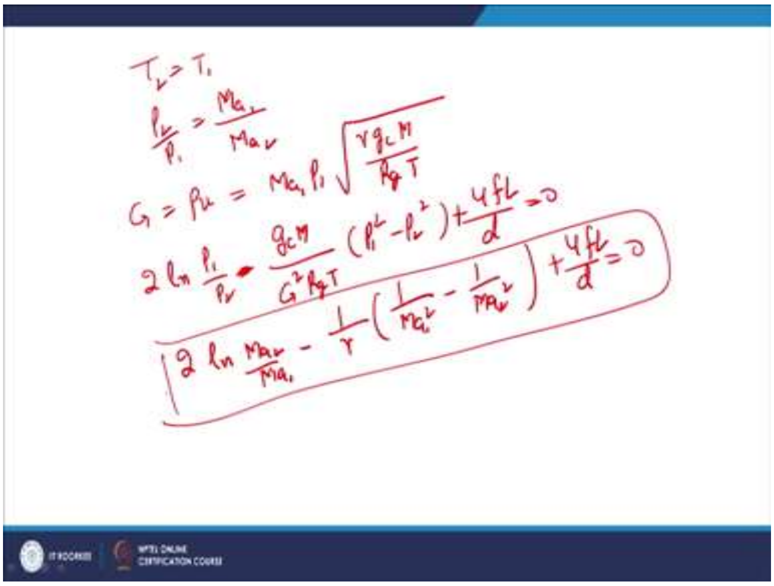

So, we may use the following conditions

$$T_2 = T_1$$

with T_2 is equal to T_1 because we have assumed the isothermal condition. Then

$$\frac{P_2}{P_1} = \frac{Ma_1}{Ma_2}$$

Now,

$$G = \rho u = Ma_1 P_1 \sqrt{\frac{\gamma g_c M}{R_g T}}$$

$$2ln\frac{P_1}{P_2} - \frac{g_c M}{G^2 R_g T}\left({P_1}^2 - {P_2}^2\right) + \frac{4fL}{d} = 0$$

$$2ln\frac{Ma_2}{Ma_1} - \frac{1}{\gamma}\left(\frac{1}{{Ma_1}^2} - \frac{1}{{Ma_2}^2}\right) + \frac{4fL}{d} = 0$$

So, by this way we can model or we can calculate the appropriate things. Now, there are certain conditions related to the flashing of liquid and we may consider the four cases.

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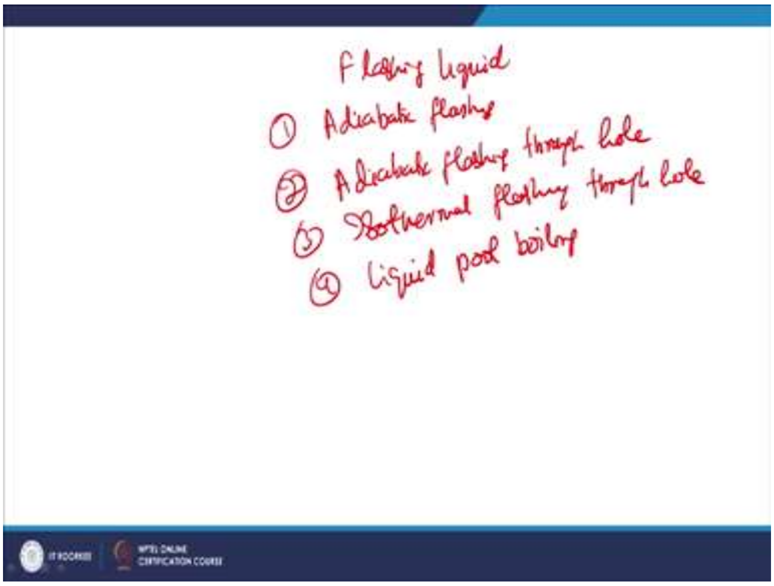

The adiabatic flashing, flashing of flashing liquids, so first thing is that we may assume the adiabatic flashing. Second is adiabatic flashing through hole, third is isothermal flashing through hole and fourth is the liquid pool boiling. So, flashing liquids, now flashing event may occur when the liquid is stored under the pressure above their normal boiling points. Now, if liquid is stored in a container or flowing in a pipe develops a leak under such condition, the liquid starts partially flashing into vapour, explosive flashing may also occur.

So, this process can be assumed as an adiabatic process, the energy of vaporization comes from the superheated liquid which results to the decrease in temperature of the liquid.

