**INDIAN INSTITUTE OF TECHNOLOGY ROORKEE**

**NPTEL**

**NPTEL ONLINE CERTIFICATION COURSE**

**Unit Operations of Particulate Matter**

**Lec- 07**
**Filtration and Batch Filtration (Part-02)**


**Dr. Shabina Khanam**
**Department of Chemical Engineering**
**Indian Institute of Technology Roorkee**


Welcome to the second lecture of week 2 where we will discuss the governing equations involved in batch filtration. So we will derive the equation for the batch filtration process as far as filtration process is concerned, here we will consider cake filtration.

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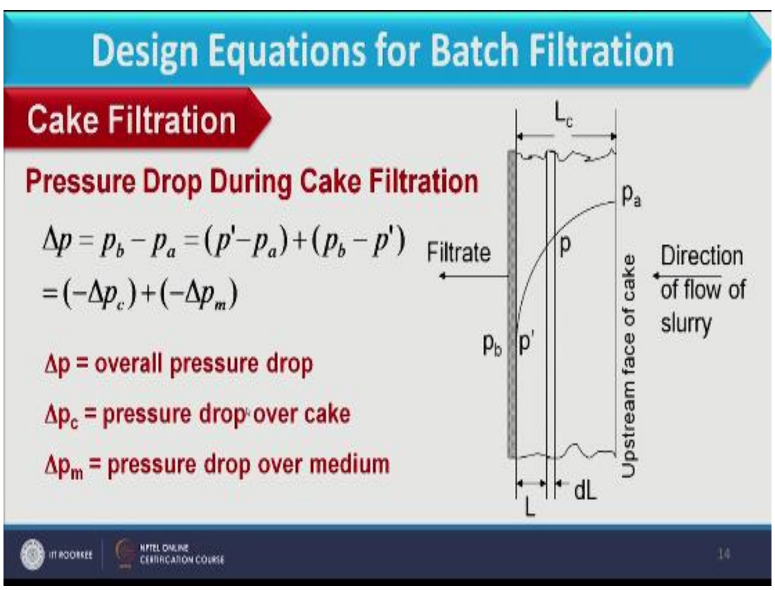


So if you consider this image here what happens this is basically the filter media and here we have, here slurry comes to the filter media and we have to accumulation of cake over the filter media. The thickness of cake is LC now this is the batch process and first of all we have to calculate the pressure drop of this. So from this side slurry enters and here we have the pressure as Pa.

Now through this cake pressure will continuously keep on decreasing, so here we have pressure P' and beyond this we have – and further we have the filter media where pressure will be decreased further, and near the filter media towards the filtrate side we have the pressure as Pb. So when we calculate the total pressure drop across this system we have to consider the pressure drop in the media and pressure drop in the cake. So pressure drop during cake formation is denoted as $\Delta p$ which is basically Pb-Pa and it is – it can ve equated to P'-Pa. Now how we have considered P'-Pa because difference is basically the final value minus initial value.

So in the cake final value is P', initial value is Pa. And similarly difference is observed in media, so P'-Pa can be represented as $-\Delta Pc$ because why we have the negative sign because we have considered final minus initial, so final pressure is lesser than the initial, so minus sign will appear over here. And similarly we have minus sign in media, so here we have defined different pressure drop that is $\Delta P$ overall pressure drop in the system, $\Delta Pc$ pressure drop over the cake and $\Delta Pm$ pressure drop across the media.

So as for calculation of pressure drop we have two pressure drop basically. So first of all we will discuss the pressure drop in cake and then pressure drop in the media. The equation, expression of these two pressure drop will be added to calculate total pressure drop.

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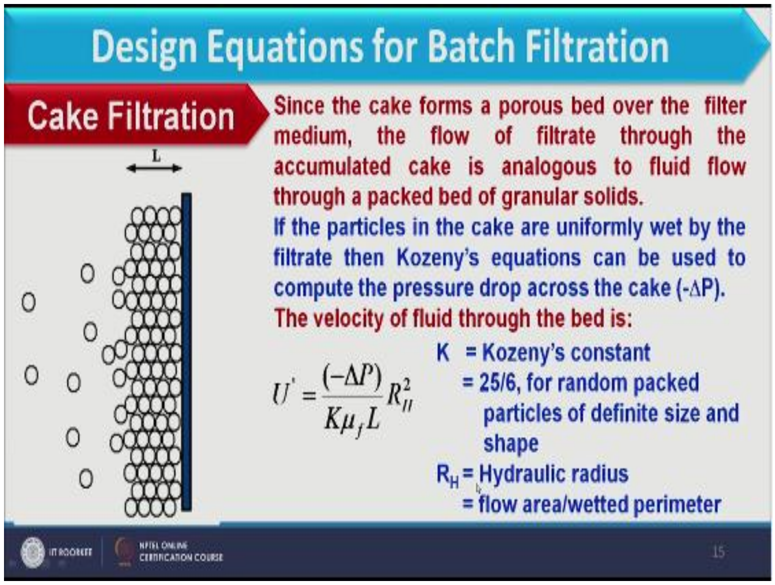


Now what happens, when we calculate the pressure drop in the cake, in the cake we have solid deposition. So since the cake forms a porous bed over the filter media, the flow of filtrate through accumulated cake is analogous to the fluid flow through a packed bed of granular solids. So when we consider the filter media where we have accumulation of solid particles, so when liquid passes through inside void in between the particle that resembles to the condition where fluid passes through the packed bed of granular solid.

So if the particles in cake are uniformly wet by filtrate then Kozeny's equation can be used to calculate the pressure drop across the cake, because here we have assumed that bed is completely wetted with the liquid, so we can use Kozeny's equations. So here you see this is the media, and the total thickness of the media is L, solids deposited over this media and through these void age you see through this void age the liquid flow and therefore it can be considered as packed bed.

So Kozeny's equation for this case is used to calculate velocity of the fluid and the equation is $U'$=pressure drop/$K\mu^f L R_H^2$, pressure drop is basically we have to calculate that should be the pressure drop across the cake $\mu_f$ viscosity of the fluid and L is total thickness of the cake. In this equation this K is basically Kozeny's constant and its value should be 25/6 for random packed particles of definite size and shape.

So here you see we are considering random packing because we are not aware for which orientation particle will settle over the media, therefore, it can be considered as random packing. $R_H$ we have denoted as the hydraulic radius which is defined as the flow area by wetted perimeter. So here we have to derive the expression of $R_H$ to calculate the expression of U'. So here I have to define the $R_H$ expression and then that expression will be put in this U' expression for further derivation, so $R_H$ we can define here as flow area by wetted perimeter.

However, in actual sense or even we consider this cake or that we have resembled that we have assumed as a packed bed, so this $R_H$ is further defined as the flow volume by wetted area. Here you see $R_H$ is defined as flow area by wetted perimeter and for this particular case we can define as flow volume by wetted area.

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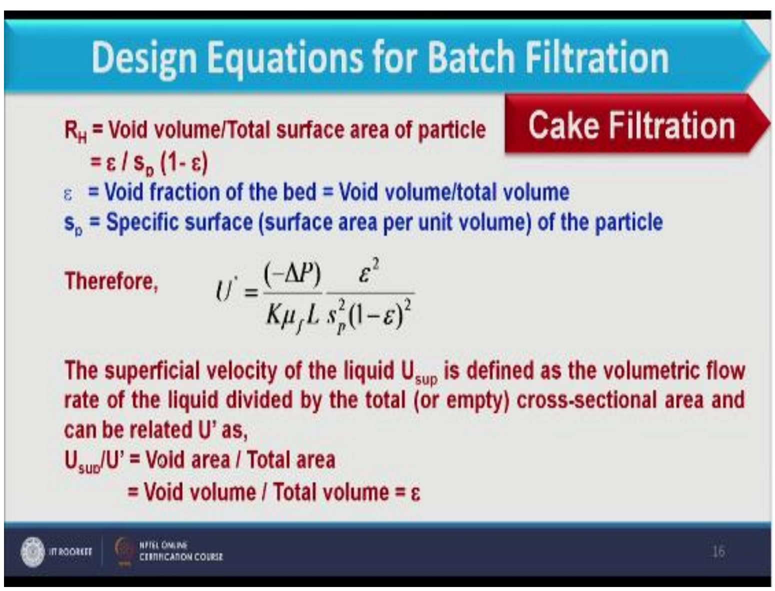


Now $R_H$ we have defined over here and its expression should be $\Sigma/S_p(1-\Sigma)$ you see this expression. So here $\Sigma$ is basically void fraction of the bed that is void volume/total volume. So $\Sigma$

is basically the void age inside the bed through which particle – through which liquid will penetrate. So when we consider Σ that is basically the – it will speak about the flow volume of the liquid. And when we consider 1-Σ that will speak about the solid

So Sp we have defined as specific surface that is the surface area per unit volume of the particle, so this is for a particle and it will be multiplied with 1-Σ to calculate total flow area of the particle. And therefore, when we put the expression of $R_H$ which is equal to Σ/Sp(1-Σ)U' we can find this equation. Now here we will define another parameter which is superficial velocity and superficial velocity can be defined as the volumetric flow rate of the liquid divided by total cross sectional area and that can be related with U'.

So how we can relate this with U'? We have defined a factor that is U superficial divided by U'. So U superficial is the superficial velocity divided by the velocity of the liquid through cake. Now this U superficial by U' is defined as void area by total area. Now how we can define this by void area by total area, how we define the superficial velocity is volumetric flow rate of the liquid divided by cross sectional area.

So volumetric flow rate of the liquid when we consider this as the meter cube of hole, because through holes or void it will flow, so that can be defined, that can be considered as $m^3$ of holes divided by second that is the rate. And further when we consider U' that would be the velocity of liquid through bed, so that we can define as meter of holes or meter of void age divided by the time. So that time factor will be cancelled out and meter of hole, where we divide $m^3$ of hole by m of hole, so that should be the MALE_2>>> of hole, so that should be – when we consider $m^2$ of hole that is nothing but the void area.

And U superficial that is, it is defined as volumetric flow by cross sectional, total cross sectional area. So this complete factor can be defined as void area by total area. Further we can consider this as void volume by total volume and that is equated with the Σ as we have defined earlier. So here we have to calculate superficial velocity. So when we multiply this Σ with U' we can calculate superficial velocity.