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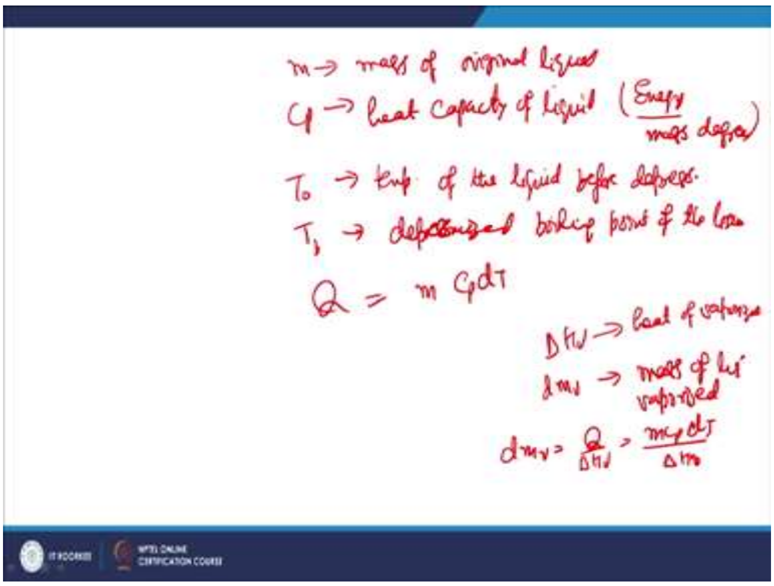

Now, if m is the mass of a original liquid, mass of original liquid and C_P is the heat capacity of the liquid energy plus mass degree and T_0 is the temperature of the liquid before (depressive) depressurization, temperature of the liquid before the depressurization, T_b is the depressurized boiling point of the liquid. So, excess energy contained the superheated liquid may be calculated by this original thermodynamic equation,

$$Q = mC_p dT$$

This excess energy provides the heat of vaporization ΔH_v for the mass of liquid vaporized that is dm_v, this is the mass of liquid vaporized and this is the heat of vaporization.

So,

$$\mathrm{d}m_v = \frac{Q}{\Delta H_v} = \frac{mC_P dT}{\Delta H_v}.$$

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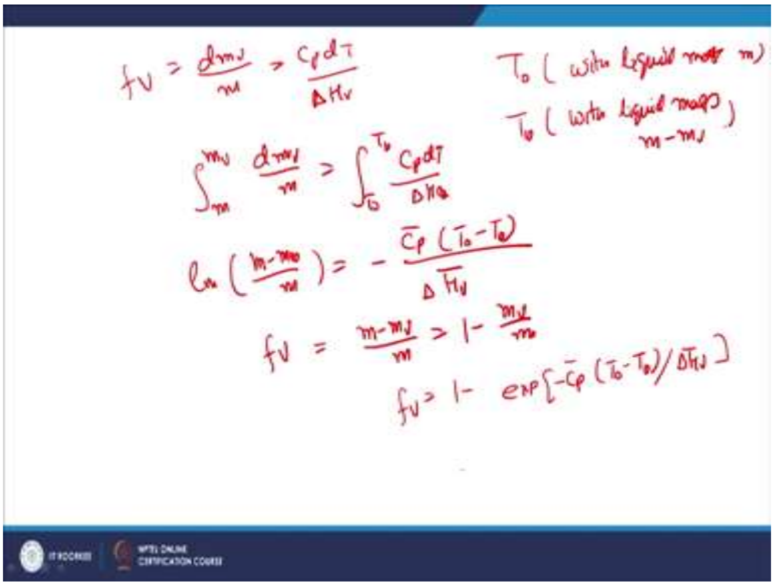

So, we can calculate the fraction of liquid vaporized like

$$f_v = \frac{dm_v}{m} = \frac{C_P dT}{\Delta H_v}$$

Now, if you integrate between the initial temperature T_0 with the liquid mass m and the final boiling point T_b with liquid mass "m-m_v". so, if we integrate between this then

$$\int_m^{m_v} \frac{dm_v}{m} = \int_{T_o}^{T_b} \frac{C_P dT}{\Delta H_v}$$

Now,

$$\ln\left(\frac{m - m_v}{m}\right) = -\frac{\overline{C_P}(T_o - T_b)}{\overline{\Delta H_V}}$$

So,

$$f_v = \frac{m - m_v}{m} = 1 - \frac{m_v}{m}$$

So, after substituting the value we get the fraction of liquid vaporized as

$$f_v = 1 - \exp[-\overline{C_P}(T_o - T_b)/\overline{\Delta H_V}]$$

Now, equilibrium flashing choking conditions, so fluid path is great fluid path length greater than 10 centimeter, now choked pressure is approximately assumed equal to the saturation vapour pressure of the fluid in question.