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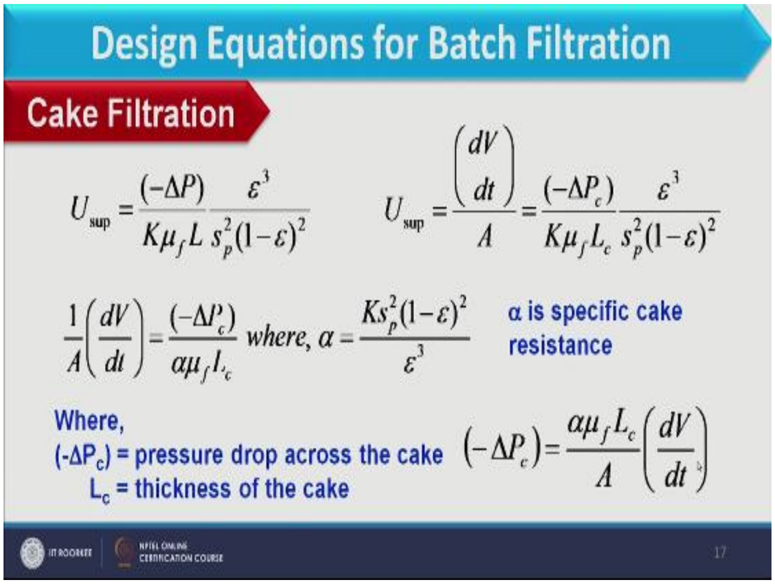


And the expression we can obtain like this. Superficial velocity according to its definition that should be (dV/dT)/A and that equated with the expression over here. Now here you see the pressure drop over here is replaced with the pressure drop across the cake, that is pressure drop of cake, because here what we are considering the particle movement through the bed. So whatever pressure drop will be obtained that will be offered by the cake itself, so therefore pressure drop will be replaced by pressure drop across the cake.

So in this way we define the superficial velocity. Further here we can rewrite this particular expression as 1/A(dV/dT) and that is equated with –ΔPc/αμfLc. Now Lc is basically total thickness of the cake, α we have introduced over here and μf we have defined as the viscosity of the fluid. So α we can further define like this where we have all constants values. So α is basically defined as the specific cake resistance.

So following this expression where ΔPc is pressure drop across the cake and Lc is the thickness of cake as we have already discussed. So ΔPc expression would be αμfLc/A(dV/dT), so here we can represent the pressure drop across the cake in terms of rate of filtration dV/dT is nothing but the rate of filtration.

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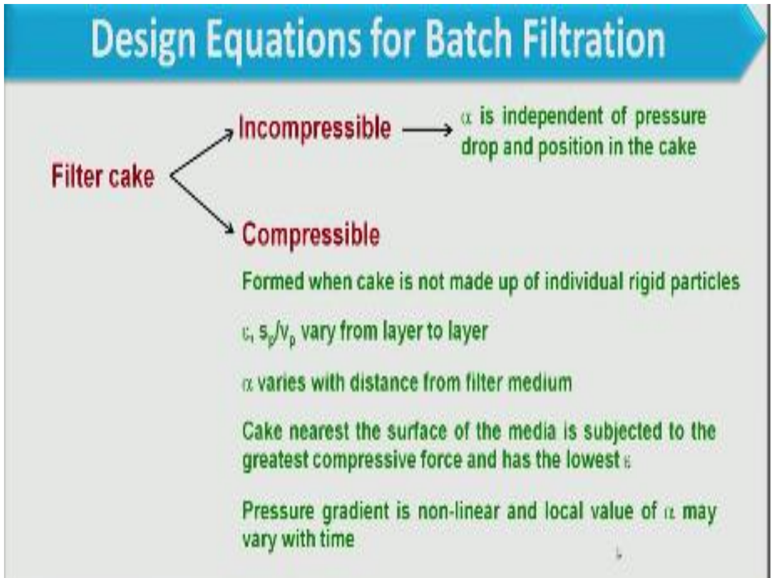


So further we have to define the condition of the cake that is what expression, what value we have to take for α because α is basically the specific cake resistance and that will be offered by the filtered cake. So here we can define as filter cake can be incompressible, you understand what is the meaning of incompressible that whatever position it has take over the bed it will remain as it is, so whatever volume, whatever void age over there, that will remain as it is.

So for incompressible α is independent of the pressure drop and in the position in the cake, because when we deal with the pressure drop, it will force the pressure, high force will force the slurry to pass through and therefore it compresses the bed also, it compresses the cake also. And here we have assumed the cake should be incompressible. So it should be free from the pressure drop or it should not dependent on the pressure drop the α value.

And here we are considering the compressible cake where we have different conditions such as the incompressible cake is formed when cake is not made up of individual rigid particles, when particles are not rigid then we can make the compressible cake where ΣSp/Vp vary from layer to layer Σ we understand the void age because when incompressibility, when compressibility will be considered the bed near to the filter media it has lesser void age in comparison to the void age available at, near the slurry.

So $\Sigma$ as well as Sp/Vp that is the specific surface area of the particle that will be varied from layer to layer, $\Sigma$ varies with distance from filter media, so here we have the resistance that is the specific case resistance so as compressibility increases, resistance increases, so $\alpha$ will be varied from initial layer to the final layer which is near to the media.

Now cake nearest the surface of the media is subjected to the greatest compressive force and has lowest $\Sigma$ as we have already discussed, because the nearest to the filter media, because slurry is coming from this side so all particles available in the cake that compresses the initial layer. So obviously void age should be minimum at media and it should be maximum when we are considering and that should be maximum when we are considering the slurry.

Pressure gradient is not linear and local value of $\alpha$ may vary with time. So all these factors considered when we are defining compressible cake and the average value of $\alpha$ is determined experimentally for each sludge. So here you see for compressibility and for incompressibility $\alpha$ can be defined, but for compressibility it is more complicated in comparison to incompressible cake. For the compressible cake $\alpha$ value determination of $\alpha$ value will be more difficult in comparison to incompressible cake.

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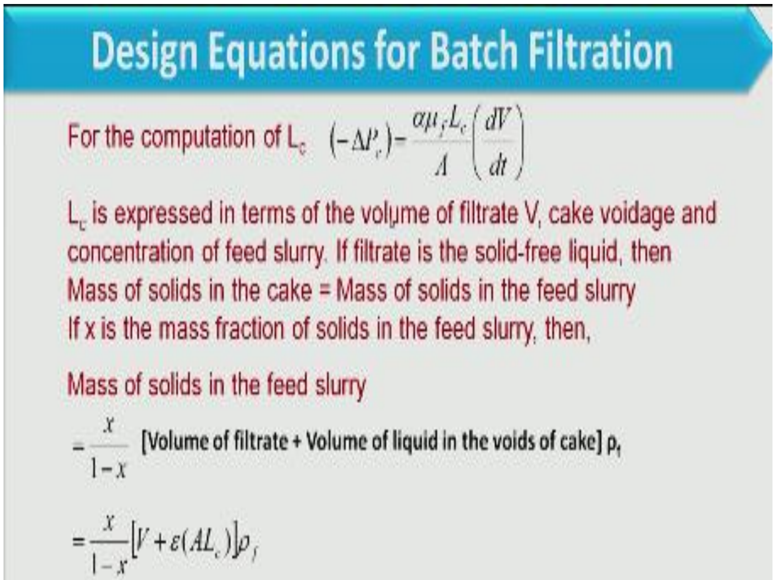


Now here in this slide we will define, we will derive the expression of the Lc which is nothing but the total thickness of cake. So Lc is expressed in terms of volume of filtrate that is V, cake void age and concentration of feed slurry. If filtrate is solid free then we can say that mass of the solid in the cake in the equal to mass of the solid in the feed slurry.

So as we have assumed that filtrate is solid free, so whatever mass is available in the slurry all that mass will be accumulated in the cake. So we can equate these two, if x is the mass fraction of solid in feed slurry then mass of the solid in feed slurry can be defined as x/1-x, volume of filtrate plus volume of liquid in void age of cake into ρf. Now how this expression is obtained, when you see the definition of x, what is x, is the mass fraction of solid in the slurry.

So x can be defined as mass of the liquid divided by mass of solid plus mass of liquid in the slurry. So from this expression when we consider when we calculate mass of the solid, so that should be, that should come as x/1-x into mass of the liquid, so how we can calculate the volume of the liquid is the volume of filtrate plus volume of liquid which is available in the bed, because volume will not available in the slurry, slurry has been filtered.

So whatever liquid is available that is either collected in terms of filtrate or it will remain in the bed itself. So here we can define as x/1-x(Vb) is the total filtrate which has been collected plus Σ(ALc) now what is ALc if you consider A is the cross sectional area of cake and Lc is the thickness of cake, so this is the volume of the cake into Σ, because Σ is the void age. So in this we can define the volume of the liquid in the cake. So whole factor is multiplied with ρf, because we have to consider over here the mass of the liquid, so volume collector into density of the fluid.

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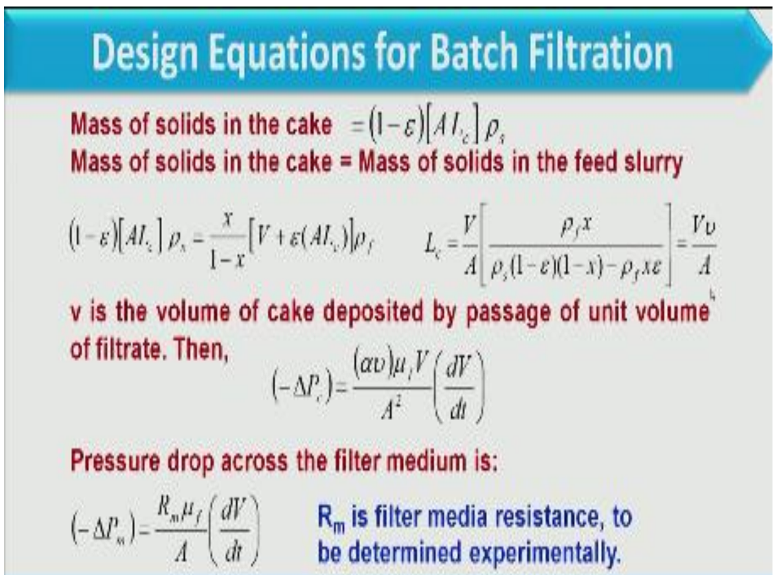


Further mass of the solid in cake is 1-Σ(ALc)ρs again ALc is the volume of the cake 1-Σ is the total volume of the solid in cake into ρs so it will give mass of solid in cake. So after equating mass of the solids in cake and mass of the solid in feed slurry we can find this expression from where we can calculate this Lc all constant value will be collected in terms of v. So in v how we can define this if we consider this ALc that is total volume of the cake, and v is the volume of the filtrate.

So how we can define in the unit volume of the filtrate, so we can see the small value of the cake in the Pc in the expression we can in the use and then this the final expression and the pressure drop, and then cake and the pressure drop in the filter and the you see the reply in the all the alpha in the $R_M$ that is the total resistance in the media in the dV/dT.