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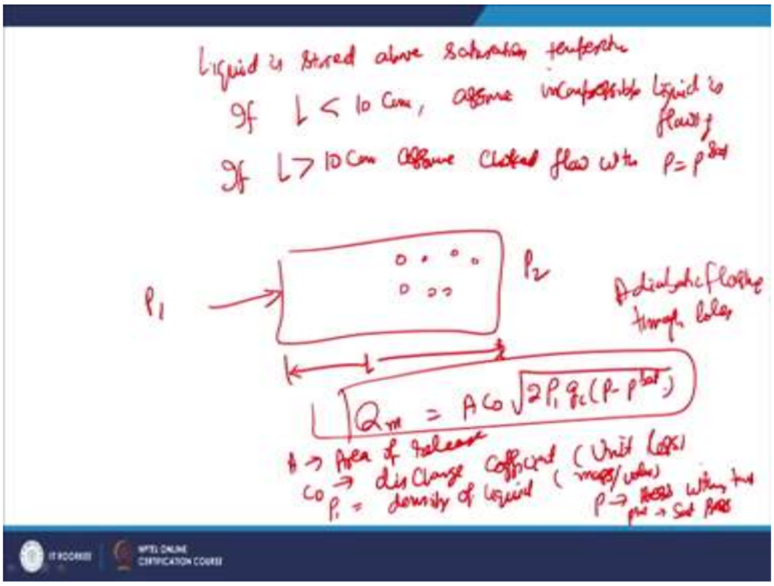

So, the another aspect is that the liquid is stored above saturation pressure, if we take liquid is stored above saturation temperature, so if L is less than 10 centimeter, assume incompressible liquid is flowing. Now, if L is greater than 10 centimeter assume choked flow with P is equal to P saturation. So, like this here the P1 this is the length L and this is the P2, so this is the case of adiabatic flashing through holes.

So, mass flow rate can be given as

$$Q_m = AC_0\sqrt{2\rho_l g_c(P - P^{sat})}$$

so this gives you the mass flow rate where A is area of release, C_0 is the discharge of coefficient which is unitless, ρ_l is the density of liquid mass per volume, P is the pressure within tank and P^{sat} (saturation) is the saturation pressure.

Now, let us consider the isothermal flashing through a hole.

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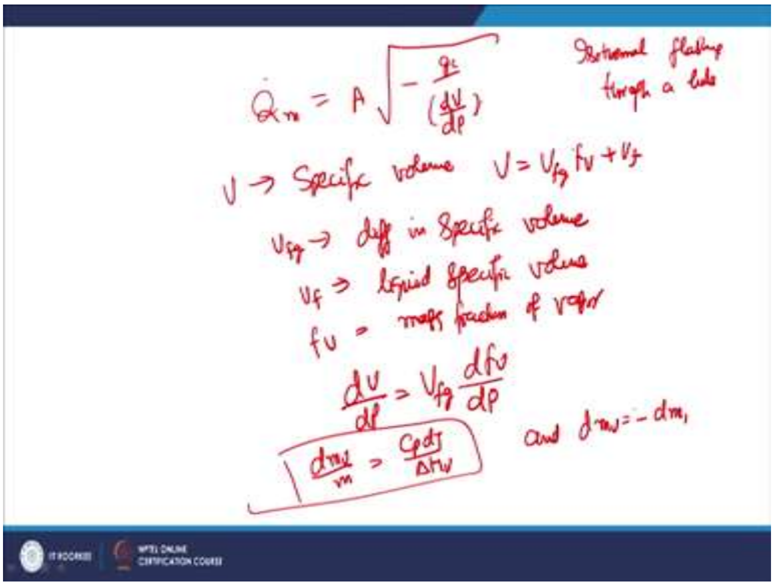

Now, if we assume the choked flow of a two phase fluid then

$$\dot{Q_m} = A\sqrt{-\frac{g_c}{(\frac{dV}{dP})}}$$

this is the isothermal flashing a hole. Now, here V is the specific volume,

$$V = V_{fg}f_v + V_f$$

V_{fg} is the difference in specific volume,

V_f is the liquid specific volume,

f_V is the mass fraction of vapour.