And this expression that will calculate the media, so the $R_M$ will be in the resistance in the media experimentally in the further we will see the factor about the resistance in the p v/ pressure drop as the solid particles can be in the forced in to the filter and the media in the in the filter in the media it will be affected. So what we can say that the $R_M$ value will be effected by the pressure drop.

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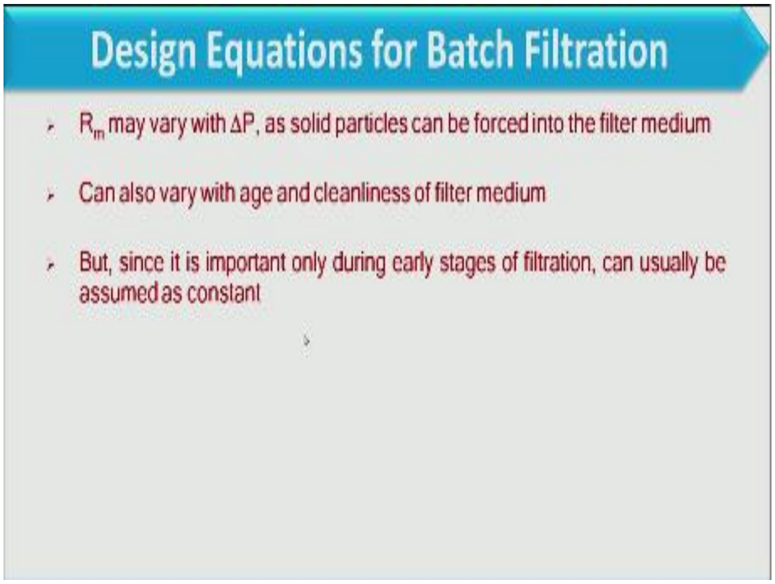


And in the high pressure in the center in to the media so it can n the inter media in that is in during in the stages of the filtration will usually assumed as constant. And in the constant we usually in the assumed in the constant in the register in the total in the filtration is we have clubbed over here the pressure in the crossed the cake and in the media in the re arranging in the contrition and the in the filtration we can dT/dV = to Uf pressure drop in to the $R_M$.

This equation is basically the governing equation for the filtration process and this we use as the further and the calculation. And there I will be in the defend cases and in the different expression and we will see in the expression propose the calculate in the cake resistance that is the α in the in the and it will be in the, and the alpha are the imperially coefficient that we have defined as in the that it should 0 the incompressible cake it should have the compress one usually it varied from 20, 22.

And this is the particular expression it has the limitation that the pressure drop will be 0 it will predict the in the alpha. Now to overcome this is the limitation and this will be the coordinating and the alpha should be = to α 01 + m and in the in the coordination of the, α in the coordination in the w in the compressible slurry in the also in the constant.

If the filtration can be conducted in the constant pressure so you can see the constant pressure in the weight filtration that cannot be accounted by these equation so these are communicating.

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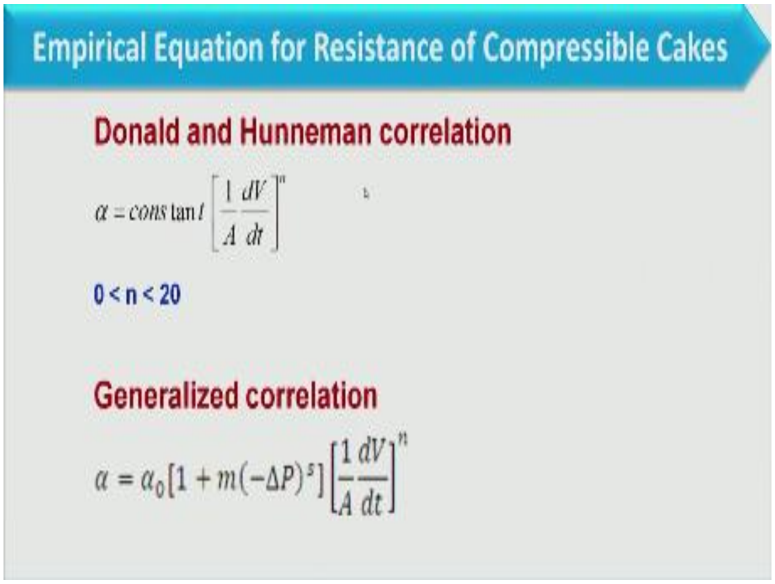


For the dominant in the equal to the constant n the over the D so that will be defined as the great expression it will not consider variable pressure drop condition and here in this expression and in the 0 to 20. So cleaving all these condition together generalize in that propose and it consider it common date the expression and in the expression and in the as well as in the constant in these we have calculate.

The expression for α now here we will consider the different for the cases for theme the like we have the constant in the pressure constant and the rate as well as the compressible and in compressible. So all though we have different cases and for these cases we will defined as the filtration definition.