Now, if you differentiate with respect to pressure then we will get

$$\frac{dV}{dP} = V_{fg}\frac{df_v}{dP}$$

So, we derived that

$$\frac{dm_v}{m} = \frac{C_P dT}{\Delta H_v}$$

and

$$dm_v = - dm_l$$

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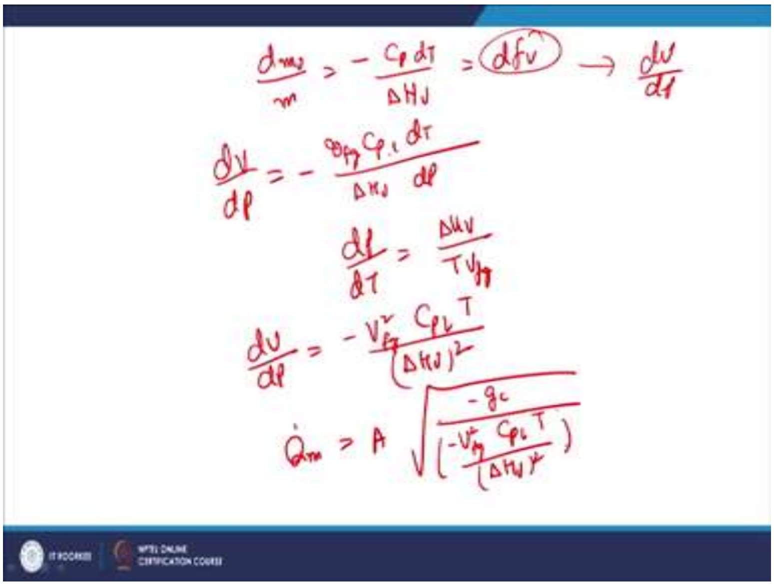

So, applying all these conditions we have therefore

$$\frac{dm_v}{m} = -\frac{C_{Pl}dT}{\Delta H_v} = \mathrm{d}f_v$$

Now, if we substitute df in this particular thing into dV/ dP we have

$$\frac{dV}{dP} = -\frac{v_{fg}C_{p.l}}{\Delta h_v}\frac{dT}{dP}$$

So, the Clausius-Clapeyron equation can be used for dT over dP, so

$$\frac{dP}{dT} = \frac{\Delta h_v}{Tv_{fg}}$$

So, if we combine the equations then we will get

$$\frac{dv}{dP} = \frac{-v^2{}_{fg}C_{p,l}T}{(\Delta h_v)^2}$$

So, we can substitute the into the final relationship that is

$$\dot{Q_m} = A\sqrt{\frac{-g_c}{\left(\frac{-v^2{}_{fg}C_{p,l}T}{(\Delta h_v)^2}\right)}}$$

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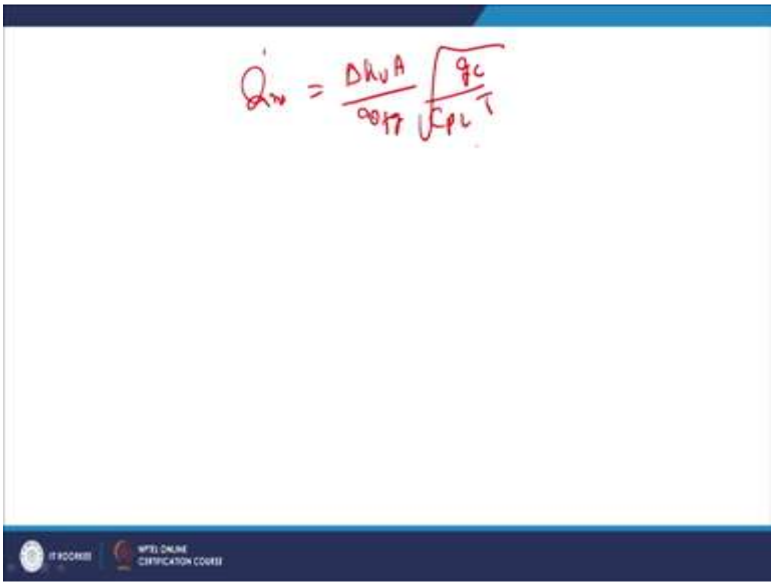

Now, reducing the equation for vapour mass flow rate then

$$\dot{Q_m} = \frac{\Delta h_v A}{v_{fg}} \sqrt{\frac{g_c}{C_{P,l} T}}$$

So when flashing process is done or done at or a near a P^{sat} (saturation), small droplets of liquid are also entrained with vapour. So, design assumption is taken as the mass of the liquid entrained is equal to the mass of vapour formed from the (from) flashing. So, in this particular module we have discussed a different aspect of leakage through hole and orifice.

And in the subsequent module we will discuss the liquid pool evaporation and other models for applicable for gaseous fluid, thank you.