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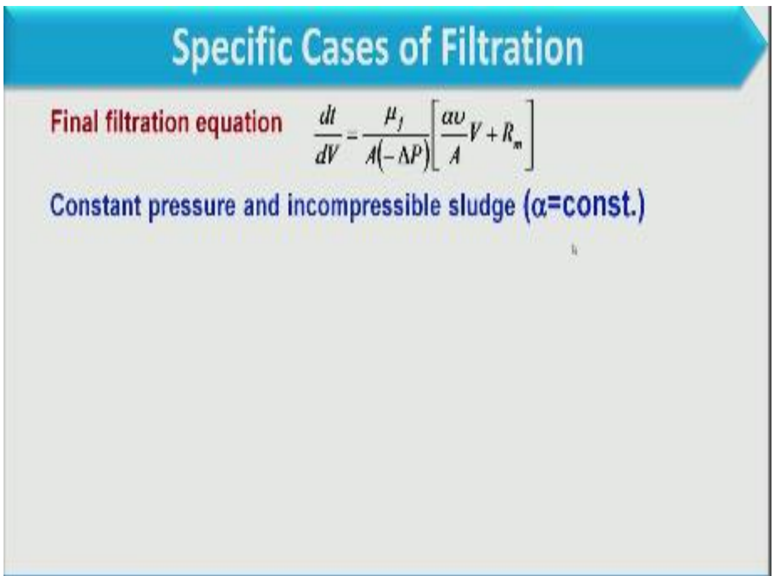


And it will also just derived in the different case's so the final filtrations and we also derived in this form we will use for the further case and in the first case is constant pressure and in the incompressible and in the splurge in it stands for the common sludge should be constant.

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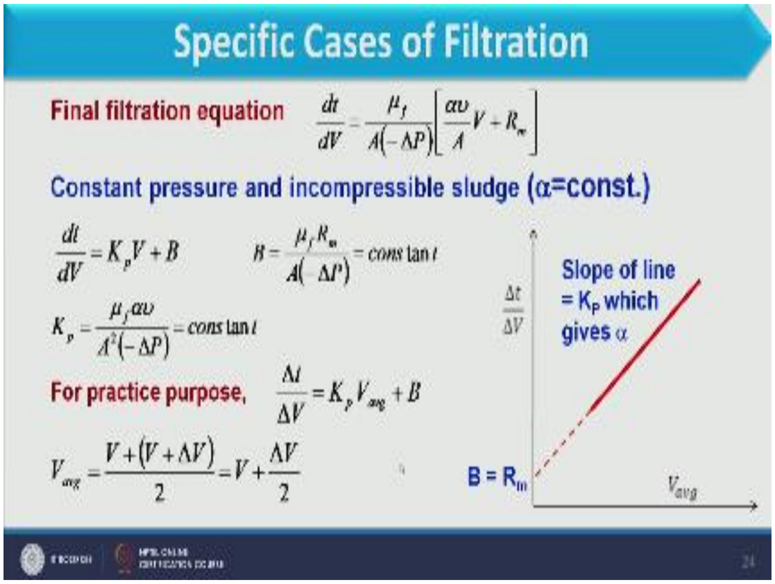


So what we can do over here and that will the pressure drop is constant all these factor except the volume we can considerate constant the whole expressing of the filtration will be denoted in the D T /D V =to KPV+B so B will be the constant and k p would should be the constant and you see B considers the expression for V considers the value of $R_m$ and $K_p$ considers the value of α, for practical purpose what happens instead of dT we consider δ, so δ t / δ v = $K_p$V + B.

Now what is V average, V average we can calculate as v1 + v2 /2 that is very simple v1 that is the initial volume v2 when we consider so that we can write as v1 + the differential volume collected. So v average we can define as v + v + δ v / 2 so it should be v+ δv / 2, so here you see this equation δt/ δv = $K_p V_{avg}$ + B so here we can plot δt/ δv with v average and it is the straight line equation so we can get this line and once extra polite this it has the intercept and it has a slope the slope of the line give the value of kp, which consequently gives the value of α.

And similarly the intercept will give value of B, and therefore we can calculate value of $R_m$ from this. So in this way all constants value will be calculated for this particular case.

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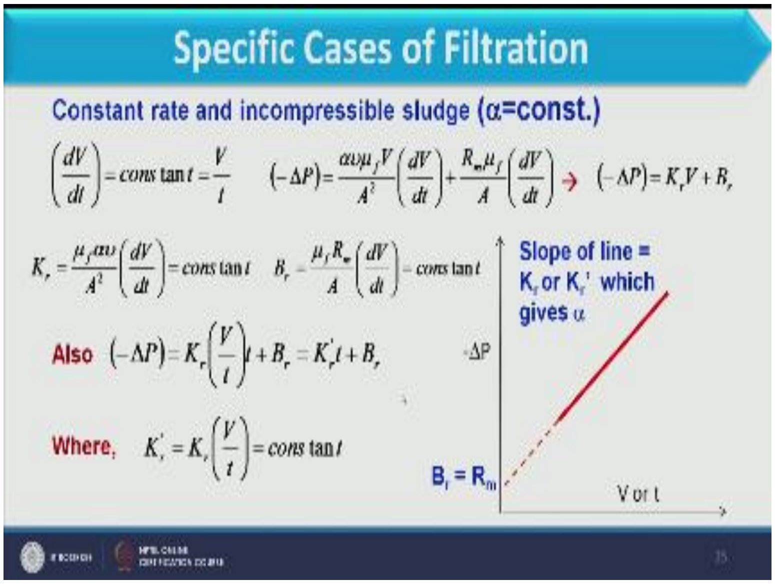


Similarly we have constant rate and incompressible sludge that is α is constant so as rate is constant dv / dt we can denoted as v/t which should be a constant. So pressure drop equation is this and here we can write pressure droop in terms of volume + Br as dv / dt is also a constant all these factors all these parameters will clubbed in to one and that we have denoted with Kr so Kr is this and Br is shown by this expression.

So further what we can do here we have Kr V + Br so when we divide and multiply this v with time so that should be v/t x t so v/t will would be a constant so collecting these constant that is Kr, and v/ t we have denoted another constant that is Kr’ nx t + Br. So here you see we have pressure drop with respect to volume also when we have not consider t and we have pressure drop with respect to time also when we have consider v/t.

So here once we drop pressure we drop with respect to volume or time we can straight line and in the similar method as we have discuss in the last slide when we extra polite this we have the intercept and it this is the particular line has a slope. So the intercept will give the value of Br and through which we can calculate value of $R_m$ slope of the line gives value of kr or kr’ and through which we can calculate value of α. So in this way we can calculate α as well as $R_m$ for constant rate and incompressible sludge.

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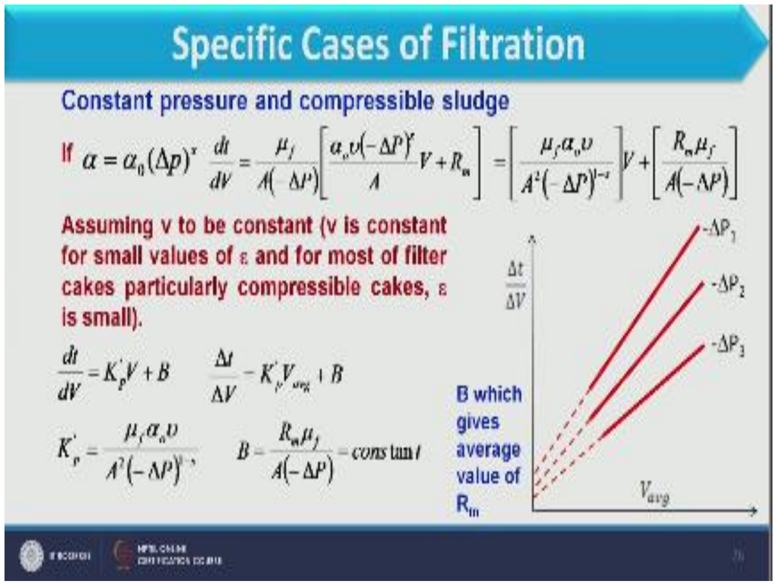


Now third condition we have this constant pressure and compressible sludge, now for constant pressure and compressible sludge as the compressible sludge we are considering α should not be a constant so for constant pressure and for compressible sludge α value should not be a constant it will be a function so that we have consider, so here we have consider a Sperry co relation to define α value this is basically the final filtration equation.

Where α would be replace by $\alpha_0$ Ps Δp – $\Delta p^s$, so that Δ p we have replaced over here with – sign because y-sign is there that we have already discussed. So collecting the constant we have one constant which will be multiplied with volume and here we have another constant. Now usually a V vary, here we are assuming V as constant for a small value of ε and for most of the filter cakes particularly compressible cake ε value is very small so we can consider v as a small.

So all these constant will give the value of, will be collected as Kp’ and all these constants values will be collected as B so again you see δt/δv would be the function of v average we can defined like this, so again δt/δv will be a function of $v_{avg}$ which we can define like this here you see single expression is there but we have three liner why we have three lines is a different pressure drop.

Now here we have consider constant pressure drop so how these three pressure drop will appear and in one pressure drop we have carried out whole filtration process and further we change the pressure drop at that particular pressure droop whole filtration process will be carried out and similarly for third pressure drop. So though we have consider three different pressure drop value but this will be a constant pressure drop operation why we are doing this it will be clear sometime.

So once we have three or four value of pressure drop we have three or four straight line in this graph extending the values extending these graph up to y axis we have three different intercept. So we have three different value of B three different value of $R_m$ will be calculated through this B values. So average if these $R_m$ value will give the value of $R_m$, further slope of this line will give Kp' value so here we have three lines so three Kp' values we can obtained.

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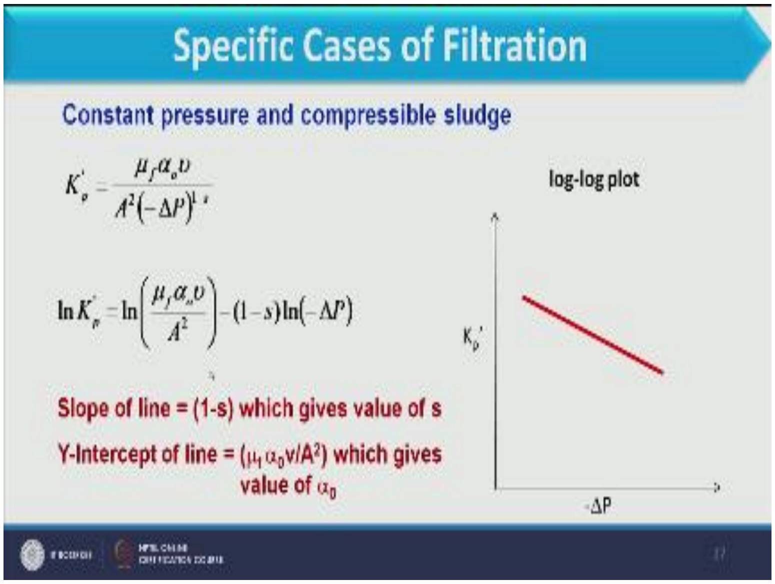

Further if you see the expression of Kp' that is $\mu_f\ \alpha_0$ v/ $a^2$, pressure droop 1- s so when we take log of this expression that should be log Kp' = log of this expression – 1 – s log pressure drop. So here we have you can see Kp' vary with a pressure drop so we can draw the line we can draw the graph between Kp' and pressure drop 1-s with – would be the slope of this so obviously the slope would be negative slope of this line will give value of s because that should be equal to 1- s from here we can calculate s and intercept of this line will give the value of this particular constant and through which we can calculate $\alpha_0$.

So here you see for constant pressure and compressible sludge we have three different constant $R_m$ value s value and $\alpha_0$, so therefore we have gone through two steps for calculating these constants.

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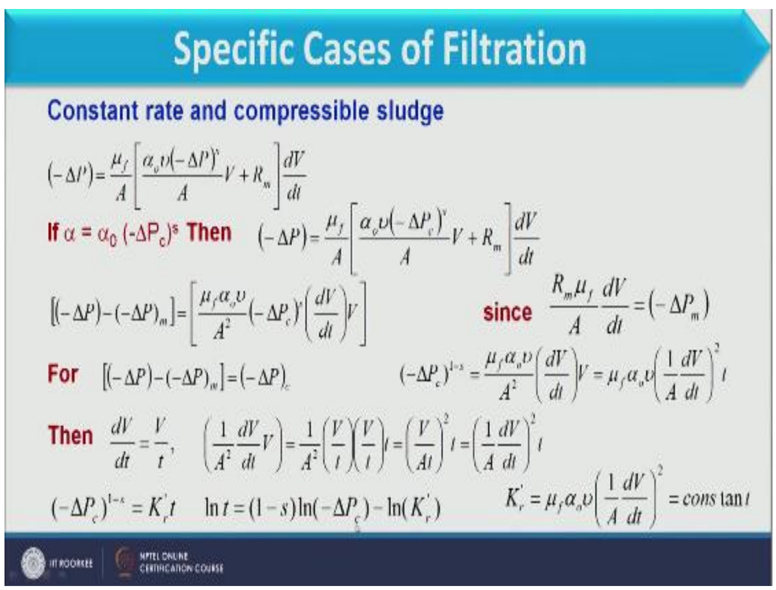


And finally we have constant rate and compressible sludge this is the pressure drop equation dv/dt is constant because that is constant rate and here α is that is a Sperry correlation and that consider as the function of δPc. Now usually it is defined as total pressure drop but here we are considering only the function of δpc because total pressure drop will be the pressure drop across the bed and pressure drop across the media.

Usually pressure drop across the media is constant so α will be the function of pressure drop across the cake therefore here α is consider like this now once we definer α like this then the pressure drop expression where we replace α value over here and when we elaborate this particular expression when we consider this $R_m$ dv/dt μf / a so that is nothing but the pressure drop of media if we club all these parameter it will be nothing but pressure drop across the media.

So that we can take to left side so that is –δ p – δp m = this expression so when we consider this particular expression this would be nothing but pressure drop across the cake. So pressure drop across the cake we can keep and here also we have pressure drop across the cake so final equation would be pressure drop across the cake power 1- s = μf $\alpha_0$ V 1/ adv / $dt^2$ x t, now how this expression is obtained because here we have the expression dv /dt /$a^2$ x v, so as this is the constant rate case dv/ dt would be a constant that should be v/t.

Now this is the expression as you can see here we have the expression that we have to rewrite how we can rewrite this when it is equal to 1/ $a^2$ dv/ dt will replaced by v/ t and here we have the v value that we multiply and divide by t, so here you se v/ $at^2$ x t which is we can write as 1/a dv/$dt^2$ so this dv/dt $^2$ t we can put over here, so all these value would be constant and δ $pc^{1-s}$ should be equal to kr' x t taking log of this we can calculate we can rewrite the expression.

Now here you see time varies with – δpc so to calculate s value as well as kr' we have to draw the line of pressure drop across the cake versus time. Now how I can obtain practical value of pressure drop across the cake like when we draw the graph between pressure drop that is total pressure drop versus time and when we extend this curve up to t = 0, so at t = 0 whatever would be the pressure drop that would be the pressure drop of due to media not the pressure drop due to cake because at t = 0 cake will not be formed.

So in that way we can calculate pressure drop across the media and when we deduct this pressure drop across the media from total pressure drop we can calculate pressure drop across the cake. So in this way we can calculate the pressure drop across the cake and then δpc versus time we can plot and we can calculate value of s as well as Kr', so in this way we can define or we can use the govern equation for filtration for these four cases.

So here I am stopping lecture two we will discuss filtration process in lecture three also so that is all for now, thank you.

**For Further Details Contact**

**Coordinator, Educational Technology Cell**

**Indian Institute of Technology Roorkee**

**Roorkee – 247667**

**E Mail: etcell.iitrke@gmail.com. etcell@iitr.ernet.in**

**Website: www.iitr.ac.in/centers/ETC, www.nptel.ac.in**

**Web operations**

**Dr. Nibedita Bisoyi**

**Neetesh Kumar**

**Jitender Kumar**

**Vivek Kumar**

**Camera & Editing**

**Pankaj Saini**

**Graphics**

**Binoy. V.**

**Production Team**

**Jithin K.**

**Mohan Raj. S**

**Arun S**

**Sarath K V**